Relative quantification and equative scope-taking

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Abstract

Relative measure (RM) expressions, e.g., *60% of the olives*, are known to be amenable to a non-conservative interpretation, e.g., *60% olives$_F$*, since Ahn & Sauerland (2015a). Herein, I propose an analysis for modified RM phrases with equatives (*as much as 60%*) and with non-increasing restriction (*exactly 60%*) that combines a quantificational semantics for proportions (Pasternak & Sauerland 2022) with entity negation for pluralities (Bledin 2024; Elliott 2024). I apply this theory to the novel observation that non-conservative RM phrases can yield a cumulative reading (*exactly 2 suppliers sold exactly 60% olives$_F$*), as in van Benthem's puzzle (Benthem 1986; Brasoveanu 2013; Charlow 2021; a.o.).

1 Interpreting proportions

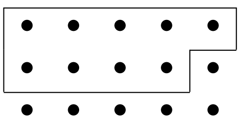
Similar in character to the proportional (1-a) and relative proportional (1-b) readings of vague *many* and *few* (Westerståhl 1985, p. 403; Herburger 1997; Partee 1989), relative measure (RM) expressions (2) can admit non-conservative readings (following Ahn & Sauerland 2015a,b, 2017).

(1) a. Many [$_S$ Scandinavians] [$_N$ have won the Nobel Prize in literature].
$\rightsquigarrow |S \cap N| \geqslant n$ (cardinal)
$\rightsquigarrow {}^{|S \cap N|}/_{|S|} \geqslant n$ (proportional)
b. Many [$_S$ Scandinavians$_F$] [$_N$ have won the Nobel Prize in literature].
$\rightsquigarrow {}^{|S \cap N|}/_{|N|} \geqslant n$ (relative proportional)

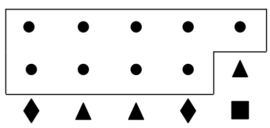
(2) a. The fruit supplier sold 60% of the olives.
b. The fruit supplier sold 60% olives$_F$.

With the removal of overt partitive material and the addition of focus-marking on the substance noun (*olives*), (2-b) yields reversed quantification. It asserts that, out of all things a particular fruit supplier sold (restrictor), 60% are olives (scope). We might visualize the state of affairs between the two readings as follows:

(3) a. 60% of the olives



b. 60% olives$_F$



Descriptively, what is partitioned in the former is the set denoted by the genitive substance noun, while it is the set denoting the sold items (of which, olives are included) for the latter.

In this paper, I address RM constructions with modified numerals in contexts that have been argued to potentially implicate other degree-related, scope-bearing elements — namely, as phrasal standards in quantity equatives and as components of non-increasing cardinality in cumulative readings (van Benthem's puzzle, Benthem 1986). The solution undertaken for the latter, which will inform the former, comes by way of extending the nascent proposal of negative individuals (Bledin 2024; Elliott 2024) to precise proportions.

2 A quantificational semantics

2.1 Pluralities with negation

Bledin (2024) observes that coordination with non-upward entailing quantifiers (*Mary and nobody else*) can follow from a compositional semantics where entity domains are 'polarized' to differentiate between positive and negative elements (Def. 2.1). Elliott (2024) picks up this proposal to develop a candidate theory of modified numerals, in comparison to post-suppositional (Brasoveanu 2010, 2013) and dynamic update (Charlow 2021) accounts.

Meta-language expressions and types are recorded in sans serif, with Function Application ($\ulcorner\alpha\ y\ x\urcorner$) left-associative and type complexity ($\ulcorner\alpha :: \phi \to \psi\urcorner$) right-associative by default.

Definition 2.1. Following Bledin (2024), we say that the *orthogonal counterpart* to an entity $x :: \mathsf{e}$ is $\neg x :: \mathsf{e}$ ('not-x'), to which *entity negation* has applied. Entity negation is assumed to apply on the atoms of the domain of entities D_{e}, but sum-closure of D_{e} excludes those with both an atom and its orthogonal counterpart as subparts, e.g., $x \oplus \neg x$ ('incoherence', Elliott 2024).

