Steedman's Temporality Proposal and Finite Automata

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Abstract. The proposal from Steedman 2005 that "the formal devices" required for temporality in linguistic semantics "are those related to representation of causality and goal-directed action" is developed using finite automata, implicit in which are notions of causality (labelled transitions) and goal-directed action (final/accepting states). Three strands in theories of aspect isolated in Binnick 2006 are examined: temporal relations, phases, and boundedness. The commonly held dichotomy between states and events is linked to that between programs and their runs, as strings representing events are extended to automata.

1 Introduction

In a wide-ranging study, Steedman (2005) proposes that "the so-called temporal semantics of natural language is not primarily to do with time at all" (as given say, by the real line $\mathbb{R}$) but that "the formal devices we need are those related to representation of causality and goal-directed action." Finite automata are simple candidates for such devices, with notions of causality and goal-directed action implicit in an automaton's transition table and accepting (final) states. Stepping back from automata, the strings which such automata may or may not accept are employed in Fernando 2004, 2008 to represent events of various kinds (for linguistic semantics). Does this step (back) not downplay and obscure, however, the notions of causality and goal-directed action offered by a finite-state approach? The present paper is an attempt to clarify these notions, linking the relationship between strings and automata to the dichotomy between events and states widely recognized (e.g. Kamp and Reyle 1993). The more general aim is to explain why Steedman's proposal – henceforth ST – is interesting, and how it might be implemented through finite-state methods – ST_{fa}.

Motivation for ST can be found from discourse coherence, (1), down to tense and aspect, (2).

(1) Max fell. Mary pushed him.
(2) John [*has] left, but is back.

In (1), the push is most naturally understood as preceding the fall because (1) suggests a causal connection (e.g. Asher and Lascarides 2003), while in (2), the difference between the simple past and the present perfect (*has left*) is that under the latter, the result of John's departure persists through (2)'s utterance (incompatible with him back). In (1) and (2), time and its modelling by $\mathbb{R}$ are secondary to the changes or, as the case may be, non-changes that are communicated. Put crudely under ST_{fa}, time is the result of running automata. A more moderate position is that time is conceived in ST to be relative (as opposed to absolute), the raison d'être of which is to place some set E of events and states in some order. Fleshed out according to Russell and Wiener (e.g. Kamp and Reyle 1993), this relative conception of time can be brought in line with the ST_{fa}-view of time as runtime (Fernando 2011a). Apart from their runs, however, do the automata merit a place in semantics? Consider the habitual (3a), in contrast to the episodic (3b).

(3) a. Tess eats dal.
  b. Tess is eating dal.

Not only can we assert (3a) and at the same time deny (3b), it is not entirely clear that the truth of (3a) at a world and time can be reduced to its episodic instances at that world. Under the *rules-and-regulations* view defended in Carlson 1995, generic sentences such as (3a) are true at a world and time only if at that world and time, "structures" exist that are not "the episodic instances but rather the causal forces behind those instances" (page 225). Can automata serve as such structures?

The present work sets out to make this plausible, based on a modelling of episodic sentences through strings representing runs of an automaton. This modelling is explained in the next two sections with an eye to the subsequent introduction (in the final section) of automata as causal forces. The plot, very briefly, is as follows. From a set E of events and states (of interest), strings are formed on which a system of projections π_X is defined that captures the Russell-Wiener construction for arbitrary finite subsets X of E. Event-tokens (in E) with interval temporal extent bounded to the left by pre-states and to the right by post-states are generalized (beyond Russell-Wiener) to event-types that are analyzed in terms of temporal propositions subject to forces. Restricting the alphabet to a finite set paves the way for finite-state methods that

(i) transcend, in a precise sense recalled below, first-order logic, while keeping validity decidable, and
(ii) bounds the granularity at which meaning is described, yielding finitary fragments that contrast sharply with complete histories/possible worlds.

