On question exhaustivity and NPI licensing*

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Abstract

Guerzoni and Sharvit (2007) discovered that the licensing of weak negative polarity items (NPIs) by embedded questions requires a (strongly) exhaustive interpretation in the sense of Groenendijk and Stokhof (1982). This paper points out that under the view that wh-questions themselves are ambiguous between exhaustive and non-exhaustive meanings (George 2011, Guerzoni and Sharvit 2014, Nicolae 2015, Theiler et al. 2016), Guerzoni and Sharvit's observation falls out from an analysis of NPI licensing by questions developed in Krifka (1995) and van Rooy (2003), an analysis that construes questions' strength as their information theoretic entropy (Shannon 1948). Restrictions on NPIs in disjunctive questions (Schwarz in press) and singular *which*-questions are presented as further support for the account.

1 Introduction

Guerzoni and Sharvit (2007) propose that the licensing of weak Negative Polarity Items (NPIs) like *any* or *ever* by embedded questions requires (strong) exhaustivity in the sense of Groenendijk and Stokhof (1982). That is, they propose that an embedded question licenses an NPI only if the semantics of the embedding predicate makes reference to (strongly) exhaustive answers. As evidence for this claim, call it the *exhaustivity-licensing generalization*, Guerzoni and Sharvit point to the pattern illustrated in (1) and (2), which suggests that wh-questions function as NPI licensors when embedded under *wonder* or *know*, but not *surprise*.

(1) Dan wonders who said anything.

(2) a. Dan knows who said anything.
b. *It surprised Dan who said anything.

The exhaustivity-licensing generalization is supported by the contrast in (2) in virtue of truth conditional evidence discovered by Heim (1994) indicating that *know* but not *surprise* makes reference to the negations of so-called Hamblin answers, hence that *know* but not *surprise* makes reference to exhaustive answers to the embedded question. Fleshing out their case for the exhaustivity-licensing generalization, Guerzoni and Sharvit argue that *wonder*, like *know* and unlike *surprise*, makes reference to exhaustive answers.

Under one current approach to (non-)exhaustivity in the interpretation of embedded questions, it is wh-questions themselves that are semantically ambiguous between exhaustive and non-exhaustive interpretations (George 2011, Guerzoni and Sharvit 2014, Nicolae 2015, Theiler et al. 2016). On this view, a question embedding structure is intuited to make reference to (non-)exhaustive answers in virtue of the embedding predicate selecting for the (non-)exhaustive

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reading of the question. The exhaustivity-licensing generalization then holds in virtue of questions being able to license NPIs only if they are exhaustive.

The task is then to account for the latter condition, hereafter the *exhaustivity-licensing condition*. The main objective is to demonstrate that the exhaustivity-licensing condition can be made to fall out as an immediate consequence of an analysis of NPIs in questions developed in Krifka (1995) and van Rooy (2003). This analysis that refers to question's information theoretic entropy (Shannon 1948), and below is dubbed *strength-as-entropy analysis*.

Before proceeding, note that analyzing exhaustivity and non-exhaustivity as properties of question meanings themselves leads to the prediction that, barring any semantic or pragmatic factors that obviate exhaustivity, unembedded questions too can be NPI licensers. As is well known (Klima 1964), this prediction is correct, borne out by examples like (3), where the complement clause featured in (1) and (2) appears as a matrix question.

(3) Who said anything?

After outlining a classic baseline analysis of NPI licensing in terms of strength reversal (Ladusaw 1979, Kadmon and Landman 1993) in section 2, and its elaboration for the case of questions under the strength-as-entropy analysis in section 3, section 4 shows how this account derives the exhaustivity licensing generalization. Section 5 offers additional support for this account from disjunctive questions and singular *which*-questions.

