How Conditionals Restrict

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Conditionals restrict modal domains: in sentences of the form $A > MODAL\ B$, the modal quantifies only over worlds where the antecedent is true. This paper has two aims. The first is to derive these data axiomatically for basic modals, giving a logic for the conditional which has as theorems the axiomatic regimentations of the restrictor data. The second is to show that this logic does not require the conditional to be a *pure* restrictor, as in Kratzer (1981) and Kratzer (1991); it is sound on a *restricting operator* semantics, where conditionals shift worlds and accessibility relations. The restricting operator semantics offers a picture whereby conditionals can restrict without that being their only semantic function, thus sidestepping a notorious difficulty for Kratzer's semantics, the existence of bare conditionals.

1 Restrictor Data for Basic Modals

This paper focuses on restrictor data for *basic modals*, possibility and necessity modals of epistemic or historical flavour.

For basic modals, the following object language principle expresses the restrictor data for necessity modals:

Boxiness. $(A > \Box C) \leftrightarrow \Box(A \supset C)$

This is easy to see with epistemic *must.* Take a conditional like:

(1) If the butler didn't commit the crime, the gardener must have done it.

The intuitive truth-conditions here, reported by Kratzer and others, are that in all the epistemic possible worlds where the butler didn't commit crime, the gardener committed the crime. But this is just to say that in all epistemically possible worlds it is true that $\neg butler \supset gardener$.

To motivate Boxiness for historical modals, we focus on cases where it is known that what is historically necessary goes beyond what is epistemically necessary:

> *The bet.* You have a coin which is either double-heads or double-tails. You may or may not toss the coin at 12pm. I make two bets. First I bet either that the coin won't land heads: that is, I bet it won't be flipped or it will land tails. I also bet it won't land tails, that it won't be flipped or will land heads. Right now it is not settled whether the coin will be flipped.

I could describe my situation as follows:

(2) Either if the coin is flipped, I can't win the first bet; or if the coin is flipped, I can't win the second bet. (I just don't know which.)

Here the modal appears to be restricted; and it does not appear to be epistemic.

The following principle captures the restrictor data for possibility modals:[1]

[1] See Gillies (2010) for a similar principle.

Conjunctivitis. $\Diamond A \supset ((A > \Diamond\ B) \leftrightarrow \Diamond(A \wedge B))$

Why the rider that A is possible? Because when A is not possible, $A > \Diamond B$ will be trivial true but $\Diamond(A \wedge B)$ will be false.

Again, take an epistemic example first. The following seem interchangeable:

(3) a. If the butler is the murderer, the gardener might have helped him.
b. Maybe the butler is the murderer and the gardener helped him.

(3-a) says that among the epistemically possible worlds where the butler is the murderer there is some world where the gardener helped him; which is just to say there is an epistemically possible world where the butler is the murderer and the gardener helped him (given the rider that it is possible the butler is the murderer).

With historical modality, we need to again focus on cases where epistemic and historical possibility come apart. One such case is that of action. Deciding I will leave work early to go to a party this evening enables me to now know that I will not be there; but it does not yet mean that it is historically impossible for me to go. Now consider:

(4) a. If I leave work early, I could go to the party. (But I know I won't.)
b. I could leave work early and go to the party. (But I know won't.)

Here we see the same Conjunctivitis pattern; but the modal cannot be epistemic.

2 The R Logic

I show how to derive both principles in a minimal logic which I take to be a regimentation of the restrictor theory, a logic I call R.

The first principle of R is Boolean Persistence:[2]

Boolean Persistence. $A > (B > A)$, when A is Boolean.

In update terms, Persistence says that successive updates do not undo earlier ones (for Booleans at least). The plausibility of Persistence is also witnessed by the status of claims like:

(5) If it rained, then if it was cold, then it rained and it was cold.

The second principle is Flattening:[3]

Boolean Flattening. $((A \wedge B) > C)) \leftrightarrow (A > ((A \wedge B) > C))$ when A, B, C are Boolean

Given the rest of R, this principle boils down to a simpler one: when A is logically stronger than B then $A > (B > C)$ is equivalent to $B > C$. This seems correct:

(6) a. If the first coin is flipped, then if it lands head, the second coin will also land heads.
b. If the first coin lands heads, the second coin will also land heads.

The final principle is Modal CEM:

Modal CEM. $(A > (\neg C > \bot)) \vee (A > \neg(C > \bot))$

As we will shortly see, $\neg C > \bot$ is a normal modal operator and $\neg(C > \bot)$ its dual. This principle thus says, in this proprietary sense, either if A, C is necessary or if A, then C is possible.

