

When Maximality Is Not Maximally Informative: A Compositional Analysis of the Depth-Charge Illusion*

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Abstract

Building on existing proposals for the semantic analysis of negative antonyms [8, 19], intensional comparatives with *too* [11, 12], and monotonicity in comparison constructions [1, 2], we propose a compositional analysis of so-called depth-charge sentences like *No head injury is too trivial to be ignored*: The alleged illusion derives from the interaction of maximal informativity with the scalar properties of the degree set that underlies the standard of the comparison.

1 Introduction and Background

So-called depth-charge sentences like (1) have featured prominently in the psycholinguistics literature [3, 14, 16, 20] and are generally construed to entail (1-b): Regardless of their severity, all head injuries must receive medical attention. A large body of literature has taken this interpretation, while shown to be robustly available both in English and across languages [9, 13], to be an illusion that obscures the literal and compositional interpretation of the sentence in (1-a), under which all head injuries can or even should be ignored [4, 7, 22].

- (1) *No head injury is too trivial to be ignored.* “The Head Injury”
- a. %Ignore all head injuries!
- b. ✓Do not ignore any head injuries!

Evidence for a view under which the prominent reading of (1) is an illusion comes from the structurally parallel case in (2), which is taken to convey that all missiles should be banned, as in (2-a), rather than legalised, as in (2-b).

- (2) *No missile is too small to be banned.* “The Missile Ban”
- a. ✓Ban all missiles!
- b. #Do not ban any missiles!

However, with the exception of [14], no rigorous formal semantic analysis of the difference between the two examples has been attempted. The modest goal of this paper is to show that such an analysis is possible. We argue that the difference in interpretation is not an illusion and show that both readings can be derived compositionally. Building on existing formal analyses of negative-polar antonyms [19], modal comparison with *too* [11, 12], and monotonicity effects in the comparative constructions [1, 2], we propose that the alleged illusion derives from the interaction of maximal informativity with the scalar properties of the set of degree intervals that feeds into the standard of the intensional comparison.

*For helpful feedback and discussion, we would like to thank Dario Paape, Viola Schmitt, Ryan Walter Smith, Frank Sode, Siena Weingartz, Valerie Wurm, Malte Zimmermann, and Richard Zimmermann.

2 Towards the Analysis: Intensional Comparison and Negative Polarity

In this section, we review two of the key pieces of a compositional analysis of depth-charge sentences— the analysis of the syntax and semantics of intensional comparison with *too* [11, 12], and the semantic analysis of negative antonymity [8, 19].

Intensional Comparison. We take as our starting point one prominent analysis of *too* [11, 12] (but see also [5, 6, 21]), under which it encodes a greater-than comparison of two degree maxima, where the standard involves a covert possibility modal, as in (3). A sentence such as (4), under this analysis, has the Logical Form¹ sketched in (5) and is interpreted as in (6). Here, the comparison is between the maximum of the degree interval on the quality scale associated with the food *F* in the actual world $w_{@}$ and a degree interval whose right boundary corresponds to the highest quality at which it would be allowed to throw *F* away, a result of applying MAX to the set of degrees such there is a world in which *F* is thrown away; compare also Figure 1.

$$(3) \quad \llbracket too \rrbracket = [\lambda Q \in D_{\langle\langle s,t \rangle, \langle s,t \rangle\rangle} . [\lambda R \in D_{\langle d, \langle s,t \rangle \rangle} . [\lambda w \in D_s . \text{MAX}(\lambda d. R(d)(w) = 1) > \text{MAX}(\lambda d*. Q(R(d*))(w) = 1)]]], \quad \text{-- to be revised --}$$

where, for any set of degree intervals $P \in D_{\langle d, \langle s,t \rangle \rangle}$,
 $\text{MAX}(P) = \iota d [P(d) = 1 \ \& \ \forall d' [P(d') = 1 \rightarrow d \geq d']]$

(4) *The food is too good to be thrown away.*

(5) Logical Form for (4):

$$\begin{aligned} & [w_{@} [[\text{the food}] [\lambda 1, e [[\text{DegP too} \\ & [\textcircled{1} (\text{modal component}) \text{CAN}_{\langle\langle s,t \rangle, \langle\langle s,t \rangle, \langle s,t \rangle\rangle} [\text{PRO}_{1,e} \text{be thrown away}]] \\ & [\textcircled{2} (\text{degree set}) \lambda 2, d [t_{1,e} [\text{VP is } t_{2,d} \text{good}_{\langle d, \langle e, \langle s,t \rangle \rangle}]]]]]]]] \end{aligned}$$

