Computing quantifier scope with witness sets

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1 Introduction

One of the basic claims of Generalized Quantifier Theory[1] is that natural language determiners like *most, every, some but not all, ...* denote binary functions from subsets of the domain of individuals D into the set of truth-values (or equivalently, binary relations between subsets of D). In this paper I challenge this view by developing an alternative theory of quantification where determiners denote unary functions from subsets of D. The basic idea, developed in section 2, is that if a determiner denotation is applied to a subset R (mnemonic for restrictor set) of D, the value is a pair $\langle R, W \rangle$, where W is the set of subsets of R which satisfy the condition imposed by the determiner. These subsets will be called witness sets or witnesses. For example, $[\![\textit{every man}]\!]$ has exactly one witness, namely the set $[\![\textit{man}]\!]$; $[\![\textit{no man}]\!]$ has only the empty set as a witness; and the witnesses of $[\![\textit{at least two men}]\!]$ are all subsets of $[\![\textit{men}]\!]$ whose cardinality is bigger than one.

Next, we need to specify (i) how the semantic composition of predicates and determiner phrase denotations proceeds, and (ii) how the different scope dependencies are computed. Since semantic composition is not driven by the denotation types, determiner phrase denotations can compose with the predicate in any order. The specification of the scope dependencies can be done in two different ways. The scope of a determiner phrase can either be fixed in the process of semantic composition, or it can be decoupled from semantic composition, by leaving the scope relations unspecified during semantic composition, and specifying them afterwards by means of so-called expansion operations. In section 3 I briefly sketch the first option, before turning in section 4 to the much more challenging second option. Section 5 concludes.

Given the determiner denotation type and the proposed principles of semantic composition, this theory predicts that the set of entities which are in the scope set but not the restrictor set is always irrelevant for the truth-conditions, providing a novel explanation for conservativity. Further, if scope dependencies are decoupled from semantic composition, and determined instead by applying so-called expansion operations to underspecified denotations, we capture (unlike most other theories of scope underspecification) (i) what the possible readings all have in common, and (ii) the intuitive idea that specifying the scope dependencies should amount to adding information, as opposed to choosing an element (a fully specified reading) from a set of fully specified readings (the denotation of the underspecified reading).

[1] Cf. Peters and Westerståhl (2006) for a comprehensive treatment of Generalized Quantifier Theory.

2 Restricting determiner denotation type

In Generalized Quantifier Theory, determiners like for example *every*, *most*, *no* denote binary functions from subsets of the domain to truth-values (or equivalently relations between subsets of the domain). Determiner phrases like *every man*, *most girls*, *no student* denote unary functions from subsets of the domain to truth-values. Here we shall depart from this analysis, and propose instead that (i) determiners are unary functions, taking as their single argument the restrictor set, and (ii) determiner phrases refer to pairs $\langle R, W\rangle$ consisting of a restrictor set R, and the set W of subsets of R, called witness sets. To illustrate, let the domain be $D = \{p_1, p_2, p_3, p_4, p_5, r_1, r_2, r_3, r_4\}$, and let the denotations of *paper* and *referees* be $[\![paper(s)]\!] = \{p_1, p_2, p_3, p_4, p_5\}$ and $[\![referees]\!] = \{r_1, r_2, r_3, r_4\}$, respectively.

$$[\![every]\!](A) := \langle A, \{X : X \subseteq A \wedge X = A\}\rangle = \langle A, \{A\}\rangle$$
$$[\![every]\!]([\![referee]\!]) = \langle\{r_1, r_2, r_3, r_4\}, \{\{r_1, r_2, r_3, r_4\}\}\rangle$$

$$[\![more\ than\ half\ the]\!](A) := \langle A, \{X : X \subseteq A \wedge |A - X| < |A \cap X|\}\rangle$$
$$[\![more\ than\ half\ the]\!](\{r_1, r_2, r_3, r_4\}) = \langle\{r_1, r_2, r_3, r_4\}, \{\{r_1, r_2, r_3\}, \{r_1, r_2, r_4\}, \{r_1, r_3, r_4\}, \{r_2, r_3, r_4\}, \{r_1, r_2, r_3, r_4\}\}\rangle$$

Given a determiner phrase *DP* with $[\![DP]\!] = \langle R, W\rangle$, the negation $[\![not\ DP]\!]$ consists of the same restrictor set R and the set of subsets of R which are not in W, i.e. $\langle R, \wp(R) - W\rangle$, which shall be represented as $\langle R, \overline{W}\rangle$. To illustrate again:

$$[\![not\ every]\!](A) := \langle A, \{X : X \subseteq A \wedge X \neq A\}\rangle$$
$$[\![not\ every]\!]([\![referee]\!]) = \langle\{r_1, r_2, r_3, r_4\}, \{\varnothing, \{r_1\}, \{r_2\}, \{r_3\}, \{r_4\}, \{r_1, r_2\}, \{r_1, r_3\}, \{r_1, r_4\}, \{r_2, r_3\}, \{r_2, r_4\}, \{r_3, r_4\}, \{r_1, r_2, r_3\}, \{r_1, r_2, r_4\}, \{r_1, r_3, r_4\}, \{r_2, r_3, r_4\}\}\rangle$$