2 NPI licensing under strength reversal

The account of Krifka (1995) and van Rooy (2003) assumes a baseline theory of NPI licensing of the sort pioneered in Kadmon and Landman (1993), which builds on Ladusaw (1979) and is further developed in Krifka (1995), Lahiri (1998), and Chierchia (2013). In a current elaboration, the theory assumes that, in addition to the actual denotation $[\![\phi]\!]$, grammar assigns a linguistic expression ϕ a set of alternative semantic values $[\![\phi]\!]^{ALT}$. Alternative sets feed into the theory of NPIs and their licensing as outlined in (4).

(4) a. $\forall f' \, [f' \in [\![\text{NPI}]\!]^{ALT} \rightarrow f' \subset [\![\text{NPI}]\!]]$ NPI semantics

b. $\forall f' \, [f' \in [\![\ldots\text{NPI} \ldots]\!]^{ALT} \rightarrow [\![\ldots\text{NPI} \ldots]\!] \subset f']$ NPI condition

According to the *NPI semantics* in (4a), the actual denotation of a NPI is strictly weaker than any of the alternatives, while the *NPI condition* in (4b) requires that in some larger syntactic domain *[…NPI …]*, this strength relation be reversed, with the domain's actual denotation being strictly stronger than each of the alternatives.

As shown in (5), alternatives for NPI *anything* are taken to be existential quantifiers that differ from *anything*'s actual denotation in that they have a narrower domain. This ensures that, as stated in (6), each alternative value strictly entails (in a generalized sense) the actual denotation, thereby instantiating the NPI semantics in (4a).

(5) a. $[\![\text{anything}]\!] = \lambda P.\lambda w.\ \exists x{\in}D\ [P(x)(w)]$

b. $[\![\text{anything}]\!]^{ALT} = \{\lambda P.\lambda w.\ \exists x{\in}D'\ [P(x)(w)]\text{: } D'{\subset}D\}$

(6) $\forall f' \, [f' \in [\![\text{anything}]\!]^{ALT} \rightarrow f' \subset [\![\text{anything}]\!]]$

Alternatives are assumed to expand through point-wise composition (Hamblin 1973, Rooth 1985), yielding sets of alternative properties for a verb phrase like *said anything*, as in (7).

As recorded in (8), in this syntactic domain, the direction of entailment between the actual denotation and the alternatives is of course preserved, rather than reversed.

(7) a. $[\![\text{said anything}]\!] = \lambda y.\lambda w.\ \exists x{\in}D\ [y \text{ saw } x \text{ in } w]$
b. $[\![\text{said anything}]\!]^{ALT} = \{\lambda y.\lambda w.\ \exists x{\in}D'\ [y \text{ saw } x \text{ in } w]{:}\ D'{\subset}D\}$

(8) $\forall f'\ [f' \in [\![\text{said anything}]\!]^{ALT} \rightarrow f' \subset [\![\text{said anything}]\!]]$

How, then, is the NPI condition (4b) met in a question like (3)? The answer given in Kadmon and Landman (1990), Krifka (1995), and van Rooy (2003) is that the NPI condition can be satisfied by the question as a whole. That is, as stated in (9), the proposal is that, under a suitable notion of strength, the actual question denotation can be stronger than each of the alternatives.

(9) $\forall Q'\ [Q' \in [\![\text{who said anything?}]\!]^{ALT} \rightarrow [\![\text{who said anything?}]\!] \subset Q']$

Naturally, this answer requires a proper construal of the strength relation $\subset$, one that is applicable to questions. The construal of the strength relation, in turn, is dependant on the semantic analysis of questions. These issues are addressed in the next section.

3 Question strength as entropy

The notion of question strength proposed in van Rooy (2003) builds on the classic question semantics of Groenendijk and Stokhof (1982). Groenendijk and Stokhof posit the (strongly) exhaustive notion of question meaning referred to in section 1. Such a question meaning determines a set of propositions that partitions the set of possible worlds. For concreteness, suppose the domain contains just two individuals, a and b. If **S** is the property denoted by *said anything*, the *Hamblin answers* to (3) are then the propositions **S**(a) and **S**(b). The denotation of (3) is the partition shown in (10a), whose cells can be obtained by conjoining the two Hamblin answers and their negations. Pointwise composition yields the set of alternative question meanings shown in (10b).

