Numerous relative clauses
permutation invariance, anti-restrictiveness, triviality*

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Abstract

In this paper we study the different patterns that *gather*-like and *numerous*-like predicates present in restrictive relative clauses. We argue that *numerous* is generally bad in restrictive relative clauses, because when it applies to a predicate that denotes a complete join semilattice, triviality arises with the definite plural. As a consequence, we predict that restrictive *numerous* is fine if it applies to predicates that do not give rise to complete join semilattices. This is indeed the case for complex predicates like *gathered students*, whenever the collective predicate *gather* is not pluralized.

1 Introduction

The present paper aims at providing an analysis to some puzzles linked to restrictive uses of *gather*(-like) and *numerous*(-like) predicates. The first puzzle arises from the contrast shown by the following sentences:[1]

(1) a. Jack only talked to the students that gathered.
b. #Jack only talked to the students that were numerous.

Whereas (1a) is a felicitous sentence in which the predicate *students* has been modified by *gathered* by means of a relative clause, (1b) - the result of substituting the predicate *gather* with *be numerous* - is somehow deviant.[2] A preliminary question we have to ask is whether the general infelicity of restrictive relative clauses involving the predicate *numerous* is specific to relative clauses or is actually linked to restrictiveness *per se*. With respect to this issue, the following data from Italian is particularly helpful:

(2) a. Ho parlato con i numerosi studenti.
have.1SG talked with the.PL numerous.PL student.PL
I have talked with the numerous students.
b. ??Ho parlato con gli studenti numerosi.
have.1SG talked with the.PL student.PL numerous.PL
I have talked with the numerous students.

*The main examples and the problems they constitute, together with important suggestions toward a solution, were proposed by Martin Hackl in a problem set for the course 'Introduction to Semantics'. Thanks also to all the participants to 'Introduction to Semantics', 'Workshop in Linguistics' and the 33rd European Summer School in Logic, Language and Information for crucial feedback on an early version of this work, in particular Martin Hackl, Keely New, Jad Wehbe, Omri Doron, Dean McHugh, Patrick Elliott, and six anonymous reviewers.

[1] All our examples include *only* in the attempt to make sure that the relative clause is interpreted as a restrictive relative clause.

[2] There are some speakers that upon reflection manage to find (1b) acceptable 'imagining a scenario in which there are different groups of students'. We claim that in this case *students* is contextually interpreted as *gathered students*. We unfortunately don't have space to discuss whether this interpretation can be due to a cover analysis or to an implicit *gathered*. But we believe that for the intended purposes they are equivalent, even though the cover analysis might result in additional assumptions on the application of relative clauses.

The minimal pair above exploits a typical Romance phenomenon: modifiers in pre-nominal positions are linked to non-restrictive interpretations, while post-nominal modifiers are ambiguous between the two readings [Mor08, Cin03].[3] The contrast between (2a) and (2b) signals then that the infelicity of the predicate *essere numeroso / to be numerous* is really tied to restrictive interpretations in general, rather than limited to (restrictive) relative clauses. There must be something that blocks *numerous* from being restrictive, namely from selecting a specific subset of a given set.[4]

The contrast observed in (1) becomes even more compelling when compared with the second puzzle we will discuss, which arises from the following sentence:

(3) Jack only talked to the gathered students that were numerous.

This crucial data that seems to contradict the generalization we stated above: (3) shows that *numerous*-restrictive relative clauses can be felicitous; in particular, they are perfectly fine if the predicate they combine with is a complex predicate, in our case if *student* has already been modified by some other predicate, *gather*. We will show that a specific internal structure is required for these predicates, and we will derive the felicity of (3) as a prediction of our analysis for the infelicity of (1b).

To summarize the goal of this paper, we can take away two main observations and two questions, respectively, from the examples presented insofar:

1. the modifier *numerous* is generally fine as non-restrictive but not as restrictive; what is preventing it from being restrictive?

2. there exist cases in which *numerous* is fine in restrictive position; what is the difference with the other cases?

To answer the first question, we claim that restrictive readings are blocked whenever a *permutation invariant* predicate like *numerous* applies to complete join semilattices. This property leads to redundancy: the individual resulting from the application of the relative clause has no potential to differ (be restricted) from the individual that would have been denoted without the relative clause.[5]

To answer the second question, we will build on the intuition that (3) involves several groups of gathered students. We will argue that while most complex predicates (like *smiling students*) preserve a complete join semilattice structure, others (like *gathered students*) have the possibility to give rise to other structures. In particular, we will claim that *gathered* changes the complete join semilattice structure of *students* when it is not pluralized. We will assume that pluralization comes from a syntactic operator, the star operator [Lin87], in line with [Ste98] and [BS00]. On the one hand, we assume that pluralization can apply to every base-generated and complex predicate; on the other hand, we assume pluralization to be optional, or, more precisely, omitted if its application leads to triviality. Let us now start from an introduction of the relevant properties of *gather*-like and *numerous*-like predicates that we will borrow from the existing literature to provide an analysis to our puzzles.