$$[\![not\ exactly\ two]\!](A) := \langle A, \{X : X \subseteq A \wedge |X| \neq 2\}\rangle$$
$$[\![not\ exactly\ two]\!]([\![referees]\!]) = \langle\{r_1, r_2, r_3, r_4\}, \{\emptyset, \{r_1\}, \{r_2\}, \{r_3\}, \{r_4\}, \{r_1, r_2, r_3\}, \{r_1, r_2, r_4\}, \{r_1, r_3, r_4\}, \{r_2, r_3, r_4\}, \{r_1, r_2, r_3, r_4\}\}\rangle$$

The conjunction of two DP denotations $\langle R, W\rangle$ and $\langle R', W'\rangle$ is the pair consisting of the union $R \cup R'$ of the two restrictor sets, and the set of pairwise unions of witness sets in W and W', i.e. $\langle R, W\rangle \wedge \langle R', W'\rangle = \langle R \cup R', \{w \cup w' : w \in W \wedge w' \in W'\}\rangle$. For example, the denotation of *every referee and at most one paper* is:

$$\begin{aligned}[\![every\ referee\ and\ at\ most\ one\ paper]\!] &= [\![every\ referee]\!] \wedge [\![at\ most\ one\ paper]\!] \\ &= \langle[\![paper]\!], \{\emptyset, \{p_1\}, \{p_2\}, \{p_3\}, \{p_4\}, \{p_5\}\}\rangle \\ &= \langle[\![referee]\!] \cup [\![paper]\!], \\ &\quad \{\{r_1, r_2, r_3, r_4\}, \{r_1, r_2, r_3, r_4, p_1\}, \{r_1, r_2, r_3, r_4, p_2\}, \\ &\quad \{r_1, r_2, r_3, r_4, p_3\}, \{r_1, r_2, r_3, r_4, p_4\}, \\ &\quad \{r_1, r_2, r_3, r_4, p_5\}\}\rangle\end{aligned}$$

The exclusive disjunction of two DP denotations $\langle R, W\rangle$ and $\langle R', W'\rangle$ is the pair consisting of the union $R \cup R'$ of the two restrictor sets, and the union of the sets W and W' of witness sets, i.e. $\langle R, W\rangle \wedge \langle R', W'\rangle = \langle R \cup R', W \cup W'\rangle$. The denotation of *every referee or at most one paper* is:

$$\begin{aligned}
[\![\textit{every referee or at most one paper}]\!] &= [\![\textit{every referee}]\!] \vee [\![\textit{at most one paper}]\!] \\
&= \langle [\![\textit{referee}]\!], \{r_1, r_2, r_3, r_4\}\rangle \vee \\
&\quad \langle [\![\textit{paper}]\!], \{\emptyset, \{p_1\}, \{p_2\}, \{p_3\}, \{p_4\}, \{p_5\}\}\rangle \\
&= \langle [\![referee]\!] \cup [\![paper]\!], \\
&\quad \{\{r_1, r_2, r_3, r_4\}, \emptyset, \{p_1\}, \{p_2\}, \{p_3\}, \{p_4\}, \{p_5\}\}\rangle
\end{aligned}$$

Unlike in Generalised Quantifier Theory, the semantics of the determiner does not make reference to the scope set but only to the restrictor set. This has an important consequence, namely that the number of possible determiner denotations is considerably reduced, compared to Generalised Quantifier Theory. Given a universe of discourse D containing exactly two entities and the set $T = \{0, 1\}$ of truth-values, there are four subsets of D, $2^4 = 16$ functions from subsets of D into T (i.e. 16 possible determiner phrase denotations), and $2^{16} = 65536$ functions from subsets of D into DP denotations (i.e. 65536 possible determiner denotations). Out of these only 512 determiner denotations are conservative, where a determiner denotation D is conservative iff $D(A)(B) \leftrightarrow D(A)(A \cap B)$. To illustrate, the determiner denotation $[\![most]\!]$ is a conservative function, since the sentence *Most students smoke* is true (in a model M) if and only if *Most students are students who smoke.* is true (in M). Based on the fact that no clear example of a determiner *DET* has yet been found, where the equivalence $[\![DET]\!](A)(B) \leftrightarrow D(A)(A \cap B)$ does not hold, Barwise and Cooper (1981) and Keenan and Stavi (1986) have hypothesized that all natural language determiner denotations are conservative. If, on the other hand, determiners are functions from subsets R of D into pairs $\langle R, W\rangle$, where W is a set of subsets of R, i.e. $W \subseteq \wp(R)$, then there are exactly 512 possible determiner denotations. To see this, note that given the same universe of discourse $D = \{a, b\}$ there are $n_1 \times n_2 \times n_4 \times n_4$ possible determiner denotations, where $n_1 = 2$ is the number of possible witnesses given the restrictor set $\varnothing$ (the first witness is $\varnothing$, and the second is $\{\varnothing\}$), $n_2 = 4$ is the number of possible witnesses given the restrictor set $\{a\}$ (namely $\varnothing$, $\{\varnothing\}$, $\{\{a\}\}$ and $\{\varnothing, \{a\}\}$), $n_3 = 4$ is the number of possible witnesses given the restrictor set $\{b\}$ (namely $\varnothing$, $\{\varnothing\}$, $\{\{b\}\}$ and $\{\varnothing, \{b\}\}$), and $n_4 = 16$ is the number of possible witnesses given the restrictor set $\{a, b\}$ (namely all the subsets of $\{\varnothing, \{a\}, \{b\}, \{a, b\}\}$. So there are $2 \times 4 \times 4 \times 16 = 512$ possible DET denotations, showing that the present proposal considerably reduces the class of possible DET denotations. Assuming that the denotation type fixes the search space for the learner, the reduction of the class of possible DP denotations is attractive, as it narrows down the search space and thus simplifies the learnability problem (other things being equal).

