

# Dynamic Effects of Modalized Questions

Valentin D. Richard

ILLC, University of Amsterdam

Amsterdam, The Netherlands

Université de Lorraine, CNRS, Inria, LORIA, F-54000

Nancy, France

`valentin.richard@loria.fr`

## Abstract

A dynamic analysis of modalized questions faces several difficulties. When an existential modality scopes over the *wh*-phrase, two kinds of phenomena are observed. First, the external staticity of the modal operator prevents the *wh*-phrase from being referred back to in indicative follow-up sentences. Anaphoric reference to the *wh*-phrase should yet be allowed through modal subordination. Second, exhaustivity and global presuppositions are weakened in modalized *which* questions, leading to mention-some readings and local presuppositions. We design a model capturing these two desiderata, based on dynamic inquisitive semantics and stack-based semantics for local contexts. Our model provides a uniform analysis of modalized and conditional questions using the Kratzerian theory of modality. It also correctly predicts discrepancies in presupposition projection between modalized and conditional questions.

## 1 Modalized Questions

### 1.1 Possibility Weakens Questions

Singular *which* questions have a uniqueness presupposition (Higginbotham and May 1981; Dayal 1996). For example, (1-b) carries the presupposition that a single letter is hidden in the actual world. Either the true answer is *R* (for FORM), or it is *A* (for FOAM).

- (1) a. SITUATION: *Alice and Bob are playing a game. Alice wrote a mystery English word of 4 or 5 letters on the board, but her hand hides a part of it. The skeleton is FO\_\_M.*  
b. ALICE: Which letter is hidden here?  
c. ALICE: Which letter could be hidden here?

Hirsch and Schwarz 2019 observed that this presupposition is weakened with an existential modality operator. Modalized question (1-c) does not presuppose that there exists a single letter that is actually hidden. But it still presupposes something, which could be paraphrased as: *For each possible answer, a single letter is hidden*. Roelofsen and Dotlačil 2023 call this a local presupposition.

Moreover, as opposed to (1-b), where only one true answer exists in every world, several true answers are possible for (1-c) in the actual world. We could answer (1-c) by saying *A*, *R*, or a distributive conjunction or disjunction *A and/or R*. Ultimately, question (1-c) requires some information about the content of the English lexicon, not about the actual mystery word. In particular, the hidden letters might actually be *RU* (for FORUM). But (1-c) presupposes that the English lexicon does not *only* have *RU* as possible completion.

Plural *which* questions are strongly exhaustive (Groenendijk and Stokhof 1984). For example, in (2-b), Mary requires the whole set of eczema patients.

- (2) a. SITUATION: *Mary would like to experiment with two or three patients with eczema. She asks Ann, who knows the medical files of all patients with skin conditions in the hospital, a question.*

- b. Which patients have eczema?
- c. Which patients could I invite for my experiment?

However, in modalized question (2-c), Mary is open to a non-exhaustive set of eczema patients. Modalized plural *which* questions allow mention-some readings. Xiang 2016 observed this phenomenon, and Xiang and Cremers 2017 confirmed it through quantitative data.

We follow the analysis of Hirsch and Schwarz 2019 claiming that these phenomena are explained by the modality operator scoping over the *which* phrase.

## 1.2 Modal Subordination with Modalized Questions

In declarative sentences, a modal scoping over an indefinite blocks its anaphoric potential (Karttunen 1969). However, modally subordinated sentences have access to the embedded antecedent (Roberts 1989). We observe the same with modalized questions.

Under the reading of (3-b) where Hansel is not asking *What is the specific animal which might eat us?*, but where he focuses on imagining a scenario, response (3-c) feels off. The discourse referent  $u$  raised by *which* <sup>$u$</sup>  under the scope of *might* cannot be accessed in this indicative sentence. However, in sentence (3-d),  $u$  can be referred to by  $it_u$  because of modal subordination.

- (3) a. SITUATION: *Hansel and Gretel are walking in the woods. They saw scary shadows and heard noises. They believe that child-eating animals are hiding in these woods.*
- b. HANSEL: Which <sup>$u$</sup>  animal might eat us? (*might* > *which* reading)
- c. GRETTEL: ?I don't know, but  $it_u$  is very large.
- d. GRETTEL: I don't know, but  $it_u$  would eat you first.

## 2 Previous works

Our goal is to define an existential modal operator in dynamic semantics that captures all these properties. It has (1.) to be externally static while (2.) still allowing modal subordination. It has (3.) to obviate the global uniqueness presupposition while still providing a local one. Finally, it has (4.) to be able to turn a mention-all question into a mention-some one.

Previous dynamic semantic models do not account for all four desired properties. The dynamic model of questions and modalities of Groenendijk 1998; Groenendijk, Stokhof, and Veltman 1996 satisfies (1.) without allowing (2.). van Rooij 1998 manages to encode modal subordination (2.), but the DPPL substrate (from van den Berg 1996) does not structurally provide external staticity (1.). Similarly, Brasoveanu's (2010) account of modal subordination fails to model in which worlds individual discourse referents are *not* accessible.

In appendix A, I briefly discuss another issue of previous definitions of modals in dynamic semantics, which is solved by using Kratzer 1991 theory.

