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# Logic, Language and Meaning

17th Amsterdam Colloquium  
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Revised Selected Papers



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# Logic, Language and Meaning

17th Amsterdam Colloquium  
Amsterdam, The Netherlands, December 16-18, 2009  
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# Preface

The 2009 edition of the Amsterdam Colloquium was the 17th in a series which started in 1976. The Amsterdam Colloquia aim at bringing together linguists, philosophers, logicians and computer scientists who share an interest in the formal study of the semantics and pragmatics of natural and formal languages. Originally an initiative of the Department of Philosophy, the colloquium is now organized by the Institute for Logic, Language and Computation (ILLC) of the University of Amsterdam.

These proceedings contain revised extended abstracts of most of the articles presented at the 17th Amsterdam Colloquium.

The first section contains extended abstracts of the talks given by the invited speakers of the general program. The second, third and fourth sections contain invited and submitted contributions to the three thematic workshops that were hosted by the colloquium: the workshop on *Implicature and Grammar* organized by Maria Aloni and Katrin Schulz; the workshop on *Natural Logic* organized by Jan van Eijck; and the workshop on *Vagueness*, organized by Robert van Rooij and Frank Veltman. The final section consists of the submitted contributions to the general program.

For the organization of the 17th Amsterdam Colloquium financial support was received from: the Royal Dutch Academy of Sciences (KNAW); the Institute for Logic, Language and Computation (ILLC); the NWO-funded project ‘Indefinites and Beyond: Evolutionary pragmatics and typological semantics’ (coordinator: Maria Aloni); the NWO-funded project ‘Vagueness — and how to be precise enough’ (coordinators: Robert van Rooij and Frank Veltman); and the Municipality of Amsterdam. This support is gratefully acknowledged.

The editors would like to thank the members of the Program Committee and the anonymous reviewers for their help in the preparation of this volume. Many thanks also to Paul Dekker, Fengkui Ju, Peter van Ormondt, Angelika Port, Floris Roelofsen, Margaux Smets, Joel Uckelman, and Lucian Zagan, for help with the organization of the conference.

February 2010

Maria Aloni  
Harald Bastiaanse  
Tikitu de Jager  
Katrin Schulz

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# Empirical Evidence for Embodied Semantics

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**Abstract.** This paper addresses the question whether and under which conditions hearers take into account the perspective of the speaker, and vice versa. Empirical evidence from computational modeling, psycholinguistic experimentation and corpus research suggests that a distinction should be made between speaker meanings and hearer meanings. Literal sentence meanings result from the hearer's failure to calculate the speaker meaning in situations where the hearer's selected meaning and the speaker meaning differ. Similarly, non-recoverable forms result from the speaker's failure to calculate the hearer meaning in situations where the speaker's intended meaning and the hearer meaning differ.

**Keywords:** Bidirectional Optimality Theory, Embodied Semantics, Perspective Taking, Processing Efficiency, Pronouns, Word Order.

## 1 Introduction

If we were to interpret all sentences literally, we would frequently misunderstand others. We wouldn't understand metaphors such as *The car died on me*, we would have trouble responding appropriately to indirect speech acts like *Can you tell me the time?*, and we would fail to understand the implicated meaning 'Not all people like soccer' for the utterance *Some people like soccer*. Fortunately, many hearers are quite capable of going beyond the literal meaning of these utterances to grasp the meaning that was intended by the speaker. However, despite hearers' remarkable ability to avoid misunderstanding, how hearers arrive at the intended meaning is still the subject of a lively debate.

Traditionally, a sharp distinction is made between sentence meaning (i.e., the literal meaning of the sentence) and speaker meaning (i.e., what the speaker intended to communicate) (see, e.g., [1] for discussion). Sentence meaning is assumed to be explained by a theory of grammar, whereas speaker meaning is assumed to be explained by a theory of pragmatics. It is thus believed that semantics and pragmatics are distinct domains, with the only uncertainty being where exactly the distinction should be drawn. Contrasting with this traditional view on meaning, this paper argues in favor of *embodied semantics*, the view that meaning does not exist independently of speakers and hearers. Consequently, the relevant distinction is argued to be between speaker meanings and hearer meanings. In this paper, empirical evidence of various sorts will be provided to support this alternative view. The central claim is

that hearers always aim at calculating the speaker meaning. However, if they fail to do so, perhaps because they have insufficient processing resources or cognitive abilities, they may assign an intermediate, for example literal, meaning instead. Similarly, speakers are argued to always aim at calculating the hearer meaning. This guarantees that the produced sentence conveys the intended meaning. If speakers fail to do so, they may produce a non-recoverable form instead.

A distinction between speaker meanings and hearer meanings presupposes a linguistic theory that distinguishes the speaker's perspective from the hearer's perspective. The next section introduces different approaches to perspective taking in semantics and pragmatics. Section 3 considers the question whether and under which conditions hearers calculate the speaker meaning. This question is addressed on the basis of experimental investigations of the pronoun interpretation problem in language acquisition. Section 4 considers the inverse question and asks whether speakers calculate the hearer meaning. This possibility is investigated by looking at semantic factors determining word order in Dutch.

## 2 Perspective Taking in Semantics and Pragmatics

In his influential William James lectures at Harvard in 1967, Grice [2] proposed that speakers are guided by a Cooperative Principle, backed by a set of Maxims of Conversation that specify speakers' proper conduct. For example, the Maxim of Relation tells speakers to be relevant, and the Maxim of Quantity tells speakers to make their contribution as informative as is required for the purposes of the exchange, but not more informative than that. By choosing a particular form to express their intentions, speakers assume that hearers will be able to infer the intended meaning on the basis of this form. Grice formulates this as follows: "[Speaker] meant something by  $x$ ' is (roughly) equivalent to '[Speaker] intended the utterance of  $x$  to produce some effect in an audience by means of the recognition of this intention.'" (p. 220).

Several later studies have sought to reduce Grice's maxims, while maintaining the division of labor between speakers and hearers in the sense that speakers choose the sentence to be uttered, while hearers must do a certain amount of inferencing to determine the speaker's intended meaning. However, given Grice's formulation of the Maxim of Quantity, speakers also have to do some inferencing, as they have to determine how much information is required for the purposes of the exchange. Are the inferences that speakers draw of the same sort as the inferences that hearers draw, or are they fundamentally different?

A fully symmetric account of conversational inference, according to which hearers and speakers make similar inferences about the effects of their choices, has been proposed within the framework of optimality theory (OT) [3]. According to Blutner's definition of bidirectional optimality theory (biOT) [4], speakers select the best form for a given meaning, thereby taking into account the hearer's perspective, and hearers select the best meaning for a given form, thereby taking into account the speaker's perspective. Contrasting with Blutner's symmetric conception of bidirectional optimization, various asymmetric models have been proposed within OT. For example, Zeevat proposes an asymmetric model according to which hearers take into account the speaker's perspective, while speakers do not take into account the

hearer's perspective to the same degree [5]. A similar position is adopted by Franke in his game theoretic model of conversational inference [6]. Jäger, on the other hand, develops a bidirectional learning algorithm in which speakers take into account hearers when evaluating form-meaning pairs, but not vice versa [7]. These different positions are mainly based on theoretical arguments and have not been tested by looking at the actual processes of speaking and understanding. Therefore, a relevant question is whether it is possible to find empirical evidence for the symmetry or asymmetry of conversational inference by considering how actual hearers and speakers comprehend and generate sentences.

A second question is whether the proposed conversational inferences are automatic word-by-word interpretational processes (as is believed to be the case for grammatical processes) or additional end-of-sentence processes (as is assumed by some to be true for pragmatic processes). This question is independent of the symmetry or asymmetry of perspective taking and conversational inference, but is relevant in relation to the distinction between semantics and pragmatics. According to Blutner and Zeevat, (weak) bidirectional optimization, and in fact the whole domain of pragmatics, should be seen as reflecting offline interpretation mechanisms [8]. In their view, perspective taking through bidirectional optimization only occurs at the end of the utterance. An alternative view on bidirectional optimization is that it is a grammatical mechanism that is applied in an online fashion during incremental sentence interpretation.

The remainder of this paper aims to shed new light on these two issues by discussing empirical evidence from computational modeling, psycholinguistic experimentation, and corpus research. Section 3 considers a phenomenon that has been argued to require hearers to take into account the speaker's perspective, and addresses the question whether this conversational inference is a local and online interpretational process, or a global and offline process. Whether speakers also take into account hearers is the topic of Section 4.

### 3 Speaker Effects on the Hearer

A well-studied phenomenon in language acquisition is the interpretation of pronouns and reflexives. Many studies have found that children make errors interpreting pronouns in sentence sequences such as *This is Mama Bear and this is Goldilocks. Mama Bear is washing her* until the age of five or six (see, e.g., [9]). This contrasts with children's interpretation of reflexives, which is adult-like from the age of four onward. Many explanations of children's pronoun interpretation delay appeal to non-syntactic factors, such as children's inability to compare the various interpretational possibilities for pronouns (see [10] for an influential approach).

#### 3.1 A Bidirectional Account of Pronoun Interpretation

In [11], an explanation is proposed of children's pronoun interpretation delay in terms of biOT. Whereas the distribution of reflexives is subject to Principle A, which requires reflexives to corefer with the local subject, it is argued that pronouns are *not* subject to a complementary Principle B which forbids pronouns to corefer with the local subject. Rather, pronouns are essentially free in their interpretation. As a

consequence, children allow both a coreferential and a disjoint interpretation for pronouns. This would explain children's guessing behavior with pronouns in experimental tasks. In contrast to children, adults are argued to optimize bidirectionally ([11], cf. [12]) and hence block the coreferential meaning for the pronoun. Adults reason that a speaker, due to a weaker constraint preferring reflexives to pronouns, would have used a reflexive to express a coreferential meaning. As a consequence, for a hearer the coreferential meaning is blocked for the pronoun. This leaves only the disjoint meaning as the meaning of the pronoun.

The biOT explanation of children's errors in pronoun interpretation proposed in [11] predicts children's production of pronouns, in contrast to their comprehension, to be adult-like. If Principle A is stronger than the preference for reflexives over pronouns, a disjoint meaning is expressed best by a pronoun. Choosing a reflexive to express a disjoint meaning would violate the stronger constraint Principle A. On the other hand, if the meaning to be expressed is a coreferential meaning, the optimal form is a reflexive. In this situation, choosing a reflexive satisfies both constraints, whereas choosing a pronoun would violate the weaker constraint. This remarkable prediction of a guessing pattern in comprehension but correct performance in production was confirmed in a psycholinguistic experiment. Testing comprehension and production of the same type of sentences with pronouns and reflexives in the same children, Spender et al. [13] found that children who made errors interpreting pronouns performed correctly on pronoun production.

### 3.2 A Cognitive Model of Pronoun Interpretation

Although the biOT explanation proposed in [11] accounts for children's delay in pronoun interpretation, it is compatible with a local as well as a global view on bidirectional optimization. Children may compare the pronoun to the alternative reflexive form as soon as the pronoun is encountered, or they may wait until the end of the sentence to compare the sentence containing the pronoun with the alternative sentence containing a reflexive.

To test the biOT explanation and to compare it to non-OT accounts of children's delay in pronoun interpretation, the biOT explanation was implemented in the cognitive architecture ACT-R [14][15]. The cognitive architecture ACT-R is both a theory of cognition and a computational modeling environment. The cognitive architecture imposes cognitive constraints on the computational models, based on a wide range of experimental data on information processing, storage and retrieval. By constructing a cognitive model, concrete and testable predictions can be generated regarding children's development and online comprehension of pronouns.

Two aspects of ACT-R are of crucial importance to constructing a cognitive model of pronoun interpretation. First, every operation in ACT-R takes a certain amount of time. Because operations can be executed in parallel if they belong to different modules of the architecture, the total time that is necessary to perform a cognitive process is not simply the sum of the durations of all constituting operations. Rather, the total time critically depends on the timing of the serial operations within a module, and how the various modules interact. To generate predictions about the timing of cognitive processes, computational simulation models can be constructed and run.

A second aspect of ACT-R that is essential to constructing a cognitive model of pronoun interpretation is that higher processing efficiency can be obtained through the mechanism of production compilation. If two cognitive operations are repeatedly executed in sequence, production compilation integrates these two operations into one new operation. This new operation will be faster than the two old operations together. This process of production compilation can continue until the cognitive process has been integrated into a single operation. As a consequence of production compilation, cognitive processes become faster with experience.

Bidirectional optimization combines the speaker's direction of optimization with the hearer's direction of optimization. In the cognitive model, bidirectional optimization is therefore implemented as two serial processes of unidirectional optimization:

$$(1) \quad f \rightarrow m \rightarrow f'$$

Interpreting a pronoun thus consists of a first step of interpretation ( $f \rightarrow m$ ), followed by a second step of production ( $m \rightarrow f'$ ), in which the output of the first step (the unidirectionally optimal meaning) is taken as the input. If the output of production  $f'$  is identical to the initial input in interpretation  $f$ , a bidirectionally optimal pair results. If the output of the production step is different, the unidirectionally optimal meaning  $m$  must be discarded and another meaning  $m'$  must be selected in the first optimization step. Because pronouns are ambiguous according to the biOT explanation discussed in Section 3.1, discarding the coreferential meaning results in selection of the disjoint meaning. The interpretation process in (1) formalizes the assumption that hearers take into account the choices of the speaker.

If unidirectional optimization needs a given amount of time, the serial version of bidirectional optimization in (1) will initially need about twice this amount of time. When time for interpretation is limited, the model will initially fail to complete the process of bidirectional optimization. So at first, the output of the model will be a unidirectionally optimal meaning rather than a bidirectionally optimal meaning. However, over time the model's performance will become more and more efficient as a result of the mechanism of production compilation. As soon as processing efficiency is high enough to perform bidirectional optimization within the given amount of time, the model will do so, resulting in a bidirectionally optimal meaning as the output. As production compilation results from the repeated sequential execution of particular operations, such as retrieval of particular lexical items from declarative memory, it is dependent on the frequency of these lexical items in the language spoken to the child. As a consequence, the speed of development of bidirectional optimization is different for different lexical items.

Simulations of the cognitive model show a pattern of interpretation that is similar to the pattern displayed by English- and Dutch-speaking children [14]. Already from the beginning of the simulated learning period, when the constraints are already in place but bidirectional optimization is not mastered yet, the interpretation of reflexives is correct. In contrast, the proportion of correct interpretations for pronouns hovers around 50% during the first half of the simulated learning period, and then gradually increases to correct performance. The model's correct performance on

reflexives is not surprising because unidirectional and bidirectional optimization both yield the correct meaning. The model's performance on pronouns follows from the gradual increase in processing efficiency, as a result of which bidirectional optimization can be performed more frequently.

### 3.3 Testing the Cognitive Model Experimentally

In incremental interpretation, time limitations arise from the speed at which the next word of the sentence arrives. In the previous section it was argued that children need less time for interpretation if their processing has become more efficient. If bidirectional optimization is a local process which takes place as soon as the pronoun is encountered, there is a second way to facilitate bidirectional optimization: by slowing down the speech rate, so that it takes longer for the next word to arrive. This prediction was tested in a study with 4- to 6-year-old Dutch children, who were presented with sentence sequences such as *Look, a penguin and a sheep are on the sidewalk. The penguin is hitting him with a pan* (translated from Dutch) at a normal speech rate as well as at a speech rate that was artificially slowed down [15]. Crucially, the pronoun does not appear at the end of the sentence but is always followed by a prepositional phrase. It was found that slower speech improved children's performance with pronouns but not with reflexives, and only improved children's performance with pronouns if they made errors with pronouns at normal speech rate. In all other situations, slowing down the speech rate had a negative effect. Because children who make errors in pronoun interpretation succeed in arriving at the correct interpretation when they are given more time, the experimental results suggest that insufficient processing speed is the limiting factor in children's comprehension of pronouns. If this is true, children's interpretation is already aimed at computing the speaker's meaning before they have acquired sufficient processing speed to actually do so.

Apparently, taking into account the speaker as a hearer requires sufficient processing efficiency. If, initially, children's processing is too slow, they may fail to optimize bidirectionally, and select a unidirectionally optimal meaning instead. With experience in pronoun interpretation, children's processing of pronouns becomes more efficient until correct performance is reached. These results suggest that bidirectional interpretation of pronouns must be viewed as a local rather than a global process, since slowing down the speech rate gave the child participants in the experiment more time within the sentence (immediately after they heard the pronoun), while they still had the same amount of time at the end of the sentence. Thus, these results seem incompatible with approaches to bidirectional optimization and perspective taking advocating a purely global view (e.g., [8]). Also, the results provide a challenge to alternative accounts of children's pronoun interpretation delay that attribute children's errors with pronouns to their lack of pragmatic knowledge, limitations in perspective taking, or task effects (see [15] for discussion). The results even seem to undermine the processing explanation proposed by Reinhart [10], who attributes children's errors to their insufficient working memory capacity to perform a global comparison operation, as child hearers compare the pronoun to the alternative reflexive form as soon as the pronoun is encountered.

## 4 Hearer Effects on the Speaker

In the previous section, empirical evidence was presented for the view that hearers take into account the speaker's perspective to arrive at the intended meaning for object pronouns. If conversational inference is fully symmetric, we expect speakers to also take into account the hearer's perspective, perhaps in the following way:

$$(2) \quad m \rightarrow f \rightarrow m'$$

According to (2), producing a form  $f$  consists of a first step of production ( $m \rightarrow f$ ), followed by a second step of interpretation ( $f \rightarrow m'$ ), in which it is checked whether the initial meaning  $m$  is recoverable on the basis of form  $f$ . In the next section, the issue is addressed whether sentence generation actually proceeds in this manner.

### 4.1 Constituent Fronting in Dutch

Word order in Dutch is characterized by the fact that in declarative main clauses the finite verb must occur in second position. In addition, however, Dutch allows for a moderate amount of word order variation with respect to what can appear in front of this finite verb. Although the first position of the sentence is most frequently (in roughly 70% of cases, according to an estimation [16]) occupied by the subject, this position can also be occupied by direct objects, indirect objects and other constituents.

In a large scale corpus study, Bouma [16] investigated the factors determining what constituent comes first in a Dutch main clause. To this end, Bouma conducted a logistic regression analysis of data from the spoken Dutch corpus *Corpus Gesproken Nederlands* (CGN). The factors grammatical function, definiteness and grammatical complexity were found to independently influence the choice of constituent in first position. Regarding grammatical function, subjects have the strongest tendency to occur in first position, followed by indirect objects and direct objects. Regarding definiteness, definite full NPs are more likely to appear in first position than indefinite full NPs. Although pronouns as a group show a strong tendency to appear in first position, this is only visible in the fronting behavior of demonstrative pronouns, which front more often than definite full NPs. Reduced personal pronouns are strongly discouraged from appearing in first position, perhaps because they express highly predictable material. Finally, more complex material is preferably placed at the right periphery of the clause, thus resulting in an avoidance of the first position.

### 4.2 Partial Word Order Freezing

Although speakers of Dutch may place non-subjects in first position under the influence of factors such as the ones mentioned above, in certain situations placing a non-subject in first position makes it difficult for the hearer to infer the intended meaning. If a hearer encounters a sentence such as *Fitz zag Ella* ('Fitz saw Ella'), he can in principle assign an SVO interpretation or an OVS interpretation to this sentence, as both word orders are possible in Dutch. Under the first interpretation, Fitz is the subject. Under the second interpretation, Fitz is the object. However, presented out of context and in the absence of any intonational clues, most hearers

will interpret this sentence as conveying an SVO interpretation. Their preferred interpretation thus reflects the observation that the first constituent most likely is the subject. This observation about hearers' preference may have consequences for speakers' freedom of word order variation. If the speaker wishes to convey the meaning that Ella did the seeing, the sentence *Fitz zag Ella* is a poor choice because hearers will have a preference for Fitz as the subject.

This type of conversational inference is implicit in the biOT model of word order variation proposed by Bouma [16] (cf. [17]). In this model, the speaker's choice for a particular word order is influenced by the hearer's ability to recover the subject and object. If speakers take into account the perspective of the hearer, they are expected to limit the freedom of word order variation in situations such as the one sketched above, where subject and object can only be distinguished on the basis of word order. On the other hand, if other clues are present that allow the hearer to distinguish the subject from the object, speakers are expected to have more freedom of word order variation. Such clues may include definiteness. Subjects tend to be highly definite, whereas direct objects tend to be indefinite. Indeed, Bouma's analysis of the transitive sentences in the CGN confirmed the prediction that a non-canonical word order occurs more frequently in sentences with a definite subject and an indefinite object [16]. A preliminary analysis of a manually annotated subset of the CGN suggests that animacy may have a similar effect, as a non-canonical word order occurs more frequently in sentences with an animate subject and an inanimate object [16]. These hearer effects on the speaker's choice of word order were found on top of the factors discussed in Section 4.1. So the possibility of word order variation increases if subject and object can be distinguished on the basis of other clues than word order. Speakers limit word order variation in situations where a non-canonical word order would make it more difficult for the hearer to recover the intended meaning.

Bouma's corpus study thus provides evidence for a tendency toward partial freezing of word order variation in spoken Dutch discourse, parallel to the observation of partial blocking in the domain of interpretation. In the previous section, we saw that hearers restrict the interpretational possibilities of pronouns in situations where a better form is available to the speaker for expressing one of the meanings. The corpus study provides evidence for the assumption that speakers take into account the hearer's perspective, and limit word order variation in situations that would result in unrecoverability of the expressed meaning.

## 5 Embodied Semantics

Section 3 addressed the question whether hearers take into account the speaker's perspective in interpretation. A biOT account of conversational inference in pronoun interpretation, according to which hearers also consider alternative forms the speaker could have used but did not use, was shown to be supported by results from cognitive modeling and psycholinguistic experimentation. Section 4 addressed the inverse question whether speakers take into account the hearer's perspective when producing a sentence. Bouma's corpus study of word order in Dutch seems to provide evidence that speakers consider how hearers will interpret potential forms. Empirical evidence of various sorts thus suggests that hearers take into account speakers, and vice versa.

Hence, the empirical evidence discussed in Section 3 and Section 4 supports a symmetric conception of biOT. Moreover, hearers appear to consider the speaker's choices locally, as soon as they encounter the relevant linguistic form, and do not wait until the end of the utterance.

This paper argued for a distinction between speaker meaning and hearer meaning. Hearers select an initial meaning for the utterance they hear on the basis of the constraints of the grammar. This hearer meaning usually is the literal meaning of the utterance (although other interpretations are possible if the hearer still has a non-adult constraint ranking, or under strong contextual pressure). Hearers then go on to compute the speaker meaning (i.e., the intended meaning) on the basis of the hearer meaning and the constraints of the grammar. These two steps can be formally modeled by bidirectional optimization. In some situations the speaker meaning differs from the hearer meaning. If the hearer is not (yet) able to optimize bidirectionally, he will assign a non-intended meaning to the utterance in this situation. Note that the speaker meaning not necessarily is the meaning that is actually intended by the speaker. Rather, it is the meaning that the hearer assumes is intended by the speaker by considering the speaker's perspective. The same two meanings, speaker meaning and hearer meaning, also play a role in production, with speakers aiming to compute the hearer meaning but sometimes failing to do so. Given this distinction between speaker meaning and hearer meaning, there is no need to distinguish a separate sentence meaning. In fact, under the proposed view sentences do not have meanings by themselves. Sentences have meanings only in so far as these meanings are assigned to them by speakers and hearers. This view of semantics as embodied in speakers and hearers and their tasks of speaking and understanding is a departure from traditional thinking about meaning.

If no distinction is made between sentence meaning and speaker meaning, it becomes difficult to distinguish semantics and pragmatics. The difference between assigning a literal meaning to a sentence and assigning a speaker meaning to this sentence is argued to lie in the hearer's processing efficiency. As acquiring higher processing efficiency is a gradual process, the distinction between semantics and pragmatics (if there is any) must also be gradual. The traditional distinction between semantics and pragmatics is blurred even more by the fact that this paper addressed two phenomena that are not immediately associated with conversational inference, namely pronoun interpretation and constituent fronting. Nevertheless, evidence was presented that supports analyses of these phenomena in terms of conversational inference. Cognitive modeling of the development of pronoun interpretation illustrates that it is possible for these processes of conversational inference to become automatic in such a way that their output cannot be distinguished from the output of regular grammatical processes. This suggests that at least some conversational inferences that start out as slow and effortful processes can become automatic and indefeasible over time, not only in the course of diachronic language change (cf. [8]) but also in synchronic language development.

**Acknowledgments.** This investigation is supported by a grant from the Netherlands Organization for Scientific Research (NWO grant no. 277-70-005). I thank the audience of the Amsterdam Colloquium for their useful comments.

## References

1. Levinson, S.: *Presumptive Meanings: The Theory of Generalized Conversational Implicatures*. MIT Press, Cambridge (2000)
2. Grice, P.H.: *Studies in the Ways of Words*. Harvard University Press, Cambridge (1989)
3. Prince, A., Smolensky, P.: *Optimality Theory: Constraint Interaction in Generative Grammar*. Blackwell, Malden (2004)
4. Blutner, R.: Some Aspects of Optimality in Natural Language Interpretation. *Journal of Semantics* 17, 189–216 (2000)
5. Zeevat, H.: The Asymmetry of Optimality Theoretic Syntax and Semantics. *Journal of Semantics* 17, 243–262 (2000)
6. Franke, M.: *Signal to Act: Game Theory in Pragmatics*. PhD Thesis, University of Amsterdam (2009)
7. Jäger, G.: Learning Constraint Sub-hierarchies: The Bidirectional Gradual Learning Algorithm. In: Blutner, R., Zeevat, H. (eds.) *Optimality Theory and Pragmatics*, pp. 251–287. Palgrave Macmillan, Hampshire (2004)
8. Blutner, R., Zeevat, H.: Optimality-Theoretic Pragmatics. In: Maienborn, C., von Stechow, K., Portner, P. (eds.) *Semantics: An International Handbook of Natural Language Meaning*. Mouton de Gruyter, Berlin (to appear)
9. Chien, Y.C., Wexler, K.: Children's Knowledge of Locality Conditions on Binding as Evidence for the Modularity of Syntax and Pragmatics. *Language Acquisition* 1, 225–295 (1990)
10. Reinhart, T.: *Interface Strategies: Optimal and Costly Computations*. MIT Press, Cambridge (2006)
11. Hendriks, P., Spenader, J.: When Production Precedes Comprehension: An Optimization Approach to the Acquisition of Pronouns. *Language Acquisition* 13, 319–348 (2005)
12. De Hoop, H., Krämer, I.: Children's optimal interpretations of indefinite subjects and objects. *Language Acquisition* 13, 103–123 (2005)
13. Spenader, J., Smits, E.J., Hendriks, P.: Coherent Discourse Solves the Pronoun Interpretation Problem. *Journal of Child Language* 36, 23–52 (2009)
14. Hendriks, P., van Rijn, H., Valkenier, B.: Learning to Reason about Speakers' Alternatives in Sentence Comprehension: A Computational Account. *Lingua* 117, 1879–1896 (2007)
15. Van Rij, J., van Rijn, H., Hendriks, P.: Cognitive Architectures and Language Acquisition: A Case Study in Pronoun Comprehension. *Journal of Child Language* (in press)
16. Bouma, G.: *Starting a Sentence in Dutch: A Corpus Study of Subject- and Object-Fronting*. PhD Thesis, University of Groningen (2008)
17. Lee, H.: *Optimization in Argument Expression and Interpretation: A Unified Approach*. PhD Thesis, Stanford University (2001)

# Natural Color Categories Are Convex Sets

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**Abstract.** The paper presents a statistical evaluation of the typological data about color naming systems across the languages of the world that have been obtained by the World Color Survey. In a first step, we discuss a principal component analysis of the categorization data that led to a small set of easily interpretable features dominant in color categorization. These features were used for a dimensionality reduction of the categorization data.

Using the thus preprocessed categorization data, we proceed to show that available typological data support the hypothesis by Peter Gärdenfors that the extension of color category are convex sets in the CIELab space in all languages of the world.

## 1 Introduction: The World Color Survey

In their seminal study from 1969, Berlin and Kay investigated the color naming systems of twenty typologically distinct languages. They showed that there are strong universal tendencies both regarding the extension and the prototypical examples for the meaning of the basic color terms in these languages.

This work sparked a controversial discussion. To counter the methodological criticism raised in this context, Kay and several co-workers started the **World Color Survey** project (WCS, see Cook et al. 2005 for details), a systematic large-scale collection of color categorization data from a sizeable amount of typologically distinct languages across the world.

To be more precise, the WCS researchers collected field research data for 110 unwritten languages, working with an average of 24 native speakers for each of these languages. During this study, the Munsell chips were used, a set of 330 chips of different colors covering 322 colors of maximal saturation plus eight shades of gray. Figure 6 (on the last page of this contribution) displays them in form of the Munsell chart.

The main chart is a  $8 \times 40$  grid, with eight rows for different levels of lightness, and 40 columns for different hues. Additionally there is a ten-level column of achromatic colors, ranging from white via different shades of gray to black. The level of granularity is chosen such that the difference between two neighboring chips is minimally perceivable.

For the WCS, each test person was “asked (1) to name each of 330 Munsell chips, shown in a constant, random order, and (2), exposed to a palette

of these chips and asked to pick out the best example(s) (‘foci’) of the major terms elicited in the naming task” (quoted from the WCS homepage). The data from this survey are freely available from the WCS homepage <http://www.icsi.berkeley.edu/~wcs/data.html>.

This invaluable source of empirical data has been used in a series of subsequent evaluations that confirming Berlin and Kay’s hypothesis of universal tendencies in color naming systems across languages (see for instance Kay and Maffi [1999](#)), even though the controversy about universality vs. relativism continues.

## 2 Feature Extraction

For each informant, the outcome of the categorization task defines a partition of the Munsell space into disjoint sets — one for each color term from their idiolect.

An inspection of the raw data reveals — not surprisingly — a certain level of noise. This may be illustrated with the partitions of two speakers of a randomly chosen language (Central Tarahumara, which is spoken in Mexico). They are visualized in Figure [7](#) (on the last page of this contribution). In the figure, colors represent color terms of Central Tarahumara. We see striking similarities between the two speakers, but the identity is not complete. They have slightly different vocabularies, and the extensions of common terms are not identical. Furthermore, the boundaries of the extensions are unsharp and appear to be somewhat arbitrary at various places. Also, some data points, like the two blue chips within the green area in the center of the upper chart, seem to be due to plain mistakes. Similar observations apply to the data from other participants.

To separate genuine variation between categories (of the same or of different speakers, from the same or from different languages) on one hand from random variation due to the method of data collection on the other hand, I employed **principal component analysis** (PCA), a standard technique for feature extraction and dimensionality reduction that is widely used in pattern recognition and machine learning.

The extension of a given term for a given speaker is a subset of the Munsell space. This can be encoded as a 330-dimensional binary vector. Each Munsell chip corresponds to one dimension. The vector has the value 1 at a dimension if the corresponding chip belongs to the extension of the term in question, and 0 otherwise. By using this encoding I obtained a collection of 330d vectors, one for each speaker/term pair.