[3] Crucially, we do not commit here to any specific syntactic or semantic analysis of the contrast between the two interpretations, beyond the idea that restrictive interpretations select a subset of the predicate they combine with.

[4] We don't delve here into the thorny question of the proper semantics for restrictiveness and non-restrictiveness, and we just treat restrictiveness as intersection of predicates.

[5] Our solution is marginally linked to the *anti-monotonicity constraint* proposed in [Ami21]. However, we do believe that the interplay between the structure of plurals and *permutation invariance* is the real notion at play here, and monotonicity is just a related consequence that shows up most but not all of the times.

2 *Gather*, *be numerous*, and their relevant properties

Both *gather* and *be numerous* can be classified as collective predicates, for they only accept plural individuals in their domain. However, it has been known for a while now that a more fine-grained distinction is needed within the class of collective predicates, since *gather* and *numerous* give rise to different patterns when combined with plural quantifiers [Kro74, D$^+$87, Cha10]. These patterns resemble the contrast we presented in (1), since only so-called *gather*-type predicates can felicitously combine with plural quantifiers, as displayed in (4) and (5).

(4) a. All the students gathered.
b. #All the students are numerous.

(5) a. Most of the students gathered.
b. #Most of the students are numerous.

A couple of intuitions on the properties that are linked to this problem are generally shared by the literature, and, following [Kuh20], they can be summarized as follows:

1. *numerous*-like predicates denote properties of groups as such;
2. *gather*-like predicates license 'distributive sub-entailments'.

Property 1 states that *numerous*-like predicates do not hold (or are not guaranteed to hold) for subparts of what they are predicated of. However, the pivotal notion we want to stress here, is that these predicates are 'blind to differences', they denote properties of groups but they cannot distinguish whether a certain atom instead of another is in the plurality they are predicated of. We call this property *permutation invariance*. Take for example the case of *numerous*: it only checks whether a given plurality has a certain cardinality, regardless of the properties of the specific members that constitute such plurality. To say that $a \oplus b$ are a *numerous* plurality, we are not concerned with any of the properties of a and b, it suffices to say that the plurality $a \oplus b$ exists, i.e. we are only interested in the fact that we can group a and b together. For instance, if we substituted a with c, the numerosity of the plural individual would not change. In contrast, when *gather* is predicated of $a \oplus b$, it is crucial that a and b participated in a gathering event, and substituting b for c, will not necessarily guarantee that $a \oplus c$ gathered too. In section §3 will discuss how this property play a crucial role in blocking restrictive interpretations.

With respect to property 2, [Kuh20] traces back the distinction between the two classes of collective predicates to a mass/count distinction. In particular, in the same way every part of something that is water is water itself, *gather*-like predicates have the crucial property of holding for any plural subpart of the individual they are predicated of.[6] However, here we are interested in the reverse property, quickly mentioned by [Kuh20]. As a collective predicate, *gather* denotes pluralities by definition, but when it is not pluralized it does not hold for the sums of such pluralities. This property plays a crucial role in creating a set of plural individuals that is not a complete join semilattice, and therefore does not lead restrictive modification by *numerous* to triviality. Let us now introduce the details on how these properties explain the behaviour of *numerous*-relative clauses.

[6] [Distributive sub-entailments]If a predicate holds for a plurality x, then it holds for any y s.t. $y \leq x$ and $\neg Atom(y)$. 'ε-bounded divisiveness' is a variant of this principle also used in the literature [Kuh20]. The general idea of 'distributive sub-entailments' was first proposed in [D$^+$87] and Kuhn has recently improved the analysis to account for several over and undergenerations.

3 The analysis, starring: most verbs but not *gather*

3.1 Permutation invariance, i.e. anti-restrictiveness, i.e. triviality

Recall our initial example (1b), repeated here as (6):

(6) # Jack only talked to the students that were numerous.