[2] See Boylan (2024b) for further discussion of this principle.
[3] This first appears in Mandelkern (forthcoming) and attributed to Cian Dorr.

The complete R logic is obtained by adding these three principles to a restricted version of the B conditional logic.[4] Here is the entire R logic:

PC. All theorems of PC.

Identity. $A > A$

Boolean Cautious Mon. $((A > B) \wedge (A > C)) \supset ((A \wedge B) > C)$, when A, B, C are Boolean.

Boolean OR. $((A > C) \wedge (B > C)) \supset ((A \vee B) > C)$, when A, B, C are Boolean.

Boolean Flattening. $((A \wedge B) > C) \leftrightarrow (A > ((A \wedge B) > C))$, when A, B, C are Boolean.

Boolean Persistence. $A > (B > A)$, when A is Boolean.

Modal CEM. $(A > (\neg C > \bot)) \vee (A > \neg(C > \bot))$

LLE. If $\vdash A \leftrightarrow B$ then $\vdash (A > C) \leftrightarrow (B > C)$

Agglomeration. If $\vdash (B_1 \wedge ... \wedge B_n) \supset C$ then $\vdash ((A > B_1) \wedge ... \wedge (A > B_n)) \supset (A > C)$

Detachment. $A, A \supset C \vdash C$

An important theorem of this logic is a restricted version of the Import-Export Principle:

Boolean Import-Export. $((A \wedge B) > C) \leftrightarrow (A > (B > C))$, when A, B, C are Boolean.

The various restrictions throughout are to avoid triviality: as Boylan (2024b) shows, triviality results can be proved from unrestricted Persistence plus Identity and Agglomeration; and from Boolean Persistence with unrestricted Cautious Monotonicity and OR. Triviality results can also be obtained from unrestricted Flattening, given the rest of R. And of course, famously, various triviality results can be proved from unrestricted Import-Export.

3 Deriving Boxiness and Conjunctivitis

We now show how to derive both Boxiness and Conjunctivitis for each flavour of modality.

First of all, it's helpful to introduce by stipulation a conditional necessity operator: $\Box_> A$ says that $\neg A > \bot$; its dual, $\Diamond A$, says that $\neg(A > \bot)$. The B conditional logic suffices to show that $\Box_>$ is a normal modal operator. We can then prove versions of Boxiness and Conjunctivitis stated in terms of this operator:

Boxiness$_{\Box_>}$. $(A > \Box_> C) \leftrightarrow \Box_>(A \supset C)$

Conjunctivitis$_{\Diamond_>}$. $\Diamond_> A \supset ((A > \Diamond_> B) \leftrightarrow \Diamond_>(A \wedge B))$

I sketch the proofs here. For Boxiness, suppose first that $A > \Box_> B$. Given the definition of $\Box_>$, this is equivalent to a purely conditional claim, namely $A > (\neg B > \bot)$. Applying Import-Export, we derive $(A \wedge \neg B) > \bot$ and then, applying LLE, $\neg(A \supset B) > \bot$. But this is just equivalent to $\Box_>(A \supset B)$. In fact, the whole argument is a chain of equivalences: simply run it in reverse to obtain the other direction of Boxiness.

For Conjunctivis, first suppose that we have $A > \Diamond_> B$ and $\Diamond_> A$. By definition of $\Diamond_>$, the former is equivalent to $A > \neg(B > \bot)$. When $\Diamond_> A$, then $A > \neg C$ holds only if $\neg(A > C)$ holds. Thus from $A > \neg(B > \bot)$ and $\Diamond_> A$ it follows that $\neg(A > (B > \bot))$. From Import-Export it then follows that $\neg((A \wedge B) > \bot)$. This is just equivalent to $\Diamond_>(A \wedge B)$. To obtain the

[4] The B logic is due to Burgess (1981).

other direction, we run the argument in reverse except Modal CEM is used to transition from $\neg(A > (B > \bot))$ to $A > \neg(B > \bot)$

To extend these arguments to the epistemic case, note that the default reading of the indicative is usually taken to be epistemic. For instance, indicatives usually presuppose the epistemic possibility of their antecedents:

(7) a. If the butler didn't do it, the gardener did.
b. $\rightsquigarrow$ The butler might have done it.

In general, such patterns make sense if the conditional has an epistemic accessibility relation; which amounts to assuming that the modality defined from the conditional as above is epistemic necessity, as suggested by Dorr and Hawthorne (ms.):

Identity of Accessibility, Epistemic. $\Box_E A \leftrightarrow (\neg A >_E \bot)$.

