Monotonicity restored: *more* never means *purer*[*]

Alan Bale[1], Bernhard Schwarz[2], and David Shanks[2]

[1] Concordia University, Montreal, Quebec, Canada
alan.bale@concordia.ca
[2] McGill University, Montreal, Quebec, Canada
bernhard.schwarz@mcgill.ca
david.shanks4@mail.mcgill.ca

Abstract

Bale and Barner (2009) suggested that comparatives with *more* plus mass noun, such as *There is more gold in the ring than there is in the bracelet*, can be interpreted as comparing degrees of purity of material. As Wellwood (2015) observes, this suggestion is incompatible with Schwarzschild's (2006) *Monotonicity Constraint* which puts limits on the types of measure functions that can enter into the evaluation of such comparisons. Scrutinizing the truth conditions of a range of relevant cases, we resolve this tension by showing that comparisons of purity with *more* are only apparent, and that the correct analysis of such cases is one that respects the Monotonicity Constraint.

1 Introduction

To block unattested readings of comparatives and other degree constructions, Schwarzschild (2006) posits a Monotonicity Constraint (MC): measure functions that enter into the evaluation of certain constructions must be monotonic with respect to the part-whole ordering inherent in noun phrases. Despite the descriptive utility of MC, Bale and Barner (2009) discuss readings of comparative constructions which, in their (and Wellwood's 2015) characterization, involve non-monotonic measurements of *purity*. We argue, however, that reference to such MC-violating functions is only apparent. We demonstrate that the cases that purportedly require purity measurements instead involve proportional measure functions that critically conform to MC.

Section 2 reviews MC and some of the empirical evidence that motivates it. Section 3 introduces apparent comparisons of *purity* first observed in Bale and Barner 2009, spelling out the MC-incompatible analysis that Bale and Barner hinted at. Section 4 presents an alternative account that is compatible with MC. Sections 5 and 6 argue that, independently of MC, this alternative is more adequate than Bale and Barner's proposal. Section 7 concludes.

2 Measure functions and monotonicity

As discussed in Schwarzschild 2006, different types of modification seem to license different types of measurements. For example, although it is perfectly acceptable to talk about *three litres of water*—where the modification of *water* by *three litres* happens in a pseudo-partitive construction—it is very odd to talk about *three degrees of water*. In contrast, although one can talk about *three-degree water*—where *three-degree* modifies *water* by some kind of compounding

*For comments and discussion, we thank the members of the Semantics Research Group at McGill University, especially Aron Hirsch, and three anonymous reviewers for *Amsterdam Colloquium 22*. The authors acknowledge support provided by the Social Sciences and Humanities Research Council of Canada (SSHRC; Bale, Schwarz: Insight Grant #435-2016-1376; Schwarz, Shanks: Insight Grants #435-2013-0592, #435-2016-1448) and by McGill University's Faculty of Arts (Shanks: Arts Research Internship Award).

operation—it is very awkward to talk about *three-litre water*. Schwarzschild hypothesized that this contrast correlates with how measurement systems relate to certain types of partial orders.

In our examples with *water* above, there are two types of measurement systems: volume and temperature. The elements that *water* is true of are partially ordered by the material part-whole relation. The two measurement systems differ in terms of how they interact with this part-whole relation: if a portion of water x is a proper part of another portion of water y, then it necessarily follows that x is less in volume than y, however it does not necessarily follow that x is less in temperature than y. A function that measures volume is a *monotonic measure function*, in the sense defined in (1), while a function that measures temperature is not.

(1) **Monotonic Measure Functions**: A measure function μ is monotonic with respect to a strict partial order $\sqsubset$ if and only if for all x and y, $x \sqsubset y \rightarrow \mu(x) <_\mu \mu(y)$, where $<_\mu$ is the strict ordering of degrees in the range of μ.

According to Schwarzschild, determining what counts as being "three litres" or "three degrees" requires measuring members of the nominal denotation. Different measure terms require different measure functions: *three litre(s)* requires a function that maps individuals to a measure of volume, whereas *three degrees* requires a function that maps individuals to a measure of temperature. Certain types of grammatical constructions, such as the pseudo-partitive above, only permit monotonic measure functions, whereas others exclude such functions.