(6) Compositional Interpretation of (4),

where $w_{@}$ is the actual world and $\llbracket \text{the food} \rrbracket^g =_{\text{simplified}} F$:

- a. $[\lambda x : x \in D_e . \llbracket too \rrbracket^g (\llbracket \textcircled{1} \rrbracket^g) (\llbracket \textcircled{2} \rrbracket^g)] (\llbracket \text{the food} \rrbracket^g) (w_{@}) = 1$
 iff $\text{MAX}(\lambda d. \text{QUALITY}_{w_{@}}(F) \geq d) > \text{MAX}(\lambda d*. \exists w* [w* \text{ is accessible from } w_{@} \ \& \ \text{QUALITY}_{w*}(F) \geq d* \ \& \ F \text{ is thrown away in } w*])$
- b. $\llbracket \textcircled{1} \rrbracket^g = [\lambda p \in D_{\langle s,t \rangle} . [\lambda w' \in D_s . \exists w* [w* \text{ is accessible from } w' \ \& \ p(w*) = 1 \ \& \ g(1, e) \text{ is thrown away in } w*]]]$
- c. $\llbracket \textcircled{2} \rrbracket^g = [\lambda d \in D_d . [\lambda w \in D_s . \text{QUALITY}_w(g(1, e)) \geq d]]$

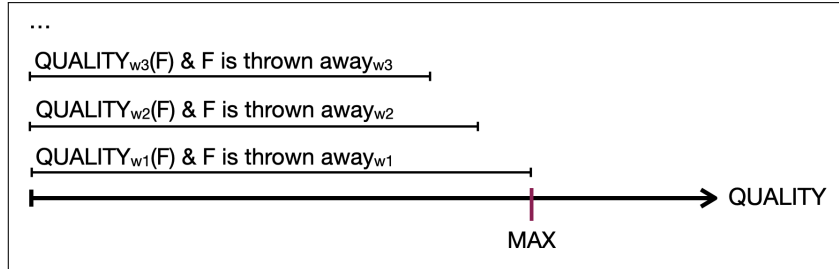


Figure 1: $\text{MAX} \gg \exists$ in the interpretation of the modal standard

¹See also the more explicit tree structure for “The Missile Ban” in (9) below.

Negative Polarity. In the case of “The Head Injury” in (1) and “The Missile Ban” in (2), the comparison involves a negative antonym, however. Under an extent-based semantics [8, 19], these access the same scale as their positive-polar counterparts. In the antonym pair *old* and *young*, for instance, both adjectives relate entities to their extents on the age scale, as shown in Figure 2. The negative-polar *young* thus picks out those negative extents to which the entity is not old, as also indicated notationally by the $^-$ superscript.

- (7) a. $\llbracket \textit{old} \rrbracket = [\lambda d \in D_d \ \& \ d \in \text{AGE}^+. [\lambda x \in D_e. [\lambda w \in D_s. \text{AGE}_w(x) \geq d]]]$
 b. $\llbracket \textit{young} \rrbracket = [\lambda d \in D_d \ \& \ d \in \text{AGE}^-. [\lambda x \in D_e. [\lambda w \in D_s. \text{AGE}_w(x) \geq d]]]$

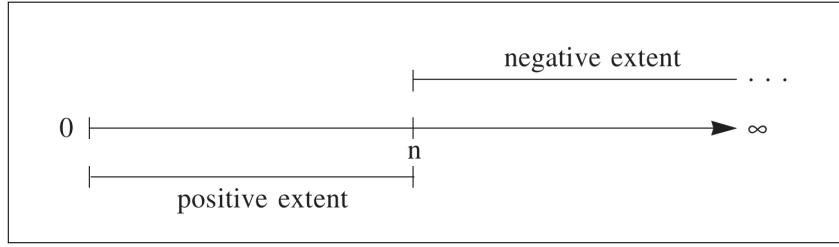


Figure 2: Negative extents [12]

A simple comparative like (8) with a negative-polar adjective will be true if and only if in the world of evaluation, the left boundary of Siena’s longest negative age interval is lower on the age scale than the left boundary of Eve’s longest negative age interval.² For an illustration of the mechanism behind determining the maximum of a negative extent, see also Figure 3.