(10) a. $[\![\text{who said anything?}]\!] =$
$\{\mathbf{S}(a){\cap}\mathbf{S}(b),\ {-}\mathbf{S}(a){\cap}\mathbf{S}(b),\ \mathbf{S}(a){\cap}{-}\mathbf{S}(b),\ {-}\mathbf{S}(a){\cap}{-}\mathbf{S}(b)\}$
b. $[\![\text{who said anything?}]\!]^{ALT} =$
$\{\ \{\mathbf{S'}(a){\cap}\mathbf{S'}(b),\ {-}\mathbf{S'}(a){\cap}\mathbf{S'}(b),\ \mathbf{S'}(a){\cap}{-}\mathbf{S'}(b),\ {-}\mathbf{S'}(a){\cap}{-}\mathbf{S'}(b)\}{:}$
$\mathbf{S'} \in [\![\text{said anything}]\!]^{ALT}\}$

Assuming this question semantics, van Rooy (2003), generalizing a proposal in Kadmon and Landman (1990) and developing suggestions in Krifka (1995), defines the strength relation between question meanings Q and Q' as in (11): Q is stronger than Q' just in case Q has greater information theoretic entropy (Shannon 1948) than Q'.

(11) $Q \subset Q' :\Leftrightarrow \mathrm{Ent}_{\mathrm{Pr_s}}(Q) > \mathrm{Ent}_{\mathrm{Pr_s}}(Q')$
where $\mathrm{Ent}_{\mathrm{Pr}}(Q) = \sum_{q \in Q} \mathrm{Pr}(q) \times \log_2(\frac{1}{\mathrm{Pr}(q)})$

The subscripts that Ent carries in (11) indicate that the entropy of a set of a proposition Q is defined relative to a probability mass function Pr with domain Q, that is, a function that maps each member of Q to a probability such that the probabilities in the range of Pr sum up to 1. The subscript s in $\mathrm{Pr_s}$ indicates that the ordering of questions is intended to be relative to

$\Pr(p\cap q)$	$\Pr(\neg p\cap q)$	$\Pr(p\cap\neg q)$	$\Pr(\neg p\cap\neg q)$	$\mathrm{Ent}_{\Pr}(Q1)$
0	0	0	1	0
$\frac{1}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{9}{16}$	1.62
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	2
$\frac{9}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	1.62
1	0	0	0	0

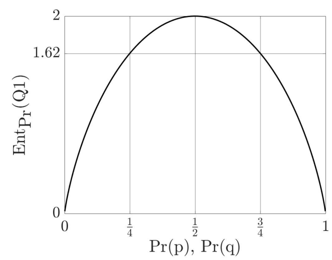

Figure 1: Question entropy as a function of the probabilities of exhaustive answers (table) and Hamblin answers (graph)

a probability mass function determined by the speaker's information state s. The entropy of a question partition Q relative to any given probability mass function Pr is a measure of how evenly Pr distributes the probability mass over the propositions in Q. Accordingly, the entropy of Q relative to $\Pr_s$ is a measure of the speaker's uncertainty about which member of Q is true.

To illustrate, the table in Figure 1 specifies the entropy relative to a few selected probabilities mass functions for the question partition Q1 defined in (12) below. The entropy is minimal (at 0) if all the probability mass is in one of the cells of the partition (first and fifth row) and maximal (here 2) if all the cells of the partition have equal probability (third row). For a probability mass function determined by the speaker's information state, the former case amounts to the speaker's beliefs entailing a complete answer to the question, while the latter case amounts to the speaker being maximally uncertainty or unbiased as to the question's true answer. Probability mass functions that are between those two extremes determine entropy values between 0 and the maximal entropy value (here 1.62, second and fourth row).

