A localist approach to the semantics and distribution of X-marking*

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Abstract

I propose an account that assigns a uniform semantics to each local occurrence of X-marking. The proposed account correctly captures the distribution of X-marking in conditionals and desire reports in English and Greek (and possibly many other languages).

1 Introduction

X-marking (von Fintel and Iatridou 2023; vFI) refers to grammatical items that imply suspension of some established facts, which is necessary, for instance, to discuss counterfactual situations.[1] Such items include Past (Imperfective) that occurs in conditionals (1) and desire reports (2).[2]

(1) [*An o archigos* ***pethene***$_{\text{X-MARKED}}$ *avrio*], [*tha ton* ***thavame***$_{\text{X-MARKED}}$ *eki*]. (Greek)
if the chief die.PAST.IMPF tomorrow FUT him buy.PAST.IMPF there
'If the chief **died**$_{\text{X-MARKED}}$ tomorrow, we **would**$_{\text{X-MARKED}}$ bury him there.' (vFI: 12)

(2) *Tha* ***ithela***$_{\text{X-MARKED}}$ [*na* ***erxotan***$_{\text{X-MARKED}}$ *avrio*]. (Greek)
FUT want.PAST.IMPF NA come.PAST.IMPF tomorrow
'I wish he **would**$_{\text{X-MARKED}}$ come tomorrow.'

There are multiple occurrences of X-marking in (1) and (2): in (1), X-marking appears in both the antecedent and the consequent; in (2), it appears both on the desire predicate and in the complement (at least in Greek). A challenge for any account of X-marking is to correctly figure out the contribution of *each local occurrence* of X-marking. This is left open in vFI. Many authors have similarly sidestepped this issue (e.g., Iatridou 2000; Mackay 2019), or have suggested that either one of them is the result of agreement with another (e.g., Schulz 2014; Grønn 2021).

I propose an account that assigns a uniform semantics to each local occurrence of X-marking in (1)-(2), thus dispensing with the assumption of agreement to explain the same facts.

2 Proposal

The idea is that each occurrence of X-marking presupposes that the local domain with which it is associated has undergone a *domain expansion* (i.e., an operation that suspends some facts/beliefs; see vFI). (3) is the proposed entry. $\mathcal{D}$ takes a world w and returns a set of worlds (a domain) for evaluating modal expressions with respect to w. Which world and domain function are default may be affected by contextual or grammatical factors; this will be clarified along the way.

(3) Let w^* be a default world and $\mathcal{D}^*$ be a default domain function. For any world w and domain function $\mathcal{D}$, $[\![\text{X } \phi]\!]^{w,\mathcal{D}}$ is defined only if $\mathcal{D}^*(w^*) \subset \mathcal{D}(w)$; if defined, $[\![\text{X } \phi]\!]^{w,\mathcal{D}} = [\![\phi]\!]^{w,\mathcal{D}}$.

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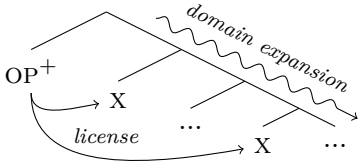
[1] This way of characterizing the purported contribution of X-marking, vFI note, is traced to Stalnaker 1975.

[2] Also known as "Fake Past" (see Iatridou 2000).

I assume that domain expansion is enforced by an operator $\textsc{op}^+$, as defined in (4). Crucial about the proposed account is the system where the effect of domain expansion "trickles down" and licenses each local occurrence of X-marking one by one, as illustrated in (5).[3]

(4) $[\![\textsc{op}^+\phi]\!]^{w,\mathcal{D}} = [\![\phi]\!]^{w,\mathcal{D}_+}$ (where $\mathcal{D}(w) \subset \mathcal{D}_+(w)$)

(5)



I assume that domains are determined by accessibility relations: for any $w, \mathcal{D}$, $\mathcal{D}(w)$ consists of worlds that are accessible from w through an accessibility relation R (i.e., $\mathcal{D}(w) = \{w' \mid wRw'\}$). I focus on those that are serial, transitive and euclidean, using "$\sim$" to refer to any such relations.[4] Domains created by $\sim$ may represent one's doxastic state (Stalnaker 2002; S. Kaufmann 2005) or context sets in non-defective cases (Stalnaker 2014; see also Yalcin 2024); my account generalizes to any of these.[5] An expanded domain $\mathcal{D}_+(w)$ is created from $\sim^+$, which is defined on $\sim$: $\sim^+$ is serial, transitive, euclidean, and such that $\{w' \mid w \sim w'\} \subset \{w' \mid w \sim^+ w'\}$.[6]

3 Conditionals

I assume that conditionals universally quantify over a domain that is restricted by the antecedent (Kratzer 1986, 1991), with the presupposition that the antecedent is compatible with the domain.