The resulting approach is allied with the "toolbox" conception of natural languages propounded in Cooper and Ranta 2008, centered around local use (and action), away from a monolithic truth (as in the representation of time through the real line $\mathbb{R}$).

2 Strings for Russell-Wiener and beyond

For quick orientation, it is instructive to represent a calendar year by the string

$$s_{mo} \;:=\; \boxed{\mathsf{Jan}}\boxed{\mathsf{Feb}}\cdots\boxed{\mathsf{Dec}}$$

of length 12 (with a month in each box), or (adding one of 31 days d1, d2,...d31) the string

$$s_{mo,dy} \;\; := \;\; \fbox{Jan,d1}\fbox{Jan,d2}\cdots\fbox{Dec,d31}$$

of length 365 (for a non-leap year). Unlike the points in the real line $\mathbb{R}$, a box can be split by enlarging the set E of symbols we can put in it, as illustrated by the step from $\fbox{Jan}$ in s_{mo} to $\fbox{Jan,d1}\fbox{Jan,d2}\cdots\fbox{Jan,d31}$ in $s_{mo,dy}$. Or, going the opposite direction, from $s_{mo,dy}$ to s_{mo}, let us define two functions ρ_{mo} and bc that respectively, restricts E to the months $mo = \{\mathsf{Jan},\mathsf{Feb},\ldots\mathsf{Dec}\}$

$$\rho_{mo}(s_{mo,dy}) \;\; = \;\; \fbox{Jan}^{31}\fbox{Feb}^{28}\cdots\fbox{Dec}^{31}$$

and compresses a block α^n to α

$$\mathit{bc}(\fbox{Jan}^{31}\fbox{Feb}^{28}\cdots\fbox{Dec}^{31}) \;\; = \;\; \fbox{Jan}\fbox{Feb}\cdots\fbox{Dec} \;\; = \;\; s_{mo}$$

so that $\mathit{bc}(\rho_{mo}(s_{mo,dy})) = s_{mo}$. More precisely, for $X \subseteq E$, ρ_X sees only the elements of X (discarding non-X's)

$$\rho_X(\alpha_1\alpha_2\cdots\alpha_n) \;\; := \;\; (\alpha_1 \cap X)(\alpha_2 \cap X)\cdots(\alpha_n \cap X)$$

whereas *block compression* bc sees only change (discarding repetitions/stuttering)

$$\mathit{bc}(s) \;\; := \;\; \begin{cases} \mathit{bc}(\alpha s') & \text{if } s = \alpha\alpha s' \\ \alpha\,\mathit{bc}(\alpha' s') & \text{if } s = \alpha\alpha' s' \text{ with } \alpha \neq \alpha' \\ s & \text{otherwise.} \end{cases}$$

Let bc_X be the composition $\rho_X; \mathit{bc}$ mapping s to

$$\mathit{bc}_X(s) \;\; := \;\; \mathit{bc}(\rho_X(s)),$$

so that

$$\mathit{bc}_{\{\mathsf{Jan}\}}(s_{mo,dy}) = \mathit{bc}_{\{\mathsf{Jan}\}}(s_{mo}) = \fbox{Jan}\fbox{} \qquad \mathit{bc}_{\{\mathsf{d1}\}}(s_{mo,dy}) = (\fbox{d1}\fbox{})^{12}$$

$$\mathit{bc}_{\{\mathsf{Feb}\}}(s_{mo,dy}) = \mathit{bc}_{\{\mathsf{Feb}\}}(s_{mo}) = \fbox{}\fbox{Feb}\fbox{} \qquad \mathit{bc}_{\{\mathsf{d2}\}}(s_{mo,dy}) = (\fbox{}\fbox{d2})^{12}\fbox{}\ .$$