Given entity negation, it follows that the conventional denotation of a plural noun, e.g., $[\![\text{olives}]\!] = \lambda x.{}^*\mathsf{olive}\ x$, includes negative, non-negative, and mixed-polarity sums (4). What is the effect of composing a mixed-polarity predicate with a numeral n? We'll obtain the set of relevant entities with at least n-many non-negative atoms (5). If parthood and the sum relation are themselves set-based, $|x| = 3$ for $x = a \oplus b \oplus c$. Similarly, $|\cdot|^+$ counts only non-negative atoms.

(4) If $[\![\mathsf{olive}]\!] = \{a, b, c, \neg a, \neg b, \neg c\}$, $[\![{}^*\mathsf{olive}]\!] = \{a, b, c, \neg a, \neg b, \neg c, a \oplus \neg b, \ldots, a \oplus b \oplus \neg c, \ldots\}$

(5) a. $[\![\text{one olive}]\!] = \lambda x.{}^*\mathsf{olive}\ x \wedge |x|^+ \geqslant 1 \rightsquigarrow \{a, b, c, a \oplus b, a \oplus \neg b, \ldots\}$ $\qquad \mathsf{e} \to \mathsf{t}$

b. $[\![\text{two olives}]\!] = \lambda x.{}^*\mathsf{olive}\ x \wedge |x|^+ \geqslant 2 \rightsquigarrow \{a \oplus b, \ldots, a \oplus b \oplus \neg c, \ldots\}$ $\qquad \mathsf{e} \to \mathsf{t}$

2.2 *Percent* semantics

With this basic sketch, we can return to a semantics for proportions. The RM noun *percent* is typed as a degree quantifier given the denotation in (6) (adapted from Pasternak & Sauerland 2022, p. 251; Spathas 2022, p. 279), where $\mathsf{D} ::= (\mathsf{d} \to \mathsf{t}) \to \mathsf{t}$.[1] The **max** operator (7-a) returns the highest degree from a given degree predicate (Heim 2000; Buccola & Spector 2016; a.o.).

(6) $$[\![\text{percent}]\!] := \lambda d \lambda D.\frac{\mathbf{max}\ D}{\mathbf{max}(\mathbf{dom}\ D)} \geqslant \frac{d}{100} \qquad \mathsf{d} \to \mathsf{D}$$

(7) a. $[\![\mathbf{max}]\!] := \lambda D \iota d.D\ d \wedge \forall d'[D\ d' \to d' \leqslant d]$ $\qquad (\mathsf{d} \to \mathsf{t}) \to \mathsf{d}$

b. $[\![\mathbf{size}]\!] := \lambda d \lambda x.|x|^+ \geqslant d$ $\qquad \mathsf{d} \to \mathsf{e} \to \mathsf{t}$

To mesh an atomic-degree denotation for numerals (as needed for (6)) with their intersective meaning from (5), I'll assume a type-shifter from the former to the latter (7-b). I notate it as **size**, though it goes under different guises across the literature (see Bylinina & Nouwen 2020 for an overview). Arriving at the Predicate Modification in (5) is thus preceded by applying (7-b).

(8) 60% λd the fruit supplier sold d-MEAS olives$_F$ (cf. (2-b))

With *percent* and its numeral forming a separate constituent, attempting to compose with a gradable predicate ($\mathsf{d} \to \mathsf{e} \to \mathsf{t}$) would fail. The natural solution is for the RM noun to occupy a higher scope (QR) and saturate the degree argument with its trace, i.e, (8). The substance noun is shifted to a gradable denotation with an off-the-shelf measure operator (Solt's (2015)

[1] For space, I restrict attention to *percent* (which I use interchangeably with %) and set aside other RM nouns, e.g., *half*, *third*, *quarter*, which may warrant separate semantic (cf. Benbaji-Elhadad & Wehbe 2024) or morphosyntactic (cf. Pasternak & Sauerland 2022, pp. 236–237) treatments.