(6) a. $[\![\textsc{if}\ \phi, \psi]\!]^{w,\mathcal{D}}$ is defined only if $[\![\phi]\!]^{w',\mathcal{D}} = 1$ for some $w' \in \mathcal{D}(w)$;
b. if defined, $[\![\textsc{if}\ \phi, \psi]\!]^{w,\mathcal{D}} = 1$ iff $\forall w'' \in \{w' \in \mathcal{D}(w) \mid [\![\phi]\!]^{w',\mathcal{D}}\}.\ [\![\psi]\!]^{w'',\mathcal{D}}$

The conditionals in (1) are analyzed as involving the structure "$\textsc{op}^+[\ \textsc{if}\ [\textsc{x}\ \phi], [\textsc{x}\ \psi]]$". Let w and $\mathcal{D}$ be the default world and the default domain function: w is, say, the actual world, and $\mathcal{D}(w)$ the context set or the speaker's doxastic state at w, depending on utterance context.

The derivation goes as in (7). At (a), the compatibility presupposition is triggered for ϕ and $\mathcal{D}_+(w)$. At (b), in the scope of a universal quantifier, the two instances of X-marking are assumed to trigger a *universal presupposition* at their respective loci (i.e., for all $w' \in \mathcal{D}_+(w)$, $\mathcal{D}(w) \subset \mathcal{D}_+(w')$). This universal presupposition is satisfied trivially as $\mathcal{D}_+(w)$ forms an *equivalence class* of worlds (see Appendix A.1 for proofs). Since $\mathcal{D}_+(w)$ forms an equivalence class, it holds that for all $w' \in \mathcal{D}_+(w)$, $\mathcal{D}_+(w) = \mathcal{D}_+(w')$. This means that the universal presupposition is satisfied as long as $\mathcal{D}(w) \subset \mathcal{D}_+(w)$ holds, and it is indeed satisfied since $\mathcal{D}(w) \subset \mathcal{D}_+(w)$ holds trivially.

(7)
$[\![\ \textsc{op}^+\ [\ \textsc{if}\ [\textsc{x}\ \phi], [\textsc{x}\ \psi]]]\!]^{w,\mathcal{D}}$
$= [\![\ \textsc{if}\ [\textsc{x}\ \phi], [\textsc{x}\ \psi]]\!]^{w,\mathcal{D}_+}$ $\cdots (a)\ [\![\phi]\!]^{w',\mathcal{D}_+} = 1$ for some $w' \in \mathcal{D}_+(w)$
$= 1$ iff $\forall w'' \in \{w' \in \mathcal{D}_+(w) \mid [\![\textsc{x}\ \phi]\!]^{w',\mathcal{D}_+}\}.\ [\![\textsc{x}\ \psi]\!]^{w'',\mathcal{D}_+}$ $\cdots (b)\ \forall w' \in \mathcal{D}_+(w).\ \mathcal{D}(w) \subset \mathcal{D}_+(w')$ (Local)
$= 1$ iff $\forall w'' \in \{w' \in \mathcal{D}_+(w) \mid [\![\phi]\!]^{w',\mathcal{D}_+}\}.\ [\![\psi]\!]^{w'',\mathcal{D}_+}$

The presupposition of each occurrence of X-marking is locally satisfied and filtered, and thus does not globally project; what globally projects in (7) is only the compatibility presupposition.

[3] My account thus takes a "reflective" view in the sense introduced by vFI; each occurrence of X-marking does not enforce domain expansion by itself but *reflects* the existence of an expansion operator higher in the structure.

