Temporal propositions as vague predicates*

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Abstract. The notion that temporal propositions are vague predicates is examined with an eye to the nature of the objects over which the predicates range. These objects should not, it is argued, be identified once and for all with points or intervals in the real line (or any fixed linear order). Context has an important role to play not only in sidestepping the Sorites paradox (Gaifman 2002) but also in shaping temporal moments/extent (Landman 1991). The Russell-Wiener construction of time from events (Kamp 1979) is related to a notion of context given by a string of observations, the vagueness in which is brought out by grounding the observations in the real line. Moreover, that notion of context suggests a slight modification of the context dependency functions in Gaifman 2002 to interpret temporal propositions.

1 Introduction

Fluents, as temporal propositions are commonly known in AI, have in recent years made headway in studies of events and temporality in natural language semantics (e.g. Steedman 2000, van Lambalgen and Hamm 2005). The present paper concerns the bounded precision implicit in sentences such as (†).

(†) Pat reached the summit of K2 at noon, and not a moment earlier.

Presumably, a moment in (†) is less than an hour but greater than a picosecond. Whether or not determining the exact size of a moment is necessary to interpret or generate (†), there are pitfalls well known to philosophers that lurk. One such danger is the Sorites paradox, which is commonly associated not so much with time as with vagueness. Focusing on time, Landman has the following to say.

> It is not the abstract underlying time structure that is semantically crucial, but the system of temporal measurements. We shouldn't ask just 'what is a moment of time', because that is a context dependent question. We can assume that context determines how precisely we are measuring time: it chooses in the hierarchy of temporal measurements one measurement that is taken as 'time as finely grained as this context requires it to be.' The elements of that measurement are then regarded as moments *in that context.* (Landman 1991, page 138)

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The present paper considers some notions of context that can be deployed to flesh out this suggestion. We start in section 2 with the use of context in Gaifman 2002 to sidestep the Sorites paradox before returning to the special case of time. A basic aim is to critically examine the intuition that the temporal extent of an event is an interval — an intuition developed in Kamp 1979, Allen 1983 and Thomason 1989, among other works.

2 Sorites and appropriate contexts

The tolerance of a unary predicate P to small changes is expressed in Gaifman 2002 through conditionals of the form (1).

$$N_P(x, y) \rightarrow (P(x) \ \rightarrow \ P(y)) \tag{1}$$

P is asserted in (1) to be tolerant insofar as P holds of y whenever P holds of an x that is N_p-near y. Repeatedly applying (1), we conclude $P(z)$, given any finite sequence $y_1, \ldots, y_n$ such that $y_n = z$, $P(y_1)$ and $N_P(y_i, y_{i+1})$ for $1 \leq i < n$. A *Sorites chain* is a sequence $y_1, \ldots, y_n$ such that P holds of y_1 but not y_n, even though $N_P(y_i, y_{i+1})$ for $1 \leq i < n$. Gaifman's way out of the Sorites paradox is to interpret P against a *context dependency function* f mapping a finite set C (of objects in a first-order model) to a subset $f(C)$ of C, understood to be the extension of P at "context" C. (In effect, the predication $P(x)$ becomes $P(x, C)$, for some comparison class C that contains x.) The idea then is to pick out finite sets C that do *not* contain a Sorites chain

$$\text{for every Sorites chain } y_1, \ldots, y_n, \ \{y_1, \ldots, y_n\} \not\subseteq C \ .$$

Such sets are called *feasible contexts*. Formally, Gaifman sets up a *Contextual Logic* preserving classical logic in which tolerance conditionals (1) can be sharpened to (2), using a construct $[C]$ to constrain the contexts relative to which $P(x)$ and $P(y)$ are interpreted.

$$[C] \ (N_P(x, y) \rightarrow (P(x) \rightarrow P(y))) \tag{2}$$

As contexts in Contextual Logic need not be feasible, (2) must be refined further to restrict C to feasible contexts

$$\text{feasible}(C) \rightarrow [C] \ (N_P(x, y) \rightarrow (P(x) \rightarrow P(y))).$$

The formal notation gets quite heavy, but the point is simple enough:

> sentences and proofs have associated contexts. Those whose contexts are feasible form the feasible portion of the language; and it is within this portion that a tolerant predicate is meant to be used. The proof of the Sorites contradiction fails, because it requires an unfeasible context and in unfeasible contexts a tolerant predicate looses [*sic*] its tolerance; it has some sharp cutoff. Unfeasible contexts do not arise in practice. (Gaifman 2002, pages 23, 24)

