

Tropes, Intensional Relative Clauses, and the Notion of a Variable Object

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abstract. NPs with intensional relative clauses such as *the impact of the book John needs to write* pose a significant challenge for trope theory (the theory of particularized properties), since they seem to refer to tropes that lack an actual bearer. I will propose a novel semantic analysis of such NPs on the basis of the notion of a variable object. This analysis avoids a range of difficulties that an alternative analysis based on the notion of an individual concept would face.

keyword: tropes, intensional verbs, individual concepts, relative clauses, situations

1. Introduction

It is a common view, since Aristotle, that terms of the sort in (1) refer to tropes or particularized properties, that is, particular, non-sharable features of individuals (Williams 1953, Strawson 1959, Woltersdorff 1977, Campbell 1990, Lowe 2006, Mertz 1996):

- (1) a. the wisdom of Socrates
- b. the originality of the book
- c. the simplicity of the dress

According to that view, (1a) refers to the particular manifestation of wisdom in Socrates, that is, a wisdom trope with Socrates as its bearer.

Given general diagnostics for trope reference, there are equally good reasons to take the terms below to be terms referring to tropes, namely quantitative tropes (Campbell 1990, Moltmann 2009, to appear a):

- (2) a. the number of planets
- b. the height of the building
- c. the length of the vacation

According to that view (2a) refers to the instantiation of the property of being eight in the plurality of planets, a feature not shared by any equally numbered plurality.

There are closely related terms, however, that present a significant challenge to trope theory. These are NPs of the sort below with relative clauses containing an intensional verb:

- (3) a. the impact of the book John needs to write
 - b. the simplicity of the dress Mary needs for the occasion
 - c. the wisdom of the director that the institutes should hire
- (4) a. the number of people that fit into the car
 - b. the height of the desk John needs
 - c. the length of the time John might be away

I will call apparent trope-referring NPs with intensional relative clauses of this kind *IR-NPs*.

Tropes as discussed in philosophy are meant to be real entities, involving a real object as bearers. In fact, tropes generally are taken to depend for their existence and their identity on their bearer. But the tropes the terms in (3) and (4) seem to refer to lack an actual bearer. In this paper, I will argue that nonetheless the terms in (3) and (4) refer to tropes, or rather, in most cases, what I will call *variable tropes*. Central on this account is the notion of a *variable object*, a particular case of the notion of a variable embodiment of Fine (1999). IR-NPs either refer to tropes with a variable object as bearer or else they themselves refer to variable tropes whose bearer is driven by the variability of the bearer. I will argue that making use of variable objects avoids a range of serious difficulties for the more standard alternative account that would make use of individual concepts.

2. Trope reference with NPs containing intensional relative clauses

The NPs in (3) and (4) share a range of diagnostics for trope reference with the NPs in (1) and (2). Tropes, unlike properties, generally are taken to be perceivable and causally efficacious (Williams 1953, Lowe 2006). In fact both sorts of NPs allow for the application of perceptual predicates, as in (5), and predicates describing causal relations, as in (6):

- (5) a. John observed Mary's politeness.
 - b. John noticed the small number of women that were present.
 - c. John noticed the number of screws that were missing.
- (6) a. The weight of the lamp caused the table to break.
 - b. The great number of screws that were missing caused the table to fall apart.

Moreover both sorts of terms accept predicates of neutral evaluation such as *exceed*, *great*, *high*, or *negligible*, predicates which are not naturally applicable to the corresponding abstract objects (properties or numbers):

- (7) a. The number of women exceeds the number of men.
 - b. The number of people that fit into the bus exceeds the number of people that fit into the car.
 - c. ?? Eight exceeds seven.
 - d. The impact of John's book is great / negligible.
 - e. The impact of the book John needs to write has to be great / negligible.
 - f. ?? The property of having an impact is great / negligible.