Given this principle we can move between claims like $\neg A >_E \bot$ and $\neg(A >_E \bot)$ on the one hand and $\Box_E$ and $\Diamond_E$ on the other. This allows us to run the same arguments as above to obtain Boxiness and Conjunctivitis for the epistemic modal operators $\Box_E$ and $\Diamond_E$ and $>_E$.

To obtain Boxiness and Conjunctivitis for historical modal operators, I postulate a second, historical reading of the indicative, $>_H$. I assume that, on this reading, the conditional has a historical accessibility relation, giving us:

Identity of Accessibility, Epistemic. $\Box_H A \leftrightarrow (\neg A >_H \bot)$.

This allows us to move between $\neg A >_H \bot$ and $\neg(A >_H \bot)$ on the one hand and $\Box_H$ and $\Diamond_H$. We can then once more run the arguments above to obtain Boxiness and Conjunctivitis for the historical modal operators.

This historical reading of the indicative is in fact independently plausible. Consider:

> *Don't Jump!* Jones, the office worker standing on the roof of her 20 story office building, looks down over the edge towards the ground. When we looked half an hour ago, there was no net anywhere near the building; that being said, we cannot be completely certain that there isn't one hanging now. Jones is a known thrill-seeker, but she has no death wish. (Adapted from Moss (2016), §4.3)

Here we can hear true (or highly likely) readings of both of the following:

(8) If Jones jumps, she will die.

(9) If Jones jumps, she will live.

As Ciardelli and Ommundsen (2024) note, it is plausible that (8) is a historical and (9) is an epistemic reading of the conditional.

4 A Restricting Operator Semantics

I now outline a *restricting operator* semantics, on which the R logic is sound.

This semantics has three core ideas. First, sentences are evaluated relative to a pair of a world and an accessibility relation. Second, conditionals have a selection function semantics: $A > C$ is true just in case C is true at the "closest" indices where A is true. Finally, conditionals shift the accessibility relation, as well as the world: the closest accessibility relation is one that *accepts* the antecedent.[5]

[5]For important precedents, see Yalcin (2007) and Goldstein and Santorio (2021).

Here is the semantic entry for the conditional presented more precisely. The selection function f is a function from a set of world-accessibility relation pairs (i.e. a set of indices) and a world-accessibility relation pair (i.e. an index) to a set of world-accessibility relation pairs. Then we have:

Def of $>$. $\langle w, R\rangle \models A > C$ iff for all $\langle w', R'\rangle \in f(\llbracket A \rrbracket, \langle w, R\rangle) : \langle w', R'\rangle \models C$

In order to validate the axioms of R, various constraints are imposed on f.

A particularly important constraint is the *Update Constraint*. Say that a pair $\langle w, R\rangle$ accepts A iff for all w' such that wRw' $w', R \models A$; that is, holding the original accessibility relation fixed, A is true at all worlds accessible from w. The Update Constraint ensures that, for all elements of $f(\mathbf{A}, \langle w, R\rangle)$, the accessibility relation parameter accepts $\mathbf{A}$. Say that $R^{\mathbf{A}}$ is the set of maximal subrelations R' of R which see only points where A is accepted, relative to R' itself.

Def. of $R^{\boldsymbol{A}}$. $R^{\mathbf{A}} = max\{R' \subseteq R : \text{if } wR'w' \text{ then } \langle w', R'\rangle \in \mathbf{A}\}$

The Update Constraint says that at all the "closest" pairs where A is true, the accessibility parameter is always one of the maximal subrelations of the original which accepts A:

Update Constraint. If $\langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ then $R' \in R^{\mathbf{A}}$.

Note that usually the relations in $R^{\mathbf{A}}$ are non-reflexive: for instance, if p is false at w, we have counterexamples to reflexivity at w. To ensure that no such points are selected by f we impose:

Propriety. If $\langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ then $w' \in R'(w')$.

We can now see that both Boolean Persistence and Identity will be valid, for essentially the same reason. Identity is valid because if $\langle w', R'\rangle \in f(\llbracket A \rrbracket, \langle w, R\rangle)$ then, by the Update Constraint, R' is such that if $w''R'w'''$ then $w''', R' \models A$; and by Propriety $w'R'w'$. Boolean Persistence is valid because Booleans persist through multiple updates: if A is Boolean, then $(R^{\llbracket A \rrbracket})^{\llbracket C \rrbracket}$ still accepts A; and so for all $\langle w', R'\rangle \in f(\llbracket A \rrbracket, \langle w, R\rangle)$ if $\langle w'', R''\rangle \in f(\llbracket C \rrbracket, \langle w', R'\rangle)$ then $R'' \in (R^{\llbracket A \rrbracket})^{\llbracket C \rrbracket}$ and, by Propriety, $w''R''w''$.

