Neg-raising interacts with implicatures: The case of *doubt*[*]

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Abstract

The verb *doubt* is a downward-monotonic attitude predicate. I argue that it should be analyzed as a negated existential modal, and that the inferences it comes with result from the interaction between neg-raising and a possibility implicature. In doing so, I provide a discussion of neg-raising and scalar implicatures arising from one and the same predicate.

1 Introduction

The verb *doubt* is an attitude predicate that creates a downward monotonic environment in its clausal complement; this licenses weak negative polarity items (NPIs) like *ever*.

(1)   a.   Abigail doubts that Taro eats meat.
            ∴ Abigail doubts that Taro eats chicken.
       b.   Abigail doubts that Taro will ever pass the exam.

This predicate has received some attention in the literature. It is generally considered a negative form of *believe* (see for instance Heim 1992, p. 190, Gajewski 2007, p. 297, Chierchia 2013, p. 200, or Abrusán 2016, p. 185), but some work focusing on this predicate has noticed peculiarities in the inferences it leads to and the types of clauses it can embed (Karttunen 1977; Fischer 2003; Biezma and Rawlins 2012; Anand and Hacquard 2013; Uegaki 2023). A shared intuition is that the meaning of *doubt* is more weakly negative than *not doubt* is positive. The examples in (2) show that Moorean contradictions arise with *not doubt*, but not with non-negated *doubt*.

(2)   From Uegaki 2023, p.c. with F. Staniszewski

       a.   I doubt that Jo's sick, but she might be.
       b.   #I don't doubt that Jo's sick, but she might not be.

The main explanandum of this paper is why *not doubt* conveys a strong doxastic attitude towards the embedded proposition, whereas non-negated *doubt* has a weak meaning, such that the attitude holder can still entertain the possibility that the embedded proposition is true. Throughout the paper, I will make reference to English judgments by my informants as well as my own judgments on Italian, which align with English for all the examples considered.[1]

2 Not so weak

Uegaki (2023) proposes that *doubt* has a weak assertive meaning: it is a negated universal quantifier over the attitude holder's doxastic alternatives, in a Hintikkan semantics. When embedding a declarative CP (the case I'll focus on in this paper), it has the denotation in (3).

(3) $[\![\mathsf{doubt}]\!]^w = \lambda p_{\langle s,t \rangle} . \lambda x_e : \exists w' \in \mathrm{DOX}_x^w \ p(w') . \ \neg \forall w'' \in \mathrm{DOX}_x^w \ p(w'')$ (Uegaki 2023)

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[1]Unlike English, Italian *dubitare* 'doubt' does not generally embed finite polar questions.

Under this account, a strong meaning with universal quantificational force is immediately returned when *doubt* is embedded under negation. This explains (2-b). To prevent that non-negated *doubt* has too weak a meaning, Uegaki proposes that an implicature is calculated on domain alternatives (following Bar-Lev 2021) in upward-entailing environments. However, the meaning of *doubt* cannot be strengthened to that of a negated existential quantifier: it is presupposed that at least one accessible world makes the embedded proposition true. For the presupposition of the assertion to be satisfied, some alternatives must be pruned, namely at least those that consider singleton domains. This minimal pruning moderates the strength of *doubt* to less than negated existential, thus guaranteeing that discourses like (2-a) are coherent.

A first concern with this analysis is the meaning that *doubt* has if only minimal pruning applies. Following Bar-Lev (2021), the exhaustification operator $\text{EXH}^{\text{IE+II}}$, which considers both exclusion and inclusion of alternatives (Bar-Lev and Fox 2020), asserts all propositions obtained by replacing the domain of quantification DOX_x^w with its subdomains D. Uegaki seems to implicitly assume that domain alternatives are not presuppositional. The resulting meaning in (4-a), where $|D| \geq 2$ because of the presupposition, is equivalent to the one in (4-b).