MEAS/Rett's (2014) M-OP). The basic picture of (2-b) is depicted as follows, where (9) shows the launch site of the degree quantifier, and (10), the landing site (intensional descriptions omitted; subject a proper name for simplicity).[2]

(9)
MeasP :: $\mathsf{e} \to \mathsf{t}$
$\lambda x.{}^{*}\mathsf{olive}\ x \wedge \mu\ x \geqslant d$
- d
- Meas′ :: $\mathsf{d} \to \mathsf{e} \to \mathsf{t}$
  $\lambda d \lambda x.{}^{*}\mathsf{olive}\ x \wedge \mu\ x \geqslant d$
  - MEAS
  - olives$_F$

(10)
TP :: t
$\frac{\textbf{max}\ [\![\mathrm{TP}]\!]}{\textbf{max}(\textbf{dom}\ [\![\mathrm{TP}]\!])} \geqslant \frac{60}{100}$
- 60% :: D
- TP :: $\mathsf{d} \to \mathsf{t}$
  $\lambda d.\exists x[{}^{*}\mathsf{olive}\ x \wedge \mu\ x \geqslant d \wedge \mathsf{buy}\ x\ \mathsf{Aldi}]$

For count nouns, e.g., units of fruit, the (contextual) measure function μ will amount to the non-negative cardinality from earlier.[3] For the regular, conservative use (e.g., (2-a)), I assume *percent* is shifted to a function that remains in-situ and expresses a parthood relation with the e-type genitive noun (Pasternak & Sauerland 2022).

3 Scopal interactions

We now turn to the issue of cumulativity (11) that §2's machinery has built up to. (11) is true when the maximum number of interviewing recruiters is 2, and the maximum proportion of women interviewed by recruiters (out of all interviewees) is 60%.

(11) Exactly two recruiters interviewed exactly 60% women$_F$ (between them).

This is, when excluding *between them* in (11), in addition to a distributive reading with surface scope — such that two (and only two) recruiters each interviewed 60% women and 40% non-women. The cumulative intuition does not fall out from the assumption that *exactly two recruiters* is simply existential, i.e., that there exists a two-member plurality of recruiters (since (11) should be false if there are, e.g., three interviewing recruiters). The challenge, moreover, is how to prevent (11) from yielding truth when there are multiple different combinations of interviewing recruiters such that each combination has the 60%-40% split.

This challenge is 'van Benthem's problem' (following Benthem 1986), which has elicited a variety of proposals about the scopal properties of non-increasing modified numerals (Krifka 1999; Brasoveanu 2013; Alxatib & Ivlieva 2018; Haslinger & Schmitt 2020; Charlow 2021; a.o.). RM nouns serve as a unique angle into this problem — since they can be employed in modified numeral constructions (unlike, e.g., **{exactly, less than, at most} many recruiters*) while also being degree-quantificational.

Enrichment with entity negation as sketched in §2 allows for a straightforward approach to this phenomenon following Elliott (2024): *exactly two recruiters* will denote a maximal plurality of the set of recruiters P with two non-negative members and all other elements from P encoded as negative members. To get there, Elliott (2024) leans crucially on an assumption that plural nouns must themselves only denote maximal sums, e.g., where $^{*}\mathsf{olive}$ in (4) would only contain the three-member elements. Without it, we'd be back at the original challenge, since sums without negative members for all other recruiters would also be admitted (no maximality effect would be predicted). Rather than generalize such an assumption to all plural nouns, I'll encode it into the

[2]Accessing Roothian focus alternatives to the substance noun will allow for the denominator to make reference to a domain larger than the noun itself. See Pasternak & Sauerland (2022, pp. 257–261) for additional discussion.

[3]Another strategy for measuring cardinality would be to gather the non-negative subparts (if any) first, and then feed them as a set or distinct sum to a non-polarity-specific $|\cdot|$.