We can delete any initial or final empty boxes by a function *unpad*, which we apply after bc_X to form π_X

$$\pi_X(s) \;\; := \;\; \mathit{unpad}(\mathit{bc}_X(s))\ .$$

We can then say e is an *s-interval*, and write $s \models \text{interval}(e)$, if $\pi_{\{e\}}(s)$ is $\fbox{$e$}$

$$s \models \text{interval}(e) \;\; \overset{\text{def}}{\Leftrightarrow} \;\; \pi_{\{e\}}(s) = \fbox{e}\ .$$

Strings over the alphabet $\mathit{Pow}(E)$ of subsets of E relative to which each $e \in E$ is an interval have π_E-outputs in the language $\pi_E[\bigcap_{e\in E} \pi^{-1}_{\{e\}}\fbox{e}]$, which we abbreviate $\mathcal{L}_\pi(E)$

$$\mathcal{L}_\pi(E) \;\; := \;\; \{\pi_E(s) \mid s \in \mathit{Pow}(E)^* \text{ and } (\forall e \in E)\ \pi_{\{e\}}(s) = \fbox{e}\}\ .$$

For example, $\mathcal{L}_\pi(\{e\})$ consists of the single string $\boxed{e}$, whereas $\mathcal{L}_\pi(\{e, e'\})$ consists of 13 strings, one per interval relation in Allen 1983. Partitioning $\mathcal{L}_\pi(\{e, e'\})$ in terms of the relations $\prec$ of *(complete) precedence* and $\bigcirc$ of *overlap* used in the Russell-Wiener construction of time from E, we have

$$\mathcal{L}_\pi(\{e, e'\}) \quad = \quad \text{Allen}(e \bigcirc e') + \text{Allen}(e \prec e') + \text{Allen}(e' \prec e)$$

where Allen($e \bigcirc e'$) consists of the 9 strings in which e overlaps e'

$$\text{Allen}(e \bigcirc e') \quad := \quad (\boxed{e} + \boxed{e'} + \epsilon)\boxed{e, e'}(\boxed{e} + \boxed{e'} + \epsilon)$$

(with empty string ϵ), and Allen($e \prec e'$) consists of the 2 strings in which e precedes e'

$$\text{Allen}(e \prec e') \quad := \quad \boxed{e}\boxed{e'} + \boxed{e}\boxed{}\boxed{e'}$$

and similarly for Allen($e' \prec e$). In general, $\mathcal{L}_\pi(E)$ is larger than the set RW(E) of Russell-Wiener time orders over E because no two Russell-Wiener times (within a single time order) can be related by inclusion (subset). For finite E, the discrepancy between RW(E) and $\mathcal{L}_\pi(E)$ can be traced to the language $\mathcal{M}(E)$ of strings over $Pow(E)^*$, no *two* symbols in which are related by inclusion

$$\mathcal{M}(E) \quad := \quad \bigcup_{n \geq 1} \{\alpha_1 \cdots \alpha_n \in Pow(E)^n \mid \text{not } (\exists i, j \in \{1, \ldots, n\})\ \alpha_i \subset \alpha_j\} \ .$$

RW(E) is the part of $\mathcal{L}_\pi(E)$ in $\mathcal{M}(E)$

$$\text{RW}(E) \ \cong \ \mathcal{L}_\pi(E) \cap \mathcal{M}(E) \qquad \text{(for finite } E\text{)}.$$

To reconcile Russell-Wiener with $\mathcal{L}_\pi(E)$, an alternative to reducing $\mathcal{L}_\pi(E)$ is to neutralize the $\subseteq$-maximality requirement on RW-times by

(i) introducing for each $e \in E$, fresh "events" *pre*(e) and *post*(e) into an expansion $E_\pm$ of E

$$E_\pm \quad := \quad E \ \cup \ \{pre(e) \mid e \in E\} \ \cup \ \{post(e) \mid e \in E\}$$

and

(ii) delimiting occurrences of e in a string by *pre*(e) to the left and by *post*(e) to the right.

