Scope interactions between modals and modified numerals*

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1 Introduction

It is a widely observed phenomenon in the literature about modified numerals that sentences with an existential modal and *at most* cannot be used to merely give permission to do something (e.g. Geurts & Nouwen, 2007; Nouwen, 2010; Schwarz, 2011; Coppock & Brochhagen, 2013; Penka, 2014; Kennedy, 2015). (1-a), for instance, forbids the hearer from taking more than two biscuits. It cannot be used only to convey that she has permission to take one or two biscuits. When *at most* is replaced by *fewer than*, as in (1-b), we do get this 'pure permission' reading.

(1) a. You're allowed to take at most two biscuits.
 b. You're allowed to take fewer than three biscuits.

This difference is not restricted to *fewer than* and *at most*. It is a general difference that holds between all so-called class A and class B modified numerals that set an upper bound. Class B numeral modifiers are characterised by the fact that they, unlike class A modifiers, give rise to ignorance inferences in unembedded contexts (Nouwen, 2010).[1]

When class B modified numerals are combined with a modal, two readings can arise: the *ignorance reading* and the *authoritative reading*. For example, (1-a) can be uttered when the speaker does not know the maximum number of biscuits the hearer is allowed to take (but she knows it is not more than two) or when the speaker has full knowledge of what is allowed.

The examples in (1) raise the question of why upper-bounded class B modifiers lack the pure permission reading that their class A counterparts do display when they are combined with an existential modal. Most current theories wrongly predict that sentences like (1-a) have the same truth conditions as sentences like (1-b). One notable exception is Penka (2014), who tackles the problem by positing a decomposition account for *at most*. However, as I will discuss in this paper, her account misses an important generalisation. As I aim to show here, class B numeral modifiers obligatorily outscope existential modals. This generalisation holds for upper-bounded

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[1] To see this, consider (i).

(i) a. I know exactly how many books there are in this bookshop, #and it's { at most / maximally / no more than } 10,000.
 b. I know exactly how many books there are in this bookshop, and it's { fewer than / less than / under } 10,000.

The class B modifiers in the a-sentence are incompatible with exact knowledge of the number of books in the bookshop. The class A modifiers in the b-sentences do not give rise to ignorance inferences and are felicitous in a context in which a speaker claims to have knowledge of the precise number of books.

modifiers like *at most* and *maximally* but also for their lower-bounded counterparts such as *at least* and *minimally*. As will become clear in this paper, this generalisation accounts for a variety of data involving lower-bounded and upper-bounded class B modifiers, root modals, epistemic modals, and islands. It also raises problems for existing theories of modified numerals.

In the following section I briefly discuss one of these existing theories (Schwarz, 2011) and Penka's (2014) modification of the theory. In section 3 I review the data with class B modifiers and existential modals that have led me to propose the generalisation mentioned above. I also explore the consequences of this generalisation for current theories of modified numerals. Section 4 concludes.

2 Previous accounts

2.1 The neo-Gricean approach

Current theories of modified numerals generally derive the ignorance and authoritative readings of sentences with numeral modifiers and modals with an implicature mechanism. I will use the account proposed in Schwarz (2011) to illustrate this. Schwarz assumes the denotations of *at least* and *at most* given in (2).

(2) a. $[\![\text{at least}]\!] = \lambda d_d \lambda P_{\langle d,t\rangle}.\textsc{max}\{n \mid P(n)\} \geq d$
b. $[\![\text{at most}]\!] = \lambda d_d \lambda P_{\langle d,t\rangle}.\textsc{max}\{n \mid P(n)\} \leq d$

At least and *at most* take a degree of type d and a degree predicate of type $\langle d,t\rangle$ and express that the maximal degree n such that P holds of that degree is at least as high or at most as high as d. Schwarz assumes that there are two Horn sets that are used for the calculation of implicatures: the set of natural numbers; alternatives to the modified number, and a set containing the modifiers *at least*, *exactly*, and *at most*; alternatives to the modifier.

(3) a. { 1, 2, 3, 4, 5, ... }
b. { at least, exactly, at most }

2.1.1 Lower-bounded modified numerals

Let us first turn to modified numerals that set a lower bound, like *at least*. A sentence with *at least* and a universal modal such as (4) has two possible LFs depending on where the modified numeral takes scope. These are given in (5). The narrow scope reading in (5-a) says that in all permissible worlds, Mary submits one or more abstracts. The wide scope reading in (5-b) says that the maximum number of abstracts such that Mary submits that many abstracts in all permissible worlds is one or higher.

