Strengthening Principles and Counterfactual Semantics

David Boylan and Ginger Schultheis*

Massachusetts Institute of Technology, Cambridge MA, USA
dboylan@mit.edu, vks@mit.edu

1 Introduction

There are two leading theories about the meaning of counterfactuals like (1):

(1) If David's alarm hadn't gone off this morning, he would have missed class.

Both say that (1) means, roughly, that David misses class in all of the closest worlds where his alarm doesn't go off. They disagree about how this set of worlds is determined. The *Variably Strict Analysis* (VSA) says that the domain *varies* from antecedent to antecedent. The *Strict Analysis* (SA) says it doesn't.[1] VSA and SA validate different inference patterns. For example, VSA validates *Antecedent Strengthening*, whereas SA does not. Early VSA theorists, such as Lewis (1973) and Stalnaker (1968), believed that certain apparent counterexamples to Antecedent Strengthening, which are now known as *Sobel Sequences*, refuted SA. More recently, defenders of SA have responded by enriching SA with certain *dynamic* principles governing how context evolves. They argue that Sobel sequences are not counterexamples to a *Dynamic Strict Analysis* (Dynamic SA).

But Antecedent Strengthening is just one of a family of strengthening principles. We focus on a weaker principle—*Strengthening with a Possibility*—and give a counterexample to it. The move to Dynamic SA is of no help when it comes to counterexamples to Strengthening with a Possibility. We show that these counterexamples are easily accommodated in a VSA framework, and we explain how to model our case and others like it using a Kratzerian ordering source.

2 Two Theories of Counterfactuals

Both VSA and SA assume that context supplies a comparative closeness ordering on worlds, represented by $\preceq_{c,w}$, which compares any two worlds with respect to their similarity to a world w.[2]

VSA uses a *selection function*, a contextually-determined function $f_{\preceq_c}$ that takes an antecedent A, and a world w, and returns the set of closest A-worlds to w, according to $\preceq_{c,w}$. A world w' is among the closest A-worlds to w just if there's no other A-world w'' that's closer to w than w' is. VSA's semantic entry for the counterfactual runs as follows:

VSA $[\![A \,\Box\!\!\rightarrow C]\!]^{c,w} = 1$ iff $\forall w' \in f_{\preceq_c}(A, w) : [\![C]\!]^{c,w'} = 1$.[3]

---

*The authors made equal contributions to the paper. Thanks to Justin Khoo, Matt Mandelkern, Milo Phillips-Brown, Bob Stalnaker, Kai von Fintel, and Steve Yablo for helpful discussion.

[1] For defenses of VSA, see Stalnaker (1968), Lewis (1973), Kratzer (1981a), Kratzer (1981b), Moss (2012), and Lewis (2017). For defenses of SA, see von Fintel (2001) and Gillies (2007).

[2] $\preceq_{c,w}$ is transitive, reflexive, antisymmetric, and at least weakly centered.

[3] This statement of VSA makes the limit assumption. (This is purely for ease of presentation.) It is neutral about the uniqueness assumption.

SA replaces the selection function with an accessibility relation $min_{\preceq_c}$ that takes a world w and returns the set of closest worlds to w, according to $\preceq_{c,w}$. (Unlike the selection function $f_{\preceq_c}$, $min_{\preceq_c}$ does not take the antecedent as argument.) Here's SA's semantic entry:

SA $[\![A \,\Box\!\!\rightarrow C]\!]^{c,w} = 1$ iff $\forall w' \in min_{\preceq_c}(w) \cap [\![A]\!]^c$: $[\![C]\!]^{c,w'} = 1$.

We said that VSA and SA do not validate all the same inference patterns. Here's one principle they disagree about:

(2) **Antecedent Strengthening**. $A \,\Box\!\!\rightarrow C \vDash (A \wedge B) \,\Box\!\!\rightarrow C$

SA validates Antecedent Strengthening, whereas VSA does not. It's not hard to see why. SA doesn't allow the evaluation domain to vary. If all the closest worlds where A is true are worlds where C is true, then *a fortiori*, all of the closest worlds where A *and* B are true are worlds where C is true. On the other hand, VSA allows the evaluation domain to vary from antecedent to antecedent: The closest AB-worlds need not be the closest A-worlds. So, what's true in all of the closest A-worlds may be false in some of the closest AB-worlds.

3 The State of Play

Antecedent Strengthening seems subject to counterexample. Consider this Sobel sequence:

(3) a. If I had struck the match, it would have lit.
b. But of course, if I had struck the match *and* it had been soaked in water last night, it wouldn't have lit.