For instance, $\boxed{e}\boxed{}\boxed{e'}$ becomes

$$\boxed{e, pre(e')}\boxed{post(e), pre(e')}\boxed{post(e), e'} \ .$$

In general, given a string $s = \alpha_1 \cdots \alpha_n$ of subsets α_i of E, we define its *delimitation* to be the string $s_\pm = \alpha'_1 \cdots \alpha'_n$ of subsets α'_i of $E_\pm$ where

$$\begin{aligned} \alpha'_i \quad := \quad & \alpha_i \ \cup \ \{pre(e) \mid e \in (\bigcup_{j=i+1}^{n} \alpha_j) - \bigcup_{j=1}^{i} \alpha_j\} \\ & \cup \ \{post(e) \mid e \in (\bigcup_{j=1}^{i-1} \alpha_j) - \bigcup_{j=i}^{n} \alpha_j\} \ . \end{aligned}$$

As illustrated by $\boxed{e}\boxed{\ }\boxed{e'}$ above, *not* every element of $E_\pm$ need occur in $s_\pm$. That said, for every $s \in \mathcal{L}_\pi(E)$, if

$$s_\pm = \alpha'_1 \cdots \alpha'_n \ \text{ and } \ E_s := \bigcup_{i=1}^{n} \alpha'_i$$

then for finite E,

(4) $\quad s_\pm \in \ \mathcal{L}_\pi(E_s) \cap \mathcal{M}(E_s) \ \ (\cong \ \text{RW}(E_s))$

as exactly one of $e, \mathit{pre}(e)$ and $\mathit{post}(e)$ belongs to α'_i

$$|\{e, \mathit{pre}(e), \mathit{post}(e)\} \cap \alpha'_i| \ = \ 1$$

for all $e \in E$ and $1 \leq i \leq n$.

What if E is infinite? In this case, we glue together approximations of E given by the family $\mathit{Fin}(E)$ of finite subsets of E. More precisely, $\mathit{Fin}(E)$-indexed strings $(s_X)_{X \in \mathit{Fin}(E)}$ in which s_X can be calculated as $\pi_X(s_{X'})$ for any $X' \supseteq X$ are collected in the *inverse limit of* the system $(\pi_X)_{X \in \mathit{Fin}(E)}$ of functions

$$\varprojlim\, (\pi_X)_{X \in \mathit{Fin}(E)} \ := \ \{(s_X)_{X \in \mathit{Fin}(E)} \ \in \prod_{X \in \mathit{Fin}(E)} \mathit{Pow}(X)^* \ | \ s_X \ = \ \pi_X(s_{X'}) \text{ whenever } X \subseteq X' \in \mathit{Fin}(E)\} \ .$$

Every Russell-Wiener precedence relation $\prec$ on E has a representation $(s^{\prec}_X)_{X \in \mathit{Fin}(E)}$ in this inverse limit with

$$\prec \ = \ \{\langle e, e' \rangle \in E \times E \mid s^{\prec}_{\{e,e'\}} \in \text{Allen}(e \prec e')\}$$

and $s^{\prec}_X$ approximating the set $T_\prec$ of RW-times of $\prec$ on E as follows. A finite subset X of E induces a notion $\approx_X$ of equivalence on $T_\prec$ that holds between RW-times t and t' precisely if for all $e \in X$,

(i) $e \in t$ iff $e \in t'$, and
(ii) $(\exists e' \in t)\ e' \prec e$ iff $(\exists e' \in t')\ e' \prec e$, and
(iii) $(\exists e' \in t)\ e \prec e'$ iff $(\exists e' \in t')\ e \prec e'$.

