Attitudes, Conditionals and Margins for Error

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This paper is about *The Qualitative Thesis*, the thesis that if you are not sure that φ is false, then you are sure of the indicative conditional $\varphi > \psi$ just in case you are sure of the material conditional $\varphi \supset \psi$. Following contextualists about indicative conditionals like Bacon [2015], we will understand this thesis in a *local* way—roughly as saying that if you are not sure that φ is false, then you are sure of the proposition expressed by $\varphi > \psi$ *in your context* just in case you are sure of the material conditional $\varphi \supset \psi$. To state this precisely, let $S^{c,w}(\llbracket\varphi\rrbracket^c)$ mean that the speakers in c are sure of $\llbracket\varphi\rrbracket^c$ in w. Then:

> **The Local Qualitative Thesis**. For any world w and context c, if $\neg S^{c,w}(\llbracket\neg\varphi\rrbracket^c)$, then: $S^{c,w}(\llbracket\text{if } \varphi, \text{ then } \psi\rrbracket^c)$ if and only if $S^{c,w}\llbracket\varphi \supset \psi\rrbracket^c$.

We investigate the epistemological consequences of The Qualitative Thesis. We characterize The Qualitative Thesis in standard formal frameworks for studying the logic of attitudes and conditionals. With these characterization results in hand, we develop a connection first observed by Ben Holguín (p.c.) between The Qualitative Thesis and a plausible margin-for-error requirement on rational sureness. We show that The Qualitative Thesis is inconsistent with the margin-for-error principle. We propose a new *shifty* semantics for indicative conditionals. We say that the meaning of an indicative conditional is partly determined by the conditional's local informational environment—the conditional's *local context*—which, in turn, is systematically shifted by attitude operators. Our account validates The Qualitative Thesis, but dispenses with its undesirable epistemological consequences.

1 Motivating The Qualitative Thesis

The first argument for the Qualitative Thesis is that it follows from the conjunction of two standard claims about reasoning with conditionals. The first claim is that Modus Ponens is valid. This entails one half of the Qualitative Thesis—if you are sure of the indicative conditional $\varphi > \psi$, then you are sure of the corresponding material conditional $\varphi \supset \psi$ (regardless of whether you are sure of $\neg\varphi$). The second claim is that Stalnaker's *Direct Argument* is a reasonable inference. This entails the second half of the Qualitative Thesis, namely, that if you are not sure that $\neg\varphi$ and you're sure of the material conditional $\varphi \supset \psi$, then you are also sure of the indicative conditional $\varphi > \psi$.

The Direct Argument is the argument from the disjunction $\varphi \vee \psi$ to the indicative conditional $\neg\varphi > \psi$. The argument is compelling, as the following example shows.

(1) Matt is either in Los Angeles or London.

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(2) So, if Matt is not in Los Angeles, he is in London.

We should not say that the Direct Argument is a *valid* inference. For (1) is equivalent to the material conditional *Matt's not in Los Angeles* $\supset$ *Matt's in London*. So to say that (1) entails (2) would be to say that the material conditional entails the indicative conditional, a notoriously unacceptable consequence. Following Stalnaker, we should instead say that the Direct Argument is a *reasonable inference*—roughly, if you are sure of the disjunction $\varphi \vee \psi$, and are not sure that φ, then you are sure that $\neg\varphi > \psi$. This claim is equivalent to the second half of the Qualitative Thesis: if you are sure of the material conditional $\varphi \supset \psi$ and you are not sure that $\neg\varphi$, then you are sure of the indicative conditional $\varphi > \psi$.[1]

The second argument for The Qualitative Thesis is that, given plausible assumptions, it follows from Stalnaker's Thesis, stated informally below.

> **Stalnaker's Thesis**. The probability of $\varphi > \psi$ is equal to the probability of ψ conditional on φ.

Stalnaker's Thesis is strongly supported both by intuition and experimental data. Take an example. You are holding a standard 52-card deck of cards, and you draw one at random. Ask yourself how confident you are in the following conditional.

(3) The selected card is a jack if it's a red card.