Monotonicity not only plays a role in pseudo-partitives, but also in constructions where *much*, or its comparative counterpart *more*, combines directly with a noun. As discussed in Cresswell 1976 and Bale and Barner 2009, when *much* and *more* combine with mass nouns, the dimension of comparison can vary. For example, *having too much string* or *having more string than needed* usually means having too much in terms of length (but not weight or volume), whereas *having too much pudding* or *having more pudding than needed* usually means having too much in terms of weight or volume (but not length). In contrast, *witnessing too much celebration* or *witnessing more celebration than expected* usually means witnessing too much in terms of the time-span, number of participants, or intensity of emotion (but definitely not too much in terms of weight, length, or volume). Thus, *much* permits variable types of measurement. Nevertheless, not all measurement types are available. Consider the sentences in (2).

(2) a. This tub has too much water in it.
b. This tub has more water in it than that tub.

The sentence in (2a) cannot be true in a context where the weight/volume of the water is not *too much*, but where the water is too hot. Similarly, (2b) cannot be true in a context where both tubs have an equal amount of water in terms of weight/volume, but where the water in one tub is hotter than the water in the other. In other words, just like the pseudo-partitive constructions, the sentences in (2) are required to be evaluated using measurement systems that are monotonic with respect to the material part-whole relation of *water*.

To explain why sentences like (2a) cannot be construed as involving measurements of temperature, Schwarzschild (2006) adapted the constraint that he hypothesized for pseudo-partitive constructions so that it could also apply to the combination of *much* with mass nouns.

(3) **Monotonicity Constraint (MC)** (Schwarzschild 2006): Only monotonic functions are available as values for the measure function in the meaning of *much*.

MC also applies to the comparative *more* as in (2b) if it is decomposed into *much+-er* (cf. Bresnan 1973). We will in fact focus mainly on comparatives, as their truth conditions are less vague and thus easier to assess. For concreteness, we will adopt a specific analysis of *much* and *-er*, although the generalizations discussed above hold for any reasonable analysis involving degrees. As (4a) shows, we assume that *much* is a determiner that applies to a degree and two sets of individuals (cf. Hackl 2000). Critically, the output that *much* returns depends on the value of an underspecified measure function (symbolized with the variable μ), which measures the supremum (i.e., the sum, symbolized with $\sqcup$) of the elements in the input sets' intersection.[1]

(4) a. $[\![\textit{much}]\!] = \lambda \mathrm{d_d}.\, \lambda \mathrm{X_{et}}.\, \lambda \mathrm{Y_{et}}.\, \mu(\sqcup(\mathrm{X} \cap \mathrm{Y})) \geqslant \mathrm{d}$
b. $[\![\textit{-er}]\!] = \lambda \mathrm{f_{dt}}.\, \lambda \mathrm{g_{dt}}.\, \textsc{max}(\mathrm{g}) > \textsc{max}(\mathrm{f})$

As (4b) shows, we take comparative *-er* to relate two sets of degrees. This meaning assumes that there are two instances of *much*, one in the main clause and another (which is usually silent because of ellipsis) in the *than*-clause. The degree argument in the *than*-clause is abstracted to form a degree predicate (the first argument to *-er*). The comparative morpheme combines with the *than*-clause creating a generalized degree quantifier. This entire phrase (-*er* plus *than*-clause) covertly moves to a propositional level binding a degree argument within the parameterized determiner (cf. Heim 2000). Thus, a sentence like (2b) would have an LF representation like the one in (5), resulting in truth conditions like those in (6).

