THE EPISTEMICS OF PRESUPPOSITION PROJECTION

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We carry out (formalize) the Karttunen-Stalnaker pragmatic account of presupposition projection within a state-of-the art version of dynamic epistemic logic. This sheds light on a recent controversy on the appropriateness of dynamic semantics as a tool for analysing presupposition.

1. Introduction

Pragmatic accounts of presupposition projection go back to work of Karttunen and Stalnaker, who proposed that presuppositions are requirements on the common ground and that their projection behaviour should follow from the way this common ground is updated in a discourse. This idea has been worked out by various authors who made the idea of shifting context precise, most notably Heim 1983, and presupposition projection has been a major topic in dynamic semantics of natural language ever since, although there have been dissenting voices, see e.g. Schlenker 2007.

Recent advances in the logic of announcements and knowledge — the logic of public announcements of Plaza 1989, the action style dynamics of Baltag et al. 1999, and the axiomatisation of a very general logic of communication and change in van Benthem et al. 2006 — make it possible to have another go at formalizing the intuitions of Karttunen and Stalnaker. This task is taken up in this paper. Context is represented (not as a set of propositions but) as a multimodal Kripke model, utterances are (public) announcements, sequencing is uttering one announcement after another, context shift is epistemic updating, common ground is common knowledge between discourse participants (or, more subtly, knowledge that the speaker believes to have in common with the audience), and basic presuppositions are checks on common knowledge. We will show that the core of presupposition projection facts then follows from the way in which announcements are composed in dynamic epistemic logic.

2. The State of Knowledge of an Audience

The state of knowledge of an audience (or: set of agents I) is given by a multimodal Kripke model $\mathbf{M} = (W, V, R)$ where W is a non-empty set of possible worlds, $V : P \to \mathcal{P}(W)$ is a valuation that assigns to every basic proposition from a set P the subset of all worlds where that proposition is true, and R is a function that assigns to every agent $i \in I$ an epistemic indistinguishability relation $\sim_i$, where $w \sim_i w'$ indicates that agent i cannot see the difference between worlds w and w'.

Our language will have basic propositions from P, and boolean combinations plus epistemic operations on these. We know from Baltag et al. 1999 that an axiomatisation of public announcement logic in terms of reduction axioms is impossible in the presence of common knowledge; the reason is that $[!\phi]C\psi$ (after public announcement of ϕ it holds that ψ is common knowledge) cannot be expressed in terms of common knowledge alone. In van Benthem et al. 2006 a reduction axiom for restricted common knowledge is given, and it is also shown how public announcements are reduced in the presence of composite epistemic operators, where the composition uses the regular operations. The appropriate logic for this is epistemic PDL, which is what we will use in what follows. If p ranges over P and i over I, the language of epistemic PDL with public announcements is given by:

$$\begin{array}{lcl} \phi & ::= & \top \mid p \mid \neg\phi \mid \phi \wedge \phi' \mid [\pi]\phi \mid [!\phi]\phi' \\ \pi & ::= & i \mid ?\phi \mid \pi;\pi' \mid \pi \cup \pi' \mid \pi^* \end{array}$$

The interpretation of boolean formulas is as usual, that of $[\pi]\phi$ is given by:

$$\mathbf{M}, w \models [\pi]\phi \text{ iff for all } v \text{ with } (w,v) \in [\![\pi]\!]^{\mathbf{M}} \colon \mathbf{M}, v \models \phi,$$

where $[\![\pi]\!]^{\mathbf{M}}$ is the interpretation of the epistemic construct π. This is defined as in PDL, with $[\![i]\!]^{\mathbf{M}} = \sim_i$, $[\![?\phi]\!]^{\mathbf{M}} = \{(v,v) \mid v \in W \text{ and } \mathbf{M}, w \models \phi\}$, $[\![\pi;\pi']\!]^{\mathbf{M}} = [\![\pi]\!]^{\mathbf{M}} \circ [\![\pi']\!]^{\mathbf{M}}$ (where $\circ$ is relational composition), $[\![\pi \cup \pi']\!]^{\mathbf{M}} = [\![\pi]\!]^{\mathbf{M}} \cup [\![\pi']\!]^{\mathbf{M}}$, and $[\![\pi^*]\!]^{\mathbf{M}} = ([\![\pi]\!]^{\mathbf{M}})^*$ (where * is reflexive transitive closure).