### 2.1 Stack-based Treatment of Local Contexts

Implementing externally static modal operators prompts us to be able to retrieve modal local contexts somehow (Stalnaker 1981). Many authors resolve local context as anaphora, using world referents (Brasoveanu 2010; Hofmann 2019) or context referents (Frank 1997; Geurts 1999). Kibble 1998 and Asher and McCready 2007 claim that, unlike individual anaphora, modal local contexts are more restricted and have no real resolution ambiguity. Here, we follow their arguments and opt for a different system.

We adopt a stack-based semantics (Kaufmann 2000). Sentences are denoted by functions updating macro-contexts  $\tau$ . A stack  $\tau$  contains local contexts. The bottom element  $c_0$  corresponds to the common ground. Modal operators push on top of  $\tau$  a new local context, which can contain

more discourse referents than  $c_0$ . A follow-up modal sentence is evaluated in that topmost local context. Indicative sentences presuppose that the topmost element is  $c_0$ . We assume here that popping elements out of  $\tau$  is performed by pragmatics (e.g. via discourse relations (Asher and Lascarides 2003)).

Isaacs and Rawlins 2008 use Kaufmann’s (2000) system to model modal subordination of conditional questions within partition semantics (Groenendijk and Stokhof 1984). Hara and Sano 2017 revise their model to provide better predictions for conditional questions, using inquisitive semantics (Ciardelli, Roelofsen, and Theiler 2017; Ciardelli, Groenendijk, and Roelofsen 2018). We extend their approach by (i.) adding referents in the model, by upgrading to dynamic inquisitive semantics  $\text{Inq}_B^D$  (Dotlačil and Roelofsen 2019; Dotlačil and Roelofsen 2021); (ii.) defining the denotation of modals in  $\text{Inq}_B^D$  based on Kratzerian theory; and (iii.) specifying their presuppositions and actions on stacks.

The choice of  $\text{Inq}_B^D$  is motivated by another factor. Socolof, Hirsch, and Schwarz 2020 showed that exhaustivity and presupposition weakening also happen with disjunction.  $\text{Inq}_B^D$  was proven to be able to model desiderata (3.) and (4.) for disjunction (Roelofsen and Dotlačil 2023).

### 3 Modal Dynamic Inquisitive Semantics

#### 3.1 Inquisitive Possibility

In Kratzerian theory, every world is equipped with a modal base (a set of classical propositions) and an ordering source. Due to the lack of space, we only focus on the modal base  $f$  here. For each world  $w \in W$ , call  $\mu(w) = \bigcap f(w)$  the modal set at  $w$ . The intensional denotation of  $\Diamond$  is (1), with  $R_\mu$  being the accessibility relation associated with  $\mu$ .

$$\begin{aligned} \llbracket \Diamond \varphi \rrbracket_{\text{Int}} &= \{w \in W \mid \mu(w) \cap \llbracket \varphi \rrbracket_{\text{Int}} \neq \emptyset\} = R_\mu^{-1}(\llbracket \varphi \rrbracket) \\ \text{where } R_\mu^{-1}(s) &:= \{w \in W \mid \mu(w) \cap s \neq \emptyset\} \end{aligned} \quad (1)$$

The inquisitive existential modality (2) proposed by Ciardelli 2016, § 7.1 is sensitive to the inquisitiveness of its complement but produces non-inquisitive propositions. Yet, a modalized question is questioning. Therefore, we propose another definition (3). By inverting quantifiers  $\forall$  and  $\exists$ , we let inquisitiveness project from  $\varphi$  to  $\Diamond \varphi$ . In this different modal inquisitive semantics ( $\text{MInq}_B$ ), a set of worlds  $s$  resolves  $\Diamond \varphi$  iff  $s$  resolves some  $\Diamond \psi$ , where  $\psi$  is a proposition resolving  $\varphi$ .

$$\llbracket \Diamond \varphi \rrbracket_{\text{InqBK}} = \{s \subseteq W \mid \forall w \in s. \exists s' \in \llbracket \varphi \rrbracket_{\text{InqBK}}. \mu(w) \cap s' \neq \emptyset\} \quad (2)$$

$$\llbracket \Diamond \varphi \rrbracket_{\text{MInqB}} := \{s \subseteq W \mid \exists s' \in \llbracket \varphi \rrbracket_{\text{MInqB}}. \forall w \in s. \mu(w) \cap s' \neq \emptyset\} \quad (3)$$

$$= \{s \subseteq W \mid \exists s' \in \llbracket \varphi \rrbracket_{\text{MInqB}}. s \subseteq R_\mu^{-1}(s')\} \quad (4)$$

To illustrate this definition consider example (4).

- (4) a. SITUATION: *Mary supervised an exam for students a and b. Mary and John know that exactly one student cheated. John knows that Mary saw one or both of them suspiciously looking at their pencil case during the exam. This counts as a cheat suspicion. He asks her about that.*
- b. JOHN: Which student may have cheated?

Let’s model this situation with four worlds. The subscript  $x$  of world  $w_x^z$  is the real cheater:  $a$  or  $b$ . The superscript  $z$  is the set of students Mary saw suspiciously looking at their pencil case:  $a$ ,  $b$  or  $ab$  (i.e.  $a$  and  $b$ ). World  $w_x^z$  is accessible from  $w_{x'}^{z'}$  iff.  $z = z'$ . The semantics of (4-b) in this model is the issue on the right in Fig. 1.