(4) Mary is required to submit at least one abstract.

(5) a. $\Box$ [$\textsc{max}$ { n | Mary submits n abstracts } ≥ 1]
b. $\textsc{max}$ { n | $\Box$ [Mary submits n abstracts] } ≥ 1

The stronger alternatives to (5-a) and (5-b) are given in (6) and (7) respectively.

(6) a. $\Box$ [$\textsc{max}$ { n | Mary submits n abstracts } $= 1$]
b. $\Box$ [$\textsc{max}$ { n | Mary submits n abstracts } ≥ 2]

(7) a. $\textsc{max}$ { n | $\Box$ [Mary submits n abstracts] } $= 1$
b. $\textsc{max}$ { n | $\Box$ [Mary submits n abstracts] } ≥ 2

The alternatives in (6) are not symmetric, so the primary implicatures that arise from them can be strengthened to secondary implicatures (Sauerland, 2004). The meaning of the assertion in (5-a) combined with these implicatures is that the speaker believes Mary is required to submit one or more abstracts but she is not required to submit exactly one and she is not required to submit two or more.[2] This reading does not involve any ignorance on the part of the speaker and is referred to as the authoritative reading.

The alternatives in (7), on the other hand, are symmetric, so only primary implicatures can be derived. The primary implicatures together with the assertion generate ignorance implicatures: the speaker believes that Mary is required to submit at least one abstract, but she is not sure whether Mary is required to submit exactly one and she is not sure whether Mary is required to submit at least one. Thus, Schwarz's account derives the authoritative reading when the modified numeral takes narrow scope and the ignorance reading when the modified numeral takes wide scope.

Now let us turn to a similar example with an existential modal. A sentence like (8) is taken to have the scope configurations given in (9). The narrow scope reading in (9-a) merely says that there is a permissible world in which Mary submits one or more abstracts. The wide scope reading in (9-b) conveys that there is an upper bound to the number of abstracts Mary is allowed to submit, and that upper bound is one or higher.

(8) Mary is allowed to submit at least one abstract.

(9) a. $\Diamond$ [$\textsc{max}$ { n | Mary submits n abstracts } ≥ 1]
b. $\textsc{max}$ { n | $\Diamond$ [Mary submits n abstracts] } ≥ 1

The stronger alternatives to both (9-a) and (9-b), are symmetric, which leads to the ignorance implicatures in (10) for (9-a) and in (11) for (9-b) (ignorance is indicated with a question mark).

(10) a. ?$\Diamond$ [$\textsc{max}$ { n | Mary submits n abstracts } $= 1$]
b. ?$\Diamond$ [$\textsc{max}$ { n | Mary submits n abstracts } ≥ 2]

(11) a. ?$\textsc{max}$ { n | $\Diamond$ [Mary submits n abstracts] } $= 1$
b. ?$\textsc{max}$ { n | $\Diamond$ [Mary submits n abstracts] } ≥ 2

The narrow scope ignorance implicatures in (10) say that the speaker does not know whether submitting one abstract is allowed or whether submitting at least two abstracts is allowed. The wide scope ignorance implicatures in (11) do not indicate ignorance merely about which numbers are allowed, but about the upper bound of the allowed numbers. Thus, the wide scope reading is that the maximum allowed number of pages is at least one, but the speaker does not know if this maximum is exactly one or higher than one. As Schwarz admits, only the wide scope reading and not the narrow scope reading appears to be attested.

2.1.2 Upper-bounded modified numerals

Now let us turn our attention to upper-bounded modified numerals. Parallel to (8), (12) has the two denotations given in (13).[3] The narrow scope reading in (13-a) says that submitting one or two abstracts is allowed without excluding the possibility of submitting more abstracts. This is the unattested pure permission reading. The wide scope reading in (13-b) expresses that the maximum number of abstracts Mary is allowed to submit is two or less.

[2] This is the so-called *free choice* reading: Mary can choose freely between submitting one or more than one abstract.

[3] I do not discuss examples with upper-bounded modified numerals and universal modals here as they are not relevant for the current discussion.

(12) Mary is allowed to submit at most two abstracts.

