# Inverse Linking: Taking Scope with Dependent Types

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#### Abstract

Inverse linking constructions (ILCs) refer to complex DPs which contain a quantified NP (QP) which is selected by a preposition (e.g. a representative of every country). ILCs have been known to be ambiguous between a surface-scope reading and an inverse-scope reading. One puzzling difficulty for the existing accounts of ILCs is that some prepositions (e.g. with) block inverse-scope interpretations. In this paper, we propose a new dependent type analysis of the two readings of ILCs. In our analysis, we follow Zimmermann (2002) and assume that ILCs are structurally ambiguous at surface structure: the two readings of ILCs derive from the two (string identical) surface structures. The advantage of our dependent type account over Zimmermann's analysis is that it interprets the surface structure for the inverse-scope reading in a fully compositional way. Other compositional non-movement accounts of ILCs have been proposed; however, our dependent type account is the first to offer a principled solution to the puzzle of why inverse-scope readings are blocked with certain prepositions.

### 1 Introduction

ILCs refer to complex DPs which contain a quantified NP (QP) which is selected by a preposition, as illustrated in (1):

(1) A representative of every country is bald.

ILCs like (1) are considered to be ambiguous between a surface-scope reading and an inverse-scope reading.<sup>1</sup> On the surface-scope reading, (1) is understood to mean that there is some one person who represents every country and who is bald. On the inverse-scope reading, (1) is understood to mean that a different representative of each country is bald in each case. Puzzlingly, some prepositions (e.g. with) resist inverse-scope interpretations, as illustrated in (2).

(2) Someone with every known skeleton key opened this door.

Sentence (2) can only mean that there is some one person who happens to have every known skeleton key and who opened the door (for the discussion of the puzzle, see [27]). In this paper, we propose a new dependent type analysis of the two readings of ILCs.<sup>2</sup> Standard analyses of ILCs involve LF-movement [25, 26, 18, 16]. A more recent account is Zimmermann's surface-structure analysis [33]. In our analysis, we follow Zimmermann and assume that ILCs are

<sup>&</sup>lt;sup>1</sup>In May's [25], ILCs were thought to allow inverse-scope readings only but this has changed since May's [26], and both surface-scope and inverse-scope readings are now considered to be available for ILCs (see e.g., [14, 15, 5, 32]).

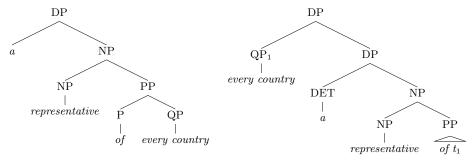
<sup>&</sup>lt;sup>2</sup>Another characteristic property of ILCs is that, on the inverse-scope reading, the DP-internal QP (every country) can bind a pronoun outside the ILC, as in: A representative of every country despises it. The discussion of this property goes beyond the scope of this paper (but see [12] for our dependent type analysis of bound readings)

structurally ambiguous at surface structure: the two readings of ILCs derive from the two (string identical) surface structures. But whereas the surface structure for the inverse-scope reading has been interpreted using a non-fully compositional procedure in [33], we will show that it can be interpreted in a fully compositional way in our semantic framework with dependent types [10, 11]. Other compositional non-movement accounts of ILCs have been proposed: Hendriks's type-shifting approach [13] and Barker and Shan's continuation-based strategies [2, 3]. However, to the best of our knowledge, our dependent type account is the first to offer a principled solution to the puzzle of why inverse-scope readings are blocked with certain prepositions.

The structure of the paper is as follows. In section 2, we review Zimmermann's surface-structure analysis. Section 3 introduces the main features of our semantic framework with dependent types. In 4, we present a new dependent type analysis of the two reading of ILCs and show that the analysis proposed can account for the preposition puzzle, before concluding in 5.