The obvious question is why not build into Contextual Logic only contexts that *do* "arise in practice" — viz. the feasible ones? For tolerant predicates in general, such a restriction may, as Gaifman claims, well result in a "cumbersome system." Fluents are, however, a very particular case of vague predicates, and insofar as practice is what matters, it is of interest to restrict time to practice. That said, Contextual Logic leaves open the question of what the stuff of time is — integers, real numbers or events — or how times are formed from that stuff — points, intervals or some other sets. Whatever the underlying first-order model might be, the crucial point is to pick out, for every fluent P, finite sets C of times that validate (2), for a suitable interpretation of N_P. Such feasible contexts C avoid the sharp cutoffs characteristic of unfeasible contexts, and allow us to sidestep the difficulty of pinning down the precise moment of change by bounding the granularity. Bounded granularity is crucial for making sense of talk about the first (or last) moment a fluent is true (or of claims that a fluent true at an interval is true at every non-null part of that interval).

3 Contexts for temporal extent

The context-dependent conception of time outlined in pages 138-140 of Landman 1991 features a discrete order at every context, subject to refinement by more fine-grained contexts. Contexts become more fine-grained as we consider further fluents side by side, not only through the variations in the truth of the fluents over time, but through the additional nearness predicates in (2). Refinements should, as pointed out in page 139, be carried out only "as long as it is sensible," as "there may be points after which refinement is no longer practically or even physically possible (these would be points where our measurement systems are not fine-grained enough to measure)." It would appear that dense linear orders such as the set of rational numbers or the set of real numbers outstrip the bounded precision of fluents in ordinary natural language discourse. Instead of such numbers, one might construct time from fluents — an approach that suggests the "actual usage" of vague predicates that Gaifman claims for feasible contexts pertains to Contextual Logic's proof system more than to any of its particular model-theoretic interpretations. The remainder of this section builds on Kamp 1979 to explore the view that as predicates, fluents range not so much over time, but over eventuality-occurrences.

The point is to make time just fine grained enough to order certain events of interest. Kamp 1979 collects such events in a set E, and adds binary relations (on E) of temporal overlap $\bigcirc$ and complete precedence $\prec$ to form an *event structure* $\langle E, \bigcirc, \prec \rangle$ satisfying (A_1) – (A_5).

(A_1) $e \bigcirc e$
(A_2) $e \bigcirc e'$ implies $e' \bigcirc e$
(A_3) $e \prec e'$ implies not $e \bigcirc e'$
(A_4) $e \prec e' \bigcirc e'' \prec e'''$ implies $e \prec e'''$
(A_5) $e \prec e'$ or $e \bigcirc e'$ or $e' \prec e$

(Seven postulates are given in Kamp 1979, but two are superfluous.) Before extracting temporal moments from $\langle E, \bigcirc, \prec\rangle$, it is useful for orientation to proceed in the opposite direction, forming event structures from a relation $s \subseteq T \times E$ associating a time $t \in T$ with an event $e \in E$ according to the intuition that

$$s(t,e) \ \text{ says } \ \text{`}e\ s\text{-occurs at } t\text{'}.$$

That is, s is a schedule, for which it is natural to define temporal overlap $ov(s)$ between events e and e' that s-occur at some time in common

$$e\ ov(s)\ e' \ \stackrel{\text{def}}{\Leftrightarrow}\ (\exists t)\ s(t,e) \text{ and } s(t,e')$$

and to apply a linear order $<$ on T to relate an event e to another e' if e s-occurs only $<$-before e

$$e <_s e' \ \stackrel{\text{def}}{\Leftrightarrow}\ (\forall t,t' \text{ such that } s(t,e) \text{ and } s(t',e'))\ t < t' \ .$$

Proposition 1. *$\langle E, ov(s), <_s\rangle$ is an event structure provided*

(i) *$<$ is a linear order on T*
(ii) *$(\forall e \in E)(\exists t \in T)\ s(t,e)$, and*
(iii) *$s(t,e)$ whenever $s(t_0,e)$ and $s(t_1,e)$ for some $t_0 < t$ and $t_1 > t$.*

Let us call $\langle s, T, <\rangle$ an *interval schedule* if it satisfies (i) – (iii).