In fact the exceed-relation appears to be inherited from the underlying pluralities in both cases in the very same way. Thus (7a) and (7b) are equivalent to (7a') and (7b') with the specifier of numerical respect *in number*:

- (7) a'. The women exceed the men in number.
b'. The people that fit into the bus exceed the people that fit into the car in number.

Finally, trope reference is reflected in the application of the *be* of identity as opposed to the predicate *is the same as*. The observation is that whereas (8a) and (8c) can be true, (8b) and (8e) cannot:

- (8) a. The number of men is the same as the number of women.
b. ?? The number of men is the number of women.
c. The number of people that fit into the bus is the same as the number of people that fit into the car.
d. ?? The number of people that fit into the bus is the number of people that fit into the car.

The same as in fact expresses close or exact similarity not numerical identity. Numerical identity is expressed only by identity *be* (Moltmann 2009, to appear). Tropes with different bearers that instantiate the same property are similar but not identical. Tropes that instantiate the same 'natural' property (for example the same number property) are exactly similar, and thus 'the same'.

3. Approaches based on individual concepts

Given standard semantics, an obvious approach to the terms in (3)-(4) would be to consider them terms referring to tropes with individual concepts as bearers, that is, (partial) functions from worlds and times ('circumstances' for short) to individuals (or collections of individuals) (Montague 1974). That individual concepts of some sort are the denotations of certain types of NPs with intensional relative clauses has in fact been argued by Moltmann (2008) for NPs as in (9a) and, for the closely related construction in (9b), by Grosu/Krifka (2007):

- (9) a. The assistant John needs must speak French fluently.
b. The gifted mathematician that you claim to be could solve this problem in no time.

However, using reference to individual concepts and of individual concepts as bearers of tropes raises a range of problems, ontologically, conceptually, empirically, and regarding the compositional semantics of IR-NPs.

The ontological problem concerns the notion of a trope itself: tropes are entities in the world that are potentially causally efficacious and perceivable. This means that tropes have objects as bearers, not intensions or functions (unless of course the tropes are features of abstract objects, but this is not what is at stake).

The conceptual problem concerns substitution problems that reference to individual concepts in general give rise to: an abstract function has quite different properties (that is, is a bearer of quite different tropes) than 'the book that John needs to write'. Thus neither (5a') nor (6e') are valid inferences from (5b) and (6e):

- (5) a'. John noticed the number of some function.

(6) e'. The number of some function caused the table to fall apart.

The empirical problem concerns the particular behavior of NPs as in (3)-(4) with respect to the requirement that the predicate contain a modal. Sometimes IR-NPs are subject to the requirement, as in (10a), sometimes they are not, as in (10b):

- (10) a. The impact of the paper John needs to write ?? exceeds /ok must exceed the impact of the papers he has so far written.
 b. The number of people that fit into the bus exceeds the number of people that fit into the car.

It is obscure how an account based on individual concepts could explain the difference.

Finally, there are problems regarding the compositional semantics of IR-NPs on the basis of individual concepts. There are two options of analyzing *the book John needs to write* as standing for an individual concept. For reasons of space, I can discuss those only briefly and only in their roughest outline.

The first option would be an extension of Grosu/Krifka's (2007) analysis of *the gifted mathematician that John claims to be*. Their analysis involves several assumptions. First, it involves type-lifting of all predicates to predicates of individual concepts and all singular terms (including proper names) to terms for individual concepts. Second, it requires treating all intensional verbs as operators quantifying over possible worlds. Finally, it interprets the head noun *book* in the upper position, rather than reconstructing it into the lower position inside the relative clause. Greatly simplifying, this analysis would yield the following as the denotation of *the book John needs to write*:

$$(11) \min(\{f \mid \text{book}(f)\} \cap \{f \mid \text{John need to write } (f)\})$$

(The second set would be the set of functions mapping a world *w* compatible with the satisfaction of John's needs to an object John writes in *w*.) This analysis raises a range of problems. First of all; it involves an excessive use of individual concepts as well as the assumption that all intensional verbs be analyzable as operators quantifying over words, an assumption that a great number of philosophers will find problematic. Furthermore, it poses a problem of uniqueness (a problem that did not arise for the construction for which Grosu and Krifka's analysis was originally developed¹). In a given world in which John's needs are satisfied, John may have written more than one book meeting his need. To account for uniqueness, not entire worlds should be taken into account in which John's needs are satisfied, but rather situations exactly satisfying the need. A given world in which John's needs are satisfied may then contain several situations satisfying his need. A given world may contain several such satisfaction situations. It is far from clear how the analysis in (11) would be able to accommodate the dependence on situations satisfying a need.