(4)   Given (3), $[\![\text{EXH}^{\text{IE+II}} \ x \ \textsf{doubts}_{\text{DOX}_x^w} \ p]\!]^w = \texttt{T}$ if and only if:

    a. $[\![x \ \textsf{doubts}_{\text{DOX}_x^w} \ p]\!]^w = \texttt{T}$ and $[\![x \ \textsf{doubts}_D \ p]\!]^w = \texttt{T}$ for all $D \subseteq \text{DOX}_x^w$ such $|D| \geq 2$

    b.   Equivalently, $\exists! w' \in \text{DOX}_x^w \ p(w')$

Proof. Assume (4-a). By the presupposition of (3), there is at least one $w' \in \text{DOX}_x^w$ such that $p(w') = \texttt{T}$. Suppose there is more than one: then, there is a subdomain D of two elements that both make p true. This contradicts (4-a): so, there is just one such w'. Conversely, assume (4-b). The presupposition and the assertion of (3) are satisfied, and no subdomain of two elements will contain more than one world that makes p true. So, all the asserted alternatives are true.   □

A meaning that requires a proposition to be true in exactly one world of a domain is implausible. Moreover, Uegaki admits that pruning of more alternatives should be context dependent, but this returns an undesirable form of context dependency. First, it is possible to see, following the proof above, that if all alternatives on domains of cardinality less or equal to n are pruned, the meaning asserts that p is true of $n - 1$ worlds in DOX_x^w. If pruning is licensed by a question under discussion that makes certain alternatives irrelevant (following Bar-Lev 2021), a question that affects pruning for *doubt* must ask whether the number of worlds in DOX_x^w that make the embedded proposition true is higher or lower than a certain n. As argued by von Fintel and Iatridou (2008, p. 118), meanings that "count possible worlds" might be implausible.

A second reason to reject (3) is that entailments of sentences containing *doubt* are not captured. The inferences in (5) are intuitively valid, and (5-b) shows that (5-a) can be generalized to the conjunction of an indefinite number of proposition (here via universal quantification).

(5)   a.   I doubt that Taro has a red hat, and I doubt that he has a green hat.
            ∴ I doubt that Taro has a red or a green hat.

    b.   For every student in the class, I doubt that s/he passed the exam.
            ∴ I doubt that any student in the class passed the exam.

In standard modal logic (K), these entailments are valid for a strong, but not for a weak negative modal, as in (6). Countermodels exist for hypothetical modals of intermediate strength as well: the symbol $\square_{/2}$ below means that the proposition is true in at least half of the accessible worlds. Countermodels are notated with sets $R[w^0]$ of the worlds $w^1, \ldots, w^n$ accessible from the world of evaluation w^0, subscripts indicating what propositions are true at each world.

(6)   a. $\neg \Diamond p \wedge \neg \Diamond q \models \neg \Diamond (p \vee q)$

    b. $\neg \square p \wedge \neg \square q \not\models \neg \square (p \vee q)$ *countermodel*: $R[w^0] = \{w^1_{p, \neg q}, w^2_{\neg p, q}\}$

c. $\neg\square_{/2}p \wedge \neg\square_{/2}q \not\models \neg\square_{/2}(p \vee q)$ *countermodel*: $R[w^0] = \{w^1_{p,\neg q}, w^2_{\neg p,\neg q}, w^3_{\neg p,q}\}$

Given (5), *doubt* should be analyzed as a negated existential quantifier on a modal domain. In Section 4, the intuitive weakness of *doubt* and its felicity in (2-b) are argued to be a function of the domain of quantification. Before moving on to that, consider that *not doubt* is strong, too: the relevant inferences are in (7), and (8) shows that universal quantificational force is necessary.

(7)   I don't doubt that Taro has a red hat, and I don't doubt that he has a green hat.
        ∴ I don't doubt that Taro has a red and a green hat.