PCA takes a set of data points in a vector space as input and linearly transforms the coordinate system such that (a) the origin of the new coordinate system is at the mean of the set of points, and (b) the new dimensions are mutually stochastically independent regarding the variation within the data points. The new dimensions, called **principal components**, can be ordered according to the variance of the data points along that dimension.

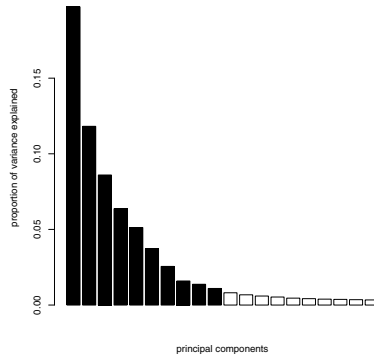
One motivation for performing a PCA is **dimensionality reduction**. Suppose the observed data points are the product of superimposing two sources of variation — a large degree of “genuine” or “interesting” variation and a small

degree of irrelevant noise (and the latter is independent of the former). Then PCA is a way to separate the former from the latter. If the observed data live in an  $n$ -dimensional vector space but the genuine variation is  $m$ -dimensional (for  $m < n$ ), then the first  $m$  principal components can serve as an approximation of this genuine variation.

In our domain of application, “interesting” variation is the variation between the extensions of different categories, like the difference between the extensions of English “red” and English “green” or between the extensions of English “blue” and Russian “galubòj” (which denotes a certain light blue). Inessential variation is the variation between the extensions that two speakers (of the same dialect of) the same language assign to the same term. It is plausible to assume the latter to be small in comparison to the former. So as a heuristic, we can assume that the first  $m$  principal components (for some  $m < 330$  that is yet to be determined) capture the essence of the “interesting” variation.

Figure 1 depicts the proportion of the total variance in the data explained by the principal components. The graphics does not motivate a specific choice of  $m$ . For the time being, I will choose  $m = 10$  because, as we will see shortly, the first ten principal components can be interpreted straightforward, while the others can’t. The main result of the paper does not depend on this choice though. The first ten principal components jointly explain about 62.0% of the total variance in the data. Each of the following 320 principal components only explains a small additional proportion of variance of less than 1%.

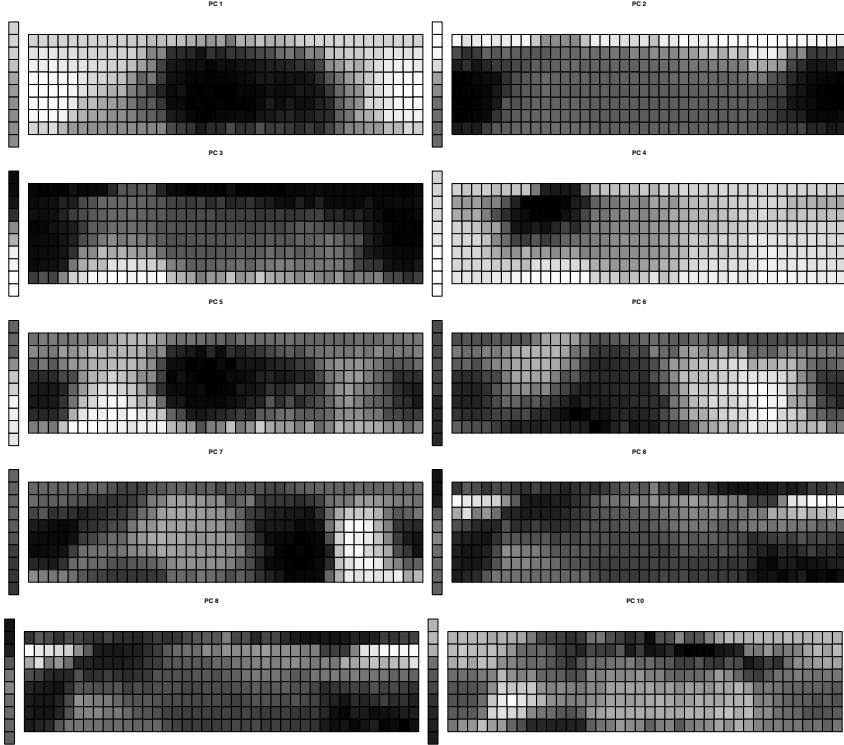
It is worthwhile to look at the first ten principal components in some detail. Figure 2 gives a visualization. Please note that each principal component is a vector in the 330d space defined by the Munsell chips. The degree of lightness of each chip in the visualization corresponds to the value of the principal component in question in the corresponding dimension. The values are scaled such that black stands for the maximal and white for the minimal value, whatever their absolute numerical value may be. Also note the directionality of principal components is arbitrary — so inverting a chart would result in a visualization of the same principal component. The important information is where the regions



**Fig. 1.** Proportion of total variance explained by principal components

of extreme values (black or white) are located, in opposition to gray, i.e. the non-extreme values.

In all ten charts, we find clearly identifiable regions of extreme values. They are listed in Table [1](#).



**Fig. 2.** Visualization of the first ten principal components

**Table 1.** Oppositions defined by the first ten principal components

PC	extreme negative values	extreme positive values
1	red, yellow	green, blue
2	white	red
3	black	white, red
4	black, red, blue, purple	yellow
5	black, brown	red, green, blue
6	blue	red, black, green
7	purple	red, orange, blue
8	pink	red, orange, yellow, white, purple
9	pink, orange	black
10	brown	black, light green, light blue

With very few exceptions, the thus identified regions approximately correspond to (unions of) ten of the eleven universal basic color terms identified by Berlin and Kay (1969). (The only universal basic color that does not occur is gray. This is likely due to the fact that shades of gray are under-represented in the Munsell chart in comparison to shades of other basic colors. The absence of gray is thus likely an artefact of the way the data in the WCS were collected.) Remarkably, the first six principal components jointly define exactly the six primary colors black, white, red, green, blue and yellow. (Purple has extreme values for PC4, but it is not distinguished from the neighboring red and blue.) The 7th – 10th principal components additionally identify the composite colors purple, brown, orange and pink. The 10th principal component furthermore identifies another composite color between green/blue and white.

As can be seen from this discussion, the 10th principal component is less clearly interpretable than the first nine. The remaining principal components starting with the 11th lend themselves even less to an intuitive interpretation.

### 3 Dimensionality Reduction

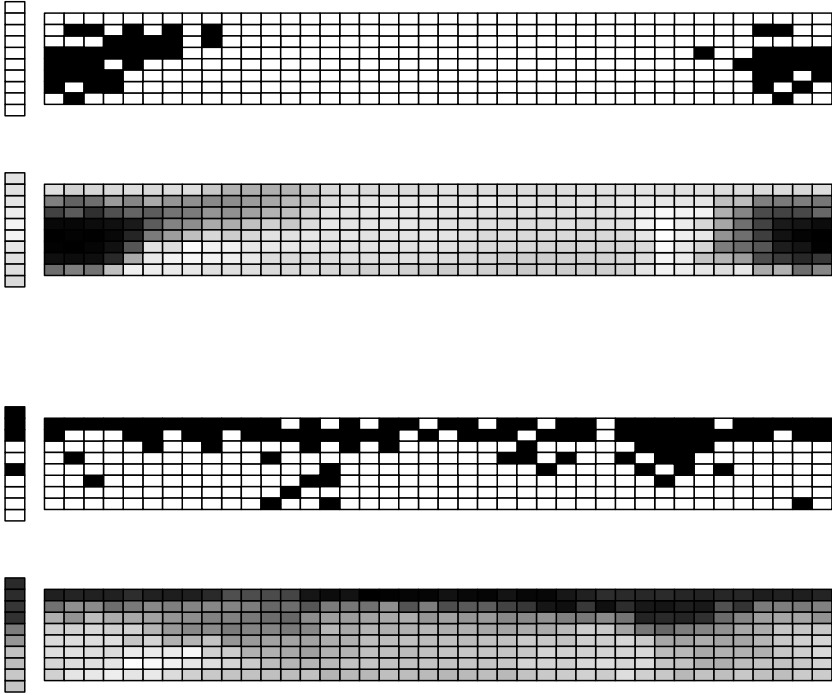
The first ten principal components define a linear 10d subspace of the original 330d space. We are operating under the assumption now that most of the “interesting” variation between color categories takes place within this low-dimensional subspace, while variation outside this subspace is essentially noise. As the next step, I projected the original 330d data points to that subspace. Technically this means that in the transformed coordinate system defined by PCA, only the first ten dimensions are considered, and the values of all data points for the other 320 dimensions are set to 0. The resulting vectors are transformed back into the original coordinate system.

If visualized as a chart of gray values, the original data points correspond to black-and-white pictures where the extension of the corresponding category is a black region with jagged edges. After dimensionality reduction, we get dark regions with smooth and fuzzy gray borders. Put differently, while the original data points are classical binary sets with sharp and jagged boundaries, the projected data points are fuzzy sets with smooth boundaries.<sup>1</sup> (Technically speaking this is not entirely true because the values of the vectors after dimensionality reduction may fall slightly outside the interval  $[0, 1]$ , but the notion of a fuzzy set is still a good conceptual description.) Figure 3 contains two randomly chosen examples of data points before and after dimensionality reduction.

For a given speaker, we can now determine for each Munsell chip which category has the highest value (after dimensionality reduction). In this way we can assign a unique category to each chip, and we end up with a partition of the color space again. The boundaries of the categories are sharp again, but in most cases not jagged but smooth. As an illustration, the cleaned-up versions

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<sup>1</sup> The idea that the extensions of color categories are best modeled as fuzzy sets has been argued for on the basis of theoretical considerations by Kay and MacDaniel (1978).



**Fig. 3.** Dimensionality reduction

of the partitions from Figure 7 are given in Figure 8 (on the last page of this contribution).

## 4 Convexity in the CIELab Space

The visualizations discussed so far suggest the generalization that after dimensionality reduction, category extensions are usually contiguous regions in the 2d Munsell space. This impression becomes even more striking if we study the extensions of categories in a geometrical representation of the color space with a psychologically meaningful distance metric. The CIELab space has this property. It is a 3d space with the dimension  $L^*$  (for lightness),  $a^*$  (the green-red axis) and  $b^*$  (the yellow-blue axis). The set of perceivable colors forms a three-dimensional solid with approximately spherical shape. Figuratively speaking, white is at the north pole, black at the south pole, the rainbow colors form the equator, and the gray axis cuts through the center of the sphere. The CIELab space has been standardized by the “Commission Internationale d’Eclairage” such that Euclidean distances between pairs of colors are monotonically related to their perceived dissimilarity.

The 320 chromatic Munsell colors cover the surface of the color solid, while the ten achromatic chips are located at the vertical axis. Visually inspecting CIELab

representations of the (dimensionality-reduced) partitions led to the hypothesis that boundaries between categories are in most cases approximately linear, and extensions of categories are convex regions. This is in line with the main claim of Gärdenfors’ (2000) book “Conceptual Spaces”. Gärdenfors suggests that meanings can always be represented geometrically, and that “natural categories” must be convex regions in such a conceptual space. The three-dimensional color space is one of his key examples.

We tested to what degree this prediction holds for the partitions obtained via dimensionality reduction. The algorithm we used can be described as follows. Suppose a partition  $p_1, \dots, p_k$  of the Munsell colors into  $k$  categories is given.

1. For each pair of distinct categories  $p_i, p_j$  (with  $1 \leq i, j \leq k$ ), find a linear separator in the CIELab space (i.e. a plane) that optimally separates  $p_i$  from  $p_j$ . This means that the set of Munsell chips is partitioned into two sets  $\tilde{p}_{i/j}$  and  $\tilde{p}_{j/i}$ , that are linearly separable, such that the number of items in  $p_i \cap p_{j/i}$  and in  $p_j \cap p_{i/j}$  is minimized.
2. For each category  $p_i$ , define

$$\tilde{p}_i \doteq \bigcap_{j \neq i} p_{i/j}$$

As every  $p_{i/j}$  is a half-space and thus convex, and the property of convexity is preserved under set intersection, each  $\tilde{p}_i$  is a convex set (more precisely: the set of Munsell coordinates within a convex subset of  $R^3$ ).

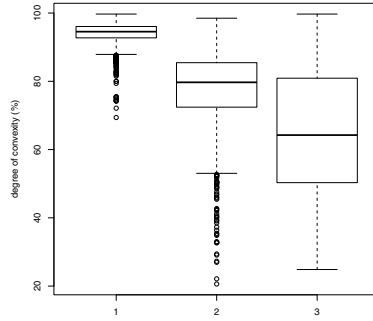
To perform the linear separation in a first step, I used a soft-margin Support Vector Machine (SVM). An SVM (Vapnik and Chervonenkis 1974) is an algorithm that finds a linear separator between two sets of labeled vectors in an  $n$ -dimensional space. An SVM is soft-margin if it tolerates misclassifications in the training data.<sup>2</sup> As SVMs are designed to optimize generalization performance rather than misclassification of training data, it is not guaranteed that the linear separators found in step 1 are really optimal in the described sense. Therefore the numerical results to be reported below provide only a lower bound for the degree of success of Gärdenfors’ prediction.

The output of this algorithm is a re-classification of the Munsell chips into convex sets (that need not be exhaustive). The *degree of convexity* “conv” of a partition is defined as the proportion of Munsell chips not re-classified in this process. If  $p(c)$  and  $\tilde{p}(c)$  are the class indices of chip  $c$  before and after re-classification, and if  $\tilde{p}(c) = 0$  if  $c \notin \bigcup_{1 \leq i \leq n} \tilde{p}_i$ , we can define formally:

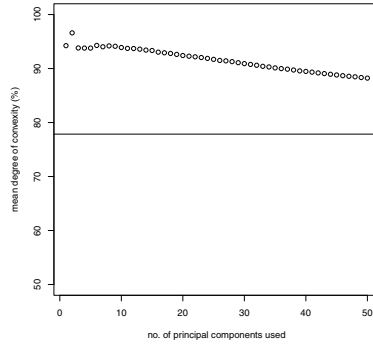
$$\text{conv} \doteq |\{c | p(c) = \tilde{p}(c)\}| / 330$$

The mean degree of convexity of the partitions obtained via PCA and dimensionality reduction is 93.9%, and the median is 94.5% (see the first boxplot in

<sup>2</sup> The main reasons for the popularity of SVMs in statistical learning are that they are easily adaptable to non-linear classification tasks and that they find separators that generalize well to unseen data. These features are of lesser importance here. See (Schölkopf and Smola, 2002) for a comprehensive account.



**Fig. 4.** Degrees of convexity (in %) of 1. cleaned-up partitions, 2. raw partitions, and 3. randomized partitions



**Fig. 5.** Mean degree of convexity as a function of  $m$

Figure 4). If the above algorithm is applied to the raw partitions rather than to those obtained via dimensionality reduction, the mean degree of convexity is 77.9%.

Since the difference between these values is considerable, one might suspect the high degree of convexity for the cleaned-up data actually to be an artifact of the PCA algorithm and not a genuine property of the data. This is not very plausible, however, because the input for PCA were exclusively categorization data from the WCS, while the degree of convexity depends on information about the CILab space. Nevertheless, to test this hypothesis, I applied a random permutation of the category labels for each original partition and applied the same analysis (PCA, dimensionality reduction, computation of the degree of convexity) to the thus obtained data. The mean degree of convexity for these data is as low as 65.3% (see the third boxplot in Figure 4). The fact that this value is so low indicates the high average degree of convexity to be a genuine property of natural color category systems.

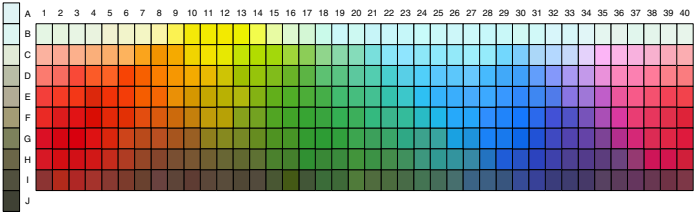


Fig. 6. The Munsell chart

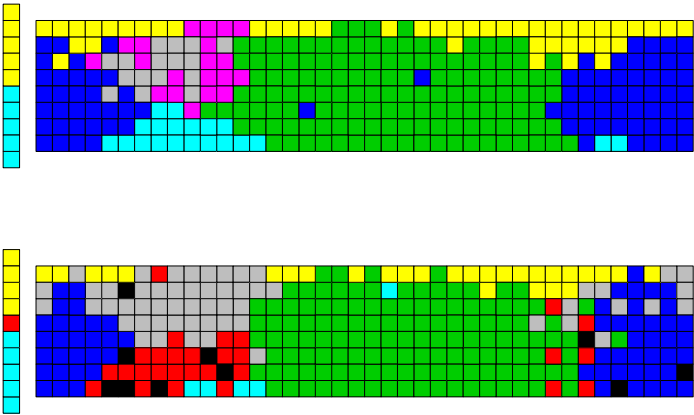


Fig. 7. Partitions for two speakers of Central Tarahumara

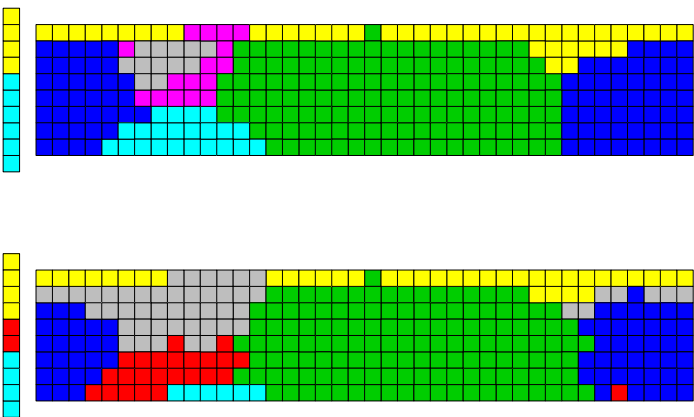


Fig. 8. Cleaned-up partitions for the two speakers of Central Tarahumara

The choice of  $m = 10$  as the number of relevant principal component was motivated by the fact that only the first ten principal components were easily interpretable. As this is a subjective criterion, it is important to test to what degree the results from this section depend on this choice.

Therefore I performed the same analysis with the original data for all values of  $m$  between 1 and 50. The dependency of the mean degree of convexity on  $m$  is displayed in figure 5. It can be seen that the degree of convexity is not very sensitive to the choice of  $m$ . For all values of  $m \leq 35$ , mean convexity is above 90%. The baseline is the degree of convexity of 77.9% for the raw data (or, equivalently, for  $m = 330$ ), which is indicated by the horizontal line.

So I conclude that the data from the WCS provide robust support for Gärdenfors' thesis.

## References

- Berlin, B., Kay, P.: Basic color terms: their universality and evolution. University of California Press, Chicago (1969)
- Cook, R., Kay, P., Regier, T.: The world color survey database: History and use. In: Cohen, H., Lefebvre, C. (eds.) *Handbook of Categorisation in the Cognitive Sciences*, pp. 223–242. Elsevier, Amsterdam (2005)
- Kay, P., Maffi, L.: Color appearance and the emergence and evolution of basic color lexicons. *American Anthropologist*, 743–760 (1999)
- Kay, P., McDaniel, C.K.: The linguistic significance of the meanings of basic color terms. *Language* 54(3), 610–646 (1978)
- Gärdenfors, P.: *Conceptual Spaces*. The MIT Press, Cambridge (2000)
- Vapnik, V., Chervonenkis, A.: *Theory of pattern recognition*. Nauka, Moscow (1974) (in Russian)
- Schölkopf, B., Smola, A.J.: *Learning with Kernels. Support Vector Machines, Regularization, Optimization, and Beyond*. MIT Press, Cambridge (2002)

# Concealed Questions with Quantifiers

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**Abstract.** Concealed question noun phrases headed by the definite article have been analysed as contributing the intension of the noun phrase –an individual concept– as semantic argument of the verb. Concealed questions with quantifiers challenge this analysis. Several empirical observations will be presented and an analysis will be sketched that treats this quantification as external to the concealed question itself, making it parallel to quantificational adverbs with interrogative clauses and plural individuals. This way, the basic individual concept analysis is maintained.

**Keywords:** Concealed question, quantifiers, quantificational variability effect, quantificational adverb, question, interrogative clause, plurality.

## 1 Introduction

Concealed Question (CQ) noun phrases –italicized in (1)–(2)– combine with a question-taking verb to yield roughly the same meaning as an interrogative:

- (1) Mary knows ( guessed / revealed / forgot) *the capital of Italy*.
- (2) The waiter (knows / remembers / forgot) *the dishes you ordered*.

Simple CQs headed by the definite article have been analysed as contributing the regular intension of the NP –an individual concept, e.g. (4a)–(5a)– as the argument of the verb ([14], [1]). The verb is defined crosscategorially to combine both with questions and with individual concepts: ([3]). This analysis correctly generates the truth-conditions in (4b)–(5b) for the sentences above:<sup>1</sup>

- (3) a.  $\llbracket know_{qu} \rrbracket(q_{\langle s, \langle st, t \rangle \rangle})(z)(w) = 1$  iff  $\forall w' \in Dom_z(w)[q(w') = q(w)]$   
b.  $\llbracket know_{CQ} \rrbracket(x_{\langle s, e \rangle})(z)(w) = 1$  iff  $\forall w' \in Dom_z(w)[x(w') = x(w)]$
- (4) a.  $\lambda w'' . \iota x_e[\text{capital-of}(x, \text{italy}, w'')]$   
b.  $\lambda w . \forall w' \in Dom_m(w)[\iota x_e[\text{capital-of}(x, i, w')]] = \iota x_e[\text{capital-of}(x, i, w)]$

\* I would like to thank the audiences of the workshop “Frequently Asked Concealed Questions” (U. Göttingen, June 5-7, 2009), Semantik und Pragmatik im Südwesten I, the 17th Amsterdam Colloquium, and the departments of Linguistics at the universities of Geneva and Düsseldorf. Special thanks to Irene Heim, Hans Kamp, Sebastian Löbner and Ur Schlonsky for their insightful comments.

<sup>1</sup> CQs questions of higher type and the A/B ambiguity are left out of this paper.

- (5) a.  $\lambda w''. \sigma x_e[*dish(x, w'') \wedge order(you, x, w'')]$   
 b.  $\lambda w. \forall w' \in Dox_{waiter}(w) [ \sigma x_e[*dish(x, w') \wedge order(you, x, w')] = \sigma x_e[*dish(x, w) \wedge order(you, x, w)] ]$

The individual concept-based analysis runs into trouble when the CQ is headed by a quantificational determiner, as in (6). As the intension of the quantificational noun phrase is not an individual concept ( $\langle s, e \rangle$ ) but a generalized quantifier ( $\langle \langle e, st \rangle, \langle s, t \rangle \rangle$ ), something needs to be said. A first possibility is to make the argument slot of the embedding verb of property type or higher (4). However, this would derive the wrong reading (see observation 1 below). A second approach uses conceptual covers and salient properties (15, 13). Yet a third line exploits syntactic conversion and intensionalization of traces (7 building on 5). For space reasons, we will not review these approaches here.

- (6) The waiter remembered **some** / **most** dishes you ordered.

The goal of this paper is to sketch a solution to quantificational CQs within the individual concept line that maintains the analysis of (1)-(2) above. The idea is this: in the same way that adverbials like *to some extent* and *for the most part* can quantify over subquestions of a question, the determiners *some* and *most* can quantify over sub-individual concepts of a CQ individual concept. That is, the meaning of (6) will be roughly (though not exactly) that of (7):

- (7) The waiter **to some extent** / **for the most part** remembered what dishes you ordered.

The plot of this paper is as follows. §2 reviews QVE with interrogative clauses. Three empirical observations about quantificational CQs are presented in §3, followed by a first step towards an analysis in §4. A fourth observation and a second, promisory step conclude the paper in §5.

## 2 Review of Quantificational Variability Effect (QVE)

Consider (8). Under a first approach (3), (8) involves quantification over individuals and the embedding verb takes a proposition, as in (9a). A second approach (8) maintains that the quantification is over propositions (true answers to the question) and that the embedding verb takes a proposition, as in (9b). However, 2 observe that some verbs that only embed questions, e.g. *depend* in (10), also allow for QVE. 2 propose that quantification is over subquestions –defined in (11)– and that the embedding verb takes a subquestion, as in (9c).

- (8) John knows for the most part who cheated on the final exam.  
 (9) a. Most  $x$  [ $x$  cheated on the final exam] [John knows that  $x$  cheated on the final exam]  
 b. Most  $p$  [ $p$  is an answer to ‘who cheated on the final exam’ and  $p$  is true] [John knows  $p$ ]

- c. Most  $Q'$  [ $Q'$  is a relevant subquestion of ‘who cheated on the final exam’] [John knows  $Q'$ ]
- (10) a. Who will be admitted depends for the most part (exclusively) on this committee.  
 b. \*That John will be admitted depends on this committee.
- (11)  $Q'$  is a subquestion of  $Q$  iff it is possible that the answer to  $Q'$  provides a partial answer to  $Q$ .  
 That is, iff  $\exists w' \exists p [\text{Ans-strg}(Q')(w') \rightarrow p \wedge p \text{ is a partial answer to } Q]$

### 3 Three Observations about Quantificational CQs

**Observation 1.** The quantification introduced by the quantifier of a CQ is not part of the intensional object fed into  $\llbracket \text{know} \rrbracket$ , but it is external to it. That is, as suggested above, the (roughly) correct paraphrase of (12) is not (a) but (b). Proof of it is that, in scenario (13), the sentence underlined in (14) is false.

- (12) Spy C knows most of the code.  
 a.  $\neq$  “C knows what series of digits has the property of being most of the code.”  
 b.  $=$  “C for the most part knows what series of digits the code has.”
- (13) Scenario: The code is 60 digits long. Spy C got the first 57 digits, but she does not know what proportion of the code her finding amounts to.
- (14) Look! This is what spy C knows of the code. If she knew that she is so close to having the complete code, she’d be unstoppable. Luckily, she doesn’t know most of the code, so she may get discouraged and give up.

**Observation 2.** In some languages, CQs occur quite productively with *know*-type embedding verbs (*know*, *remember*, *reveal*, *tell*, etc.), with *depend*-type embedding verbs, and with *ask*-type embedding verbs. (15)–(16) illustrate this for Spanish. (See (11) and (12) for English.)

- (15) “Señor Conde Lucanor” –dijo Patronio–, “(...) me gustaría contaros lo que sucedió a un rey moro con tres pícaros granujas que llegaron a palacio”. Y el conde le preguntó lo que había pasado.  
 ‘“Count Lucanor” –said Patronio–, “(...) I would like to tell you what (lit. the that) happened to an Arab king with three naughty urchins that arrived in the palace”. And the count asked him what (lit. the that) had happened.’  
<http://www.ciudadseva.com/textos/cuentos/esp/juanma/lucanor/32.htm>
- (16) Lo que haga                      Marga esta semana depende de ti.  
 The that does-Subjunct M      this week      depends on you  
 ‘The things Marga goes this week depend on you.’

Interestingly, in those languages, CQs admit quantificational determiners (c-examples below) and adverbs of quantification (b-examples) with *know*-type

**Table 1.** The three types of quantification and embedding verbs

	<i>know</i>	<i>depend</i>	<i>ask</i>
ADV + INTERROGATIVE	✓	✓	*
ADV + CQ	✓	✓	*
DET + CQ	✓	✓	*

embedding verbs and with *depend*-type embedding verbs, but they are very awkward with *ask*-type embedding verbs. This parallels the facts about QVE with interrogatives (a-examples). The pattern is summarized in Table 1.

(17) With *know*:

- a. Juan sabe en su mayor parte qué estudiantes copiaron en el examen.  
'Juan knows for the most part which students cheated on the exam.'
- b. J. sabe en su mayor parte los estudiantes que copiaron en el examen.  
'J. knows for the most part the students who cheated on the exam.'
- c. Juan sabe la mayoría de los estudiantes que copiaron en el examen.  
'Juan knows most students who cheated on the exam.'

(18) With *depend*:

- a. En su mayor parte, qué haga Marga esta semana depende (exclusivamente) de ti.  
'For the most part, what<sub>INTERR</sub> Marga does this week depends (exclusively) on you.'
- b. En su mayor parte, lo que haga Marga esta semana depende (exclusivamente) de ti.  
'For the most part, the things (*lit.* the that) Marga does this week depend (exclusively) on you.'
- c. La mayor parte de lo que haga Marga esta semana depende de ti.  
'Most of what Marga will do this week depends on you.'

(19) With *ask*:

- a. #En su mayor parte, me preguntó qué había comido.  
'For the most part, s/he asked me what<sub>INTERR</sub> I/she/he had eaten.'
- b. #En su mayor parte, me preguntó lo que había comido.  
'For the most part, s/he asked me the things (*lit.* the that) I/she/he had eaten.'
- c. \*Me preguntó la mayor parte de lo que había comido.  
'S/he asked me most of what I/she/he had eaten.'

**Observation 3.** CQs with adverbs of quantification have more freedom in choosing the relevant set of sub-questions (or of sub-individual concepts) than CQs headed by quantificational determiners. A first way to divide a question like (20a) into subquestions is by using the domain of the *wh*-phrase as the sorting key and building the relevant *whether*-subquestions about its members. Under this division of the question, the sub-questions quantified over in (21) are the

ones in the set (22). This reading is also available for the combination of an adverb and a CQ, as in (20b), and for a quantificational CQ, as in (20c)<sup>2</sup>

- (20) a. For the most part, John knows which students cheated.  
       b. For the most part, John knows the students who cheated on the exam.  
       c. John knows most students who cheated on the exam.
- (21) Most  $Q'$  [ $Q'$  is a relevant subquestion of ‘which students cheated’] [John knows  $Q'$ ]
- (22) Set of *whether*-subquestions:  
       {Did student 1 cheat?, Did student 2 cheat?, Did student 3 cheat?, ... }

A second way to divide a question is by using a distributive plural NP –e.g. *these children* in (23a)– as the sorting key and forming the corresponding sub-questions about its atoms. Under this division, the logical representation of (23a) in (24) quantifies over the sub-questions in (25). This reading is also available for (23b), with an adverb and a CQ. In contrast, this reading is not possible if the CQ is directly headed by a quantificational determiner: sentence (23c), if acceptable, does not involve quantification over children.

- (23) a. For the most part, how well these children do depends (exclusively) on their families.  
       b. En su mayor parte, el rendimiento diario de estos niños depende exclusivamente del ambiente familiar.  
           ‘For the most part, the daily performance of these children depends exclusively on the family atmosphere.’  
       c. (#) La mayor parte del rendimiento diario de estos niños depende exclusivamente del ambiente familiar.  
           (#) ‘Most of the daily performance of these children depends exclusively on the family atmosphere.’
- (24) Most  $Q'$  [ $Q'$  is a relevant subquestion of ‘how well these children do’] [ $Q'$  depends (exclusively) on the family atmosphere]
- (25) Set of subquestions induced by a distributive plural NP: {How well does child 1 do?, How well does child 2 do?, How well does child 3 do?, ... }

Similarly to the case of distributive plurals, a third strategy uses a plural NP with a cumulative reading as the sorting key, e.g. *these professors* in (26a). In this case, the quantification in (27) ranges over the sub-questions in (28). This reading is also available for the adverb plus CQ variant (26b), but it is unavailable for the CQ with a quantificational determiner in (26c).

