Knowledge, Justification and Reason-Based Belief *

Paul Egré[1]†, Paul Marty[2] and Bryan Renne[3]‡

[1] Institut Jean-Nicod, CNRS, ENS, PSL
paul.egre@ens.fr
[2] MIT, Linguistics Department
pmarty@mit.edu
[3] ILLC, Amsterdam
bryan@renne.org

Abstract

Can the ordinary concept of knowledge be defined in terms of justified true belief ('JTB')? We argue that the Gettier cases need not invalidate every analysis of knowledge in terms of justified true belief, depending on how the notion of justification is understood. We present a framework for the representation of reason-based belief and use it to define a notion of true belief supported by adequate reasons. An adequate reason is characterized as being infallible, namely as a justification supporting only true propositions. We give a tentative definition of knowledge in those terms. We present two variants of our axiomatics for belief, which differ mostly on how subjective and accessible the notions of reason and support are taken to be. A more detailed version of our approach and its model theory appears in an expanded version of this paper.

1 Knowledge and Justified True Belief

Can the ordinary concept of knowledge be defined in terms of justified true belief ('JTB')? Since Gettier (1963), the answer to this question is widely considered negative. Gettier produced two cases convincingly suggesting that a belief can be true, justified, and yet fall short of knowledge.

Our point of departure in this paper is the following: even though we agree with the force of Gettier's cases (see Machery et al. 2015), we share with others (in particular Chisholm 1977; Dretske 1971; Goldman 1979; Sosa 1974, 1979; Turri 2012) the intuition that those examples do not invalidate every analysis of knowledge in terms of justified true belief, depending on how the notion of justification is understood. What Gettier's cases teach us is that an agent can have a justification for believing a proposition that is *plausible* on internal grounds, without that justification being properly *adequate* to the truth of the proposition in question. But if so, then Gettier cases only show that knowledge is not identical with JTB under an internalist conception of justification.

In the wake of work done in Justification Logic (Artemov 2008; Artemov and Fitting 2011; Baltag et al. 2014), we propose an explicit treatment of reasons for belief, which we use to tease

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apart two notions of justified true belief, and to defend an externalist version of the equation between knowledge and JTB. The gist of our account lies in the distinction between reasons that *(merely) support* belief in a proposition and reasons that are not only supportive but are also what we call *adequate*. Fundamentally, we view a reason as adequate only if every proposition it supports is true. We thereby endorse a form of infallibilism about knowledge (see Dutant 2010, 2015 for similar views).

In this paper, we give mostly an outline of our system: we refer to Egré et al. (2015) for more details, both technical and philosophical. To make our paper informative relative to the extended version, we present two versions of the axiomatic treatment of the notion of reason-based belief. In the more extended version, we introduce only the second of those versions, the system QRBB, but we find it worthwhile to also present the first system we came up with, in order to better situate our approach. Below we also briefly mention two objections that can be made against our approach: (i) the objection that agents have knowledge only if every reason they have for a proposition is adequate, and (ii) the objection that there appears to be knowledge from false lemmas (Warfield 2005).

2 Reason-based belief

2.1 Motivations

The main motivation behind our approach is the observation that there are two kinds of justified true beliefs: on the one hand true propositions that are believed on the basis of good or adequate reasons, and on the other true propositions believed on the basis of bad or inadequate reasons. A Gettierized belief is when a proposition φ is true and believed on the basis of some reason that is not adequate. A Gettier-proof belief is when a proposition φ is true and believed on the basis of an adequate reason. To represent both notions, we need to quantify over reasons, and first of all to represent the notion of an adequate reason as well as the notion of believing a proposition on the basis of some supporting reason.

Adequacy, as we use it, is a term of art: instead of saying that a reason is adequate, we could say that a reason is *good*, or *infallible*, provided goodness again is understood in externalist terms, independently of how good the reason appears to the agent. An important remark is that the notion of adequacy of a reason is not relative to a specific proposition. This is an important difference with Dretske (1971)'s account of knowledge in terms of *conclusive* reasons. For Dretske, a reason is only conclusive *for* a given proposition. In our approach, reasons are adequate or not, without this explicit relativity to a specific proposition.