Sentences (3-a) and (3-b) seem consistent. Indeed, in most ordinary match-striking scenarios, (3-a) and (3-b) are both true. But if Antecedent Strengthening is valid, (3) is not consistent. If it's true that if I'd struck the match, it would have lit, then, by Antecedent Strengthening, it follows that if I'd struck the match *and* it had been soaked in water, it would (still) have lit.

That it validates Antecedent Strengthening would seem to be a clear strike against SA. But things aren't quite so simple. As von Fintel (2001) shows, a suitably sophisticated strict conditional analysis *can* account for the sequence in (3). His strategy is to appeal to the dynamic effects of counterfactuals in conversation: Though (3-b) isn't true when (3-a) is uttered, *asserting* (3-b) changes the context so that it comes out true. More precisely: Counterfactuals presuppose that their domains contain some worlds where the antecedent is true. When that presupposition is not met, the context is minimally altered to ensure that it is. Suppose a speaker utters a counterfactual $A \,\Box\!\!\rightarrow C$. If the domain contains A-worlds, nothing changes; if it doesn't, it *expands* to include the closest A-worlds. $A \,\Box\!\!\rightarrow C$ is true in this *new* context just in case all of the A-worlds in the expanded set are C-worlds.

Let's apply von Fintel's Dynamic SA to our example. When the speaker asserts (3-a), the domain expands to include the closest worlds where she strikes the match. (3-a) is true. So in all of these worlds, the match lights. But any world where the match lights is one where the match is *dry*. So the presupposition of (3-b) isn't satisfied. When the speaker *utters* (3-b), the domain expands to include the closest worlds where she strikes the match and the match is wet. Since all of these worlds are ones where the match doesn't light, (3-b) comes out true.

Note that Antecedent Strengthening is not *classically* valid on Dynamic SA. An inference is classically valid just in case its conclusion is true whenever its premises are. Dynamic SA says that Antecedent Strengthening is merely *Strawson valid*: Whenever $A \,\Box\!\!\rightarrow C$ is true, and

$(A \wedge B) \mathrel{\Box\!\!\rightarrow} C$ is *defined*, $(A \wedge B) \mathrel{\Box\!\!\rightarrow} C$ is true, too.[4] Dynamic SA allows contexts where $A \mathrel{\Box\!\!\rightarrow} C$ is true yet $(A \wedge B) \mathrel{\Box\!\!\rightarrow} C$ is undefined. This is critical to Dynamic SA's account of Sobel sequences. It is the fact that (3-b) is undefined, rather than simply false, that forces the context to change when (3-b) is asserted so that (3-b) comes out true.

We aim to advance the debate between VSA and Dynamic SA by looking at a broader range of data. Antecedent Strengthening is the strongest of a family of strengthening principles. By Strawson-validating Antecedent Strengthening, Dynamic SA predicts that a whole host of strengthening principles are Strawson-valid. We argue that this prediction is unwelcome. We focus on one strengthening principle—*Strengthening with a Possibility*—and present a counterexample to it. Dynamic SA *classically* validates this principle, rather than (merely) Strawson-validating it. This means that Dynamic SA's dynamic resources are of no help when it comes to counterexamples to Strengthening with a Possibility.

4 Strengthening with a Possibility

We can think of a strengthening principle as a principle that allows us to move from a counterfactual $A \mathrel{\Box\!\!\rightarrow} C$, along with certain auxiliary premises, to a counterfactual with a strengthened antecedent $(A \wedge B) \mathrel{\Box\!\!\rightarrow} C$. More formally, where $n \geq 0$, we have:

(4) **Strengthening Principle**. $A \mathrel{\Box\!\!\rightarrow} C, P_1, ..., P_n \vDash (A \wedge B) \mathrel{\Box\!\!\rightarrow} C$

Antecedent Strengthening is the instance of (4) where $n = 0$. It says that *no* further premises are needed to strengthen the antecedent of a counterfactual. This makes it the strongest strengthening principle: A semantics that validates it validates *every* strengthening principle. Classical validity is monotonic: *Adding* premises never turns a valid inference into an invalid one. Similar reasoning shows that *Strawson*-validating Antecedent Strengthening Strawson-validates every other strengthening principle—Strawson-entailment is monotonic.[5]

There are weaker strengthening principles that allow us to strengthen an antecedent not with just *any* conjunct, but only those that satisfy some auxiliary premises. We're interested in Strengthening with a Possibility:[6]

---

[4] The inference from $A, P_1, ..., P_n$ to C is Strawson-valid iff for any c such that $[\![A]\!]^{c,w_c}$, $[\![P_1]\!]^{c,w_c}$, ..., $[\![P_n]\!]^{c,w_c}$ and $[\![C]\!]^{c,w_c}$ are all defined and such that $[\![A]\!]^{c,w_c}= [\![P_1]\!]^{c,w_c}= ... =[\![P_n]\!]^{c,w_c}=1$, $[\![C]\!]^{c,w_c}=1$ also.