The second option of analysing *the book John needs to write* as standing for an individual concept would involve reconstructing the head noun into the lower position inside the relative clause yielding the analysis indicated below:

¹ Grosu/Krifka (2007) have no problem of uniqueness because they analyse *the gifted mathematician John claims to be* as involving identity *be*, which for them takes two individual concepts as arguments.

(12) The f [for any world w compatible with the satisfaction of John's needs, $\text{write}_w(\text{John}, f(w))$ & $\text{book}_w(f(w))$]

This analysis raises the very same problem of uniqueness as the first analysis. Also, just like the first analysis, it is forced to treat all intensional verbs as modal operators quantifying over worlds. Moreover, in its attempt of avoiding type-shifting the analysis cannot go very far. Even though it is plausible that the head noun reconstructs into the lower position, reconstruction of the functional trope noun into a position inside the relative clause is in general impossible: there is no place inside the relative clause for a noun like *impact* in (3a), repeated below:

(3) a. the impact of the book John needs to write

Impact will have to be interpreted in the upper position. But this means that it will have to denote a function applying to individual concepts.

4. The variable-objects approach

The account I would like to propose is based both on the notion of a variable object and the notion of a variable trope. Variable objects are entities that fall under Fine's (1999) more general notion of a variable embodiment (see also Koslicki 2008). The notion of a variable embodiment for Fine is a central notion in metaphysics and accounts for a great variety of 'ordinary' objects. But Fine himself (p.c.) also meant to apply the notion of a variable embodiment to the semantic values of functional NPs as in (13) as well as NPs with intensional relative clauses such as *the book John needs to write*:

- (13) a. John changed his trainer.
 b. The temperature is rising.
 c. The number of students has increased.

According to the standard Montagovian view, functional NPs such as (13a)-(13c) are of a different type than singular terms: they are of type $\langle e, t \rangle$ rather than of type $\langle e \rangle$. Functional NPs thus stand for individual concepts: functions from world-time pairs to objects. Some predicates such as *change*, *rise*, *increase* will apply to individual concepts directly. Other predicates will be type-shifted to predicates of individual concepts while imposing the following meaning postulate:

- (14) For any predicate of individuals P and any individual concept f ,
 $P^{w,t}(f) = 1$ iff $P^{w,t}(f(w, t)) = 1$.

There are various reasons to consider NPs of the sort in (13) as standing for objects (variable objects) rather than being of a different type than singular terms and as such denoting functions. For example, object-related predicates can apply to such NPs just as they apply to individuals (such as the predicates *change*, *rise*, and

increase). This also holds for NPs with intensional relative clauses. Most strikingly, the predicate *count* can apply with such NPs just as it applies with ordinary singular terms:

- (15) a. John counted the books he needs to write.
 b. John counted the screws that are missing.
 c. John counted the things he needed to buy.

Moreover, functional NPs can naturally provide the bearers of tropes, as in the sentences below:

- (16) a. The impact of the increasing number of students is noticeable.
 b. The rise of the temperature caused the drought.

The notion of a variable embodiment allows an account of functional NPs that avoids type-shifting of predicates and also avoids treating their referents as abstract functions.