2.2 Syntax and quantificational axioms

The two systems of reason-based belief to be presented rest on the following syntax. F is the set of formulas φ defined by the following grammar, in which R is a set of reason symbols, and P a set of propositional symbols, both sets being disjoint:

$$\varphi \quad ::= \quad p \mid \neg\varphi \mid (\varphi \vee \varphi) \mid (r : \varphi) \mid r \mid B\varphi \mid r = r \mid (\forall r)\varphi$$
$$p \in P,\ r \in R$$

Other Boolean connectives we shall use are defined as abbreviations, and brackets are removed when no ambiguity results. The main innovation relative to a system of standard epistemic logic is the use of reason variables. $(r : \varphi)$ is shorthand for: 'reason r supports the proposition

φ'; r alone is shorthand for 'reason r is adequate'. The quantificational core of both systems involves the axioms in Table 1.

Quantificational axioms
(UD) $(\forall r)(\varphi \to \psi) \to (\varphi \to (\forall r)\psi)$, where r is not free in φ
(UI) $(\forall r)\varphi \to \varphi[s/r]$, where s is free for r in φ
(EP) $r = r$
(EN) $\neg(r = s)$, where r and s are syntactically different
Quantificational Rule
$\dfrac{\varphi}{(\forall r)\varphi}$ (Gen)

Table 1. Quantificational axioms and rule

As an illustration of the syntax, consider the sentence: 'John believes that 2+2=4 on the basis of his calculations. We would represent this as: $B(r : p) \wedge Br$, letting p stand for '2+2=4' and r refer to John's calculating evidence. $B(r : p)$ says that John believes that r supports φ, that is John is inclined to believe p on the basis of r, and Br says that John actually endorses his reason or justification. We use the paraphrase "John is inclined to believe p on the basis of r" as another paraphrase for $B(r : p)$, since John may believe that r supports p without thinking of r as being an adequate reason.

Consider an agent, John, who concludes that 2+2=4 on the basis of his calculations, but who does not quite trust his calculating capacities; nevertheless, John would trust his teacher telling him that his calculations are correct. We may represent this by: $B(r : p) \wedge \neg Br \wedge \neg B \neg r$: John is inclined to believe that 2+2=4 on the basis of his calculations, but is not sure whether his calculations are correct. However, if s were the evidence of his friend confirming, John would believe that s supports that his calculations are adequate. We would then have: $B(s : r) \wedge Bs$, to mean that John believes that his teacher's testimony supports the adequacy of his calculations, and furthermore John believes that s is adequate, as a result of which John would believe r to be adequate.

Our account is close in spirit to Dutant's account of knowledge in terms of method-based belief (Dutant 2010, 2015). Dutant conceives of justification primarily as methods of belief formation, and we likewise think of reasons not fundamentally as propositions but as processes or experiences by which those propositions come to be believed.[1] There is, however, a noteworthy difference: semantically, we treat $(r : p)$ as a proposition while Dutant does not; instead, Dutant proposes to handle belief as a binary operator, involving a method-argument and a propositional argument. An objection that can be made to our approach is that it fails to separate the content from the method. We acknowledge this limitation of our system, though we believe that it is possible to impose an interpretation of our syntax compatible with Dutant's idea, namely such that the first reason-argument appearing in the scope of a belief operator is to refer to the method by which the beliefs are produced.

[1] We are indebted to J. Dutant for bringing us to endorse that view. In a preliminary version of this work, we left open the possibility that reason symbols could denote propositions directly.

2.3 The system RBBS

The first system we propose is a quantifier-free system of 'subjective' reason-based belief. We call it subjective because to say that a reason supports a proposition is to say that it is *ipso facto* a reason to believe that proposition, and conversely (see the axiom (IS) below). Axiom (IS) is very strong in ensuring that all reasons are accessible, and that if a reason is believed to support a proposition, it is thereby supportive. In this system, $r : \varphi$, which we said is shorthand for 'r supports φ', may therefore also be read as: 'r is a reason to believe φ'.