[5] **Proof**: Suppose that $A, P_1, ..., P_n \nvDash_{Str} C$. There there must be some c such that $[\![A]\!]^{c,w_c} = [\![P_1]\!]^{c,w_c} = ... = [\![P_n]\!]^{c,w_c} = 1$ but $[\![C]\!]^{c,w_c} = 0$. But then, since c itself is a context where $[\![A]\!]^{c,w_c} = 1$ but $[\![C]\!]^{c,w_c} = 0$, we have $A \nvDash_{Str} C$. Contraposing, if $A \vDash_{Str} C$ then $A, P_1, ...P_n \vDash_{Str} C$.

[6] Here we assume that $A \mathrel{\Diamond\!\!\rightarrow} B$ is the dual of $A \mathrel{\Box\!\!\rightarrow} B$. So, according to Dynamic SA, it has the following semantics:

(i) $[\![A \mathrel{\Diamond\!\!\rightarrow} B]\!]^{c,w}=1$ iff $\exists w' \in min_{\preceq_c}(w) : [\![A]\!]^{c,w'}=1$ and $[\![B]\!]^{c,w'}=1$.

And according to VSA it has the following semantics:

(ii) $[\![A \mathrel{\Diamond\!\!\rightarrow} B]\!]^{c,w}=1$ iff $\exists w' \in f_{\preceq_c}(A, w) : [\![B]\!]^{c,w'}=1$.

Throughout we also assume that English *might*-counterfactuals have the semantics of $A \mathrel{\Diamond\!\!\rightarrow} B$. This assumption is called *Duality*. Gillies and von Fintel seem to accept Duality. Note also that Duality falls out of the widely-accepted restrictor analysis of conditionals in Kratzer (1986): on this analysis, the 'might' will only quantify over worlds that make the antecedent true and so *might*-counterfactuals will have the truth-conditions of $\mathrel{\Diamond\!\!\rightarrow}$.

That being said, Duality has been denied by some in the wider literature on counterfactuals (in particular, by various defenders of Counterfactual Excluded Middle like Stalnaker (1981) and Williams (2010)). We assume Duality merely for ease of exposition. Our central counterexample can be stated without it. See footnote 8 for further details.

(5) **Strengthening with a Possibility**. $(A \boxright C) \wedge (A \diamondright B) \vDash (A \wedge B) \boxright C$

(7) says that one can strengthen an antecedent with any proposition with which that antecedent is *counterfactually consistent*. Suppose it's true that if I'd taken modal logic next semester, I would have passed. Does that mean that I would have passed had I taken the class and the class was taught by Joe? According to Strengthening with a Possibility, that depends on whether Joe *might* have been the teacher, had I taken the class. If Joe couldn't have taught the class—say, because he was on leave—I can truly say that I would have passed even if I would have bombed a class taught by Joe. On the other hand, if Joe might have taught the class, then I can't truly say that I would have passed unless I would have passed Joe's class, too.

We said that Antecedent Strengthening is the strongest strengthening principle. So, by Strawson-validating Antecedent Strengthening, Dynamic SA Strawson-validates Strengthening with a Possibility. But we can show something stronger: By Strawson-validating Antecedent Strengthening, Dynamic SA *classically* validates Strengthening with a Possibility.[7] This is important. If Strengthening with a Possibility is classically valid, we can't appeal to the dynamic resources of Dynamic SA to account for apparent counterexamples. By the definition of classical validity, $(A \wedge B) \boxright C$ is *defined* (and true) in any context in which $A \boxright C$ and $A \diamondright B$ are true. But if $(A \wedge B) \boxright C$ is defined, then *asserting* $(A \wedge B) \boxright C$ won't change the context. The domain will not expand to make $(A \wedge B) \boxright C$ false, as we would hope; $(A \wedge B) \boxright C$ will simply come out true in the original context in which $A \boxright C$ and $A \diamondright B$ are uttered.

5 Against Dynamic SA

In this section, we present an apparent counterexample to Strengthening with a Possibility. Here it is:

> *Dice*: Alice, Billy, and Carol are playing a simple game of dice. Anyone who gets an odd number wins \$10; anyone who gets even loses \$10. Each player throws their dice. Alice gets odd; Billy gets even; and Carol gets odd.