Now for the Russell-Wiener construction in Kamp 1979 of time from an event structure $\langle E, \bigcirc, \prec\rangle$. We collect subsets of E any two in which $\bigcirc$-overlap in

$$O_\bigcirc \ \stackrel{\text{def}}{=}\ \{t \subseteq E \mid (\forall e,e' \in t)\ e \bigcirc e'\}$$

and equate temporal moments with $\subseteq$-maximal elements of $O_\bigcirc$

$$T_\bigcirc \ \stackrel{\text{def}}{=}\ \{t \in O_\bigcirc \mid (\forall t' \in O_\bigcirc)\ t \subseteq t' \text{ implies } t = t'\} \ .$$

We then pass $\prec$ on to $T_\bigcirc$ existentially

$$t \prec_\bigcirc t' \ \stackrel{\text{def}}{\Leftrightarrow}\ (\exists e \in t)(\exists e' \in t')\ e \prec e'$$

and define $sched_\bigcirc \subseteq T_\bigcirc \times E$ as the converse of membership

$$sched_\bigcirc(t,e) \ \stackrel{\text{def}}{\Leftrightarrow}\ e \in t$$

for all $t \in T_\bigcirc$ and $e \in E$.

Theorem 2 (Kamp). *$\langle sched_\bigcirc, T_\bigcirc, \prec_\bigcirc\rangle$ is an interval schedule if $\langle E, \bigcirc, \prec\rangle$ is an event structure.*

Applying the transformations in Theorem 2 and Proposition 1 in sequence to an event structure leads to the same event structure, but applying the transformations in reverse to an interval schedule may result in a different (reduced) interval schedule. The notion of time obtained from events is fine enough just to determine overlap $\bigcirc$ and complete precedence $\prec$ between events. But are there not other temporal relations to consider?

Thirteen different relations between intervals are catalogued in Allen 1983, strung out below as sequences of snapshots (enclosed in boxes).

e equals e'	$\boxed{e,e'}$		
e meets e'	$\boxed{e}\boxed{e'}$	e met-by e'	$\boxed{e'}\boxed{e}$
e overlaps e'	$\boxed{e}\boxed{e,e'}\boxed{e'}$	e overlapped-by e'	$\boxed{e'}\boxed{e,e'}\boxed{e}$
e before e'	$\boxed{e}\boxed{\ }\boxed{e'}$	e after e'	$\boxed{e'}\boxed{\ }\boxed{e}$
e contains e'	$\boxed{e}\boxed{e,e'}\boxed{e}$	e during e'	$\boxed{e'}\boxed{e,e'}\boxed{e'}$
e finishes e'	$\boxed{e'}\boxed{e,e'}$	e finished-by e'	$\boxed{e}\boxed{e,e'}$
e starts e'	$\boxed{e,e'}\boxed{e'}$	e started-by e'	$\boxed{e,e'}\boxed{e}$

If a box must be $\subseteq$-maximal (as required by $T_{\bigcirc}$), only three of the thirteen strings above survive

$$\boxed{e}\boxed{e'} + \boxed{e'}\boxed{e} + \boxed{e,e'} \ .$$

To recover the ten other strings, it is useful to equip an event e with pre- and post-events $pre(e)$ and $post(e)$, marrying (and mangling) ideas from Allen and Ferguson 1994 and Walker instants (Thomason 1984 as well as van Lambalgen and Hamm 2005) to take the steps

$$\boxed{\ }\boxed{e}\boxed{\ } \ \leadsto \ \boxed{pre(e)}\boxed{e}\boxed{post(e)}$$

$$\boxed{e}\boxed{\ }\boxed{e'} \ \leadsto \ \boxed{e, pre(e')}\boxed{post(e), pre(e')}\boxed{post(e), e'}$$

or, in terms of a schedule s, to enrich s

$$s^{<} \ \stackrel{\text{def}}{=} \ s \ \cup \ s^{<}_{-} \ \cup \ s^{<}_{+}$$

by

$$s^{<}_{-} \ \stackrel{\text{def}}{=} \ \{\langle t, pre(e)\rangle \mid (\exists t' > t) \ s(t', e) \text{ and } (\forall t' \leq t) \text{ not } s(t, e)\}$$

$$s^{<}_{+} \ \stackrel{\text{def}}{=} \ \{\langle t, post(e)\rangle \mid (\exists t' < t) \ s(t', e) \text{ and } (\forall t' \geq t) \text{ not } s(t, e)\}$$

where for all $e, e', e'' \in E$, the set $\{e, pre(e'), post(e'')\}$ has cardinality 3. It is easy to see that if $\langle s, T, <\rangle$ is an interval schedule on E, then so is $\langle s^{<}, T, <\rangle$ on the extended set

$$E^{<}_{s} \ \stackrel{\text{def}}{=} \ \{y \mid (\exists t) \ s^{<}(t, y)\}$$

of events, and moreover each of the thirteen relations between e and $e' \in E$ can be determined from the overlap relation $ov(s^{<})$ induced by $s^{<}$ — e.g.