Figure 1: Illustration of the semantics of (4-b) in MInqB. Arrows represent the accessibility relation  $R_\mu$ .

The alternatives of the prejacent  $\varphi$  are projected to  $\Diamond\varphi$  by mediation of  $R_\mu^{-1}$ . If the accessibility relation  $R_\mu$  is an equivalence relation,  $R_\mu^{-1}(s)$  contains  $s$ . Hence, output resolving proposition  $s$  is wider than prejacent resolving proposition  $s'$ . This widening is what weakens the question.<sup>1</sup>

### 3.2 Dynamic Inquisitive Possibility

In GSV, the dynamic semantics of Groenendijk, Stokhof, and Veltman 1996, information states  $s$  are sets of pairs  $\langle w, g \rangle$  of a world  $w$  and an assignment function  $g$ . A Kratzerian externally static denotation of  $\Diamond$  in GSV would be (5).<sup>2</sup> It restricts the input information states  $s$  to the possibilities whose world content is in relation  $R_\mu$  with the world content of some possibility in  $\llbracket \varphi \rrbracket_{\text{MGSV}}$ .

$$\llbracket \Diamond\varphi \rrbracket_{\text{MGSV}} := s \mapsto \{ \langle w, g \rangle \in s \mid \exists \langle w', g' \rangle \in \llbracket \varphi \rrbracket_{\text{MGSV}}. w' \in \mu(w) \} \quad (5)$$

$$= s \mapsto \{ \langle w, g \rangle \mid w \in R_\mu^{-1}(\text{WC}(\llbracket \varphi \rrbracket_{\text{MGSV}})) \} \quad (6)$$

where the world content of an information state is defined as  $\text{WC}(s) = \{w \mid \exists \langle w, g \rangle \in s\}$ .

Putting all together, we obtain a denotation of  $\Diamond$  in Modal Dynamic Inquisitive Semantics  $\text{MInqB}^D$  as (7), where a context  $c$  is a downward-closed nonempty set of information states.<sup>3</sup>

$$\Diamond\mathcal{U} := c \mapsto \{ s \in c \mid \exists s' \in \mathcal{U}(c). \forall \langle w, g \rangle \in s. \exists \langle w', g' \rangle \in s'. w' \in \mu(w) \} \quad (7)$$

$$= c \mapsto \{ s \in c \mid \exists s' \in \mathcal{U}(c). \text{WC}(s) \subseteq R_\mu^{-1}(\text{WC}(s')) \} \quad (8)$$

Formula (8) has the same form as the definition of  $\Diamond$  in MInqB (4). The only difference is the presence of the world content projection  $\text{WC}$ , so that the inclusion is an inclusion between sets of worlds.

### 3.3 Pushing and Percolating

The syntactic language of our model  $\text{MInqB}^D$  has two formula sorts. The first formula sort is type  $k \rightarrow k$ , where  $k$  is the type of contexts. Formulas of dynamic inquisitive semantics  $\text{InqB}^D$  have this type. By taking  $\ell$  the type of stack of contexts  $\tau = \langle c_0, \dots, c_n \rangle$ , the second sort of formula is of

<sup>1</sup>Information state widening was also advocated by Giannakidou and Mari 2019 for questions with an overt possibility modal.

<sup>2</sup>In formula (5), quantified assignment  $g'$  is independent from  $g$ . This allows us to solve the modal identity problem (Beaver 2001, § 8.3.1). For example, in the following discourse, interpreting both  $it_u$  in (i-b) and (i-c) with the same assignment  $g$  leads to a contradiction because  $g(u)$  cannot equal  $A$  and  $R$ . The modality operator must thus make it possible to evaluate  $it_u$  in (i-b) with another (accessible) assignment. This is unnecessary for universal modality (13).

- (i) a.  $A^u$  letter is hidden.
- b.  $It_u$  might be  $R$ .
- c. But  $it_u$  is  $A$ .

<sup>3</sup>Here, for simplicity, we do not take assignment matrices as in Dotlačil and Roelofsen 2021, but simple assignments. Note that it would work the same.

type  $\ell \rightarrow \ell$ , i.e. macro-context update functions. Like Kaufmann 2000, we take modals to be operators of type  $(k \rightarrow k) \rightarrow (\ell \rightarrow \ell)$ . After pushing a new local context, modals percolate the information (but not referents) to lower contexts, viz. (9).

$$\begin{aligned} \llbracket \text{if} \rrbracket_{\text{MInq}_B^D} &:= \mathcal{U} \mapsto \text{PUSH } \mathcal{U} \\ \llbracket \text{then} \rrbracket_{\text{MInq}_B^D} = \llbracket \text{would} \rrbracket_{\text{MInq}_B^D} &:= \mathcal{U} \mapsto \text{PUSH } \mathcal{U}; \text{PERC } \mathcal{U} \\ \llbracket \text{might} \rrbracket_{\text{MInq}_B^D} = \llbracket \text{could} \rrbracket_{\text{MInq}_B^D} &:= \mathcal{U} \mapsto \text{PUSH } \mathcal{U}; \text{PERC } \Diamond \mathcal{U} \end{aligned} \quad (9)$$

where  $\ell \rightarrow \ell$  conjunction is defined as  $\mathcal{T}; \mathcal{S} := \tau \mapsto \mathcal{S}(\mathcal{T}(\tau))$ .