Now consider this sequence of counterfactuals:

(6) a. If Alice and Billy had thrown the same type of number, then at least one person would still have won \$10.
b. If Alice and Billy had thrown the same type of number, then Alice, Billy and Carol could have *all* thrown the same type of number. (So they could have all won \$10.)
c. If Alice, Billy and Carol had all thrown the same type of number, then at least one person would still have won \$10.

(6-a) and (6-b) seem true, but (6-c) is dubious. (6-a) seems right because if Alice and Billy had thrown the same type of number, nothing would have changed with respect to *Carol*—she'd still have rolled odd. So someone would still have won \$10.

(6-b) seems right, too. If Alice and Billy had thrown the same type of number, either Alice or Billy would have gotten a different number from the one they actually got. But there's no reason to think it would have been Alice rather than Billy: Billy might have thrown odd, along with Alice and Carol.

[7] The proof is straightforward. Suppose that for a given context c, (i) $A \boxright C$ is true in c, and (ii) $A \diamondright B$ is true in c. It follows from (ii) and Dynamic SA that the domain in c contains worlds where A and B are both true. But that's just to say (iii) that $(A \wedge B) \boxright C$ is *defined* in c. Since Antecedent Strengthening is Strawson-valid, (i) and (iii) entail that $(A \wedge B) \boxright C$ is true in c.

But (6-c) seems wrong. There are two ways for Alice, Billy, and Carol to throw the same type of number. They could all roll odd or they could all roll even. And we can't just rule out the latter. If Alice, Billy, and Carol had thrown the same type of number, they might have all thrown even, so there might have been no winner: (6-c) is false.

Dynamic SA wrongly predicts that (6-c) follows from (6-a) and (6-b). For (6-a), (6-b), and (6-c) are respectively equivalent to:[8]

(7) a. *Alice Billy same* $\Box\!\!\rightarrow$ *someone wins* \$*10*
b. *Alice Billy same* $\Diamond\!\!\rightarrow$ (*Alice Billy same* $\wedge$ *Billy Carol same*)
c. (*Alice Billy same* $\wedge$ *Billy Carol same*) $\Box\!\!\rightarrow$ *someone wins* \$*10*

Suppose (7-a) and (7-b) are true. Since (7-b) is true, some worlds in the domain are ones where its antecedent and consequent are true—that is, where Alice, Billy, and Carol all throw the same type of number. But that's just to say that (7-c) is *defined*. Dynamic SA Strawson-validates Antecedent Strengthening. So, if (7-a) is true, and (7-c) is defined, then (7-c) must be true, too. That's wrong. (7-a) and (7-b) are true, and (7-c) is not.

6 A Way Out?

In its current form, Dynamic SA cannot account for our judgments about these sentences. Is there a way to modify Dynamic SA so that it can? We don't think so. Let us explain.

We know that someone wins just in case someone rolls odd. To predict that (6-a) is true, there must be someone who rolls odd in all domain-worlds where Alice and Billy roll the same type of number. And to predict that (6-c) is false, some domain-worlds where Alice and Billy (and Carol) roll the same type of number must be ones where everyone rolls *even*. So, the domain must expand between utterances of (6-a) and (6-c). It must start out containing no worlds where Alice, Billy, and Carol roll even, but acquire some by the time we get to (6-c). There are only two ways for this to happen. Either asserting (6-b) expands the domain, or asserting (6-c) does. We already ruled out the latter—if (6-a) and (6-b) are true, (6-c) is true, and thereby defined, so asserting (6-c) will not expand the domain. So if *anything* expands the domain, it must be asserting (6-b).

(6-b) is a *might*-counterfactual. We haven't yet said how they update the domain. One possibility is that they work just like *would*-counterfactuals do: (6-b) presupposes that the domain contains worlds Alice and Billy roll the same type of number. But this account won't help with our data. (6-a) and (6-b) have the same antecedent, so there can be no shifting that is triggered by the latter that isn't already triggered by the former. A different idea can be found in Gillies (2007). Gillies argues that $A \Diamond\!\!\rightarrow B$ presupposes that the domain contains worlds where A and B are both true.[9] For example, (6-b) presupposes that the domain contains

[8] In assuming that (6-b) is equivalent to (7-b), we assume Duality. However, as we noted, the counterexample does not ultimately rely upon it. We can state the dual of the *would*-counterfactual using wide-scope negation:

(i) a. If Alice and Billy had thrown the same type of number, then at least one person would still have won \$10.
b. It's not true that if Alice and Billy had thrown the same type of number, then Alice, Billy and Carol *wouldn't* have all thrown the same type of number.
c. If Alice, Billy and Carol had all thrown the same type of number, then at least one person would still won \$10.