(5) -er λd[than [[*d* much] water] λ*x*[that tub has x in it]]
λd[[[*d* much] water] λ*x*[this tub has x in it]]

(6) $\textsc{max}(\{d : \mu(\sqcup\{x : \textsc{water}(x)\ \&\ \textsc{in-this-tub}(x)\}) \geqslant d\}) >$
$\textsc{max}(\{d : \mu(\sqcup\{x : \textsc{water}(x)\ \&\ \textsc{in-that-tub}(x)\}) \geqslant d\})$

We will adopt this general analysis of the comparative throughout, although it is important to remember that the compositional details are moot. What is critical for our purposes is how the measure function variable is valued. The correct truth conditions are obtained for (5) by setting the value of μ in both the main-clause and *than*-clause to either of the monotonic values in (7), but critically not the non-monotonic value in (8).

(7) a. $\mu = \lambda \mathrm{x}.\, \textsc{volume}(\mathrm{x})$ (*than*-clause & main clause)
b. $\mu = \lambda \mathrm{x}.\, \textsc{weight}(\mathrm{x})$ (*than*-clause & main clause)

(8) $\mu = \lambda \mathrm{x}.\, \textsc{temperature}(\mathrm{x})$ (*than*-clause & main clause)

In summary, MC in (3) permits *much* to have a semantic value that is not fully fixed lexically (which is empirically required to account for its behaviour with respect to various mass nouns) while imposing limits on permissible values that prevent empirically unattested readings.

3 Relative readings: non-monotonic proportionality?

Given the data reviewed in the previous section, it is clear that something akin to MC is needed to prevent unattested meanings. However, Wellwood (2015) portrays certain comparatives with mass nouns like *gold* and *silver* as a potential challenge to MC. In this section, after first discussing data with such mass nouns that MC applies to correctly, we will present cases of readings, we will call them *relative*, that could be analyzed as inconsistent with this constraint.

[1] A familiar alternative to analyzing *much* as a projecting a determiner is to assume that it projects an intersective adjectival modifier (e.g., Schwarzschild 2006, Wellwood 2015). While lack of space prevents us from elaborating, we note that the existence of this alternative does not affect our overall conclusions, even though some of the details of our argumentation would change if couched in this alternative setting.

Intuitions indicate that there are a variety of different ways one could compare pieces of precious metals like gold and silver. One could compare them by weight, volume, beauty or shininess (a.k.a., gloss). If one worked with shaping these metals (where gauging temperature is relevant), it would even be natural to compare them in terms of heat. However, as discussed in Schwarzschild 2006 and Wellwood 2015, in many cases only certain types of comparisons are permitted. For example, (9a) can only be true in a situation where the gold is "too much" in terms of weight or volume; it cannot be true by virtue of the gold being too hot, shiny, or beautiful. Similarly, (9b) can only be true if Al's gold has a greater weight or volume than Bill's, independent of whether Al's gold is hotter, shinier, or more beautiful.

(9) a. He put too much gold in the ring. (Schwarzschild 2006)
b. Al has more gold than Bill does. (Wellwood 2015)

As Schwarzschild and Wellwood point out, MC applies again correctly, regulating the availability of these various dimensions of comparison. Like *water*, *gold* is implicitly associated with the material part-whole relation; and as before, *weight* and *volume* are monotonic with respect to this relation: for any piece of gold x that is a proper part of a piece y, it necessarily follows that x has less weight and less volume than y. In contrast, *beauty*, *shininess*, and *temperature* are not monotonic: for any piece of gold x that is a proper part of a piece of gold y, it does not necessarily follow that x is colder than y or that x has less beauty/shininess than y.

Although the range of readings in (9) is captured by underspecification restrained by MC, there are other constructions which may appear problematic. To illustrate, consider Bale and Barner's (2009) example (10) below. Bale and Barner report that among the readings that (10) permits, there is one that they portray as a comparison of the silver and gold in terms of *purity*. Indeed, (10) can be true in a scenario where Esme's ring is quite small but made of high quality, pure gold and Seymour's necklace is quite large but made of a low-grade, impure silver. That is, the sentence can be true even when the total amount of gold in Esme's ring is less in absolute weight and volume than the total amount of silver in Seymour's necklace. This reading becomes even more prominent when *relatively speaking* is added to the front of the sentence, and for this reason we will refer to this type of reading as a *relative* reading.

