PARTITION SEMANTICS OF *AT LEAST* & *AT MOST*

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1. Introduction

Geurts & Nouwen (2007) convincingly demonstrated that there is a fundamental difference between *at least* and *at most* ("superlative modifiers") and *more than* and *less/fewer than* ("comparative modifiers") and that the differences have to do with modality. Then, they proposed for superlative modifiers, meanings that explicitly involved modal operators. However, as will be seen, their analysis does have some obvious, substantial problems. In the current work, we will argue that the modal properties characteristic of the superlative modifiers are due to their interactions with modal operators around, explicit or implicit, not due to their internal meanings, and propose alternative meanings for them, which are partitions of sets of possible worlds. The current analysis will be demonstrated to be empirically more adequate than Geurts & Nouwen's in accounting for the modality-related properties of superlative modifiers and more. In the following, because of space limitations, we will mostly discuss on *at least*, being followed by a short discussion on *at most*.

2. Geurts & Nouwen (2007)

Interpreted out of context, the equivalence between (1a) and (1b) seems to be unquestionable, from which one would be tempted to conclude that *at least n* and *more than n-1* are synonymous.

(1) a. John has at least $[\text{three}]_F$ cars.
b. John has more than two cars.

However, as Geurts & Nouwen (2007) pointed out, embedded in context, superlative and comparative modifiers suddenly behave differently in inference (pattern) as illustrated in (2):

(2) Mary believes John has exactly four cars.
a. $\Rightarrow$ Mary believes John has more than two cars.
b. $\not\Rightarrow$ Mary believes John has at least $[\text{three}]_F$ cars.

What is going on here, in intuitive terms, that an *at least*-sentence is felicitous only in a context where all the "cases" or "sub-propositions" compatible with the sentence are deemed possible; in the case of (2), in Mary's belief, it is NOT possible that John has exactly three cars, it is NOT possible that John has exactly five cars although it is possible or must be the case that John has exactly four cars. This kind of observation involving modality prompted Geurts & Nouwen to suggest that superlative modifiers are modal expressions and propose a modal meaning for *at least*, as in (3).

(3) If α is of type $\langle a, t\rangle$, then $[\![$at least $\alpha]\!] = \lambda X[\Box\alpha(X) \wedge \exists\beta[\beta \rhd \alpha \wedge \Diamond\beta(X)]$, where *a* is any type and '$\rhd$' symbolizes the "higher than in a scale" relation.

With the analysis there are problems, among which the most serious and obvious one, as they themselves admitted, is that their analysis predicts that an *at least*-sentence would always have a modal reading irrespective of its environment. For example, (4) will be wrongly predicted to have the reading it does not have, 'If it must be the case that Betty had three martinis and it may be that she had more than three, then she must have been drunk'.

(4) If Betty had at least three martinis, she must have been drunk.

2. Alternative Approach: Partition-Based Analysis

Alternatively, we will propose for *at least*, a meaning free of modal operators. Before that, a word is in order about the assumed framework and some notations to be adopted. Given that *at least* is a focus-sensitive expression (Krifka, 1999), we propose that *at least* is an operator taking a structured proposition as its argument along the lines of the structured-meaning approach to focus; a structured meaning of a sentence, or a structured proposition is an ordered pair of the background meaning and the focus meaning denoted $\langle$B, F$\rangle$, where the focus meaning, F is the ordinary meaning of the focused constituent and the background meaning, B is the result of λ-abstracting the focus meaning from the ordinary meaning of the sentence; for example, the logical form of (1a) will be something as in (5).

(5) at-least´($\langle\lambda$x[have´(j, x) $\wedge$ car´(x)], λP.|P| $\geq$ 3 $\rangle$).

And, a partial order $\leq_B$ and an equivalence relation $\sim_B$, over possible worlds with respect to the extension of a background meaning are defined as follows:

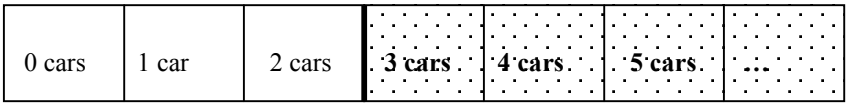
(6) (Definitions of $\leq_B$ and $\sim_B$)
Let w and w´ be possible worlds and <B, F> be a background-focus meaning.
- $w \leq_B w'$ iff w is exactly like w´ except that $[B]^M(w) \subseteq [B]^M(w')$. [1]
- $w \sim_B w'$ iff $|[B]^M(w)| = |[B]^M(w')|$.

Here is our analysis of the meaning of *at least*; an *at least*-sentence denotes a partition of the set of possible worlds whose cells correspond to the "sub-propositions" collectively constituting the propositional meaning of the sentence.

(7) $[\![\text{at-least}'(\langle B, F\rangle)]\!]^M =$
$\{w \in W: \exists v \in W_M[F(v)(B(v)) = 1 \ \& \ \neg\exists u \in W_M[F(u)(B(u)) = 1 \ \& \ u <_B v] \ \& \ v \leq_B w]\}/\sim_B$.