We notice no difference in our judgements here.

[9] Gilies does not think this assertability condition is a genuine presupposition, even though he calls it an 'entertainability presupposition'. We take no view on how to cash out entertainability presuppositions.

worlds where Alice, Billy, and Carol all roll the same.

How might Gillies' theory help with *Dice*? Suppose that the initial context is such that, in all domain-worlds, Alice and Carol roll odd, and Billy rolls even. (6-a)'s presupposition isn't met in this context, so asserting (6-a) expands the domain, adding worlds where Alice and Billy roll the same. Suppose we include worlds where Alice and Billy roll even, but none where they roll odd. (We can't include any worlds where *Carol* rolls even, lest we render (6-c) false.) But in that case, (6-b)'s presupposition won't be met. (6-b)'s consequent is true only if Alice, Billy, and Carol roll the same. But, as we've set things up, the domain doesn't contain any worlds where they all roll the same. This means that asserting (6-b) will add worlds where Alice, Billy, and Carol all throw the same type of number. If we include worlds where they all throw even, (6-c) comes out false.

So far things are looking better for Dynamic SA. But trouble is near. If (6-b) introduces worlds where Alice, Billy, and Carol all throw even, we will indeed make (6-c) false, but there are other, less welcome consequences. Consider the sequence:

(6-b) If Alice and Billy had rolled the same type of number, Alice, Billy, and Carol might have *all* rolled the same type of number.

(8) If Alice and Billy had rolled the same type of number, Carol might not have rolled odd.

(6-b) is true, but (8) is false. Indeed, (8) is false for the same reason that (6-a) is true—there's no reason to suppose that, if Alice and Billy had rolled the same type of number, things might have changed with respect to *Carol*. She would have still rolled odd. But if (6-b) adds to the domain worlds where Alice, Billy, and Carol throw even, (8) will come out true.[10]

We don't want (6-b) to add worlds where Alice, Billy, and Carol all roll even. When we evaluate (6-b), we're still holding fixed that Carol rolls odd—that's why we judge (8) false. (We judge (6-b) true not because we think they might have all thrown even, but because we think they might have all thrown odd.) To be sure, things change by the time we get to (6-c). At that point, we are considering worlds where they all throw even—we judge (6-c) false because they might have all thrown even and lost. But it isn't (6-b) that makes those worlds relevant. It is only when we hear (6-c) that we consider worlds where Carol rolls even.

We've now seen that asserting (6-b) doesn't expand the domain, and neither does asserting (6-c). But if there's no domain expansion between (6-a) and (6-c), Dynamic SA cannot predict a false reading of (6-c).

7 Variably Strict Semantics

By its very structure, SA is committed to Strengthening with a Possibility. No assumptions about its underlying closeness relation were needed to prove this. Not so for VSA. Strengthening with a Possibility is not written into the semantics of VSA; rather, it corresponds to a certain formal constraint on the closeness ordering $\preceq_{c,w}$, *almost-connectedness*. Some of VSA's proponents, including Stalnaker (1968) and Lewis (1973), do enforce this constraint.[11] We show

[10] We can make this same point with the following *would*-counterfactual:

(i) If Alice and Billy had rolled the same type of number, Carol would still have rolled odd.

(i) is intuitively true. But if (6-b) introduces worlds where Alice, Billy, and Carol roll even, (i) will be false.

[11] In particular, both Stalnaker (1968) and Lewis (1973) say that whatever else is true about the ordering on worlds, it is total. Total orderings rule out incomparabilities of any kind, and so do not allow for failures of almost-connectedness.

that by adding a Kratzerian *ordering source* to the semantics we naturally generate an ordering without this constraint, allowing us to predict the counterexamples in a principled way.[12]

7.1 Predicting the counterexamples

Say that the closeness ordering $\preceq_w$ is *almost-connected* just in case $\forall w_1, w_2, w_3 : (w_1 \prec_w w_2 \rightarrow (w_1 \prec_w w_3) \vee (w_3 \prec_w w_2))$. If w_1 is closer to w than w_2 is, then for any third world w_3, either w_1 is closer to w than w_3 is, or w_3 is closer to w than w_2 is. Simplifying, if w_1 beats w_2, then either w_1 beats w_3, or w_3 beats w_2. Where $\preceq_w$ is a partial order, Strengthening with a Possibility is valid just in case $\preceq_w$ is almost-connected.[13]

To predict the data in *Dice*, it's not enough that the ordering simply fail to be almost-connected; it must fail to be almost-connected *in the right ways*. To predict (6-a), we need worlds where Alice and Billy get even and Carol gets odd to be closer to the actual world than worlds where they all get even. To predict (6-b), we need worlds where all three get odd to be among the closest worlds to actuality where Alice and Billy get the same of type of number. And, finally, to predict (6-c), we need worlds where they all get odd to *not* be closer to actuality than worlds where they all get even. These three jointly hold just in case worlds where they all get odd are neither closer to actuality than worlds where they all get even, nor further away from actuality than worlds where Carol gets odd but Alice and Billy get even.