(10) Esme has more gold in her ring than Seymour has silver in his necklace.

As noted, Bale and Barner hint at an analysis, let us now label it *purity-based*, on which (10) in its relative reading is a comparison of purity measures. It will be useful for us to spell out this analysis in some detail. We begin by observing that mass nouns like *gold* and *silver* have both *loose* and *strict* usages. Under a strict usage, the nouns are only true of aggregates consisting of pure gold (the element Au) and pure silver (the element Ag) respectively. However, under a loose usage—the more common one by far—the nouns are not only true of pure gold and pure silver but also alloys consisting of these elements. It is this loose meaning that plausibly allows the phrase *12 karat gold* to be non-contradictory (only 24 karat gold is pure gold) or that permits the question *How pure is this silver?* to be non-trivial.

We can exploit this ambiguity to define a measure of purity: the purity of an aggregate of gold/silver, understood in its loose sense, is the proportion of pure gold/silver within that aggregate. Thus, in order to be consistent with the purity analysis, the occurrences of *gold* and *silver* in (10) must be understood in the loose sense. At the same time, the measure functions operative in the two occurrences of *much* must reference the strict sense of these nouns. (11) spells out such purity measures as functions mapping an aggregate of matter to the weight proportion of Au/Ag contained in it.

(11) a. $\text{PURITY}_{\text{Au}} := \lambda \text{x}.\ \text{WEIGHT}(\sqcup\{\text{y: y}\sqsubseteq\text{x} \wedge \text{Au(y)}\})/\text{WEIGHT(x)}$
b. $\text{PURITY}_{\text{Ag}} := \lambda \text{x}.\ \text{WEIGHT}(\sqcup\{\text{y: y}\sqsubseteq\text{x} \wedge \text{Ag(y)}\})/\text{WEIGHT(x)}$

These measure functions can be used to derive plausible truth conditions for (10). The expected LF of (10) is shown in (12). Given the meaning of *much* and *-er* specified in the previous section, this LF ends up producing the general truth conditions in (13). There, we label the underspecified measure functions μ_1 and μ_2 respectively, using subscripts to indicate the connection between the measure functions and the two instances of *much*. The purity reading could then arise from the setting in (14). We note that this analysis predicts, correctly it seems, that the relative reading of (10) can receive a paraphrase like (15).

(12) -er λd[than [[*d* much] silver] λx[Seymour has x in is necklace]]
λd[[[*d* much] gold] λx[Esme has x in her ring]]

(13) $\text{MAX}(\{d : \mu_1(\sqcup\{x : \text{GOLD}(x)\ \&\ \text{IN-ESME'S-RING}(x)\}) \geqslant d\}) >$
$\text{MAX}(\{d : \mu_2(\sqcup\{x : \text{SILVER}(x)\ \&\ \text{IN-SEYMOUR'S-NECKLACE}(x)\}) \geqslant d\})$

(14) a. $\mu_1 = \lambda \text{x}.\, \text{PURITY}_{\text{Ag}}(\text{x})$ (*than*-clause)
b. $\mu_2 = \lambda \text{x}.\, \text{PURITY}_{\text{Au}}(\text{x})$ (main clause)

(15) The gold in Esme's ring is purer than the silver in Seymour's necklace.

This purity-based analysis looks like an initially plausible account of the relative reading of (10). One of its benefits is that it naturally extends to other instances of relative readings. To illustrate with just one additional case, consider the sentence in (16).

(16) This bottle has more alcohol in it than that bottle does.

This sentence too permits a relative reading, that is, one that does not involve a comparison of absolute weights or volumes. It can be judged true if the percentage of ethanol is higher in this bottle than in that bottle, but the absolute volume/weight of ethanol is not higher, for example in the following specific scenario: this bottle contains 30% ethanol within 750 millilitres of whisky but that bottle contains only 25% ethanol within 2 litres of whisky.