(13) a. $\Diamond$ [MAX { n | Mary submits n abstracts } ≤ 2]
b. MAX { n | $\Diamond$ [Mary submits n abstracts] } ≤ 2

The stronger alternatives for (13-a) and (13-b) are given in (14) and (15) respectively.

(14) a. $\Diamond$ [MAX { n | Mary submits n abstracts } $= 2$]
b. $\Diamond$ [MAX { n | Mary submits n abstracts } ≤ 1]

(15) a. MAX { n | $\Diamond$ [Mary submits n abstracts] } $= 2$
b. MAX { n | $\Diamond$ [Mary submits n abstracts] } ≤ 1

Again, symmetry arises in both cases. This means that only primary implicatures are derived, which leads to ignorance inferences. Parallel to the *at least* cases, the weak narrow scope ignorance inferences (corresponding to (13-a)) convey that the speaker does not know if two is an allowed number or if one or less is allowed, whereas the stronger wide scope ignorance implicatures (corresponding to (13-b)) are about the upper bound: the speaker does not know if the maximum number of abstracts Mary is allowed to submit is two or lower than two.

There are two problems here. The first is that, as mentioned in the introduction of this paper, the weak narrow scope reading that this account derives is not attested. The second is that there is no way to derive the correct authoritative reading. This account has no hope of deriving any kind of an authoritative reading for (12) because existential modals lead to symmetric alternatives regardless of where they take scope, and this blocks secondary implicatures. As we have seen, primary implicatures lead to ignorance inferences, so only ignorance readings can be obtained. The chances of deriving the particular authoritative reading that is attested for (12) are even lower. As the neo-Gricean accounts derive ignorance readings when the modified numeral takes wide scope and authoritative readings when the modified numeral takes narrow scope, *at most* would have to scope under the existential modal in the authoritative LF. This kind of a scope configuration can never lead to a strong upper bound because it merely states that the upper bound is there in one permissible world and remains silent about other permissible worlds. In the next section, I show how Penka (2014) addresses these issues.

2.2 Penka's solution

Penka (2014) posits an account that derives the missing authoritative reading for sentences with *at most* and an existential modal. Her proposal is to decompose *at most* into an antonymising operator ANT and *at least*, as in (16).

(16) [at most n] = [[n ANT] at least]

At least is defined as in (2-a), repeated here as (17-a), and ANT is defined as in (17-b).[4]

(17) a. $[\![\text{at least}]\!] = \lambda d_d \lambda P_{\langle d,t \rangle}.\text{MAX}\{n \mid P(n)\} \geq d$
b. $[\![\text{ANT}]\!] = \lambda d_d \lambda P_{\langle d,t \rangle}.\forall d' : d' > d \rightarrow \neg D(d')$

In this decomposition account, a sentence like (12) has the three LFs in (18).

(18) a. [allowed [ANT 2 abstracts [λd [at least d [$\lambda d'$ [Mary submits d' abstracts]]]]]]
b. [ANT 2 abstracts [λd [at least d [$\lambda d'$ [allowed [Mary submits d' abstracts]]]]]]
c. [ANT 2 abstracts [λd [allowed [at least d [$\lambda d'$ [Mary submits d' abstracts]]]]]]

[4]The ANT operator is thus equivalent to *at most* as defined in (2-b).

(18-a) and (18-b) have the same truth conditions as (13-a) and (13-b) respectively, but (18-c), where the modal takes scope between the antonymising operator and *at least*, is an LF that is not available in the orignial neo-Gricean account. This LF has the truth conditions in (19).

(19) $\forall d' : d' > 2 \rightarrow \neg\Diamond$ [$\textsc{max}\{$ d | Mary submits d abstracts $\} \geq d'$]
$= \neg\Diamond$ [$\textsc{max}$ $\{d$ | Mary submits d abstracts $\} > 2$]

These are the desired strong truth conditions for the authoritative reading: it is not allowed for Mary to submit more than two abstracts. For the calculation of the implicatures, Penka assumes the same Horn sets as Schwarz along with the two Horn sets given in (20).

(20) a. $\{$ $\textsc{ant}$, $\varnothing$ $\}$
b. $\{$ $\Diamond$, $\Box$ $\}$

The stronger scalar alternatives derived using these four Horn sets are the ones in (21).