How do we guarantee that context supplies an ordering with this structure? Our suggestion is to follow Kratzer (1981a) and Kratzer (1981b) and posit an extra contextual parameter—an *ordering source*, a function that takes a world w and returns a set of propositions. This set of propositions represents the facts about w that the speakers judge relevant to determining similarity. We then define our ordering in terms of those propositions. w_1 is at least as close to w as w_2 is just if it makes true all the same ordering source propositions as w_2, and possibly more. Formally:

(9) $w_1 \preceq_w w_2$ iff $\{p \in g(w) : w_1 \in p\} \supseteq \{p \in g(w) : w_2 \in p\}$

w_1 is at least as close to w as w_2 is just in case every proposition in $g(w)$ that is true in w_2 is also true in w_1. w_1 is strictly closer to w than w_2 is just in case, every proposition in $g(w)$

[12] Kratzer also adds to her semantics a modal base which is shifted by the antecedent. We omit this in what follows for ease of exposition, and make the simplifying assumption that it is true in all worlds that one wins the game if and only if one rolls odd.

We also depart from Kratzer with respect to which facts we think the ordering source holds fixed. For Kratzer, the ordering source is totally realistic: the intersection of $g(w)$ is just $\{w\}$. We do not make this assumption; instead, we only include the facts that are relevant, in the sense spelled out in 7.2. Were we to spell out the semantics in full, we would say that all other relevant details about the case, such as the rules of the game, go in the modal base, rather than in the ordering source.

[13] **Proof**: $\Rightarrow$: Our model that follows demonstrates that if Strengthening with a Possibility is valid, then $\preceq$ is almost-connected. If a frame is not almost-connected, then we can build a model on it like the one in the text.

$\Leftarrow$: Suppose that $\preceq$ is almost-connected and suppose that, for contradiction, that Strengthening with a Possibility is not valid. Then there is some world w_1 such that $A \mathrel{\Box\!\!\rightarrow} C$ and $A \mathrel{\Diamond\!\!\rightarrow} B$ are true there but $A \wedge B \mathrel{\Box\!\!\rightarrow} C$ is not. This means that $f(A, w_1) \subseteq C$, there is a world $w_2 \in f(A, w_1)$ such that B is true at w_2 and there is a world $w_3 \in f(A \wedge B, w)$ such that $\neg C$ is true there. w_3 cannot be in $f(A, w)$: unlike w_3 all worlds in $f(A, w)$ are C worlds. By definition of f, this means that there must be some world w_4 in $f(A, w)$ such that $w_4 \prec_{w_1} w_3$.

Now consider whether either $w_4 \prec_{w_1} w_2$ or $w_2 \prec_{w_1} w_3$. In fact, the first disjunct cannot hold: by definition of f, if it did then w_2 would not be in $f(A, w_1)$ after all. But the second disjunct cannot be true either. Again by definition of f, if it were then w_3 would not be in $f(A \wedge B, w_1)$. But now we have proved that, contrary to our supposition that $\preceq$ is not almost connected: $w_4 \prec_{w_1} w_3$ but neither $w_4 \prec_{w_1} w_2$ nor $w_2 \prec_{w_1} w_3$. So if $\preceq$ is almost-connected Strengthening with a Possibility must be valid. (To the best of our knowledge, this result was first shown by Veltman (1985).)

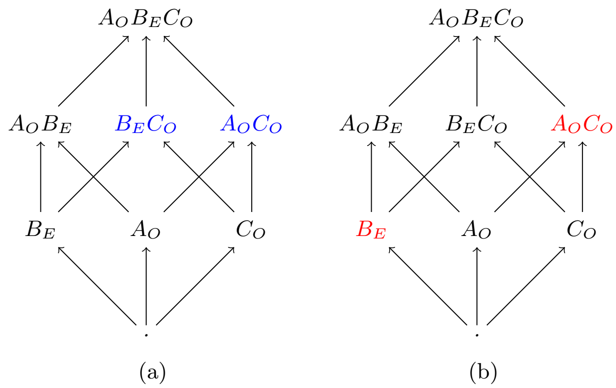

Figure 1

that's true in w_2 is true in w_1, and some proposition in $g(w)$ that's true in w_1 is false in w_2.