Many of the remaining constraints are simply analogues of the usual variably strict ones. The complete list is as follows, where $f_w(\mathbf{A}, \langle w, R\rangle) = \{w' : \exists R' : \langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)\}$:[6]

Update Constraint. If $\langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ then $R' \in R^{\mathbf{A}}$.

Propriety. If $\langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ then $w' \in R'(w')$.

Visibility. If $\langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ then $w' \in R(w)$.

Non-Vacuity. If for some $R' \in R^{\mathbf{A}}$ $R'(w) \neq \emptyset$, then $f(\mathbf{A}, \langle w, R\rangle) \neq \emptyset$.

CMon. If $f(\mathbf{A}, \langle w, R\rangle) \subseteq \mathbf{B}$, then $f_w(\mathbf{A} \cap \mathbf{B}, \langle w, R\rangle) \subseteq f_w(\mathbf{A}, \langle w, R\rangle)$.

Union. $f_w(\mathbf{A} \cup \mathbf{B}, \langle w, R\rangle) \subseteq f_w(\mathbf{A}, \langle w, R\rangle) \cup f_w(\mathbf{B}, \langle w, R\rangle)$.

Flat. $f_w(\mathbf{A} \cap \mathbf{B}, f(\mathbf{A}, \langle w, R\rangle)) = f_w(\mathbf{A} \cap \mathbf{B}, \langle w, R\rangle)$.

Relational Uniqueness. If $\langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ and $\langle w'', R''\rangle \in f(\mathbf{A}, \langle w, R\rangle)$, then $R' = R''$ and $R'(w') = R'(w'')$.

[6]Soundness for R is proved in Boylan (2024a).

5 Comparison to the Pure Restrictor Semantics

I close by comparing this semantics to its famous cousin, Kratzer's pure restrictor semantics:

Kratzer Semantics. $w, R \models A > C$ iff $w, R + [\![A]\!] \models C$, where $R + \mathbf{A}(w) = \{w : \langle w, R \rangle \in \mathbf{A}\}$.

The main difference in these views lies in how many parameters are shifted:[7] the Kratzer semantics shifts *only* the accessibility relation; the restricting operator semantics shifts both the world and the accessibility parameter. Does this make a difference?

It does. Consider bare conditionals like:

(10) If it rained, there was no party.

As Kratzer immediately noted, such conditionals pose a problem, prima facie at least, for a pure restrictor semantics: there is no modal to restrict and so the if-clause is vacuous. Kratzer posits a covert modal in sentences like (10), suggesting their true structure is something like:

(11) If it rained, MODAL there was no party.

The restricting operator semantics needs no covert modals here. When there is no modal, although the shift in accessibility relation is vacuous, the shift in world is not: (10) simply receives the standard variably strict truth-conditions for indicatives.

I think there are at least two things to be said in favour of the restricting operator approach. The first is that its explanation reflects better the structure of the data. Our earlier examples, like (1), (2), (3-a) and (4-a), show that conditionals can restrict modals; bare conditionals like (10) suggest this is not the *only* thing conditionals do. This is exactly the account given by the restricting operator semantics.

Second, the assumption of covert modals raises its own problems. A variety of authors, including Schulz (2010) and Ciardelli (2021), note that, without heavily restricting the places where this modal can appear, the theory massively overgenerates modal readings of non-conditionals. For instance, take claims like:

(12) Either it's raining or it's snowing.

(13) I doubt that it's raining.

Neither can be read respectively as:

(14) Either it must be raining or it must be snowing

(15) I doubt that it must be raining.

I thus conclude that its simple account of bare conditionals is a genuine advantage of the restricting operator theory. Further debate will examine how extensively the view can be developed. For the pure restrictor view has been applied to a large range of data, in particular, to both q-adverbs and deontic conditionals. In Boylan (2024a) I extend the semantics in this paper to capture those cases too.

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[7] Not the only difference though: as Mandelkern (2021) shows, Kratzer's semantics fails to validate Identity. This however could be avoided by shifting instead to an element of $R^{\mathbf{A}}$, rather than simply intersecting.

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