(8)   a. $\square p \wedge \square q \models \square(p \wedge q)$
        b. $\lozenge p \wedge \lozenge q \not\models \lozenge(p \wedge q)$ *countermodel*: $R[w^0] = \{w^1_{p,\neg q}, w^2_{\neg p,q}\}$

3   A negative neg-raiser

If we model natural language attitude predicates and negation as modal operators and negation in modal logic, the observations above raise a symmetry problem. If ⌜doubt that p⌝ is translated as $\neg\lozenge p$, then ⌜not doubt that p⌝ should be $\lozenge p$; but this fails to capture (7). On the other hand, if the latter is translated as $\square p$, the former should be $\neg\square p$; but this fails to capture (5). A standard solution to this type of puzzle is to assume that one set of inferences is due to the lexical meaning of the predicate, whereas the other is the result of a meaning strengthening operation. I propose that the lexical meaning of *doubt* has strong quantificational force, as in (9).[2] Here, $\mathcal{B}^w_{\mathsf{d}}(x)$ returns the quantification domain of *doubt* for an agent x and world w.

(9) $[\![\mathsf{doubt}]\!]^w = \lambda p_{\langle s,t\rangle} \,.\, \lambda x_e \,.\, \neg\exists w' \in \mathcal{B}^w_{\mathsf{d}}(x)\ p(w')$

Instead, when *doubt* is embedded under negation, a meaning strengthening operation applies. This amounts to saying that *doubt* is a neg-raising predicate (Horn 1971; Horn 1978). Neg-raising alone does not explain why *not doubt* expresses certainty, but its role will become clear in Section 4. However, it crucially guarantees the validity of inferences like (7). Evidence that *doubt* indeed behaves like a neg-raiser comes from the disappearance of the strong reading when negation is introduced by a higher clause, as shown by the contrast in (10). Similarly, negating an utterance containing *doubt*, with an appropriate prosodic contour, can lead to a non-strengthened reading, as in (11). This resembles what has been labeled local accommodation under negation (Heim 1983), metalinguistic negation (Horn 1985), or contrastive negation (Bruno 2017), and the intonation profile is that of a contrastive topic (Büring 2003; Meyer 2016).

(10)   a. #Abigail doesn't doubt that Taro eats meat; she thinks he might eat it.
        b. It's not true that Abigail doubts that Taro eats meat; she thinks he might eat it.

(11)   –   I've heard that you doubt that Taro will pass the exam.
        –   I don't DOUBT that Taro will pass the exam. I'm merely agnostic.

Clause-mate negation anti-licenses an NPI in the complement of *doubt* (Homer 2021), as shown in (12). The effect seems to persist if negation comes from a superordinate *not believe*, but not *not want*, as shown in (13). If *doubt* is a neg-raiser, this contrast might be explainable because only *believe*, but not *want*, allows for cyclic neg-raising (Horn 1971; Gajewski 2007; Homer 2015).

(12)   ??Abigail doesn't doubt that Taro will ever pass the exam.

(13)   a. ??I don't believe Abigail doubts that Taro will ever pass the exam.
        b. I don't want Abigail to doubt that Taro will ever pass the exam.

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[2]This lexicalization, which gives *doubt* the meaning of a negated existential modal, is consistent with the generalization in Horn (1990) that natural language tends not to lexicalize negated universal quantifiers.
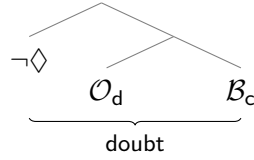
Figure 1: Decompositional analysis of the verbal head for *doubt*

4   A possibility implicature

Doubt being a neg-raiser is not enough to explain the difference in strength that arises under negation and the contrast in (2-b). My proposal builds on the following observation: the intuitive uncertainty that *doubt* carries can be reinforced or canceled like an implicature, as in (14).

(14)   a.   Abigail doubts that Taro will pass the exam, but she isn't certain that he won't.
        b.   Abigail doubts that Taro will pass the exam; in fact, she's certain that he won't.