The analyses of Roberts 1989 and Gillies 2004 for conditionals and modal subordination are given in (10).

$$\begin{aligned} \text{if } \varphi, \text{ then } \psi &\rightsquigarrow \Box(\varphi \rightarrow \psi) \\ \text{if } \varphi, \text{ then might } \psi &\rightsquigarrow \Box(\varphi \rightarrow \Diamond \psi) \\ \text{might } \varphi, \text{ would } \psi &\rightsquigarrow \Diamond \varphi \wedge \Box(\varphi \rightarrow \psi) \end{aligned} \quad (10)$$

To obtain these predictions, we define the percolation of a context update function  $\mathcal{U}$  as follows.  $\text{PERC } \mathcal{U}$  applies  $\mathcal{U}$  on the penultimate context  $c_{n-1}$  and modal conditionalization wrt.  $c_{n-1}$  and  $\mathcal{U}$  on lower ones (11). Intuitively,  $c[c' \vdash \mathcal{U}]$  is context  $c$  after learning that *if*  $c'$  *then*  $\mathcal{U}$ .

$$\begin{aligned} \text{PUSH } \mathcal{U} &:= \lambda \langle c_0, \dots, c_n \rangle. \langle c_0, \dots, c_n, \mathcal{U}(c_n) \rangle \\ \text{PERC } \mathcal{U} &:= \lambda \langle c_0, \dots, c_{n-2}, c_{n-1}, c_n \rangle. \langle c_0[c_{n-1} \vdash \mathcal{U}], \dots, c_{n-2}[c_{n-1} \vdash \mathcal{U}], \mathcal{U}(c_{n-1}), c_n \rangle \end{aligned} \quad (11)$$

with  $c[c' \vdash \mathcal{U}]$  defined as  $(\Box(c' \rightarrow \mathcal{U}))(c)$ . Universal modality and implication (13) are consistent with  $\text{Inq}_B^D$  and (7).<sup>4</sup>

$$\begin{aligned} c' \rightarrow \mathcal{U} &:= c \mapsto \{s \in c \mid \forall t \subseteq s. \forall t' \in c'. t \subseteq t' \rightarrow t' \in \mathcal{U}(c')\} \\ \Box \mathcal{U} &:= c \mapsto \{s \in c \mid \exists s' \in \mathcal{U}(c). \forall \langle w, g \rangle \in s. \forall w'. w' \in \mu(w) \rightarrow \langle w', g \rangle \in s'\} \end{aligned} \quad (13)$$

### 3.4 Meeting the Requirements

As an illustration, let's interpret sentence (5-a) in our model. Like Roelofsen and Dotlačil 2023, I assume functional heads. Focus head  $\text{Foc}_u$  provides the witness request operator  $?u$ . Interrogative clause type head  $\text{Int}$  ensures non-informativeness, by providing the presuppositional closure operator  $\dagger$  (Roelofsen 2015). To weaken the uniqueness presupposition, we require the modality to raise above  $\text{Foc}_u$  but below  $\text{Int}$ , hence logical form (5-b).

- (5) a. Which<sup>u</sup> letter could be hidden in  $\text{FO\_M}$ ?  
 b.  $\text{Int}(\text{could}(\text{Foc}_u(\text{which}^u \text{ letter hidden})))$   
 c.  $\mathcal{T} = \dagger(\text{PUSH } \mathcal{V}; \text{PERC } \Diamond \mathcal{V})$ , with  $\mathcal{V} = [u]; \mathbf{atom}\{u\}; \mathbf{letter}\{u\}; \mathbf{hidden}\{u\}; \mathbf{max}\{u\}; ?u$

The denotation of formula (5-c) is illustrated in Fig. 2.

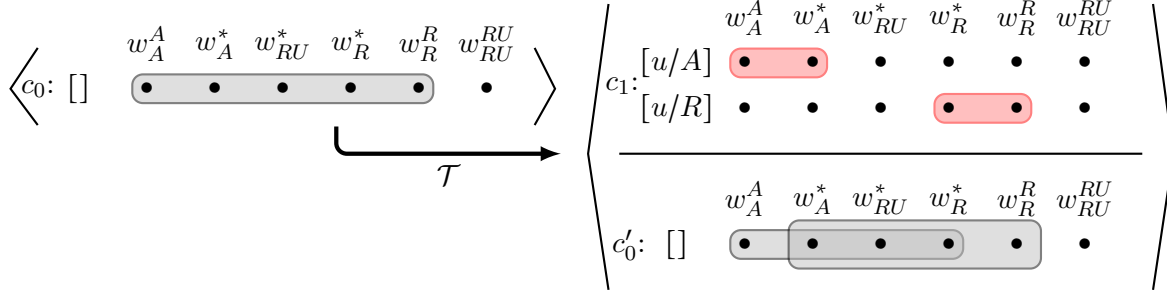
Referent  $u$  introduced by  $\mathcal{U}$  is not projected to the common ground  $c_0$ , ensuring external staticity (1.). A subsequent modal operator has access to the topmost local context  $c_1$ , containing assignments defined on  $u$ , thus ensuring modal subordination (2.).