(12) $Q1 = \{p\cap q, \neg p\cap q, p\cap\neg q, \neg p\cap\neg q\}$

We are now interested in how the entropy of this question denotation depends on the probabilities of the Hamblin answers p and q. Assuming that p and q are independent and have equal probability, this dependency is as shown by the graph on the right-hand side of Figure 1. It can be read off this graph that the NPI condition is satisfiable for (3). To see why, suppose again that the domain is comprised of just a and b, and hence that the question semantics is as in (10) above. This semantics guarantees the truth of (13a), that is, that any two alternative Hamblin answers **S'**(a) and **S'**(b) are strictly stronger than the actual Hamblin answers **S**(a) and **S**(b), respectively. Probability theory in turn guarantees the truth of (13b), that is, that the probabilities of any two alternative Hamblin answers **S'**(a) and **S'**(b) are no greater than those of the actual Hamblin answers **S**(a) and **S**(b), respectively.

(13) a. $\forall\, \mathbf{S'}[\mathbf{S'} \in [\![\text{said anything}]\!]^{ALT} \rightarrow \mathbf{S'}(a) \subset \mathbf{S}(a)\ \&\ \mathbf{S'}(b) \subset \mathbf{S}(b)]$
b. $\forall\, \mathbf{S'}[\mathbf{S'} \in [\![\text{said anything}]\!]^{ALT} \rightarrow \Pr(\mathbf{S}(a)) \geq \Pr(\mathbf{S'}(a))\ \&\ \Pr(\mathbf{S}(b)) \geq \Pr(\mathbf{S'}(b))\,]$

Suppose now that $\Pr_s(\mathbf{S}(a))$ and $\Pr_s(\mathbf{S}(b))$ are equal and are both no greater than $\frac{1}{2}$ but greater than 0. Given (13b), since the actual Hamblin answers **S**(a) and **S**(b) are independent, we can then read off the graph in Figure 1 that any alternative question will have no greater entropy than the actual question. So, if the probabilities of all alternative Hamblin answers **S'**(a) and **S'**(b) are different from the probabilities of **S**(a) and **S**(b), (9) above will be true, ensuring that the NPI condition (4b) is met. Hence the NPI condition is satisfiable, correctly permitting the acceptability of questions like (3).

4 The exhaustivity-licensing generalization derived

The analysis of NPI licensing by questions reviewed above, call it the *strength-as-entropy analysis*, captures one half of Guerzoni and Sharvit's (2007) exhaustivity-licensing generalization. Under the assumption that *wonder* and *know* select for the (strongly) exhaustive question meaning, the acceptable embedding cases in (1) and (2a) can satisfy the NPI condition (4b) in much the same way as the matrix question (3). By choosing the embedded question as the relevant syntactic domain, the demonstration of satisfiability for the matrix question (3) carries over. The only possible adjustment concerns the nature of the probability function relative to which entropy is calculated. In section 3, entropy was taken to be relative to a probability function determined by the speaker's information state s, while (1) and (2a) might instead make reference to attitude holder's information state.

But what about the other half of the exhaustivity-licensing generalization? What accounts for the unacceptability of (2b)? In brief commentary, Guerzoni and Sharvit (2007, 370) suggest that a strength-as-entropy analysis does not shed any light on the exhaustivity-licensing generalization. However, that assessment can be questioned. As noted in section 1, Heim (1994) presented observations indicating that the meaning of *surprise*, unlike the meaning of *know* (and presumably *wonder*), does not make reference to negated Hamblin answers. Under the present setup, this suggests that, instead of the (strongly) exhaustive semantics in (10), the embedded question in (2b) has the non-exhaustive semantics in (14), where negated Hamblin answers do not contribute to the membership of the answer set (Hamblin 1973, Karttunen 1977).