3 Fixing quantifier scope during semantic composition

To illustrate the basic idea, note that the truth-condition of the sentence *Most dogs sleep.* can be formulated as follows: there is a witness set X of $[\![most\ dogs]\!]$ such that for all elements x in $[\![dogs]\!]$ it holds that x is an element of the witness set X if and only if x sleeps. And the truth-condition for the subject-wide-scope reading of *Most dogs chased some cat.* is that there is a witness set X of $[\![most\ dogs]\!]$ such that for all elements x in $[\![dogs]\!]$ it holds that $x \in X$ if and only if there is a witness set X' of $[\![some\ cat]\!]$ such that for all elements x' of $[\![cat]\!]$ it holds that $x' \in X'$ if and only if x chased x'.

We shall assume that the denotation of *chased* is the basic (open) formula $chase'(x_1, x_2)$, and that the determiner phrase denotations can compose with it in any order. The scope relations are determined by the order in which the determiner phrases compose, with the phrase composing first having narrowest scope. To 'record' this information, we assume a so-called scope dependency sequence, which in the case of basic formulas is the empty string, and which gets prefixed with the index of the variable that gets replaced by a determiner phrase.

(1) **Terms**:

a. If t is a variable, then t is a term.
b. If $t = det(cn)$, where $det \in \{most, no, every, exactly\ two, \ldots\}$ and $cn \in \{man, boy, girl, \ldots\}$, then t is a (quantified) term.
c. If t is a term, then so is $not(t)$.
d. If t_1, t_2 are terms, then so is $and(t_1, t_2)$

(2) **Basic formulas**:

a. Let R be an n-ary relation symbol and let $x_1, \ldots, x_n$ be pairwise distinct variables. Then $\langle R(x_1, \ldots, x_n), \epsilon\rangle$ is a formula relative to the empty scope dependency ϵ (where ϵ is the empty string).

b. Let $\langle R(x_1, \ldots, x_n), \epsilon\rangle$ be a formula. Then the semantic value of $\langle R(x_1, \ldots, x_n), \epsilon\rangle$ relative to model M and assignment function g is:

$$[\![\langle R(x_1, \ldots, x_n), \epsilon\rangle]\!]^{M,g} = 1 \text{ iff } \langle g(x_1), \ldots, g(x_n)\rangle \in I(R)$$

(3) **Quantified formulas**:

a. Let $\langle R(t_1, \ldots, t_{t-1}, x_i, t_{t+1}, \ldots, t_n), \delta\rangle$ be a formula, and let q be a quantified term. Then $\langle R(t_1, \ldots, t_{t-1}, q, t_{t+1}, \ldots, t_n), \langle i, \delta\rangle\rangle$ is a formula for any $1 \leq i \leq n$.

b. Let $[\![q]\!]^{M,g} = \langle R, W\rangle$. Then:
$[\![\langle R(t_1, \ldots, q, \ldots, t_n), \langle i, \delta\rangle\rangle]\!]^{M,g} = 1$ iff
$\exists Y \in W.\forall y \in R.[y \in Y \leftrightarrow [\![\langle R(t_1, \ldots, x_i, \ldots, t_n), \delta\rangle]\!]^{M,g[x_i/y]} = 1]$

4 Decoupling scope determination from semantic composition

4.1 Reformulating verb denotation and semantic composition

In what follows we shall separate the analysis of semantic role assignment from the analysis of the resolution of scope dependencies. The main motivation for this is the observation that in many languages the assignment of semantic roles is (i) encoded grammatically and therefore (ii) independent of the context of use, whereas the resolution of scope dependencies is often (i) not encoded grammatically, and moreover (ii) dependent on the particular lexical items involved as well as the particular context of use.[2] Semantic role assignment will be achieved by SEMANTIC COMPOSITION RULES, whereas scope dependencies are determined by the way in which EXPANSION RULES apply.