The idea is that (i) says t and t' do not differ on e, (ii) on $\mathit{pre}(e)$, and (iii) on $\mathit{post}(e)$, as $\prec$ extends to RW-times t and t' by existential quantification

$$t \prec^T t' \ \stackrel{\text{def}}{\Leftrightarrow} \ (\exists e \in t)(\exists e' \in t')\ e \prec e' \ .$$

The number of $\approx_X$-equivalence classes cannot exceed the number $3^{|X|}$ of functions mapping $e \in X$ to one of $e, \mathit{pre}(e)$ and $\mathit{post}(e)$. Whether or not $T_\prec$ is isomorphic to the real line $\mathbb{R}$, the relation $\prec_X$ on $\approx_X$-equivalence classes U, U' given by

$$U \prec_X U' \ \stackrel{\text{def}}{\Leftrightarrow} \ (\exists t \in U)(\exists t' \in U')\ t \prec^T t'$$

is discrete (for X finite), and representable as a string $\hat{s}$ over the alphabet $Pow(X)$. Unpadding $\hat{s}$ yields the X-approximation $s_X^{\prec} := unpad(\hat{s})$.[1]

The **bounded** granularity of $\approx_X$ (for finite X) is at odds with the infinite precision of real numbers, arbitrarily small increments in which lead to Sorites chains/arguments problematic for the vagueness of natural language. For the inverse limit construction above, the maps $\pi_X : Pow(E)^* \rightarrow Pow(X)^*$ can be restricted to $\pi_{X',X} : Pow(X')^* \rightarrow Pow(X)^*$, for $X \subseteq X' \in Fin(E)$, each of which is computable by a finite-state transducer. In other words, Russell-Wiener time can be pictured (in accordance with ST_{fa}) as runtime up to a granularity bounded by the choice of a finite set of events. Enlarging the set of events refines the granularity. For instance, adding *pre*(e) to the left of an event e (to mark its past) and *post*(e) to its right (to mark its future) leads to refinements (suggested by Allen 1983) of the **temporal relations** $\bigcirc$ and $\prec$.

Given an event e, can we legitimately count *pre*(e) or *post*(e) as an event, as opposed to a state? Whether or not we can, their addition poses no obstacle to the Russell-Wiener construction. Where Russell-Wiener breaks down is over non-intervals. Not so, however, for the functions $\pi_{X',X}$. Although $s_X^{\prec} \in \mathcal{L}_\pi(X)$ for all $X \in Fin(E)$ and Russell-Wiener precedence $\prec$ on E, the inverse limit above also has $Fin(E)$-indexed strings $(s_X)_{X \in Fin(E)}$ for which an $e \in X$ may fail to be an s_X-interval. Consider again the example of $s_{mo,dy}$ above, for which it is perfectly reasonable that $\mathsf{d}n$ is not an $s_{mo,dy}$-interval. Working with strings, we may relax the interval requirement to extend our event-based account of time into one of event-types, with events understood broadly to include states (delimiting events). In terms of Kripke semantics, $Pow(X)^*$ specifies not only Kripke frames (for event-tokens/time) but Kripke models with valuations for elements of X construed as temporal propositions, also known as *fluents*. Indeed, the delimiting *pre*(e)-e-*post*(e) configurations become the *event nuclei* of Moens and Steedman 1988 once we sharpen *pre*(e) into e's *preparatory process*, and *post*(e) into e's *consequent state*. Can we flesh out these **phases** with finite automata?

Regular languages (accepted by finite automata) enjoy extensive closure properties, perhaps the most striking expression of which is the Büchi-Elgot theorem characterizing regular languages in terms of Monadic Second-Order Logic (MSO; e.g. Thomas 1997). Various modal logics are expressible in MSO, includ-

[1] Because π_X unpads (unlike bc_X), the string $s_X^{\prec}$ leaves out RW-times before all events in X, as well as after. We can repair this blemish by replacing π_X with bc_X, as

(4) $s_\pm \in \ \mathcal{L}_\pi(E_s) \cap \mathcal{M}(E_s) \ \ (\cong \ \mathrm{RW}(E_s))$

holds for s drawn not just from $\mathcal{L}_\pi(E)$ but from the larger language

$$\{bc_E(s) \mid s \in Pow(E)^* \text{ and } (\forall e \in E)\ s \models \mathrm{interval}(e)\} \ = \ (\Box + \epsilon)\mathcal{L}_\pi(E)(\Box + \epsilon).$$