A variable embodiment, according to Fine, is an entity that allows for the replacement of constituting material or parts, and more generally that may have different manifestations in different circumstances. Organisms and artifacts are variable embodiments, but also entities like ‘the water in the river’. ‘The water in the river’ is a variable embodiment that has different manifestations as different quantities of water at different times. Variable embodiments differ from ‘rigid embodiments’, entities which do not allow for a replacement of their immediate parts. An example is a token of the word *be*, which has as its immediate parts a token of *band* and a token of *e*, neither of which can be replaced.

Fine’s theory of variable embodiments as formulated in Fine (1999) applies to variable embodiments that may have different manifestations at different times. But the theory is also meant to apply to entities that have different manifestations in different worlds and in fact may lack a manifestation in the actual world. ‘The book John needs to write’ is such an entity. It is an entity that has different manifestations as different objects in various counterfactual worlds. My term of a variable object is meant to apply to entities that have different manifestations as different objects at different times or in different worlds.

Let us then adopt the following conditions from Fine (1999) for variable objects:

- (17) a. Existence: A variable object *e* exists in a circumstance *i* iff *e* has a manifestation in *i*.
 b. Location: If a variable object *e* exists in a circumstance *i*, then *e*’s location in *i* is that of its manifestation in *i*.
 c. Property inheritance 1: A variable object *e* has a (world- or time-relative) property *P* in a circumstance *i* if *e*’s manifestation in *i* has *P*.

In addition to local properties, which they obtain in the way of (13b), variable objects may have global properties, properties that they may have on the basis of several of their manifestations at different times (for example properties of change, rise, or increase). Variable objects moreover may have properties that are not time- or world-relative (though may be attributed at a time or in a world). Thus we can add a second condition of property inheritance:

- (17) d. Property inheritance 2: A variable object has a (world- and time-independent) property P if all its manifestations in any circumstances have P.

These two conditions can be reformulated as follows when the property in consideration is understood as a particularized property (a trope):

- (18) a. Trope ‘inheritance’ 1: A variable object e bears a trope t relative to a circumstance i if e ’s manifestation in i bears a trope in i that is exactly similar to t .²
 b. Trope ‘inheritance’ 2: A variable object bears a trope t if for any circumstance i , e ’s manifestation in i bears a trope exactly similar to t in i .

Using variable objects in this sense has a significant advantage over the individual-concept approach to the compositional semantics of functional NPs and NPs with intensional relative clauses by avoiding a type ambiguity among predicates entirely. Let us first apply the account to (19a) and (19b):

- (19) a. the impact of the number of students
 b. the increase of the number of students

The functional trope noun in the upper position applies to a variable object and maps it onto a trope that is of either of two sorts: a local trope based on a single circumstance, as in (19a), or a global trope based on a series of circumstances, as in (19b). The two functional trope nouns in (19a) and (19b) denote different functions from variable objects to tropes, as indicated below, where F is the function mapping a variable object e and a circumstance i to the manifestation of e in i :

- (20) a. For a variable object e , $\text{impact}^{w,t}(e) = \text{the trope that has } e \text{ as its bearer and is exactly similar to } \text{impact}^{w,t}(F(e, (w, t)))$.
 b. For a variable object e , $\text{increase}^{w,t}(e) = \text{the trope that has } e \text{ as its bearer and is the instantiation of the property } \lambda e'[(F(e', w, t_1) \text{ less than } F(e', w, t_2) \text{ less than } \dots \text{ for subintervals } t_1, t_2, \text{ and } t_3 \text{ of } t \text{ and } t_1 < t_2 < t_3 \dots]$

A different case is that of *the impact of the book John needs to write*, which refers not to a single trope but rather to a variable trope. Let us first focus on what the variable object is that *the book John needs to write* stands for. Assuming that the head noun *book* is interpreted in the lower position inside the relative clause, the lower variable will stand for a variable object, an object to which the relative clause attributes certain properties in particular circumstances. But this variable object cannot be the variable object each of whose manifestations is a paper John writes in a world in which John’s needs are satisfied. A world in which John’s needs are satisfied may contain several papers that John writes in that world. Moreover, some of those papers may not qualify as

² In Moltmann (to appear b), I argue that in such cases the very same trope is inherited. This result is a trope with multiple bearers. This is of course a nonstandard view about tropes.