If you are like most, you will judge the probability of (3) to be 1/13. There are 26 red cards, and 2 of them are jacks. So the probability that the selected card is a jack given that it is red is 1/13. That is the probability that you assign to to (3), in conformity with Stalnaker's Thesis.[2] It is easy to multiply examples like this. In general, we calculate the probability of a conditional $\varphi > \psi$ by calculating the probability of ψ conditional on φ. This is just what we would expect if Stalnaker's Thesis were true. If we assume a plausible probabilistic account of being sure—specifically, that one is sure of some proposition just in case one assigns credence 1 to that proposition—then Stalnaker's Thesis entails The Qualitative Thesis.

2 The Qualitative Thesis in the Standard Framework

Here we present a standard formal framework for thinking about The Qualitative Thesis. This framework gives sureness ascriptions a Hintikka semantics. And, following Kratzer [2012] and Stalnaker [1975a], it gives the conditional a variably strict semantics, where $\ulcorner$if ϕ, then $\psi\urcorner$ says, roughly, that ψ is true in the closest ϕ-worlds.[3] We characterize the Qualitative Thesis in this framework and then use this result to show that the Qualitative Thesis puts a significant constraint on the logic of sureness, entailing a principle we call *No Opposite Materials*.

[1] Note that it doesn't follow from the Qualitative Thesis that *whenever* the you are sure of (1), you are in position to infer (2). You might be sure of (1) without leaving open that Matt is in Los Angeles, and the Qualitative Thesis is silent about that case. But, as Stalnaker points out, it is felicitous to assert (1) only if the context leaves open that Matt is not in Los Angeles, and so whenever (1) is felicitously asserted, the posterior context will entail that Matt is in Los Angeles or London, but leave open that Matt is in Los Angeles. This means that The Qualitative Thesis predicts that the speakers can infer (2) from (1) whenever they have become sure of (1) on the basis of a successful assertion of (1).

[2] See Adams (1975), Jeffrey and Edgington (1991), Stalnaker (1970), and van Fraassen (1976) for semantic theories that are designed to predict Stalnaker's Thesis. See Douven and Verbugge (2013) and Evans and Over (2004) for empirical work supporting Stalnaker's Thesis.

[3] In Boylan and Schultheis [2019], we prove that analogous results hold in a strict conditional framework, defended by Gillies [2004], Gillies [2009], Rothschild [2013], and Willer [2017], where $\phi > \psi$ says that $\phi \supset \psi$ holds throughout some fixed set of closest worlds.

2.1 A Standard Framework

We begin by constructing a propositional modal language that we can use to describe what a subject is sure of. The set of sentences of the language $\mathcal{L}$ is the set of sentences generated by the following grammar:

- $\phi ::= p \mid \neg\phi \mid \phi \wedge \psi \mid \phi > \psi \mid S\phi$

The propositional connectives $\supset$, $\equiv$, and $\vee$ are defined as usual; $>$ is our conditional operator. We read $S\phi$ as *the subject is sure of* ϕ.

Next, the interpretation of the language. We assume that we are in some fixed arbitrary context with some relevant speaker who determines the particular interpretation of the conditional; that is, our semantic evaluation function, $[\![\cdot]\!]$, specifies only the *content* of the sentences in our language in this context.

We interpret the logical connectives in the standard way. To give the truth-conditions of the conditional, we use a *selection function*, which we assume is supplied by the background context. Where $f(\mathbf{A}, w)$ is the set of selected $\mathbf{A}$-worlds at w and $[\![\phi]\!] = \{w : [\![\phi]\!]^w = 1\}$, we say:

Standard Variably Strict Semantics. $[\![\phi > \psi]\!]^w = 1$ iff $f(w, [\![\phi]\!]) \subseteq [\![\psi]\!]$

This clause says that $\varphi > \psi$ is true at a world w just in case all of the selected φ-worlds at w are ψ-worlds. We stipulate that the selection function has the following natural properties:

Success. $f(w, \mathbf{A}) \subseteq \mathbf{A}$

Minimality. If $w \in \mathbf{A}$, then $w \in f(w, \mathbf{A})$

Non-Vacuity. If $R(w) \cap \mathbf{A} \neq \emptyset$ then $f(w, \mathbf{A}) \neq \emptyset$