In this section, we aim at showing how, because of the algebraic properties of plurals, whenever *numerous* tries to restrict a pluralized predicate, it inevitably ends up receiving a non-restrictive interpretation, which results in ungrammaticality when this interpretation cannot be derived by any parse of a given sentence. Consider the denotation for *students* as the complete join semilattice structure used to analyse plurals due to [Lin87] (obtained interpreting the plural morpheme *-s* as the star operator[7]), together with the following definition for *numerous*:

(7) $[\![numerous]\!]^{g_c} = \lambda x : x \in dom(|\ |).|x| \geq$ the contextually supplied numerosity standard

In simple terms, *numerous* are all those plural individuals with a cardinality above a certain threshold. A visual representation is showed in Fig.1, where the numerosity threshold has been set to 2, for the sake of simplicity.

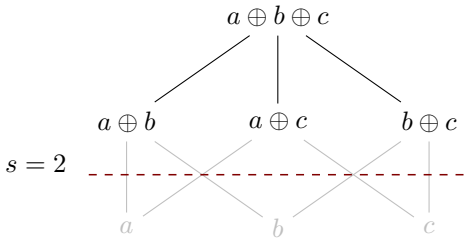

Figure 1: $[\![numerous\ {}^*students]\!]^{g_c} = \{a \oplus b, a \oplus c, b \oplus c, a \oplus b \oplus c\}$

With no additional contextual restriction on how we can pick plural individuals, *any* group with cardinality 2 or more is going to satisfy *be numerous*.[8] This means that for any x such that x is an atom, there is at least one y such that y is a numerous individual and $y \succeq x$. We call this property *anti-restrictiveness*, and it is a property that predicates like *be numerous* / *be few in number* / *be exactly 3 in number* / etc. carry whenever they are applied to a complete join semilattice. The composite predicate resulting from the application of the *numerous*-relative clause to the original predicate of the NP (*students* in our example) has then to be pluralized (i.e. closed under sum formation via star), to ensure composition with the definite article, to satisfy the existence of a unique maximal element.[9] However, if any atom is part of at least one *numerous* individual, the maximal element resulting from the application of star to *students*

[7] Where ${}^*h$ is the smallest function s.t. $\forall x[h(x) = 1 \rightarrow^* h(x) = 1]$ and $\forall x, y[{}^*h(x) =^* h(y) = 1 \rightarrow^* h(x \oplus y) = 1]$ (Martin Hackl, lecture notes).

[8] If we wanted to claim that $a \oplus b$ from the previous figure is a numerous individual we wouldn't be able to hold that $a \oplus b$ is a numerous individual without admitting that $a \oplus c$, $b \oplus c$, and $a \oplus b \oplus c$ are numerous individuals too.

[9] Assuming for the definite article the following meaning adapted from [Sha80]: $[\![the]\!]^{g_c} = \lambda F : F_{\langle e,t\rangle} \& \exists x[F(x) \& \forall y[F(y) \rightarrow y \preceq x]. \iota z[F(z) \& \forall y[F(y) \rightarrow y \preceq z]$. Pluralization after the relative clause composition is vacuous for *numerous*, since it already includes the maximal element (because of monotonicity with respect to parthood), but it would be required for other predicates of this sort like *be few in number* and *be exactly 3 in number*.

that are numerous will just be the maximal element of *students*, i.e. the sum of every atomic individual in the predicate set. The supposed restriction via *numerous* results therefore in *triviality*.

3.2 Collective predicates and structural modifications

As stated above, *numerous*-like predicates will end up in triviality whenever they are applied to complete join semilattices. This means that they would result in ungrammaticality whenever they are applied to pluralized predicates (*students)*, or to intersections of pluralized predicates (*smiling students*). Note that, if we try to not pluralize these distributive predicates, *numerous* would still fail to apply, this time because, it being a plural predicate, it cannot be applied to atomic entities.[10] On the other hand, we do expect *numerous*-like predicates to be compatible with predicates that can provide them with suitable individuals (plural entities), without generating a complete join semilattice. This is the case for collective predicates when they are not pluralized: they apply to plural entities by definition but not necessarily to their sums. In the same way that a non-pluralized distributive predicate like *smile* would have in its denotation only the atomic individuals that smile, a predicate like *gather* would only have the groups that gathered together (and their sub-entailments). For instance, if the students gathered in one room and the professors in another, the sum of students and professors would not be in the denotation of unpluralized *gather*. In other words, modifications with unpluralized *gather*-like predicates is not semilattice preserving. As a consequence, the maximality problem with restrictive *numerous* does not arise. This is why (3), repeated here as (8), is felicitous:

(8) Jack only talked to the gathered students that were numerous.