'the paper John's needs to write': the complement of *need* gives only a partial characterization of the exact need. Rather to obtain uniqueness, use must be made of situations exactly satisfying John's needs. That is, 'the paper John needs to write' stands for the variable object each of whose manifestations is a paper John writes in a situation exactly satisfying John's needs. Uniqueness then holds relative to a situation of satisfaction of the need. The situation may impose various constraints on the paper John writes in it (constraints the speaker in fact need not know about). Given its dependence on satisfaction situations, the variable object that is 'the paper John needs to write' is an object that itself depends on a need.

But what is a need? A need is not a state of needing, and thus not a Davidsonian event argument. Only a need, but not a state of needing, can be 'satisfied' by a situation. How then do we obtain a 'need' in the interpretation of the sentence so that the variable object in question could depend on it? Without going into a greater discussion, I would simply like appeal to a particular syntactic proposal concerning the verb *need* by Harves/Kayne (to appear). According to their view, the verb *need* is the result of incorporating the copula *have* and the noun *need*. Given this proposal, an entity that is a need would be made available as part of the compositional semantics of the complex predicate *have+need*. Thus the analysis of *the book John needs to write* would be roughly as follows, where the variable object d_e is dependent on a need e :

- (21) the book [John needs to write e] = the book [John has a need to write e] =
 $\text{the } e \text{ [John has a need to write [} e \text{ book]]} = \text{id[} \exists e (\text{have}(e, \text{John}) \ \& \ \text{need}(e, \wedge \text{write}(\text{John}, d_e) \ \& \ \text{book}(d_e)))$

5. The Modal Compatibility Requirement

Let us then turn to the question of when a modal is required in the main clause of a sentence with an IR-NP. Following Grosu/Krifka (2007), who noticed the requirement for a related construction (see also Moltmann 2008), I will call this the *Modal Compatibility Requirement*:

(22) The Modal Compatibility Requirement (MCR)

IR-NPs require an appropriate modal in the main clause to 'access' the entities in the counterfactual circumstances.

The MCR does not hold for all sentences with IR-NPs. It does not hold in (22a) and (22b), though it does hold in (23a) and (23b):

- (22) a.. The number of people that can fit into the bus exceeds the number of people that can fit into the car.
 b. The length of the vacation John is allowed to take exceeds the length of the vacation Mary is allowed to take.
 (23) a. The impact of the book John needs to write ?? exceeds / ok must exceed / ok might exceed the impact of the book he has already written.
 b. The elegance of the dress the bridesmaid should wear ?? does not exceed / ok should not exceed the elegance of the dress that the bride will wear.

This might suggest that IR-NPs referring to quantitative tropes are not subject of the MCR. But this is not right. The MCR is in place below:

(24) ?? The number of people John might invite exceeds the number of people Mary might invite.

Yet the distinction between quantitative and qualitative trope does matter. This can again be seen from the difference between (23a) and (23b) with a one-place evaluative predicate:

- (25) a. The number of papers a student has to write during this program is too high.
b. The quality of the paper John must write ?? is very high / must be very high.

I propose an explanation of the MCR and exceptions to it based on general conditions on when a variable object can bear a trope on the basis of its instances. The cases in which the MCR is in place are cases in which the head noun applies to a variable object and maps it onto a variable trope. A variable trope driven by the variability of its bearer e has as its manifestation in a circumstance i the trope t that has as its bearer the manifestation of e in i . A variable trope that has manifestations only in counterfactual circumstances requires a modal in the main clause in order to be attributed local properties in the first place. The noun *impact* in the *impact of the book John needs to write* thus denotes a function mapping a variable object onto a variable trope, as below:

- (26) For a variable object e ,
 $\text{impact}^{w,t}(e)$ = the variable trope o such that for any circumstance s in which e has a
 manifestation $F(e, s)$, $\text{impact}^{w,t}(F(e, s))$ = the manifestation of o in s .