The semantic composition rules combining DP and verb (phrase) denotations assign a semantic role of the verb (phrase) denotation to the DP denotation. Given a verb like *give*, we shall assume that it assigns three (generalised) semantic roles, which we simply name $1, 2$ and 3, so that the person being assigned the role 1 gives the entity assigned the role 2 to the person assigned the role 3. If a DP denotation $[\![DP]\!]$ is assigned the (generalised) semantic role 2, we shall record this information by adding the pair $\langle 2, [\![DP]\!]\rangle$ to the verb (phrase) denotation. This requires the verb (phrase) denotation to be analyzed as a pair consisting of a relation R and a store S. To illustrate, consider:

(4) *More than half the referees read at least three papers.*

The denotation of this clause can be computed in two different ways. One possibility is to combine the verb and direct object denotations first, and then combine the resulting denotation with the subject denotation.

$$\mathcal{O}_2(\langle [\![read]\!], \emptyset\rangle, [\![at\ least\ three\ papers]\!]) = \langle [\![read]\!], \{\langle 2, [\![at\ least\ three\ papers]\!]\rangle\}\rangle$$

$$\mathcal{O}_1(\langle [\![read]\!], \{\langle 2, [\![at\ least\ three\ papers]\!]\rangle\}\rangle, [\![more\ than\ half\ the\ referees]\!]) =$$
$$\langle [\![read]\!], \{\langle 1, [\![more\ than\ half\ the\ referees]\!]\rangle, \langle 2, [\![at\ least\ three\ papers]\!]\rangle\}\rangle$$

The second possibility is to combine the subject with the verb first, and then combine the resulting denotation with the object denotation:

$$\mathcal{O}_1(\langle [\![read]\!], \emptyset\rangle, [\![more\ than\ half\ the\ referees]\!]) = \langle [\![read]\!], \{\langle 1, [\![more\ than\ half\ the\ referees]\!]\rangle\}\rangle$$

$$\mathcal{O}_2(\langle [\![read]\!], \{\langle 1, [\![more\ than\ half\ the\ referees]\!]\rangle\}\rangle, [\![at\ least\ three\ papers]\!]) =$$
$$\langle [\![read]\!], \{\langle 1, [\![more\ than\ half\ the\ referees]\!]\rangle, \langle 2, [\![at\ least\ three\ papers]\!]\rangle\}\rangle$$

[2] Despite this observation, there are a number of theories of quantification which do not separate semantic role assignment from the resolution of scope dependencies. For example, in Heim and Kratzer (1998) both the semantic role of a DP denotation as well as its scope is fixed at the same point in the derivation, namely when the DP denotation is combined with a property.

Given a (finite) set S of semantic roles (which, for convenience, are represented by means of integers), the SEMANTIC COMPOSITION RULE (schema) is defined as follows:

$$\mathcal{O}_i([\![DP]\!], \langle R, S\rangle) = \langle R, S \cup \{\langle i, [\![DP]\!]\rangle\}\rangle$$

for every semantic role $i \in$ S.

Given these semantic composition rules, the DP denotations can combine with the verb denotation in any order, without having to type-lift either DP or verb denotation (as required in type-driven approaches to semantic composition). Semantic composition theories that are equally flexible without requiring type-shifting include among others the recent approaches in Beaver and Condoravi (2007) and Eckardt (2010). Unlike e.g. in Hendriks (1993), the combination of verb and DP denotations does not require (i) the DP denotations to combine in a certain order, (ii) the scope dependencies to be fixed beforehand, and (iii) the verb denotation to be lifted/expanded to the worst case (in order to anticipate all possible DP denotations). Unlike in Cooper (1983), the result of combining DP and verb denotations is not the set of all possible readings, but a relation-store pair capturing what all possible expansions have in common, namely the verb denotation and the assignment of semantic roles to DP denotations.

4.2 Expansion

To complete our toy model, assume that

$$\begin{aligned}[\![read]\!] = \{ & \langle r_1, p_1\rangle, \langle r_1, p_2\rangle, \langle r_1, p_3\rangle, \langle r_1, p_5\rangle, \langle r_1, b_1\rangle, \langle r_1, b_2\rangle,\\ & \langle r_2, p_1\rangle, \langle r_2, p_2\rangle, \langle r_2, p_4\rangle, \langle r_2, b_2\rangle,\\ & \langle r_3, p_2\rangle, \langle r_3, p_3\rangle, \langle r_3, p_4\rangle, \langle r_3, b_1\rangle,\\ & \langle r_4, p_4\rangle, \langle r_4, b_1\rangle, \langle r_4, b_2\rangle,\\ & \langle s_1, p_1\rangle\}\end{aligned}$$

which, for ease of readability, will be represented as:

$$\begin{array}{lcl} r_1 \rightarrow p_1, p_2, p_3, p_5, b_1, b_2 & \text{or} & r_1, r_2, s_1 \rightarrow p_1 \\ r_2 \rightarrow p_1, p_2, p_4, b_2 & & r_1, r_2, r_3 \rightarrow p_2 \\ r_3 \rightarrow p_2, p_3, p_4, b_1 & & r_1, r_3 \quad\ \rightarrow p_3 \\ r_4 \rightarrow p_4, b_1, b_2 & & r_2, r_3, r_4 \rightarrow p_4 \\ s_1 \rightarrow p_1 & & r_1 \qquad\ \rightarrow p_5 \\ & & r_1, r_3, r_4 \rightarrow b_1 \\ & & r_1, r_2, r_4 \rightarrow b_2 \end{array}$$