- (26) a. Louise knows for the most part which books these professors recommended.

<sup>2</sup> It remains to be determined whether all the variants in (20) allow for weakly and strongly exhaustive readings.

- b. For the most part, Louise knows the books that these professors recommended.
  - c. Louise knows most books that these professors recommended.
- (27) Most  $Q'$  [ $Q'$  is a relevant subquestion of ‘which books these professors recommended’] [ Louise knows  $Q'$  ]
- (28) Set of subquestions induced by a cumulative plural NP:  
 {Which books did professor 1 recommend?, Which books did professor 2 recommend?, Which books did professor 3 recommend?, ... }

In sum, adverbial quantification with interrogative clauses and with CQs is quite permissible as to how the original question or individual concept (IC) can be divided into smaller parts. In contrast, when the quantificational element is the determiner heading the CQ, only the head noun can be used as the sorting key, yielding a set of “whether” sub-ICs. This is summarized in Table 2.

**Table 2.** The three types of quantification and possible divisions

	“whether” sub-questions/ICs	sub-questions/ICs based on distr. NP	sub-questions/ICs based on cum. NP
ADV + INTERROG	✓	✓	✓
ADV + CQ	✓	✓	✓
DET + CQ	✓	*	*

## 4 First Step towards an Analysis

[2] propose the definitions [11] and (roughly) [29] to handle adverbial quantification with interrogative clauses. As the reader can check for herself, the definitions allow for the three types of divisions illustrated in the previous section.

- (29) A set  $Part(Q)(w)$  of questions  $Q'$  is a division of  $Q$  into subquestions in  $w$  iff these subquestions taken together exhaust the original question. More formally: iff
- i. For each  $Q' \in Part(Q)(w)$ ,  $Q'$  is a subquestion of  $Q$ ; and
  - ii Either a.  $\cap\{Ans-wk(Q')(w) : Q' \in Part(Q)(w)\} = Ans-wk(Q)(w)$   
 or b.  $\cap\{Ans-strg(Q')(w) : Q' \in Part(Q)(w)\} = Ans-strg(Q)(w)$

Now we need something similar for adverbial quantification with CQs. The idea is to take the original IC expressed by the CQ noun phrase and divide it into natural sub-individual concepts that, taken together, exhaust the original IC. The proposed definition is the following.<sup>3</sup>

<sup>3</sup> The restriction to natural ICs in [30] aims to eliminate the empty  $\langle s, e \rangle$  function from the division set and to prevent spurious splitting of e.g. the IC  $\langle < w_{100}, 2 >, < w_{101}, 2 > \rangle$  in [33] into two separate unnatural ICs  $\langle < w_{100}, 2 > \rangle$  and  $\langle < w_{101}, 2 > \rangle$ .

- (30) A set  $Part(x_{<s,e>})$  of natural individual concepts  $y_{<s,e>}$  is a **division** of  $x_{<s,e>}$  into sub-individual concepts iff:  
 For all  $w \in \text{Dom}(x) : \sqcup\{y(w) : y \in Part(x)\} = x(w)$ .

Let us apply this definition to CQs with adverbial quantification. Consider first example (31a) and its truth conditions (31b). The (partial) IC function expressed by the NP is formulated and illustrated in (32) (assuming for simplicity that there are only three students under consideration: student 1, student 2 and student 3). How can we divide this original IC into a set of sub-ICs? One possible avenue allowed by the definition above is to divide it by individual students, that is, to carve out sub-ICs like “the student 1 who cheated on the exam”. This produces the set (33), which derives the intended reading of the sentence.

- (31) a. For the most part, John knows the students who cheated on the exam.  
 (= (20b))  
 b.  $\lambda w. \text{Most } y_{<s,e>}$   
 $[y_{<s,e>} \in Part([the\ students\ that\ cheated\ on\ exam])]$   
 $[John\ knows\ y_{<s,e>}\ in\ w]$
- (32)  $[the\ students\ who\ cheated\ on\ the\ exam] =$   
 $\lambda w'. \sigma z_e [*student(z, w') \wedge cheated(z, w')]$   
 E.g.:  $\begin{bmatrix} w_{100} \rightarrow 1+2+3 \\ w_{101} \rightarrow 2 \\ w_{102} \rightarrow \# \end{bmatrix}$
- (33)  $\{ \lambda w. \iota z_e [student(z, w) \wedge z = stud1 \wedge cheated(z, w)],$   
 $\lambda w. \iota z_e [student(z, w) \wedge z = stud2 \wedge cheated(z, w)],$   
 $\lambda w. \iota z_e [student(z, w) \wedge z = stud3 \wedge cheated(z, w)] \}$   
 E.g.:  $\left\{ \begin{bmatrix} w_{100} \rightarrow 1 \\ w_{101} \rightarrow \# \\ w_{102} \rightarrow \# \end{bmatrix}, \begin{bmatrix} w_{100} \rightarrow 2 \\ w_{101} \rightarrow 2 \\ w_{102} \rightarrow \# \end{bmatrix}, \begin{bmatrix} w_{100} \rightarrow 3 \\ w_{101} \rightarrow \# \\ w_{102} \rightarrow \# \end{bmatrix} \right\}$

Consider now example (34a) and its logical representation in (34b). The intension of the NP is spelled out and exemplified in (35). Recall that, under the intended reading, the division of the original individual concept uses as sorting key the cumulative NP *these professors*. This means that we have to carve out “book” sub-ICs based on the professors, e.g. the sub-IC “the books that professor 1 recommended”. This yields the division set (36), which meets definition (30):

- (34) a. For the most part, Louise knows the books that these professors recommended. (= (26b))  
 b.  $\lambda w. \text{Most } y_{<s,e>}$   
 $[y_{<s,e>} \in Part([the\ books\ that\ these\ profs\ recommended])]$   
 $[Louise\ knows\ y_{<s,e>}\ in\ w]$
- (35)  $[the\ books\ that\ these\ professors\ recommended] =$   
 $\lambda w'. \sigma z_e [*book(z, w') \wedge **recommend(these.profs, z, w')]$

$$\begin{aligned}
& \text{E.g.: } \begin{bmatrix} w_{100} \rightarrow a+b+c+d+e \\ w_{101} \rightarrow e+f+g \\ w_{102} \rightarrow \# \end{bmatrix} \\
(36) \quad & \{ \lambda w. \sigma z_e [*book(z, w) \wedge recommend(\text{prof1}, z, w)], \\
& \lambda w. \sigma z_e [*book(z, w) \wedge recommend(\text{prof2}, z, w)], \\
& \lambda w. \sigma z_e [*book(z, w) \wedge recommend(\text{prof3}, z, w)] \} \\
& \text{E.g.: } \left\{ \begin{bmatrix} w_{100} \rightarrow a+b+c \\ w_{101} \rightarrow e \\ w_{102} \rightarrow \# \end{bmatrix}, \begin{bmatrix} w_{100} \rightarrow b \\ w_{101} \rightarrow f \\ w_{102} \rightarrow \# \end{bmatrix}, \begin{bmatrix} w_{100} \rightarrow d+e \\ w_{101} \rightarrow g \\ w_{102} \rightarrow \# \end{bmatrix} \right\}
\end{aligned}$$

Let us turn to CQs with quantificational determiners. Now only divisions into “whether” sub-ICs should be allowed. This can be achieved by defining a lexical entry for the determiner that builds the original IC out of the N'-property and requires it to be divided into (possibly partial) *constant* functions, as follows:

- (37)  $\lambda P_{\langle e, st \rangle}. \lambda Q_{\langle se, st \rangle}. \lambda w.$   
 $\text{MOST } y_{\langle s, e \rangle} [y_{\langle s, e \rangle} \in CPart(\lambda w'. \sigma z_e [P(z)(w')])] [Q(y_{\langle s, e \rangle})(w)]$
- (38) A set  $CPart(x_{\langle s, e \rangle})$  of natural individual concepts  $y_{\langle s, e \rangle}$  is a **constant-based division** of  $x_{\langle s, e \rangle}$  into sub-individual concepts iff:
- For all  $y_{\langle s, e \rangle} \in CPart$ :  $y_{\langle s, e \rangle}$  is a constant function, and
  - For all  $w \in \text{Dom}(x) : \sqcup \{y(w) : y \in Part(x)\} = x(w)$ .

For (39), we will obtain (32) as the original IC and will be able to use the set (33) as the constant-based division of it, hence deriving the desired reading. For (40), the IC in (35) will be built, but we will not be able to use the set (36), as this set contains non-constant functions. Hence, the intended reading is unavailable.<sup>4</sup>

- (39) a. John knows most students who cheated on the exam. (= (20c))  
 b.  $\lambda w. \text{MOST } y_{\langle s, e \rangle}$   
 $[y_{\langle s, e \rangle} \in CPart(\lambda w'. \sigma z_e [\llbracket \text{students that cheated on exam} \rrbracket(z)(w')])]$   
 $[\text{John knows } y_{\langle s, e \rangle} \text{ in } w]$
- (40) a. Louise knows most books that these professors recommended. (= (26c))  
 b.  $\lambda w. \text{MOST } y_{\langle s, e \rangle}$   
 $[y_{\langle s, e \rangle} \in CPart(\lambda w'. \sigma z_e [\llbracket \text{books that these profs recomm.} \rrbracket(z)(w')])]$   
 $[\text{Louise knows } y_{\langle s, e \rangle} \text{ in } w]$

To sum up, the quantification induced by a determiner in a CQ is similar to adverbial quantification with interrogative clauses and CQs in that it is external to the content of the question/IC (observation 1) and in that it is compatible

<sup>4</sup> In this paper, only name-like answers like (iA) are considered and intensional answers like (iA') are left aside. The latter would require a more permissible type of division than (38) (H. Kamp, p.c.). I leave this issue for future research.

(i) Q: Which students cheated on the final exam?

A: Bill Smith and Paul Taylor.

A': The dumbest student in the class and the busiest student in the class.

only with certain embedding verbs (observation 2). However, a determiner in a CQ allows only for a division of the original individual concept into constant sub-concepts, whereas adverbial quantification allows for divisions into non-constant functions. The question remains, why should adnominal and adverbial quantification differ this way. This is briefly addressed in §5.

## 5 A Fourth Observation and a Second, Promisory Step

It is well-known that adverbials producing QVE over individuals (type e) can “target” different NPs in the clause ([9], [10]). We note that they can also target NPs in an *embedded* clause, e.g. the NPs underlined in ([41]). In other words, the difference between Adv+CQ and Det+CQ in §4 is just an instance of a more general contrast between Adv+Plural, as in ([41]), and Det+Plural, as in ([42]).

- (41) a. For the most part, John read the books that these professors recommended.  
 b. For the most part, John can achieve the performance that those brilliant students achieved on the test.
- (42) a. John read most (of the) books that these professors recommended.  
 b. (#) John can achieve most of the performance that those brilliant students achieved on the test.

[10] analyse two constructions with *most* involving plural individuals. Determiner *most* directly quantifies over the set of individuals denoted by its sister NP, as ([43]). In contrast, *for the most part* directly quantifies over a set of events, as in ([44]), only indirectly is QVE over individuals derived, through a contextual mapping from events to individuals (see [10] for justification and details).

- (43) Truth conditions for ‘Most of NP VP’:  
 $\exists x' [x' \leq \llbracket \text{NP} \rrbracket \wedge |x'| > 1/2 \llbracket \llbracket \text{NP} \rrbracket \rrbracket \wedge \forall x'' [x'' \leq x' \rightarrow \llbracket \text{VP} \rrbracket (x'')]$   
 Consider the plural individual  $\llbracket \text{NP} \rrbracket$ , e.g.  $x$ . There is an  $x'$  that is a major part of  $x$  such that, for all subindividuals  $x''$  of  $x'$ ,  $\llbracket \text{VP} \rrbracket (x'')$  holds.
- (44) Truth conditions of ‘For the most part NP VP’:  
 $\exists e [p(e) \wedge \exists e' [e' \leq e \wedge |e'| > 1/2 |e| \wedge \forall e'' [e'' \leq e' \rightarrow q(e'')]]]$   
 There is a plural event  $e$  for which  $p(e)$  holds and there an event  $e'$  that is a major part of  $e$  such that, for all subevents  $e''$  of  $e'$ ,  $q(e'')$  holds.  
 ( $p$  = non-focused material;  $q$  = focused material)

This idea applies to our examples as follows. In the case of plural individuals, determiner quantification yields the truth conditions in ([45]), where the relevant set of books is univocally quantified over. Adverbial quantification generates the truth conditions in ([46]), where we quantify over subevents of  $e$ . If those subevents are “carved out” in a one-to-one mapping with the individual books, we get the same reading as in ([45]); if they are “carved out” in a one-to-one mapping with the professors, we get a QVE reading over professors.

- (45) John read most of the books that these professors recommended.  
 $\lambda w. \exists x' [ x' \leq \llbracket \text{the books that these profs recommended} \rrbracket(w) \wedge$   
 $|x'| > 1/2 | \llbracket \text{the books that these profs recommended} \rrbracket(w) | \wedge$   
 $\forall x'' [x'' \leq x' \rightarrow \text{John read } x'' \text{ in } w] ]$
- (46) For the most part, John read<sub>F</sub> the books that these profs. recommended.  
 $\lambda w. \exists e [ \text{Agent}(e, \text{john}) \wedge \text{Theme}(e, \llbracket \text{the books these profs rec.} \rrbracket(w)) \wedge e \text{ is}$   
 $\text{in } w \wedge \exists e' [ e' \leq e \wedge |e'| > 1/2 |e| \wedge \forall e'' [e'' \leq e' \rightarrow \text{read}(e'')] ] ]$

I tentatively propose that quantification with CQs is done in a parallel way. Determiner quantification leads to truth conditions like (48). The original IC is converted into the corresponding plural sum of *constant* sub-ICs according to the definition of PL-DIV in (47), and *most* univocally quantifies over those sub-ICs. In contrast, adverbial quantification leads to truth conditions like (49). If the subevents quantified over are in a one-to-one mapping with ICs of the shape “the book X that (some of) these professors recommended”, we obtained the same reading as in (48); if the subevents are in one-to-one mapping with ICs like “the books that professor X recommended”, we derive the intended reading.

- (47) A sum of individual concepts  $y_{1,<s,e>} + y_{2,<s,e>} + \dots + y_{n,<s,e>}$  is a plural division of an individual concept  $x_{<s,e>}$ , PL-DIV( $x$ ), iff:
- For all atomic  $y \leq y_1 + y_2 + \dots + y_n$ ,  $y$  is a (possibly partial) constant function, and
  - For all  $w \in \text{Dom}(x)$ :  $y_1 + y_2 + \dots + y_n = x(w)$ .
- (48) John knows most (of the) books that these professors recommended.  
 $\lambda w. \exists x' [ x' \leq \text{PL-DIV}(\llbracket \text{the books that these profs recommended} \rrbracket) \wedge$   
 $|x'| > 1/2 | \text{PL-DIV}(\llbracket \text{the books that these profs recommended} \rrbracket) | \wedge$   
 $\forall x'' [x'' \leq x' \rightarrow \text{John knows } x'' \text{ in } w] ]$
- (49) For the most part, John knows<sub>F</sub> the books these profs. recommended.  
 $\lambda w. \exists e [ \text{Agent}(e, \text{john}) \wedge \text{Theme}(e, \llbracket \text{the books these profs rec.} \rrbracket) \wedge e \text{ is}$   
 $\text{in } w \wedge \exists e' [ e' \leq e \wedge |e'| > 1/2 |e| \wedge \forall e'' [e'' \leq e' \rightarrow \text{know}(e'')] ] ]$

## References

- Aloni, M.: Concealed questions under cover. *Grazer Ph. St.* 77, 191–216 (2008)
- Beck, S., Sharvit, Y.: Pluralities of questions. *J. Sem.* 19, 105–157 (2002)
- Berman, S.: On the semantics and LF of Wh-clauses, UMass Ph.D. diss. (1991)
- Frana, I.: The de re analysis of concealed questions. In: *Proc. SALT XVI* (2006)
- Frana, I.: Concealed questions: in search of answers. UMass Ph.D. diss. (2009)
- Heim, I.: Concealed Questions. In: Bäuerle, R., Egli, U., von Stechow, A. (eds.) *Semantics from different points of view*, pp. 51–60. Springer, Berlin (1979)
- Heim, I.: Nouns as predicates of individual concepts? In: *Talk at the Workshop Frequently Asked Concealed Questions*, U. Göttingen (2009)
- Lahiri, U.: Questions and answers in embedded contexts. Oxford U. Press, Oxford (2002)
- Löbner, S.: Polarity in natural language: predication, quantification and negation in particular and characterizing sentences. *L&P* 23, 213–308 (2000)

10. Nakanishi, K., Romero, M.: Two Constructions with *Most* and their Semantic Properties. In: Proceedings of NELS, vol. 34, pp. 453–467. GLSA, Amherst (2004)
11. Nathan, L.: On the interpretation of concealed questions. MIT Ph.D. diss. (2006)
12. Percus, O.: Concealed questions as concealed questions. In: Talk at the Workshop Frequently Asked Concealed Questions, U. Göttingen (2009)
13. Roelofsen, F., Aloni, M.: Perspectives on concealed questions. In: Proceedings of SALT XVIII (2008) (Revised version November 2009)
14. Romero, M.: Concealed questions and specificational subjects. L&P 28, 687–737 (2005)
15. Schwager, M.: Keeping prices low: an answer to a concealed question. In: Proceedings of Sinn und Bedeutung XII (2007)

# Specific, Yet Opaque

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**Abstract.** In her dissertation, Janet Fodor has argued that the quantificational force and the intensional status of certain quantifier phrases can be evaluated independently. The proposal was only halfway accepted: the existence of non-specific transparent readings is well-established today, but specific opaque readings are deemed illusory. I argue that they are real and outline a semantic framework that can generate them. The idea is to permit two types of quantifier raising: one that carries the restrictor of the determiner along and another that does not. When the second is applied the restrictor can be stranded within the scope of an intensional operator as the quantificational determiner itself takes wider scope.

## 1 Fodor's Readings

Consider the following inference:

- (1) Alex, Bart, and Chloe are three distinct individuals  
Ralph thinks Alex is an American who lives in Paris  
Ralph thinks Bart is an American who lives in Paris  
Ralph thinks Chloe is an American who lives in Paris  
Ralph thinks at least three Americans live in Paris

This looks valid but under standard assumptions it can't be. The conclusion, under its *de re* reading, entails the existence of Americans who live in Paris. But the premises don't: they are compatible with Ralph having false beliefs about Alex, Bart, or Chloe. Under the *de dicto* reading, the conclusion entails that Ralph has a general belief about the number of Americans who live in Paris. But the premises don't: they are compatible with him thinking nothing more than that Alex is an American who lives in Paris, and Bart is, and Chloe is. It is certainly likely that if Ralph has these three specific beliefs he will also come to have the general one. But logic alone can't force him to live up to his commitments.

It would not be fair to dismiss this problem by pointing out that if propositions are taken to be sets of possible worlds then the inference is valid on the *de dicto* construal. True enough, if in all of Ralph's belief worlds Alex, Bart, and Chloe are Americans who live in Paris then all those worlds contain at least three Americans who live in Paris. But we should not forget that given the high-flying idealization of a Hintikka-style semantics for attitude verbs, (2) is also supposed to be a valid inference:

- (2) Alex, Bart, and Chloe are three distinct individuals  
 Ralph thinks Alex is an American who lives in Paris  
 Ralph thinks Bart is an American who lives in Paris  
Ralph thinks Chloe is an American who lives in Paris  
 Ralph thinks arithmetic is incomplete

Semanticists tend to concede that (2) is not valid. Attitude verbs are hyper-intensional but for many purposes they can be treated as if they were merely intensional. Fair enough, we all like to keep things simple when we can. But if we don't want to take the blame when the simplifying assumption of logical omniscience leads to unacceptable predictions we should not take credit when it accidentally delivers the right result – as it happens in (1).

The real reason (1) is valid has nothing to do with logical omniscience. It is rather that the conclusion can be read in a way that differs from both the *de re* and *de dicto* interpretations. This reading can be paraphrased as (3):

- (3) There are at least three people Ralph thinks are Americans who live in Paris.

That intensional constructions may give rise to such readings has been conjectured before. Fodor [4] argued that the quantificational force and the intensional status of certain quantified phrases can be evaluated independently. Thus, she claimed that a sentence like (4) has four distinct readings:

- (4) Mary wants to buy an inexpensive coat.
- a. *Non-specific, opaque (de dicto)*: Mary wants this: that she buys an inexpensive coat.
  - b. *Specific, transparent (de re)*: There is an inexpensive coat which Mary wants to buy.
  - c. *Non-specific, transparent*: There are inexpensive coats of which Mary wants to buy one.
  - d. *Specific, opaque*: There is a thing which Mary wants to buy as an inexpensive coat.

It is easy to imagine conditions under which the non-specific transparent reading is true but the *de re* and *de dicto* readings are false. Mary could have a certain type of coat in mind and have the desire to purchase an instance of that type, while being completely unaware of the fact that such coats are inexpensive. That (4-c) is a genuine reading of (4) has been generally recognized by semanticists; many have taken it as evidence that the scope theory of intensionality is either completely mistaken or in need of a thoroughgoing revision.<sup>1</sup> But the reading that corresponds to (4) is (4-d) – it is specific (in the sense that its truth-value depends on how things stand with objects in the actual world) yet opaque (in the sense that it characterizes these objects not as they are but as Mary takes them to be).

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<sup>1</sup> The presence of the non-specific transparent readings is attested by other examples as well; cf. Bäuerle [2], Abusch [1], Percus [9].

Alas, the consensus these days is that the existence of the specific opaque reading is an illusion. One reason for this is the difficulty of paraphrase: ‘There is a thing which Mary wants to buy as an inexpensive coat’ is quite artificial. Perhaps the best we can do is (5):

(5) There is a coat Mary wants to buy. She thinks it is inexpensive.

Can (4) have a reading like (5)? Here is a widely-accepted argument that it cannot.<sup>2</sup> (4) and (5) do not permit the same continuations; while (5+) is coherent (4+) is not:

(4+) Mary wants to buy an inexpensive coat. # But it is actually quite expensive.

(5+) There is a coat Mary wants to buy. She thinks it is inexpensive. But it is actually quite expensive.

But then (4) and (5) cannot be synonyms, and thus, (4) lacks a specific opaque reading. I think the argument is too quick. The presence of an anaphoric pronoun forces a specific reading on the preceding sentences in both (4+) and (5+). In (5+) the anaphoric pronoun can pick out the coat Mary wants to buy and thinks is inexpensive. In (4+) the the anaphoric pronoun must pick out the inexpensive coat Mary wants to buy, which leads to inconsistency. But this contrast could be explained by the fact that the word *thinks* is present in (5) but missing from (4). Thus, we should not jump to the conclusion that (4) and (5) cannot have the same truth-conditions.<sup>3</sup> Consider (6):

(6) Mary thinks she bought an inexpensive coat. It is actually quite expensive.

I think this sequence is perfectly consistent; it is certainly much better than (4+). If there is no such thing as a specific opaque reading, the contrast is a bit of a mystery.

The intuitive validity of (1) and the intuitive coherence of (6) suggest that the dismissal of Fodor’s specific opaque reading is a mistake. But generating such a reading within the standard quantificational framework using QR is far from trivial. To get a non-specific transparent reading we need to find a way to evaluate the restrictor of the DP “higher up” while interpreting the quantificational force “downstairs”. There are various mechanisms that have this effect

<sup>2</sup> This argument goes back to Ioup [6]. My presentation follows Keshet [7].

<sup>3</sup> Compare this with Partee’s marble example. The reason (i) *I lost ten marbles and found all but one* can but (ii) *I lost ten marbles and found nine* cannot be felicitously continued with (iii) *It must be under the sofa* has to do with the fact that (i) does and (ii) does not contain the word *one*. One might conclude from this (as proponents of dynamic approaches have) that (i) and (ii) are not synonyms. But one would certainly not want to say that (i) and (ii) differ in their truth-conditions. Similarly, I am inclined to accept that (4) and (5) cannot mean the same while I reject the suggestion that they have different truth-conditions.

– we can, for example, use overt world-variables<sup>4</sup>. Then the simplified logical forms of the two non-specific readings of (4) would differ only in the choice of the world variable associated with the DP:

- (4) a'.  $\lambda w$  Mary wants [ $\lambda w'$  to buy [an inexpensive coat in  $w'$ ]]  
 c'.  $\lambda w$  Mary wants [ $\lambda w'$  to buy [an inexpensive coat in  $w$ ]]

To get the corresponding specific readings we would need to raise the DP, which results in the following logical forms:

- (4) b'.  $\lambda w$  [an inexpensive coat in  $w$ ] <sub>$i$</sub>  Mary wants [ $\lambda w'$  to buy  $i$ ])  
 d'.  $\lambda w$  [an inexpensive coat in  $w'$ ] <sub>$i$</sub>  Mary wants [ $\lambda w'$  to buy  $i$ ])

(4b') is a perfectly adequate way to capture the specific transparent (*de re*) reading, but (4d') says nothing like (4d). It would if the world-variable within the raised DP could be bound “from below” – but that is not how variable binding works.

To bypass this problem, we need to change the standard framework more radically. Before proposing such a change in section 4, I will try to provide more robust evidence that the specific opaque readings are real.

## 2 Summative Reports

Let's start with the core example. Alex is a somewhat paranoid – he thinks that his neighborhood is full of terrorists. He spends much of his time observing comings and goings, following people around, and making inquiries. One day he goes to the police. The police officer who interviews Alex hands him a pile of photographs of people who live in his neighborhood. When Alex looks at a photograph he is asked first whether the person is a terrorist and if he answers affirmatively he is then asked where the person lives. When he is done looking through the photographs he is asked whether there are terrorists in the neighborhood who are not on any of the photographs he has seen. He says that there are not. He is also asked whether he knows how many terrorists he has identified. He says that there were quite a few but he does not know precisely how many. Fortunately, the police officer took tally. It turns out that Alex has identified 17 photographs as showing terrorists, and of those 11 as showing ones that live in the apartment building across the street from him. When the police officer who conducted the interview later reports this to his superiors he says the following:

- (7) Alex believes that eleven terrorists live across the street from him.

Assuming Alex was honest in expressing his beliefs this seems like a true report. It is neither a *de re* claim (the people Alex accused need not be terrorists) nor a *de dicto* one (Alex did not count up his accusations). Rather, it is what I will call a *summative report*. Alex's answers express a number of *de re* beliefs regarding the people on the photographs and the police officer summarizes those

<sup>4</sup> See Percus [9], von Stechow and Heim [3], and Keshet [7].

beliefs in his report. The words ‘terrorist’ and ‘lives across the street’ show up in Alex’s answers, so they are to be taken to reflect how Alex thinks of the people on the pictures. The police officer need not think that either of these predicates applies to any of those people. By contrast, the word ‘eleven’ is the police officer’s contribution to the report. He is the one keeping tally. Alex need not have any belief about the number of people he takes to be terrorists across the street. The summative reading of (7) is what Fodor called specific opaque.

We could replace ‘eleven’ in (7) with any other numerical determiner and preserve the summative reading. Other intersective determiners work as well. (8), for example, can be used to make a true report under the circumstances described above, even if Alex thinks that eleven terrorists across the street are but a pittance. (Perhaps he thinks most neighborhoods have a lot more terrorists than his own.)

(8) Alex believes that many terrorists live across the street from him.

When the report is summative ‘many’ is the officer’s contribution, and the report is true because Alex in fact identified eleven people as terrorists living across the street from him and because eleven terrorists across the street are in fact many.

Here is another example, this time using a non-intersective quantifier. Imagine that Bob, who lives in the same neighborhood, also comes to the police and claims that there are a number of terrorists living there. The police officer goes to his supervisor and they discuss the new development, comparing Bob’s accusations with those made by Alex. The police officer observes that there is not much agreement between Alex and Bob about where the terrorists are concentrated in the neighborhood. He says:

(9) Alex believes that most terrorists live across the street from him.

Given that Alex has identified 17 people as terrorists and 11 of them as living across the street from him and that he also said that there are no terrorists in the neighborhood who are not on any of the photographs he has seen, this report seems true. The report quantifies restrictedly – the context makes clear that only people in Alex’s neighborhood are at issue. With the obvious changes in the pattern of responses Alex gave, we can confirm the existence of summative readings involving other non-intersective quantifiers, such as ‘every’, ‘two thirds of’, or ‘no’.

Given the character of summative reports one might expect that we can lump together not only *de re* beliefs of a single person, but also *de re* beliefs of multiple people. This expectation is borne out. Imagine that besides the 17 people Alex accuses of terrorism, Bob accuses another 9. The police officer could report the outcome of the two interviews as (10):

(10) Alex and Bob believe that twenty-six terrorists live in their neighborhood.

The summative reading of (10) is cumulative – it is like saying that Alex and Bob ate twenty-six cookies if they jointly devoured that much. The difference is that those twenty-six things Alex and Bob ate had to be really cookies while the twenty-six people they have beliefs about needn't be really terrorists.

I conclude that summative readings are available for quantified belief reports no matter what quantifier is used. This strengthens the evidence for the existence of specific opaque readings provided by the inference under (1).

### 3 Modals, Tense, and Aspect

Similar readings arise with modals, tense, and aspect as well. They are relatively easy to find, once one knows where to look.

Imagine that Anna is taking a course and the term paper is due next Monday. She has three outlines and she is trying to decide which one to work on. She doesn't have time to write more than one paper before next Monday. Under these circumstances (11) seems true.

- (11) Anna could write three papers. Now she has to decide which one to write.

Note that it is false that three papers are such that it is possible that Anna writes them and equally false that it is possible that Anna writes three papers. The true reading is summative: three things (i.e. the outlines) are such that each could be a paper written by Anna.

Ben is in the same class but his situation is different. He has been working steadily for a long time but he tends to be unhappy with what he writes. On Friday he finishes a paper and burns it, on Saturday he finishes another and burns that too, and on Sunday he finishes the third but that one doesn't make it to Monday either. Still, when Monday comes (12) seems true:

- (12) Ben wrote three papers. Unfortunately, he burned them all.

On the *de re* construal, *three papers* would scope above the tense and thus there would have to be three papers in existence on Monday for the sentence to be true then. On the *de dicto* construal, *three papers* is interpreted within the scope of the past tense and thus there would have to be some time before Monday when three papers were in existence. But there never were three papers Ben wrote in existence. The true reading is again summative: three things (i.e. the past papers) are such that each was a paper written by Ben.<sup>5</sup>

<sup>5</sup> The puzzle of summative readings for tense was discovered a long time ago. Sextus Empiricus in *Against the Physicists* 2.98. attributes a puzzle to Diodoros Chronos. The puzzle concerns Helen of Troy who was consecutively married to three different men – Menelaus, Paris, and Deiphobus. Thus, it seems like we can use *Helen had three husbands* to say something true. Since the husbands are no more, it is false that three husbands are such that each was at some time in the past Helen's. Since Helen is not guilty of trigamy it is false that at some time in the past Helen had three husbands.