An empty box $\Box$ may then be attached to the start and/or end of a string in Allen($e \prec e'$), and get filled upon delimiting. The projections π_X abstract away information about boundedness retained by bc_X. This information is redundant if as in Allen 1983, all intervals are assumed bounded.

ing well-known Priorean tense logics (with *nominals* interpretable as first-order objects, and other fluents as sets). Beyond the languages defined by truth $\models$ in these logics, there are relations such as π_X that characterize not only s-intervals but also the temporal relations $\bullet \in \{\prec, \bigcirc, \succ\}$

$$s \models e \bullet e' \quad \stackrel{\text{def}}{\Leftrightarrow} \quad \pi_{\{e,e'\}}(s) \in \text{Allen}(e \bullet e')$$

(for s-intervals e, e'). More in the next section, where we generalize π_X to relations with outputs that may fall outside that of π_X due to underspecification.

3 Non-deterministic relations and underspecification

The idea behind the present section is to derive satisfaction $\models$ of a formula[2] φ from a relation R_φ between strings s and s' such that

$$s \; R_\varphi \; s' \quad \text{iff} \quad s' \text{ makes } \varphi \text{ true at } s$$

with inputs s (fed to R_φ) as *indices* (to the left of $\models$)

$$s \models \varphi \quad \text{iff} \quad s \in \mathit{domain}(R_\varphi)$$

and outputs s' as *extensions/denotations* (Fernando 2011). For example, if φ is $e \prec e'$ then R_φ can be $\pi_{\{e,e'\}}$ restricted to outputs from Allen$(e \prec e')$. In stringing together sets of fluents, denotations develop the view of events as "intervals cum description" (van Benthem 1983) beyond pairs $\langle i, \varphi \rangle$ of descriptions φ given by *event-atoms* (Pratt-Hartmann 2005) and intervals i, separate representations of which are rendered otiose by strings as descriptions (Fernando 2011b). The precise choice of denotations can be a delicate matter. One reason is underspecification, which in the present context takes the form of a resistance to treating disjunction as non-deterministic choice. Take overlap $\bigcirc$ on s-intervals e and e'

$$s \models e \bigcirc e' \quad \text{iff} \quad \pi_{\{e,e'\}}(s) \in \text{Allen}(e \bigcirc e')$$

which we can generalize to non-s-intervals e, e' as

$$s \models e \bigcirc e' \quad \text{iff} \quad s \sqsupseteq \boxed{e, e'}$$

where *containment* $\sqsupseteq$ is a (non-deterministic) relation that weakens componentwise inclusion $\trianglerighteq$ between strings of the same length, pronounced *subsumption*,

$$\alpha_1 \cdots \alpha_n \trianglerighteq \alpha'_1 \cdots \alpha'_m \quad \stackrel{\text{def}}{\Leftrightarrow} \quad n = m \text{ and } \alpha_i \supseteq \alpha'_i \text{ for } 1 \leq i \leq n$$

to compare strings of different lengths

$$s \sqsupseteq s' \quad \stackrel{\text{def}}{\Leftrightarrow} \quad (\exists s'') \; s \trianglerighteq s'' \text{ and } \mathit{unpad}(s') = \mathit{unpad}(s'')$$

[2] For concreteness, the formula can (by Büchi-Elgot) be assumed to be from MSO.