### 2 ILCs and Surface-Structure Ambiguity

In [33], Zimmermann argues that the two readings of ILCs derive from the two different surface structures (SS's) and that there is syntactic evidence for distinguishing the two SS's. Standard LF-based approaches assume that the prepositional phrase (PP) in ILCs has the syntactic status of a regular postnominal modifier, i.e., the PP (of every country) stands in the sister position to the head noun (representative). The inverse-scope reading of ILCs is attributed to the application of quantifier raising (QR). QR replaces the QP every country with the coindexed trace  $(t_1)$ , and adjoins it at DP [26, 18, 1]:<sup>3</sup>



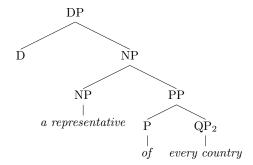
The central observation that drives Zimmermann's surface-structure analysis is that PPs containing QPs that give rise to inverse-scope readings cannot freely change places with regular postnominal modifiers (e.g. relative clauses (RCs)) but must be DP-final, as illustrated by the examples below:

- (3) One person [RC who was famous] [PP from every city] was invited.
- (4) # One person [PP from every city] [RC who was famous] was invited.

Sentence (3) can be understood to mean that every city x is such that one famous person from x was invited, while sentence (4) is semantically odd — it only allows a surface-scope reading saying that one person who came from every city and who was famous was invited. Inverse-scope readings are possible when PPs follow RCs (as in (3)), while non-final PPs give

<sup>&</sup>lt;sup>3</sup>ILCs have been also analyzed as involving adjunction of the QP at S (see e.g., [25, 9]).

rise to surface-scope readings only (as in (4)). This asymmetry is unexpected on the LF-based analysis since all postnominal modifiers (PPs, RCs) have the same syntactic status. Based on this argument, Zimmermann proposes a different structure for the inverse-scope reading, where the PP (of every country) is not a regular postnominal modifier but is right-adjoined to the whole indefinite expression a representative (for this analysis, the adjectival theory of indefinite expressions is assumed, see e.g. [17]):



On the semantic side, the PP is reinterpreted as a generalized quantifier, as in the formula below:

$$\|of\ every\ country\| = \lambda P. \forall z[z \in country(z) \rightarrow \exists X[P(X) \land of(X,z)]],$$

where P stands for the property of pluralities denoted by the indefinite expression. Under this analysis, the observed problematic asymmetry can be readily explained. In the presence of regular postnominal modifiers (e.g. RCs), the PP that gives rise to the inverse-scope reading must be DP-final because its adjunction blocks the adjunction of additional regular postnominal modifiers. If the PP is followed by regular postnominal modifiers, it must be interpreted as a regular postnominal modifier itself and can give rise to the surface-scope reading only. The main problem for this account, as pointed out by Zimmermann himself, is that the reinterpretation of the PP is not a compositional semantic operation, i.e., it does not come as a result of the composition of the meanings of the constituent parts.

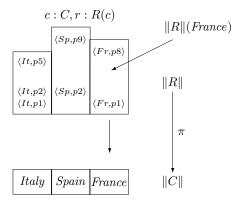
In our analysis, we adopt Zimmermann's position that ILCs are structurally ambiguous at surface structure (the two readings derive from the two surface structures distinguished by the syntactic position of the PP). The advantage of our dependent type account over Zimmermann's analysis is that it interprets the surface structure for the inverse-scope reading in a fully compositional way.

## 3 Semantics with Dependent Types

At the heart of our analysis is a dependent type theoretical framework [23, 24, 22, 6]. Whereas previous approaches adopted within the dependent type theoretical framework have been either proof-theoretic [31, 21, 4] or involved a combination of proof-theoretic and model-theoretic elements [8, 7], our semantics is model-theoretic with truth and reference being basic concepts (and no proofs). In this section, we introduce the main features of our semantic framework with dependent types [10, 11]. In 3.1, we introduce the concept of dependent types. In 3.2, it is shown how QPs and predicates are to be interpreted in this framework. In 3.3, we present a novel dependent type account of relational nouns.