What summative readings involving attitude verbs, modals and tense have in common is that they express the results of counting across certain boundaries. In (7), we count across Alex’s *de re* beliefs: he believes of this guy that he is a terrorist and also that he lives across the street, and he believes the same of this other guy, . . . , and of this eleventh guy as well, *ergo* Alex believes eleven terrorists live across the street. In (11), we count across worlds: there is a possible world where Anna writes this paper, a different possible world where she writes this other one, and a third where she writes this third one, *ergo* Anna could write three papers. In (12), we are summing up what is the case at different times: there is a time when Ben wrote this paper, another when he wrote this other one, and a third when he wrote this third one, *ergo* Ben wrote three papers.<sup>6</sup>

In the cases hitherto considered, the specific opaque reading had to be teased out by constructing the appropriate contexts in which the sentence is plausibly used with that intended reading. But there are also cases where this reading is the dominant one. Consider Chris who is in the same class as the other two. Chris is a show-off – he intends to hand in not one but three papers on Monday. On Saturday he is sitting at his computer working simultaneously on all three drafts. (13) describes the case correctly:

(13) Chris is writing three papers. All three are up on his screen.

There is a long-standing debate in the semantics literature about the status of things in progress. The establishment view is that a sentence like (13) does not entail the existence of any actual paper Chris is writing. This is the *de dicto* construal, where *three papers* is interpreted below the aspect. The anti-establishment view denies this and says that (13) entails that there are three papers such that Chris is writing them. This is the *de re* reading, where aspect takes narrow scope with regard to *three papers*.<sup>7</sup> The establishment has the upper hand – it seems clear that while Chris is working on the papers there are no papers yet. But the anti-establishment makes a good point too – it seems equally clear that there are three things that are the objects of Chris’s writing. They are actual drafts stored on the computer, not mere possibilities. Thus, the normal reading of (13) is, I think, neither *de re* nor *de dicto*. Rather, it is specific opaque one: there are three things (i.e. the drafts) such that each is becoming a paper written by Chris.

## 4 Split Quantifiers

How can specific opaque readings be generated? Within a QR-based approach to quantification, the task comes down to specifying a mechanism that splits

<sup>6</sup> I argue for the existence of such readings in Szabó (10).

<sup>7</sup> The most prominent defense of the anti-establishment view is Parsons (8). Zucchi (12) is the standard critique of this aspect of Parsons’s work. In Szabó (11) I argue for an account of the progressive that takes a middle course – it takes the sentence *Jack is building a house* to entail the existence of an actual object Jack is building without characterizing this object as a house.

quantificational determiners from their restrictors. Then the former can move above an intensional operator while the latter is evaluated “downstairs”. I will present such a mechanism within the standard framework of Heim and Kratzer [5].

The idea is that raising of a quantified DP is more akin to copying: the syntactic structure remains in its original position while an identical one is attached above a higher  $S$  node. The quantificational determiner moves to the higher position leaving an ordinary trace below. For the restrictor there are two possibilities: it can move or it can stay. The unfilled restrictor position – whether it is the higher one or the lower one – is filled by a default predicate whose extension is  $D_e$ . Finally, we need a new rule that combines the trace with a predicate and delivers what I call a *restricted trace*. The semantic value of a restricted trace is undefined whenever the trace is assigned a value that is not within the extension of the restrictor.

Here is a small fragment of a language that allows split raising. It contains just one verb (*run*), two nouns (*dog*, *thing*), three quantificational determiners (*every*, *some*, *most*) and traces indexed by natural numbers ( $t_\iota$ , where  $\iota \in \omega$ ). The semantic types of lexical items are the usual: the nouns and the verb are of type  $\langle e, t \rangle$ , the quantificational determiners of type  $\langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle$ , and the traces of type  $e$ . Semantic values of lexical items are standard (since only traces have assignment-dependent semantic values the superscript is suppressed elsewhere):

$$\begin{aligned}
\llbracket t_\iota \rrbracket^a &= a(\iota) \\
\llbracket runs \rrbracket &= \lambda x \in D_e \ x \ runs \\
\llbracket dog \rrbracket &= \lambda x \in D_e \ x \ is \ a \ dog \\
\llbracket thing \rrbracket &= \lambda x \in D_e \ x = x \\
\llbracket every \rrbracket &= \lambda f \in D_{\langle e, t \rangle} \cdot \lambda g \in D_{\langle e, t \rangle} \cdot \text{for all } x \in D_e \text{ if } f(x) = 1 \text{ then } g(x) \neq 0 \\
\llbracket some \rrbracket &= \lambda f \in D_{\langle e, t \rangle} \cdot \lambda g \in D_{\langle e, t \rangle} \cdot \text{some } x \in D_e \text{ is such that } f(x) = 1 \text{ and } g(x) = 1 \\
\llbracket most \rrbracket &= \lambda f \in D_{\langle e, t \rangle} \cdot \lambda g \in D_{\langle e, t \rangle} \cdot \text{more } x \in D_e \text{ are such that } f(x) = 1 \text{ and } g(x) = 1 \\
&\quad \text{than are such that } f(x) = 1 \text{ and } g(x) = 0
\end{aligned}$$

The only slightly unusual thing here is the interpretation of *every*; normally the clause ends with ‘ $g(x) = 1$ ’ rather than ‘ $g(x) \neq 0$ ’. The semantics allows partial functions, so this will make a difference, which will be explained below.

Concatenation is interpreted as functional application except in the following two cases (the first is Heim & Kratzer’s, the second is new).

- (PA) If  $\iota$  is an index and  $\sigma$  is a sentence then  $[\iota\sigma]$  is a *predicate abstract* of type  $\langle e, t \rangle$ , and  $\llbracket \iota\sigma \rrbracket^a = \lambda x \in D_e \llbracket \sigma \rrbracket^{a[x/\iota]}$
- (RT) If  $t_\iota$  is a trace and  $\nu$  is a noun then  $[t_\iota\nu]$  is a *restricted trace* of type  $e$ , and  $\llbracket t_\iota\nu \rrbracket^a = a(\iota)$  if  $\llbracket \nu \rrbracket(a(\iota)) = 1$ ; otherwise undefined.

The syntax has two rules of quantifier-raising: one that carries along the restrictor and another that does not. (Like indices, the noun *thing* is phonologically null.)

$$\begin{aligned}
(\text{QR}^\uparrow) \quad & [S \ \xi \ [\text{DP}[\delta][\nu]] \ \psi] \Rightarrow [S[\text{DP}[\delta][\nu]]][[\iota][S \ \xi \ [[t_\iota][\text{thing}]] \ \psi]] \\
(\text{QR}^\downarrow) \quad & [S \ \xi \ [\text{DP}[\delta][\nu]] \ \psi] \Rightarrow [S[\text{DP}[\delta][\text{thing}]]][[\iota][S \ \xi \ [[t_\iota][\nu]] \ \psi]]
\end{aligned}$$

Let me illustrate how all this works on (14). The sentence could be obviously interpreted without quantifier raising and the results of applying  $(QR^\uparrow)$  or  $(QR^\downarrow)$  will not change the truth-conditions. What they do is allow some intensional operator (attitude verb, modality, tense, aspect, etc.) to intervene between the quantificational determiner and its trace. (To keep things simple, I did not include these in the fragment but they could be introduced without complications.) If the restrictor of the DP is upstairs we get a specific transparent reading; if it is left downstairs we obtain the specific opaque one.

- (14)  $[S[DP[D \textit{every}][N \textit{dog}]]_{VP \textit{runs}}]$   
 (14<sup>↑</sup>)  $[S[DP[D \textit{every}][N \textit{dog}]] [5[S[t_5][N \textit{thing}]]_{VP \textit{runs}}]]]$   
 (14<sup>↓</sup>)  $[S[DP[D \textit{every}][N \textit{thing}]] [8[S[t_8][N \textit{dog}]]_{VP \textit{runs}}]]]$

Since  $\llbracket \textit{thing} \rrbracket$  is the total identity function on  $D_e$ , according to (RT)  $\llbracket [t_5][N \textit{thing}] \rrbracket^a = a(5)$ , and according to (PA)  $\llbracket [5[S[t_5][N \textit{thing}]]_{VP \textit{runs}}]] \rrbracket = \llbracket \textit{runs} \rrbracket$ . So, obviously,  $\llbracket (14) \rrbracket = \llbracket (14^\uparrow) \rrbracket$ . By contrast,  $\llbracket [t_8][N \textit{dog}] \rrbracket^a$  is only defined for those values of the assignment function that are dogs in  $D_e$ . So,  $\llbracket [8[S[t_8][N \textit{dog}]]_{VP \textit{runs}}]] \rrbracket = \llbracket \textit{runs} \rrbracket$  only if  $a(8)$  is a dog; otherwise it is undefined. Now it is clear why the interpretation of *every* had to be modified: had we used the standard one (14<sup>↓</sup>) would come out as false when there are no dogs in  $D_e$ . But with the modified rule we have  $\llbracket (14) \rrbracket = \llbracket (14^\downarrow) \rrbracket$ , as desired.

The interpretations of *some* and *most* did not have to be adjusted. In order for the predicate abstract to yield a truth-value for some assignment, the assignment must map its index to a member of  $D_e$  that satisfies the restrictor below. This requirement becomes part of the truth-conditions. Thus, the lower reading of ‘Some dog runs’ (i.e. the one obtained via  $(QR^\downarrow)$ ) is true iff there is some  $x \in D_e$  such that  $\llbracket \textit{thing} \rrbracket(x)$  and  $\llbracket [8[S[t_8][N \textit{dog}]]_{VP \textit{runs}}]](x) = 1$ , where the latter requirement boils down to  $\llbracket \textit{dog} \rrbracket(x) = 1$  and  $\llbracket \textit{runs} \rrbracket(x) = 1$ . The lower reading of ‘Most dogs run’ is true just in case there are more  $x$ ’s  $\in D_e$  such that  $\llbracket \textit{thing} \rrbracket(x) = 1$  and  $\llbracket [8[S[t_8][N \textit{dog}]]_{VP \textit{runs}}]](x) = 1$  than  $x$ ’s  $\in D_e$  such that  $\llbracket \textit{thing} \rrbracket(x) = 1$  and  $\llbracket [8[S[t_8][N \textit{dog}]]_{VP \textit{runs}}]](x) = 0$ . This is equivalent to the condition that there be more  $x$ ’s  $\in D_e$  such that  $\llbracket \textit{dog} \rrbracket(x) = 1$  and  $\llbracket \textit{runs} \rrbracket(x) = 1$  than  $x$ ’s  $\in D_e$  such that  $\llbracket \textit{dog} \rrbracket(x) = 1$  and  $\llbracket \textit{runs} \rrbracket(x) = 0$ . All as it should be.

If there is an intensional operator that intervenes between a raised determiner and a stranded restrictor we can get a specific opaque reading. But in an extensional setting  $(QR^\uparrow)$  and  $(QR^\downarrow)$  are semantically indistinguishable. Here is a sketch of a proof.

Let  $\sigma$  be a sentence in an extensional language containing the restricted quantifier  $\delta\varrho$  (where  $\delta$  is the determiner and  $\varrho$  is the restrictor). Let’s say that the output of  $(QR^\uparrow)$  applied to an occurrence of  $\delta\varrho$  is  $\sigma^\uparrow$  and that an application of  $(QR^\downarrow)$  yields  $\sigma^\downarrow$ ; let the index of the resulting restricted trace in both cases be  $\iota$ . I want to show that  $\llbracket \sigma^\uparrow \rrbracket = \llbracket \sigma^\downarrow \rrbracket$ . Suppose  $\varepsilon^\uparrow$  is an arbitrary constituent of  $\sigma^\uparrow$  and  $\varepsilon^\downarrow$  the corresponding constituent of  $\sigma^\downarrow$ . (The two sentences have the same syntactic structure.) Call an assignment function that assigns a member

of  $D_e$  to  $\iota$  that satisfies  $\varrho$  *good*. I claim that if  $a$  is a good assignment then  $\llbracket \varepsilon^\uparrow \rrbracket^a = \llbracket \varepsilon^\downarrow \rrbracket^a$ . This is enough to prove what we want because (assuming  $\delta$  satisfies conservativity and extension) assignments that aren't good make no difference when it comes to the truth-conditions of  $\sigma$ . That  $\llbracket \varepsilon^\uparrow \rrbracket^a = \llbracket \varepsilon^\downarrow \rrbracket^a$  for all good assignments can be proved by induction. When  $\varepsilon$  is the restricted trace left behind as a result of raising  $\delta\varrho$  this follows from (RT). When  $\varepsilon$  is a lexical constituent of  $\sigma$  that is not part of the restricted trace  $\varepsilon^\uparrow = \varepsilon^\downarrow$ . And the inductive steps involving functional application, predicate abstraction, and restricted trace formation (using a different index) are trivial. (It matters here that we don't have intensional operators in the language.)<sup>8</sup>

## References

1. Abusch, D.: The scope of indefinites. *Natural Language Semantics* 2, 83–135 (1994)
2. Bäuerle, R.: Pragmatisch-semantische aspekte der NP-interpretation. In: Faust, M., Harweg, R., Lehfeldt, W., Wienold, G. (eds.) *Allgemeine Sprachwissenschaft, Sprachtypologie und Textlinguistik: Festschrift für Peter Hratmann*, pp. 121–131. Narr, Tübingen (1983)
3. von Fintel, K., Heim, I.: *Intensional Semantics*. Lecture Notes (2008), <http://mit.edu/fintel/IntensionalSemantics.pdf>
4. Fodor, J.: *The Linguistic Description of Opaque Contexts*. PhD thesis, Massachusetts Institute of Technology (1970)
5. Heim, I., Kratzer, A.: *Semantics in Generative Grammar*. Blackwell, Oxford (1998)
6. Ioup, G.: Some universal for quantifier scope. *Syntax and Semantics* 4, 37–58 (1975)
7. Keshet, E.: *Good Intentions: Paving Two Roads to a Theory of the De Re/De Dicto Distinction*. PhD thesis, Massachusetts Institute of Technology (2008)
8. Parsons, T.: *Events in the Semantics of English*. MIT Press, Cambridge (1990)
9. Percus, O.: Constraints on some other variables in syntax. *Natural Language Semantics* 8, 173–229 (2000)
10. Szabó, Z.G.: Counting across times. *Philosophical Perspectives* 20, 399–426 (2007)
11. Szabó, Z.G.: Things in progress. *Philosophical Perspectives* 22, 499–525 (2008)
12. Zucchi, S.: Incomplete events, intensionality, and imperfective aspect. *Natural Language Semantics* 7, 179–215 (1999)

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<sup>8</sup> Thanks to Itamar Francez, Tamar Szabó Gendler, Justin Khoo, and Anna Szabolcsi for comments.

# Affective Demonstratives and the Division of Pragmatic Labor<sup>\*</sup>

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## 1 Introduction

Building on [1] and [2, 3] argues for a ‘division of pragmatic labor’: as a result of general pragmatic interactions, unmarked expressions are generally used to convey unmarked messages and marked expressions are generally used to convey marked messages (see also [4, 5]). [6] explicitly splits this into two separate pressures (“What is expressed simply is stereotypically identified” and “What’s said in an abnormal way isn’t normal”), and [7, 8, 9], and [10] seek to characterize the opposition in terms how form–meaning pairs are optimally chosen.

In [3], Horn argues that the division of pragmatic labor is at work in a wide range of places: pronoun choice, lexicalization, indirect speech acts, and clause-mate negations, as well as issues in language change. Since then, the field has largely stayed within these empirical confines, exploring in more detail the specific pragmatic interactions Horn identified. With the present paper, we seek to branch out, by finding an important role for Horn’s division of pragmatic labor in *affective* (uses of) demonstratives [11, 12, 13, 14, 15, 16, 17, 18]. We focus on proximal demonstratives in Japanese, German and English, and begin to make the case that our generalizations are cross-linguistically robust.

Our evidence comes largely from a newly expanded version of the UMass Amherst Sentiment Corpora [19]. These are collections of informal online product reviews, in Chinese, English, German, and Japanese. The English and Japanese portions contain a total of 643,603 reviews and 72,861,543 words. We use these corpora to sharpen our empirical understanding of affective demonstratives and to substantiate the claims about markedness, for forms and meanings, that underlie our treatment in terms of the division of pragmatic labor.

Section 2 introduces Lakoff’s [11] notion of affective demonstratives, arguing that the basic claims are true for English and Japanese. Section 3 presents our corpora and experiments, which address not only demonstratives but also a wide range of exclamatives and related items, as a way of building a general picture of the kinds of pragmatic generalizations that the data support. Finally, in section 4, we reconnect with pragmatic theory, arguing that the division of pragmatic labor is responsible for the patterns we see in our large corpora.

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<sup>\*</sup> Our thanks to David Clausen, Noah Constant, Marie-Catherine de Marneffe, Sven Lauer, Florian Schwarz, and Jess Spencer for discussion.

## 2 Affective Demonstratives Cross-Linguistically

Lakoff [11] identifies a range of uses of English demonstratives that involve ‘emotional deixis’, as in (II).

- (1) a. This Henry Kissinger is really something!
- b. How’s that toe?
- c. There was this travelling salesman, and he ...

Lakoff’s central generalization is that affective demonstratives are markers of solidarity, indicating the speaker’s desire to involve the listener emotionally and foster a sense of shared sentiment. She also ventures a direct connection with exclamativity. [13] argue that similar effects arise for generic demonstratives, which “mark the kind being referred to as a relatively subordinate or homogeneous kind located among the speaker’s and hearer’s private shared knowledge”. [17] (and commentators) apply some of these findings to then-U. S. Vice-Presidential candidate Sarah Palin’s noteworthy demonstrative use.

Lakoff does not really take a stand on whether affective uses represent an ambiguity in demonstrative phrases or some kind of pragmatic extension of the basic meanings, and her own characterization suggests that the issue could be decided either way. Cross-linguistic investigation could provide important evidence in deciding the matter. If it is an ambiguity, then we have no expectation for it to arise consistently out of the more basic demonstrative meanings. Conversely, if it is a natural extension of deixis into the emotive realm, then it should turn up again and again in language. [18] address precisely this question, arguing, on the basis of parallels between English and German, that this is not an accidental lexical ambiguity, but rather an emergent property of deixis.

Correspondences between German and English are perhaps unsurprising. Even stronger evidence comes from work on Japanese demonstratives. [12][14][15] argue that demonstratives in Japanese can contribute an affective or expressive meaning component by indexing a kind of emotional deixis, echoing the Lakoff’s suggestions for affective uses of the English demonstrative.

These characterizations of affective demonstratives are intriguing, but we have so far seen limited evidence in favor of them. So, the first task before us is to see if we can find more robust and extensive empirical evidence for affectivity in the demonstrative realm across languages. The next section seeks to provide such evidence, building on the methods of [18]. Following that, we address the question of where these effects come from, arguing that they follow from Horn’s division of pragmatic labor.

## 3 Corpus Experiments

### 3.1 Data

Our data for this paper come from an expanded (and soon to be released) version of the UMass Amherst Sentiment Corpora [19]. The English texts are online

**Table 1.** Summary of the data sets used in this paper

English						
	1 star	2 star	3 star	4 star	5 star	total
Reviews	39,383	48,455	90,528	148,260	237,839	564,465
Words	3,419,923	3,912,625	6,011,388	10,187,257	16,202,230	39,733,423
Japanese						
	1 star	2 star	3 star	4 star	5 star	total
Reviews	3,973	4,166	8,708	18,960	43,331	79,138
Words	1,612,942	1,744,004	3,908,200	8,477,758	17,385,216	33,128,120

reviews at Amazon.com (reviews of a wide range of products), Tripadvisor.com (hotel reviews), and GoodReads.com (book reviews). The Japanese collection comes from Amazon.co.jp (reviews of a wide range of products). Every review included in this collection has some text and an associated star rating, which the author of the text has assigned to the product in question. Table 1 breaks down the corpora into categories based on these star-ratings, a perspective that we rely on heavily throughout this paper. (The substantial five star bias is a common feature of informal, user-supplied product reviews; see section 3.2 for our way to manage it.)

In contrast to professional reviews, these texts are informal and heavily emotive. Authors writing 1-star or 5-star reviews either loved or loathed the product they are reviewing, and this shines forth from their language, which tends to emphasize subjective experience. This makes the texts ideal for studying perspectival and emotional information of the sort that is at issue for affective demonstratives. Reviews in the middle of the scale (2-4 stars) tend to be more balanced and objective, which further helps to bring out the linguistic contrasts we are after. For more on the nature of corpora like this, as well as examples, we refer to [20,21,22,23].

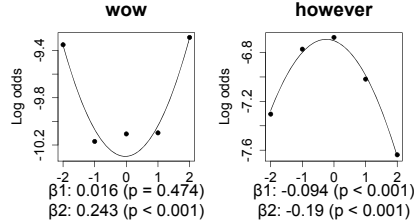
### 3.2 Methods

Our statistical method is simple: we track the frequency of words, phrases, and constructions across the five star-rating categories and study the resulting distributions. Because the rating categories are so imbalanced, with the bulk of the reviews assigning 5-stars, we always consider relative frequencies: let  $\text{count}(w_n, R)$  be the number of tokens of  $n$ -gram  $w$  in reviews in rating category  $R \in \{1, 2, 3, 4, 5\}$ , and let  $\text{count}_n(R)$  be the total count for all  $n$ -grams in rating category  $R$ . Then the frequency of  $w_n$  in rating category  $R$  is  $\text{count}(w_n, R) / \text{count}_n(R)$ . We center the rating values, so that a rating of 3 corresponds to a value on the  $x$  axis of 0, and other rating values are shifted appropriately, so that a rating of 1 maps to  $-2$  and a rating of 5 maps to 2. Centering the data in this way allows us to test positive and negative biases in words and constructions, as explained just below.

We also fit logistic regression models to the log-odds versions of these distributions, in order to gain further insight into their structure. There is not space here to review the technical details of these statistical models (we refer to [24,25] for gentle introductions and [26,27] for motivation and extension to mixed-effects models). However, we think it is worth giving the basic mathematical form for the model we use, (2), and we offer many graphical depictions in later sections to try to bring out the intuitions behind them.

$$(2) \quad P(y) = \text{logit}^{-1}(\alpha + \beta_1 x + \beta_2 x^2)$$

These profiles are curved. Where  $\beta_2$  is large and positive, we have U-shaped profiles, i.e., profiles in which the bulk of the usage is at the extreme ends of the rating scale. Where  $\beta_2$  is large and negative, we have Turned-U profiles, i.e., profiles in which the bulk of the usage is in the middle of the scale. Figure 1 illustrates each of these basic cases. The coefficient  $\beta_1$  tells us the slope of the curve when  $x = 0$ ; since we have centered our rating values so that a middle value of 3 is mapped to 0, we can use the value of  $\beta_1$  to test the size and significance of any positive or negative bias in an item’s distribution. A significant positive value of  $\beta_1$  indicates a positive rating bias, while a significant positive value of  $\beta_1$  indicates a negative rating bias, as discussed in [22].



**Fig. 1.** Example of (2) with the rating scale centered around 0. The fitted models are  $P(\text{wow}) = \text{logit}^{-1}(-10.30 - .016 + .24x^2)$ , and  $P(\text{however}) = \text{logit}^{-1}(-6.70 - .094x - .19x^2)$ . The sign of the quadratic coefficients (.24 and  $-.19$ ) determine the direction of the turn (U or Turned U) as well as its depth.

In figure 1 and throughout, we have included  $p$  values for the coefficients. However, given the large amount of data we have and the small number of empirical points involved,  $p$  values are not all that informative about the quality of the models. For present purposes, it is often more useful to compare the empirical points (black) against the models’ predictions.

### 3.3 Exclamatives and Anti-exclamatives

By way of building towards our results for demonstratives, we now present, in figure 2, the statistical profiles for a series of markers of exclamation, as well as some of their ‘anti-exclamative’ counterparts. Exclamatives are much more frequent at the extreme ends of the rating scale than in the middle. This is

consistent with the notion that they are generalized markers of unusualness or surprise [28,29,30]. Whatever lexical polarity they have comes from the predicates around them (*What a pleasure/disappointment!*). With the intensives (e.g., *absolutely, total*; [31]), we seem also to be seeing a connection between the rating scale’s endpoints and endpoint modification, as well as a potential argument for the degree-based approach to exlativity that underlies most treatments of exclamative constructions.

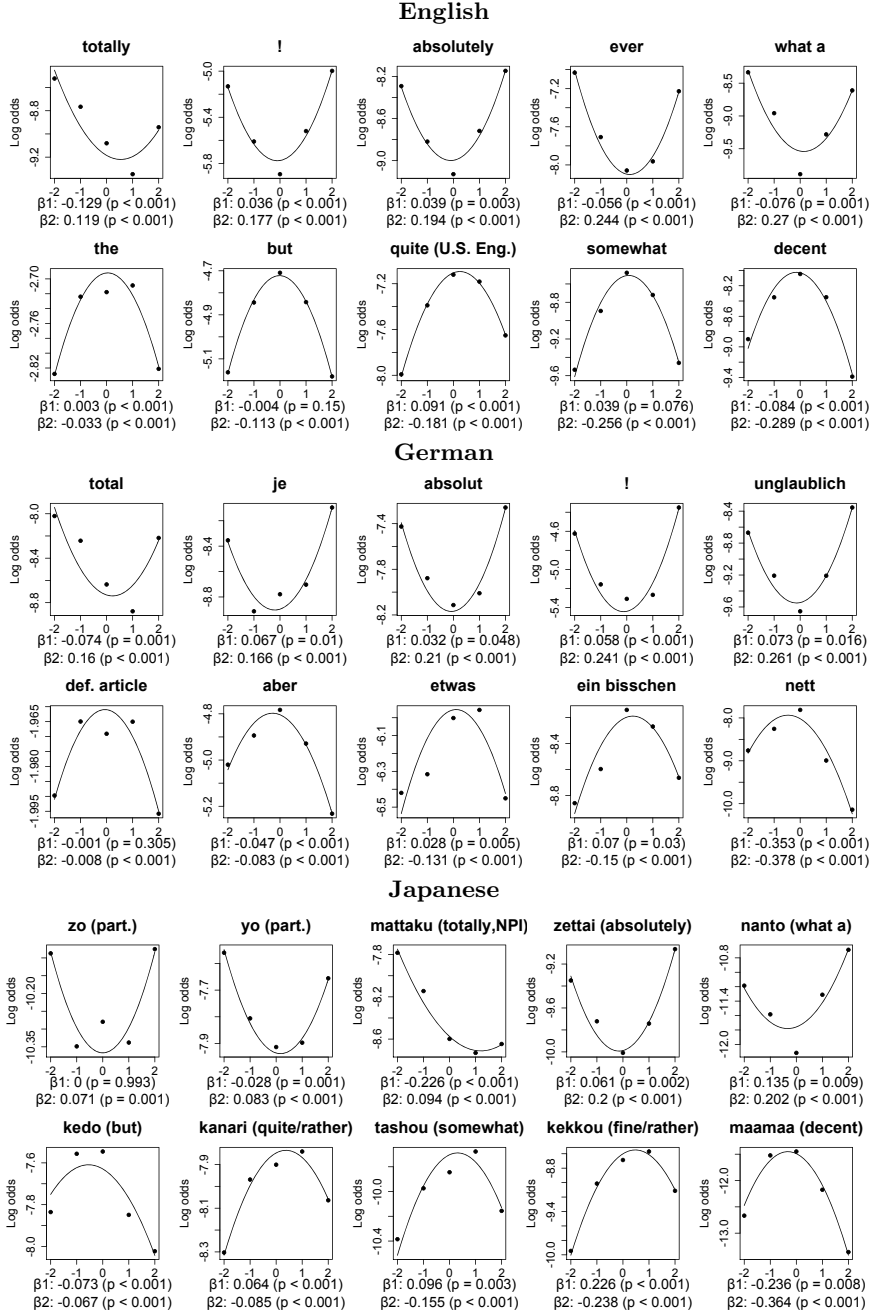
### 3.4 Affective Demonstratives

The above picture of exclamatives in our corpora strongly suggests that our statistical approach can detect affectivity. The rating scale brings out their generalized heightened emotion, placing them in opposition to more balanced expressions like *somewhat* and *but*. The approach can also detect rich modifier scales [32] and a wide range of expressive meanings [23]. Thus, if [11,12,13,14,15,16,17] are correct in claiming that demonstratives have (at least in English and Japanese) affective uses, then we should see this in our corpora. And this is in fact what we find for proximate demonstrative markers; figure 3 again gives results for English, German, and Japanese. The English data are for a 18,659,017 word, 118,395 review, subset of our data that we have part-of-speech tagged using [33] and NP chunked using [34] in order to get at the distinction between determiner *this* (*I’ll have this cake*) and pronominal *this* (*I’ll have this.*) For Japanese, we have the three morphologically complex proximal demonstratives, formed from the proximal demonstrative morpheme *ko-* combining with *-re* to form a pronominal demonstrative, *-no* to form an adnominal demonstrative determiner, and *-nna* to form an adnominal demonstrative determiner meaning “this kind of”. The proximal demonstratives form part of the paradigm summarized in Table 2.

**Table 2.** The Japanese demonstrative paradigm

	pronominal <i>-re</i>	determiner <i>-no</i>	kind determiner <i>-nna</i>
proximal <i>ko-</i>	<i>kore</i>	<i>kono</i>	<i>konna</i>
distant from speaker <i>so-</i>	<i>sore</i>	<i>sono</i>	<i>sonna</i>
distant from both <i>a-</i>	<i>are</i>	<i>ano</i>	<i>anna</i>
indefinite (‘which’) <i>do-</i>	<i>dore</i>	<i>dono</i>	<i>donna</i>

We should emphasize that the U shapes for these demonstratives are not nearly as deep as those for prototypical exclamatives; the quadratic coefficient for *what a*, for example, is 0.27 (figure 2), which is more than three times bigger than the coefficient for the English determiner *this* (0.078). Thus, it is not as though the model is (wrongly) predicting that proximal demonstratives typically pack as much of an emotive punch as exclamatives. We believe that they can do that, but the majority of uses do not, so the overall effect is relatively mild.



**Fig. 2.** Exclamatives and anti-exclamatives. Exclamatives are given in the top row of each language's panel, anti-exclamatives in the bottom row. As we move from left to right, the exclamativity (anti-exclamativity) grows more pronounced as measured by the absolute size of the quadratic coefficient ( $\beta_2$ ).

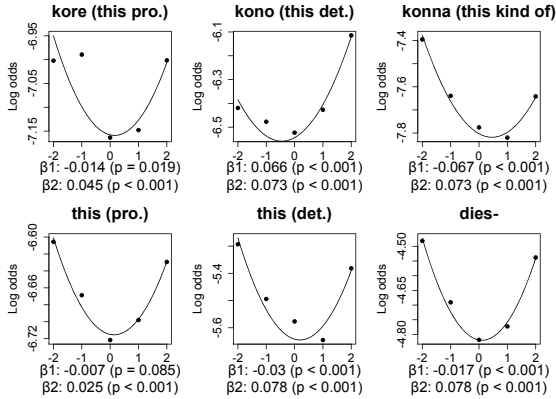


Fig. 3. Proximal demonstratives in Japanese, English, and German

## 4 (Un)Marked Forms and Meanings

Now that we have some quantitative evidence for the reality of affective demonstratives (for proximates) we can move to asking why such meanings arise. The consistency of the effects across languages seems to rule out a treatment in terms of lexical ambiguity. As Lakoff observes, the affectivity has a metaphoric connection with the more basic meaning; it is perhaps no surprise that a marker of physical closeness would be extended into the emotive realm where it would foster, or gesture at, shared sentiment. However, while this makes intuitive sense, it is hard to make the argument rigorously. It has the feel of a ‘just-so’ story.