(21) a. $\neg\Diamond$ [$\textsc{max}$ $\{d$ | Mary submits d abstracts $\} > 1$]
b. $\Box$ [$\textsc{max}$ $\{d$ | Mary submits d abstracts $\} = 2$][5]

These alternatives are not symmetric, so the secondary implicatures in (22) are derived. The implicatures are that the speaker believes Mary is allowed to submit more than one abstract and she is not required to submit exactly two. Leaving aside the question of whether these specific implicatures are empirically correct, the derived reading is clearly not an ignorance reading but an authoritative reading that sets a strong upper bound, as desired.

(22) a. B $\Diamond$ [$\textsc{max}$ $\{d$ | Mary submits d abstracts $\} > 1$]
b. B $\neg\Box$ [$\textsc{max}$ $\{d$ | Mary submits d abstracts $\} = 2$]

Although this analysis solves one of the issues the neo-Gricean account suffers from, some problems remain. In the following section I will discuss these problems and argue that they stem from a common core: the fact that class B modifiers always outscope existential modals.

3 A new generalisation

3.1 The scopal behaviour of class B modifiers

At the end of section 2.1 I mentioned that the neo-Gricean account has two problems with examples with existential modals and *at most*: it yields an unattested weak ignorance reading (when the modal takes wide scope) and it does not generate a strong authority reading. While Penka solves the second issue, her account inherits the first one. Penka's LF in (18-a) corresponds to a weak reading that merely says that submitting zero, one, or two abstracts is allowed.

Kennedy (2015), whose account has the same problem, argues that the strong upper bound of the authoritative reading is a scalar implicature. In other words, (23) has the weak truth conditions that the neo-Gricean account predicts and the upper bound is calculated pragmatically because the speaker chose not to say *at most three*.

(23) Mary is allowed to submit at most two abstracts.

However, as the upper bound is not cancellable, as illustrated in (24), this does not seem likely.

[5] Here $\textsc{ant}$ has been replaced by $\varnothing$, $\Diamond$ has been replaced by $\Box$, and *at least* has been replaced by *exactly*.

(24) Mary is allowed to submit at most two abstracts. #In fact, she can submit three.

I believe that (24) strongly suggests that the authoritative upper bounded reading should be accounted for in the semantics. The only way to arrive at a reading with a strong upper bound is to say that *at most* has to take scope over the modal. As we have seen, letting it take scope under an existential modal inevitably leads to truth conditions that are too weak.

A way to get rid of the reading where *at most* takes narrow scope in Penka's account is to say that ANT has negative features.[6] It has been observed in the literature (Iatridou & Zeijlstra, 2010) that negation outscopes existential modals. If ANT displays the same behaviour, this would rule out the LF in (18-a), where ANT occurs in the scope of the modal. Penka's antonymising operator does not have the semantics of negation in that it does not yield the complement of its prejacent, but Penka could still stipulate that ANT is an operator that displays the same syntactic behaviour as negation.

If she takes this route, a potential problem is the fact that *at least* is a PPI, as illustrated in (25) (Spector, 2014).

(25) ??Mary didn't solve at least three problems.

Taking this into consideration, it would be curious if *at most* were decomposed into an operator with negative features and *at least*, where *at least* consistently occurs in the immediate scope of the negative operator. However, Penka could again argue that the semantics and the syntactic behaviour of the elements under discussion should be teased apart: perhaps the *at least* part of the decomposed *at most* is not the same *at least* as the *at least* that occurs by itself. It could be an operator that has the same denotation as *at least* but does not share its syntactic features, one of those features being its PPI-hood.[7]

While a few stipulations are needed, it seems as though Penka's theory can account for the way *at most* interacts with existential modals. However, there is one crucial fact that has been overlooked thus far: the data with *at least* and existential modals suggest that upper-bounded numeral modifiers like *at most* do not actually behave differently from lower-bounded numeral modifiers like *at least*. To see this, let us reconsider (8), repeated here as (26-a).

(26) a. Mary is allowed to submit at least one abstract.
b. Mary is allowed to submit more than one abstract.

As we saw in section 2.1, these kinds of examples are predicted to give rise to two different types of ignorance readings in Schwarz's neo-Gricean account (cf. (9)-(11)): one where the speaker claims that the maximal allowed number of abstracts Mary submits is at least one (but she does not know the exact maximum), and one where the speaker merely claims that submitting at least one abstract is allowed (but she does not know if submitting exactly one is allowed or if submitting more than one is allowed). This weaker reading seems absent for (26-a); one clearly gets the impression that there is a maximum number of abstracts Mary is allowed to submit. The weaker reading does seem to be available when we use *more than*, as in (26-b).