The scenario described by speakers when confronted with (8) is one in which there are multiple student groups, some of them being numerous, and some of them being not. Consider then an instance of this scenario in which a, b, c, e and f are students that gathered in two groups: $a \oplus b \oplus c$ and $e \oplus f$ and take '3' as the contextually supplied cardinality necessary for *numerous* to hold. If we then apply *numerous* to the meaning of *gathered students* (in Fig.2 on the next page), where *gather* is unpluralized, the result will give us $a \oplus b \oplus c$ as the only numerous individual in the set.

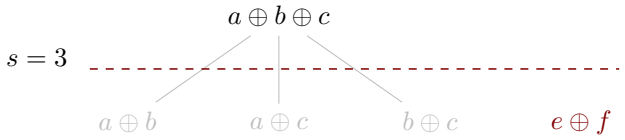

Figure 2: $[\![\textit{gathered *students}]\!]^{g_c} = \{a \oplus b, a \oplus c, b \oplus c, e \oplus f, a \oplus b \oplus c\}$
$[\![\textit{gathered *students that are numerous}]\!]^{g_c} = \{a \oplus b \oplus c\}$

In this structure restriction manages to go through, selecting a numerous individual that excludes $e \oplus f$: there is no numerous y such that $e \preceq y$ or $f \preceq y$. As a consequence, when we pluralize *gathered students that are numerous* to make sure we meet the definite article presupposition, *the* will pick up $a \oplus b \oplus c$, and not $a \oplus b \oplus c \oplus e \oplus f$. This time the relative clause successfully restricts the predicate: the maximal individual post-relative clause is strictly smaller

[10] And this might be the reason why speakers object that 'singular students cannot be numerous' when confronted with (1b). Probably pluralization is removed to avoid triviality, but a mismatch between categories of entities (singular vs plural) arises.

than the maximal individual pre-relative clause. The assumption that permits this machinery is that pluralization can apply to any node in which a new predicate is formed (e.g. *gather*, *gathered students*, *gathered students that are numerous*, ...), and can be freely omitted to avoid triviality. The LF of (8) would therefore be the following, where pluralization takes place on *students*, to compose with the collective predicate *gather*, and before the definite article, to meet its presupposition in case there are multiple groups of gathered students that were numerous.

(9) Jack only talked to [the [* [[[gathered] [* [students]]] [that were numerous]]]].

4 Conclusions and Future Work

In this work we provided an explanation to the contrast observed in (10a) and (10b):

(10) a. #Jack only talked to the students that were numerous.
b. Jack only talked to the gathered students that were numerous.

We claimed that the infelicity of (10a) is due to the permutation invariance of *numerous*-like predicates when applied to certain structures. Any atom of the predicate to which *numerous* applies cannot be excluded if such atom appears in every layer of the complete join semilattice generated by the predicate's pluralization. This implies that any proper restriction is forbidden, given that the maximal element necessary for the definite article *the* will be the sum of every single atom of the original predicate. We then explained how (10b) is a correct prediction of our theory, provided that unpluralized collective predicates can denote pluralities without being closed under sum formation. The underlying assumption of our system is that pluralization can apply to any node in which a new predicate is formed. The application of star in felicitous sentences, like (10b), is driven by the simultaneous satisfaction of two semantic and logical properties: meeting the presupposition of the definite article, while avoiding triviality. Such reading is impossible for infelicitous sentences like (10a), since removing pluralization to avoid triviality leads to a mismatch between the atomic individuals of distributive predicates and the plural individuals to which *numerous*-like predicates can apply. This solution has certain repercussions on the relation between the star operator and the plural morpheme that we would like to explore more in the future. For example, felicitous restrictive applications of *numerous*-relative clauses are found (as expected) with collective nouns. Consider the following example:

(11) We organized a competition only for [the [* [groups [that were numerous]]]].

It is clear that either we abandon the identification of the star operator with the plural morpheme -s, or we assume that the plural on *groups* (and possibily on *were*) morphologically attaches at a later stage than the one on *students* in (9). Along with this question, for future research we would like to fully explore the advantages of our approach compared to a possible alternative analysis based on covers, in the style of [Sch96]. While covers might be an appealing way to model the reading in which *gathered students* denotes multiple groups of students, it seems that such an approach would still require assumptions on pluralization similar to those made here. In particular, the cover approach could face some challenges in explaining how relative clauses compose with denotations resulting from non-pluralized covers, while still passing on pluralized meanings to the definite article. Finally, in the near future we would like to extend our solution to the other partitive constructions that present a contrast between *numerous*-like and *gather*-like predicates: plural quantifiers and non-proportional partitive constructions.

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