Success and Minimality are standard assumptions.[4] Success says that the selected $\mathbf{A}$-worlds at w must be $\mathbf{A}$-worlds; it's needed to validate $\phi > \phi$. Minimality says that if w is an $\mathbf{A}$-world, then it must be among the selected $\mathbf{A}$-worlds at w; it's needed to validate Modus Ponens. Non-Vacuity says that if there are accessible $\mathbf{A}$-worlds at w, then the set of selected $\mathbf{A}$-worlds at w isn't empty. It's needed to validate a form of Conditional Non-Contradiction, specifically:

Weak Conditional Non-Contradiction. $\neg S\neg\phi \supset \neg((\phi > \psi) \wedge (\phi > \neg\psi))$

Weak Conditional Non-Contradiction says that if φ is a live possibility, then $\varphi > \psi$ and $\varphi > \neg\psi$ are not consistent. This is a standard—and desirable—principle in conditional logic.[5] In general, there is something very wrong with asserting both $\varphi > \psi$ and $\varphi > \neg\psi$.

Truth for the sureness operator S is defined in terms of an accessibility relation R: wRw' means that w' is compatible with what the subject is sure of in w.[6]

Standard Hintikka Semantics. $[\![S\phi]\!]^w = 1$ iff $\forall w' \in R(w) : [\![\phi]\!]^{w'} = 1$

[4] See, for example, Stalnaker [1968] and Lewis [1973].

[5] Why not a stronger version of Conditional Non-Contradiction that just says $\varphi > \psi$ and $\phi > \neg\psi$ are not consistent? This stronger principle is inconsistent with Logical Implication, which says that $\varphi > \psi$ is always true when φ entails ψ. Weak Conditional Non-Contradiction, by contrast, is consistent with Logical Implication. See Stalnaker [1968] and Lewis [1973] for theories that validate a version of Conditional Non-Contradiction that is at least as strong as Weak Conditional Non-Contradiction.

[6] We use the term *doxastic accessibility* to mean compatibility with what the subject *is sure of*, not what she believes.

We assume only that R is serial: at every world the subject has consistent beliefs and so what they know is compatible with *some* world. We assume that the accessibility relation R is that of the relevant agent in the arbitrary context we interpret our language in.

Given how we understand the interpretation of our language, we can characterize The Qualitative Thesis by characterizing the following object language principle:

QT. $\neg S\neg\phi \supset (S(\phi > \psi) \equiv S(\phi \supset \psi))$

Our interpretation of the language forces us to understand QT *locally*—specifically, as saying that if the speaker of a given context c leaves open $[\![\varphi]\!]$, then she is sure of the proposition expressed by $\phi > \psi$ relative to the information in *her* context just in case she is sure of $[\![\varphi \supset \psi]\!]$.

2.2 Characterizing the Qualitative Thesis

We will now characterize QT and show that it requires a strong constraint on the logic of sureness. Consider Stalnaker's *Indicative Constraint*:

Indicative Constraint. If $R(w) \cap \mathbf{A} \neq \emptyset$, then if $w' \in R(w)$, then $f(w', \mathbf{A}) \subseteq R(w)$.[7]

This says that if $\mathbf{A}$ is compatible with what the speaker is sure of in a world w, then for any world w' that is compatible with what the speaker is sure of in w, the selected $\mathbf{A}$-worlds at w' are a subset of the worlds compatible with what the subject is sure of at w. We prove:

Fact 1. QT is valid iff the Indicative Constraint holds.

Proof. $\Leftarrow$: We split QT into the following two principles and show that both must be valid on $\mathcal{F}$, if it meets the Indicative Constraint:

$\mathbf{QT}_{\Rightarrow}$ $\neg S\neg\phi \supset (S(\phi > \psi) \supset S(\phi \supset \psi))$
$\mathbf{QT}_{\Leftarrow}$ $\neg S\neg\phi \supset (S(\phi \supset \psi) \supset S(\phi > \psi))$

First we show $\text{QT}_{\Rightarrow}$ cannot fail.. Suppose for contradiction it did. Then, for some w, $[\![\neg S\neg\phi]\!]^w = [\![S(\phi > \psi)]\!]^w = 1$ but $[\![S(\phi \supset \psi)]\!]^w = 0$. So, for some $w' \in R(w)$: $[\![\phi]\!]^{w'} = 1$ but $[\![\psi]\!]^{w'} = 0$. But, by Minimality, $w' \in f([\![\phi]\!], w')$. So $[\![\phi > \psi]\!]^{w'} = 0$ and $[\![S(\phi > \psi)]\!]^w = 0$ after all; contradiction. So $\text{QT}_{\Rightarrow}$ holds on any normal frame; and in particular it holds on any normal frame that meets the Indicative Constraint.