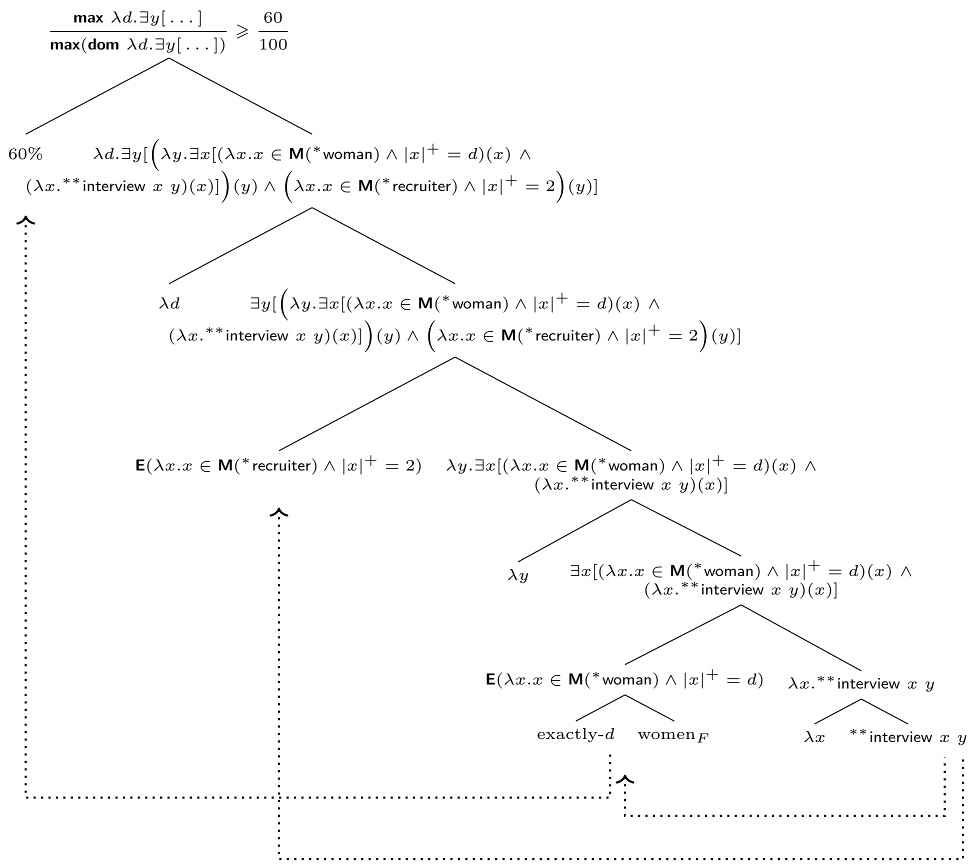


Figure 1: Scoping a modified proportion

denotation of the modifier. Whereas the predicate meaning of *two* is the set of sums with two or more non-negative members (12-a), *exactly two recruiters* denotes the set of maximal sums in $^*\mathsf{recruiter}$ with exactly two non-negative members (12-b)-(12-c). The operator gathering the maximal sums (notated as $\mathbf{M}$ to differentiate from $\mathbf{max}$) will return a set with multiple elements, since sets with entity negation will not have a unique maximum, e.g., $\mathbf{M}(\{a, \neg a\}) = \{a, \neg a\}$.

(12) a. $[\![\mathbf{size}\ \text{two}]\!] = \lambda x.|x|^+ \geqslant 2 \rightsquigarrow \{a{\oplus}b{\oplus}c, a{\oplus}b{\oplus}\neg c, a{\oplus}\neg b{\oplus}c, \neg a{\oplus}b{\oplus}c, a{\oplus}b, a{\oplus}c, b{\oplus}c\}$
b. $[\![\text{exactly}]\!]([\![\text{two}]\!]) = \lambda P \lambda x.x \in \mathbf{M}(P) \wedge |x|^+ = 2 \rightsquigarrow \{a \oplus b \oplus \neg c, a \oplus \neg b \oplus c, \neg a \oplus b \oplus c\}$
c. $[\![\text{exactly}]\!]([\![\text{two}]\!])([\![\text{recruiters}]\!]) = \lambda x.x \in \mathbf{M}(^*\mathsf{recruiter}) \wedge |x|^+ = 2$

Extending entity negation to modified precise proportions from (11), the degree trace that *60%* abstracts over (due to type-clash) serves as an argument to the modifier (Fig. 1).[4] Aside from this, composition proceeds in the same fashion as (12-c). The modifier undertakes the task that would normally be attributed to a measure operator, so no MEAS applies in the lower e-type quantifier.[5] The object quantifier will need to scope under the RM so the d-trace is bound.[6]

We can also consider cases besides non-monotone quantifiers, as in quantity equatives (13), without a RM modifier that introduces maximality or an 'exact'-reading. As one example,

[4]For a polarity-sensitive variant of the cumulation operator ** for verbal predicates (Sternefeld 1998; Beck & Sauerland 2000), see Elliott (2024). It stipulates a given relation must hold only of non-negative subparts.

[5]See (19) in the Appendix for a denotation that permits MEAS to always apply to the substance noun by accepting a gradable predicate.

[6]$\mathbf{E}$ is the type-shifter from predicates to quantifiers that introduces existential force (i) (following Partee 1987).

(i) $\mathbf{E}\ P = \lambda Q.\exists x[P\ x \wedge Q\ x]$.