(14) a. $[\![\text{who said anything?}]\!] =$
$\{\mathbf{S}(\text{b}), \mathbf{S}(\text{a}), \mathbf{S}(\text{a})\cap\mathbf{S}(\text{b})\}$

b. $[\![\text{who said anything?}]\!]^{\text{ALT}} =$
$\{\ \{\mathbf{S'}(\text{b}), \mathbf{S'}(\text{a}), \mathbf{S'}(\text{a})\cap\mathbf{S'}(\text{b})\}\text{: } \mathbf{S'} \in [\![\text{said anything}]\!]^{\text{ALT}}\}$

How does a strength-as-entropy analysis apply under such a non-exhaustive question semantics? The first question is whether the notion of entropy is well-defined for the relevant meanings. The answer is that it can be. There exist probability mass functions that have non-exhaustive question meanings as their domain relative to which entropy can be calculated. For example, assuming that $\mathbf{S}(\text{a})$ and $\mathbf{S}(\text{b})$ are independent, so that $\Pr(\mathbf{S}(\text{a})\cap\mathbf{S}(\text{b})) = \Pr(\mathbf{S}(\text{a}))\times\Pr(\mathbf{S}(\text{b}))$, Pr is a probability mass function with domain (14a) if $\Pr(\mathbf{S}(\text{a})) = \frac{1}{2}$ and $\Pr(\mathbf{S}(\text{b})) = \frac{1}{3}$, since $\frac{1}{2} + \frac{1}{3} + \frac{1}{2} \times \frac{1}{3} = 1$. Hence (14a) has a well-defined entropy relative to this function (viz. 1.46).

Even so, however, under the strength-as-entropy analysis, such a non-exhaustive question semantics makes it impossible for the NPI condition to be met at the question level. That is, the strength-as-entropy analysis derives the exhaustivity-licensing condition introduced in section 1. To see why, note that the consequence of the NPI semantics stated in (13a) above entails (15a); therefore, in addition to (13b), probability theory guarantees (15b).

(15) a. $\forall\ \mathbf{S'}[\mathbf{S'} \in [\![\text{said anything}]\!]^{\text{ALT}} \rightarrow \mathbf{S'}(\text{a})\cap\mathbf{S'}(\text{b}) \subseteq \mathbf{S}(\text{a})\cap\mathbf{S}(\text{b})]$

b. $\forall\ \mathbf{S'}[\mathbf{S'} \in [\![\text{said anything}]\!]^{\text{ALT}} \rightarrow \Pr(\mathbf{S}(\text{a})\cap\mathbf{S}(\text{b})) \geq \Pr(\mathbf{S'}(\text{a})\cap\mathbf{S'}(\text{b}))\ ]$

So each member of the actual non-exhaustive question meaning (14a) is at least as likely as its counterpart in any of the alternatives. This has the following consequence. Suppose that the probabilities of the members of the actual question meaning sum to 1. For the probabilities of members of an alternative question to sum to 1 as well, each of those members must have the very same probability as its actual counterpart. That is, in virtue of entailing (15), (13) also entails (16).

(16) $\mathrm{Pr}(\mathbf{S}(a)) + \mathrm{Pr}(\mathbf{S}(b)) + \mathrm{Pr}(\mathbf{S}(a)\cap\mathbf{S}(b)) = 1 \rightarrow \forall\mathbf{S'}[\mathbf{S'} \in [\![\text{said anything}]\!]^{\mathrm{ALT}}\ \&\ \mathrm{Pr}(\mathbf{S'}(a)) + \mathrm{Pr}(\mathbf{S'}(b)) + \mathrm{Pr}(\mathbf{S'}(a)\cap\mathbf{S'}(b)) = 1 \rightarrow [\mathrm{Pr}(\mathbf{S'}(a)) = \mathrm{Pr}(\mathbf{S}(a))\ \&\ \mathrm{Pr}(\mathbf{S'}(b)) = \mathrm{Pr}(\mathbf{S}(b))\ \&\ \mathrm{Pr}(\mathbf{S'}(a)\cap\mathbf{S'}(b)) = \mathrm{Pr}(\mathbf{S}(a)\cap\mathbf{S}(b))\ ]]$

Once again, the entropy of a question meaning is only defined relative to a probability mass function that has the question meaning as its domain, and this requires that the probabilities assigned to the members of a question meaning sum to 1. Therefore, since the entropy of a question meaning is determined by the probabilities assigned to the answers it contains, in the case at hand the entropy of any of the alternative question meaning, if defined, must equal the entropy of the actual question meaning. That is, (16) entails (17).