Now suppose that $\text{QT}_{\Leftarrow}$ fails. Then, for some w, $[\![\neg S\neg\phi]\!]^w = [\![S(\phi \supset \psi)]\!]^w = 1$ but $[\![S(\phi > \psi)]\!]^w = 0$. So, for some $w' \in R(w)$, $[\![\phi > \psi]\!]^{w'} = 0$. This means there is some w'' such that $w'' \in f([\![\phi]\!], w')$ and $w'' \notin [\![\psi]\!]$. So, by Success, $w'' \notin [\![\phi \supset \psi]\!]$. But, since $[\![S(\phi \supset \psi)]\!]^w$ it follows $R(w) \subseteq [\![\phi \supset \psi]\!]$. So $w'' \notin R(w)$; the Indicative Constraint fails.

$\Rightarrow$: Suppose that the Indicative Constraint does not hold. Then for some $\mathbf{A}$, there's some w and w' such that $R(w) \cap \mathbf{A} \neq \emptyset$, $w' \in R(w)$ but $f(\mathbf{A}, w') \nsubseteq R(w)$. So there's some $w'' \in f(\mathbf{A}, w)$ such that $w'' \notin R(w)$. But now we can build a model where QT fails. Let $V(p) = \mathbf{A}$ and $V(q) = \{w''\}$. We can see that for all $w' \in R(w)$ $[\![p \supset \neg q]\!]^{w'} = 1$, as $w'' \notin R(w)$. So $[\![S(p \supset q)]\!]^w = 1$. But $[\![p > q]\!]^{w'} = 0$, since $w'' \in f([\![p]\!], w')$. But $w' \in R(w)$, so $[\![S(p > q)]\!]^w = 0$. □

[7] Versions of the Indicative Constraint are defended by von Fintel [1998], Bacon [2015], Khoo [2019], Mandelkern and Khoo [2019] and Mandelkern [2019b].

Given Fact 1, we can show that the Qualitative Thesis has important epistemological upshots. Consider the following property on frames:

No Opposite Materials. For any two worlds w_1, w_2, if there's some w_3 such that $w_1 R w_3$ and $w_2 R w_3$, then, for any $\mathbf{A} \subseteq W$: if $R(w_1) \cap \mathbf{A} \neq \emptyset$, $R(w_2) \cap \mathbf{A} \neq \emptyset$ and $R(w_3) \cap \mathbf{A} \neq \emptyset$, then there's no $\mathbf{C} \subseteq W$ such that $R(w_1) \subseteq \mathbf{A} \supset \mathbf{C}$ and $R(w_2) \subseteq \mathbf{A} \supset \neg\mathbf{C}$.

No Opposite Materials says that for certain pairs of worlds, and certain propositions $\mathbf{A}$, you can't be sure of a material conditional $\mathbf{A} \supset \mathbf{C}$ at the first world and sure of the 'opposite' material conditional, $\mathbf{A} \supset \neg\mathbf{C}$, at the second. Which pairs of worlds? Any two worlds that see a world in common. And for which propositions? Any proposition that is consistent with what you're sure of at all three worlds.

We prove that No Opposite Materials is a consequence of QT:

Fact 2. QT is valid only if No Opposite Materials holds.