Axiom Schemes	
(CL)	Axiom Schemes of Classical Propositional Logic
(RK)	$r : (\varphi \to \psi) \to (r : \varphi \to r : \psi)$
(A)	$r : \varphi \to (r \to \varphi)$
(RB)	$r : \varphi \to (Br \to B\varphi)$
(IS)	$B(r : \varphi) \leftrightarrow (r : \varphi)$
(D)	$B\varphi \to \neg B \neg \varphi$

Rules

$$\frac{\varphi \to \psi \quad \varphi}{\psi}\ \text{(MP)} \qquad \frac{\varphi}{r : \varphi}\ \text{(RN)} \qquad \frac{\varphi \leftrightarrow \psi}{B\varphi \leftrightarrow B\psi}\ \text{(E)}$$

Table 2. The theory RBBS

As for mnemonics, (CL) is 'Classical Logic', (MP) is 'Modus Ponens', (RK) is Kripke's axiom K of modal logic (used here for reasons), (RN) is 'Reason Necessitation', (IS) is 'Internal Support', (A) is 'Adequacy',' (RB) is 'Reasons to Believe', (D) is a consistency requirement on belief, and (E) is a well-known rule from minimal modal logic (Chellas 1980).

(CL) and (MP) say that RBBS is an extension of classical propositional logic. (D) says that the agent's beliefs are consistent: the agent cannot have contradictory beliefs (i.e., believe both φ and $\neg\varphi$ for some φ). (E) says that the agent's beliefs do not distinguish between provably equivalent formulas. (RK) says that reasons are closed under material implication, and (RN) says that reasons support all derivable formulas.

(A) says that if r is a reason to believe φ and r is an adequate reason, then φ is true. (RB) says that if r is a reason to believe φ and the agent believes that r is an adequate reason, then the agent believes φ. (IS) says that an agent believes that r is a reason to believe φ if and only if r is a reason to believe φ. The following two closure properties are noteworthy consequences of (IS) and (RB) in our system:

$$\text{(RS)} \quad B(r : \varphi) \to (Br \to B\varphi)$$
$$\text{(RB+)} \quad Br \to (r : \varphi \to B(r : \varphi))$$

Technically, only the left-to-right direction of (IS) is needed to derive (RS) from (RB), but the biconditional version of (IS) also allows us to get (RB+) (directly, by weakening). (RB+) may appear implausibly strong, but so is (RB), and it appears natural to have both as soon as one of them is accepted.

2.4 The system RBB

	Axiom Schemes
(CL)	Axiom Schemes of Classical Propositional Logic
(RK)	$r:(\varphi \to \psi) \to (r:\varphi \to r:\psi)$
(A)	$r:\varphi \to (r \to \varphi)$
(BRK)	$B(r:\varphi) \to (B(r:(\varphi \to \psi)) \to B(r:\psi))$
(BA)	$B(r:\varphi) \to (Br \to B\varphi)$
(AS)	$B(r:\varphi) \to (r \to (r:\varphi))$
(D)	$B\varphi \to \neg B\neg\varphi$
	Rules
	$\dfrac{\varphi \to \psi \quad \varphi}{\psi}$ (MP) $\quad \dfrac{\varphi}{r:\varphi}$ (RN) $\quad \dfrac{\varphi \leftrightarrow \psi}{B\varphi \leftrightarrow B\psi}$ (E)

Table 3. The theory RBB

The system RBB is almost exactly like RBBS except for the three axioms (BRK), (BA), and (AS), which we have instead of (RB) and (IS). In RBBS, there is a single axiom constraining the notion of adequacy for reasons, namely axiom (A), and the two axioms (RB) and (IS) only concern the interaction between belief and support for reasons. In RBB, we have an additional axiom (AS) about adequacy, and the two axioms (BRK) and (BA) concern the interaction between belief and support for reasons.

RBB is not a 'subjective' system of reason-based belief since we give up both directions of (IS): a reason r can be a supporting reason for a proposition φ without being believed to support φ, and conversely, one can believe a reason r to support a proposition φ without there being support (we are thinking, here, of cases of delusion or hallucination). This means that in RBB the support relation is allowed to be independent of one's belief. An agent can incorrectly believe a reason to support a proposition. On the other hand, (AS) says that when an agent believes a reason to support a proposition, the reason can only be adequate provided the support relation holds indeed.