In our example, the relevant facts are those that concern who got what type of number. We assume, then, that the ordering source is that in (10):

(10) $g(w) = \{$*Alice gets odd*, *Billy gets even*, *Carol gets odd*$\}$

Let A_O, B_E, and C_O be the propositions that Alice rolls odd, Billy rolls even, and Carol rolls odd, respectively. The ordering source in (10) gives rise to the ordering in Figure 1. In the top-ranked worlds, things are just as they actually are—Alice and Carol roll odd, and Billy rolls even. Next we have worlds where things differ in *one* respect—worlds where either Alice or Carol rolls even instead of odd, or Billy rolls odd instead of even. Then we have worlds differing in *two* respects, and finally, worlds where *everything* is different—Alice and Carol roll even, and Billy rolls odd.

Let's see how VSA predicts the right judgments in *Dice* using this ordering. The closest worlds where Alice and Billy throw the same type of number are in blue in Figure 1(a). In both worlds, Carol rolls odd and wins \$10, so (6-a) is true: If Alice and Billy had rolled the same, one person would still have won \$10. Moreover, in one of the closest worlds where Alice and Billy roll the same, Alice, Carol, and Billy all roll odd. So (6-b) is also true: If Alice and Billy had rolled the same, Alice, Billy, and Carol might have all rolled the same.

Finally, turn to (6-c), which says that if Alice, Billy, and Carol had rolled the same, someone would still have won \$10. We find the worlds where Alice, Billy, and Carol roll the same type of number. They are highlighted in red in Figure 1(b). We have worlds where Alice, Billy, and Carol all throw odd (top right) and worlds where they all throw even (bottom left). These are incomparable—neither is closer to actuality than the other is. (The reason they are incomparable is that the sets of ordering source propositions true at each are disjoint.) Both are among the closest worlds where Alice, Billy, and Carol roll the same. So (6-c) is false: In some of the closest worlds where they throw the same, they throw even, and nobody wins.

7.2 Other Cases

We've argued that Strengthening with a Possibility has counterexamples, and we've offered a Kratzerian premise semantics that doesn't validate it. Still, the inference often *seems* valid. Suppose I say, with confidence, that if I had taken modal logic last semester, I would have passed. You reply that if I had taken the course, it might have been taught by Joe, who's notorious for his difficult problem sets and harsh grading. If I accept your response, I seem to have two options. I could stand firm, insisting that I would have passed even Joe's challenging course, or I could retreat, rescinding my earlier claim that I would have passed the class. What I *can't* do is maintain that I would have passed the course, even though I might not have passed a course taught by Joe. That I don't have this option is only explained if Strengthening with a Possibility does not fail in this particular case. We must place certain constraints on when the inference can fail. We want it to fail in *Dice*, but not here.

Our idea is to place a constraint on how our ordering sources relate to salient questions in context. Say that Q_c is the most refined salient (non-counterfactual) question in c. Now let $g^{\neg}$ be the following function: $g^{\neg}(w) = \{p : \neg p \in g(w)\}$; that is, $g^{\neg}(w)$ contains all and only the negations of propositions in $g(w)$. Finally consider all the sets of maximal consistent propositions built out of $g(w) \cup g^{\neg}(w)$; call it G_w. We propose the following constraint on ordering sources: whatever g_c is, it must be the case that $G_{w_c} = Q_c$. This constraint tells us that the ordering source cannot distinguish between worlds in ways that are not already present in the most refined salient question.

With this constraint in hand, we can prove that we get failures of Strengthening with a Possibility only if there are two distinct answers to Q_c that realise the antecedent of the final strengthened conditional.

> There are A, B, C such that $[\![A \;\Box\!\!\rightarrow C]\!]^{c,w,g} = 1$, $[\![A \;\Diamond\!\!\rightarrow B]\!]^{c,w,g} = 1$ and $[\![A \wedge B \;\Box\!\!\rightarrow C]\!]^{c,w,g} = 0$ only if $\exists p, q \in Q_c : p \vDash A \wedge B$ and $q \vDash A \wedge B$ and $p \neq q$.[14]

Put informally, the reason for this is as follows: if there is only one partition cell, call it p, that entails $A \wedge B$, then either p is a subset of the closest A-worlds or not. If it is, then, if $A \;\Box\!\!\rightarrow C$ is true, all the closest $A \wedge B$-worlds will have to be C-worlds. If it isn't, then, since all worlds in p are equally close, no B-worlds will be among the closest A-worlds and so $A \;\Diamond\!\!\rightarrow B$ will be false.