#### 3.1 Dependent Types

The two key features of our semantic framework are: many-sorted (many-typed) analysis and type dependency. Our analysis is many-sorted in the sense that it includes many basic types (e.g., type M(an), type C(ountry), ...). The variables of our semantic system are always typed. We write c: C to denote that the variable c is of type c. Types are interpreted as sets. We write the interpretation of the type c as ||c||. In our system, types can depend on the variables of other types, e.g., if c is a variable of the type of countries c, there is a type c0 of the representatives of that country



If we interpret type C as the set ||C|| of countries, then we can interpret R as the set of the representatives of the countries in ||C||, i.e. as the set of pairs:

$$||R|| = \{\langle a, p \rangle : p \text{ is the person from the country } a\}$$

equipped with the projection  $\pi: ||R|| \to ||C||$ . The particular sets ||R||(a) of the representatives of the country a can be recovered as the fibers of this projection (the preimages of  $\{a\}$  under  $\pi$ ):

$$||R||(a) = \{r \in ||R|| : \pi(r) = a\}.$$

Our semantic system makes no use of assignment functions; variables serve to determine dependencies and act as an auxiliary syntactic tool to determine how the operations combining interpretations of QPs and predicates are to be applied.

### 3.2 QPs and Predicates

QPs and predicates are interpreted relative to the context, where context is understood typetheoretically as a sequence of type specifications of the (individual) variables:

$$x : X, y : Y(x), z : Z(x, y), \dots$$

Here, type Z depends on the variables x and y of types X and Y, respectively; type Y, on the variable x of type X; and type X is a constant type, i.e., it does not depend on any variables (for the formal definition of our type-theoretic notion of context, see [11]).

We assume a polymorphic interpretation of quantifiers. A generalized quantifier associates to every set Z a subset of the power set of Z:

$$||Q||(Z) \subseteq \mathcal{P}(Z).$$

Whereas in the standard Montagovian setting QPs are interpreted over the universe of all entities E [28], in our dependent type theoretical framework they are interpreted over types. For example, some man is interpreted over the type Man (given in the context), i.e., some man denotes the set of all non-empty subsets of the set of men:

$$\|\exists\|(\|Man\|) = \{X \subseteq \|Man\| : X \neq \emptyset\}.$$

In our semantic framework, quantification is also allowed over fibers, e.g., we can quantify existentially over the fiber of the representatives of France, as in *some representative of France*:

$$\|\exists \|(\|R\|(France)) = \{X \subseteq \|R\|(France) : X \neq \emptyset\}.$$

As a consequence of our many-sorted (many-typed) analysis, we also have a polymorphic interpretation of predicates. A predicate like love is interpreted over types (given in the context, e.g.  $Man, Book, \ldots$ ), and not over the universe of all entities.

#### 3.3 Relational Nouns

Nouns can be classified into three kinds: sortal (e.g., man, country, book), relational (e.g., representative, part, attribute) and functional (e.g., mother, head, age) (see. e.g., [19, 20]). In this paper, we only consider sortal and relational nouns. Whereas in the Montagovian setting sortal nouns are interpreted as one-place relations (expressions of type  $\langle e, t \rangle$ ), in our dependent type theoretical framework they are treated as types. For example, man is interpreted as the type M/set of men. Nouns modified by regular postnowinal modifiers are also interpreted as types. For example, man who represents every city is interpreted as the type/set of men who represent all the cities. In the Montagovian setting relational nouns (e.g. representative) are interpreted as two-place relations (expressions of type  $\langle e, \langle e, t \rangle \rangle$ ). Our framework allows us to treat them as dependent types, e.g., representative (as in a representative of a country) is interpreted as the dependent type c: C, r: R(c)/for any element a in the set of countries  $\|C\|$ , there is a set (fiber)  $\|R\|(a)$  of the representatives of that country. Our analysis can be further extended to nested relational nouns, e.g., a formulation of a solution of a problem can be interpreted as the nested dependency p: P, s: S(p), f: F(p, s).

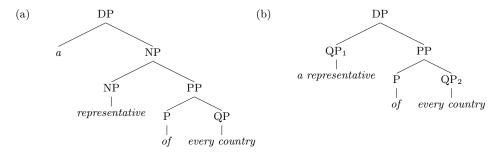
As discussed in Partee and Borschev's [29], the boundary between sortal and relational nouns is pervasively permeable. Sortal nouns can undergo 'sortal-to-relational' shifts, as in uses of sortal nouns with an overt argument (e.g., book expresses a sortal concept but its relational use can be coerced in books of ...). 'Relational-to-sortal' shifts are also possible, as in uses of relational nouns without an overt argument. Some nouns can even have both meanings, e.g. child can be understood to mean either 'direct descendant of' (relational reading) or 'nonadult' (sortal reading). For more discussion on sortal and relational nouns, see [19, 20, 29, 30]. The possibility of both 'sortal-to-relational' and 'relational-to-sortal' shifts has an important role to play in our account of ILCs.