Horn’s division of pragmatic labor gives us a richer perspective on the problem. It is fairly easy to argue that this is a case in which marked forms associate with marked meanings. [35] argues that English demonstratives are, at the meaning level, strictly more complex morphosemantically than *the*. They are also significantly less frequent than definite articles. In our data, there are 977,637 tokens of *the*, but only 171,432 of *this* and another 13,026 of *these*.<sup>1</sup> This is no quirk of our corpora, either. In the Google *n*-gram corpus, *the* is 8.5 times more frequent than *this*.

When we look at the profiles for the English and German definite determiners in our corpora (in the leftmost column of figure 2), we find that they are the mirror images of the profiles of the proximal demonstratives, exhibiting a significant inverse-U shape. We explain this finding as the result of competition between marked and unmarked meanings. The more marked proximal demonstratives generate an exclamative profile, with uses concentrated in the extreme regions of the scale. Since demonstratives compete for the same syntactic slot as the definite determiner, we get an inverse implicature arising from the use of the

<sup>1</sup> In fact, *the* is about 4.5 more frequent than all of the demonstratives combined (216,407 tokens).

definite. This is a species of upper-bounding implicature; the speaker used a form (definite) whose meaning contribution is strictly weaker than a competing form (demonstrative) [6]. This gives rise, in certain contexts, to a kind of implicature whereby the proximal emotional deixis we saw to be generated by the proximal demonstratives is negated, so that use of the definite determiner can implicate a negation of strong emotional commitment. The strength of the effect is weak, as seen in the small size of the quadratic term in our models. We are not predicting that use of the definite determiner is inconsistent with exclaimativity; instead, we argue that competition with demonstratives generates a small but significant tendency for anti-exclaimativity in the use of the definite determiner.

Japanese does not have a definite determiner to play the role of the unmarked-to-unmarked counterpart. Its demonstrative system, however, is more articulated than that of English and thus allows us to see the expressive effects of relative semantic markedness within the demonstrative paradigm itself. It is reasonable to hypothesize that the proximal demonstrative ending in *-nna* is more semantically marked than the one ending in *-no*, since the proximal demonstrative ending in *-no* refers only to the entity ultimately picked out by the construction *kono NP*, while the one ending in *-nna* makes reference not only to the entity directly indexed by the demonstrative, but also to a set of ‘similar’ entities. There is thus an intuitive sense in which a sentence including the *-nna* series proximal demonstrative *konna* is stronger, and hence more marked, than the same sentence in which *-nna* is replaced with *-no*.

[14] argues that the *-nna* series can be used to contribute both a note of “surprise” and “negative emotion”. In a discussion of the *-nna* demonstratives, she says that most researchers concentrate on the physical deictic uses, but she continues:

Conversational data, however, indicates that the usage described above is scarcely seen in informal conversation. Rather than solely referring to the characteristics of an object, most of the usage overtly expresses the following speaker’s modality: 1) negative emotion or rejection, and 2) surprise. These emotions and attitudes are toward the object, the interlocutor, or the whole utterance or action that includes the object.

We can relate the note of “surprise” that [14] identifies to the U-shaped distributions we identified earlier. In the case of proximal demonstratives, we saw that this U-shape characterized both the *-no* series proximal demonstrative *kono* and the *-nna* series proximal demonstrative *konna*. We conclude, on the basis of collaborating evidence from English and German, that this exclamative or “surprisal” value is derived by metaphorical extension of the proximity encoded by *ko-*. This leaves negativity. In our corpus, expressive negativity is reflected in a bias toward the negative end of the review scale. When we look at the distribution of *konna*, we see not only a U-shape, but also a negative bias, reflected in the significantly negative value of the linear coefficient ( $\beta_1 = -0.081$ ,  $p < 0.001$ ). This contrasts with the significant *positive* bias for *sono*, reflected in the significantly positive value of its linear coefficient ( $\beta_1 = 0.071$ ,  $p < 0.001$ ).

Graphically, the profiles of these two items appear to be mirror images of each other, in the horizontal dimension.

In line with our previous discussion of the complementary use of proximal demonstratives and the definite determiner, we posit a competition-based explanation of the contrast between *kono* and *konna*. Using *konna* tends, through the influence of *-nna*, to contribute a hint of negativity, as argued by [14]. The less marked *kono* has a complementary positive shift in its profile, as a result of competition between forms. The presence of a significant U-shape in both proximal demonstratives is due to the proximal morpheme *ko-*. This suggests an additive model of pragmatic enrichment, in which the *ko-* morpheme contributes a tendency for extremity, and the competition between unmarked *-no* and marked *-nna* is reflected in a distinct positive and negative bias.

## 5 Conclusion

Using large-scale corpus evidence, we began to make a case for the idea that affective uses of demonstratives are a robust, cross-linguistically stable phenomenon. We also addressed the question of where affective readings come from, arguing that they trace to Horn's division of pragmatic labor: the morphosyntactically complex, relatively infrequent (marked) demonstratives associate with the emotionally deictic (marked) messages. In English, we argued that the definite article plays the unmarked role for form and meaning, and the Japanese data support nuanced oppositions within the demonstrative system.

## References

1. McCawley, J.D.: Conversational implicature and the lexicon. In: Cole, P. (ed.) *Syntax and Semantics. Pragmatics*, vol. 7, pp. 245–259. Academic Press, New York (1978)
2. Kiparsky, P.: Word formation and the lexicon. In: Ingemann, F. (ed.) *Proceedings of the Mid-Atlantic Linguistic Conference*, University of Kansas (1982)
3. Horn, L.R.: Toward a new taxonomy for pragmatic inference: Q-based and R-based implicature. In: Schiffrin, D. (ed.) *Meaning, Form, and Use in Context: Linguistic Applications*, pp. 11–42. Georgetown University Press, Washington (1984)
4. Horn, L.R.: *A Natural History of Negation*. University of Chicago Press, Chicago (1989) (Reissued 2001 by CSLI)
5. Horn, L.R.: Presupposition and implicature. In: Lappin, S. (ed.) *The Handbook of Contemporary Semantic Theory*, pp. 299–319. Blackwell Publishers, Oxford (1996)
6. Levinson, S.C.: *Presumptive Meanings: The Theory of Generalized Conversational Implicature*. MIT Press, Cambridge (2000)
7. Blutner, R.: Lexical pragmatics. *Journal of Semantics* 15(2), 115–162 (1998)
8. Blutner, R.: Some aspects of optimality in natural language interpretation. *Journal of Semantics* 17(3), 189–216 (2000)
9. van Rooy, R.: Signalling games select Horn strategies. *Linguistics and Philosophy* 27(4), 493–527 (2004)

10. Franke, M.: *Signal to Act: Game Theory in Pragmatics*. ILLC Dissertation Series. Institute for Logic, Language and Computation, Universiteit van Amsterdam (2009)
11. Lakoff, R.: Remarks on 'this' and 'that'. In: *Proceedings of the Chicago Linguistics Society*, vol. 10, pp. 345–356 (1974)
12. Kitagawa, C.: A note on 'sono' and 'ano'. In: Bedell, G. (ed.) *Explorations in Linguistics: Papers in Honor of Kazuko Inoue*, Kurosio, Tokyo, pp. 232–243 (1979)
13. Bowdle, B.F., Ward, G.: Generic demonstratives. In: *Proceedings of the Twenty-First Annual Meeting of the Berkeley Linguistics Society*, pp. 32–43. Berkeley Linguistics Society (1995)
14. Naruoka, K.: Expressive functions of Japanese adnominal demonstrative 'konna/sonna/anna'. In: Hudson, M.E., Sells, P., Jun, S.A. (eds.) *Japanese/Korean Linguistics*, vol. 13, pp. 433–444. CSLI, Stanford (2008)
15. Ono, K.: Territories of information and Japanese demonstratives. *The Journal of the Association of Teachers of Japanese* 28(2), 131–155 (1994)
16. Wolter, L.K.: *That's That: The Semantics and Pragmatics of Demonstrative Noun Phrases*. PhD thesis, UC Santa Cruz (2006)
17. Liberman, M.: Affective demonstratives, *Language Log* (2009)
18. Potts, C., Schwarz, F.: Affective 'this'. *Linguistic Issues in Language Technology* (to appear)
19. Constant, N., Davis, C., Potts, C., Schwarz, F.: *UMass Amherst sentiment corpora* (2009)
20. Pang, B., Lee, L.: Seeing stars: Exploiting class relationships for sentiment categorization with respect to rating scales. In: *Proceedings of the 43rd Annual Meeting of the Association for Computational Linguistics (ACL 2005)*, Ann Arbor, Michigan, June 2005, pp. 115–124. Association for Computational Linguistics (2005)
21. Pang, B., Lee, L., Vaithyanathan, S.: Thumbs up? sentiment classification using machine learning techniques. In: *Proceedings of the Conference on Empirical Methods in Natural Language Processing (EMNLP)*, Philadelphia, July 2002, pp. 79–86. Association for Computational Linguistics (2002)
22. Potts, C., Schwarz, F.: Exclamatives and heightened emotion: Extracting pragmatic generalizations from large corpora. Ms., UMass Amherst (2008)
23. Constant, N., Davis, C., Potts, C., Schwarz, F.: The pragmatics of expressive content: Evidence from large corpora. *Sprache und Datenverarbeitung* 33(1–2), 5–21 (2009)
24. Gelman, A., Hill, J.: *Data Analysis Using Regression and Multilevel/Hierarchical Models*. Cambridge University Press, Cambridge (2007)
25. Baayen, R.H.: *Analyzing Linguistic Data: A Practical Introduction to Statistics*. Cambridge University Press, Cambridge (2008)
26. Jaeger, T.F.: Categorical data analysis: Away from ANOVAs (transformation or not) and towards logit mixed models. *Journal of Memory and Language* (2008)
27. Baayen, R.H., Davidson, D.J., Bates, D.M.: Mixed-effects modeling with crossed random effects for subjects and items. *Journal of Memory and Language* 59(4), 390–412 (2008)
28. Ginzburg, J., Sag, I.A.: *Interrogative Investigations: The Form, Meaning, and Use of English Interrogatives*. CSLI, Stanford (2001)
29. Zanuttini, R., Portner, P.: Exclamative clauses at the syntax–semantics interface. *Language* 79(1), 39–81 (2003)
30. Castroviejo Miró, E.: *Wh-Exclamatives in Catalan*. PhD thesis, Universitat de Barcelona (2006)

31. Beaver, D., Clark, B.Z.: *Sense and Sensitivity: How Focus Determines Meaning*. Wiley-Blackwell, Oxford (2008)
32. de Marneffe, M., Potts, C., Manning, C.D.: “Was it good? it was provocative.” learning adjective scales from review corpora and the web. Ms., Stanford University (2009)
33. Manning, C.D., Klein, D., Morgan, William Tseng, H., Rafferty, A.N.: *Stanford log-linear part-of-speech tagger*. version 1.6 (2008)
34. Greenwood, M.: *NP chunker v1.1* (2005)
35. Elbourne, P.: Demonstratives as individual concepts. *Linguistics and Philosophy* 31(4), 409–466 (2008)

# Experimental Detection of Embedded Implicatures<sup>\*,\*\*</sup>

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## 1 Theories of Scalar Implicatures: Globalism vs. Localism

According to the Gricean approach to scalar implicatures (SIs for short), SIs are pragmatic inferences that result from a reasoning about the speaker's communicative intentions. In recent years, an alternative view of SIs (let us call it the 'grammatical view' of SIs) has been put forward, according to which they result from the optional presence of a covert so-called exhaustivity operator in the logical form of the relevant sentences and are thus reducible to standard semantic entailment (see Chierchia 2006, Fox 2007, Chierchia, Fox, and Spector in press, a.o, building on earlier grammatical approaches by, e.g., Landman 1998, Chierchia 2004).

While these two radically different approaches do not make distinct predictions in simple cases, they do for more complex ones. In particular, if the 'grammatical approach' is correct, then the exhaustivity operator should be able to occur in an embedded position (just like *only*), so that the strengthening, say, of 'some' into 'some but not all' could occur 'locally', under the scope of linguistic operators. This approach is often called 'localist', as opposed to pragmatic, so-called 'globalist' approaches.

Consider for concreteness the following example:

- (1) Every student solved some of the problems.

The standard neo-Gricean mechanism predicts that (I) should be interpreted as implying the negation of its scalar alternative, i.e. the negation of 'Every student solved all of the problems'. Hence, (I) should give rise to the following reading (henceforth, we will refer to this reading as the 'global reading'):

- (2) Every student solved some of the problems and at least one student didn't solve them all.

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\*\* Chemla and Spector (2009) is an extended presentation of this work, with many more results and discussions.

If, however, the strengthening of ‘some’ into ‘some but not all’ can occur at an embedded level, as predicted by localist approaches, one expects that another possible reading for (II) is the one expressed by (3) below (which we will henceforth call the ‘local reading’):

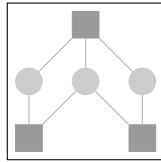
- (3) Every student solved some but not all the problems.

It thus seems that determining the possible readings of sentences like (II) should provide decisive evidence in the debate between localism and globalism. This is unfortunately not so. For several formalized *globalist* theories of SIs (e.g., Spector 2003, 2006, van Rooij and Schulz 2004, Chemla 2008, 2009b) also predict that (3) is a possible reading of (II).<sup>1</sup>

The first goal of this paper is to provide new experimental data which show, contrary to claims put forward in a recent paper by Geurts and Pouscoulous (Geurts and Pouscoulous 2009), that (3) *is* a possible reading for (II). A second goal of this paper is to examine a case where localism and globalism are bound to make different predictions, and to test it with a similar experimental paradigm.

## 2 Geurts and Pouscoulous’ Results

G&P collected truth-value judgments for sentence-picture pairs, asking subjects to evaluate the relevant sentence as true, false, or ambiguous between a true and a false reading. One of their crucial conditions consisted of the sentence ‘All the squares are connected with some of the circles’, paired with the picture in Fig. 1.



**Fig. 1.** Item from Geurts and Pouscoulous’s (2009) experiment 3 (their Fig. 2). NB: This is our own black and white reproduction of their item.

Here are the three relevant potential readings for the sentence they used:

- (4) a. Literal Reading. Every square is connected with at least one circle.
- b. Global Reading. Every square is connected with at least one circle, and it’s not the case that every square is linked with all the circles.
- c. Local Reading. Every square is connected with at least one circle, and no square is connected with all the circles.

<sup>1</sup> These theories do not derive this reading by localist means, of course. They argue instead that the proposition: ‘Some students solved all the problems’ should be added to the list of negated scalar alternatives of (II).

G&P found that virtually all the subjects considered the sentence to be true in Fig. 1, even though it is false under the local reading (the top square is linked to *all* the circles), and concluded that the local reading does not exist. We challenge this interpretation, by pointing out that there are several reasons why the strong reading, even if it existed, might have been very hard to detect:

- (i) G&P’s pictures are hard to decipher; in particular, the unique falsifier of the local reading (i.e. the top square) is hard to identify as such.

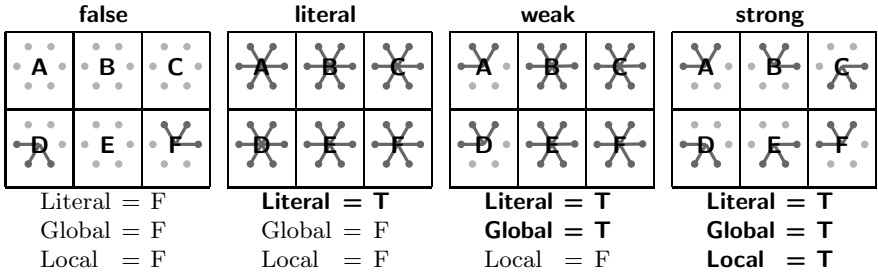
- (ii) Note that the local reading asymmetrically entails the global reading, which in turn asymmetrically entails the literal reading. Meyer and Sauerland (2009), among others, argue that subjects, due to some kind of a charity principle, tend to interpret ambiguous sentences under their weakest readings, unless a stronger available reading is particularly ‘accessible’ (see also, e.g., Crain and Thornton 2000, Abusch 1993, Reinhart 1997). If the global and the local readings are equally accessible, it follows that the local reading will be hard to detect experimentally even if it exists (see also Sauerland 2010).

### 3 Our Experimental Design

Like G&P, we used a sentence-picture matching task, but with some crucial modifications. We believe that our design improves on that of G&P’s in the following respects:

- (re i) The falsifiers of the strong reading are easy to identify (see Fig. 2 below, and in particular the **weak** condition which is the counterpart of G&P’s item represented in Fig. 1).

- (re ii) Instead of asking for absolute judgments of truth or falsity, we asked for graded judgments: subjects were asked to position a cursor on a continuous line going from ‘No’ (i.e. ‘false’) on the left, to ‘Yes’ (i.e. ‘true’) on the right.<sup>2</sup>



**Fig. 2.** Illustrative examples of the images used in the different conditions **false**, **literal**, **weak** and **strong** for the test sentence (5). We also reported below each image whether the literal, global and local readings are true (T) or false (F). In the actual images, the linked circles (with there links) appeared in red.

<sup>2</sup> See Chemla (2009a,c) for the use of a similar methodology to collect judgments in pragmatics, and the references cited therein.

By offering subjects more options, we hoped to get more fine-grained results, which could reveal differences that remained hidden when subjects were given only two or three options, and thus to overcome some of the consequences of the charity principle. More specifically, we hypothesized that given a sentence  $S$  and two distinct pictures  $P1$  and  $P2$ , if the set of available readings for  $S$  that are true in  $P1$  is a proper subset of those that are true in  $P2$ , then the degree to which  $S$  will be judged true will be lower in the case of  $P1$  than with  $P2$ .

## 4 Experiment 1: Scalar Items in Universal Sentences

In this experiment, we showed that the local reading is available for sentences like (II) above: French scalar items like ‘certain(s)’ (*some*)<sup>3</sup> and ‘ou’ (*or*), when embedded under universal quantifiers, can give rise to readings in which they seem to be equivalent to, respectively, ‘some but not all’ or an *exclusive* disjunction.

### 4.1 Experimental Items

The items explicitly discussed in the instructions were presented first to allow participants to get used to the display and to the task.<sup>4</sup> After that, participants ran a first block of items in which all target conditions were repeated several times (in pseudo-random order). Participants then could take a short break before moving to a second block of items instantiating the same experimental conditions (with superficially different pictures). In a last experimental block of items, some control conditions were administered.

**Target conditions: universal sentences.** Each item consisted of a sentence and a picture. We used the two distinct sentence-types, illustrated in (5) and (6). For each of them, we were interested in the availability of three distinct potential readings, namely the literal, the global and the local readings:

- (5) Chaque lettre est reliée à certains de ses cercles.  
*Each letter is connected to some of its circles.*
  - a. Literal Reading: Each letter is connected to at least one of its circles.
  - b. Global Reading: Each letter is connected to at least one of its circles, and it is not the case that each letter is connected to all its circles.
  - c. Local Reading: Each letter is connected to at least one of its circles, and no letter is connected to all its circles.
- (6) Chaque lettre est reliée à son cercle rouge ou à son cercle bleu.  
*Each letter is connected to its red circle or to its blue circle.*
  - a. Literal Reading: Each letter is connected to its red circle, its blue circle or both.

<sup>3</sup> Note that French *certain(s)*, unlike its singular counterpart *un certain* or English *certain*, does not force a specific reading.

<sup>4</sup> The experiment involved 16 native speakers of French, with no knowledge of linguistics, ranging in age from 19 to 29 years (10 women).

- b. Global Reading: Each letter is connected to at least one of its circles, and it is not the case that each letter is connected to both the red and the blue circle.
- c. Local Reading: Each letter is connected to its red circle or its blue circle but none is connected to both.

Each of these sentences was paired with various pictures, giving rise to the following four target conditions (see Fig. 2): **false**: no reading is true, **literal**: only the literal reading is true, **weak**: both the literal and the global readings are true but the local reading is false, **strong**: all readings are true.

**Control conditions: downward entailing (DE) environments.** When scalar items are embedded in the scope of ‘No’ as in (7a) or (8a), it is uncontroversial that the potential ‘local’ readings described in (7b) and (8b) are only marginally available at best.

- (7) a. *Aucune lettre n’est reliée à certains de ses cercles.*  
*No letter is connected to some of its circles.*
- b. Potential local reading: No letter is connected to some but not all of its circles.
- (8) a. *Aucune lettre n’est reliée à son cercle rouge ou à son cercle bleu.*  
*No letter is connected to its red circle or its blue circle.*
- b. Potential local reading: No letter is connected to exactly one of its two circles.

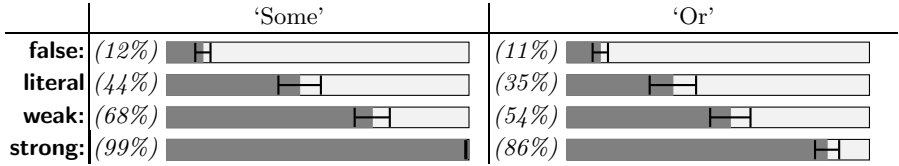
Sentences like (7a) and (8a) were thus used as controls, to check that participants do not access the ‘local’ reading for such sentences, or do so only marginally (given the marginal availability of the local reading). They were paired with pictures instantiating the following three conditions: **false**: no reading is true, **?local**: only the local reading is true, **both**: both the local and the literal readings are true.

## 4.2 Results and Interpretation

**Main result: detection of the local reading.** Fig. 3 reports the mean ratings in the target conditions. The relevant *t*-tests show that all differences between two consecutive bars are significant.<sup>5,6</sup>

<sup>5</sup> SOME: **false** vs. **literal**:  $F(1, 15) = 14, p < .01$ ; **literal** vs. **weak**:  $F(1, 15) = 27, p < .001$ ; **weak** vs. **strong**:  $F(1, 15) = 25, p < .001$ . OR: **false** vs. **literal**:  $F(1, 15) = 6.2, p < .05$ ; **literal** vs. **weak**:  $F(1, 15) = 22, p < .001$ ; **weak** vs. **strong**:  $F(1, 15) = 17, p < .001$ . Note that 4.6% of the responses were excluded as outliers or for technical reasons. Statistical analyses presented here are computed per subject, per item analyses yield similar results.

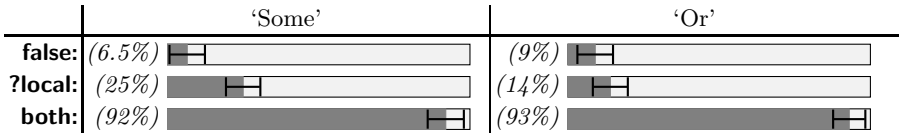
<sup>6</sup> Non-parametric Wilcoxon tests lead to the same conclusion. Application of a Bonferroni correction for multiple comparison *à la* Holm does not affect the results. We ensured that this was the case for all the statistical analyses reported here.



**Fig. 3.** Main results: Mean position of the cursor in the target conditions of Exp. 1. Error bars represent standard errors to the mean.

The crucial result is that the ratings are higher in the **strong** condition than in the **weak** condition, even though the two conditions differ only according to the truth value of the local reading. This difference provides important support for the existence of the local reading. Indeed, these results are fully explained if we assume that a) the target sentence is ambiguous between the literal reading, the global reading and the local reading, and b) the more readings are true, the higher the sentence is rated. They are not expected if only the literal and the global readings exist.

**Control result : downward-entailing environments.** Fig. 4 reports the results for the control conditions. For the scalar item ‘some’, all 2 by 2 differences are significant, while for the scalar item ‘or’, there is no difference between the **false** condition and the **?local** condition.



**Fig. 4.** Mean responses for the DE control conditions in exp. 1 (see §4.1)

In the case of ‘some’, we cannot exclude that participants perceived the ‘local’ reading, because the **?local** condition is judged a little higher than the **false** condition. But this result is not terribly disturbing for two reasons. First, it does not generalize to the scalar item ‘or’. Second, the control sentences receive a much lower rating in the condition **?local** than in conditions where it is uncontroversial that the target sentence has a true reading. Note that even with the scalar item ‘some’, the condition **?local** is rated at a radically lower level than the condition **both** (25 % vs. 92 %); more importantly, in the case of ‘some’, the condition **?local** is rated much lower than conditions in which it is uncontroversial that the target sentence has a true reading (consider for instance the important difference between this **?local** condition and the **weak** condition – which involved the target sentences. This difference is statistically significant:  $F(1, 15) = 22, p < .001$ ).

<sup>7</sup> SOME: **false** vs. **?local**:  $F(1, 15) = 6.5, p < .05$ ; **?local** vs. **both**:  $F(1, 15) = 43, p < .001$ . OR:  $F(1, 15) = .45, p = .51$  and  $F(1, 15) = 60, p < .001$ , respectively.

## 5 Experiment 2: Non-monotonic Environments

In this second experiment, we tested cases for which pragmatic and grammatical theories are bound to make different predictions. This happens with sentences where a scalar item like ‘some’ or ‘or’ occurs in a non-monotonic environment:

- (9) Exactly one letter is connected to some of its circles.
- (10) Exactly one letter is connected to its blue circle or its red circle.

The relevant potential readings (i.e. those that the sentence could in principle have according to various theories) can be paraphrased as follows:<sup>8</sup>

- (11) Potential readings of (9)
  - a. Literal meaning: one letter is connected to some or all of its circles, the other letters are connected to no circle.
  - b. Global reading: one letter is connected to some but not all of its circles, the other letters are connected to no circle.
  - c. Local reading: one letter is connected to some but not all of its circles, the other letters may be connected to either none or all of their circles.
- (12) Potential readings of (10)
  - a. Literal meaning: one letter is connected to its blue circle or its red circle or to both, the other letters are connected to no circle.
  - b. Global reading: one letter is connected to exactly one of its two circles, the other letters are connected to no circle.
  - c. Local reading: one letter is connected to exactly one of its two circles, the other letters may be connected to either none or both of their circles.

Because the scalar item now occurs in a non-monotonic environment, the local reading does not entail the global reading. In fact, it does not even entail the literal reading. This is of major importance for three reasons. First, globalist theories are bound to predict readings that entail the literal reading. Hence they cannot predict local readings like (11c) or (12c) in these non-monotonic cases. Second, the fact that the local reading does not entail any of the other two potential readings could automatically make it easier to detect (according to a charity principle). Finally, this very fact allowed us to construct cases where only the local reading is true and to assess its existence independently of the other readings.

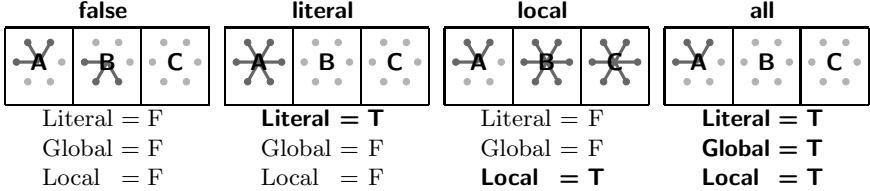
### 5.1 Experimental Items

The task and the instructions were essentially the same as in experiment 1. The items were presented just like in experiment 1: the examples from the instructions were presented first; then came two blocks of target conditions, and finally

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<sup>8</sup> The global reading (11b) is obtained by adding to the literal reading the negation of the alternative sentence “Exactly one letter is connected to all its circles”.

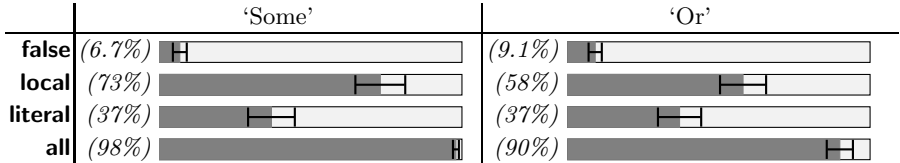
came a block with exactly the same control conditions as in experiment 1. The target conditions involved French translations of (9) and (10). Each of these sentences was paired with various pictures, giving rise to the following four target conditions, which represent all the possible combinations of true and false readings, and are illustrated in Fig. 5: **false**: no reading is true, **literal**: only the literal reading is true, **local**: only the local reading is true and **all**: all three readings – literal, global and local – are true.



**Fig. 5.** Illustrative examples of the images used to illustrate the different conditions **false**, **literal**, **local** and **all** for the test sentence (9). We also reported below each image whether the literal, global and local readings are true (T) or false (F).

## 5.2 Results

**Main result: the local reading exists.** Fig. 6 reports the mean ratings of the target conditions.<sup>9</sup> All 2 by 2 differences are significant, except for the **local** vs. **literal** conditions in the case of ‘or’.<sup>10</sup>



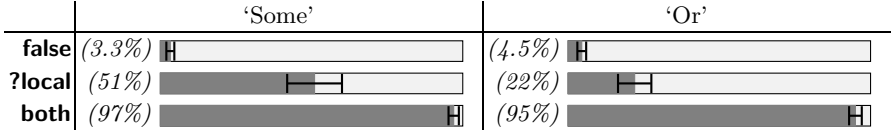
**Fig. 6.** Mean responses in the target conditions of experiment 2

This first set of data qualifies the local reading as a possible interpretation for our target sentences (involving non-monotonic operators), since i) the **local** condition is rated much higher than the **false** condition and ii) the **local** reading is rated significantly higher than the **literal** condition, a fact which is totally unexpected under the globalist approach, but can be understood within the localist approach.

<sup>9</sup> This experiment involved 16 native speakers of French, with no prior exposure to linguistics, ranging in age from 18 to 35 years (9 women). 14% of the responses had to be excluded for various technical reasons. All statistical analyses presented below are computed per subject; per item analyses yielded similar results.

<sup>10</sup> SOME: **false** vs. **literal**:  $F(1, 15) = 12$ ,  $p < .01$ , **literal** vs. **local**:  $F(1, 15) = 6.7$ ,  $p < .05$ , **local** vs. **all**:  $F(1, 15) = 10$ ,  $p < .01$ . OR: **false** vs. **literal**:  $F(1, 15) = 11$ ,  $p < .01$ , **literal** vs. **local**:  $F(1, 15) = 2.3$ ,  $p = .15$ , **local** vs. **all**:  $F(1, 15) = 18$ ,  $p < .001$ .

**Control result: downward entailing environments.** Fig. 7 reports the results for the DE control conditions (which were the same as in Exp. 1). All 2 by 2 differences are statistically significant with both ‘some’ and ‘or’<sup>11</sup>



**Fig. 7.** Mean responses for the control conditions when administered at the end of experiment 2

Surprisingly, the rates for the **?local** condition are higher than they were in the first experiment (compare Fig. 7 to Fig. 4), which calls for an explanation. A possible hypothesis is the following: subjects become much better at perceiving ‘local’ readings even in cases where they are normally dispreferred *once* they have experienced cases in which the local reading is salient. The target conditions of the second experiment seem to have precisely this property, given the results we have just presented.