Proof. Suppose for contradiction that QT holds but No Opposite Materials does not. Then there are w_1 w_2, w_3 and $\mathbf{A}$ such that (i) $R(w_1) \cap \mathbf{A} \neq \emptyset$, $R(w_2) \cap \mathbf{A} \neq \emptyset$ and $R(w_3) \cap \mathbf{A} \neq \emptyset$ but (ii) for some $\mathbf{C}$, $R(w_1) \subseteq \mathbf{A} \supset \mathbf{C}$ and $R(w_2) \subseteq \mathbf{A} \supset \neg\mathbf{C}$. Since QT is valid on $\mathcal{F}$, $\mathcal{F}$ obeys the Indicative Constraint. This means that $f(\mathbf{A}, w_3) \subseteq R(w_1)$ and $f(\mathbf{A}, w_3) \subseteq R(w_2)$. So $f(\mathbf{A}, w_3) \subseteq \mathbf{A} \supset \mathbf{C}$ and $f(\mathbf{A}, w_3) \subseteq \mathbf{A} \supset \neg\mathbf{C}$. Given Success, this means $f(\mathbf{A}, w_3) \subseteq \mathbf{C}$ and $f(\mathbf{A}, w_3) \subseteq \neg\mathbf{C}$. But this can only happen if $f(\mathbf{A}, w_3) = \emptyset$. But this is already ruled out by Non-Vacuity. Contradiction. □

In the next section we develop a connection noted first by Ben Holguín (p.c.) and show that No Opposite Materials is inconsistent with a plausible *margin for error* requirement on rational sureness.[8] Fact 2 tells us that QT entails No Opposite Materials. It follows that QT is itself inconsistent with the margin for error requirement.

3 No Opposite Materials and Margin for Error Principles

To illustrate the margin for error requirement, we begin with a case from Timothy Williamson.[9] Mr. Magoo is staring out the window at a tree some distance off, wondering how tall it is. Assuming his only sources of information are reflection and present perception of the tree, what should he believe? That depends on how tall the tree actually is. If the tree is 100 inches tall, Mr. Magoo's visual information rules out possibilities in which the tree is 200 inches tall, or so we can imagine. So it would be reasonable for Magoo to be sure that the tree is not 200 inches tall. On the other hand, Magoo's visual information does not rule out possibilities in which the tree is 101 inches tall; his eyesight is simply nowhere near that good. It would not be reasonable for Magoo to be sure that the tree is not 101 inches tall. There's a general principle underlying these observations. Mr. Magoo's beliefs about the height of the tree are rational only if they leave a *margin for error*.[10]

[8] Holguín [Forthcoming] draws a very different moral from his argument, concluding that if you accept the margin for error principle you should reject The Qualitative Thesis. We think these can be reconciled.

[9] See Williamson [2000].

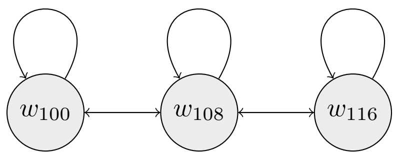
[10] Williamson introduces the margin for error principle as a requirement on knowledge, but as Hawthorne and Magidor [2009],Hawthorne and Magidor [2010] suggest, the principle is equally plausible for other attitudes. Hawthorne and Magidor focus on Stalnaker's attitude of *presupposition*, but similar considerations apply to rational sureness.

To state the margin for error requirement, we introduce a *margin for error frame* $\langle W, R\rangle$. W is a set of worlds representing possible tree heights. Where i is the height in inches of the tree in w, $W = \{w_i : i \in \mathbb{R} \text{ and } i > 0\}$. R is a binary doxastic accessibility relation on W: $w_i R w_j$ means that, in a world where the tree is i inches tall, it is compatible with everything Magoo is rationally sure of that the tree is j inches tall. R is defined as follows, relative to an arbitrarily chosen positive constant h.

Magoo's Margin. $w_i R w_j$ if and only if $|j - i| < h$.

h is Magoo's margin for error; h is positive, for otherwise his discrimination would be perfect.

No Opposite Materials fails on every margin for error frame. To see this, suppose that $h = 10$, and consider three worlds in W: w_{100}, w_{108}, and w_{116}. Here is a diagram depicting Mr. Magoo's beliefs in these three worlds.