(paralleling the construction of π_X from bc_X in the the previous section, the deterministic analog of $\sqsupseteq$, $\trianglerighteq$). Formulas φ for which satisfaction $\models$ is defined by the positive connectives $\wedge$, $\vee$ and $\exists$ can be expected to be $\sqsupseteq$-*persistent* in that

$$\text{whenever } s \sqsupseteq s' \text{ and } s' \models \varphi,\ \ s \models \varphi$$

(e.g., let φ be $e \bigcirc e'$[3]). On the other hand, the formula $e \prec e'$ is demonstrably *not* $\sqsupseteq$-persistent (let s' be $\boxed{e}\boxed{e'}$ and s be $\boxed{e}\boxed{e, e'}$). Even then, however, we might appeal to additional fluents $\mathit{pre}(e'), \mathit{post}(e)$ and stipulate

(5) $\quad s \models e \prec e' \quad \text{iff} \quad s_\pm \sqsupseteq \boxed{\mathit{pre}(e')}\boxed{\mathit{post}(e)}$

provided we agree to equate s with its delimitation $s_\pm$. But the step from s to $s_\pm$ goes against the partiality of information (behind underspecification). An alternative to (5) is to drop the delimitation $s_\pm$, asserting

$$s \models e \prec e' \quad \text{iff} \quad s \sqsupseteq \boxed{\mathit{pre}(e')}\boxed{\mathit{post}(e)}$$

for s satisfying suitable constraints on *pre* and *post*, short (to allow for underspecification) of setting $s = s_\pm$. (These constraints *ex*clude strings that $\sqsupseteq$-contain $\boxed{\mathit{pre}(e), e}$ or $\boxed{\mathit{post}(e), e}$ or $\boxed{\mathit{post}(e), \mathit{pre}(e)}$ or any of the strings in

$$\boxed{e}\boxed{}^{*}\boxed{\mathit{pre}(e)} + \boxed{\mathit{post}(e)}\boxed{}^{*}\boxed{e} + \boxed{\mathit{post}(e)}\boxed{}^{*}\boxed{\mathit{pre}(e)}$$

for any e.) Making $\boxed{\mathit{pre}(e')}\boxed{\mathit{post}(e)}$ the denotation of $e \prec e'$ at such an s steers clear of the trivial solutions $R_\varphi := \{\langle s, \boxed{\varphi}\rangle \mid s \models \varphi\}$ or $\{\langle s, s\rangle \mid s \models \varphi\}$, to unwind $\boxed{\varphi}$ without simply returning the full input. An argument for defining $R_{e \prec e'}$ one way or another may have less to do with the local concern for denotations of $e \prec e'$ than with the more global picture of how the relations R_φ fit together, for a range of different φ's. Insofar as denotations are proper parts of indices/inputs, they point to a piecewise approach to constructing indices using constraints (sometimes described as presuppositions).

4 Peering inside the box(es)

Having disputed the soundness of delimiting a string s (to recognize underspecification in s), we face the question of how to bound fluents φ (under pressure to draw inferences). One approach is to suppose there are forces acting for and against φ, and (similar to fluents $\mathit{pre}(e)$ and $\mathit{post}(e)$ delimiting e) fluents $\mathrm{f}\varphi$ decreeing "there is a force for φ" — at least, that is, in strings such that whenever $\mathrm{f}\varphi$ occurs, φ holds at the next moment unless some force opposes it. More precisely, let $\mathcal{F}_\varphi$ be the set of strings $s = \alpha_1 \cdots \alpha_n$ such that

[3] Or indeed any φ with a set $\mathcal{A}_\varphi$ of strings such that $s \models \varphi$ iff $(\exists s' \in \mathcal{A}_\varphi)\ s \sqsupseteq s'$. As the relation $\sqsupseteq$ (like $\trianglerighteq$ and *unpad*) can be computed by a finite-state transducer, the set of strings satisfying φ is regular provided $\mathcal{A}[\varphi]$ is.