I propose a way of deriving the possibility inference as an implicature. I assume that the modal domain of *doubt* competes and forms a scale with the modal domain of *be certain*; this scale is given by a subset relation between the former and the latter, and it is revealed by the asymmetric entailment between ⌜be certain that $\neg p$⌝ and ⌜doubt that p⌝ illustrated in (15).

(15)   Abigail is certain that Taro doesn't eat meat.
        $\therefore$ Abigail doubts that Taro eats meat.

I adopt a decompositional analysis of verbal heads for attitude predicates along the lines of Theiler, Roelofsen, and Aloni (2019), such that the quantificational component is detached from the domain of quantification in syntax. This reproduces the intuition that Kratzer (1977) had for modals. Unlike modals, attitude predicates in English are rigid in flavor, which might suggest that such a decomposition is unmotivated. However, recent research on other languages (see for example Bogal-Allbritten 2016; Močnik and Abramovitz 2019) showed that this difference is not cross-linguistically stable, and that attitude predicates can vary in their flavors just like modals.

The logical component of *doubt*, which I notate as $\neg\Diamond$, provides negated existential quantificational force. The domain of quantification is derived by applying an ordering operator $\mathcal{O}_{\mathsf{d}}$ to the modal domain of quantification for the predicate *be certain*, notated as $\mathcal{B}_{\mathsf{c}}$. Denotations of terminals are given in (16), which compose as in Figure 1. For every world w and agent x, let $<_x^w$ be a strict partial ordering relation such that for all w', w'', it holds that $w' <_x^w w''$ if and only if w' is more plausible than w'' according to x in w. This is evocative of plausibility orderings from the literature on belief revision (Blumberg and Lederman 2021; Hansson 2022).

(16)   a. $[\![\mathcal{B}_{\mathsf{c}}]\!]^w = \lambda x_e . \lambda w' . w'$ is compatible with what x is certain of at w
        b. $[\![\mathcal{O}_{\mathsf{d}}]\!]^w = \lambda B_{\langle e, \langle s, t \rangle \rangle} . \lambda x_e . \lambda w' . w' \in B(x) \wedge \neg \exists w'' \in B(x) \; w'' <_x^w w'$
        c. $[\![\neg\Diamond]\!]^w = \lambda B_{\langle e, \langle s, t \rangle \rangle} . \lambda p_{\langle s, t \rangle} . \lambda x_e . \neg \exists w' \in B(x) \; p(w')$

Given (16) and Figure 1, it is possible to check that the domain of quantification for *doubt* is a subset of that for *be certain*: for all x, w, it holds that $[\![\mathcal{O}_{\mathsf{d}}]\!]^w([\![\mathcal{B}_{\mathsf{c}}]\!]^w)(x) \subseteq [\![\mathcal{B}_{\mathsf{c}}]\!]^w(x)$. By adopting deletion alternatives from Fox and Katzir 2011, an alternative for a sentence containing *doubt* can be formally derived by deleting the ordering operator $\mathcal{O}_{\mathsf{d}}$ from the structure. If this strictly stronger alternative is negated by exhaustification, the result is that there is a world in the domain of quantification for *be certain* where the embedded proposition is true (see (20-b)).

The domain of *doubt* competes with the domain of *be certain*, but the reverse does not happen. Given that the latter corresponds to a syntactically simpler structure, it does not have an alternative where $\mathcal{O}_{\mathsf{d}}$ is inserted (Fox and Katzir 2011). This asymmetry in structural

complexity guarantees that the implicature in (17) is not generated.

(17)   Abigail is not certain that Taro eats meat.
         ⇝̸ Abigail doubts that Taro doesn't eat meat.