A purity-based analysis of (16) is fully parallel to the purity-based analysis of (10). Like *gold* and *silver*, the mass noun *alcohol* has both a strict and loose usage. In its strict usage, it is only true of ethanol. In its loose usage, it is true of liquids that have ethanol as a principal component. It is in this loose sense that statements like *this whisky is alcohol* can be true or questions like *How pure is this alcohol?* are non-trivial. The purity of an aggregate of alcohol in the loose sense can be characterized as the proportion of alcohol in the strict sense that this aggregate contains. Hence (16) can be analyzed by interpreting *alcohol* in the loose sense, and by positing a measure function that references alcohol in the strict sense. Specifically, the relevant purity function can be defined as in (17), as a function that maps any aggregate to the volume proportion of ethanol contained in it. The relative reading of (16) can now be captured assuming the LF in (18), deriving the general truth conditions in (19), and by setting the value of μ, for both occurrences of *much*, to the purity function defined in (17).

(17) $\text{PURITY}_{\text{ethanol}} := \lambda \text{x}.\ \text{VOL}(\sqcup\{\text{y: y}\sqsubseteq\text{x} \wedge \text{ETHANOL(y)}\})/\text{VOL(x)}$

(18) -er λd[than [[*d* much] alcohol] λx[that bottle has x in it]]
λd[[[*d* much] alcohol] λx[that bottle has x in it]]

(19) $\text{MAX}(\{d : \mu(\sqcup\{x : \text{ALCOHOL}(x)\ \&\ \text{IN-THIS-BOTTLE}(x)\}) \geqslant d\}) >$
$\text{MAX}(\{d : \mu(\sqcup\{x : \text{ALCOHOL}(x)\ \&\ \text{IN-THAT-BOTTLE}(x)\}) \geqslant d\})$

So, positing that *much* can reference purity measure functions like those in (11) and (17) enables a seemingly natural analysis of relative readings with mass nouns like *gold*, *silver* and *alcohol*. However, as Wellwood (2015) notes, this analysis of relative readings is inconsistent with Schwarzschild's MC. In virtue of capturing the intuitive notion of purity, the functions in (11) and (17) are non-monotonic. After all, if y is a piece of gold or silver alloy that has a piece x as a proper part, it does not necessarily follow that x is less pure than y in the sense of containing a lower proportion of Au or Ag. Similarly, a portion of whisky x that is a proper part of a portion of whisky y does not necessarily have a smaller ethanol proportion than y.

Does this mean that the existence of relative readings calls for a weakening of MC? We think not. The remainder of this paper is dedicated to detailing our justification for this answer.

4 Relative readings: monotonic proportionality?

This section articulates an account of relative readings which, minimally, appears like a credible alternative to the purity-based analysis. Given the availability of such an alternative, the mere existence of readings that qualify as relative (in virtue of not comparing absolute weights or volumes) is insufficient to establish that the interpretation of *much* can make reference to the non-monotonic measure functions like those in (11) and (17).

Returning to the relative reading of (10), we retain the LF in (12) and the resulting general truth conditions (13), but we now propose the value assignments in (20) as an alternative to (14). (In a somewhat different context, similar functions are posited in Bale and Schwarz 2019.)

(20) a. $\mu_1 = \lambda x.\ \textsc{weight}(x)/\textsc{weight}(\text{Seymour's necklace})$ (*than*-clause)
b. $\mu_2 = \lambda x.\ \textsc{weight}(x)/\textsc{weight}(\text{Esme's ring})$ (main clause)

For any input x, the functions in (20) output the proportion of x's weight to the weight of Seymour's necklace and Esme's ring, respectively. Given our assumptions, in the interpretation of (10) these functions take as inputs the supremum (i.e., the sum) of all the silver aggregates in Seymour's necklace and of all the gold aggregates in Esme's ring, respectively. (20) is compatible with interpreting *gold* and *silver* in either a strict or loose sense. If the strict reading is chosen, the truth conditions resulting from (20) can be stated as in (21). Similarly for the loose reading.