(17) $\mathrm{Ent}_{\mathrm{Pr}}([\![\text{who said anything?}]\!])$ is defined $\rightarrow [\forall Q'[Q' \in [\![\text{who said anything?}]\!]^{\mathrm{ALT}}\ \&\ \mathrm{Ent}_{\mathrm{Pr}}(Q')$ is defined $\rightarrow \mathrm{Ent}_{\mathrm{Pr}}(Q') = \mathrm{Ent}_{\mathrm{Pr}}([\![\text{who said anything?}]\!])\ ]]$

An obvious consequence of (17), stated in (18), is that the actual question meaning in (14b) cannot be stronger than any of the alternative question meanings in (14b). Given that the set of alternative question meanings in (14b) is non-empty, (18) contradicts the requirement (9).

(18) $\forall Q'\ [Q' \in [\![\text{who said anything?}]\!]^{\mathrm{ALT}} \rightarrow [\![\text{who said anything?}]\!] \not\subset Q'\ ]$

The non-exhaustive question meaning, in other words, cannot possibly reverse strength. This completes the argument that it is impossible for the non-exhaustive question meaning in (14) to satisfy the NPI condition (4b). The exhaustivity-licensing condition on question meanings, and hence Guerzoni and Sharvit's (2007) exhaustivity-licensing generalization, has been derived.

The strength-as-entropy analysis, then, is a possible account of the exhaustivity-licensing generalization. Is it the right account? In support of the analysis, the next section demonstrates that it makes additional welcome predictions about NPI licensing by question.

5 Support from presuppositional questions

Guerzoni and Sharvit (2014) report that in contrast to wh-questions, disjunctive questions (also known as alternative questions) never license NPIs. This generalization is illustrated by the unacceptability of (19).

(19) #Did Al say anything or Ben?

Schwarz (in press) observes that this is expected under the strength-as-entropy analysis, given independent observations about the interpretation of disjunctive questions. Such questions have been observed to carry a presupposition of existence and uniqueness (e.g., Karttunen and Peters 1976). For example, *Did Al talk or Ben?* presupposes that exactly one of Al and Ben talked. Under a partition semantics, this presupposition can be captured in the question meaning by expunging from it those answers that are inconsistent with the presupposition. This leads to the question meaning for (19) shown in (20). (20) excludes the answer that neither of Al and Ben said something as well as the answer that both did, encoding the existence and uniqueness presupposition as the disjunction of the remaining two propositions.

(20) a. $[\![$did Al say anything or Ben?$]\!]$ =
$\{-\mathbf{S}(a)\cap\mathbf{S}(b),\ \mathbf{S}(a)\cap-\mathbf{S}(b)\}$

b. $[\![$did Al say anything or Ben?$]\!]^{ALT}$ =
$\{\ \{-\mathbf{S'}(a)\cap\mathbf{S'}(b),\ \mathbf{S'}(a)\cap-\mathbf{S'}(b)\}\!:\ \mathbf{S'} \in [\![$say anything$]\!]^{ALT}\}$

Schwarz shows that, if the probabilities of the members of the actual question in (20a) sum to 1 and the same holds for the members of any alternative question in (20b), then the probabilities of the members of the alternative question must equal the probabilities of the corresponding propositions in the actual question. That is, (21) holds for (20).