What this says is that the basic epistemic operations i are interpreted by means of $\sim_i$, and the composed ones by means of the regular operations on relations. Common knowledge is given by the reflexive transitive closure of the set of all individual accessibilities lumped together: $[(i \cup j \cup \cdots)^*]\phi$ expresses that ϕ is common knowledge, and $(i \cup j \cup \cdots)^*$ is interpreted as the relation $(\bigcup_{i \in I} \sim_i)^*$.

The communicative effect of a public announcement of ϕ is given by a restriction operation on epistemic models. If $\mathbf{M} = (W, V, R)$ is an epistemic model, then $\mathbf{M} \mid \phi$, the restriction of $\mathbf{M}$ with ϕ, is the epistemic model $\mathbf{M}' = (W', V', R')$ where $W' = \{v \in W \mid \mathbf{M}, v \models \phi\}$, V' is V restricted to W' (for each $p \in P$, $V'(p) = V(p) \cap W'$), and R' is the result of restricting each $\sim_i$ to $W' \times W'$. Note that the restriction operation is a partial function: if $\{v \in W \mid \mathbf{M}, v \models \phi\} = \emptyset$, then $\mathbf{M} \mid \phi$ is undefined. The interpretation of $[!\phi]\psi$ is given by:

$$\mathbf{M}, w \models [!\phi]\psi \text{ iff } \mathbf{M}, w \models \phi \text{ implies } \mathbf{M} \mid \phi, w \models \psi.$$

This logic has a sound and complete axiomatisation which consists of the axioms for PDL (see Segerberg 1982), the axioms for individual S5 knowledge, and the rules Modus Ponens, Necessitation for epistemic constructs π, and Necessitation for public announcements, plus a set of reduction axioms of the general form $[!\phi][\pi]\phi' \leftrightarrow$

$[\pi'][!\phi]\phi'$, where π' is the result of transforming π with $!\phi$. E.g., if π is the epistemic construct $(i \cup j)^*$ that expresses common knowledge between i and j, then the reduction axiom for π takes the following shape: $[!\phi][(i \cup j)^*]\psi \leftrightarrow [(?\phi; i \cup j)^*][!\phi]\psi$. The transformed epistemic construct $(?\phi; i \cup j)^*$ expresses so-called relativized common knowledge. See van Benthem et al. 2006 for further details.

3. Making Announcements to an Audience

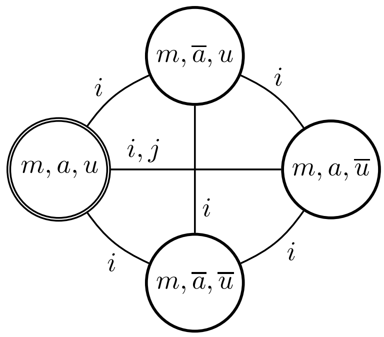
When a couple announces the birth of their child in the evening paper, this is an assertion with the epistemic effect of creating common knowledge. The readers now know about the birth and, moreover, also know that other readers know as well.

Our basic communicative actions are public announcements to a given audience. Let's investigate the special case of announcements of the form $C\phi$, where C is an abbreviation for the common knowledge operator. First of all, notice that it does not matter if we restrict attention to point-generated epistemic models, i.e., to models $\mathbf{M} = (W, V, R)$ with a distinguished world $w \in W$ (the actual world), and with the property that every $v \in W$ is reachable from w by a sequence of accessibility steps. For pointed models $(\mathbf{M}, w)$, we say that an update with $!\phi$ aborts if $\mathbf{M}, w \not\models \phi$. We get the following:

Proposition 1 *If $\mathbf{M}$ is generated from w, then for all announcements of the form $C\phi$, either $\mathbf{M} \mid C\phi = \mathbf{M}$ or the update operation with $C\phi$ aborts in w.*

Proof: Assume $\mathbf{M}, w \models C\phi$. Then ϕ is the case at every world that can be reached through a sequence of accessibility steps. But then ϕ is the case everywhere in the model, since $\mathbf{M}$ is generated from w. Hence $C\phi$ is also true everywhere, and the restriction operation does not remove any worlds. If, on the other hand, $\mathbf{M}, w \not\models C\phi$, the update operation aborts. □

To give an example of this, assume three atomic propositions m, a, u (m for 'male', a for 'adult' and u for 'unmarried'), and consider the following Kripke model.