In  $\text{Inq}_B^D$ , mention-all reading is triggered by the interaction between the exhaustivity operator **max**, provided by *which*, and the witness request operator  $?u$ , creating the alternatives. Context  $c_1 = \mathcal{V}(c_0)$  contains the alternatives “*d is hidden*” for every single letter  $d$ . Crucially,  $\Diamond$  projects this inquisitiveness onto  $c'_0$ . In  $c'_0$ , the alternatives “*A can be hidden*” and “*R can be hidden*” overlap on  $\{w_A^*, w_{RU}^*, w_R^*\}$ , creating a mention-some reading (4.).

<sup>4</sup>Extension  $\leq$  and subsistence are  $\in, \sqsubseteq$  are defined as follows. Notation  $\in, \sqsubseteq$  is taken from Dekker 1992.

$$\begin{aligned} \langle w, g \rangle &\leq \langle w', g' \rangle && \text{if } w = w' \wedge g \subseteq g' && i \in s' && \text{if } \exists j \in s'. i \leq j \\ s &\leq s' && \text{if } \forall j \in s', \exists i \in s. i \leq j && s \sqsubseteq s' && \text{if } s \leq s' \wedge (\forall i \in s. i \in s') \\ &&& && s \in c' && \text{if } \exists s' \in c'. s \sqsubseteq s' \end{aligned} \quad (12)$$

World $w$	$w_A^A$	$w_A^*$	$w_{RU}^*$	$w_R^*$	$w_R^R$	$w_{RU}^{RU}$
Words with skeleton FO__M in English	FOAM	FOAM, FORM, FORUM	FOAM, FORM, FORUM	FOAM, FORM, FORUM	FORM	FORUM
Actual word on the board	FOAM	FOAM	FORUM	FORM	FORM	FORUM
Modal set $\mu(w)$	$\{w_A^A\}$	$\{w_A^*, w_R^*, w_{RU}^*\}$	$\{w_A^*, w_R^*, w_{RU}^*\}$	$\{w_A^*, w_R^*, w_{RU}^*\}$	$\{w_R^R\}$	$\{w_{RU}^{RU}\}$

Table 1: Illustration model  $\mathcal{M}$  made of the set of worlds  $W = \{w_A^A, w_A^*, w_{RU}^*, w_R^*, w_R^R, w_{RU}^{RU}\}$ .Figure 2: Diagram of the actions of the semantics of question (5-a) in model  $\mathcal{M}$  on initial stack  $\tau_0 = \langle c_0 \rangle$ . We only represent alternatives of contexts, as sets of dots. A dot represents a possibility, i.e. a pair of a possible word (in abscissa) and an individual assignment function (in ordinate).

World  $w_{RU}^*$  is included in the alternatives of  $c'_0$ . Thus, global presupposition is obviated. However, if we take  $c''_0 = \wp(\{\langle w, [] \rangle \mid w \in W\})$ <sup>5</sup>,  $\mathcal{T}$  is undefined on stack  $\langle c''_0 \rangle$ , because of world  $w_{RU}^{RU}$ , where no single letter can fit in skeleton FO\_\_M. The  $\dagger$  operator (14), by applying on every element  $\tau_i$  of stack  $\tau$ , ensures local uniqueness presupposition (3.).

$$\dagger \mathcal{S} := \tau \mapsto \begin{cases} \mathcal{S}(\tau) & \text{if } \forall i < |\tau|. \cup \tau_i \subseteq \cup \mathcal{S}(\tau)_i \\ \text{undefined} & \text{otherwise} \end{cases} \quad (14)$$

Finally, our model allows us to compare (5-a) with (6). The conditional antecedent introduces a context where  $w_{RU}^{RU}$  is excluded. Thus,  $\dagger$ , provided in the consequent, acts vacuously. It correctly predicts that both (5-a) and (6) have the same effect, except for one point: (6) has no presupposition.

(6) If a<sup>v</sup> single letter is hidden, which<sup>u</sup> letter is it<sub>v</sub>?

## 4 Conclusion

We designed a modal dynamic inquisitive semantics which accounts for exhaustivity and presupposition weakening of modalized questions. It also captures modal subordination with an externally static existential modal. To do so, we used stack-based semantics, allowing us to treat conditional and modalized questions uniformly.

**Acknowledgements.** I am grateful to Floris Roelofsen, Reinhard Muskens, and Philippe de Groote for precious feedback about this work.

<sup>5</sup> $[]$  is the empty assignment, and  $\wp$  the powerset operator.



## A Non-Distributive Test Semantics for Modals

A side issue of previous dynamic semantics of modals concerns their non-distributive test interpretation. The definition advocated by Veltman 1996, and used by Dekker 1992; Groenendijk, Stokhof, and Veltman 1996; van Rooij 1998; Kaufmann 2000; Beaver 2001; Asher and McCready 2007, considers that epistemic possibility  $\Diamond\varphi$  is a test: if  $\varphi$  is true at some world  $w \in s$ , then  $\llbracket \Diamond\varphi \rrbracket(s) = s$ , otherwise  $\llbracket \Diamond\varphi \rrbracket(s) = \emptyset$ . This has several drawbacks.