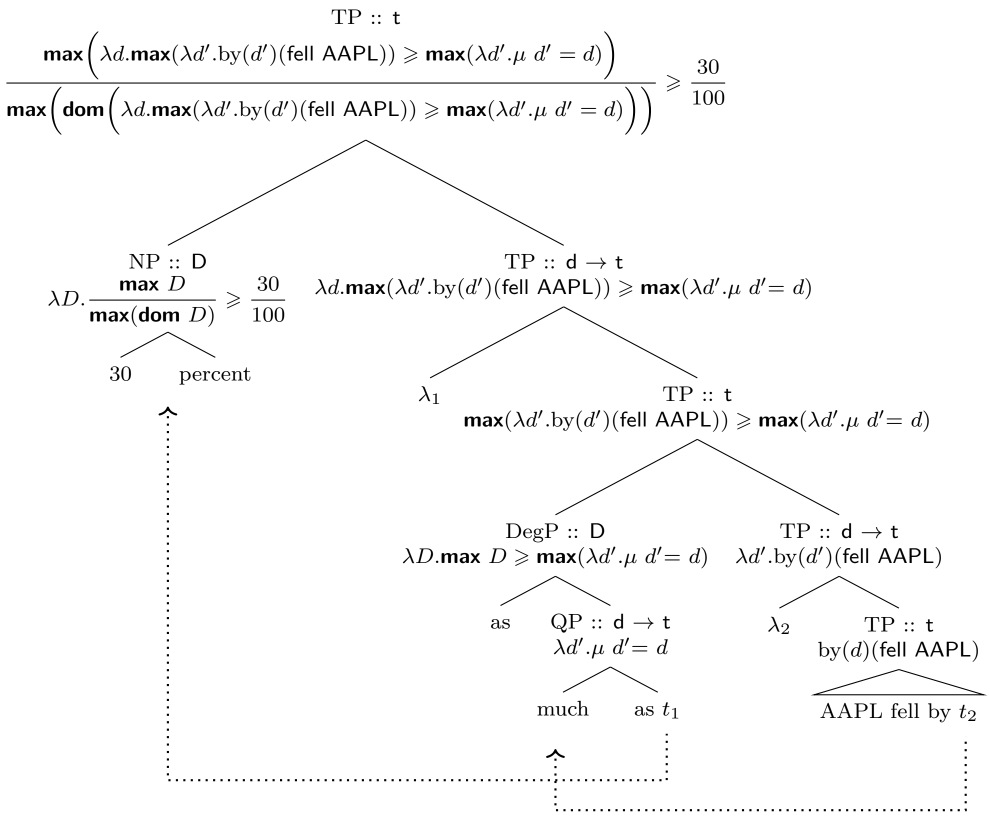


Figure 2: A proportion without a non-monotone quantifier

equatives are themselves typically understood to implicate degree quantification, as with the entry for the equative operator (the first *as* in, e.g., *as much as*) in (14) (e.g., Heim 2000; Rett 2010, 2015, 2020). In this case, the resulting reading for the RM phrase is still an 'at-least' one.

(13) a. Fanta contains as much as 30% sugar$_F$.
b. The price fell by as much as 30%.

(14) $[\![\text{as}]\!] := \lambda D \lambda D'.\textbf{max}\ D' \geqslant \textbf{max}\ D$ $\qquad (\mathsf{d} \to \mathsf{t}) \to \mathsf{D}$

Accounting for the LF of (13), then, means considering the scope-taking potential of both the equative operator and the RM noun, as in Fig. 2.[7] While a degree-quantifier approach to percentages is not the only one available to us — e.g., the degree-relational denotation (16-a) from Gobeski & Morzycki (2017), who primarily focus on reduced clausal equatives — we must avoid type-conflicts with the quantity adjective (16-b) (entry adapted from Coppock & Bogal-Allbritten 2018). Evident in (15), however, the modifier is vacuous in its contribution, hence the persistence of the 'at-least' reading.