So in this toy model, the sentence (5) denotes (6):

(5) *More than half the referees read at least three papers.*

(6) $\langle [\![read]\!], \{\langle 1, [\![more\ \ than\ \ half\ \ the\ \ referees]\!]\rangle, \langle 2, [\![at\ \ least\ \ three\ \ papers]\!]\rangle\}\rangle$

(6) is an underspecified denotation which (i) expresses what all the possible specified readings have in common, while (ii) leaving the scope dependencies

unspecified. The scope dependencies are specified by applying expansion operations to this underspecified denotation. The idea behind the sequential expansion operation, which was initially inspired by Akiba (2009), is to add tuples to the relation R of a clause denotation $\langle R, S\rangle$. To illustrate, the expansion of the second projection of the relation $[\![read]\!]$ in (6) adds the pair $\langle r_1, [\![at\ least\ three\ papers]\!]\rangle$ to $[\![read]\!]$, since (i) r_1 stands in the reading relation to the papers p_1, p_2, p_3, p_5, (ii) the set $\{p_1, p_2, p_3, p_5\}$ is a witness set of $[\![at\ least\ three\ papers]\!]$, and (iii) the semantic role 2 is assigned to the DP denotation $[\![at\ least\ three\ papers]\!]$. Further, r_2 reads the papers p_1, p_2, p_4, and since $\{p_1, p_2, p_4\}$ is also a witness set of $[\![at\ least\ three\ papers]\!]$, the expansion of the second projection also adds $\langle r_2, [\![at\ least\ three\ papers]\!]\rangle$ to $[\![read]\!]$. Finally, r_3 read the papers p_2, p_3, p_4, and since $\{p_2, p_3, p_4\}$ is a witness set of $[\![at\ least\ three\ papers]\!]$ the operation expanding the second projection adds $\langle r_3, [\![at\ least\ three\ papers]\!]\rangle$ to $[\![read]\!]$. In other words, the expansion of the second projection makes explicit the information that r_1, r_2 and r_3 read at least three papers each. Based on this, the expansion of the first projection adds the pair $\langle[\![more\ than\ half\ the\ referees]\!], [\![at\ least\ three\ papers]\!]\rangle$, since (i) the only referees who read at least three papers are r_1, r_2, r_3, (ii) the set r_1, r_2, r_3 is a witness set of the subject DP denotation $[\![more\ than\ half\ the\ referees]\!]$, and (iii) the semantic role 1 is assigned to $[\![more\ than\ half\ the\ referees]\!]$. The resulting denotation, then, includes the following pairs:

$$
\begin{array}{ll}
r_1 & \to p_1, p_2, p_3, p_5, b_1, b_2 \\
r_2 & \to p_1, p_2, p_4, b_2 \\
r_3 & \to p_2, p_3, p_4, b_1 \\
r_4 & \to p_4, b_1, b_2 \\
s_1 & \to p_1 \\
r_1 & \to [\![at\ least\ three\ papers]\!] \\
r_2 & \to [\![at\ least\ three\ papers]\!] \\
r_3 & \to [\![at\ least\ three\ papers]\!] \\
[\![more\ than\ half\ the\ referees]\!] & \to [\![at\ least\ three\ papers]\!]
\end{array}
$$

So expanding first the second projection and then the first projection resulted in the addition of the pair $\langle[\![more\ than\ half\ the\ referees]\!], [\![at\ least\ three\ papers]\!]\rangle$ to $[\![read]\!]$. In this case we shall say that the sentence is true in the given model M relative to expanding the second projection and then the first projection, i.e. under the reading where the subject has wide scope.

However, if we expand the first projection before the second projection, this pair cannot be added, as shown in the next subsection, and therefore the sentence is false in this model relative to the inverse order of expansion.

The evaluation of the inverse reading begins with the expansion of the first projection of $[\![read]\!]$. To see more clearly which entities stands in the reading relation to a given entity, we use the following representation of $[\![read]\!]$:

$$\begin{array}{ll} r_1, r_2, s_1 & \to p_1 \\ r_1, r_2, r_3 & \to p_2 \\ r_1, r_3 & \to p_3 \\ r_2, r_3, r_4 & \to p_4 \\ r_1 & \to p_5 \\ r_1, r_3, r_4, s_1 & \to b_1 \\ r_1, r_2, r_4, r_2 & \to b_2 \end{array}$$