First, it fails to model other modal flavours, like circumstantial or deontic modalities. Second, it fails to model knowledge variations because it assumes that all worlds are accessible from any world. Finally, it implies that a modalized declarative either brings no at-issue information or leads to a contradiction. As a consequence, Veltman’s (1996) existential modality cannot account for weakenings (3.) and (4.). Worse, extending this definition to inquisitive semantics would prevent an interrogative formula  $\psi$  from raising alternatives because  $\llbracket \psi \rrbracket$  would either output the input context or the contradiction state.

We argue against modelling modals as tests, for both declaratives and interrogatives. Modalities can safely remove possible worlds from the global context. For example, sentence  $S$  in (7) does provide some new (non-contradictory) at-issue information about the content of the English lexicon. Suppose we are in an initial information state  $s$  which contains the actual world plus a world  $w_R^R$  (see Tab. 1) where, in English, the only word of the form FO\_\_M is FORM. Then, updating  $s$  with  $\llbracket S \rrbracket$  should remove  $w_R^R$  from  $s$ . With a test, on the contrary, updating  $s$  with  $\llbracket S \rrbracket$  would not remove  $w_R^R$ . This is unwanted.

(7) A vowel can be inserted in FO\_\_M to make an English word.

We can solve these problems by adopting a Kripkean accessibility semantics for modals (Kripke 1959), and more generally, Kratzerian theory of modals (Kratzer 1977; Kratzer 1991). Our account is both eliminative and distributive.<sup>6</sup>

## References

- Asher, Nicholas and Alex Lascarides (2003). *Logics of Conversation*. Cambridge University Press.
- Asher, Nicholas and Elin McCready (May 1, 2007). “Were, Would, Might and a Compositional Account of Counterfactuals”. In: *Journal of Semantics* 24.2, pp. 93–129. ISSN: 0167-5133. DOI: 10.1093/jos/ffl013. URL: <https://doi.org/10.1093/jos/ffl013> (visited on 04/23/2024).
- Beaver, David I. (June 2001). *Presupposition and Assertion in Dynamic Semantics: A Critical Review of Linguistic Theories of Presupposition and a Dynamic Alternative*. Studies in Logic, Language, and Information. Center for the Study of Language and Information. 250 pp. ISBN: 978-1-57586-120-3. URL: <https://press.uchicago.edu/ucp/books/book/distributed/P/bo3639443.html> (visited on 11/04/2024).
- Brasoveanu, Adrian (Nov. 1, 2010). “Decomposing Modal Quantification”. In: *Journal of Semantics* 27.4, pp. 437–527. ISSN: 0167-5133. DOI: 10.1093/jos/ffq008. URL: <https://doi.org/10.1093/jos/ffq008> (visited on 07/11/2023).
- Ciardelli, Ivano (2016). “Questions in Logic”. PhD thesis. Amsterdam: University of Amsterdam. 398 pp.
- Ciardelli, Ivano, Jeroen Groenendijk, and Floris Roelofsen (Nov. 27, 2018). *Inquisitive Semantics*. Oxford Surveys in Semantics and Pragmatics. Oxford, New York: Oxford University Press. 240 pp. ISBN: 978-0-19-881478-8.

<sup>6</sup>The epistemic behaviour which motivated Veltman 1996 can be retrieved by requiring the accessibility relation to be an equivalence relation.