Another important difference between RBB and RBBS is that RBBS predicts stronger closure conditions on belief. In both RBB and RBBS, reasons are strong, in virtue of the closure axiom (RK) and of rule (RN): in particular, every reason supports every logical truth. Consider an agent who holds a belief that some reason is adequate, namely for whom Br holds. Then, because r supports every logical truth, it follows by (RB) that the agent believes every logical truth. An agent who believes a single reason to be adequate is thereby logically omniscient. This result is not welcome. In that regard, (BA) is a natural weakening of (RB).

For those various reasons, we think RBB is a better system than RBBS: it predicts fewer closure properties on belief, and it separates out a weak notion of belief from a strong notion of reason more neatly. In what follows, we therefore call QRBB the system that results from the combination of RBB with the quantificational axioms and rule of Table 3, and likewise, we call QRBBS the system resulting from RBBS. Irrespective of the choice between those systems, axiom (A) remains in a sense the central characterization of adequacy for reasons in our approach.

2.5 Semantics

We refer to Egré et al. (2015) for a systematic exploration of the system QRBB and of its model theory. Here, we only point out some essential facts about the interpretation of formulae. The models are structures $M = (W, [\cdot], N, V)$ where W is a nonempty set of possible worlds, N is a neighborhood function associating sets of worlds to each world, and $[\cdot]$ is a function mapping each reason $r \in R$ to a binary relation $[r] \subseteq W \times W$ on the set of possible worlds. Let $[r](w) := \{v \in W; w[r]v\}$, that is $[r](w)$ is the set of r-accessible worlds from w. The main clauses of the semantics are:

- $M, w \models B\varphi$ iff the set $[\![\varphi]\!]$ of φ-worlds belongs to $N(w)$.
- $M, w \models r$ iff $w \in [r](w)$.
- $M, w \models r : \varphi$ iff $[r](w) \subseteq [\![\varphi]\!]$.
- $M, w \models \forall r\varphi$ iff $M, w \models \varphi[s/r]$ for each s free for r in φ.

By imposing appropriate constraints on the function N and on the cardinality of R, the logic can be shown to be sound and complete for the semantics.

One aspect of the semantics worth pointing out is that reasons are handled by means of accessibility relations. Basically, a reason is adequate at a world w provided that w is accessible from itself via the relation, that is if the relation is reflexive at w. Thus, we model adequacy for reasons in a way that is similar to the way in which factivity is usually handled for knowledge when knowledge is taken as a primitive operator. In what follows, however, we propose to define knowledge as a special type of justified true belief, namely as true belief supported by adequate reasons.

3 Two notions of JTB

Whether in QRBB or QRBBS, there are (at least) two natural ways to define JTB, which we call 'external' vs. 'internal'.[2]

- $\mathsf{JTB}^e_r(\varphi) := B(r : \varphi) \wedge Br \wedge r$ (external JTB)
- $\mathsf{JTB}^i_r(\varphi) := B(r : \varphi) \wedge Br \wedge \varphi$ (internal JTB)

Both notions of JTB are factive, but the notions do not have the same properties. To illustrate the difference between the two cases, consider Gettier's case II: Smith wrongly believes Jones owns a Ford (p) on the basis of various plausible inductive evidence (represented by the symbol r), and 'realizes the entailment' (Gettier 1963) that either Jones owns a Ford or Brown is in Barcelona. As it turns out, Brown is in Barcelona (q). This is a case in which: $B(r : p) \wedge Br \wedge B(r : (p \to p \vee q)) \wedge q \wedge \neg p$. It follows in QRBB that the agent has $\mathsf{JTB}^i_r(p \vee q)$ without $\mathsf{JTB}^e_r(p \vee q)$, for by (A) the falsity of p implies $\neg r$.

Our analysis can be extended to cover more complex cases, such as Goldman-Ginet cases (Goldman 1976), in which an agent has a correct belief in a proposition, but in which the justification is arguably not adequate (because lucky). For instance, consider an agent traveling in the country of fake barns, and thinking of the only true barn that it is a barn (p), based on his visual experience. We can describe the situation as: $B(r : p) \wedge (r : p) \wedge Br \wedge p$, without assuming

[2] In QRBBS the definitions are the same, except that we can write $r : \phi$ instead of $B(r : \phi)$, due to axiom (IS).

r to hold, namely the reason to be adequate. This means that the agent's belief would be true, but not adequately true, given that it is lucky. We do not capture this connection between adequacy and luck in our axioms, however. This implies that axioms (A) and (AS) are not jointly sufficient for a reason to be adequate, but are only necessary conditions. This comports with our externalist inspiration, for our semantics basically treats adequacy as a property of a reason and a world: two worlds could be exactly alike relative to an agent's belief, without the reason being adequate in both, or inadequate in both (see Williamson 2000 on *good* vs. *bad* cases).