As can be seen, the paper p_1 is read by exactly two referees, namely r_1 and r_2. Since in our model $\{r_1, r_2\}$ is not a witness of ⟦*more than half the referees*⟧, the expansion of the first projection cannot add the pair ⟨⟦*more than half the referees*⟧, p_1⟩ to ⟦*read*⟧. However, the paper p_2 is read by the referees r_1, r_2 and r_3, and since (in our model) $\{r_1, r_2, r_3\}$ is a witness of ⟦*more than half the referees*⟧, the expansion of the first projection adds the pair ⟨⟦*more than half the referees*⟧, p_2⟩ to ⟦*read*⟧. Further, p_4, b_1 and b_2 are read by $\{r_2, r_3, r_4\}$, $\{r_1, r_3, r_4\}$ and $\{r_1, r_2, r_4\}$ respectively. Since these sets are witnesses of ⟦*more than half the referees*⟧, the three pairs ⟨⟦*more than half the referees*⟧, p_4⟩, ⟨⟦*more than half the referees*⟧, b_1⟩ and ⟨⟦*more than half the referees*⟧, b_2⟩ are added to ⟦*read*⟧, resulting in:

$$\begin{array}{ll} r_1, r_2, s_1 & \to p_1 \\ r_1, r_2, r_3 & \to p_2 \\ r_1, r_3 & \to p_3 \\ r_2, r_3, r_4 & \to p_4 \\ r_1 & \to p_5 \\ r_1, r_3, r_4, s_1 & \to b_1 \\ r_1, r_2, r_4, r_2 & \to b_2 \\ [\![\textit{more than half the referees}]\!] & \to p_2 \\ [\![\textit{more than half the referees}]\!] & \to p_4 \\ [\![\textit{more than half the referees}]\!] & \to b_1 \\ [\![\textit{more than half the referees}]\!] & \to b_2 \end{array}$$

As can now be seen, if the first projection is expanded first, there are only two papers which are being read by ⟦*more than half the referees*⟧, namely p_2 and p_4. But the set $\{p_2, p_4\}$ is not a witness of ⟦*at least three papers*⟧, and therefore the expansion of the second projection cannot add the pair ⟨⟦*more than half the referees*⟧, ⟦*at least three papers*⟧⟩. This shows that the sentence (5) is not true in this model relative to the inverse scope reading (analyzed as expanding the first projection of ⟦*read*⟧ before the second projection).

Consider next the following sentence containing the downward entailing expression *no referee.*[3]

(7) *No referee read every paper.*

Note that in our toy model this sentence is true under the direct scope reading. For this to come out right, the expansion of the second projection before the first

[3] Within the present theory, a DP denotation is downward entailing iff the set of witness sets contains ∅.

projection should result in the addition of the pair $\langle [\![\textit{no referee}]\!], [\![\textit{every paper}]\!] \rangle$ to $[\![\textit{read}]\!]$. However, since in our model no individual stands in the reading relation to every paper, the expansion strategy employed so far does not add any pair containing $[\![\textit{every paper}]\!]$ as its second element. What is missing is that the expansion operator also adds negative information, i.e. information about what is not the case. We shall deal with this in two steps. First, if $[\![\textit{every paper}]\!]$ is assigned the semantic role 2, and an individual x stands in the reading relation to a set P of papers which is not a witness of $[\![\textit{every paper}]\!]$, we shall let (sequential) expansion add the pair $\langle x, [\![\textit{not every paper}]\!] \rangle$.

If every referee stands in the reading relation to $[\![\textit{not every paper}]\!]$, that is just another way of saying that no referee read every paper. Therefore, the second step in dealing with negative information in our example is to let the expansion of the first projection add the pair $\langle [\![\textit{no referee}]\!], [\![\textit{every paper}]\!] \rangle$, provided that the following condition on the expansion of the first projection is met: (i) the set of referees standing in the reading relation to $[\![\textit{not every paper}]\!]$ is not a witness set of $[\![\textit{no referee}]\!]$, (ii) no pair $\langle x, [\![\textit{every paper}]\!] \rangle$ with x a referee has been added, and (iii) the semantic role 1 is assigned to a downward entailing DP denotation. Since this condition is indeed satisfied, expansion of the first projection results in the addition of the pair $\langle [\![no\ referee]\!], [\![\textit{every paper}]\!] \rangle$, showing that the sentence (7) is true in our model under the direct scope reading.

4.3 Formalizing expansion and truth

(8) **Definition (sequential i-expansion):**
Let $[\![\phi]\!] = \langle V, \{\langle i, \langle R_i, W_i \rangle\rangle : 1 \leq i \leq n\} \rangle$ be the denotation of an n-ary formula ϕ, $\sigma[i/Y]$ be the result of replacing the i-the element of σ by Y, let $\sigma_i^V = \{\pi_i(\tau) : \tau \sim_i \sigma \wedge \tau \in V\}$, and let $X_\sigma = \sigma_i^V \cap R_i$ for any n-ary sequence σ.[4] Then the **sequential i-expansion $\mathbf{EXP}_i^{SEQ}([\![\phi]\!])$** of $[\![\phi]\!]$ is the pair $\langle V', S \rangle$ where V' is the smallest set satisfying the following conditions:

1.$V \subseteq V'$

2.for all $\sigma \in D^n$:

(a)$X_\sigma \in W_i \rightarrow \sigma[i/\langle R_i, W_i \rangle] \in V'$

(b)$X_\sigma \notin W_i \rightarrow \sigma[i/\langle R_i, \overline{W_i} \rangle] \in V'$

3.for all $\sigma \in V$ with $\pi_i(\sigma) \in D$ and $\pi_j(\sigma) \notin D$ ($1 \leq j \leq n$ and $j \neq i$)

(a)$X_\sigma \in W_i \rightarrow \sigma[i/\langle R_i, W_i \rangle] \in V'$

(b)$(X_\sigma \notin W_i \wedge X_\sigma \neq R_i) \rightarrow \sigma[i/\langle R_i, \overline{W_i} \rangle] \in V'$

(c)$(X_\sigma \notin W_i \wedge X_\sigma = R_i \wedge \emptyset \in W_i) \rightarrow \sigma[i/\langle R_i, W_i \rangle, j/\langle R_j, \overline{W_j} \rangle] \in V'$

(d)$(X_\sigma \notin W_i \wedge X_\sigma = R_i \wedge \emptyset \notin W_i) \rightarrow \sigma[i/\langle R_i, \overline{W_i} \rangle, j/\langle R_j, \overline{W_j} \rangle] \in V'$

So if X_σ (the set of entities in R_i for which there is a $\tau \in V$ with $\sigma \sim_i \tau$) is a witness set of the denotation assigned the i-th semantic role, then the sequence $\sigma[i/\langle R_i, W_i \rangle]$ is added to V (clauses (2a) and (3a)). If, however, X_i is

[4] $\tau \sim_i \sigma$ iff τ differs from σ at most in the i-th element; $\pi_i(\sigma)$ is the i-th element of the sequence σ.

not a witness set (i.e. $X_i \notin W_i$), we need to distinguish four cases. Clause (2b) accounts for the addition of $\langle r_1, [\![\textit{not every paper}]\!]\rangle$ in (7), when the second projection of the clause denotation is expanded. Clause (3c) accounts for the addition of $\langle [\![no\ referee]\!], [\![\textit{every paper}]\!]\rangle$ when the first projection is expanded. To illustrate the need for the remaining clauses, consider expanding the third projection, then the second projection and finally the first projection of the denotation of *No boy gave exactly two girls exactly three flowers.*. If $[\![gave]\!] = \emptyset$, then by clause (2b), 3-expansion adds $\langle x, y, [\![\textit{not exactly three flowers}]\!]\rangle$ for every $x, y \in D$. If, further, the model does not contain exactly two girls, then by clause (3d) the 2-expansion will add $\langle x, [\![\textit{not exactly two girls}]\!], [\![\textit{exactly three flowers}]\!]\rangle$ for every $x \in D$.[5] Finally, by clause (3c), the 1-expansion adds the triple $\langle [\![\textit{no boy}]\!], [\![\textit{exactly two girls}]\!], [\![\textit{exactly three flowers}]\!]\rangle$. Clause (3b) is applied in the 2-expansion of the same sentence in a model where one boy (say b_1) gives one girl (g_2) three flowers, and another boy (b_2) gives another girl (g_2) three flowers. In this case, 3-expansion adds both $\langle b_1, g_1, [\![\textit{exactly three flowers}]\!]\rangle$ and $\langle b_2, g_2, [\![\textit{exactly three flowers}]\!]\rangle$, and 2-expansion adds by clause (3b) the triples $\langle b_1, [\![\textit{not exactly two girls}]\!], [\![\textit{exactly three flowers}]\!]\rangle$ and $\langle b_2, [\![\textit{not exactly two girls}]\!], [\![\textit{exactly three flowers}]\!]\rangle$.

(9) **Definition (truth relative to sequence of expansions)**:
An n-ary formula ϕ with $[\![\phi]\!] = \langle V, \{\langle i, X_i\rangle : 1 \leq i \leq n\}\rangle$ is **true relative to a sequence of expansions** $\langle \alpha_1, \ldots, \alpha_m\rangle, 0 \leq m$ iff

$$\langle X_1, \ldots, X_n\rangle \in \pi_1(\alpha_1(\alpha_2(\ldots \alpha_m([\![\phi]\!]))))$$

5 Conclusion

To conclude, I briefly highlight two important features of the present proposal and relate them to other proposals.

Theories of semantic scope underspecification can be divided into two types, depending on whether they are based on the notion of satisfaction or on the notion of derivation.[6] The central tenet of satisfaction-based theories of scope underspecification is that the relation between a representation SR with fully specified scope relations and a representation UR with underspecified scope relations is one of satisfaction: SR is a possible specification of underspecified representation UR iff SR satisfies UR.[7] On the other hand, the central idea of

[5] If the model contains exactly two girls, then by clause (3a) the 2-expansion adds $\langle x, [\![\textit{exactly two girls}]\!], [\![\textit{not exactly three flowers}]\!]\rangle$ for every $x \in D$.

[6] See Egg (2011) for a survey of theories of underspecification.