- Ciardelli, Ivano, Floris Roelofsen, and Nadine Theiler (Feb. 1, 2017). “Composing Alternatives”. In: *Linguistics and Philosophy* 40.1, pp. 1–36. ISSN: 1573-0549. DOI: 10.1007/s10988-016-9195-2. URL: <https://doi.org/10.1007/s10988-016-9195-2> (visited on 03/15/2021).
- Dayal, Veneeta (1996). *Locality in WH Quantification*. Red. by Gennaro Chierchia, Pauline Jacobson, and Francis J. Pelletier. Vol. 62. Studies in Linguistics and Philosophy. Dordrecht: Springer Netherlands. ISBN: 978-0-7923-5478-9. DOI: 10.1007/978-94-011-4808-5. URL: <http://link.springer.com/10.1007/978-94-011-4808-5> (visited on 07/11/2023).
- Dekker, Paul (Jan. 1, 1992). *An Update Semantics for Dynamic Predicate Logic*. Report. URL: <https://eprints.illc.uva.nl/id/eprint/1177/> (visited on 11/07/2024).
- Dotlačil, Jakub and Floris Roelofsen (2019). “Dynamic Inquisitive Semantics: Anaphora and Questions”. In: *Proceedings of Sinn Und Bedeutung 23*. Universitat Autònoma de Barcelona. DOI: 10.18148/sub/2019.v23i1.538. URL: <https://dare.uva.nl/search?identificator=e767de8b-ac55-4848-af4f-7bf63dfbfb95> (visited on 06/25/2021).
- (Mar. 2, 2021). “A Dynamic Semantics of Single-Wh and Multiple-Wh Questions”. In: *Semantics and Linguistic Theory* 30.0, pp. 376–395. ISSN: 2163-5951. DOI: 10.3765/salt.v30i0.4839. URL: <https://journals.linguisticsociety.org/proceedings/index.php/SALT/article/view/30.376> (visited on 08/09/2021).
- Frank, Anette (1997). “Context Dependence in Modal Constructions”. PhD thesis. Stuttgart: Universität Stuttgart. 411 pp.
- Geurts, Bart (1999). *Presuppositions and Pronouns*. Current Research in the Semantics / Pragmatics Interface 3. Leiden, The Netherlands: Brill. ISBN: 978-0-08-043592-3.
- Giannakidou, Anastasia and Alda Mari (2019). *Modalization and Bias in Questions*. Unpublished manuscript. University of Chicago and Institut Jean Nicod Paris.
- Gillies, Anthony S. (2004). “Epistemic Conditionals and Conditional Epistemics”. In: *Noûs* 38.4, pp. 585–616. DOI: 10.1111/j.0029-4624.2004.00485.x.
- Groenendijk, Jeroen (1998). “Questions in Update Semantics”. In: *Formal Semantics and Pragmatics of Dialogue: Proceedings of the Thirteenth Twente Workshop on Language Technology (Twendial ’98)*. Twente: Universiteit Twente, Faculteit Informatica, pp. 125–137. URL: <https://dare.uva.nl/search?identificator=e7f44061-4fc2-4924-8811-848a0c9630ed> (visited on 07/24/2023).
- Groenendijk, Jeroen and Martin Stokhof (Jan. 1, 1984). “Studies on the Semantics of Questions and the Pragmatics of Answers”. PhD thesis. Amsterdam, The Netherlands: Institute for Logic, Language and Computation (ILLC), University of Amsterdam.
- Groenendijk, Jeroen, Martin Stokhof, and Frank Veltman (1996). “Coreference and Modality”. In: *The Handbook of Contemporary Semantic Theory*. Ed. by Shalom Lappin. Cambridge, Mass., USA: Blackwell Reference, pp. 179–216. ISBN: 978-0-631-18752-3. URL: <https://dare.uva.nl/search?identificator=c655089e-9fae-4842-90da-30e46f37825b> (visited on 01/31/2024).
- Hara, Yurie and Katsuhiko Sano (2017). “Conditional Questions Revisited”. In: *InqBnB2: Inquisitiveness Below and Beyond the Sentence Boundary*. Amsterdam. URL: [https://yuriehara.github.io/website/download/CQR\\_NINJALpdf.pdf](https://yuriehara.github.io/website/download/CQR_NINJALpdf.pdf).
- Higginbotham, James and Robert May (Jan. 1, 1981). “Questions, Quantifiers and Crossing”. In: *The Linguistic Review* 1.1, pp. 41–80. ISSN: 1613-3676. DOI: 10.1515/tlir.1981.1.1.41. URL: <https://www.degruyter.com/document/doi/10.1515/tlir.1981.1.1.41/html> (visited on 07/27/2023).
- Hirsch, Aron and Bernhard Schwarz (2019). “Singular Which, Mention-Some, and Variable Scope Uniqueness”. In: *Semantics and Linguistic Theory* 29.0, pp. 748–767. ISSN: 2163-5951. DOI: 10.3765/salt.v29i0.4637. URL: <https://journals.linguisticsociety.org/proceedings/index.php/SALT/article/view/29.748> (visited on 09/05/2023).