[4] I adopt the symbol "$\sim$" from S. Kaufmann 2005. A relation R is *serial* iff for any x, there is y such that xRy; *transitive* iff for any x, y, z, if xRy and yRz, then xRz; *euclidean* iff for any x, y, z, if xRy and xRz, then yRz. Seriality implies consistency; transitivity and euclideanness imply positive and negative introspection respectively.

[5] Adding reflexivity results in "factive" domains. My account applies to these domains too.

[6] Expansion will be sensitive to, e.g., similarity ordering (Stalnaker 1968; Lewis 1973), which I gloss over here.

4 Desire reports

I assume that WANT universally quantifies over domains of worlds that are optimal with respect to the desirability ranking of an agent **a** (i.e., $\mathcal{D}^{\preccurlyeq_{\mathbf{a}}}(w)$; e.g., von Fintel 1999).[7] I also assume that WANT triggers unsettledness presupposition for its complement (Heim 1992; von Fintel 1999).

(8) a. $[\![\text{WANT}_a\ \phi]\!]^{w,\mathcal{D}}$ is defined only if the truth or falsity of ϕ is not settled in $\mathcal{D}(w)$;
b. if defined, $[\![\text{WANT}_a\ \phi]\!]^{w,\mathcal{D}} = 1$ iff $\forall w' \in \mathcal{D}^{\preccurlyeq_{\mathbf{a}}}(w).\ [\![\phi]\!]^{w',\mathcal{D}}$

The domain that WANT quantifies over is usually identified with the attitude holder's doxastic state (von Fintel 1999; see also Heim 1992).[8] I implement this idea by positing the covert belief operator as defined in (9) above the occurrence of WANT. (10) shows a sample derivation. $\Box_a^{\text{BEL}}$ introduces as a new domain the attitude holder's doxastic state. The outer layer of universal quantification at (*b*) is reduced, as the two quantifiers quantify over the same doxastic state.

(9) $[\![\Box_a^{\text{BEL}}\ \phi]\!]^{w,\mathcal{D}} = 1$ iff $\forall w' \in \mathcal{D}'(w).\ [\![\phi]\!]^{w',\mathcal{D}'}$ (where $\mathcal{D}' = \lambda w.\ \{w' \mid w \sim_{\mathbf{a}} w'\}$)

(10) $[\![\ \Box_a^{\text{BEL}}\ [\ \text{WANT}_a\ \phi\ ]\ ]\!]^{w,\mathcal{D}} = 1$
iff $\forall w' \in \mathcal{D}'(w).\ [\![\text{WANT}_a\ \phi]\!]^{w',\mathcal{D}'}$ ⋯(*a*) $\mathcal{D}' = \lambda w.\ \{w' \mid w \sim_{\mathbf{a}} w'\}$
iff $\forall w' \in \mathcal{D}'(w).\ \forall w'' \in \mathcal{D}'^{\preccurlyeq_{\mathbf{a}}}(w').\ [\![\phi]\!]^{w'',\mathcal{D}'}$ ⋯(*b*) ϕ or $\neg\phi$ is unsettled in $\mathcal{D}'(w)$
iff $\forall w' \in \mathcal{D}'^{\preccurlyeq_{\mathbf{a}}}(w).\ [\![\phi]\!]^{w',\mathcal{D}'}$

With these in place, the Greek sentence in (2) above is analyzed as involving the structure "$\Box_a^{\text{BEL}}$ [OP$^+$ [X [WANT$_a$ [X ϕ]]]]" (I assume for simplicity that English *wish* is semantically the composite of X and WANT; it is undefined unless its domain is expanded). In the scope of $\Box_a^{\text{BEL}}$, the default domain function for X-marking is assumed to be identified with the attitude holder's doxastic state. The default world is assumed again to be the actual world, for which I use w.

The derivation goes as in (11). OP$^+$ applies below $\Box_a^{\text{BEL}}$, thus expanding the attitude holder's doxastic state. For proofs for the reduction from (∗) to (*d*), see Appendix A.2. As above, in the scope of universal quantifiers, the two occurrences of X-marking are assumed to trigger universal presuppositions at their respective positions (see (*b*) and (*d*)). The one triggered at (*d*) is trivially satisfied since $\mathcal{D}'(w) \subset \mathcal{D}'_+(w)$ and for all $w' \in \mathcal{D}'_+(w)$, $\mathcal{D}'_+(w) = \mathcal{D}'_+(w')$. Since $\mathcal{D}'(w) \subset \mathcal{D}'_+(w)$ and the restrictor of a universal quantifier is a downward-monotonic environment, the universal presupposition triggered at (*b*) is also satisfied. The two presuppositions are filtered locally.