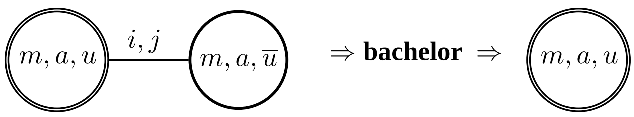


The double circle indicates the actual world. Reflexive arrows are not drawn; a connection between two worlds with label i means that i confuses these worlds. What the model says is that m, a, u all hold in the actual world, but j does not know about u, and i does not know about a and u. It is not difficult to see that in the actual world (and in fact in all worlds) Cm holds, whereas Ca and Cu do not hold. Here C is shorthand for $[(i \cup j)^*]$, the common knowledge operator for i and j. Updating with $!Cm$ (the announcement that m is common knowledge) does not change the model. The updates with $!Ca$ or $!Cu$, however, are undefined.

To handle lexical presuppositions in terms of public announcements, add the following shorthand to the logical language:

$$!(\phi, \phi') \text{ abbreviates } !(C\phi \wedge \phi').$$

Take the case of stating that someone is a bachelor, slightly simplified to fit our propositional framework. This statement presupposes that $m \wedge a$ is common knowledge, and asserts that u. The corresponding update **bachelor** equals $!(m \wedge a, u)$, or written out in full: $!(C(m \wedge a) \wedge u)$. Updating the previous example model with **bachelor** results in undefinedness, because, as we have seen, $C(m \wedge a)$ does not hold in the model. In the following example, where $C(m \wedge a)$ holds in the initial model, the update succeeds:



The basic presupposition projection facts now fall out of our set-up, because the logic provides a natural interpretation for 'being presuppositional update', namely being a public announcement of the form $!C\phi$, and for 'saying things in order', namely making one public announcement after another.

Immediate from proposition 1 we get an illuminating fact about updates with common knowledge:

Proposition 2 $\mathbf{M}, w \models [!C\phi]\psi$ *iff* $\mathbf{M}, w \models C\phi \rightarrow \psi$.

Another thing we get from proposition 1 is that putting a presupposition before an assertion has the same update effect as lumping them together:

Proposition 3 $\mathbf{M}, w \models [!(C\phi \wedge \phi')]\psi$ *iff* $\mathbf{M}, w \models [!C\phi][!\phi']\psi$.

Proof: $\mathbf{M}, w \models [!(C\phi \wedge \phi')]\psi$ iff (proposition 1) $\mathbf{M}, w \models C\phi$ and ($\mathbf{M}, w \models \phi'$ implies $\mathbf{M} \mid \phi', w \models \psi$) iff $\mathbf{M}, w \models C\phi$ and $\mathbf{M}, w \models [!\phi']\psi$ iff (proposition 1) $\mathbf{M}, w \models [!C\phi][!\phi']\psi$. □

For the analysis of presupposition projection we need a slight generalization of proposition 1:

Proposition 4 *If* $\mathbf{M}$ *is generated from* w*, then for all announcements of the form* $[!\phi]C\psi$*, either* $\mathbf{M} \mid [!\phi]C\psi = \mathbf{M}$ *or the update operation with* $[!\phi]C\psi$ *aborts in* w*.*

Proof: Assume $\mathbf{M}, w \models [!\phi]C\psi$. Then $\mathbf{M}, w \models \phi$ implies $\mathbf{M} \mid \phi, w \models C\psi$. Therefore, it holds for every v with $\mathbf{M}, v \models \phi$ that $\mathbf{M} \mid \phi, v \models \psi$. It follows that for all v with $\mathbf{M}, v \models \phi$ we have $\mathbf{M} \mid \phi, v \models C\psi$. Therefore $\mathbf{M}, v \models [!\phi]C\psi$ for all v in the domain of $\mathbf{M}$, and the restriction operation does not remove any worlds. Alternatively, if $\mathbf{M}, w \not\models [!\phi]C\psi$, the update operation aborts. □

The logic tells us that a formula of the form $[!\phi]C\psi$ reduces to a relativized common knowledge statement. We will abbreviate this as $C(\phi, \psi)$. Proposition 4 tells us that updates with relativized common knowledge formulas express presuppositions.