Schwarz claims that the weaker reading is not visible because it is blocked by the existence of the stronger one. To test this claim, let us consider a scenario where only the weak reading would be felicitous. Say that there is a high demand for instant formula, and the manifacturers are unable to keep up with this demand. For this reason, a particular retail chain has a rule that customers are only allowed to buy one box of instant formula at a time. After some

[6] I would like to thank Yaron McNabb for suggesting this to me.

[7] This would tie Penka to a syntactic story of PPI anti-licensing. Under a semantic account, it is not possible for two expressions to share the same truth conditions and for only one of those expressions to be a PPI.

time, the manifacturers manage to catch up with the high demand, so the rules are loosened. However, there is some confusion about what the new rules are. One store manager has said that customers are now allowed to buy a maximum of two boxes at a time, and that the boxes will be sold in packages of two to facilitate transport. In other words, it is only possible to buy exactly two boxes. Another store manager has announced that there no longer is a maximum, so customers can buy as many boxes of instant formula as they want. She has also said that the boxes will be sold in packages of three, which means customers can buy three or more boxes. David is talking to someone who is not aware of the recent changes; this person thinks that customers are still allowed to buy only one box of formula. David does know that the rules have been changed, but he does not know which store manager to believe. David says:

(27) I'm not entirely sure about the current rules, but I know you're allowed to buy { more than one box / # at least two boxes } now.

In this scenario, David uses *at least two* in a situation where he is not sure whether buying exactly two boxes is allowed or whether buying at least three boxes is allowed, which corresponds exactly to the weak narrow scope reading. The fact that this sentence is not felicitous in the context indicates that the weak reading that arises when *at least* takes scope under an existential modal is simply not there. By extension, the data point towards the conclusion that lower-bounded class B modified numerals, like their upper-bounded counterparts, take scope over existential modals.

What about universal modals? *At most* seems to be able to scope both over and under universal modals. (28-a) has the reading in (28-b), where *at most* takes narrow scope. This says that Mary submits two or fewer abstracts in all permissible worlds. It also appears to have the wide scope reading in (28-c), which says that the number of abstracts Mary submits in every permissible world is two or lower (i.e. the minimum requirement is two or less). This is an ignorance reading that is compatible with Mary being allowed to submit more than two abstracts, as illustrated in (29).

(28) a. Mary is required to submit at most two abstracts.
b. $\Box$ [MAX $\{ n \mid$ Mary submits n abstracts $\} \leq 2$]
c. MAX $\{ n \mid \Box$ [Mary submits n abstracts] $\} \leq 2$

(29) I'm not sure how many abstracts Mary has to submit, but I know she's required to submit at most two. She may actually choose to submit three abstracts though.

Unfortunately it is not possible to see where *at least* takes scope with respect to universal modals as both scope configurations lead to the same truth conditions.[8] Since *at most* and *at least* behave the same way in their interactions with existential modals, the null hypothesis should be that they also display the same behaviour in their interactions with universal modals.[9]

[8] The denotations corresponding to the two scope configurations of (i-a) are given in (i-b) and (i-c) (repeated from (4)-(5)). (i-b) says that Mary submits one or more abstracts in all permissible worlds. This is equivalent to (i-c), which says that the maximum number such that Mary submits that many abstracts in all permissible worlds is one or higher.

(i) a. Mary is required to submit at least one abstract.
b. $\Box$ [MAX $\{ n \mid$ Mary submits n abstracts $\} \geq 1$]
c. MAX $\{ n \mid \Box$ [Mary submits n abstracts] $\} \geq 1$

[9] The only differences one would expect have to do with the fact that *at most* is downward entailing. As some universal modals are said to be PPIs (Iatridou & Zeijlstra, 2013; Homer, 2015), this could mean that these modals have to outscope *at most* but not the upward entailing *at least*.