(11) $[\![\ \Box_a^{\text{BEL}}\ [\ \text{OP}^+\ [\ \text{X}\ [\ \text{WANT}_a\ [\ \text{X}\ \phi\ ]]]]\ ]\!]^{w,\mathcal{D}} = 1$
iff $\forall w' \in \mathcal{D}'(w).\ [\![\ \text{OP}^+\ [\ \text{X}\ [\ \text{WANT}_a\ [\ \text{X}\ \phi\ ]]]]\ ]\!]^{w',\mathcal{D}'}$ ⋯(*a*) $\mathcal{D}' = \lambda w.\ \{w' \mid w \sim_{\mathbf{a}} w'\}$
iff $\forall w' \in \mathcal{D}'(w).\ [\![\ \text{X}\ [\ \text{WANT}_a\ [\ \text{X}\ \phi\ ]]]\ ]\!]^{w',\mathcal{D}'_+}$ ⋯(*b*) $\forall w' \in \mathcal{D}'(w).\ \mathcal{D}'(w) \subset \mathcal{D}'_+(w')$ (Local)
iff $\forall w' \in \mathcal{D}'(w).\ [\![\ \text{WANT}_a\ [\ \text{X}\ \phi\ ]\ ]\!]^{w',\mathcal{D}'_+}$ ⋯(*c*) ϕ or $\neg\phi$ is unsettled in $\mathcal{D}'_+(w)$
iff $\forall w' \in \mathcal{D}'(w).\ \forall w'' \in \mathcal{D}'^{\preccurlyeq_{\mathbf{a}}}_+(w').\ [\![\text{X}\ \phi]\!]^{w'',\mathcal{D}'_+}$ ⋯(∗)
iff $\forall w' \in \mathcal{D}'^{\preccurlyeq_{\mathbf{a}}}_+(w).\ [\![\text{X}\ \phi]\!]^{w',\mathcal{D}'_+}$ ⋯(*d*) $\forall w' \in \mathcal{D}'_+(w).\ \mathcal{D}'(w) \subset \mathcal{D}'_+(w')$ (Local)
iff $\forall w' \in \mathcal{D}'^{\preccurlyeq_{\mathbf{a}}}_+(w).\ [\![\phi]\!]^{w',\mathcal{D}'_+}$

5 Interaction with O-marking

My account can explain illicit combinations of X-marking and *O-marking*, such as Greek (12). The semantics of O-marking is defined as in (13): it presupposes that the local domain is identical

[7] Let $\preccurlyeq_{\mathbf{a}}$ be a (world-relative) desirability ranking attributed to an agent **a**. For any w and $\mathcal{D}$, $\mathcal{D}^{\preccurlyeq_{\mathbf{a}}}(w)$ is defined as follows: for all $w' \in \mathcal{D}(w)$, $w' \in \mathcal{D}^{\preccurlyeq_{\mathbf{a}}}(w)$ iff for all $w'' \in \mathcal{D}(w)$, if $w'' \preccurlyeq_{\mathbf{a},w} w'$, then $w' \preccurlyeq_{\mathbf{a},w} w''$.

[8] See Rubinstein 2012, 2017 for an alternative possibility. See also Villalta 2008; Anand and Hacquard 2013.

to the default domain (i.e., that no expansion has taken place).

(12) * ***Thelo***$_{\text{O-MARKED}}$ [*na* ***erxotan***$_{\text{X-MARKED}}$ *avrio*] (Greek)
want.NONPAST NA come.PAST.IMPF tomorrow
Intended: 'I wish he would come tomorrow.'

(13) Let w^* be a default world and $\mathcal{D}^*$ be a default domain function. For any $w, \mathcal{D}$, $[\![\text{O } \phi]\!]^{w,\mathcal{D}}$ is defined only if $\mathcal{D}^*(w^*) = \mathcal{D}(w)$; if defined, $[\![\text{O } \phi]\!]^{w,\mathcal{D}} = [\![\phi]\!]^{w,\mathcal{D}}$.