Mr. Magoo's belief worlds at w_{116} overlap with his belief worlds at w_{100}: w_{108} is consistent with what he is sure of in w_{116} and consistent with what he is sure of in w_{100}. Moreover, it's consistent with what Magoo is sure of at each world that the tree is either 100 inches tall or 116 inches tall. This means that the antecedent of No Opposite Materials is satisfied. The right and left worlds see a world in common, w_{108}. And the proposition that the tree is either 100 inches tall or 116 inches tall is consistent with what Magoo is sure of at all three worlds. But the consequent of No Opposite Materials is not satisfied. Since Magoo's margin for error is 10, w_{100} does not see w_{116} and w_{116} does not see w_{100}. As a result, Mr. Magoo is sure of 'opposite' material conditionals at w_{100} and w_{116}. At w_{100}, Mr. Magoo is sure that (4) is true; at w_{116}, Mr. Magoo is sure that (5) is true:

(4) $(116 \vee 100) \supset 100$

(5) $(116 \vee 100) \supset 116$

This shows that No Opposite Materials fails on every margin for error frame when $h = 10$. But the choice of 10 inches for h was arbitrary. It is not hard to see that No Opposite Materials will fail on every margin for error frame, regardless of the value of h.

In models that violate No Opposite Materials, The Qualitative Thesis places inconsistent demands on the selection function. At w_{100}, Magoo is sure of (4) and so by the Qualitative Thesis it follows that he is sure of the corresponding indicative conditional. Hence, at w_{108}, the selected $(116 \vee 100)$-worlds are worlds where the tree is 100 inches tall. On the other hand, at w_{116}, Magoo is sure of (5) and so by the Qualitative Thesis it follows that he is sure of the corresponding indicative conditional. Thus, at w_{108}, the selected $(116 \vee 100)$-worlds are worlds where the tree is 116 inches tall. But the selection function cannot meet both of these demands on pain of violating Non-Vacuity. Putting the problem this way suggests a solution. Instead of just one selection function, which we use to evaluate an indicative relative to just any belief state, we have multiple selection functions, indexed to different belief states. We develop this idea in the next section, showing how it allows us to validate The Qualitative Thesis in models like Williamson's Tree.

4 The Local Shifty Account of Conditionals

Starting in the early 1970s, theorists such as such as Stalnaker [1975b], Karttunen [1974], and Heim [1992] noticed that a sentence's *local* informational environment can also influence its interpretation. Specifically, how we interpret an expression in a sentence is partly determined by the information contained in the rest of the sentence, its *local context*. This idea has been applied, in both static and dynamic frameworks, to both presupposition projection (Heim [1992] and Schlenker [2009]) and the phenomenon of epistemic contradictions (Veltman [1996], Gillies [2001], Yalcin [2007] and Mandelkern [2019a]).

We develop the idea sketched in the previous section by making the conditional's contribution sensitive to its local context. Following Schlenker [2009], we add a local context parameter to the index of the semantic evaluation function. This ensures that when a conditional occurs under an attitude verb, the conditional is evaluated relative to the local context introduced by the attitude verb. We then validate The Qualitative Thesis using a version of the Indicative Constraint. But importantly, our account is not subject to the problem of conflicting demands. That is because the selection function for the conditional is indexed to the conditional's local context. When the local context changes, the selection function does, too.

4.1 The Theory

We state our theory in a static, variably strict framework. Where κ is the conditional's local context, here's our semantic entry.

> **Local Shifty Variably Strict Semantics**. $[\![\text{if } \varphi, \text{ then } \psi]\!]^{\kappa,w} = 1$ if and only if: $\forall w' \in f_\kappa(w, [\![\phi]\!]^\kappa) : [\![\psi]\!]^{\kappa,w'} = 1$

The Local Shifty Variably Strict Semantics is similar to the Standard Variably Strict Semantics. The difference is that there is a new parameter—a local context parameter—and the selection function is indexed to that parameter. Since selection functions are indexed to local contexts, we can impose constraints on selection functions that make reference to local contexts. We propose to replace Stalnaker's Indicative Constraint with the following *Localized Indicative Constraint*:

> **Localized Indicative Constraint**. If $\mathbf{A} \cap \kappa \neq \emptyset$, then $\forall w' \in \kappa : f_\kappa(w', \mathbf{A}) \subseteq \kappa$

The Localized Indicative Constraint tells us that the selected antecedent worlds relative to a world w in the local context for the conditional must be a subset of the local context (so long as the antecedent is compatible with the local context).