## 6 Conclusions

Our first experiment showed that sentences in which a scalar item is embedded under a universal quantifier can be interpreted according to what we called the ‘local’ reading, contrary to Geurts and Pouscoulous’ (2009) conclusions. We pointed out that this result is nevertheless not sufficient to establish the existence of embedded scalar implicatures (because the local reading in such a case can be predicted by a *globalist* account). In our second experiment, we focussed on a case where the local reading cannot be derived by globalist means – sentences where a scalar item occurs in a non-monotonic environment –, and we were able to detect experimentally genuinely local readings. The existence of embedded scalar implicatures is unexpected from a Gricean perspective. The grammatical approach to SIs provides one possible way of making sense of these data.

## References

- Abusch, D.: The scope of indefinites. *Natural Language Semantics* 2(2), 83–135 (1993)  
 Chemla, E.: Présuppositions et implicatures scalaires: études formelles et expérimentales. Ph. D. thesis, EHESS (2008)  
 Chemla, E.: Presuppositions of quantified sentences: experimental data. *Natural Language Semantics* 17(4), 299–340 (2009a)

<sup>11</sup> SOME: **false** vs. **?local**:  $F(1, 14) = 20$ ,  $p < .001$ ; **?local** vs. **literal**:  $F(1, 14) = 28$ ,  $p < .001$ . OR:  $F(1, 15) = 6.1$ ,  $p < .05$  and  $F(1, 15) = 190$ ,  $p < .001$ , respectively.

- Chemla, E.: Similarity: towards a unified account of scalar implicatures, free choice permission and presupposition projection. In: *Under revision for Semantics and Pragmatics* (2009b)
- Chemla, E.: Universal implicatures and free choice effects: Experimental data. *Semantics and Pragmatics* 2(2), 1–33 (2009c)
- Chemla, E., Spector, B.: Experimental evidence for embedded implicatures. *Under revision for Journal of Semantics* (2009)
- Chierchia, G.: Scalar implicatures, polarity phenomena, and the syntax/pragmatics interface. In: Belletti, A. (ed.) *Structures and Beyond*, OUP (2004)
- Chierchia, G.: Broaden Your Views: Implicatures of Domain Widening and the ‘Logicality’ of Language. *Linguistic Inquiry* 37(4), 535–590 (2006)
- Chierchia, G., Fox, D., Spector, B.: The grammatical view of scalar implicatures and the relationship between semantics and pragmatics (in press)
- Crain, S., Thornton, R.: *Investigations in Universal Grammar: A guide to experiments on the acquisition of syntax and semantics*. MIT Press, Cambridge (2000)
- Fox, D.: Free Choice and the theory of Scalar Implicatures. In: Sauerland, U., Stateva, P. (eds.) *Presupposition and Implicature in Compositional Semantics*, pp. 537–586. Palgrave Macmillan, New York (2007)
- Geurts, B., Pouscoulous, N.: Embedded implicatures?!? *Semantics and Pragmatics* 2(4), 1–34 (2009)
- Landman, F.: Plurals and Maximalization. In: Rothstein, S. (ed.) *Events and Grammar*, pp. 237–271. Kluwer, Dordrecht (1998)
- Meyer, M.-C., Sauerland, U.: A pragmatic constraint on ambiguity detection. *Natural Language & Linguistic Theory* 27(1), 139–150 (2009)
- Reinhart, T.: Quantifier scope: How labor is divided between QR and choice functions. *Linguistics and Philosophy* 20(4), 335–397 (1997)
- van Rooij, R., Schulz, K.: Exhaustive Interpretation of Complex Sentences. *Journal of Logic, Language and Information* 13(4), 491–519 (2004)
- Sauerland, U.: Embedded implicatures and experimental constraints: A Reply to Geurts & Pouscoulous and Chemla. *Semantics and Pragmatics* 3(2), 1–13 (2010)
- Spector, B.: Scalar implicatures: Exhaustivity and Gricean reasoning. In: ten Cate, B. (ed.) *Proceedings of the Eighth ESSLI Student Session*, Vienna, Austria (2003); Revised version in Aloni, M., Dekker, P., Butler, A. (eds.): *Questions in Dynamic Semantics*. Elsevier, Amsterdam (2007)
- Spector, B.: *Aspects de la pragmatique des opérateurs logiques*. Ph.D. thesis, Université Paris 7 (2006)

# Local and Global Implicatures in *Wh*-Question Disjunctions

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**Abstract.** It has been observed that *wh*-questions cannot be joined disjunctively, the suggested reasons being semantic or pragmatic deviance. We argue that *wh*-question disjunctions are semantically well-formed but are pragmatically deviant outside contexts that license polarity-sensitive (PS) items. In these contexts the pragmatic inadequacy disappears due to a pragmatically induced recalibration of the implicature triggered by *or* (as argued in [1]). We propose that the alternative-inducing property of *or* has as its syntactic correlate the feature  $[+\sigma]$  (cf. [2]), thus forcing the insertion of the operator  $O_{ALT}$ , which is responsible for the computation of implicatures at different scope sites. Importantly, the licensing of the PS property of *wh*-question disjunctions cannot be reduced to the licensing of a lexical property of *or* but also depends on the semantics of the disjoined questions.

## 1 The Deviance of *Wh*-Question Disjunctions

*Wh*-question disjunctions have been observed to be deviant, e.g. [3,4]. Whereas a conjunction of two *wh*-questions is fine, a disjunction is unacceptable:

Which dish did Al make and which dish did Bill make? (1)

\*Which dish did Al make or which dish did Bill make? (2)

The unacceptability of *wh*-question disjunctions can be given a semantic explanation if we take the partition theory of questions as a basis ([5,6]). According to this theory, a question defines a partition of the logical space. A disjunction of two questions is the union of two partitions, which is not again a partition because of overlapping cells. Thus the disjunction of two questions is not a question. According to [4], the reason for the deviance of *wh*-question disjunctions is also pragmatic, the underlying assumption being that speech acts cannot be coordinated disjunctively. Speech acts are operations that, when applied to a commitment state, deliver the commitments that characterize the resulting state. Speech act disjunction would lead to disjunctive sets of commitments, which are difficult to keep track of. [4] argues that a question like (2) could

only<sup>1</sup> be interpreted in the way indicated in (3), where the speaker retracts the first question and replaces it by the second. As a result there is only one question to be answered.

Which dish did Al make? Or, which dish did Bill make? (3)

In this paper we propose that *wh*-question disjunctions do denote proper semantic questions but are pragmatically deviant outside specific contexts. We identify these specific contexts as contexts that license polarity-sensitive items (PSIs). In PSI-licensing contexts, the pragmatic inadequacy disappears due to a pragmatically induced recalibration of the implicature triggered by *or* (cf. [1]). The account developed here does not carry over to alternative questions that have the form of *yes/no*-question disjunctions such as e.g. (4).

Are you coming or are you going? (4)

See [7] for an analysis of such questions.

## 2 The Semantics of *Wh*-Question Disjunctions

For the semantics of *wh*-questions we follow [8] and assume that a question denotes the set of its true answers. For instance, the question *How did Paul get home?* denotes the set in (5) (where **a** is the index of the actual world). Assuming that in the actual world Paul got home by bus and by train, this is the set in (6). The weakly exhaustive answer defined by (6) is the conjunction of all the propositions in this set, i.e. the proposition in (7).

$$\begin{aligned} \llbracket \text{How did Paul get home?} \rrbracket^g = \\ = \{p \mid \exists m(p(\mathbf{a}) \wedge p = \lambda i(\text{Paul got home in manner } m \text{ in } i))\} \end{aligned} \quad (5)$$

$$\{\lambda i(\text{Paul got home by bus in } i), \lambda i(\text{Paul got home by train in } i)\} \quad (6)$$

$$\lambda i(\text{Paul got home by bus in } i \wedge \text{Paul got home by train in } i) \quad (7)$$

For the disjunction of *wh*-questions we propose that such a disjunction denotes the set of propositions that results from the pairwise disjunction of any two propositions from the respective disjuncts, see (8).<sup>2</sup> Thus every proposition in the

<sup>1</sup> For some speakers, the question disjunction in (2) seems to be felicitous under a reading where it is understood as a directive to choose one of the questions and answer it (thanks to Jeroen Groenendijk and Stefan Kaufmann for pointing this out to us). This reading corresponds to the so-called *choice reading* of questions like *What did someone read?* discussed in [6]. This question can be understood as a directive to the answerer to choose a person and say for that person what s/he read, e.g. *John read 'War and Peace'*. We show in section 3 that there is a true question-disjunction reading that is different from the choice reading.

<sup>2</sup> In (8) and below, we write  $p_1 \vee p_2$  as a shorthand for  $\lambda i(p_1(i) \vee p_2(i))$ . Corresponding conventions hold for other truth functions when applied to propositional objects.

answer set of the first question is conjoined disjunctively with every proposition in the answer set of the second question. Hence, the *wh*-question disjunction in (9) denotes the singleton set in (10) if in fact Paul got home by bus at 3 a.m. and in no other way and at no other time.<sup>3</sup> The weakly exhaustive answer defined by (10) is then the proposition in (11).

$$\llbracket Q_1 \text{ or } Q_2 \rrbracket^g = \{p_1 \vee p_2 \mid p_1 \in \llbracket Q_1 \rrbracket^g \wedge p_2 \in \llbracket Q_2 \rrbracket^g\} \quad (8)$$

$$[Q_1 \text{ How did Paul get home?}] \text{ or } [Q_2 \text{ When did Paul get home?}] \quad (9)$$

$$\{\lambda i(\text{Paul got home by bus in } i \vee \text{Paul got home at 3 a.m. in } i)\} \quad (10)$$

$$\lambda i(\text{Paul got home by bus in } i \vee \text{Paul got home at 3 a.m. in } i) \quad (11)$$

The deviance of the question disjunction in (9) can be explained if we consider its pragmatics, more specifically, if we look at it from the point of view of Gricean reasoning (9). When trying to give the true exhaustive answer to (9) (= the proposition in (11), if we consider the weakly exhaustive answer) the answerer cannot avoid violating Grice's Maxim of Quantity. Both disjuncts are true in the evaluation world since they are *true* answers to the disjoined questions. The use of *and* would be more informative and would not violate the Maxim of Quality. We suggest that this is the reason for the unacceptability of *wh*-question disjunctions. *Wh*-question disjunctions are unanswerable and therefore deviant.

Before closing this section, we would like to point out that our proposal might be rejected on the assumption that the over-informative *and*-answer should pose no problems because it is generally possible to give over-informative answers to questions, see (12). So this should be possible for disjoined *wh*-questions as well.

$$Q: \text{Has someone called for me?} - A: \text{Yes, Paul did.} \quad (12)$$

We argue in section 4 that *wh*-question disjunctions do not have a strongly exhaustive (= maximally informative) answer and therefore they neither have over-informative answers.

### 3 Non-deviant *Wh*-Question Disjunctions

In the previous section we discussed the observation that *wh*-question disjunctions are deviant and gave a preliminary account for why this should be. We only

<sup>3</sup> For easier exposition we only consider singleton sets in what follows. This assumption is without loss of generality because of the distributivity of ' $\vee$ ' over ' $\wedge$ ', i.e.  $\text{ans}(\llbracket Q_1 \text{ or } Q_2 \rrbracket^g) = \lambda i(\text{ans}(\llbracket Q_1 \rrbracket^g)(i) \vee \text{ans}(\llbracket Q_2 \rrbracket^g)(i))$ . See (32) for a definition of the answer operator ' $\text{ans}$ '.

considered matrix questions in that section. Moving on to embedded questions at first sight does not change the picture: Speakers judge (13) to be unacceptable.

\*The police found out how or when Paul got home that night. (13)

For some speakers (13) improves if the question words are heavily accented and if there also is an intonational phrase break after the first question word, as indicated in (14). These phonological means, we suggest, indicate the readings in (15) or (16):

%The police found out HOW, or WHEN Paul got home that night. (14)

The police found out HOW, or rather WHEN Paul got home that night. (15)

The police found out HOW, or ~~the police found out~~ WHEN Paul got home that night. (16)

(15) is a retraction reading, similar to the one in (3) discussed in section 1. (16) involves ellipsis of matrix clause material, so that we are not dealing with a question disjunction here but with a disjunction of the matrix clause assertions. Under both readings, truth obtains if the police can answer one of the embedded questions. These readings are irrelevant for the present discussion. The relevant reading, which (13) does not have,<sup>4</sup> is the proposition that the police have attained a belief state that is a subset of the set of indices characterized by (11). Note that this belief state would not necessarily allow the police to answer either of the embedded questions.

Now, upon further inquiry we find that there are quite a number of contexts that license embedded disjoined questions:

The police did not find out how or when Paul got home that night.  
(*negation*) (17)

If the police find out how or when Paul got home that night  
they can solve the crime. (*antecedent of conditional*) (18)

Few detectives found out how or when Paul got home that night.  
(*downward-entailing quantifier*) (19)

The police hoped to find out how or when Paul got home that  
night. (*strong intensional predicate*) (20)

The police might have found out how or when Paul got home that  
night. (*modalized context*) (21)

The police refuse to find out how or when Paul got home that  
night. (*adversative predicate*) (22)

<sup>4</sup> But see [10] for a discussion of contexts that seems to improve the acceptability of such a reading in comparable cases.

Have the police found out how or when Paul got home that night?  
(*question*) (23)

Find out how or when Paul came home that night! (*imperative*) (24)

These contexts are all contexts that license PS items. Thus, *wh*-question disjunctions can be classified as polarity-sensitive.

### The PS Property of *Wh*-Question Disjunctions

*Wh*-question disjunctions are licensed in downward-entailing contexts (25) and in non-downward-entailing contexts that are non-veridical.<sup>5</sup>

Before we proceed we would like to point out that the question word disjunctions considered above indeed correspond to the disjunction of full questions. This can be seen quite easily from the fact that it is possible to coordinate disjunctively the complementizer *if* with a *wh*-word, see (26). Such a disjunction must involve ellipsis as it cannot be derived semantically as a term disjunction.

The police did not find out if or when Paul got home that night. (26)

What about matrix clause ellipsis? For the unacceptable example in (13) above, which involved a matrix context that did not license PSIs, we considered the possibility that it might improve for some speakers if the intonational means signal matrix clause ellipsis. For the felicitous examples in (17) through (24) this option is not available. Let us illustrate this for the negation context in (17). If this sentence is assumed to be derived from matrix clause ellipsis its meaning is different:

The police did not find out how or when Paul got home that night.

$\nrightarrow$

The police did not find out how ~~Paul got home that night~~ or  
~~the police did not find out~~ when Paul got home that night.

(27)

We conclude from this that ellipsis of the matrix clause is not available as a general point of departure for a unified analysis of embedded *wh*-questions disjunctions.

Another structural option we have to consider involves ellipsis of matrix clause material below the operator that creates the PSI-licensing context, e.g.:<sup>6</sup>

The police did [not [<sub>orP</sub> [<sub>VP</sub> find out [<sub>Q<sub>1</sub></sub> how ~~Paul got home that night~~]]  
[<sub>or'</sub> or [<sub>VP</sub> ~~find out~~ [<sub>Q<sub>2</sub></sub> when Paul got home that night]]]]] (28)

<sup>5</sup> A context is non-veridical if for any sentence  $\phi$ :  $C(\phi) \nrightarrow \phi$  (= if  $\phi$  occurs in a non-veridical context the truth of  $\phi$  does not follow). Some non-veridical contexts, like negation, are also anti-veridical, which means that if  $\phi$  occurs in such a context the falsity of  $\phi$  follows (11).

<sup>6</sup> We are grateful to Rajesh Bhatt and Danny Fox for pointing this out to us.

The structure in (28) gives rise to a choice reading: The police did not find out the answer to either question but they may have found out that Paul got home by bus or at 3 a.m. without knowing which is the case. This, however, is not the only available reading (17) has. This is evidenced by the fact that it can be contradicted by (29).

Not true! The police found out that Paul got home by bus  
or at 3 a.m. (They just don't know which.) (29)

This shows that (17) has a reading where the police did not find out anything at all, which, we suggest, is the reading that involves a disjunction of questions. It arises from a structure where the ellipsis is confined to the embedded clauses:

The police did [not [find out [<sub>orP</sub> [<sub>Q<sub>1</sub></sub> how ~~Paul got home that night~~]  
[<sub>or'</sub> or [<sub>Q<sub>2</sub></sub> when Paul got home that night]]]]]] (30)

The unenriched meaning of (30) is given in (31).<sup>7</sup> We follow a standard assumption in the Hamblin/Karttunen framework and assume that a predicate like *find out* embeds the true exhaustive answer to a question and not the question directly. The embedding is mediated by the operator 'ans', defined in (32): 'ans' delivers the intersection of the propositions in the set of true answers denoted by the embedded question, cf. (12).

$$\neg \text{find\_out}(\text{the\_police}, \text{ans}(\{p_1 \vee p_2 \mid p_1 \in \llbracket Q_1 \rrbracket^g \wedge p_2 \in \llbracket Q_2 \rrbracket^g\})) \quad (31)$$

$$\text{ans}(Q) = \lambda i. \forall p(p \in Q \rightarrow p(i)) \quad (32)$$

If we assume, as before, that Paul in fact got home only by bus and only at 3 a.m., (31) is the following sentence:

$$\neg \text{find\_out}(\text{the\_police}, \lambda i(\text{Paul got home by bus in } i \vee \\ \vee \text{ Paul got home at 3 a.m. in } i)) \quad (33)$$

This is the reading where the police have not attained any knowledge with respect to the actual time and manner of Paul's getting home. They do not even have the limited knowledge that the choice reading would allow.

Comparable evidence for the availability of the question disjunction reading can be drawn from conditional sentences like (18) above, see (10).

## 4 Computing Local and Global Implicatures: Explaining the PS Property of *Wh*-Disjunctions

In section 2 we explained the deviance of matrix *wh*-question disjunctions by appealing to Gicean reasoning: The disjunctive operator *or* gives rise to a scalar

<sup>7</sup> Here and below we give the extension of the considered sentence. We omit the index of the evaluation world.

alternative – the conjunctive operator *and* –, which would have been the better choice by the Maxims of Quantity and Quality (when trying to give an answer to the question disjunction). In the previous section we proposed that *wh*-question disjunctions are polarity-sensitive. Now, scalar implicatures have also been argued to play an important role in the licensing of PS items like *any*. [13] suggest that *any*-NPs are indefinites which come with an instruction to the hearer to consider domains of individuals that are broader than what one would usually consider, i.e. *any*-NPs are domain wideners. In downward-entailing contexts like negation, domain widening strengthens a statement because excluding a larger domain of individuals leads to a more informative statement than excluding a smaller domain of individuals. [14] links these consideration directly to quantity implicatures and suggests that a NPI like *any* activates alternatives with smaller domains, which triggers the implicature that the alternative selected is the strongest one the speaker has evidence for. The fact that *wh*-question disjunctions are licensed in exactly those contexts that license PS items is thus very suggestive of a close link along these lines of reasoning.

What will be important for the data we consider here is the observation that implicatures can also arise in embedded contexts. This is somewhat unexpected if pragmatic reasoning is assumed to follow all syntactic and semantic computations, and it has led [1] to argue for a “more grammatical” view of implicatures, which we take our findings to be supporting evidence for. To start with, consider the following embedded disjunction of declaratives:

The police found out that Paul got home by bus or that he got home at 3 a.m. (34)

The preferred reading of *or* in (34) is the exclusive one: (34) could describe the findings of the police if the busses stop at 12 p.m. Paul would have been home by 12 if he took the bus, or later (such as at 3 a.m.) if he did not take the bus. The implicature in (34) is a local scalar implicature which leads to the interpretation in (35).<sup>8</sup> Here and below,  $p_{\text{bus}}$  and  $p_{3\text{am}}$  are the propositions given in (36).

$$\text{find\_out}(\text{the\_police}, (p_{\text{bus}} \vee p_{3\text{am}}) \wedge \neg(p_{\text{bus}} \wedge p_{3\text{am}})) \quad (35)$$

$$\begin{aligned} p_{\text{bus}} &= \lambda i(\text{Paul got home by bus in } i) \\ p_{3\text{am}} &= \lambda i(\text{Paul got home at 3 a.m. in } i) \end{aligned} \quad (36)$$

The global implicature would be the one captured by (37), and it leads to a weaker interpretation than the local implicature: (37) is compatible with the police attaining the knowledge that it is possible that Paul came home by bus at 3 a.m. That is, the second conjunct in (37) expresses that the police did not find out anything with respect to  $p_{\text{bus}} \wedge p_{3\text{am}}$ .

$$\text{find\_out}(\text{the\_police}, p_{\text{bus}} \vee p_{3\text{am}}) \wedge \neg \text{find\_out}(\text{the\_police}, p_{\text{bus}} \wedge p_{3\text{am}}) \quad (37)$$

<sup>8</sup> Recall that the notation of the embedded proposition is a shorthand for  $\lambda i((p_{\text{bus}}(i) \vee p_{3\text{am}}(i)) \wedge \neg(p_{\text{bus}}(i) \wedge p_{3\text{am}}(i)))$ .

[1], [2] suggest that the difference between local and global implicatures can be put down to an operator  $O_{ALT}$  for scalar enrichment that can attach at various scope sites:<sup>9</sup>

$$O_{ALT}(p) = \lambda i(p(i) \wedge \forall q \in ALT(q(i) \rightarrow \forall j(p(j) \rightarrow q(j)))) \quad (38)$$

For a disjunctive proposition  $p_{dis} = p_1 \vee p_2$ ,  $ALT$  contains (exactly)  $p_{dis}$  and its conjunctive alternative as elements:  $ALT = \{p_1 \vee p_2, p_1 \wedge p_2\}$ . It can be easily verified that  $O_{ALT}(p_{dis}) = (p_1 \vee p_2) \wedge \neg(p_1 \wedge p_2)$ . This means that the local scalar implicature captured by (35) results from enriching the meaning of (34) by inserting  $O_{ALT}$  at the level of the embedded proposition, see (39).

$$\text{find\_out}(\text{the\_police}, O_{ALT}(p_{bus} \vee p_{3am})) \quad (39)$$

Turning to embedded *wh*-question disjunctions like (13) from section 3 above, the insertion of  $O_{ALT}$  at the level of the embedded proposition yields the enriched meaning in (40).

$$\text{find\_out}(\text{the\_police}, O_{ALT}(\text{ans}(\{p_1 \vee p_2 \mid p_1 \in \llbracket Q_1 \rrbracket^g \wedge p_2 \in \llbracket Q_2 \rrbracket^g\}))) \quad (40)$$

If we assume again that Paul in fact came home by bus at 3 a.m., (40) can be rendered in the same way as the locally enriched meaning of (34), i.e. as in (35). However, in the question disjunction case  $p_{bus}$  and  $p_{3am}$  are true in the actual world: They are elements of  $\llbracket Q_1 \rrbracket^g$  and  $\llbracket Q_2 \rrbracket^g$ , respectively, which are sets of true answers. This means that the enriched embedded proposition is false in the actual world. This produces a presupposition failure under the factive verb *find out*, and more generally, a failure of the existence presupposition of the embedded *wh*-question. That is we assume that a *wh*-question  $Q$  presupposes that there is a true answer, which is not satisfied by the pragmatically enriched answer to  $Q$ . This also explains why *wh*-disjunctions neither can be embedded under non-factive verbs like *tell* (not illustrated).

If the local insertion of  $O_{ALT}$  produces an unacceptable sentence we might wonder, of course, why it is not global insertion that is applied. In our scenario, the resulting enriched meaning would be the following (cf. (37) above).

$$\text{find\_out}(\text{the\_police}, p_{bus} \vee p_{3am}) \wedge \neg \text{find\_out}(\text{the\_police}, p_{bus} \wedge p_{3am}) \quad (41)$$

Inserting  $O_{ALT}$  at the root level leads to a rather weak strengthening – the police did not acquire knowledge about  $p_{bus} \wedge p_{3am}$  – but it does not lead to deviance. Still, this reading does not seem to be available. This is surprising given that  $O_{ALT}$  generally can be inserted at any scope site (cf. [15]). We have some preliminary evidence that under very specific contextual conditions the preference for the

<sup>9</sup> ‘O’ is an adequate mnemonic for ‘only’:  $p$  and its entailments are the only members of the set of scalar alternatives that hold.

local implicature can be overridden. Unfortunately we do not have the space to discuss this here (but see [10]).

Let us turn next to felicitous embedded *wh*-question disjunctions starting with downward-entailing contexts, e.g. (17) with negation, repeated below:

The police did not find out how or when Paul got home that night. (42)

[1] observes that the downward-entailing property of an operator like negation in the matrix clause typically induces a recalibration of the implicature because local enrichment would lead to weakening in these contexts. Thus,  $O_{ALT}$  applies to the matrix clause, s. (43).

$$O_{ALT}(\neg \text{find\_out}(\text{the\_police}, \text{ans}(\{p_1 \vee p_2 \mid p_1 \in \llbracket Q_1 \rrbracket^g \wedge p_2 \in \llbracket Q_2 \rrbracket^g\}))) \quad (43)$$

In our scenario, (43) is equivalent to (44) because of the following entailment:  $\neg \text{find\_out}(x, p_1 \vee p_2) \rightarrow \neg \text{find\_out}(x, p_1 \wedge p_2)$

$$\neg \text{find\_out}(\text{the\_police}, p_{\text{bus}} \vee p_{\text{3am}}) \quad (44)$$

In the present case, application of  $O_{ALT}$  to the matrix clause does not produce an implicature and hence no deviance. This result carries over to all other downward-entailing contexts.

Turning to contexts that are not downward-entailing but nevertheless license embedded *wh*-question disjunctions, let us consider questions. The positive answer to an *or*-question is entailed by the positive answer to the corresponding *and*-question. So the *or*-question is actually weaker than its scalar alternative. Importantly, asking weaker questions often is pragmatically advantageous [14]. First observe that positive *yes-no* questions come with no particular bias as to the expected answer, *yes* or *no*. In order to optimize the information gain from both possible answers, the speaker will try to maintain an equilibrium between the informational value of the positive and the negative answer ([14], also cf. [16]’s notion of *entropy*). The weaker a question is the more balanced the answers are, and the better the information gain is in proportion to the likelihood of the answer. This can be seen quite easily when considering guessing games where participants must guess e.g. the occupation of an invited person. In such a game, asking the rather weak question in (45) maximizes the information gain because the likelihood of receiving the *yes*- vs. the *no*-answer is roughly the same. This is different in a strong question like (46), where the *no*-answer would yield hardly any information gain.

Are you involved in the production/ distribution of a product? (45)

Are you a hearing aid audiologist? (46)

For questions as licensing contexts, inserting  $O_{ALT}$  at the root level rather than at the embedded level yields the weaker question.

## 5 Conclusion

Our analysis lends strong support to the central claim of [1] that the syntactic distribution of PS items is determined by grammatically conditioned pragmatic principles. The PS property of *wh*-disjunctions is semantically composed of two independent properties: the semantic/pragmatic property of *or* to induce (scalar) alternatives, and the semantics of the disjoined questions. This means that the licensing of the PS property cannot be reduced to the licensing of a lexical property of a single item. If there is a syntactic feature involved in the licensing of the PS property it must be the syntactic correlate of the alternative-inducing property of an element like *or*, cf. the feature  $[+\sigma]$  in [1]. This is what we assume here: *or* always comes with  $[+\sigma]$ , which forces the insertion of  $O_{ALT}$  as discussed above.

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## References

1. Chierchia, G.: Scalar implicatures, polarity phenomena, and the syntax/pragmatics interface. In: Belletti, A. (ed.) *The Cartography of Syntactic Structures. Structures and Beyond*, vol. 3, pp. 39–103. Oxford University Press, Oxford (2004)
2. Chierchia, G.: Broaden your views. implicatures of domain widening and the “logicality” of language. *Linguistic Inquiry* 37, 535–590 (2006)
3. Szabolcsi, A.: Quantifiers in pair-list readings. In: Szabolcsi, A. (ed.) *Ways of Scope Taking*, pp. 311–347. Kluwer, Dordrecht (1997)
4. Krifka, M.: Quantifying into question acts. *Natural Language Semantics* 9, 1–40 (2001)
5. Groenendijk, J., Stokhof, M.: Semantic analyses of *Wh*-complements. *Linguistics and Philosophy* 5, 175–233 (1982)
6. Groenendijk, J., Stokhof, M.: *Studies on the Semantics of Questions and Pragmatics of Answers*. PhD thesis, University of Amsterdam (1984)
7. Haida, A.: The syntax and semantics of alternative questions: Evidence from Chadic. In: *Proceedings of Sinn und Bedeutung 14*, Vienna (to appear)
8. Karttunen, L.: Syntax and semantics of questions. *Linguistics and Philosophy* 1, 3–44 (1977)
9. Grice, P.: *Studies in the Way of Words*. Harvard University Press, Cambridge (1989)
10. Haida, A., Repp, S.: Disjunction in *Wh*-questions. In: *Proceedings of NELS 40* (to appear)

11. Giannakidou, A.: Polarity Sensitivity as (Non)Veridical Dependency. Benjamins, Amsterdam (1998)
12. Heim, I.: Interrogative semantics and karttunen's semantics for *Know*. In: Proceedings of IATL 1. Jerusalem, Akademon, pp. 128–144 (1994)
13. Kadmon, N., Landmann, F.: Any. *Linguistics and Philosophy* 15, 353–422 (1993)
14. Krifka, M.: The semantics and pragmatics of polarity items. *Linguistic Analysis* 25, 209–257 (1995)
15. Chierchia, G., Fox, D., Spector, B.: The grammatical view of scalar implicatures and the relationship between semantics and pragmatics. In: Maienborn, C., von Stechow, K., Portner, P. (eds.) *Semantics: An International Handbook of Natural Language Meaning*. De Gruyter, Berlin (to appear)
16. van Rooij, R.: Negative polarity items in questions: Strength as relevance. *Journal of Semantics* 20, 239–273 (2003)

# Supplements within a Unidimensional Semantics I: Scope<sup>\*</sup>

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**Abstract.** Potts (2005, 2007) claims that Grice's 'conventional implicatures' offer a powerful argument in favor of a multidimensional semantics, one in which certain expressions fail to interact scopally with various operators because their meaning is located in a separate dimension. Focusing on Non-Restrictive Relative Clauses (= NRRs), we explore an alternative to Potts's bidimensional account. In our analysis, (1) NRRs can be syntactically attached with matrix scope, despite their appearance in embedded positions; (2) NRRs can in some cases be syntactically attached within the scope of other operators (whether attitudinal or not), in which case they semantically interact with them; (3) NRRs are semantically conjoined with the rest of the sentence, but (4) they are subject to a pragmatic rule that requires that their content be relatively easy to accommodate – hence some non-trivial projection facts when NRRs do not have matrix scope. In this paper, we only develop (1) and (2), which pertain to the scopal behavior of NRRs. (1), which is in full agreement with the classic 'high attachment' analysis of NRRs, shows that Potts's semantic machinery is not necessary: its effects follow from more conservative semantic assumptions once an adequate syntax is postulated. Because of (2), Potts's machinery makes incorrect predictions when NRRs have a non-matrix attachment and interact scopally with other operators. Semantic arguments for (2) were given in Wang et al. 2005 and Amaral et al. 2007, but were re-analyzed in pragmatic terms in Harris and Potts 2009a, b; we provide new evidence that suggests that in some cases the latter analysis is implausible.