If (12) involves OP$^+$, it clashes with the presupposition of O on WANT (14a); if (12) does not involve OP$^+$, the presupposition of X in the complement cannot be satisfied (14b). Thus, the presupposition of either one *always* fails to be satisfied, hence the inescapable badness of (12).[9]

(14) a. OP$^+$ [O [WANT$_a$ [X ϕ]]] ⇝ fails to satisfy O's presupposition
b. O [WANT$_a$ [X ϕ]] ⇝ fails to satisfy X's presupposition

Assuming that *wish* lexically triggers the same presupposition as X-marking does, my account also predicts the contrast in (15): if the sentence involves OP$^+$, the presupposition of O is not satisfied (16a); if it does not involve OP$^+$, the presupposition of *wish* is not satisfied (16b).

(15) I **wish** it {***is**$_{\text{O-MARKED}}$ / **was**$_{\text{X-MARKED}}$} raining now. (Crowley 2024: 49-50)

(16) a. OP$^+$ [WISH$_a$ [O ϕ]] ⇝ fails to satisfy O's presupposition
b. WISH$_a$ [O ϕ] ⇝ fails to satisfy WISH's presupposition

My account also predicts the badness of the conditional in (17a) and the conditional in (17b): in either case, if the sentence involves OP$^+$, the presupposition of O fails to be satisfied (18a); if the sentence does not involve OP$^+$, the presupposition of X fails to be satisfied (18b).[10]

(17) a. *If John **were**$_{\text{X-MARKED}}$ here now, Mary {**is**$_{\text{O-MARKED}}$ / **will**$_{\text{O-MARKED}}$ be} here too.
b. *If John **is**$_{\text{O-MARKED}}$ here now, Mary **would**$_{\text{X-MARKED}}$ be here too.

(18) a. OP$^+$ [IF [X ϕ], [O ψ]] / OP$^+$ [IF [O ϕ], [X ψ]] ⇝ fails to satisfy O's presupposition
b. IF [X ϕ], [O ψ] / IF [O ϕ], [X ψ] ⇝ fails to satisfy X's presupposition

6 Comparison with Crowley 2024

Building on Leahy 2011, 2018, Crowley 2024 proposed a localist account that treats each instance of X-marking as semantically *vacuous*.[11] (19) is a rendition of his account in the current setup.

[9] As shown in (i), the opposite pattern (i.e., X-marking appears only on WANT) is acceptable in Greek (Iatridou 2000 noted that (i) expresses an indirect, still fulfillable desire). I note that, as shown in (ii), these two patterns of mixed combination are both unacceptable in Mexican Spanish (Aarón Sánchez, p.c.), confirming the prediction of the proposed account. The apparent cross-linguistic variation observed here must be left for future research.

(i) *Tha* ***ithela***$_{\text{X-MARKED}}$ [*na* ***ine*** *psilos*].
FUT want.PAST.IMPF NA be.NPST tall
'I would like him to be tall.' (Iatridou 2000: 33)

(ii) a. ***Quiero*** [*que* {I. ***venga*** / II. ****viniera***$_{\text{X-MARKED}}$} *mañana*].
want.PRES.IND.1SG that come.PRES.SUBJ.3SG come.PAST.IMPF.SUBJ.3SG tomorrow
(I) 'I want him to come tomorrow'. (II) Intended: 'I wish he would come tomorrow.'
b. ***Quisiera***$_{\text{X-MARKED}}$ [*que* {I. ****venga*** / II. ***viniera***$_{\text{X-MARKED}}$} *mañana*].
want.PAST.IMPF.SUBJ.1SG that come.PRES.SUBJ.3SG come.PAST.IMPF.SUBJ.3SG tomorrow
'I wish he would come tomorrow.'

[10] (17b) improves if focus is placed on *would* (Si Kai Lee, p.c.). I leave this for future investigation.
[11] See also Wimmer 2024.

(19) For any world w and domain function $\mathcal{D}$, $[\![\text{X}_{\text{CROWLEY}}\ \phi]\!]^{w,\mathcal{D}} = [\![\phi]\!]^{w,\mathcal{D}}$.