$$\begin{array}{lll} e \text{ before } e' & \text{iff} & post(e) \ ov(s^{<}) \ pre(e') \\ e \text{ meets } e' & \text{iff} & post(e) \ ov(s^{<}) \ e' \text{ but neither} \\ & & e \ ov(s^{<}) \ e' \text{ nor } post(e) \ ov(s^{<}) \ pre(e') \ . \end{array}$$

For the record,

Proposition 3. *For every interval schedule $\langle s, T, < \rangle$, $\langle E_s^<, ov(s^<), \prec \rangle$ is an event structure where $\prec$ is the precedence $<_{s^<}$ induced by $s^<$ and $<$, and for $\bigcirc \stackrel{\text{def}}{=} ov(s^<)$,*

$$T_\bigcirc \;=\; \{\{y \mid s^<(t,y)\} \mid t \in T\} \; . \tag{3}$$

According to (3), $T_\bigcirc$ does *not* discard a time $t \in T$ (as may be the case were $\bigcirc = ov(s)$) but merely identifies it with $t' \in T$ such that for all y,

$$s^<(t,y) \;\text{ iff }\; s^<(t',y) \; .$$

(This equivalence is, in general, stronger than one with s in place of $s^<$.) The effect of $T_\bigcirc$ can be pictured on strings as block compression π reducing adjacent identical boxes $\alpha\alpha$ to one α so that, for example, a string

$$\fbox{rain,dawn}\,\fbox{rain}^n\,\fbox{rain,dusk}$$

of length $n+2$ recording $n+2$ observations of rain from dawn to dusk is reduced to $\fbox{rain,dawn}\fbox{rain}\fbox{rain,dusk}$ for $n \geq 1$ in accordance with the slogan "no time without change" (Kamp and Reyle 1993, page 674)

$$\pi(\fbox{rain,dawn}\,\fbox{rain}^n\,\fbox{rain,dusk}) = \fbox{rain,dawn}\fbox{rain}\fbox{rain,dusk} \quad \text{for } n \geq 1$$

suppressing the pre and post-events. To count observations, we need only introduce them as events (e.g. ticks of a clock).

4 Contexts as strings

A schedule $s \subseteq T \times E$ that is finite (as a set) can always be represented as a string, given a linear order $<$ on T, whether or not it meets the interval requirement (iii) in Proposition 1. For example, we can picture the schedule

$$\{(0,e),\; (1,e),\; (1,e'),\; (2,e'),\; (3,e)\}$$

under the usual ordering $0 < 1 < 2 < 3$ as

$$\fbox{$0,e$}\fbox{$1,e,e'$}\fbox{$2,e'$}\fbox{$3,e$} \tag{4}$$

which restricted to $E \cup \{0\}$, projects to

$$\fbox{$0,e$}\fbox{e,e'}\fbox{e'}\fbox{e} \tag{5}$$

obtained by discarding 1,2,3. Restricted to $\{e\}$, (4) and (5) both become

$$\fbox{e}\fbox{e}\square\fbox{e}$$

that π reduces to

$$\boxed{e}\Box\boxed{e}\ .$$

To be more precise, some notation will be helpful to picture the construction in Proposition 3 in terms of strings. For any subset X of E (such as E), let

$$X_+ \stackrel{\text{def}}{=} X \cup \{pre(e) \mid e \in X\} \cup \{post(e) \mid e \in X\}$$

and $r^X : {E_+}^* \to X^*$ be given by componentwise intersections with X

$$r^X(\alpha_1 \cdots \alpha_n) \stackrel{\text{def}}{=} (\alpha_1 \cap X) \cdots (\alpha_n \cap X)$$

and $\pi_X : {E_+}^* \to X^*$ be given by applying r^{X_+}, then π and then r^E (to suppress pre- and post-events)

$$\pi_X(\alpha_1 \cdots \alpha_n) \stackrel{\text{def}}{=} r^E(\pi(r^{X_+}(\alpha_1 \cdots \alpha_n)))\ .$$