$$\text{for all } i \in \{2, 3, \ldots, n\}, \quad \mathrm{f}\varphi \in \alpha_{i-1} \ \text{ implies } \ \varphi \in \alpha_i \text{ or } \mathrm{f}\overline{\varphi} \in \alpha_{i-1}$$

where $\overline{\varphi}$ is the negation of φ (whence $\mathrm{f}\overline{\varphi}$ says "there is a force opposed to φ"). $\mathcal{F}_\varphi$ can be expressed succinctly as

$$\boxed{\mathrm{f}\varphi}\boxed{} \ \Rightarrow \ (\boxed{}\boxed{\varphi} + \boxed{\mathrm{f}\overline{\varphi}}\boxed{})$$

using a binary operation $\Rightarrow$ that maps a pair of regular languages to a regular language (Fernando 2008, 2011). Closely related to $\mathcal{F}_\varphi$ is the regular language

$$\boxed{\varphi}\boxed{} \ \Rightarrow \ (\boxed{}\boxed{\varphi} + \boxed{\mathrm{f}\overline{\varphi}}\boxed{})$$

which intersected with

$$\boxed{}\boxed{\varphi} \ \Rightarrow \ (\boxed{\varphi}\boxed{} + \boxed{\mathrm{f}\varphi}\boxed{})$$

gives a regular language $\mathcal{I}_\varphi$ in which φ is *inertial* — i.e., φ persists forwards and backwards in the absence of forces on it. It is natural to view the languages $\mathcal{F}_\varphi$ and $\mathcal{I}_\varphi$ as constraints (satisfied by the strings belonging to them), associated with states (as opposed to events) represented by φ.

An example of a state that an inertial fluent may represent is given by the habitual (3a), repeated below, followed by the episodic (3b).

(3) a. Tess eats dal.
    b. Tess is eating dal.

We must be careful not to confuse a string s representing an instance of Tess eating dal with the fluent φ representing the habit of Tess eating dal. We may, however, expect the string s to belong to a language $\mathcal{L}_\varphi$ associated with φ. That is, $\mathcal{L}_\varphi$ sits alongside the sets $\mathcal{F}_\varphi$ and $\mathcal{I}_\varphi$ above as languages associated (in different ways) with φ. The obvious question is what is the status of such languages in our semantic theory?

I close with the suggestion that such languages are specifications of causal structures required by the *rules-and-regulations* view of Carlson 1995 to ground the truth of generic sentences. (These languages are underspecifications insofar as the causal structures are automata, but at present, I see no reason for that additional specificity.) Beyond $\mathcal{F}_\varphi, \mathcal{I}_\varphi$ and $\mathcal{L}_\varphi$, there are *episodes* in the sense of Moens and Steedman 1988 consisting of "sequences of causally or otherwise contingently related sequences of events" that (lest we confuse these with episodic instances at the world) are "more related to the notion of a plan of action or an explanation of an event's occurrence than to anything to do with time itself" (page 26). The intuition is that these languages are "resources for constructing local languages for *use* in particular situations," to quote Cooper and Ranta 2008 slightly out of context. A shift is made here from monolithic truth to use (or action) that is very much in line with (i) the focus in section 2 on maps for putting strings together, and (ii) the use in section 3 of input/output relations to interpret formulas. More questions are certainly left open than answered, an important issue being inference. For entailments between formulas φ and φ' based on satisfaction $\models$ (i.e., the domains of R_φ and $R_{\varphi'}$), we can use a language L to relativize the inclusion

$$\begin{aligned} \varphi \vdash_L \varphi' &\overset{\text{def}}{\Leftrightarrow} \quad (\forall s \in L)\ s \models \varphi \text{ implies } s \models \varphi' \\ &\text{iff} \quad L \cap domain(R_\varphi) \ \subseteq \ domain(R_{\varphi'}) \end{aligned}$$

which is decidable, provided all of R_φ, $R_{\varphi'}$ and L are regular. In fact, the inclusion remains decidable for L context-free, as observed by Makoto Kanazawa. Regular or not, L may consist of strings that represent episodic instances beyond those of any single world. It is one thing to say a causal force for L exists at a world and time, another to spell out the consequences for the episodic instances at a world and time. This much is clear from (3a) *versus* (3b). And once causal forces are recognized as structures underlying generic sentences such as (3a), can we keep them out of the semantics of episodic sentences such as (3b)?

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