[7] For example, in Hole Semantics introduced in Bos (1996) an object-level formula SR (e.g. a formula of Predicate Logic) is a possible specification of an underspecified meta-level representation UR (consisting of a set of metavariables called holes, a set of labeled formulas possibly containing holes, and a set of constraints on the relation between holes and labeled formulas) iff the bijective mapping between holes and labels that characterizes the object-level formula SR satisfies the constraints of the underspecified representation UR.

derivation-based theories of underspecification is that the relation between a fully specified representation SR and an underspecified representation UR is one of derivation: SR is a possible specification of an underspecified representation UR iff SR can be derived from UR.[8] What both types of theories have in common is that ultimately the denotation $[\![\text{UR}]\!]$ of an underspecified representation UR is the set of denotations $\{[\![\text{SR}_1]\!], \ldots, [\![\text{SR}_n]\!]\}$ of those fully specified representations $\text{SR}_1, \ldots, \text{SR}_n$ which (depending on the type of underspecification theory) either (i) satisfy UR or (ii) can be derived from UR.

According to this perspective on underspecification, then, an underspecified representation denotes a set of interpretations (readings), while the (partial or full) specification of an underspecified denotation amounts to narrowing down the set of interpretations. Arguably, this perspective on underspecification has two shortcomings. First, since the denotation of the underspecified representation is a set of fully specified readings, it does not capture what the different readings have in common. That is, this type of denotation for underspecified representations does not capture the fact that all readings involve the same verb denotation, and the same assignment of semantic roles to the determiner phrase denotations. Secondly, the denotation of the underspecified representation is not in any sense part of the denotation of each specified representation; on the contrary, there is a clear sense in which the denotations of the specified representations are part of the denotation of the underspecified representation, being elements of the underspecified denotation. In this paper I have propose an alternative theory of quantification and scope underspecification, where (i) the denotation of the underspecified representation does indeed capture what the possible readings all have in common, and (ii) the specification of an underspecified denotation amounts to adding information, as opposed to choosing an element (a specific reading) out of a set of specific readings (the underspecified denotation).

Unlike in GQT, conservativity follows (i) from the restriction of the denotation type of determiners, and (ii) from definition of quantified formulas in (3). To see this, note that the truth-conditions for sentences containing non-conservative determiners cannot be derived in the present theory. To illustrate, assume the non-conservative determiner *rouf* (mnemonic for the inverse of '*four*'), which denotes the following binary function:

$$[\![rouf]\!](A)(B) = 1 \text{ iff } |B - A| = 4$$

In the present theory every determiner can be seen as imposing a condition on which subsets of the restrictor set are witnesses. However, the semantic contribution of *rouf* makes essential reference to two sets, and cannot therefore be formulated as a condition on which subsets of the restrictor set are witnesses.

[8] For example, Ambiguous Predicate Logic (APL) introduced in Jaspars and van Eijck (1996) allows for formulas of Predicate Logic to be prefixed with a structured list of scope-bearing operators, which essentially represent a partial ordering on the scope-bearing operators. Further, APL defines a rewrite relation on formulas, such that SR is a possible specification of an underspecified representation UR iff it is possible to rewrite UR into SR by means of the specified rewrite relation.

Bibliography

Akiba, Ken. 2009. A New Theory of Quantifiers and Term Connectives. *Journal of Logic Language and Information* 18:403–431.

Barwise, Jon, and Robin Cooper. 1981. Generalized quantifiers and natural language. *Linguistics and Philosophy* 4:159–219.

Beaver, David, and Cleo Condoravi. 2007. On the logic of verbal modification. In *Proceedings of the 16th amsterdam Colloquium*, ed. Maria Aloni, Paul Dekker, and Floris Roelofsen, 6–12. Amsterdam: University of Amsterdam.

Bos, Johan. 1996. Predicate logic unplugged. In *Proceedings of the tenth amsterdam colloquium*, ed. Paul Dekker and Martin Stokhof, 133–143. Amsterdam, Netherlands.

Cooper, Robin. 1983. *Quantification and syntactic theory*. Dordrecht: Reidel.

Eckardt, Regine. 2010. A logic for easy linking semantics. In *Proceedings of the 17th amsterdam colloquium*, 274–283. Berlin: Springer-Verlag.

Egg, Markus. 2011. Semantic underspecification. In *Semantics: an international handbook of natural language meaning*, ed. Claudia Maienborn, Klaus von Heusinger, and Paul Portner, 533–572. Berlin: de Gruyter.

Heim, Irene, and Angelika Kratzer. 1998. *Semantics in generative grammar*. Oxford: Blackwell.

Hendriks, Herman. 1993. Studied flexibility. categories and types in syntax and semantics. Ph.D. thesis, University of Amsterdam.

Jaspars, Jan, and Jan van Eijck. 1996. Underspecification and reasoning. In *Building the Framework*, 266–287. FraCaS Deliverable D15.

Keenan, Edward, and Jonathan Stavi. 1986. A semantic characterization of natural language determiners. *Linguistics and Philosophy* 9:253–326.

Peters, Stanley, and Dag Westerståhl. 2006. *Quantifiers in Language and Logic*. Oxford: Oxford University Press.