Thus, the most plausible hypothesis seems to be that class B numeral modifiers outscope existential modals and can scope both over and under universal modals. More evidence for these claims come from finite clause islands. It is known that quantifier raising over finite clause boundaries is not possible (e.g. Fox, 2000) so these kinds of islands are a good way to test if certain scope configurations are possible. Let us see what happens when we force modified numerals to take scope below existential modals by putting them in a finite clause island. As can be observed in (30-a), class A modifiers seem quite comfortable in this position. The class B modifiers in (30-b), on the other hand, do not accept being forced to take scope under existential modals.[10]

(30) a. It is allowed that you write { fewer than / more than } five pages.
b. #It is allowed that you write { at least / at most } five pages.

The examples in (30-b) are only acceptable in echoic contexts, for example as a reply to the question if writing at least or at most five pages is allowed. As exemplified by the dialogue in (31), where a PPI is licensed in the scope of negation, echoic contexts allow all sorts of constructions that are normally ruled out. Therefore, the fact that echoic contexts may license the examples in (30-b) does not say much.

(31) A: Did you see someone?
B: No, I didn't see someone.

Now let us turn to cases where modified numerals are trapped in finite clause islands under universal modals. These examples are all felicitous and do not require an echoic context to be licensed. This shows that class B numeral modifiers do not mind occurring in the scope of a universal modal.

(32) a. It is required that you write { fewer than / more than } five pages.
b. It is required that you write { at least / at most } five pages.

A final piece of evidence for the generalisation I am defending comes from interactions with epistemic modals. (33) shows that *at most* also takes scope over epistemic existential modals (pace Kennedy, 2015), yielding a reading where the evidence rules out the possibility that there are more than four burglars in the building.

(33) Police evidence suggests there may be at most four burglars in the building.

In sum, the data discussed in this section point towards the conclusion that class B modifiers must outscope existential modals but can occur in the scope of universal modals. As lower-bounded and upper-bounded class B modifiers appear to take scope the same way, a theory like Penka's that decomposes *at most* but not *at least* is probably not on the right track. In the next section I discuss some more consequences of these observations.

[10] Some of my anglophone informants found 'it is allowed that φ' a bad construction. In Dutch, however, the construction is perfectly fine, and the Dutch data are the same as the English data provided by those speakers who did accept the 'it is allowed that' construction:

(i) a. Het is toegestaan dat je { minder / meer } dan vijf pagina's schrijft.
It is permitted that you { fewer / more } than five pages write.
'It is permitted that you write { fewer / more } than five pages.'
b. #Het is toegestaan dat je { minstens / hoogstens } vijf pagina's schrijft.
It is permitted that you { at least / at most } five pages write.
'It is permitted that you write { at least / at most } five pages.'

3.2 Further consequences

As far as I am aware, all current theories of modified numerals use scope to derive the two readings that arise when a modified numeral interacts with a modal (the neo-Gricean accounts in Büring, 2008; Schwarz, 2011, 2013; Kennedy, 2015 but also accounts in other frameworks such as Nouwen, 2010; Coppock & Brochhagen, 2013). If class B modifiers must indeed take scope over existential modals, this presents a problem for all these accounts. While some authors argue that sentences with *at least* and an existential modal only give rise to ignorance readings (e.g. Schwarz, 2011; Penka, 2014), it is clear that this is not so for cases with *at most* and an existential modal. Examples like (12), repeated here as (34), have both an authoritative reading and an ignorance reading.

(34) Mary is allowed to submit at most two abstracts.

If the modified numeral always takes scope over the modal, these two readings cannot be said to be derived from two different scope configurations. We must therefore find another way of accounting for the two readings of examples like (34).

As for cases with universal modals, which display the same ambiguity as (34), there are two theoretical possibilities. Either we propose that their two readings, too, must be accounted for in a different way, or we maintain that their ambiguity arises from scope interactions. The latter option involves proposing two different analyses for authoritative and ignorance readings: one for cases with universal modals and another for cases with existential modals. For this reason, the former option seems preferable. This would mean that we need a new theory of modified numerals that treats the scope interactions between modified numerals and modals and the authoritative and ignorance readings they give rise to as two separate phenomena.

Another argument for such a theory comes from the Heim-Kennedy generalisation, which states that degree quantifiers can bind their trace across modals but not across nominal quantifiers (Heim, 2000). Assuming, as I have done in this paper, that modified numerals are indeed degree quantifiers, this would mean that they should be unable to outscope universal quantifiers in examples such as (35).

(35) Every student submitted { at least / at most } two abstracts.