There is additional evidence that this inference has grammatical reality. The denotation proposed in (9) alone makes *doubt* an anti-additive function. Although theories of NPI licensing are very diverse, anti-additivity would generally lead to the expectation that not only weak, but also strong NPIs are licensed in the scope of *doubt* (Zwarts 1998; Gajewski 2005; Gajewski 2011; Chierchia 2013; Homer 2021). However, this is not the case for NPIs like *either*, puctual *until*, and the perfective modifier *in weeks*, or for the Italian *neppure* '(not) even', *fino a* 'until', and *affatto* 'at all'.[3] An example is given in (18). If *doubt* comes with a possibility implicature, the overall meaning is non-monotonic, and the degradedness of certain NPIs can be explained.[4]

(18)   Will Taro call Abigail before Sunday?
         a.   I don't think that he will call her until Monday.
         b.   *I doubt that he will call her until Monday.

Going back to the original explanandum, it is possible to derive the strong meaning of *not doubt* as the interaction between the possibility implicature presented above and neg-raising. For concreteness, I will follow a proposal by Romoli (2013), who analyzes neg-raising as an implicature that arises from the negation of an excluded middle alternative. To make it compliant with the architecture in Fox and Katzir 2011, I introduce a modal quantifier, defined in (19), that competes as a scalemate item with the logical component of *doubt*, in (16-c).

(19) $[\![\neg\Diamond \vee \Box]\!]^w = \lambda B_{\langle e, \langle s,t\rangle\rangle} \cdot \lambda p_{\langle s,t\rangle} \cdot \lambda x_e \cdot [\neg\exists w' \in B(x)\ p(w')] \vee [\forall w' \in B(x)\ p(w')]$

Alternative computation can replace $\neg\Diamond$ with $\neg\Diamond \vee \Box$. If multiple replacements are possible, as argued by Bar-Lev and Fox (2020), an alternative is available where both substitution of the logical component and deletion of $\mathcal{O}_d$ apply. Implicature computation is informally summarized in (20), where the subscripts d and c specify whether the domain of quantification is that of *doubt* or *be certain*. I use the EXH operator defined in Fox 2007 and adopted in much subsequent literature; the set of alternatives is in (20-a). It is shown in (20-b) that in positive environments, alternatives with the excluded middle operator do not add to the meaning, and the implicature is as described above. Instead, when *doubt* is embedded under negation, negating the excluded middle alternatives strengthens the meaning as in (20-c). The result asserts that the embedded proposition is true in all worlds compatible with what the attitude holder is certain of.

(20)   a. $Alt(\neg\Diamond_d p) = \{\neg\Diamond_d p, \neg\Diamond_c p, (\neg\Diamond_d p \vee \Box_d p), (\neg\Diamond_c p \vee \Box_c p)\}$
         b. $\textsc{exh}(\neg\Diamond_d p) = \neg\Diamond_d p \wedge \Diamond_c p \wedge \neg(\neg\Diamond_c p \vee \Box_c p) = \neg\Diamond_d p \wedge \Diamond_c p$
         c. $\textsc{exh}(\neg\neg\Diamond_d p) = \neg\neg\Diamond_d p \wedge \neg\neg(\neg\Diamond_d p \vee \Box_d p) \wedge \neg\neg(\neg\Diamond_c p \vee \Box_c p) = \Box_c p$

5   Conclusions

The analysis provided above accounts for all the inferences presented in Sections 1 and 2: *doubt* validates inferences as a strong negative modal, but it conveys uncertainty; *not doubt*, which

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[3]Certain strong NPIs that are licensed in questions, like *a single thing* (Homer 2021, p. 15) or Italian *nessuno* 'nobody', instead seem to be possible in the scope of *doubt*.