- (8) *Siena is younger than Eve.*
 $\text{MAX}(\lambda d_1^- . \text{AGE}_{w@}(S) \geq d_1) > \text{MAX}(\lambda d_2^- . \text{AGE}_{w@}(E) \geq d_2)$
 ‘The maximal age that Siena does not have exceeds the maximal age that Eve does not have.’

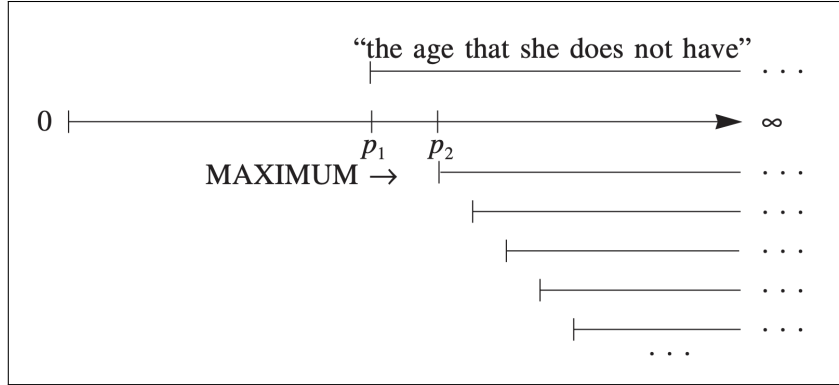


Figure 3: Maximal negative extents [12]

We put these two pieces (that is, the analysis of *too* as an intensional comparative, and the extent-based treatment of negative antonyms) together in the next section. We show that they derive the intended interpretation for “The Missile Ban”, but only predict the largely unavailable reading for “The Head Injury”, rather than the illusion. We then revise the analysis of *too* to incorporate insights from the literature [1, 2] on how to define maximality in the light

²Under a point-based view of degrees, this translates to Siena’s age < Eve’s age.

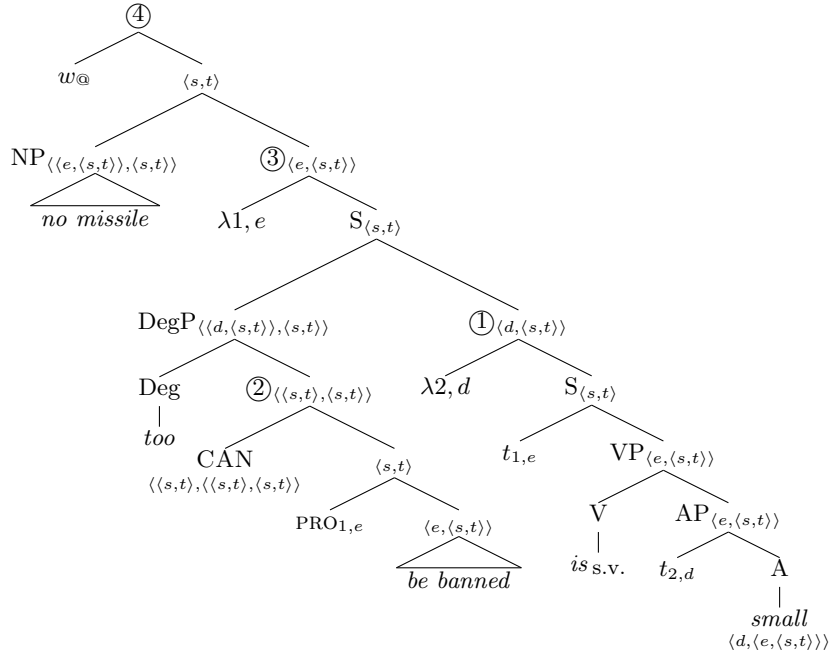
of the varying scalar properties of degree sets. We argue that the two key examples also vary in this respect, hence the different interpretations they make available.

3 Putting the Pieces Together, and a Revision

An analysis of “The Missile Ban” in (2) as a negative extent-based intensional comparison derives the observed interpretation, under which a ban is a possibility for all missiles, regardless of their size. We provide a Logical Form in (9) and sketch its interpretation in (10). The sentence is true if and only if, in the world of evaluation, there is no missile whose maximal negative size extent is greater than the standard of the comparison. The degree set whose maximal interval constitutes the standard of the comparison in this case is the set of negative size extent of $g(1, e)$ such that there is a world where $g(1, e)$ is banned.³

- (2) *No missile is too small to be banned.*
 (✓ Ban all missiles! #Do not ban any missiles!)