To see how this helps, let us return to the case of Joe. Here, quite plausibly the most salient question is *Did I take logic? And did Joe teach?*, which gives us the following partition:

> {*I take logic and Joe teaches*, *I take logic and Joe doesn't teach*,
> *I don't take logic and Joe teaches*, *I don't take logic and Joe doesn't teach*}

There is only one cell of the partition which makes true our strengthened antecedent, namely, *I take logic and Joe teaches*. Given our result from above, we can see that the relevant instance of Strengthening with a Possibility will go through.

Here we see yet another advantage of our premise semantics. Not only does it offer an account of when Strengthening with a Possibility fails, it also offers an explanation of why it

[14]**Proof.** Suppose that $A \;\Box\!\!\rightarrow C$, $A \;\Diamond\!\!\rightarrow B$, but $A \wedge B \;\Box\!\!\rightarrow C$ is false. For contradiction, suppose there's just one cell that makes $A \wedge B$ true. Call it Q. We appeal to three facts:

1. All worlds in a partition cell are equally good. This is because they all make the same ordering source propositions true.
2. $Q = Q \cap A = Q \cap (A \wedge B)$ This is because Q already contains only $A \wedge B$ worlds.
3. $Q \cap (A \wedge B) = f(A \wedge B, w)$ This follows from the definition of f plus the fact that Q is the only $A \wedge B$ cell.

Either $Q \cap A \subseteq f(A, w)$ or it isn't. Suppose it is. Then, by facts 2 and 3, $f(A \wedge B, w) \subseteq f(A, w)$. And since $f(A, w) \subseteq C$, $A \wedge B \;\Box\!\!\rightarrow C$ is true, contrary to our supposition. Suppose it isn't. Then $f(A, w)$ contains *no* B-worlds: Q contains the only $A \wedge B$ worlds and, by fact 1, they are all equally good. So $A \;\Diamond\!\!\rightarrow B$ is false, contrary to our supposition.

often seems to go through. In cases like ours where Strengthening with a Possibility fails, we are interested in different ways in which the antecedent could be true. But in normal, simple cases, we do not make such fine distinctions and so the inference seems valid.

8 Conclusion

We suggested that the debate between SA and VSA could be clarified by looking at a wider range of strengthening principles. This suggestion has been borne out. Dynamic SA validates Strengthening with a Possibility. But this inference is not valid. Counterexamples to Strengthening with a Possibility pose a much more serious problem for Dynamic SA than counterexamples to Antecedent Strengthening itself. While Antecedent Strengthening is merely Strawson-valid, Strengthening with a Possibility is *classically* valid. Counterexamples to it do not involve presupposition failure, so the dynamic principles that drive context change do not apply. But if that's right, Dynamic SA has no way to account for counterexamples to Strengthening with a Possibility. VSA, on the other hand, can easily model failures of Strengthening with a Possibility. We conclude that the failure of Strengthening with a Possibility tells strongly against Dynamic SA and in favor of an ordering source-based version of VSA.

References

Anthony Gillies. Counterfactual scorekeeping. *Linguistics and Philosophy*, 30(3):329–360, 2007.

Angelika Kratzer. The notional category of modality. *Words, worlds, and contexts*, pages 38–74, 1981a.

Angelika Kratzer. Partition and revision: The semantics of counterfactuals. *Journal of Philosophical Logic*, 10(2):201–216, 1981b.

Angelika Kratzer. Conditionals. *Chicago Linguistics Society*, 22(2):1–15, 1986.

David Lewis. *Counterfactuals.* Blackwell, 1973.

Karen Lewis. Counterfactual discourse in context. *Noûs*, 50(4), 2017.

Sarah Moss. On the pragmatics of counterfactuals. *Noûs*, 46(3):561–586, 2012.

Robert Stalnaker. A theory of conditionals. *Americal Philosophical Quarterly*, pages 98–112, 1968.

Robert Stalnaker. A defense of conditional excluded middle. In William Harper, Robert C. Stalnaker, and Glenn Pearce, editors, *Ifs*, pages 87–104. Reidel, 1981.

Frank Veltman. *Logics for Conditionals.* PhD thesis, University of Amsterdam, 1985.

Jonathan Vogel. Are there counterexamples to the closure principle? In Michael David Roth and Glenn Ross, editors, *Doubting: Contemporary Perspectives on Skepticism*, pages 13–29. Dordrecht: Kluwer Academic Publishers, 1990.

Kai von Fintel. Counterfactuals in a dynamic context. *Current Studies in Linguistics Series*, 36:123–152, 2001.

J. Robert G. Williams. Defending conditional excluded middle. *Noûs*, 44(4):650–668, 2010.