- Hofmann, Lisa (Dec. 20, 2019). “The Anaphoric Potential of Indefinites under Negation and Disjunction”. In: *Proceedings of the 22nd Amsterdam Colloquium*. Amsterdam, pp. 181–190.
- Isaacs, James and Kyle Rawlins (Aug. 1, 2008). “Conditional Questions”. In: *Journal of Semantics* 25.3, pp. 269–319. ISSN: 0167-5133. DOI: 10.1093/jos/ffn003. URL: <https://doi.org/10.1093/jos/ffn003> (visited on 04/18/2024).
- Karttunen, Lauri (1969). “Discourse Referents”. In: *Proceedings of the 1969 Conference on Computational Linguistics (COLING ’69)*. Sång-Säby, Sweden: Association for Computational Linguistics, pp. 1–38. DOI: 10.3115/990403.990490. URL: <http://portal.acm.org/citation.cfm?doid=990403.990490> (visited on 07/07/2023).
- Kaufmann, Stefan (2000). “Dynamic Context Management”. In: Faller, Martina, Stefan Kaufmann, and Marc Pauly. *Formalizing the Dynamics of Information*. CSLI Lecture Notes 92. Stanford, CA: CSLI Publications, pp. 171–181. ISBN: 1-68400-007-6. URL: [https://scholar.google.com/citations?view\\_op=view\\_citation&hl=fr&user=gffJf60AAAAJ&citation\\_for\\_view=gffJf60AAAAJ:aqlVkm33-oC](https://scholar.google.com/citations?view_op=view_citation&hl=fr&user=gffJf60AAAAJ&citation_for_view=gffJf60AAAAJ:aqlVkm33-oC) (visited on 04/18/2024).
- Kibble, Rodger (1998). “Modal Subordination, Focus, and Complement Anaphora”. In: *The Tblisi Symposium on Logic, Language and Computation: Selected Papers*. Ed. by Jonathan Ginzburg et al. Stanford, USA: CSLI Publications, pp. 73–86. ISBN: 978-1-57586-098-5.
- Kratzer, Angelika (Jan. 1, 1977). “What ‘Must’ and ‘Can’ Must and Can Mean”. In: *Linguistics and Philosophy* 1.3, pp. 337–355. ISSN: 1573-0549. DOI: 10.1007/BF00353453. URL: <https://doi.org/10.1007/BF00353453> (visited on 01/26/2024).
- (1991). “Modality”. In: *Semantik / Semantics: Ein internationales Handbuch zeitgenössischer Forschung*. Ed. by Arnim van Stechow and Dieter Wunderlich, pp. 639–650. DOI: 10.1515/9783110126969.7.639.
- Kripke, Saul A. (1959). “A Completeness Theorem in Modal Logic”. In: *The Journal of Symbolic Logic* 24.1, pp. 1–14. ISSN: 0022-4812. DOI: 10.2307/2964568. JSTOR: 2964568. URL: <https://www.jstor.org/stable/2964568> (visited on 05/21/2021).
- Roberts, Craige (Dec. 1, 1989). “Modal Subordination and Pronominal Anaphora in Discourse”. In: *Linguistics and Philosophy* 12.6, pp. 683–721. ISSN: 1573-0549. DOI: 10.1007/BF00632602. URL: <https://doi.org/10.1007/BF00632602> (visited on 07/26/2023).
- Roelofsen, Floris (2015). “The Semantics of Declarative and Interrogative Lists”. Report. URL: <http://semanticsarchive.net/Archive/jdiNjlj0/paper.pdf>.
- Roelofsen, Floris and Jakub Dotlačil (June 1, 2023). “Wh-Questions in Dynamic Inquisitive Semantics”. In: *Theoretical Linguistics* 49.1-2, pp. 1–91. ISSN: 1613-4060. DOI: 10.1515/tl-2023-2001. URL: <https://www.degruyter.com/document/doi/10.1515/tl-2023-2001/html> (visited on 12/13/2023).
- Socolof, Michaela, Aron Hirsch, and Bernhard Schwarz (2020). “Which-Questions, Uniqueness, and Answerhood: Evidence from Disjunction”. In: *Proceedings of NELS*. Vol. 50.
- Stalnaker, Robert C. (1981). “A Theory of Conditionals”. In: *IFS: Conditionals, Belief, Decision, Chance and Time*. Ed. by William L. Harper, Robert Stalnaker, and Glenn Pearce. Dordrecht: Springer Netherlands, pp. 41–55. ISBN: 978-94-009-9117-0. DOI: 10.1007/978-94-009-9117-0\_2. URL: [https://doi.org/10.1007/978-94-009-9117-0\\_2](https://doi.org/10.1007/978-94-009-9117-0_2) (visited on 06/06/2024).
- Van Rooij, Robert (1998). “Modal Subordination in Questions”. In: *Formal Semantics and Pragmatics of Dialogue. Proceedings of Twendial ’98*. Ed. by J. Hulstijn and A. Nijholt. Enschede: University of Twente, pp. 237–247. URL: <https://dare.uva.nl/search?identificator=31017f54-cf94-4cfd-b5c1-a4aa9f433b55> (visited on 07/07/2023).
- Van den Berg, Martin H. (Jan. 1, 1996). “Some Aspects of the Internal Structure of Discourse: The Dynamics of Nominal Anaphora”. PhD thesis. Amsterdam: University of Amsterdam. 202 pp. URL: <https://eprints.illc.uva.nl/id/eprint/1996/> (visited on 07/24/2023).

- Veltman, Frank (1996). “Defaults in Update Semantics”. In: *Journal of Philosophical Logic* 25.3, pp. 221–261. ISSN: 0022-3611. JSTOR: 30226572. URL: <https://www.jstor.org/stable/30226572> (visited on 05/07/2024).
- Xiang, Yimei (May 17, 2016). “Interpreting Questions with Non-Exhaustive Answers”. PhD thesis. Cambridge, Massachusetts, USA: Harvard University, Graduate School of Arts & Sciences. 300 pp. URL: <https://www.semanticscholar.org/paper/Interpreting-Questions-with-Non-Exhaustive-Answers-Xiang/3f3183c8150447d1cfdb5ab0faf6bc14bb1bf316> (visited on 09/05/2023).
- Xiang, Yimei and Alexandre Cremers (2017). “Mention-Some Readings of Plural-Marked Questions: Experimental Evidence”. In: *Proceedings of the Forty-Seventh Annual Meeting of the North East Linguistic Society (NELS 47)*. Ed. by Andrew Lamont and Katerina Tetzloff. Vol. 3. University of Massachusetts, Amherst, USA: GLSA (Graduate Linguistics Student Association), pp. 261–274. ISBN: 978-1-976344-60-2. URL: <https://www.semanticscholar.org/paper/Mention-some-readings-of-plural-marked-questions%3A-Xiang-Cremers/f29b8de6ff99142533c255fc4c831ac1fc4237bc> (visited on 07/10/2023).