With this new parameter, we restate the remaining constraints on the selection function.

> **Success.** $f_\kappa(w, \mathbf{A}) \subseteq \mathbf{A}$
>
> **Minimality**. If $w \in \mathbf{A}$, then $w \in f_\kappa(w, \mathbf{A})$.
>
> **Non-Vacuity.** If $\kappa \cap \mathbf{A} \neq \emptyset$ then $f_\kappa(w, \mathbf{A}) \neq \emptyset$.

Success says that the selected **A**-worlds are a subset of **A**. Minimality says that if w is an **A**-world, then w is one of the selected **A**-worlds at w. We assume Success and Minimality for the same reasons as the standard framework does. Non-Vacuity says that if there are some **A**-worlds in κ, then the set of selected **A**-worlds at w is not empty. This constraint guarantees a *local* version of Weak Conditional Non-Contradiction: whenever there are φ-worlds in κ, at most one of $\phi > \psi$ and $\phi > \neg\psi$ can be true at a point of evaluation $\langle \kappa, w \rangle$.

We said that selection functions are indexed to local contexts and obey the Localized Indicative Constraint. The reason this matters, of course, is that local contexts are *shiftable*. In particular, they can be shifted by attitude predicates, such as *believe*, *want*, and, our focus in this paper, *is sure that*. Following Schlenker [2009], we assume that the local context introduced by an attitude predicate like *is sure that* at a world w is the set of worlds compatible with what the subject is sure of in w. Where R is a doxastic accessibility relation representing what an arbitrary subject is sure of and $R(w)$ is the set of worlds compatible with what that subject is sure of in w:

Shifty Hintikka Semantics. $[\![S\varphi]\!]^{\kappa,w} = 1$ if and only if: $\forall w' \in R(w) : [\![\varphi]\!]^{R(w),w'} = 1$

Shifty Hintikka Semantics treats 'is sure that' as a necessity operator, just as the standard Hintikka semantics does. But now we've added a new parameter, a local context parameter, to the index. Shifty Hintikka Semantics says that attitude operators shift this parameter to $R(w)$, the set of worlds compatible with what the subject is sure of in w. This means that when we evaluate an attitude ascription like *Magoo is sure that if φ, then ψ* at a world w, we evaluate the embedded conditional relative to Magoo's belief state *at w*. As we show in the next section, this is exactly what we need to validate The Qualitative Thesis without falling prey to the problem of conflicting demands.

4.2 Local Shifty Indicatives and The Qualitative Thesis

We prove that, on the Local Shifty Variably Strict Semantics, the Localized Indicative constraint is sufficient for QT:[11]

Fact 3. If the Local Indicative Constraint holds, then QT is valid.

Proof. Suppose the QT fails. Then for some κ and w, one of two cases obtains: i) $[\![\neg S\neg\phi]\!]^{\kappa,w}= 1$, $[\![S(\phi > \psi)]\!]^{\kappa,w}= 1$ and $[\![S(\phi \supset \psi)]\!]^{\kappa,w}= 0$; or ii) $[\![\neg S\neg\phi]\!]^{\kappa,w}= 1$, $[\![S(\phi \supset \psi)]\!]^{\kappa,w}= 1$ and $[\![S(\phi > \psi)]\!]^{\kappa,w}= 0$.

Case i) is ruled out by Minimality. For suppose i) obtains. Since $[\![S(\phi > \psi)]\!]^{\kappa,w}= 1$, for all $w' \in R(w) : f_{R(w)}(w', [\![\phi]\!]^{R(w)}) \subseteq [\![\psi]\!]^{R(w)}$. Since $[\![S(\phi \supset \psi)]\!]^{\kappa,w}= 0$, there is some $w' \in R(w) : [\![\phi]\!]^{R(w),w'} = 1$ and $[\![\psi]\!]^{R(w),w'} = 0$. But by Minimality, this $w' \in f(w', [\![\phi]\!]^{R(w)})$. So $[\![\psi]\!]^{R(w),w'} = 1$ after all. Contradiction.