4 Knowledge and inadequate reasons

In view of the preceding, we surmise that knowledge may be viewed as a form of justified true belief, provided the justification is adequate. That is, we propose to defend that

$$K\varphi \text{ iff } \exists r(\mathsf{JTB}^e_r\varphi)$$

This definition raises two main objections. The first is that the definition is potentially too weak. It does not rule out cases in which an agent believes one and the same proposition based on at least two different reasons, one adequate, the other inadequate. We may require for agents to have knowledge only if every reason they have for a proposition is adequate, i.e. $\forall r(\mathsf{JTB}^i_r\varphi \rightarrow \varphi)$. We reject that option: such cases are better described as cases in which an agent knows a proposition, but whose knowledge is *confused* and of *lesser quality* than that of an agent having equally many reasons, all of them adequate (we refer to Egré et al. 2015 for more details). In that sense, our account of knowledge only commits us to a weak form of infallibilism about knowledge: an agent who knows p can still have misconceived beliefs pertaining to p.

A symmetric objection to our account is that it is potentially too strong: several authors (Fitelson 2010; Sorensen 2015; Warfield 2005) consider that there are cases in which we can get knowledge from false lemmas. I may know that I am not late for the meeting if I believe that it is currently 2:58pm, when in fact it is 2:56pm, assuming the meeting is at 7pm. On the present account, my reason to believe that it is currently less than 7pm is inadequate, simply because it also supports the false proposition that it is 2:58pm. This is a case in which I have JTB^i that it is less than 7pm, without having JTB^e that it is less than 7pm.

One option in the face of such examples is to bite the bullet and to resist the intuition that I know I am not late for the meeting. We are not sure that it is the best response. We think the problem concerns how much approximation is tolerated in forming beliefs based on one's evidence. My reading '2:58pm' is obviously wrong regarding the actual time, but still *close enough* to the actual time to be relevantly used. It would be different if the agent's watch indicated 6pm when it is 2:56pm, or even 9am. For the latter cases, our intuition is that I merely have a luckily true belief. If, when I see '2.58pm' (r) on my watch, I form the belief 'it is around 2:58pm' (p), and from that proposition I infer 'it is less than 7pm' (q), then my reason r now is veridical for both p and q.

A way out, therefore, might be to relativize the adequacy of a reason to the selection of an appropriate domain of propositions supported by that reason. This nevertheless puts pressure on us to clarify the relation of support between a reason and a proposition. In our statement of the axiom (A), we include no restriction on the support relation. We think it is better to be normative, and not to include any such restriction in the definition of knowledge in terms of JTB^e. On the other hand, we are ready to accept that in actual ascriptions of knowledge, the adequacy of reasons is referred to a set of relevant propositions that is contextually determined.

5 Perspectives

The two systems presented here do not offer ways of combining reasons, as each axiom or rule always involves a single reason. To that extent, both systems are mostly simplifications of more realistic systems of the sort discussed by Artemov (2008), Artemov and Fitting (2011), Baltag et al. (2014), and Dutant (2015). In Egré et al. (2015), we present variants of QRBB in which reasons can be combined. Some further issues are considered there, in particular regarding the implications of the current approach for higher-order knowledge (see Williamson 2000), and the question of the admission of basic beliefs, namely beliefs based on no further reason.

One may also wonder how our approach relates to the externalist view that knowledge is not definable in terms of belief plus other conditions, a view defended by Zagzebski (1994), and Williamson (2000). *Prima facie*, we may appear to be at odds with that conception, since we explicitly endorse a logical analysis of knowledge in terms of other concepts. As our analysis shows, however, we do not give a reductive analysis of the notion of adequacy, but only bring to light some substantive constraints on this notion.

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