**Keywords:** supplements, appositives, non-restrictive relative clauses.

## 1 Bidimensional vs. Unidimensional Analyses

The contrast between (1)a and (1)b suggests that appositive relative clauses are 'scopeless', i.e. that they do not interact semantically with operators in whose scope they appear.

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- (1) a. I doubt that John, who is smart, is competent.  
 => John is smart.  
 b. I doubt that John is smart and competent.  
 ≠> John is smart

This behavior was taken by Potts 2000, 2005 and Nouwen 2006 to argue for a *bidimensional semantics*, one in which ‘supplements’ (= the semantic content of appositives) are computed in a separate dimension from assertive content. Their analysis is sketched in (2)

- (2) **Bidimensional Analysis** (Potts 2000, 2005; Nouwen 2006)  
 (i) **Syntax:** Appositives are attached in their surface position.  
 (ii) **Semantics:** Supplements are computed in a separate dimension, which has two effects.  
 A. They appear to have ‘wide scope’.  
*Version 1* (Potts 2000): They do not interact scopally with other operators.  
*Version 2* (Nouwen 2006): They only interact scopally with operators *to the extent that unembedded E-type pronouns do* (e.g. in *John invited few people, who had a good time*, the NRR does interact with the quantifier; but the truth conditions are similar to those of the discourse *John invited few people. They had a good time*).  
 B. Supplements have a special epistemic status (they are not ‘at issue’).

We explore an alternative account within a unidimensional semantics. In brief, we suggest that syntactically NRRs are preferably attached to the matrix level, but that lower attachments are also possible; we take them to have a conjunctive semantics; and we assume that they are subject to a pragmatic constraint that requires that their content be both non-trivial and not too surprising. These assumptions are stated more precisely in (3).

- (3) **Unidimensional Analysis**  
 (i) **Syntax** (see McCawley 1988, Del Gobbo 2003)  
 -A NRR can be attached to any node of propositional type that dominates its associated NP.  
 -Preferences: highest attachment >> lower attachment – attitudinal >> lower attachment – non attitudinal  
 (ii) **Semantics** (Del Gobbo 2003)  
 a. In a NRR, the relative pronoun can be interpreted as E-type or as referential.  
 b. A NRR is interpreted conjunctively.  
 (iii) **Pragmatics**  
 The content of a NRR must be ‘easy to accommodate’, but non-trivial – which gives rise to non-trivial pattern of projection.

We provide three arguments in favor of our approach ((iii) is discussed in Part II).

(i) **Bidimensionalism is unnecessary** because there are independent arguments for postulating that high syntactic attachment is possible.

(ii) **Bidimensionalism is undesirable** because there are other cases in which low attachment is possible (though often dispreferred).

**(iii) Some supplements give rise to non-trivial patterns of projection which are formally similar to presupposition projection.** This suggests that there is a non-trivial interaction between the content of NRRs and other operators.

We do not attempt in this short piece to discuss *expressives* (e.g. the expression *honky*, which indicates that the speaker has a derogatory attitude towards white people). These were taken by Potts (2000, 2005) to provide an important argument in favor of a bidimensional analysis; a key argument was that expressives always appear to have ‘wide scope’. Other researchers (e.g. Sauerland 2007, Schlenker 2003, 2007) have argued that expressives can in fact take scope under attitude operators. Harris and Potts (2009a, b) show with experimental means that this is indeed possible, but that the phenomenon is broader: even in the absence of attitude operators, the content of an expressive can sometimes be attributed to someone other than the speaker. They conclude that all such data can be dealt with by a pragmatic mechanism of ‘perspectival shift’ that works *on top* of a bidimensional semantics: expressives are *bona fide* wide scope expressions, but sometimes perspectival shift gives the impression that they interact scopally with attitude operators. Their account also applies to NRRs that appear to take scope within attitude reports. While we leave expressives out of the present study, we will argue below that Harris and Potts’s analysis is implausible for *some* narrow scope NRRs.

In the rest of this paper, we focus on French, for two reasons (besides expediency).

First, Cinque 2008 has argued that there are two types of nonrestrictive relative clauses, only one of which is present in English:

- ‘Integrated NRRs’ are according to him ‘essentially identical to the ordinary restrictive construction (as such part of sentence grammar)’; in other words, they are closely integrated to the sentence they appear in. Such NRRs are not available in English. In French, they are exemplified by relative clauses introduced by *qui*.
- ‘Non-integrated NRR’ are ‘distinct from the ordinary restrictive construction (with characteristics of the grammar of discourse)’. All English NRRs are of this type. In French, this class is represented by relative clauses introduced by *lequel*.

By focusing on French, we will show that *even* integrated NRRs have the ability to attach syntactically at the matrix level when their surface position appears to be embedded. In this way, we show that not just NRRs that belong to the ‘grammar of discourse’, but even those that are part of ‘sentence grammar’ display unexpected attachment possibilities.

Second, French has some moods – notably, the subjunctive – which are obligatorily syntactically and semantically embedded. This will be crucial to show that some NRRs have narrow scope (both syntactically and semantically).

## 2 The Possibility of High Syntactic Attachment

Following and extending arguments developed in McCawley 1988, we start by arguing that NRRs *can* be attached to the matrix clause even when they appear to be embedded.

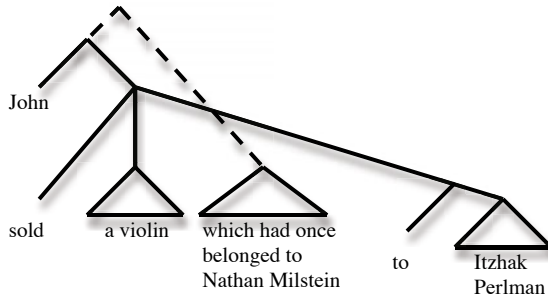
## 2.1 Ellipsis

Our first argument replicates in French a paradigm discussed by McCawley 1988 for English:

- (4) John sold a violin, which had once belonged to Nathan Milstein, to Itzhak Perlman, and Mary did too.

McCawley 1988 observed that the second sentence does not imply that the violin that Mary sold to Perlman had once belonged to Nathan Milstein. On the assumption that ellipsis targets a constituent, this suggests that the NRR can be attached outside the constituent which is the antecedent of the elided VP. This reasoning lead McCawley to posit the structure in (5), which crucially involves a discontinuous constituent. (We do not need to adopt McCawley's ternary branching structure for the VP; all that matters for present purposes is that the NRR can be attached much higher than its surface position, which implies that the VP within which it appears forms a discontinuous constituent).

(5)



The same conclusion must be reached about NRRs introduced by *qui* in French; in this respect, they contrast rather clearly with restrictive relative clauses:

- (6) *Context:* In each generation, the most famous cellist gets to meet the most talented young musicians.
- a. Yo Yo Ma a présenté ses élèves préférés, qui vivent à Cambridge, à Rostropovitch. Paul Tortelier aussi, bien sûr.  
*Yo Yo Ma introduced his favorite students, who live in Cambridge, to Rostropovich. Paul Tortelier did too, of course*  
 ≠> Tortelier has students in Cambridge.
- b. Yo Yo Ma a présenté ses élèves qui vivent à Cambridge, à Rostropovitch. Paul Tortelier aussi, bien sûr.  
*Yo Yo Ma introduced his students who live in Cambridge to Rostropovich. Paul Tortelier did too, of course.*  
 => Tortelier has students in Cambridge.<sup>1</sup>

<sup>1</sup> C. Potts (p.c.) notes that the same patterns of 'disappearance under ellipsis' hold of expressive adjectives, which certainly don't appear to be attached with matrix scope. This point is discussed in Potts et al. 2009:

## 2.2 Condition C Effects

Our second argument concerns Condition C effects, which are weakened or obviated in cases that involve NRRs, as in (7).

- (7) [Le Président]<sub>i</sub> est si compliqué qu'  
*[The President]<sub>i</sub> is so complicated that*  
 a. \*il<sub>i</sub> a donné au ministre qui n' aime pas Sarkozy<sub>i</sub> une tâche impossible.  
*he<sub>i</sub> gave the minister who doesn't like Sarkozy<sub>i</sub> an impossible task.*  
 b. (?) il<sub>i</sub> a donné au ministre de la Justice, qui n' aime pas Sarkozy<sub>i</sub>, une tâche impossible.  
*he<sub>i</sub> gave the minister the minister of Justice, who doesn't like Sarkozy<sub>i</sub>, an impossible task.*
- (8) [Le Président]<sub>i</sub> est si compliqué qu'  
*[The President]<sub>i</sub> is so complicated that*  
 a. \*il<sub>i</sub> n' a envoyé qu' à un seul journaliste qui adore Sarkozy<sub>i</sub> son<sub>i</sub> dernier livre.  
*he<sub>i</sub> sent to only one journalist who loves Sarkozy<sub>i</sub> his<sub>i</sub> latest book.*  
 b. il<sub>i</sub> n' a envoyé qu' à un seul journaliste, qui adore Sarkozy<sub>i</sub>, son<sub>i</sub> dernier livre.  
*he<sub>i</sub> sent to only one journalist, who loves Sarkozy<sub>i</sub>, his latest book.*

In (7)a and (8)a, restrictive relative clauses give rise to Condition C effects, as is expected in configurations in which a pronoun c-commands a coreferential proper name. The examples in (7)b and (8)b involve non-restrictive rather than restrictive relative clauses; the Condition C effects are weakened or obviated. This can be explained if the NRRs are attached at the matrix level, so that the pronoun *he* does *not* c-command *Sarkozy* in the end.

## 2.3 Weak Crossover Effects

I would like to suggest that Weak Crossover effects might also argue that NRRs can attach much higher than their surface position would lead one to expect. In (9)a-a', we find a standard Weak Crossover effect with a restrictive relative clause and a possessive; but this effect appears to be absent in (9)b, just as it is in discourse anaphora such as (9)b' (we gloss the pronoun *l'* as *her* because *star* is grammatically feminine in French, though it can apply to a male individual):

- (9) a. ?[Quelle star]<sub>i</sub> est-ce que le chirurgien qui l<sub>i</sub>' a opérée a failli faire mourir?  
*[Which star]<sub>i</sub> did the surgeon who operated (on) her<sub>i</sub> almost kill?*  
 a'. ?[Quelle star]<sub>i</sub> est-ce que son<sub>i</sub> chirurgien a failli faire mourir?  
*[Which star]<sub>i</sub> did her<sub>i</sub> surgeon almost kill?*

---

(i) A: I saw your fucking dog in the park.

B: No, you didn't—you couldn't have. The poor thing passed away last week.

The crucial observation is that B's reply does not commit B to the attitude expressed by the modifier *fucking*. Whatever mechanism is at play here could defuse McCawley's syntactic argument. We leave this debate for future research.

b. [Quelle star]<sub>i</sub> est-ce que ton chirurgien, qui l<sub>i</sub>'a fort mal opérée, a failli faire mourir?

[Which star]<sub>i</sub> did your surgeon, who operated very badly on her<sub>i</sub>, nearly kill?

b'. [Quelle star]<sub>i</sub> est-ce que ton chirurgien failli faire mourir? Apparemment, il l<sub>i</sub>'a fort mal opérée.

[Which star]<sub>i</sub> did your surgeon nearly kill? Apparently, he operated very badly on her<sub>i</sub>.

All Weak Crossover judgments are subtle, and the sentences at hand are also quite complicated. But this paradigm might provide additional evidence in favor of an analysis in which the *wh*-interrogative does not c-command the pronoun contained within the NRR. On a low attachment view, one might expect a Weak Crossover effect because the pronoun is directly bound by the *wh*-interrogative. On a high attachment view, no Weak Crossover effect is expected (as is desired) because the pronoun is not c-commanded by the interrogative.

## 2.4 The Similarity between NRRs and Clausal Parentheticals

Finally, we note that with respect to the tests discussed here NRRs share the behavior of clausal parentheticals. This is relevant *if* one thinks that the syntactic similarity between non-restrictive and restrictive relative clauses is an argument for postulating that the former, like the latter, must be attached at the level which is indicated by their surface position. In the case of clausal parentheticals, no such argument holds: they certainly do seem to interrupt the continuity of a syntactic structure. Whatever syntactic rule allows for discontinuous constituents in this case might be applicable to NRRs too.

As is shown in (10) and (11), the facts concerning ellipsis and Condition C are indeed the same for clausal parentheticals as for NRR<sup>2</sup>.

### (10) Ellipsis

Yo Yo Ma a présenté ses élèves préférés (ils vivent à Cambridge) à Rostropovitch. Paul Tortelier aussi, bien sûr.

*Yo Yo Ma introduced his favorite students (they live in Cambridge) to Rostropovich. Paul Tortelier did too, of course*  
 ≠> Tortelier has students in Cambridge.

### (11) Condition C effects<sup>3</sup>

(?)[Le Président]<sub>i</sub> est si compliqué qu'il<sub>i</sub> a donné à la ministre de la Santé  
 [The President]<sub>i</sub> is so complicated that he<sub>i</sub> gave the minister of Health

(elle n'aime pas Sarkozy<sub>i</sub>) une tâche impossible.  
 (she doesn't like Sarkozy<sub>i</sub>) an impossible task.

<sup>2</sup> I believe this generalization extends to Weak Crossover effects, but I find it hard to think of a context in which one would utter (i); since Weak Crossover judgments are very subtle in the first place, I leave such examples for future research.

(i) Quelle star est-ce que ton chirurgien (on dit qu'il l'a fort mal opérée) a failli faire mourir?  
 Which star did your surgeon (one says that he treated her badly) nearly kill?

<sup>3</sup> To avoid an undesirable ambiguity in the resolution of the antecedent of the masculine pronoun in the parenthetical, we make *minister* feminine.

In sum, the data involving high syntactic attachment show that an analysis that posits a separate semantic dimension in order to handle the apparent ‘wide scope’ behavior of NRRs is not necessary, since these are sometimes *syntactically* attached to the matrix level<sup>4</sup>.

### 3 The Possibility of Low Syntactic Attachment

We will now suggest that the bidimensional analysis in its usual form – which implies that NRRs *always* display wide scope behavior – is not just unnecessary, but also undesirable because there are cases in which NRRs display a *narrow* scope behavior.

Proving this is usually difficult if one accepts the hypothesis that the *wh*-pronoun of a NRR has the semantics of a donkey pronoun. This hypothesis, developed by Del Gobbo 2003, is certainly compatible with a bidimensional approach, and it was in fact implemented in great detail in Nouwen 2006. The difficulty is that E-type pronouns that have wide scope can often ‘imitate’ the behavior of variables that are bound under other operators. Thus an example such as (12)a cannot show that NRRs may scope under a quantifier, because the control sentence in (12)b doesn’t sound too bad, and suggests that some semantic or pragmatic mechanism (call it ‘quantificational subordination’) allows the pronouns in the second sentence to be interpreted as if they had scope under the universal quantifier in the first sentence.

- (12) a. On Mother’s day, every little boy calls his mother, who tells him she loves him.  
 b. On Mother’s day, every little boy calls his mother. She tells him that she loves him.

Furthermore, cases of embedding under attitude operators were taken by Harris and Potts 2009a to be explained by a pragmatic mechanism of ‘perspective shifting’, which is available even when no attitude operator is present. Thus their subjects accepted to attribute to the agent (= the roommate, rather than the speaker) the content of the supplement *a possible government spy* both in (13)a and in (13)b.

- (13) I am increasingly worried about my roommate. She seems to be growing paranoid.  
 a. The other day, she told me that we need to watch out for themailman, a possible government spy.  
 b. The other day, she refused to talk with the mailman, a possible government spy.

Special care must thus be exercised to argue that a NRR can indeed take narrow scope.

#### 3.1 Subjunctive

A helpful test-case is provided by the subjunctive, which in French is a dependent mood. The first thing to observe is that this mood normally gives rise to very sharp

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<sup>4</sup> Another question is *why* such high syntactic attachments – which violate standard syntactic constraints – are possible in the first place. We leave this question for future research.

judgments of ungrammaticality unless it is syntactically embedded under an expression that licenses it. In (14)a, both subjunctive conjuncts are in the scope of *conceivable*, which is a licenser. In (14)b, the second conjunct appears as a separate sentence, and the result is sharply ungrammatical. Importantly, (14)c shows that this is a case in which modal subordination is arguably possible – but it requires a different mood (an epistemic future, or a conditional).

(14) Context: There was incident at school.

a. Il est concevable que Jean ait appelé sa mère, et qu'elle ait appelé son avocat.

*It is conceivable that Jean has-subj called his mother, and that she has-subj called her lawyer.*

b. \*\*Il est concevable que Jean ait appelé sa mère. Elle ait appelé son avocat.  
*It is conceivable that Jean has-subj called his mother. She has-subj called her lawyer.*

c. Il est concevable que Jean ait appelé sa mère. Elle aura / aurait appelé son avocat.

*It is conceivable that Jean has-subj called his mother. She will-have / would-have called her lawyer*

With this background in mind, we turn to the paradigm in (15), which involves NRRs.

(15) Context: There was incident at school<sup>5</sup>.

a. Il est concevable que Jean ait appelé sa mère / Anne, qui ait appelé son avocat.

*It's conceivable that Jean has-sub called his mother / Anne, who had-subj called her lawyer.*

≠> If Jean had called his mother / Anne, she would have called her lawyer.

b. \*\*Il est concevable que Jean ait appelé sa mère / Anne. Elle ait appelé son avocat.

*It's conceivable that Jean has-sub called his mother. She had-subj called her lawyer.*

a'. Il est concevable que Jean ait appelé sa mère, qui aurait appelé son avocat.  
*It's conceivable that Jean has-subj called his mother, who would have called her lawyer.*

=> If Jean had called his mother, she would have called her lawyer.

b'. Il est concevable que Jean ait appelé sa mère. Elle aurait appelé son avocat.  
*It's conceivable that Jean has-subj called his mother. She would have called her lawyer.*

=> If Jean had called his mother, she would have called her lawyer.

First, we note that the subjunctive can appear in a NRR embedded under a subjunctive licenser, as in (15)a. As before, the subjunctive is sharply ungrammatical if it appears in an independent sentence, as in (15)b. This suggests that (15)a is not a case in which the NRR has wide scope. Second, the truth conditions of the sentence suggest that the NRR really is interpreted within the scope of the existential modal. This can be seen by contrasting the truth conditions of (15)a with those of (15)a'-b': the latter imply

<sup>5</sup> Thanks to B. Spector for discussion of this and related examples.

that *if John had called his mother, she would have called her lawyer*; this, in turn, is unsurprising if the mood corresponding to *would* is interpreted like an E-type world pronoun, which picks out *those (relevant) worlds in which John calls his mother*. But no such effect is obtained in (15)a, where the NRR genuinely appears to be interpreted within the scope of the existential modal.

Since French subject relative clauses sometimes have a peculiar semantics (as in: *J'ai vu Jean qui courait* 'I saw Jean who ran', which means: *I saw Jean running*), it is worth checking that the same pattern holds with non-subject relative clauses; this is indeed the case, as is shown in (16).

- (16) a. Suppose que Jean ait épousé Anne, à qui il ait fait des enfants.  
*Suppose Jean had-subj married Anne, to whom he had-subj given her children.*  
 $\nRightarrow$  Jean has children  
 b. \*\* Suppose que Jean ait épousé Anne. Il lui ait fait des enfants.  
*Suppose Jean had-subj married Anne. He had-subj given her children.*

### 3.2 NPI Licensing

We also note that if we force a NRR to be in the scope of a conditional-like construction, which creates a downward-monotonic environment, NPIs can be licensed in the NRR – as one would expect if these are indeed interpreted within the scope of their licenser. Thus the NPI in (17)b is as acceptable as its non-NPI counterpart in (17)a. If the NRR is in the indicative, which forces the NRR to have wide scope, the facts change and the NPI becomes unacceptable, as is shown in (17)c.

- (17) A supposer que Jean ait parlé à Sarkozy,  
*Assuming (= if) Jean had-subj talked to Sarkozy,*  
 a. qui lui ait dit quelque chose d'intéressant,  
*who had-subj told him something interesting,*  
 b. qui lui ait dit quoi que ce soit d'intéressant,  
*who had-subj told him anything interesting,*  
 c. \*qui lui a dit quoi que ce soit d'intéressant,  
*who has told him anything interesting,*  
 il nous en aurait dit un mot.  
*he would have told us about it.*

\*\*\*

The preceding discussion has tried to establish two facts: (i) In some cases, NRRs are syntactically attached with very wide scope – which makes a separate semantic dimension unnecessary to account for their interpretation in such cases. (ii) In other cases, NRRs are syntactically attached *and interpreted* with narrow scope. A standard bidimensional semantics makes undesirable predictions in such cases. (The special pragmatics of NRRs, and the associated patterns of projection, will be discussed in Part II).

## References

- Amaral, P., Roberts, C., Allyn Smith, E.: Review of *The Logic of Conventional Implicatures* by Chris Potts. *Linguistics and Philosophy* 30, 707–749 (2007)
- Cinque, G.: Two Types of Non-Restrictive Relatives. In: Bonami, O., Cabredo Hofherr, P. (eds.) *Empirical Issues in Syntax and Semantics*, vol. 7, pp. 99–137 (2008)
- Del Gobbo, F.: *Appositives at the Interface*. PhD dissertation, University of California (2003)
- Harris, J.A., Potts, C.: Perspective-shifting with appositives and expressives. Ms., UMass Amherst and Stanford University (2009a)
- Harris, J.A., Potts, C.: Predicting Perspectival Orientation for Appositives. To appear in *CLS* 45 (2009b)
- Macià, J.: Presuposición y significado expresivo. *Theoria: Revista de Teoría, Historia y Fundamentos de la Ciencia* 3(45), 499–513 (2002)
- McCawley, J.D.: *The Syntactic Phenomena of English*, vol. 2. The University of Chicago Press, Chicago (1988)
- Potts, C.: *The Logic of Conventional Implicatures*. Oxford Studies in Theoretical Linguistics. Oxford University Press, Oxford (2005)
- Potts, C.: The expressive dimension. *Theoretical Linguistics* 33(2), 165–197 (2007) Published with commentaries by several researchers, and replies by Potts
- Potts, C.: Conventional implicature and expressive content. In: Maienborn, C., von Stechow, K., Portner, P. (eds.) *Semantics: An International Handbook of Natural Language Meaning*. DeGruyter (to appear)
- Potts, C., et al.: Expressives and Identity Conditions. *Linguistic Inquiry* 40(2), 356–366 (2009)
- Sauerland, U.: Beyond unpluggability. *Theoretical Linguistics* 33(2), 231–236 (2007)
- Schlenker, P.: A plea for monsters. *Linguistics and Philosophy* 26(1), 29–120 (2003)
- Schlenker, P.: Expressive presuppositions. *Theoretical Linguistics* 33(2), 237–245 (2007)
- Wang, L., Reese, B., McCready, E.: The projection problem of nominal appositives. *Snippets* 10, 13–14 (2005)

# Natural Logic and Semantics

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**Abstract.** Two of the main motivations for logic and (model-theoretic) semantics overlap in the sense that both subjects are concerned with representing features of natural language meaning and inference. At the same time, the two subjects have other motivations and so are largely separate enterprises. This paper returns to the topic of language and logic, presenting to semanticists *natural logic*, the study of logics for reasoning with sentences close to their surface form. My goal is to show that the subject already has some results that natural language semanticists might find interesting. At the same time it leads to problems and perspectives that I hope will interest the community. One leading idea is that the target logics for translations should have a decidable validity problem, ruling out first-order logic. I also will present a fairly new result based on the transitivity of comparative adjective phrases that suggests that in addition to ‘meaning postulates’ in semantics, we will also need to posit ‘proof principles’.

If we were to devise a logic of ordinary language  
for direct use on sentences as they come,  
we would have to complicate our rules of inference  
in sundry unilluminating ways.

W. V. O. Quine, *Word and Object*

## 1 Natural Logic

By *natural logic*, I mean the study of inference in natural language, done as close as possible to the “surface forms”. This work has various flavors and associated projects, and my goal in this short paper based on a presentation to the Amsterdam Colloquium is to present it to semanticists who know nothing about it. I would like to make the case that natural logic should be of interest in semantics, both the results that we have so far and the problems on the research agenda. I also want to comment at various points on the quote above from Quine, as just one example of an opinion that casts doubt on the whole enterprise of natural logic in first place.

My interest in the topic began in 2005 when I taught an introductory course in semantics for graduate students mainly from our linguistics department, with a few from philosophy and other subjects as well. One motivation for semantics

found in textbooks is that it should be the study of *inference in language*: just as syntax has grammaticality judgments to account for, semantics has inference judgments. Now I happen to be mainly a logician, and this point resonates with me as a motivation for semantics. But the semantics literature, it almost never gives a full account of *any* inferences whatsoever. It is seriously concerned with truth conditions and figuring out how semantics should work in a general way. But it rarely goes back and figures out, for various fragments, what the overall *complete stock of inferences* should be. I wanted to do just this, to introduce logic as another study of inference. In particular, I wanted to give examples of *completeness theorems* that were so elementary that they could be done *without the comparatively heavy syntax* of first-order logic.

Let me give an example of this, a real “toy.” Consider sentences *All X are Y*, where *X* and *Y* are plural nouns. This is a very tiny fragment, but there certainly are inferences among sentences in it. For example,

$$\begin{array}{ll} \frac{\text{All frogs are reptiles.}}{\text{All reptiles are animals.}} & \frac{\text{All sagatricians are maltnomans.}}{\text{All sagatricians are aikims.}} \\ \text{All frogs are animals.} & \text{All maltnomans are aikims.} \end{array}$$

The inference on the left is valid, of course, and the one on the right is invalid. On the right, I have made up the nouns to hammer home the point that the validity or non-validity is not a matter of the nouns themselves, but rather comes from the form *All X are Y*. For sentences in this fragment, we can give an exact semantics: interpret each noun *X* as a subset  $\llbracket X \rrbracket$  of an underlying universe *M*. This gives models. Given such a model, say  $\mathcal{M}$ , we say that *All X are Y* is *true in M* if  $\llbracket X \rrbracket \subseteq \llbracket Y \rrbracket$ . We can go on to define  $\Gamma \models S$ , for  $\Gamma$  a set of sentences and *S* a sentence, by saying that every model of all sentences in  $\Gamma$  is again a model of *S*. We can ask whether this semantics is adequate in the sense that intuitive judgments of valid inferences, presented in English, are matched by formal statements of the form  $\Gamma \models S$ . For this fragment, the semantics is basically adequate; the main issue with it is that sentences *All X are X* come out as valid even when the speaker knows or believes that there are no *X*s. But putting this aside, the semantics is adequate<sup>1</sup>. Further, one can go on and ask for a *proof-theoretic characterization* of the relation  $\Gamma \models S$ . Here, it turns out that one can build *proof trees* using the following rules:

$$\frac{}{\text{All } X \text{ are } X} \qquad \frac{\text{All } X \text{ are } Z \quad \text{All } Z \text{ are } Y}{\text{All } X \text{ are } Y} \qquad (1)$$

We write  $\Gamma \vdash S$  if there is a tree all of whose nodes are either labeled from  $\Gamma$  or else match one of the two rules above, and whose root is labeled by *S*. Then one has the following *completeness theorem*:

<sup>1</sup> By the way, one can also change the semantics to require that  $\llbracket X \rrbracket \neq \emptyset$  in order that *All X are Y* be true. One can make similar modifications to other semantics in the area. The point is that one can work with data provided by real people ignorant of logic and mathematics and then try to find logical systems for such data.

**Theorem 1** ([10]). *For all  $\Gamma$  and  $S$ ,  $\Gamma \models S$  if and only if  $\Gamma \vdash S$*

The completeness means that every valid semantic assertion is matched by a formal proof. Nothing is missing. This is not only the simplest completeness theorem in logic, but (returning to the motivations of semantics), it is a full account of the inferential behavior in a fragment. One would think that semanticists with a backgrounds in logic would have done this early on.

Then we can ask: given both a semantic account and a proof-theoretic account, why should we prefer the former? Why would we not say that the proof-theory *is* the semantics? After all, it covers the same facts as the semantic account, and it is an account of language use to boot. In addition, it is amenable to a computational treatment.

My suspicion is that inference as such is *not* what really drives semanticists. Just as getting the raw facts of grammaticality right is not the driving force for syntacticians, there are other matters at play. At the end of the day, one wants an explanation of how meaning works in language. And one wants a field that leads to interesting questions. Finally, there are all sorts of theory-internal questions that come up, and for semantics, these questions are not so close to the matter of inference.

In any case, I am interested in asking how far one can go with natural logic. A step up from the tiny fragment of *all* are the classical syllogisms. Here one can return to Aristotle, asking whether his system for *all*, *some*, and *no* (thought of as a formal system) is *complete*. The completeness of various formulations of syllogistic logic has already been shown, for example by in Łukasiewicz [8] (in work with Śłupecki), and the basic completeness result was also rediscovered by Westerståhl [22].

## 2 Objections to Natural Logic

I want to return to the quote from Quine at the beginning, and to put forth several reasons<sup>2</sup> why one might agree with it.

The logical systems mentioned in this section are discussed a bit more in Section 3 that follows.

*A. The logical systems that one would get from looking at inference involving surface sentences would contain many copies of similar-looking rules.* Presenting things in this way would miss a lot of generalizations.

For example, consider two rules:

$$\frac{\text{Some } X \text{ are } Y \quad \text{All } Y \text{ are } Z}{\text{Some } X \text{ are } Z} \qquad \frac{\text{All } X \text{ see some } Y \quad \text{All } Y \text{ are } Z}{\text{All } X \text{ see some } Z}$$

<sup>2</sup> These objections are my formulations. I would not want to give the impression that Quine or anyone else agreed with them. In another direction, I do have to wonder how anyone could see what logic for “sentences as they come” would look like without actually working it out.

The logic for the relational syllogistic contains both.

One way around this is to use notation that displays the monotonicity type,

$$\text{All } X^{\downarrow} \text{ see some } Y^{\uparrow}.$$

and then to employ *monotonicity rules* suggested by the notation. It was Johan van Benthem who first used this notation in [2]. His work, and work influenced by it, is an important source of results and inspirations in the area, but I lack the space to discuss it. Even more importantly, it was he and Sanchez [20] who first recognized the importance of for monotonicity rules for linguistic inference. So we see that using the arrow notation, or other “meta-rules”, we can say what we want. Nevertheless, for some more complicated systems it is an open issue to present them in the “optimally informative” way. Indeed, it is not even clear what the criteria for such presentations should be.

*B. The systems would contain ‘rules’ that are not really rules at all, but instead are more like complex deduction patterns that need to be framed as rules only because one lacks the machinery to break them down into more manageable sub-deductions.* Moreover, those complex rules would be unilluminating.