- (9) Logical Form for “The Missile Ban”:



- (10) Compositional Interpretation of “The Missile Ban”:

- $\llbracket \textcircled{1} \rrbracket^g = [\lambda d : d \in D_d \ \& \ d \in \text{SIZE}^- . [\lambda w \in D_s . \text{SIZE}_w(g(1, e)) \geq d]]$
- $\llbracket \textcircled{2} \rrbracket^g = [\lambda p \in D_{\langle s, t \rangle} . [\lambda w' \in D_s . \exists w* [w* \text{ is accessible from } w' \ \& \ p(w*) = 1 \ \& \ g(1, e) \text{ is banned in } w*]]]$
- $\llbracket \textcircled{3} \rrbracket^g = [\lambda x \in D_e . [\lambda w \in D_s . \text{MAX}(\lambda d^- . \text{SIZE}_w(x)) > \text{MAX}(\lambda d*^- . \exists w* [w* \text{ is accessible from } w \ \& \ \text{SIZE}_{w*}(x) \geq d* \ \& \ x \text{ is banned in } w*]]]]]$
- $\llbracket \textcircled{4} \rrbracket^g = 1 \text{ iff } \neg \exists x [x \text{ is a missile in } w_\oplus \ \& \ \text{MAX}(\lambda d^- . \text{SIZE}_{w_\oplus}(x)) \geq d > \text{MAX}(\lambda d*^- . \exists w* [w* \text{ is accessible from } w_\oplus \ \& \ \text{SIZE}_{w*}(x) \geq d* \ \& \ x \text{ is banned in } w*]]]$

³Under a point-based view of degrees, these truth conditions in (10) amount to requiring that there are no missiles whose size is smaller than the size from which onward a ban is a possibility.

For the structurally parallel “The Head Injury” in (1), this analysis however predicts only the reading whose availability is subject to variation, in (11), which may lead one to take the view that the other reading is indeed an illusion [14].

- (1) *No head injury is too trivial to be ignored.*
 (%Ignore all head injuries! ✓Do not ignore any head injuries!)
- (11) Interpretation of “The Head Injury”: – to be revised –
 $\neg \exists x [x \text{ is a head injury in } w_{\text{@}} \ \& \ \text{MAX}(\lambda d^-. \text{SEVERITY}_{w_{\text{@}}}(x) \geq d)$
 $> \text{MAX}(\lambda d^*. \exists w^* [w^* \text{ is accessible from } w_{\text{@}} \ \& \ \text{SEVERITY}_{w^*}(x) \geq d^* \ \& \ x \text{ is ignored in } w^*])]$

We suggest here that this conclusion is not inevitable: The maximality-based analysis of comparison that is also used for *too* has been challenged by examples like (12), which have a minimality, rather than maximality interpretation [1, 2]. They crucially also involve upward, rather than downward scalarity (that is, if one pound of flour is sufficient for this cake, two pounds will be as well, and so on). Adopting a definition of maximal informativity based on [1], in (13), derives the desired interpretation: The maximal informative amount of flour in these cases is the minimal amount of flour, not the maximal amount.

- (12) *I have more flour than is sufficient to bake this cake.*
 a. # I have more flour than the maximal amount that suffices to bake this cake.
 b. ✓ I have more flour than the minimal amount that suffices to bake this cake.
- (13) For any $p \in D_{\langle d, \langle s, t \rangle \rangle}$ and world w , $\text{MAX-inf}(p)(w) = \iota d [p(d)(w) = 1 \ \& \ \neg \exists d' [d \neq d' \ \& \ p(d')(w) = 1 \ \& \ [\text{if } p(d')(w) = 1, \text{ then } p(d)(w) = 1]]]$

We follow the literature in assuming that the move away from maximality to maximal informativity is appropriate across the board, in comparison constructions [1, 2, 15], but also in the analysis of definite descriptions [17, 18], for instance. A revised lexical entry for *too* is in (14). Upon closer inspection, we find that the two key examples in (1) and (2) differ in the scalar properties of the degree set underlying the comparison; it is the informativity distinction between upward and downward scalarity which governs the available interpretations.

- (14) $\llbracket \text{too} \rrbracket = [\lambda Q \in D_{\langle \langle s, t \rangle, \langle s, t \rangle \rangle} \cdot [\lambda R \in D_{\langle d, \langle s, t \rangle \rangle} \cdot [\lambda w \in D_s \cdot \text{MAX-inf}(\lambda d. R(d)(w) = 1) > \text{MAX-inf}(\lambda d^*. Q(R(d^*))(w) = 1)]]]$ – revised version –

To see this, consider Figure 4.