This seems to be the case: the wide scope reading, given in (36), is that the maximum number of abstracts such that all students submitted that many abstracts is two or lower, which means that the number of abstracts submitted by the student who submitted the least amount of abstracts is two or lower. This lower bound reading is not attested.

(36) $\textsc{max}\ \{\ n \mid \forall x\ [\ \text{student}(x) \rightarrow x \text{ submits } n \text{ abstracts}\ ]\ \} \leq 2$

It seems to me that sentences like (35) also have two readings (pace Penka, 2014). The most obvious reading is the reading where the speaker has full knowledge of the situation (parallel to the authoritative reading) and conveys that different students submitted different numbers of abstracts, and they all submitted two or more/less. The other reading is the ignorance reading: there is a specific number such that every student submitted that many abstracts. The speaker does not know what that number is, but she knows it is two or more/less.

Class B modified numerals thus appear not to be able to outscope universal quantifiers, but they do give rise to two readings when they occur in sentences with universal quantifiers. Therefore, examples with universal quantifiers and modified numerals are another case where we do see an ignorance reading and a non-ignorance (variation) reading but where we do not want to say that scope is responsible for this. Both the examples with existential modals and the sentences with universal nominal quantifiers point towards the conclusion that the ambiguity we observe must be accounted for in a different way.

4 Conclusion

I have argued that class B modified numerals always take scope over existential modals. This means that a decomposition story à la Penka (2014), in which *at most* has different scopal properties than *at least*, does not account for the data we observe. More importantly, it means that the two readings modified numerals display when they are combined with modals and the way modified numerals take scope with respect to modals should be treated as two separate phenomena. More evidence for the latter claim comes from examples with modified numerals and universal nominal quantifiers. These sentences, like the ones with existential modals, allow only one scope configuration but do give rise to two readings. The next question is what such a 'scope-independent' theory of modified numerals might look like. I leave this issue for future research.

References

Büring, D. (2008). The least *at least* can do. In *Proceedings of the west coast conference on formal linguistics* (Vol. 26, p. 114-120).

Coppock, E., & Brochhagen, T. (2013). Raising and resolving issues with scalar modifiers. *Semantics & Pragmatics*, *6*(3), 1-57.

Fox, D. (2000). *Economy and semantic interpretation.* Cambridge, MA: MIT Press.

Geurts, B., & Nouwen, R. (2007). At least *at al.*: the semantics of scalar modifiers. *Language*, *83*(3), 533-559.

Heim, I. (2000). Degree operators and scope. In *Proceedings of SALT 10* (p. 40-64). Ithaca, NY: Cornell Linguistics Club.

Homer, V. (2015). Neg-raising and positive polarity: The view from modals. *Semantics and Pragmatics*, *8*(4), 1-88.

Iatridou, S., & Zeijlstra, H. (2010). On the scopal interaction of negation and deontic modals. In M. Aloni, H. Bastiaanse, T. de Jager, & K. Schulz (Eds.), *Proceedings of the 17th Amsterdam Colloquium.* Berlin: Springer Verlag.

Iatridou, S., & Zeijlstra, H. (2013). Negation, polarity, and deontic modals. *Linguistic Inquiry*, *44*(4), 529-568.

Kennedy, C. (2015). A 'de-Fregean' semantics (and neo-Gricean pragmatics) for modified and unmodified numerals. *Semantics and Pragmatics*, *8*(10), 1–44.

Nouwen, R. (2010). Two kinds of modified numerals. *Semantics & Pragmatics*, *3*(3), 1-41.

Penka, D. (2014). 'At most' at last. In E. Csipak & H. Zeijlstra (Eds.), *Proceedings of Sinn & Bedeutung 19.*

Sauerland, U. (2004). Scalar implicatures in complex sentences. *Linguistics & Philosophy*, *27*(3).

Schwarz, B. (2011). *Remarks on class B numeral modifiers.* Handout of workshop *Indefinites and beyond*, Universität Göttingen.

Schwarz, B. (2013). *At least* and quantity implicature: Choices and consequences. In M. Aloni, M. Franke, & F. Roelofsen (Eds.), *Proceedings of the 19th Amsterdam Colloquium.*

Spector, B. (2014). Global positive polarity items and obligatory exhaustivity. *Semantics & Pragmatics*, *7*(11), 1-61.