In words, first, there is formed a set of possible worlds such that F(B) is true in it and its extension of B is minimal or more, and the set of possible worlds is partitioned into cells with respect to the cardinality of the extension of B. Then, the partition is the meaning of 'at-least´(⟨B, F⟩)'. The meaning of (1a) by way of logical form (5) is now the set of sub-propositions: "John has (exactly) three cars", "John has (exactly) four cars", "John has (exactly) five cars", …, which is represented as the dotted area in the following diagram:

(8)



Now, the question is how the current analysis can account for differences between superlative and comparative modifiers in e.g. inference patterns as witnessed in (2). We contend that the differences can be explained by the way how an *at least*-sentence is to be interpreted in the environment of a modal operator, □ or ◇; we take epistemic and

[1] As far as the author knows, this relation between possible worlds with respective of the extension of a predicate was first introduced by Yabushita (1993) for an analysis of the exhaustive readings of (multiple-sentence) answers to wh-questions, and much more recently by Schulz & van Rooij (2006) and van Rooij & Schulz (2007).

doxastic operators are necessity operators. We will propose a semantic rule for an *at least*-sentence within the scope of a modal operator, necessity or possibility such that the truth conditions contain, on top of the standard Kripke-semantics truth conditions, (9i), the condition that for every sub-proposition there is some accessible possible world compatible with it, (9ii). The semantic rule in the case of a necessity operator is formally rendered as follows:

(9) $[\![\Box[\text{at least}(\langle B, F\rangle)]]\!]^{M, w} = 1$ iff

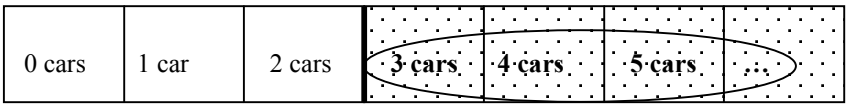
(i) $\forall w'[wRw' \rightarrow \exists c[c \in [\![\text{at-least}'(\langle B, F\rangle)]\!]^M \wedge w' \in c]$ and

(ii) $\forall c[c \in [\![\text{at-least}'(\langle B, F\rangle)]\!]^M \rightarrow \exists w'[wRw' \wedge w' \in c]]$,

where R is the accessibility relation.

Given the above semantic analysis and the standard modal-logic semantics of belief, the inference patterns of *at-least* sentences as attested in (2) can be accounted for. In (2), Mary's belief state is such that in every doxastically accessible possible world, John has exactly four cars, which does not satisfy condition (9ii), i.e. every sub-proposition is true at some doxastically accessible possible world. This accounts for the invalidity of the inference in (2b). The truth conditions of (2b) can be represented as in the following diagram, where the oval area denotes the set of Mary's doxastically accessible possible worlds:

(10)



When an *at least*-sentence is not in the scope of a modal operator as in (4), the denotation of the sentence is stipulated to be the union of the cells of the original, partitional meaning, i.e. $\cup[\![\text{at-least}'(\langle B, F\rangle)]\!]^M$; the antecedent in (4) will be synonymous with "Betty had more than two martinis". This exempts the current analysis from the problem for Geurts & Nouwen's.

One of the nice consequences of the current analysis is that it can provide a formal account for the long-standing observation about an *at least*-sentence, i.e., it resists exhaustification as an answer to a wh-question. Simply, the exhaustification mechanism, which is considered to uniquely identify one sub-proposition to be the case in the speaker's knowledge as implemented in e.g. (Schulz & van Rooij, 2006), is

incompatible with condition (9ii), which requires that all the sub-propositions should be kept as possibilities.

3. The Turn of *At Most*

From the above discussions on the case of *at least*, it should be pretty straightforward what the current analysis of *at most* will be like. The semantic rule for *at most* is defined as follows:

(11) $[\![\text{at-most}'(\langle B, F\rangle)]\!]^M =$
$\{w \in W: \exists v \in W_M[F(v)(B(v)) = 1 \ \& \ \neg\exists u \in W_M[F(u)(B(u)) = 1 \ \& \ u <_B v] \ \& \ w \leq_B u]\}/\sim_B$.

Besides the problem associated with examples like (4), Geurts & Nouwen's analysis has an apparent problem, again as was admitted by themselves. That is, it cannot account for the equivalence between sentences like (12a) and (12b).

(12) a. Betty didn't have at least three martinis.
b. Betty had at most two martinis.

In the current analysis, on the other hand, the equivalence in question will fall out from the proposed meanings of *at least* and *at most* necessarily.

4. Conclusion

We have reviewed some of the properties of *at least* which prompted Geurts & Nouwen (2007) to claim that it was a modal expression and to propose for it a meaning that involved modal operators explicitly. However, although the proposed meaning solves the problems it was designed to, it creates new problems, the most serious one of which is the presence of a modal interpretation for an *at least*-sentence irrespective of the context, as they themselves admitted. Alternatively, we have proposed a modality-free meaning for *at least* such that the meaning of an *at least*-sentence is a partition of a set of possible worlds and the modal properties of *at least* are to be derived from the interaction with a modal operator if there is one around. Besides being free from the problem in question, the current analysis has been demonstrated to account for facts like the incompatibility of an *at least*-sentence with an exhaustive reading, and the interpretation of a negative *at least*-sentence. In our current analysis, however, we have crucially assumed the "chameleon" meaning for an *at least*-sentence, i.e. a partition of a proposition in the environment of a modal operation and the proposition itself otherwise. As an anonymous reviewer pointed out, this might be considered to be an ad hoc

measure. It is left for future research to investigate if this quirk can be rectified or justified.

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