(15) $[\![\text{Fanta contains as much as 30\% sugar}_F.]\!] \rightsquigarrow$

$$\frac{\textbf{max}\Big(\lambda d.\textbf{max}(\lambda d'.\textsf{contains}(d'\text{-}\textsc{meas}\ \textsf{sugar})(\textsf{Fanta})) \geqslant \textbf{max}(\lambda d'.\mu\ d' = d)\Big)}{\textbf{max}\Big(\textbf{dom}\Big(\lambda d.\textbf{max}(\lambda d'.\textsf{contains}(d'\text{-}\textsc{meas}\ \textsf{sugar})(\textsf{Fanta})) \geqslant \textbf{max}(\lambda d'.\mu\ d' = d)\Big)\Big)} \geqslant \frac{30}{100}$$

(16) a. $[\![10\%]\!] = \lambda d.10\% \times d$ (Gobeski & Morzycki 2017, p. 727)
b. TP-[by [as $\boxed{\text{[much as 30\%]}}$]] $\rightsquigarrow [\lambda d \lambda d'.\mu\ d' = d]([\![_{\text{G\&M17}}\ 30\%]\!]) :: ✗$

[7] For a fine-grained analysis of RM constructions that properly accounts for the degree-of-change meaning that (13-b) reflects, see Spathas (2024).

4 Concluding discussion

This paper provides a first analysis of the novel observation that non-conservative proportions can be modified and receive cumulative and non-cumulative readings. The corresponding analysis combines a quantificational theory of proportions (Pasternak & Sauerland 2022) with a theory of entity negation (Bledin 2024; Elliott 2024) to derive the correct 'exact'-interpretation.

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A Bare proportions and degree pluralities

Li (2022) provides the following judgement for Mandarin (17), where the unmodified proportional DP must take narrow scope relative to the modified-numeral subject. This is taken to be an instantiation of what, e.g., Bylinina & Nouwen (2020) and Gajewski (2008) refer to as the Heim-Kennedy generalization (HKG; following Kennedy 1997; Heim 2000), which stipulates that degree quantifiers cannot scope over e-type quantifiers.

(17) Zhenghao you liang-ge xuesheng du-le sanfenzhiyi de xiaoshuo$_F$.
exactly have two-CL students read-PRF one.third DE novels

a. 'There are exactly two students such that they each read one third novels$_F$.'

b. #'One third of the books that exactly two students read were novels$_F$.'

(Li 2022, pp. 10, 26)

The narrow scope in (17) is explained as undefinedness from the failure to gather a unique minimum degree, within the context of a degree-plurality framework (see Li 2022 for analysis; see Dotlačil & Nouwen 2016 and Nouwen & Dotlačil 2017 for degree pluralities). If RM expressions modified by *exactly* indeed take scope, why do we not observe infelicity for the cumulative reading from §3? For space, I simply point out that the modified proportion would *not* violate the variant of the HKG suggested by Crnič (2017) in (18-a), if we 'pluralize' our ontology for degrees and liken *exactly* to SHIFT (both change an 'at-least' reading to 'exactly') in (18-b).

(18) a. Modified HKG: If the scope of an e-type quantifier contains the trace d of a degree quantifier, d must be an argument to SHIFT.

b. $[\![\text{SHIFT}]\!] := \lambda d \lambda A \lambda x.\mathbf{max}(\lambda d'.A\ d'\ x) \sqsubseteq d$ $\qquad \mathsf{d} \to (\mathsf{d} \to \mathsf{e} \to \mathsf{t}) \to \mathsf{e} \to \mathsf{t}$

(adapted from Crnič 2017)

Under such assumptions, we predict why the proportion can take scope over the modified-numeral subject specifically when modified by *exactly*. The resulting denotation for a modified RM phrase in (19) also preserves the structural consistency of MEAS applying to the substance noun, though the maximality condition would still need to be enforced.

(19) exactly-d women$_F$ $\rightsquigarrow \lambda x.\mathbf{max}(\lambda d'.(\text{MEAS}\ {}^*\mathsf{woman})\ d'\ x) \sqsubseteq d$ (cf. Fig. 1)

Just as we may pluralize degrees, we can also consider what explanatory possibilities arise from polarizing them (as Bledin 2024 does for entities). I leave this to future work, but note that a potential avenue may be via corrective-*but* applying to degrees (or their $\mathsf{e} \to \mathsf{t}$ equivalent), e.g., (20) — where, under a notion of 'degree negation', *not 20 but 30* might denote the sum $\neg 20 \oplus 30$.

(20) a. The fruit supplier sold not 20 but 30% olives$_F$.

b. The basketball team lost by not 20 but 30 points.

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