(21) $\textsc{weight}(\text{the Au in E's ring})/\textsc{weight}(\text{E's ring}) >$
$\textsc{weight}(\text{the Ag in S's necklace})/\textsc{weight}(\text{S's necklace})$

Such truth conditions indeed qualify as relative, as it is not absolute amounts that are being compared. In fact, under certain conditions they mimic the truth conditions based on (14), which we can state as in (22). Specifically, (21) and (22) become equivalent in a context where the ring and necklace referred to in (10) are taken to be made entirely of gold and silver (alloys), respectively — because then the weight of the gold (alloy) in Esme's ring equals the weight of Esme's ring, and likewise for Seymour's necklace and the silver (alloy) it consists of.

(22) $\textsc{weight}(\text{the Au in E's ring})/\textsc{weight}(\text{the gold (alloy) in E's ring}) >$
$\textsc{weight}(\text{the Ag in S's necklace})/\textsc{weight}(\text{the silver (alloy) in S's necklace})$

The next two sections will provide a more thorough assessment of the relative merits of measure function settings like (14) and (20). Scrutinizing truth conditions, we will present evidence confirming that *much* can reference measure functions like those in (20), as well as evidence showing that *much* cannot reference measure functions like those in (11)/(14).

Before attending to this evidence, we note that the conclusions it supports bear on the assessment of Schwarzschild's (2006) MC. The reason is that, unlike the measure functions in (11)/(14), those in (20) are monotonic. To see this, we observe that in the fractions in (20), (i) WEIGHT, the measure function that appears in the numerator, is itself monotonic, and (ii) the denominator is a constant. The claim now follows from the general fact that for any measure function m and positive real number c, if m is monotonic, then so is λx.m(x)/c. (So the crucial difference between the functions in (11)/(14) and (20) is that only the latter have this form, while the former instead have the form λx.m(x)/n(x), where the denominator can vary with the input.) The conclusion that the evidence presented below will support, then, is that the relative readings with *much* are compatible with MC, and that they in fact ultimately strengthen the case for MC, as there are truth-conditional reasons independent from MC for not letting *much* make reference to measure functions that are purity-based and hence non-monotonic.

5 Evidence for monotonic proportionality

We have said that measure functions like those in (20), which we now label *monotonic proportional*, under certain conditions replicate the effect of purity-based measure functions like those in (11). However, the two types of measure functions of course do not deliver identical truth conditions. We will now identify cases where the truth of a relative reading can be attributed to monotonic proportional measure functions but not to purity-based measure functions, thereby furnishing evidence for the availability of the former.

Returning again to (10), recall that the possible relative truth conditions in (21) and (22) become equivalent under the assumption that Esme's ring and Seymour's necklace are made up entirely of gold and silver (alloys), respectively. But it is of course not necessary to make this sort of assumption. A natural way of evaluating and comparing the adequacy of the meanings derived by different types of measure functions is to examine scenarios where the assumption does not hold. To pursue this avenue, we switch to the example in (23).

(23) There is more gold in the ring than there is in the bracelet.

This sentence permits the same sort of relative reading as (10) or (16). It is like (16), and unlike (10), in that the same mass noun featured in the main clause (viz. *gold*) is also the mass noun interpreted in the *than*-clause, which simplifies the exposition and judgments. Under our current assumptions, (23) has the LF (24) and the general truth conditions in (25).

(24) -er λd[[[*d* much] gold] λx[there is x in the bracelet]]
λd[[[*d* much] gold] λx[there is x in the ring]]

(25) $\textsc{max}(\{d : \mu_1(\sqcup\{x : \textsc{gold}(x) \ \&\ \textsc{in-the-ring}(x)\}) \geqslant d\}) >$
$\textsc{max}(\{d : \mu_2(\sqcup\{x : \textsc{gold}(x) \ \&\ \textsc{in-the-necklace}(x)\}) \geqslant d\})$

On the purity-based analysis, the relative reading of (23) is credited to the setting of the measure function variables μ_1 and μ_2 in (26). The effect of this setting is to be compared to the effect of (27), the relevant setting under the monotonic proportionality analysis.