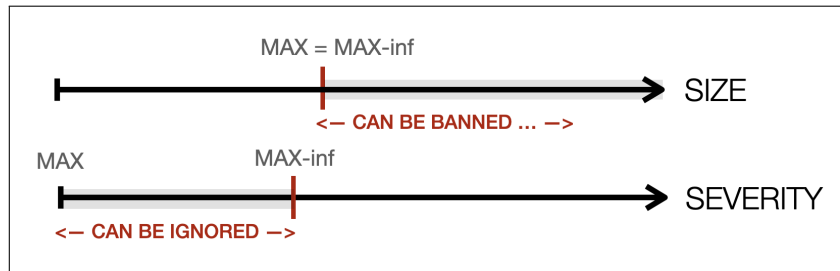


Figure 4: Upward/ downward scalarity and maximal informativity

While the negative size degrees under consideration for the comparison in “The Missile Ban” are those *from which onward* a ban is a possibility, this is not the case for “The Head Injury” under the default interpretation that has been characterised as an illusion. Here, the negative severity degrees under consideration in the standard of the comparison are those *up to which* such an

injury can be ignored. Crucially, given these differences, while in “The Missile Ban”, $\text{MAX} = \text{MAX-inf}$, this is not the case for “The Head Injury”, as shown in (15). Assuming downward monotonicity, the maximal informative negative extent is the minimal severity interval, whose lower bound is the degree up to which head injuries can be ignored (= the threshold from which treatment is required).⁴

- (15) $\neg \exists x [x \text{ is a head injury in } w_{\text{@}} \ \& \ \text{MAX-inf}(\lambda d^-. \text{SEVERITY}_{w_{\text{@}}}(x) \geq d) >$
 $\text{MAX-inf}(\lambda d^*. \exists w^* [w^* \text{ is accessible from } w_{\text{@}} \ \& \ \text{SEVERITY}_{w^*}(x) \geq d^* \ \& \ x \text{ is ignored in } w^*])]$ =
 $\neg \exists x [x \text{ is a head injury in } w_{\text{@}} \ \& \ \text{MAX-inf}(\lambda d^-. \text{SEVERITY}_{w_{\text{@}}}(x) \geq d) >$
 $\text{MIN}(\lambda d^*. \exists w^* [w^* \text{ is accessible from } w_{\text{@}} \ \& \ \text{SEVERITY}_{w^*}(x) \geq d^* \ \& \ x \text{ is ignored in } w^*])]$
 ‘No head injury has a negative severity extent that is greater
 than the minimal severity interval.’

The implausible (so-called literal) reading arises, we speculate, when the underlying degree set is re-conceptualised as upward-monotonous, that is, as having the monotonicity properties of the missile example. The negative severity degrees under consideration are then those *from which onward* no treatment is a possibility.

4 Some Concluding Remarks

Under the view presented in this paper, the depth-charge illusion is not an illusion, but rather just another case where maximality is not maximally informative. It is also another case where formal grammatical analysis can offer a systematic explanation of a phenomenon that has been labelled an illusion or fallacy [10], which highlights the importance of experimental research on meaning engaging with formal semantics research, and vice versa. The analysis presented above is compatible with many of the key observations in the literature, three of which we briefly mention here, even though we have to leave a more detailed discussion for another occasion. First, without world knowledge and lexical information, the monotonicity of the underlying degree set will end up being under-specified, and both interpretations should be equally plausible. For the example in (16), this indeed appears to be the case [20].

- (16) *No wug is too dax to be zonged.*
 (✓ Do not zong any wugs! ✓ Zong all wugs!)

Second, as it is informativity and its interaction with monotonicity that governs the available readings, similar contrasts should arise with intensional comparison with positive antonyms. Two relevant examples are in (17), which are interpreted, respectively, as encouragement to climb any mountain and as encouragement to not stop our efforts against polio [4].

- (17) a. *No mountain is too high to climb.*
 b. *No challenge is too big to stop us from saving our children from polio.*

Third, given the overall of the complexity of the key concepts involved (that is, degree comparison, negative antonymity, maximal informativity), the analysis presented here is compatible with the general observation that these sentences present a processing challenge, which may affect the availability of the different readings.

⁴Under a point-based view the interpretation in (15) corresponds to no head injury’s severity being below the relevant threshold (indicated by MAX-inf in Figure 4).

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