Crowley assumes that only O-marking is semantically non-vacuous. Recast in the present system, his analysis of O-marking triggers the presupposition that w in $[\![\cdot]\!]^{w,\mathcal{D}}$ is included in the context set of the factive common ground (conditionals) or the doxastic state of the attitude holder (desire reports); in the scope of universal quantifiers, it presupposes that the quantification domain is contained in the context set or the doxastic state. In his account, domain expansion associated with X-marking results from pragmatic competition with O-marking.

As he himself noted (fn.26), Crowley set aside cases like Greek (2) where X-marking appears both on WANT and in the complement, for which my proposal provides a straightforward account. It is in fact not clear how his account rules out the illicit combination of O/X in (12). (20) is the counterpart of (14b) that replaces my entry of X-marking with $\text{X}_{\text{CROWLEY}}$. Since it is semantically vacuous, $\text{X}_{\text{CROWLEY}}$ does not clash with O on WANT (even assuming his analysis of O-marking). His account thus does not seem to predict the badness of (12).[12]

(20) O [WANT_a [$\text{X}_{\text{CROWLEY}}$ ϕ]] ⇝ no semantic clash

In the same vein, Crowley's account does not predict the badness of the conditionals in (17): the $\text{X}_{\text{CROWLEY}}$-counterparts of (18b), as shown in (21), do not lead to semantic clash.[13]

(21) IF [$\text{X}_{\text{CROWLEY}}$ ϕ], [O ψ] / IF [O ϕ], [$\text{X}_{\text{CROWLEY}}$ ψ] ⇝ no semantic clash

7 Future outlook

I believe that the proposed account of X-marking explains similar data from many other languages beyond English and Greek, although I recognize that it alone does not capture the entirety of X-marking phenomena in natural language. Even in English, the empirical landscape is more complicated than what this paper has shown, as evidenced, e.g., by the contrast in (22) (Ippolito 2003: 41, 48). As Ippolito 2003 observed, when the existence presupposition of event participants is not satisfied at the utterance time, the conditional has to involve an *additional layer* of Past.[14]

(22) a. #Charlie is dead. If he **came** to the party tomorrow, he **would** meet Sally.
b. Charlie is dead. If he **had come** to the party tomorrow, he **would have** met Sally.

How does this fare with the present account? Recent literature has suggested that this additional layer of Past, unlike those appearing in (22a), contributes a real temporal meaning (Schulz 2017; S. Kaufmann 2023; see M. Kaufmann and Todorović 2023 for Serbian; see also Mizuno 2023 for cross-linguistic research on desire reports regarding this point). This layer of Past is assumed to make the same contribution as analyzed by the so-called "Past-as-Past" approach to Fake Past; it scopes over the conditional and expands the domain by enforcing temporal backshift.[15]

[12] Crowley proposed a way to derive the contrast in (15), but the derivation crucially hinges on his assumption that English *wish* lexically presupposes the falsity of its complement; it is less plausible that the same assumption holds for WANT+X in Greek (if so, one would have to assume that X-marking presupposes counterfactuality one way or another, which is not plausible given a range of non-counterfactual uses of X-marking; see, e.g., vFI). Note that my explanation of (15) does not hinge on whether *wish* presupposes the falsity of its complement.

[13] Crowley's account actually predicts that the two forms in (21) end up semantically equivalent to "IF [O ϕ], [O ψ]" (i.e., O-marked conditionals such as "*if John is here now, Mary is here too*"), because he assumes that O-marking, whether in the antecedent or in the consequent, triggers the same global presupposition (i.e., that the quantification domain is contained in the context set). More specifically, the two occurrences of O-marking in IF [O ϕ], [O ψ] are assumed to trigger an identical global presupposition at each locus in his account, but the net global presupposition is the same as those triggered in IF [$\text{X}_{\text{CROWLEY}}$ ϕ], [O ψ] and IF [O ϕ], [$\text{X}_{\text{CROWLEY}}$ ψ], because one occurrence of O-marking suffices to contribute the relevant presupposition. This I deem to be another undesirable prediction of his account.

[14] See also Ippolito 2013. See also S. Kaufmann 2023 for some discussion of this observation by Ippolito.

[15] I refer the reader to vFI for the debate between the Past-as-Past view and the "Past-as-Modal" view.