(26) $\mu_1 = \mu_2 = \lambda$x. WEIGHT($\sqcup$\{y: y$\sqsubseteq$x $\wedge$ Au(y)\})/WEIGHT(x) (*than*-clause & main cl.)

(27) a. $\mu_1 = \lambda$x. WEIGHT(x)/WEIGHT(the bracelet) (*than*-clause)
b. $\mu_2 = \lambda$x. WEIGHT(x)/WEIGHT(the ring) (main clause)

Consider now the following scenario. Both the ring and the bracelet mentioned in (23) are made of copper plated with white gold. The white gold, a gold alloy, consists of 90 weight percent Au and 10 weight percent nickel. The ring consists of 10g of white gold and 10g of copper, while the bracelet consists of 10g of white gold and 30g of copper.

Note that this scenario is predicted to render (23) false in a reading that compares absolute amounts. Moreover, under the setting in (26), the truth conditions in (25) come out false too. After all, the weight proportion of Au in the white gold that the ring contains equals, hence does not exceed, the weight proportion of Au in the white gold that the bracelet contains. In contrast, (25) comes out true under the setting in (27), regardless of whether *gold* is interpreted loosely or strictly. Under a loose interpretation, (24) is true in virtue of the weight proportion of white gold in the ring (50 percent) exceeding the weight proportion of white gold in the bracelet (25 percent); under a strict interpretation (24) is true in virtue of the weight proportion of Au in the ring (45 percent) exceeding the weight proportion of Au in the bracelet (22.5 percent).

Aligned with similar judgments reported in Bale and Schwarz (2019), we take it that (23) can indeed be judged true in the scenario we have given. Intuitions indicate, moreover, that (23) is judged true in the scenario given on the grounds that the white gold/Au in the ring comprises a greater weight proportion of the ring than the white gold/Au in the bracelet does of the bracelet. This judgment can be captured by the monotonic proportional measure function setting in (27) but not by the non-monotonic, purity-based, setting in (26). It therefore provides non-confounded evidence for the availability of monotonic proportional measure functions like those in (20) and (27) in the interpretation of *much*.

To confirm and sharpen the relevant judgments, it will be useful to also consider examples where the purity-based analysis is not expected to apply in the first place. Recall that the purity-based analysis exploits the strict-loose ambiguity of the mass nouns that *much* combines with. This feature of the analysis invites us to consider comparatives where the relevant mass nouns do not participate in the strict-loose ambiguity. If such examples permit relative readings, they confirm that relative readings can have a source other than purity-based proportionality.

Let us compare comparatives with the mass nouns *alcohol* and *ethanol*. As noted in Section 3, the former is ambiguous between a loose meaning, which holds of portions of liquids that have ethanol as their principal component, and a strict meaning, which holds of portions of ethanol. We now add that, in contrast, the noun *ethanol* itself does not seem to have a comparable strict-loose ambiguity. With this in mind, we first present sentence (28a) below as a baseline. It allows for a relative reading of the same sort as the the one attested for Bale and Barner's example (10). It can be read, in particular, as conveying that the volume proportion of ethanol in Sample 1 exceeds the volume proportion of ethanol in Sample 2, a reading on which the sentence's truth is independent of the absolute amounts of ethanol in the two samples. This interpretation could again be captured with reference to either a non-monotonic, purity-based, measure function (cf. (26)) or to a pair of monotonic proportional measure functions (cf. (27)).

(28) a. There is more alcohol in Sample 1 than in Sample 2.
b. There is more ethanol in Sample 1 than in Sample 2.

But now compare (28a) to (28b), where *ethanol* substitutes for *alcohol*. *Ethanol* lacking the relevant strict-loose ambiguity, the purity-based analysis does not predict (28b) to have a relative reading. In particular, since this analysis requires that in the relative reading of (28a) *alcohol* have its loose interpretation (which *ethanol* does not share), the analysis does not predict that (28b) can be judged equivalent to (28a) in a relative reading. In contrast, the monotonic proportional analysis, which can derive a relative reading of (28a) while interpreting *alcohol* strictly (or loosely), predicts that (28b) can be read as equivalent to (28a) in its relative reading.