(21) $\Pr(\mathbf{S}(a)\cap-\mathbf{S}(b)) + \Pr(-\mathbf{S}(a)\cap\mathbf{S}(b)) = 1 \rightarrow \forall\mathbf{S'}[\mathbf{S'} \in [\![\text{said anything}]\!]^{ALT}$ &
$\Pr(\mathbf{S'}(a)\cap-\mathbf{S'}(b)) + \Pr(-\mathbf{S'}(a)\cap\mathbf{S'}(b)) = 1 \rightarrow [\Pr(\mathbf{S}(a)\cap-\mathbf{S}(b)) = \Pr(\mathbf{S'}(a)\cap-\mathbf{S'}(b))$ &
$\Pr(-\mathbf{S}(a)\cap\mathbf{S}(b)) = \Pr(-\mathbf{S'}(a)\cap\mathbf{S'}(b))\ ]]$

As recorded in (22), this entails that the entropy of any of the alternative question meanings, if defined, must equal the entropy of the actual question meaning; (22) entails (23), that is, it entails that the actual question meaning in (20a) cannot be stronger than any of the alternative question meanings in (20b); and given that the set of alternative question meanings is non-empty, (23) contradicts (24). Under the present assumptions, then, the disjunctive question meaning cannot possibly reverse strength, hence cannot possibly satisfy the NPI condition (4b). So it is correctly predicted that such questions cannot license NPIs.

(22) $\mathrm{Ent}_{\mathrm{Pr}}([\![$did Al say anything or Ben?$]\!])$ is defined $\rightarrow$
$[\forall Q'[Q' \in [\![$did Al say anything or Ben?$]\!]^{ALT}$ & $\mathrm{Ent}_{\mathrm{Pr}}(Q')$ is defined $\rightarrow \mathrm{Ent}_{\mathrm{Pr}}(Q') =$
$\mathrm{Ent}_{\mathrm{Pr}}([\![$did Al say anything or Ben?$]\!])\ ]$

(23) $\forall Q'\ [Q' \in [\![$did Al say anything or Ben?$]\!]^{ALT} \rightarrow [\![$did Al say anything or Ben?$]\!] \not\subset Q']$

(24) $\forall Q'\ [Q' \in [\![$did Al say anything or Ben?$]\!]^{ALT} \rightarrow [\![$did Al say anything or Ben?$]\!] \subset Q']$

The statements in (21)–(23) are transparently parallel to those in (16)–(18) above. Under the strength-as-entropy analysis, then, questions with NPIs embedded under *surprise* and disjunctive questions with NPIs form a natural class. In both cases, the independently supported question meaning turns out to not allow any variation in entropy between the actual question and its alternatives generated by the NPI semantics, thereby rendering the satisfaction of the NPI condition impossible.

Are there other types of questions that that belong to this family? Singular *which*-questions (not discussed in Schwarz in press) are a natural candidate. Such questions, too, have been observed to carry a presupposition of existence and uniqueness (e.g., Higginbotham 1993, Dayal 1996). For example, *Which student talked*? is judged to presupposes that exactly one student talked. If the students are a and b, (25) should then have the very same semantics as (19), that is, the meaning given in (26).

(25) %Which student said anything?

(26) a. $[\![$which student said anything?$]\!]$ =
$\{\mathbf{S}(a)\cap-\mathbf{S}(b),\ -\mathbf{S}(a)\cap\mathbf{S}(b)\}$

b. $[\![$which student said anything?$]\!]^{ALT}$ =
$\{\ \{\mathbf{S'}(a)\cap-\mathbf{S'}(b),\ -\mathbf{S'}(a)\cap\mathbf{S'}(b)\}\!:\ \mathbf{S'} \in [\![$said anything$]\!]^{ALT}\}$

Under this semantics, for the very same reasons as (19), (25) necessarily violates the NPI condition. Since this holds true more generally for any non-empty set of students (as the reader is invited to confirm), (25) is predicted to pattern with (19) in being judged unacceptable.

This prediction matches the judgments of some speakers. However as the diactritic % is meant to signal, others find such questions quite acceptable. In fact, without specifically discussing singular *which*-questions, Krifka (1995, 251) presents (27) as an example of NPI licensing by (information–seeking) wh-questions. Yet some speakers, those who reject (25), also reject (27).