Next, the logic gives a precise meaning to updating with (ϕ, ϕ') followed by updating with (ψ, ψ'), namely $[!(\phi, \phi')][!(\psi, \psi')]\chi$. The latter is an abbreviation for $[!(C\phi \wedge \phi')][!(C\psi \wedge \psi')]\chi$, or equivalently $[!(C\phi \wedge \phi' \wedge [!(C\phi \wedge \phi')](C\psi \wedge \psi'))]\chi$, which is in turn equivalent to:

$$[!(C\phi \wedge \phi' \wedge [!(C\phi \wedge \phi')]C\psi \wedge (C\phi \rightarrow [\phi']\psi'))]\chi.$$

The projected presupposition is $C\phi \wedge [!(C\phi \wedge \phi')]C\psi$ and the projected assertion is $\phi' \wedge (C\phi \rightarrow [\phi']\psi')$.

Take for example the statement without presupposition $!m$ (the statement **male**) followed by the statement **bachelor**):

$$\begin{aligned}
[!m][!(C(m \wedge a) \wedge u)]\chi &\leftrightarrow [!(m \wedge [!m](C(m \wedge a) \wedge u))] \\
&\leftrightarrow [!(m \wedge [!m]Cm \wedge [!m]Ca \wedge [!m]u)]\chi \\
&\leftrightarrow [!(m \wedge [!m]Ca \wedge [!m]u)]\chi \\
&\leftrightarrow [!(m \wedge C(m, a) \wedge m \rightarrow u)]\chi \\
&\leftrightarrow [!(C(m, a) \wedge m \wedge u)]\chi
\end{aligned}$$

So the presuppositional part of the combined statement is $C(m, a)$ (common knowledge of a relativized to m) and the assertional part is $m \wedge u$.

Negating a basic statement should produce an update that tests for the same presupposition but that negates the assertion, in other words, the negation of (ϕ, ϕ') is $(\phi, \neg\phi')$. This generalizes to complex statements by means of the above separation of the presuppositional and assertional parts. For instance, implication between statements A and B where A is of the form $!(C\phi \wedge \phi')$ and B of the form $!(C\psi \wedge \psi')$ reduces to negating the sequence $!(C\phi \wedge \phi'); !(C\psi \wedge \neg\psi')$, which we know already how to do. This analysis allows us to compute the projection facts for such cases.

4. Presuppositions and Informativeness

Suppose we are in a context where the presupposition p is common knowledge. Then updating with statement $!(Cp \wedge q)$ has the same effect as updating with $!q$. If on the other hand, p is true in the actual world but not yet common knowledge, then updating with $!(Cp \wedge q)$ will lead to an inconsistent state, but updating with $!p$ followed by an update with $!(Cp \wedge q)$ will not. In other words, the logic allows the use of $!p$ followed by $!(Cp \wedge q)$ in cases where p is compatible with the context model but not yet common knowledge, but in such cases the use of just $!(Cp \wedge q)$ is ruled out. Accommodation of the presupposition would consist of replacement of $!(Cp \wedge q)$ by $[!p][!(Cp \wedge q)]$; as a matter of fact the update sequence $[!p][!(Cp \wedge q)]$ and the single update $!(p \wedge q)$ are equivalent. The logic allows the use of $!(Cp \wedge q)$ and of $!p$ followed by $!(Cp \wedge q)$ in contexts where p is common knowledge, but by invoking the Gricean maxim 'be informative' one can explain why $!p$ followed by $!(Cp \wedge q)$ is *not* appropriate in such contexts.

5. Conclusion and Further Work

We hope we have convinced the reader that the program of giving a formal pragmatic account of presuppositions can be carried out in the framework of multimodal epistemic logic with relativized common knowledge and public announcement updates. Some further work is still needed, though, to forge from this a tool for the working linguist (e.g. inclusion of quantifiers, and a dynamic treatment of anaphoric linking).

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