Now, Russell-Wiener-Kamp applied to the restriction

$$s \restriction X \stackrel{\text{def}}{=} \{(t,e) \in s \mid e \in X\}$$

of a schedule s to a finite set X becomes the π_X-approximation of s. For example, for

$$s = \{(r,r) \mid r \in \mathbb{R}\}$$

and $X = \{0,1,2\}$, the π_X-approximation of s is

$$\Box\boxed{0}\Box\boxed{1}\Box\boxed{2}\Box.$$

The reader familiar with inverse limits can carry out the construction in Proposition 3 by gluing together π_X approximations of s, for finite subsets X of E. For a more linguistic example, consider again the string $\boxed{\text{rain,dawn}}\boxed{\text{rain}}\boxed{\text{rain,dusk}}$ depicting the phrase *rain from dawn to dusk*. For $X = \{rain, dawn, dusk\}$, this string can be understood as the π_X-approximation of a schedule $s \subseteq T \times E$ such that for some $t_1, t_2 \in T$, we have for all $t \in T$,

(i) $s(t, \text{dawn})$ if $t \leq t_1$
(ii) $s(t, \text{rain})$ and
(iii) $s(t, \text{dusk})$ if $t \geq t_2$.

Weaker constraints on s can be derived by changing the π_X-approximation or perhaps by experimenting with projections other than π_X that stay away from pre- and post-borders in X_+. In the remainder of this section, we shall consider more drastic changes in the interpretation of a string, allowing us to drop the interval requirement on schedules (condition (iii) in Proposition 1).

Let us interpret a string $\alpha_1 \cdots \alpha_n$ over open intervals in the real line $\mathbb{R}$ that have length greater than some fixed real number $\epsilon > 0$. (The intuition is that an observation takes time $> \epsilon$. Let

$$\mathcal{O}_\epsilon \stackrel{\text{def}}{=} \{(a,b) \mid a,b \in \mathbb{R}_{\pm\infty} \text{ and } b > a + \epsilon\}$$

where

$$(a,b) \stackrel{\text{def}}{=} \{r \in \mathbb{R} \mid a < r < b\}$$

and for all $o, o' \in \mathcal{O}_\epsilon$,

$$o \prec_\epsilon o' \stackrel{\text{def}}{\Leftrightarrow} o, o' \in \mathcal{O}_\epsilon \text{ and } (\forall r \in o)(\forall r' \in o')\ r < r'$$

and for ϵ-successors,

$$o\ s_\epsilon\ o' \stackrel{\text{def}}{\Leftrightarrow} o \prec_\epsilon o' \text{ and not } (\exists o'' \prec_\epsilon o')\ o \prec_\epsilon o'' .$$

An ϵ-*chain* is a sequence $o_1 \ldots o_n$ in $\mathcal{O}_\epsilon$ such that

$$o_1\ s_\epsilon\ o_2\ s_\epsilon\ o_3\ \cdots\ s_\epsilon\ o_n .$$

A string $\alpha_1 \cdots \alpha_n$ *holds at* an ϵ-chain $o_1 \ldots o_n$ if for $1 \leq j \leq n$,

$$o_j \models \varphi \ \text{ for every } \varphi \in \alpha_j$$

for a suitable notion $\models$ of satisfaction. That is, a symbol α in a string is made up not just of event occurrences but of event *types*, allowing us to reconceive the string $\boxed{pre(e)}\boxed{e}\boxed{post(e)}$ as $\alpha(e)\boxed{\mathsf{oc}(e)}\omega(e)$ where the negation $\neg\mathsf{oc}(e)$ of an occurrence of e is split between

$$\alpha(e) \stackrel{\text{def}}{=} \boxed{\neg\mathsf{oc}(e), \neg \mathit{Past}(\mathsf{oc}(e)), \mathit{Future}(\mathsf{oc}(e))}$$
$$\omega(e) \stackrel{\text{def}}{=} \boxed{\neg\mathsf{oc}(e), \mathit{Past}(\mathsf{oc}(e)), \neg \mathit{Future}(\mathsf{oc}(e))}$$

not to mention

$$\text{hole}(e) \stackrel{\text{def}}{=} \boxed{\neg\mathsf{oc}(e), \mathit{Past}(\mathsf{oc}(e)), \mathit{Future}(\mathsf{oc}(e))}$$
$$\text{never}(e) \stackrel{\text{def}}{=} \boxed{\neg\mathsf{oc}(e), \neg \mathit{Past}(\mathsf{oc}(e)), \neg \mathit{Future}(\mathsf{oc}(e))}$$

where as usual,

$$\begin{array}{rcl} o \models \neg\varphi & \text{iff} & \text{not } o \models \varphi \\ o \models \mathit{Past}\ \varphi & \text{iff} & o' \models \varphi \text{ for some } o' \prec_\epsilon o \\ o \models \mathit{Future}\ \varphi & \text{iff} & o' \models \varphi \text{ for some } o' \succ_\epsilon o\ . \end{array}$$