### 4 Dependent Type Analysis of ILCs

This section puts forth our new dependent type account of ILCs. In 4.1, we show how our account produces the two readings for ILCs. In 4.2, we provide the details of our compositional analysis of the inverse reading of ILCs. In 4.3, it is shown that the analysis proposed can account for the preposition puzzle.

### 4.1 The Two Readings of ILCs

In our analysis, we follow Zimmermann in assuming that the two readings of ILCs derive from the two surface structures in (a) and (b) (we differ from Zimmermann in taking indefinites to be quantificational expressions but our proposal also can be made to work on the adjectival theory of indefinites):



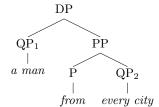
Under the perspective of our semantic framework with dependent types, the two structures in (a) and (b) (independently postulated and motivated by Zimmermann in [33]) come readily to mind — in (a) we quantify existentially over the single (complex) type/set of individuals who represent all the countries, while in (b) we quantify over two types, where the type *Representative* depends on (the variables of) the type *Country*.

More specifically, on the surface reading ((a)-structure), the PP stands in the sister position to the head nominal representative. Our proposal is that the relational noun representative undergoes a 'relational-to-sortal' shift when used with a regular postnominal modifier. The complex NP (noun modified by the postnominal PP) representative of every country is then interpreted as the type/set of individuals who represent all the countries and the DET a quantifies existentially over this set, yielding the surface ordering of quantifiers. On the inverse reading ((b)-structure), the PP is right-adjoined to the QP consisting of the head nominal (a representative). Crucially, the head nominal representative is now interpreted as the dependent type c: C, r: R(c); the preposition of signals that country is a type on which representative depends; country is interpreted as the type C. By quantifying over c: C, r: R(c), we get the inverse ordering of quantifiers (quantification over fibers is treated on a par with quantification over sets interpreting any other types):

$$\forall_{c:C} \exists_{r:R(c)}$$
.

By making the type of representatives dependent on (the variables of) the type of countries our analysis forces the inversely linked reading without positing any extra scope mechanisms or arbitrary reinterpretation procedures.

One apparent problem for our proposal is that inverse-scope readings are also available for ILCs involving sortal nouns, as in *a man from every city*. Our solution to this problem is that 'sortal-to-relational' shifts are also possible and result in (b)-structures:



The relational use of sortal nouns (e.g. man) perhaps can be coerced by the implausibility of surface-scope readings with prepositions such as from, in, on. As discussed in [33], such prepositions specify the local position or origin of an entity and since entities do not occur at more than one place simultaneously, surface-scope readings of ILCs with these prepositions become implausible. Our analysis of the above structure is then exactly like the one just described for the inverse reading of a representative of every country.

#### 4.2 The Compositional Analysis of the Inverse Reading of ILCs

This section explains the details of the compositional semantic analysis of the inverse-scope reading of ILCs on the example of sentence (1):

(1) A representative of every country is bald.

The complex DP a representative of every country is interpreted as the complex quantifier living on the set of all representatives. The interpretation of the structure:

gives us access to the sets (fibers) ||R||(a) of the representatives of the particular country a only. To form the set of all representatives, we need to use type constructor  $\Sigma$  which takes the sum of fibers of representatives over countries in ||C||. Thus the complex DP a representative of every country is interpreted as the complex quantifier living on  $||\Sigma_{c:C}R(c)||$ :

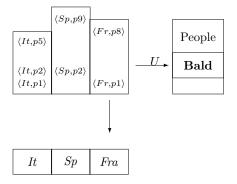
$$\|\forall_{c:C}\exists_{r:R(c)}\| = \{X \subseteq \|\Sigma_{c:C}R(c)\| : \{a \in \|C\| : \{b \in \|R\|(a) : b \in X\} \in \|\exists\|(\|R\|(a))\} \in \|\forall\|(\|C\|)\},$$

i.e., it denotes the set of the subsets of the set of representatives such that the set of countries such that each country has at least one representative in the corresponding fiber of representatives is the set of all countries.