In case ii), the Local Indicative Constraint fails. Since $[\![\neg S\neg\phi]\!]^{\kappa,w}= 1$, there is some $w' \in R(w)$ s.t. $[\![\phi]\!]^{R(w),w'} = 1$; so the antecedent of the Local Indicative Constraint is satisfied when $\kappa = R(w)$ and $\mathbf{A} = [\![\phi]\!]^{R(w)}$. Since $[\![S(\phi \supset \psi)]\!]^{\kappa,w}= 1$, for all $w' \in R(w)$: either $[\![\phi]\!]^{R(w),w'} = 0$ or $[\![\psi]\!]^{R(w),w'} = 1$. Since $[\![S(\phi > \psi)]\!]^{\kappa,w}= 0$, there is some $w' \in R(w)$ such that $f_{R(w)}(w', [\![\phi]\!]^{R(w)}) \nsubseteq [\![\psi]\!]^{R(w)}$. Since by Success $f_{R(w)}(w', [\![\phi]\!]^{R(w)}) \subseteq [\![\phi]\!]^{R(w)}$, it cannot be that $f_{R(w)}(w', [\![\phi]\!]^{R(w)}) \subseteq R(w)$. So the Indicative Constraint fails. □

Now that we've shown that the Qualitative Thesis is valid on our theory, the last thing to do is explain why we do not fall prey to the problem of conflicting demands in models where No Opposite Materials fails. Recall that in Williamson's Tree, Magoo is sure of the material conditional (4) in w_{100} and he is sure of the material conditional (5) in w_{116}.

[11] Note that the Localized Indicative Constraint is not *necessary* for validating QT: we only need the instances where $\kappa = R(w)$ for some w. But it seems to us that, from a semantic point of view, the more general principle is the more natural one.

(4) $(100 \vee 116) \supset 100$

(5) $(100 \vee 116) \supset 116$

In the standard variably strict framework, there is no way to guarantee that The Qualitative Thesis holds at both w_{100} and w_{116} without placing conflicting demands on the selection function at the overlap world w_{108}. To secure The Qualitative Thesis w_{100}, the selected $(100 \vee 116)$-worlds at w_{108} must be a subset of $\{w_{100}\}$; otherwise $(100 \vee 116) > 100$ would be false at w_{108}, and so Magoo would not be sure of it at w_{100}. To secure The Qualitative Thesis at w_{116}, the selected $(100 \vee 116)$-worlds at w_{108} must be a subset of $\{w_{116}\}$; otherwise $(100 \vee 116) > 116$ would be false at w_{108} Magoo would not be sure of it at w_{116}. The selection function cannot meet both of these demands on pain of violating Non-Vacuity.

In the local, shifty framework, by contrast, different belief states correspond to different selection functions. When we evaluate an indicative conditional relative to Magoo's belief state at w_{116}, we use one selection function; when we evaluate a conditional relative to his belief state at w_{100}, we use a different selection function. Consider (6) and (7):

(6) $[\![\text{Magoo is sure that: } 100 \vee 116 > 100]\!]^{\kappa}$

(7) $[\![\text{Magoo is sure that: } 100 \vee 116 > 116]\!]^{\kappa}$

Where R is an accessibility relation representing Magoo's beliefs, (6) is true at w_{100} just in case (8) is true at every world in $R(w_{100})$: w_{100} and w_{108}. (7) is true at w_{116} just in case (9) is true at every world in $R(w_{116}) : w_{108}$ and w_{116}.

(8) $[\![(100 \vee 116) > 100]\!]^{R(w_{100})}$

(9) $[\![(100 \vee 116) > 116]\!]^{R(w_{116})}$

But (8) and (9) do not place incompatible demands on the selection function at the overlap world w_{108}. (8) is true at w_{108} only if the selected $(100 \vee 116)$-world at w_{108}, *relative to Magoo's belief state at* w_{100}, is w_{100}, whereas (9) is true at w_{108} only if the selected $(100 \vee 116)$-world at w_{108}, *relative to Magoo's belief state at* w_{116}, is w_{116}. These are simply different demands on different selection functions, so there is no inconsistency.

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