Leaving further investigation for future work, I speculate that conditionals like (22b) involve the additional layer of Past/Perfect in the position of OP^+ and thus license the occurrences of X.[16]

(23) PAST/PERFECT [IF [X ϕ], [X ψ]]

It also remains to be seen how my account extends to X-marking appearing in weak necessity modals (von Fintel and Iatridou 2008, 2023).[17] I leave all remaining issues for future research.

A Proofs

A.1 Domains created by $\sim$ and $\sim^+$ form equivalence classes

I first show as a lemma that $\sim$ and $\sim^+$ both have the properties in (24).

(24) An accessibility relation R is
   a. *shift-reflexive* iff for all w, w', if wRw', then $w'Rw'$;
   b. *shift-symmetric* iff for all w, w', w'', if wRw' and wRw'', then $w'Rw''$ and $w''Rw'$.

Lemma A.1. *$\sim$ and $\sim^+$ are both shift-reflexive and shift-symmetric.*

Proof. Both properties follow immediately from euclideanness of $\sim$ and $\sim^+$. □

Proposition A.2. *For any w, $\{w' \mid w \sim w'\}$ and $\{w' \mid w \sim^+ w'\}$ form equivalence classes.*

Proof. Transitivity follows from the definitions of $\sim$ and $\sim^+$. Reflexivity and symmetry within these sets follow from shift-reflexivity and shift-symmetry of $\sim$ and $\sim^+$. □

A.2 Reduction from $(*)$ to (d) in (11)

$(*)$ and (d) in (11) can be restated respectively as in (25a) and (25b) (ignoring the restriction of the domain through $\preccurlyeq_{\mathbf{a}}$, which I assume does not affect the reduction here). I first show as a lemma that the two underlined parts are equivalent. Equivalence between (25a) and (25b), and consequently the reduction from $(*)$ to (d), follow immediately from the lemma.

(25) a. For any w', w'' such that $\underline{w \sim w' \text{ and } w' \sim^+ w''}$, $[\![\text{X } \phi]\!]^{w'',\mathcal{D}_+} = 1$ $\cdots (*)$ in (11)
   b. For any w'' such that $\underline{w \sim^+ w''}$, $[\![\text{X } \phi]\!]^{w'',\mathcal{D}_+} = 1$ $\cdots (d)$ in (11)

Lemma A.3. *For any w, w', w'', the following holds: $w \sim w'$ and $w' \sim^+ w''$ iff $w \sim^+ w''$.*

Proof. ($\Rightarrow$) Suppose $w \sim w'$. Since $\{w' \mid w \sim w'\} \subset \{w' \mid w \sim^+ w'\}$, it follows that $w \sim^+ w'$. By this, $w' \sim^+ w''$ and transitivity of $\sim^+$, it follows that $w \sim^+ w''$.
($\Leftarrow$) Suppose $w \sim^+ w''$. By seriality of $\sim$, there exists w' such that $w \sim w'$. Take any w' such that $w \sim w'$. Since $\{w' \mid w \sim w'\} \subset \{w' \mid w \sim^+ w'\}$, it follows that $w \sim^+ w'$. By this, $w \sim^+ w''$ and euclideanness of $\sim^+$, it follows that $w' \sim^+ w''$. □

Proposition A.4. *(25a) and (25b) are equivalent.*

Proof. This follows immediately from Lemma A.3. □

[16] See Mizuno 2023 for a formal implementation of this idea for counterfactual desire reports across languages.

[17] Another remaining issue is occurrences of X-marking in deeply embedded positions, e.g., "*were*" in (ia). While (ia) requires *were* instead of *is* in this position (Si Kai Lee, Robert Lowe, p.c.), the opposite holds for (ib).

(i) a. [If I $knew_{\text{X}}$ [this $\boldsymbol{were}_{\text{X}}$ chocolate]], I $would_{\text{X}}$ eat it. (Iatridou 2000: 63a)
   b. If New York $were_{\text{X-MARKED}}$ the capital of the United States, John $would_{\text{X-MARKED}}$ know that it {✓$\boldsymbol{is}_{\text{O-MARKED}}$ / ??$\boldsymbol{was}_{\text{X-MARKED}}$ / #$\boldsymbol{were}_{\text{X-MARKED}}$} (but Bill wouldn't).

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