Let us turn to the case of quantitative tropes not subject to the MCR. What is particular about such cases is that here the manifestations arguably are exactly similar: it is plausible to assume that the same number of people fit into the bus / the car in the various relevant circumstances, which means that the number tropes in the various circumstances are exactly similar. Recall that in such a case the variable object itself will bear an exactly similar number trope. That is, we won't have a variable trope, but an ordinary trope with a variable object as its bearer. Such cases are restricted to quantitative tropes because exact similarity among qualitative tropes is unlikely to obtain, given that natural language predicates in general do not express natural qualitative properties, but unspecific, determinable ones.

This account made use of general conditions on tropes to explain the exemption from the MCR. This in itself gives further support for the trope-based account.

There is another case of an exemption from the MCR:

- (27) The originality of the paper John wants to write exceeds the originality of the papers he has so far written.

This case is an entirely different one. In (27), the trope 'the originality of the paper John wants to write' has as its bearer an intentional object, an 'object of thought', rather than a variable object (Moltmann 1997). An

intentional object is fully present in the world in which the act generating it occurs. It is not an object that has different manifestations in worlds that may exclude the actual one. Clearly only psychological verbs allow for reference to intentional objects, modal verbs of absence like *need* do not:

- (28) a. ?? The house John needs is very grand.
 b. The house John imagines is very grand.

6. Conclusion

To summarize, the notion of a variable object allows an account of an otherwise very puzzling construction of apparent trope-referring terms. The notion of a variable object as such is not a peculiar notion, though, invoked only for the analysis of IR-NPs. Rather, it falls under the more general and ontologically central notion of a variable embodiment (in Fine's metaphysics). As subject, it is subject to the very same ontological conditions as drive variable embodiments in general.

References

1. Aristotle: *The Categories*.
2. Campbell, K. (1990): *Abstract Particulars*. Blackwell, Oxford (1990).
3. Fine, K. (1999): 'Things and Their Parts'. *Midwest Studies of Philosophy* 23, 61-74.
4. Grosu, A. / Krifka, M. (2007): 'The Gifted Mathematician that you Claim to be'. *Linguistics and Philosophy* 30, 445-485 (2007).
5. Harves, S. / R. Kayne: 'Having *need* and needing *have*'. *Linguistic Inquiry* (to appear).
6. Koslicki, K.: *The Structure of Objects*. Oxford UP (2008).
7. Lowe, J.: *The Four-Category Ontology. A Metaphysics Foundation for Natural Science*. Oxford UP (2006).
8. Mertz, D. W.: *Moderate Realism and Its Logic*. Yale UP, New Haven (1996).
9. Moltmann, F.: 'Intensional Verbs and Quantifiers'. *Natural Language Semantics* 5.1, 1-52 (1997).
10. Moltmann, F.: 'Intensional Verbs and Their Intentional Objects'. *Natural Language Semantics* 16.3., 239-270 (2008).
11. Moltmann, F.: 'Degree Structure as Trope Structure: A Trope-Based Analysis of Comparative and Positive Adjectives'. *Linguistics and Philosophy* 32, 51-94 (2009).
12. Moltmann, F.: 'Reference to Numbers in Natural Language'. *Philosophical Studies*(to appear a).
13. Moltmann, F.: 'Tropes, Bare Demonstratives, and Apparent Statements of Identity. *Nous* (to appear b).
14. Montague, R. 'The Proper Treatment of Quantification in Ordinary English'. In J. Hintikka et al.(eds.): *Approaches to Natural Language*. Reidel, Dordrecht (1973).
15. Strawson, P. *Individuals. An Essay in Descriptive Metaphysics*. Methuen, London (1959).
16. Williams, D. C.: 'On the Elements of Being'. *Review of Metaphysics* 7, 3-18 (1953).
17. Woltersdorff, N.: *On Universals*. Chicago UP, Chicago (1970).