Sentence (1) is true if and only if the set of bald representatives is in the denotation of the complex DP a representative of every country:

$$\|\forall_{c:C}\exists_{r:R(c)}Bald(r)\| = 1 \text{ iff } U^{-1}(\|Bald\|) \in \|\forall_{c:C}\exists_{r:R(c)}\|.$$

The illustration below serves to provide an intuitive understanding of the formula:



Note that a person counts as a representative only in virtue of standing in a particular relationship with some country. Now each representative has an underlying person, in fact two representatives can have the same underlying person (if this person represents two countries).

U stands for a function that forgets this part of the structure that relates people to countries and yields just the set of people. Predicate Bald is defined over the type/set of people — by taking the preimage of the set of bald people under function U,  $U^{-1}(\|Bald\|)$ , we get the set of bald representatives. For sentence (1) to be true, the set of bald representatives must be in the denotation of the complex quantifier expression a representative of every country.

#### 4.3 Preposition Puzzle

ILCs are DPs which contain a QP which is selected by a preposition. In English this includes, among others, of (connector to a relational noun) and locative prepositions such as at, from, in or on. As discussed in May and Bale's [27], one puzzling difficulty for the existing accounts of ILCs is that some prepositions like with block inverse-scope readings. To illustrate the point, May and Bale give the example in (2):

(2) Someone with every known skeleton key opened this door.

Sentence (2) can only mean that one individual who happens to have every known skeleton key opened the door. Our solution to the preposition puzzle is that inverse-scope readings are unavailable for ILCs with prepositions which induce dependencies corresponding to the surface ordering of the QPs.

Note that in the case of prepositions like of, from, in, as in a representative of (from, in) every country, the 'dependent component' (representative) comes before the component on which it is dependent (country). Thus the dependency introduced c:C,r:R(c) forces the inverse ordering of the QPs  $\forall_{c:C} \exists_{r:R(c)}$ . By contrast, in the case of prepositions like with, as in a man with every key, the 'dependent component' (key) comes after the component on which it is dependent (man). Thus the only possible dependency to be introduced is m:M,k:K(m), one that corresponds to the surface ordering of the QPs  $\exists_{m:M} \forall_{k:K(m)}$ . Hence, under our analysis, the inverse-scope interpretation is unavailable to the QP in the object position of with.

As pointed out by an anonymous reviewer, examples like a problem with every account may present a difficulty for our proposal. Note, however, that with comes with a number of meanings, including: 'having or possessing (something)' and 'accompanied by; accompanying'. If the relation expressed is one of possession, as in our example (2), then the thing possessed depends on the possessor (as described above). If, however, the relation is that of accompanying, then the accompanying entity (problem) depends on the entity to be accompanied (account). Thus the dependency introduced is a:A,p:P(a), forcing the inverse ordering of the QPs. Hence, under our analysis, the inverse-scope reading is available for the problematic example in question and, in general, for ILCs with the preposition with taken in the sense 'accompanying' (in line with intuitions reported by native speakers; consider also a slight variant of the reviewer's example found on the Internet: I had a problem with every game I played, from crashing to stupid errors).

### 5 Conclusion

In this paper, we provided a new dependent type analysis of ILCs. The advantage of our dependent type account over previous analyses (both LF-based approaches and Zimmermann's surface structure analysis) is that it is directly compositional. Using our semantic framework with dependent types, we can interpret the surface structure for the inverse-scope reading directly and in a fully compositional way. Furthermore, to the best of our knowledge, our dependent type account is the first to provide a principled solution to the preposition puzzle. Our proposal also

makes clear empirical predictions. Under Zimmermann's account, inverse-scope readings are expected to be a marked option (always a last resort), triggered by the implausibility of surface-scope readings with locative prepositions which leads to the reinterpretation procedure ([33]). Under our dependent type account, by contrast, inverse-scope readings should be a preferred option for relational nouns. In our future work, we plan to test empirically the implications of our proposal.

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