(27) %Which student has ever been to China?

How might the present line of analysis accommodate this speaker variation? How could it be reconciled with speaker judgments that allow for NPIs to be licensed by singular *which*-questions? As a tentative answer, based on informants' reports about (25) and (27), I propose that singular *which*-questions do not in fact invariably carry a presupposition of existence and uniqueness. Specifically, I propose that the existence presupposition can be suspended. Hence I hypothesize that (25) and (27) are judged acceptable by speakers who interpret those questions as consistent with no student having talked and gone to China, respectively.

For those speakers, the question semantics in (26) is to be replaced with (28), where the proposition that neither a nor b said something is admitted as a member of the question set. As a consequence, this meaning now encodes a mere presupposition of uniqueness, rather than a presupposition of existence and uniqueness.

(28) a. $[\![$ which student said anything? $]\!]$ =
$\{\mathbf{S}(a)\cap\text{–}\mathbf{S}(b),\ \text{–}\mathbf{S}(a)\cap\mathbf{S}(b),\ \text{–}\mathbf{S}(a)\cap\text{–}\mathbf{S}(b)\}$

b. $[\![$ which student said anything? $]\!]^{ALT}$ =
$\{\ \{\mathbf{S'}(a)\cap\text{–}\mathbf{S'}(b),\ \text{–}\mathbf{S'}(a)\cap\mathbf{S'}(b),\ \text{–}\mathbf{S'}(a)\cap\text{–}\mathbf{S'}(b)\}\text{: } \mathbf{S'} \in [\![\text{said anything}]\!]^{ALT}\}$

Note that this revision suffices to render the NPI condition satisfiable in the case at hand. By reasoning familiar from section 3, this can be read off the graph in Figure 2, which plots the entropy of the question meaning Q2 in (29) below as a function of the probabilities of the Hamblin answers p and q, assuming that p and q are independent and have equal probability.[1]

(29) $Q2 = \{\text{–}p\cap q,\ p\cap\text{–}q,\ \text{–}p\cap\text{–}q\}$

In sum, provided the proposed interpretation of the singular *which*-question data is correct, the evidence from disjunctive questions and singular *which*-question strengthens the case for the strength-as-entropy analysis of the exhaustivity-licensing generalization, by demonstrating that variation of entropy values between the actual question and its alternatives is a necessary condition for NPI licensing.

6 Conclusion

The main result of this paper is that van Rooy's (2003) strength-as-entropy analysis of NPI licensing by questions can derive Guerzoni and Sharvit's (2007) exhaustivity-licensing generalization. Data from disjunctive questions and singular *which*-questions are proposed to provide further independent support for the approach.

[1] Note that the graph in Figure 2 only shows entropy values for Hamblin answer probabilities in the interval $[0, \frac{1}{2}]$. The reason for this (left for the reader to verify) is that the probabilities of the propositions in Q2 will not sum to 1 (hence the question entropy will not be defined) if the probabilities of the Hamblin answers p and q are greater than $\frac{1}{2}$.

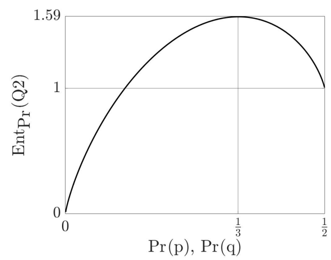

Figure 2: Entropy of a question that carries a uniqueness presupposition as a function of probabilities of Hamblin answers

A remaining question worth highlighting concerns the nature of the exhaustive/non-exhaustive distinction. The strength-as-entropy analysis explored here assumes that questions themselves are ambiguous (George 2011, Guerzoni and Sharvit 2014, Nicolae 2015, Theiler et al. 2016). However, this view has been challenged in recent work on question embedding, most notably Klinedinst and Rothschild (2011) and Uegaki (2015). It remains to be seen whether or how the arguments given in those works can be reconciled with the present proposal.

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