Again aligned with judgments reported in Bale and Schwarz 2019, speaker intuitions clearly bear out the latter prediction. That is, (28a) and (28b) are indeed intuited to have equivalent relative readings. Again, this confirms that it is at least possible for relative readings in comparatives with *more* to arise from monotonic proportional measure functions.

The next section will strengthen this conclusion by showing that the purity-based analysis, apart from undergenerating relative readings, also overgenerates.

6 Evidence against non-monotonic proportionality

We return to the example (23), repeated in (29). Still assuming the general truth conditions in (25), we will continue to examine the effects of the measure function settings in (26) and (27).

(29) There is more gold in the ring than there is in the bracelet.

Consider now the following scenario. Both the ring and the bracelet are made of silver plated with white gold, an alloy composed of gold and nickel. The ring's white gold plating weighs 5g, and the ring's total weight is 20g; the bracelet's white gold plating weighs 20g, and the bracelet's total weight is 40g. The ring's white gold plating is composed of 90 percent Au and 10 percent nickel; the bracelet's white gold plating is 80 percent Au and 20 percent nickel.

This scenario again renders (29) false in a reading that compares absolute amounts. The scenario likewise fails to render true the truth conditions in (25) under the monotonic proportional measure function setting in (27), given that the ring's white gold plating (or the Au it contains) accounts for a much smaller proportion of the ring's weight than the bracelet's white gold plating (or the Au it contains) does of the bracelet's weight. However, the scenario satisfies the truth conditions that would arise from the purity-based measure function setting in (26), on the grounds that the ring's white gold plating contains a higher weight proportion of Au than the bracelet's white gold plating does.

As for speaker judgments about (29), it seems hard or impossible to interpret the sentence in a sense that is true in the scenario given. In this respect, (29) contrasts sharply with (30), whose truth in the scenario given is uncontroversial.

(30) The gold in the ring is purer than the gold in the bracelet.

This judgment about (29), and the intuited contrast with (30), are incompatible with the assumption that *much* can be evaluated using a non-monotonic, purity-based, measure function. In conjunction with the findings in the last section, they lead us to conclude that the purity-based analysis of relative readings hinted at in Bale and Barner 2009 is incorrect, and that the relative readings that may seem to call for such an analysis are instead to be analyzed as involving monotonic proportional measure functions.

7 Conclusions

We have argued that relative readings of sentences like Bale and Barner's (2009) example (10), repeated once more in (31), invoke reference to monotonic proportional measure functions, rather than non-monotonic purity-based measure function, and therefore pose no threat to Schwarzschild's (2006) Monotonicity Constraint (MC).

(31) Esme has more gold in her ring than Seymour has silver in his necklace.

Our conclusion that the interpretation of comparatives with *more* is compatible with MC is aligned with the assessment offered in Wellwood (2015). However, Wellwood bases this assessment on the assumption that relative readings do not exist in the first place. Without discussing examples like (31), Wellwood justifies this assumption with the observation that a comparative like (9b) above, repeated here as (32), does not permit a relative reading.

(32) Al has more gold than Bill does. (Wellwood 2015)

While the existence of relative readings for cases like (31) seems undeniable, we agree with Wellwood that no such reading is available for (32). It seems that (32) indeed can only be understood as a comparison of absolute amounts. The question is then why (31) and (32) should differ in this way. More generally, how are relative readings distributed, that is, how can we characterize the general conditions under which they can and cannot appear?

While we cannot properly address this question within the confines of this paper, we will end by reframing the issue in a way that may be helpful in going forward. The monotonic proportional measure functions that we have proposed figure into the relative reading of (31) feature fractions whose denominators, the weights of Esme's ring and Seymour's necklace, are measurements of entities picked out by referring expressions in the sentence, viz. by *her ring* and *his necklace*. It seems plausible that monotonic proportional measure functions can become available for interpretation of *much* only when they can be anchored in this way to suitable referring expressions in the sentence, and hence that (32) lacks a relative reading because no suitable anchors are available there. We hope to explore this intuition in future work.

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