Next, to step from ϵ-chains to intervals, let us agree that an ϵ-chain $o_1 \ldots o_n$ ϵ-*spans* (a,b) if $a \in o_1$, $b \in o_n$ and

$$o_1 \cup o_n \subseteq (a - \epsilon,\ b + \epsilon)$$

— i.e.

$$\{a, b\} \subseteq o_1 \cup \cdots \cup o_n \subseteq (a - \epsilon,\ b + \epsilon) .$$

Now, in general, given two ϵ-chains $o_1 \ldots o_n$ and $o'_1 \ldots o'_n$ that ϵ-span (a, b), the strings that hold at $o_1 \ldots o_n$ may differ from those that hold at $o'_1 \ldots o'_n$. The door is left open for an analysis of vagueness (as opposed to tolerance) in Gaifman 2002 based on modal logic, with an accessibility relation R (for fixed ϵ) between ϵ-chains of the same length

$$o_1 \ldots o_n\ R\ o'_1 \ldots o'_n \ \stackrel{\text{def}}{\Leftrightarrow}\ (\exists a, b)\ o_1 \ldots o_n \text{ and } o'_1 \ldots o'_n \text{ both span } (a, b) .$$

Borderline cases abound in which a string $\alpha_1 \cdots \alpha_n$ that holds at an ϵ-chain $o_1 \ldots o_n$ may fail at some $o'_1 \ldots o'_n$ such that $o_1 \ldots o_n\ R\ o'_1 \ldots o'_n$. Matters change somewhat if the accessibility relation r is defined instead between intervals

$$(a, b)\ R\ (a', b') \ \stackrel{\text{def}}{\Leftrightarrow}\ |a - a'| < \epsilon \text{ and } |b - b'| < \epsilon$$

where say,

$$(a, b) \models \alpha_1 \cdots \alpha_n \ \stackrel{\text{def}}{\Leftrightarrow}\ \alpha_1 \cdots \alpha_n \text{ holds at every } \epsilon\text{-chain } o_1 \ldots o_n \text{ such that } o_1 = (a, c) \text{ and } o_n = (d, b) \text{ for some } c, d .$$

Not to mention variations in $\epsilon \ldots$

5 Conclusion

Three notions of context were considered above:

- feasible contexts in §2 for Sorites (Gaifman 2002) amounting to comparison classes
- selected events in §3 that induce temporal moments (applying the Russell-Wiener-Kamp construction on event structures with pre- and post-events), and
- strings in §4 that generalize event occurrences to event types, and are interpretable as incomplete samples from open intervals in $\mathbb{R}$ of a particular granularity $\epsilon > 0$.

The tension between strings that record sequences of observations and the real line $\mathbb{R}$ gives rise to vagueness, in the form of borderline cases analyzable in a modal logic, as outlined in Gaifman 2002. Focusing on the contexts that "arise in practice," recall that Gaifman interprets a tolerant unary predicate P via a *context dependency function* f_P mapping a context C to the extension $f_P(C) \subseteq C$ of P at C. Similarly, we might analyze a temporal proposition P as a function mapping C to the set $f_P(C)$ of parts of C that *make P true* such that

$$P \text{ is true at } C \quad \text{iff} \quad f_P(C) \neq \emptyset .$$

Building on a natural part-of relation $\sqsubseteq$ between strings where, for example,

$$\boxed{\text{a,b}}\boxed{\text{c}}\boxed{\text{d}} \;\sqsubseteq\; s\boxed{\text{a,b,c}}\boxed{\text{a,c,d}}\boxed{\text{d}}s' \ ,$$

we can put

$$f_P(C) \;=\; \{s \sqsubseteq C \mid s \in \mathcal{L}(P)\}$$

where $\mathcal{L}(P)$ is the set of strings s that make P true — for example,

$$\mathcal{L}(\text{rain from dawn to dusk}) \;=\; \boxed{\text{dawn, rain}}\boxed{\text{rain}}^{+}\boxed{\text{dusk, rain}}$$

(Fernando 2007, 2009).

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