[4]A reviewer points out that as noted in Chierchia 2013, p. 201 (p.c. with Kai von Fintel and Sabine Iatridou), universal modals generally do not give rise to indirect scalar implicatures that disrupt (even strong) NPI licensing. Although it is not clear what expectations about *doubt* this observation should lead to, it is worth noticing that the same strong NPIs as above seem to be degraded in the scope of *not certain*.

behaves like a strong positive modal, conveys certainty and can lead to Moorean contradictions. This was achieved by making neg-raising and a possibility implicature interact, given the possibility of multiple replacements in the computation of alternatives (Bar-Lev and Fox 2020).

One crucial ingredient of the analysis is the decomposition of the attitude predicate in the syntax and the consequent possibility to generate scalar alternatives by either manipulating the domain of quantification (possibility implicature) or the logical component (neg-raising). This requires alternatives that are well formed in meaning and active in the computation of implicatures even if they do not have pronounceable trees: they are *conceptual alternatives* in the sense of Buccola, Križ, and Chemla (2021). A very similar analysis can be given of *be certain* under negation, which seems to give rise to an indirect scalar implicature (called a presupposition in Uegaki 2020) despite there not being any existential attitude predicate in the English lexicon.

(21)   Abigail is not certain that Taro eats meat.
        ⤳ Abigail thinks it's possible that Taro eats meat.

This paper uses an implementation of Romoli's (2013) analysis of neg-raising as a scalar implicature. It is conceivable that another analysis of neg-raising can acheive the same results. The present account relies on the assumption that the property of being a neg-raiser is inert unless the predicate is in a negative environment (see Alxatib and Đurovic 2019 for possibly another case for the same intuition). Instead, analyzing neg-raising as a presupposition or as a scaleless implicature fails to achieve the same. If neg-raising comes from an excluded middle presupposition, and if alternatives are presuppositional, the alternative that manipulates the modal domain is never innocently excludable, regardless of polarity. A strong reading for *not doubt* can be achieved by innocently including the alternative, but the same move for ⌜doubt that p⌝ returns a meaning equivalent to ⌜be certain that not p⌝. The proofs are left to the reader.

The approach to neg-raising by Jeretic (2022) would not be suitable, either. That account, which derives neg-raising just like homogeneity in Bar-Lev 2021, uses exhaustification to assert subdomain alternatives. Given an attitude holder x and world w, if ⌜x doesn't doubt that p⌝ is strengthened to an existential statement on each subset of $[\![\mathcal{B}_{\mathsf{c}}]\!]^{w}(x)$, a strong meaning for *not doubt* can be derived. However, the same mechanism also generates innocently excludable subdomain alternatives with non-negated *doubt*, namely those for $D \subseteq [\![\mathcal{B}_{\mathsf{c}}]\!]^{w}(x) \setminus [\![\mathcal{O}_{\mathsf{d}}]\!]^{w}([\![\mathcal{B}_{\mathsf{c}}]\!]^{w})(x)$. The returned meaning asserts that the embedded proposition is true in all worlds in the domain of *be certain* that are not in the domain of *doubt*. Such a meaning is too strong and validates unacceptable inferences like (22). For reasons of space, the proofs are again left to the reader.

(22)   I doubt that Taro bought the red hat, and I doubt that he bought the green hat.
        ⇏ I'm certain that he either bought both hats or neither one.

Finally, this paper adopts a method to distinguish modals of intuitively different strengths by operating on their domain of quantification instead of their quantificational force, as has been previously done by von Fintel and Iatridou (2008). In this way, it is possible to simultaneously capture inferences that are valid because of the quantificational strength of the modal, as (5) and (7), and contrasts that arise because of the domain of quantification, as (2). Moreover, different domains of quantification that are derived from one another can compete and form a scale, thus generating implicatures about the strength of the modal. This could be explored outside of the case of *doubt*, for example with weak necessity modals. If they quantify universally on a restricted domain, compared to strong necessity modals, and if this difference is represented structurally, strength implicatures can arise, and indeed, continuations like in (23) are felicitous.

(23)   a.   Taro should bring a drink, but he doesn't have to.
        b.   Taro should bring a drink; in fact, he has to.

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