As an example, here is a rule from syllogistic logic allowing

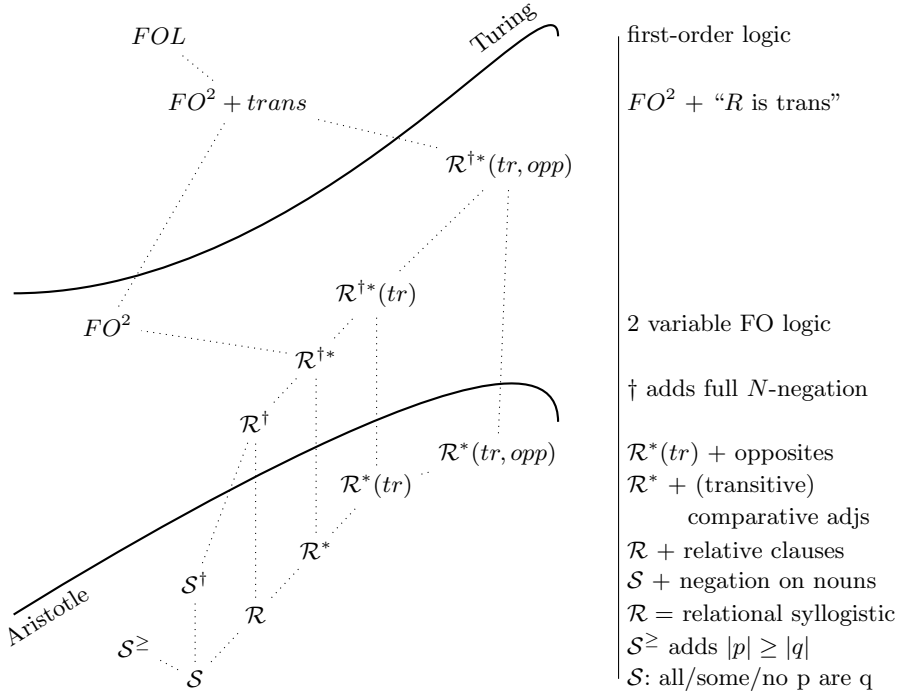
$$\frac{\text{All } X \text{ see all } A \quad \text{All } Y \text{ see all } Z \quad \text{All } Y' \text{ see all } A'}{\text{All } X \text{ see all } Z} \text{ 3pr}$$

Why is this rule sound? Well, take some  $X$ , say  $x$ . Then if this  $x$  is also a  $Y$ , then it sees all  $Z$ . Otherwise,  $x$  is a non- $Y$ . But then  $x$  sees all non- $A$ ’s. And since  $x$  was an  $X$ , it sees all  $A$  as well. Thus in this case  $x$  sees absolutely everything, a fortiori all  $Z$ .

This is not a familiar rule, and I could think of no better name for it than (3pr), since it has three premises. It is hard to take this to be a single rule, since it lacks the intuitively obvious status of some of the monotonicity rules. When presented to audiences or classes, hardly anyone believes that it is sound to begin with. Moreover, it really depends on the semantics of  $X'$  giving the *full* complement; so it might not even be the rule one always wants in the first place. But if one is committed to the semantics, one has to take it as a rule on its own because it cannot be simplified any further. In any case, I agree that the rule itself is probably not so illuminating.

I think that there a few rejoinders to this objection. First, with stronger fragments, these “weird” rules typically become provable because the informal reasoning behind them is expressible. Second a possible avenue of research could well be to *weight* the rules with numbers to model how hard it would be for a human to come up with a proof using the rule. (See Geurts [6] for a concrete proposal.) Presumably the rule (3pr) above would be weighted extremely high.

*C. The systems would lack variables, and thus they would be tedious and inelegant.*



Objection (C) is that systems for natural logic lack variables. It should be of interest in semantics to know exactly where variables really are needed, and to formulate logical systems that do involve variables. Work in the area suggests an answer. Also, it *is* possible to formulate logical systems with weak forms of variables that are (arguably) elegant and which are (provably) complete for their fragments.

*D. I would be impossible to handle quantifier-scope ambiguities in an elegant way.* It would take us too far afield in this short report to discuss this objection. But the literature contains syllogistic logics capable of handling the different scope readings in ambiguous sentences, and yet do not have much in the syntax besides the disambiguation: Nishihara et al. [13], and Moss [11].

My feeling overall is that all of these objections are to some extent apt, and to some extent miss the mark.

### 3 The Aristotle Boundary

Ian Pratt-Hartmann and I determined in [15] what I'll call the *Aristotle boundary*. This is the limit of how far one can go with purely syllogistic systems. That is, there is a formal definition of a "syllogistic proof system" in [15]; for the languages below the Aristotle boundary, we have syllogistic proof systems which match logical consequence; for the ones beyond the Aristotle boundary,

no syllogistic systems are possible. To discuss the languages a bit more we need some notation.

$\mathcal{S}$	classical syllogistic: <i>all/some/no X are Y</i>
$\mathcal{R}$	relational syllogistic: add transitive verbs to $\mathcal{S}$
$\mathcal{R}^*$	$\mathcal{R}$ , but also allowing subject NPs to be relative clauses
$\mathcal{R}^*(tr)$	add comparative adjective phrases, interpreted transitively
$\mathcal{R}^*(tr, opp)$	allows opposites of comparatives

In more detail, the syntax of  $\mathcal{R}$  is

<i>All X are Y</i>	<i>All X aren't Y</i> $\equiv$ <i>No X are Y</i>
<i>Some X are Y</i>	<i>Some X aren't Y</i>
<i>All X see all Y</i>	<i>All X don't see all Y</i> $\equiv$ <i>No X sees any Y</i>
<i>All X see some Y</i>	<i>All X don't see some Y</i> $\equiv$ <i>No X sees all Y</i>
<i>Some X see all Y</i>	<i>Some X don't see any Y</i>
<i>Some X see some Y</i>	<i>Some X don't see some Y</i>

$\mathcal{R}^*$  allows the subject noun phrases to contain relative clauses of the form

<i>who see all X</i>	<i>who see some X</i>
<i>who don't see all X</i>	<i>who don't see some X</i>

In  $\mathcal{R}^*(tr)$  we can say things like *All X are bigger than some Y*. We require that the interpretation of an adjective phrase such as *bigger than* be a transitive relation. This results in some sound principles; see (3) below. Adding opposites to  $\mathcal{R}^*(tr)$  gives the language  $\mathcal{R}^*(tr, opp)$ . In it, we require that opposites be modeled by the converse. Finally, the dagger  $\dagger$  notation attached to any of system increases that system by allowing full negation on nouns.

The figure summarizes many results: There are complete syllogistic systems for  $\mathcal{S}$  and  $\mathcal{S}^\dagger$ . There are no finite, complete syllogistic systems for  $\mathcal{R}$ . However, allowing *reductio ad absurdum*, there is a syllogistic system for  $\mathcal{R}$ . Even allowing *reductio ad absurdum*, there are no finite, complete systems for  $\mathcal{R}^\dagger$  or for  $\mathcal{R}^{*\dagger}$ .

These results begin to delimit the Aristotle boundary. It has much to do with negation, especially noun negation in connection with verbs.

Despite the negative results, the systems inside the *Turing boundary* are *decidable*. This means that in principle one could write a computer program to decide whether a purported inference was valid or not. The complete story here is that the complexity of the validity problem for these logics is known.

**Theorem 2 ([15]).** *The validity problems for  $\mathcal{S}$ ,  $\mathcal{S}^\dagger$ , and  $\mathcal{R}$  are complete for nondeterministic logspace; for  $\mathcal{R}^\dagger$ , it is complete for deterministic exponential time;  $\mathcal{R}^*$  for co-NPtime [9], and  $\mathcal{R}^{*\dagger}$  for nondeterministic exponential time.*

My feeling overall is that the Aristotle boundary should be of interest in semantics partly because of the prominence of *variables* in contemporary semantics: it would be good to pinpoint the features of language that necessitate going beyond a syllogistic presentation. This is what the results in [15] say. However, it should

1	John is a man	Hyp
2	Any woman is a mystery to any man	Hyp
3	J   Jane is a woman	Hyp
4	Any woman is a mystery to any man	R, 2
5	Jane is a mystery to any man	Any Elim, 4
6	John is a man	R, 1
7	Jane is a mystery to John	Any Elim, 6
8	Any woman is a mystery to John	Any intro, 3, 7

**Fig. 1.** An example from Fitch [5]

be noted that they do *not* say that one *must* use variables in the traditional way, only that one cannot do with a purely syllogistic presentation. In fact, one can also define logical systems for fragments like  $\mathcal{R}^*$  and  $\mathcal{R}^{*\dagger}$  which use something like variables, but with more restrictions. These restrictions correspond to the decidability of the system, a point which I return to in Section 4.

### 3.1 Fitch’s “Natural Deduction Rules for English”

I would like to digress and mention an early source on natural logic, Fitch [5]. This paper is not very well-known among people in the area, and I have seen few references to it by semanticists or anyone else for that matter. Frederic Fitch was one of the first people to present natural deduction proofs in what we call ‘Fitch style’; Stanisław Jaśkowski also did this. For a good discussion of the history, see Pelletier [14]. Fitch’s paper of 1973 presents a set of natural deduction rules for English. Figure 1 contains an example taken directly from his paper.

It should be noted that there is no formal syntax in the paper. His rules for *any* are thus ad hoc, and certainly there is more that one should say beyond his rules; they do show that he was aware of what we now call polarity phenomena. This lack of syntax is not terribly surprising, since he might not have known of Montague’s work. But in addition there is no formal semantics either. From the point of view of natural logic, one can return to Fitch’s paper and then ask whether his rules are complete. This question is open.

## 4 The Force of Decidability

I mentioned above that the Aristotle boundary should be of some interest in semantics. I want to end with discussion of the corresponding “Turing boundary”. This would be the boundary between decidable and undecidable fragments.

My feeling is that this boundary should be even more important to investigate. Formally-minded linguists should be more used to the rejection of undecidable frameworks following the Peters-Ritchie Theorem in formal language theory. There are certainly some who feel that semantics should make use of the strongest possible logical languages, presumably on the grounds that human beings can understand them anyways. But a wealth of experience stemming from computer science and cognitive science leads in the opposite direction. The feeling is that “everyday” deduction in language is not the same as mathematics, it might not call on the same mental faculty as deep reasoning in the first place. So one should investigate weak systems with an eye towards seeing what exactly can be said in them, before going on to more expressive but undecidable systems.

All of the logical systems mentioned so far in this paper have been decidable, including ones which need variables. (Incidentally, these fragments sometimes do not have the finite model property.) I am interested in finding yet stronger decidable fragments, and so this is how I end this paper. (For other work in the area, see Pratt-Hartmann [16,17] and Pratt-Hartmann and Third [18].) One source of such stronger systems is *comparative adjective phrases*, such as *bigger than*, *smaller than*, and the like. These are always interpreted by *transitive relations* on a domain:

If  $a$  is bigger than  $b$ , and  $b$  is bigger than  $c$ , then  $a$  is bigger than  $c$ . (2)

(The interpretations are also irreflexive: nobody is bigger than themselves. But this fact will not be relevant to our point in this section.) The transitivity corresponds to the validity of arguments like the following:

Every sweet fruit is bigger than every kumquat	(3)
Every fruit bigger than some sweet fruit is bigger than every kumquat	

That is, (3) is semantically valid, but only on the class of models which interpret *bigger than* by a transitive relation.

Now one might at first think that what we need is a logical system which directly expresses transitivity using variables in some version of (2). We are already heading towards the use of variables, so what is the problem with (2)? The hitch is that (2) uses *three variables*, and it is known that a logical system which can express all sentences in three variables is *undecidable*. Even more, a system which can express all of the *two-variable* sentences plus assertions of transitivity (as atomic sentences) is again undecidable, by a theorem of Grädel, Otto, and Rosen [7]. So if we believe that “simple” fragments of language should lead to decidable logics, then we cannot use a language which states (2) in a “first-class” way. Here is how this is done in [12]. The system uses variables, and also natural-deduction style rules. For transitivity, it uses

$$\frac{a(t_1, t_2) \quad a(t_2, t_3)}{a(t_1, t_3)} \text{ trans}$$

Here  $a$  is an adjective phrase (it will be **bigger** below), and the  $t$ ’s are *terms* (variables, roughly). The derivation corresponding to (3) is



2. van Benthem, J.: *Essays in Logical Semantics*. Reidel, Dordrecht (1986)
3. van Benthem, J.: *A Brief History of Natural Logic*. In: Chakraborty, M., Löwe, B., Nath Mitra, M., Sarukkai, S. (eds.) *Logic, Navya-Nyaya and Applications, Homage to Bimal Krishna Matilal*. College Publications, London (2008)
4. Englebretsen, G.: *Three Logicians*. Van Gorcum, Assen (1981)
5. Fitch, F.B.: *Natural Deduction Rules for English*. *Philosophical Studies* 24(2), 89–104 (1973)
6. Geurts, B.: *Reasoning with Quantifiers*. *Cognition* 86, 223–251 (2003)
7. Grädel, E., Otto, M., Rosen, E.: *Undecidability Results on Two-Variable Logics*. *Archive for Mathematical Logic* 38, 313–354 (1999)
8. Łukasiewicz, J.: *Aristotle's Syllogistic*, 2nd edn. Clarendon Press, Oxford (1957)
9. McAllester, D.A., Givan, R.: *Natural Language Syntax and First-Order Inference*. *Artificial Intelligence* 56, 1–20 (1992)
10. Moss, L.S.: *Completeness Theorems for Syllogistic Fragments*. In: Hamm, F., Kepser, S. (eds.) *Logics for Linguistic Structures*, pp. 143–173. Mouton de Gruyter, Berlin (2008)
11. Moss, L.S.: *Syllogistic Logics with Verbs*. *J Logic Computat* 20, 761–793 (2010)
12. Moss, L.S.: *Logics for Two Fragments Beyond the Syllogistic Boundary*. In: Blass, A., Dershowitz, N., Reisig, W. (eds.) *Studies in Honor of Yuri Gurevich*. LNCS. Springer, Heidelberg (2010)
13. Nishihara, N., Morita, K., Iwata, S.: *An Extended Syllogistic System with Verbs and Proper Nouns, and its Completeness Proof*. *Systems and Computers in Japan* 21(1), 760–771 (1990)
14. Pelletier, F.J.: *A Brief History of Natural Deduction*. *History and Philosophy of Logic* 20, 1–31 (1999)
15. Pratt-Hartmann, I., Moss, L.S.: *Logics for the Relational Syllogistic*. *Review of Symbolic Logic* 2(4), 647–683 (2009)
16. Pratt-Hartmann, I.: *A Two-Variable Fragment of English*. *J. Logic, Language and Information* 12(1), 13–45 (2003)
17. Pratt-Hartmann, I.: *Fragments of Language*. *J. Logic, Language and Information* 13, 207–223 (2004)
18. Pratt-Hartmann, I., Third, A.: *More Fragments of Language*. *Notre Dame J. Formal Logic* 47(2) (2006)
19. Purdy, W.C.: *A Logic for Natural Language*. *Notre Dame J. Formal Logic* 32(3), 409–425 (1991)
20. Sanchez Valencia, V.: *Studies on Natural Logic and Categorical Grammar*. Ph.D. thesis, Univ. of Amsterdam (1991)
21. Sommers, F.: *The Logic of Natural Language*. Clarendon Press, Oxford (1982)
22. Westerståhl, D.: *Aristotelian Syllogisms and Generalized Quantifiers*. *Studia Logica* XLVIII(4), 577–585 (1989)
23. Zamansky, A., Francez, N., Winter, Y.: *A Natural Logic Inference System Using the Lambek Calculus*. *J. Logic, Language and Information* 15(3), 273–295 (2006)

# NL from Logic: Connecting Entailment and Generation

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**Abstract.** The paper introduces an index of fully-specified logical form for unrestricted natural language. The index is argued to facilitate both computation of entailment and generation under semantic control.

## 1 Indexing Logical Form

The notion of entailment for unrestricted natural language is far from being logically validated. From a processing point of view, entailment is problematic at both the logical and the semantic side. As for the semantics: many – if not most - constructions of natural language lack a consolidated interpretation. As for the logic: even first-order representations of natural language are too rich to leave to theorem provers. In addition, Shieber has pointed out that equivalency for first-order logic being undecidable, controlled generation of (unrestricted) natural language from pure first-order logic constraints is not feasible ([15]). This means that reasoning in natural language by entailment using first-order logic cannot induce provably correct verbalization of the outcome.

It is not just a trend that in our days textual entailment is mostly computed by shallow means – not by full grammatical representations. The RTE challenges show that relatively shallow analysis combined with intelligent search and/or learning strategies may cover a fair deal of ‘common sense’ entailment ( *e.g.* [5], [16], [2]). Moreover, deepening the grammatical analysis does not improve the result by necessity ([3]). Yet, the constructional nature of unrestricted natural language calls for deep processing: the lexicon for processing is bound to be phrasal, to contain complex lambda-terms and to be submitted to subtle syntactic combinatorics in order to maintain phrasal semantic integrity. The phrasal lexicon is the source of all semantic wisdom, but in order to exploit it in processing, a lot of grammar is required. [7] argues convincingly that several tasks for natural language processing require structural semantic processing – deep structural semantic processing.

In this paper we propose a way of exploiting deeply processed and fully specified logical form for the sake of computing entailment for unrestricted natural language and for generation (*aka* realization). The target language of our system - [www.berekendnederlands.nl](http://www.berekendnederlands.nl) - is Dutch, but here, several issues will be illustrated here with English examples. The representations used are computable, and are supposed to enhance semantic inference from sentences and to feed into controlled generation. Entailment and generation are linked here since inferred statements are bound to be verbalized, and for this realization to be valid its logical form should entertain a logical relation to the input constraint. The problem – Shieber’s problem - is that to

check whether the generated sentence ‘covers’ the logical input constraint, the relation between its logical form and the input constraint must be computed, and that relation is not decidable or only semi-decidable in first-order logic.

In order to handle this problem without giving up the semantic information accumulated by deep processing of phrasal lexicons, *i.e.* without resorting to less expressive semantic representations, we propose a representation of logical form which

- facilitates semantic inference between natural language sentences
- can effectively restrain generation
- allows comparison of the input constraint and the logical form of a generated sentence.

This representation is called Flat Logical Form (FLF) and is central in the paper. In our system, FLF is derived from a purely semantic, fully-specified logical form. Neither this logical form nor its derivative FLF contains any language-specific information. Thus, if two sentences mean the same, their logical forms must be equivalent – which is undecidable – and their FLFs will converge; that is the ultimate goal of deep, construction based processing. Paraphrasing an example of [15], the following five sentences share one particular canonical and normalized semantic representation, to the extent that they share a meaning at all:

- (1)      Clapton led Derek and the Dominos  
             Derek and the Dominos was led by Clapton  
             The leader of Derek and the Dominos was Clapton  
             Clapton yazzâr ĩ Derek æd the Dominos  
             Clapton yāmôs amuzâr æn Derek æd the Dominos

According to the requirements above, their FLFs, when fed into appropriate generators for English and Tuareg respectively, should give rise to these, and maybe even more, sentences. Note that looking at logical form this way implies that it is underspecified in comparison to natural language sentences inducing it. This underspecification is inevitable, even when the logical form itself is fully specified from a logical point of view: the sentences 1 to 5 are neither equal nor equivalent, but their logical forms are.

Flat Logical Form is an index of fully-specified logical form LF (cf. [12] for a full description of its properties). The latter is the first-order spell-out of an underspecified storage of lambda-terms, Stored Logical Form (cf. [9]) This SLF, which we will not pursue here, is derived by unification of signs or complex symbols during the derivation. The FLF is derived by taking every single predicate term at LF and indexing every variable in that term for the way it is bound, for polarity and for the variables on which its valuation depends. Here are two examples.

- (2)      Every man invites a woman that he does not know  
             *standard first-order representation:*  
              $\forall x. [M(x) \Rightarrow \exists y. [W(y) \ \& \ I(x,y) \ \& \ \neg K(x,y)]]$   
             *logical form LF:*

$\exists e1 \exists e2 \exists e3 \exists e4 \forall x. [[\text{state}(e1, M) \ \& \ \text{th1}(e1, x)] \Rightarrow \exists y. [\text{state}(e2, W) \ \& \ \text{th2}(e2, y) \ \& \ \text{state}(e3, K) \ \& \ \neg[\text{th3}(e3, x) \ \& \ \text{th4}(e3, y)] \ \& \ \text{event}(e4, I) \ \& \ \text{th5}(e4, x) \ \& \ \text{th6}(e3, y)]]$

*index FLF:*

{ state(e1+↓+some+[], M),	th1(e1+↓+some+[], x+↓+all+[]),
state(e2+↑+some+[], W),	th2(e2+↑+some+[], y+↑+some+[x]),
state(e3+↓+some+[], K),	th3(e3+↓+some+[], x+↓+no+[]),
th4(e3+↓+some+[], y+↓+no+[x]),	event(e4+↑+some+[], I),
th5(e4+↑+some+[], x+↑+all+[]),	th6(e4+↑+some+[], y+↑+some+[x]) }

(3) Few monkeys dare to fly

*standard* – no first-order standard

*logical form LF:*

$\exists e1 \exists e2 \exists e3 \text{ Few}(x). [\text{state}(e1, M) \ \& \ \text{th1}(e1, x) \ \& \ \text{state}(e2, D) \ \& \ \text{th2}(e2, x) \ \& \ \text{ty}. [\text{property}(y) \ \& \ \text{th3}(e2, y) \ \& \ \text{th5}(y, e3) \ \& \ \text{event}(e3, F) \ \& \ \text{th4}(e3, x)]]$

*index FL:*

{ state(e1+↓+some+[], M),	th1(e1+↓+some+[], x+↑+few+[]),
state(e2+↑+some+[], D),	th2(e2+↑+some+[], x+↑+few+[]),
property(y+↑+the+[e2]),	th3(e2+↑+some+[], y+↑+the+[]),
event(e3+↑+some+[y], F),	th4(e3+↑+some+[y], x+↑+few+[]),
th5(y+↑+the+[e2], e3+↑+some+[y])	}

In FLF, variables are formatted as a quadruple *Variable* + *Monotony* + *Quantifier* + *Governors*. In this index, *Monotony* yields a value for upward or downward entailment for that variable with respect to its predicate, *Quantifier* identifies the binding regime and *Governors* is a (possibly empty) list of variables the valuation of which co-determines *Variable's* valuation, the parameters of the choice function for *Variable*.

Of course, the construction of LF involves numerous decisions on the semantics of natural language, many of which are not consolidated (cf. [7]). These decisions, like introducing intensional domains in (3) or assigning wide scope to state and event quantifiers, are not at stake here: whatever clause occurs in LF is indexed at FLF. The index is computed along with LF, and can be exploited for computing entailment between sentences and sentence generation.

The usefulness of the FLF index on logical form resides in its permutability. Given an FLF, one can select the subset of clauses that share a particular variable. Call this subset a *net*. For each net, a normal form can be constructed, *e.g.* by some ordering of the predicates involved. Here is the complete network for the FLF of (2):

- (4) *e1-net:* < state(e1+↓+some+[], M), th1(e1+↓+some+[], x+↓+all+[]) >  
*e2-net:* < state(e2+↑+some+[], W), th2(e2+↑+some+[], y+↑+some+[x]) >  
*e3-net:* < state(e3+↑+some+[], K), th3(e3+↑+some+[], x+↑+no+[]),  
th4(e3+↑+some+[], y+↑+no+[x]) >  
*e4-net:* < event(e4+↑+some+[], I), th5(e4+↑+some+[], x+↑+all+[]),  
th6(e4+↑+some+[], y+↑+some+[x]) >

*x-net:* < th1(e1+↓+some+[], x+↓+all+[]), th3(e3+↑+some+[], x+↑+no+[]),  
 th5(e4+↑+some+[], x+↑+all+[]) >  
*y-net:* < th2(e2+↑+some+[], y+↑+some+[x]), th4(e3+↑+some+[],  
 y+↑+no+[x]), th6(e4+↑+some+[], y+↑+some+[x]) >

The effort needed to construct the network of the FLF index is proportional to the number of variables in the LF. The network is the anchor for computation of entailment and for generation. It seems a characteristic property of natural language Logical Form, reflecting the semantic coherence of sentences, that in its FLF every variable defining a net occurs in at least one other net. It is worthwhile to point out this form of connectivity, as a conjecture.

(5) *Connectivity conjecture for FLF*

Given the LF of a well-formed and meaningful sentence *S* and the network representing it, every indexed variable that defines a net occurs in at least one other net as an indexed variable

## 2 Approaching Entailment

Our strategy is to approach the modal notion *S entails T* for Dutch sentences *S* and *T* by inspecting their respective FLF networks. We define a notion of *l-coverage* for FLF networks and suggest that *l-coverage* of FLFs is a sufficient condition for entailment.

(6) For Dutch sentences *S* and *T*, *S entails T* if FLF(*S*) *l-covers* FLF(*T*).

In order to compare the FLFs, we must assume that their respective networks can be compared *salve* alphabetic variance. This is far from trivial, but we will not pursue this here, since this problem is more general; we simply assume that an educated guess as to the line-up of variables in the two FLFs can be made.

Under this assumption, we first define the notion of *i-coverage* for individual FLF-clauses  $\phi$  and  $\psi$ . Below is a tentative inventory of its instances; each instance is illustrated with a natural language entailment relation in which that instance of *i-coverage* is supposed to be the decisive licensing factor – the phrases involved are in small caps. In this definition we take  $\langle \phi, \psi \rangle$  to be a *subnet* – the clauses share a variable by definition.  $\sigma$  stands for two-place relations between an indexed variable and a conceptual constant, like *state* and *event*;  $\tau$  abbreviates all relations between indexed variables, like thematic roles. Furthermore, we assume that between concepts and predicative constants a subsumption order may be defined, by meaning postulates and/or by ontologies such that  $X\downarrow \sqsubseteq X\uparrow$  means that  $X\downarrow$  is at least as specific as  $X\uparrow$ . Moreover, lists of governing variables are supposed to be in alphabetic variance when checked; this is indicated by the notation  $F/F'$ . Finally, an *intensional net* is a net which is headed by an intensional predicate like *proposition/1* or *property/1* the defining variable of which is dependent on some other variable (cf. the variable *y* in (3)).

(7) *I-COVERAGE*

- (a)  $\phi$  *i-covers*  $\psi$  if  $\phi$  and  $\psi$  are alphabetic variants. (identity)
- (b)  $\phi[F]$  *i-covers*  $\psi[F']$  if  $\phi$  and  $\psi$  are alphabetic variants except for the list of parameters  $F$  and  $F'$ , and  $F-F'$  does not contain variables that define an intensional net in the FLF of the covering clause and  $F'-F$  does not contain variables that define a net in the FLF of the covered clause. (strengthening)  
 (These men kissed A WOMAN *entails* SOME WOMAN is kissed;  
 He said that A MAN died *not-entails* A MAN died;  
 SOME WOMAN kissed him *not-entails* These men were kissed by A WOMAN)
- (c)  $\langle \sigma(A, C), \tau(A, x+\uparrow+Q+F) \rangle$  *i-covers*  $\langle \sigma(A, C\uparrow), \tau(A, y+\uparrow+Q+F') \rangle$  if  $C\uparrow$  is defined by subsumption (upward coverage )  
 (Every candidate is GERMAN *entails* Every candidate is EUROPEAN)
- (d)  $\langle \sigma(A, C), \tau(A, x+\downarrow+Q+F) \rangle$  *i-covers*  $\langle \sigma(A, C\downarrow), \tau(A, y+\downarrow+Q+F') \rangle$  if  $C\downarrow$  is defined by subsumption. (downward coverage )  
 (At most three candidates are EUROPEAN *entails* At most three candidates are DUTCH)
- (e)  $\langle \sigma(A, C), \tau(A, x+\downarrow+Q+F) \rangle$  *i-covers*  $\langle \sigma(A, C), \tau(A, y+\downarrow+Q+F'), \tau'(A, y+\uparrow+Q'+[]) \rangle$  if  $\tau'$  does not occur in the covering net. (downward extension)  
 (NO WOMAN GAVE flowers *entails* NO WOMAN GAVE flowers TO ME)
- (f)  $\langle \sigma(A, C), \tau(A, x+\uparrow+Q+F) \rangle$  *i-covers*  $\sigma(A, C)$ . (upward reduction)  
 (John ATE AN APPLE *entails* John ATE)
- (g)  $\tau(A, x+M+Q+F)$  *i-covers*  $\tau(A, y+M+some+F')$  unless  $Q = no$ . (existential impact)  
 (THAT STUDENT read a book *entails* SOME STUDENT read a book;  
 FEW STUDENTS read a book *entails* SOME STUDENT read a book)
- (h)  $\sigma(e1+\uparrow+Q+F, C)$  *i-covers*  $\sigma(e2+\uparrow+Q+F', C\uparrow)$  if  $C\uparrow$  is defined by subsumption. (upward concept)  
 (She is A GIRL *entails* She is FEMALE)
- (i)  $\sigma(e1+\downarrow+Q+F, C)$  *i-covers*  $\sigma(e2+\downarrow+Q+F', C\downarrow)$  if  $C\downarrow$  is defined by subsumption. (downward concept)  
 (At most three students WERE ILL *entails* At most three students HAD THE FLU)
- (j)  $\tau(A, x+\uparrow+Q+F)$  *i-covers*  $\tau\uparrow(A, y+\uparrow+Q+F')$  if  $\tau\uparrow$  is defined by subsumption. (upward relation)  
 (THIS MAN hit the woman *entails* THE MAN was involved with the woman)
- (k)  $\tau(A, x+\downarrow+Q+F)$  *i-covers*  $\tau\downarrow(A, y+\uparrow+Q+F')$  if  $\tau\downarrow$  is defined by subsumption. (downward relation)  
 (NO MEN met this woman *entails* NO MEN hit this woman)
- (l) if  $\langle \phi, \psi \rangle$  *i-covers*  $\langle \phi', \psi' \rangle$ ,  $\phi$  *i-covers*  $\phi'$  and  $\psi$  *i-covers*  $\psi'$ . (distribution)

This list of elementary coverings is rather tentative, but the ambition may be clear: the relation between FLFs can be constructed from more elementary relations at the level of nets and subnets. In linguistic terms, this means that we try to reduce, by using the FLF index, the relation between (meaning of) sentences to a relation between (meaning of) phrases, notwithstanding the complex logical and syntactic interaction between these phrases. On this basis, we can define a notion *n-coverage* for the relation between nets.

- (8) a net  $N$  is *n-covered* by a net  $M$  iff every clause in  $N$  is *i-covered* by  $M$ .

*l*-coverage needs to be almost functional in this definition: in the list of pairs of i-covering and i-covered subnets, no subnet can occur in an i-covering position more than once, with the possible exception of applications of *downward extension* (e).

This leads to the following notion of *l*-coverage between FLFs.

(9) ***L-COVERAGE***

An FLF  $\Phi = \langle \varphi_1, \dots, \varphi_k \rangle$  *l*-covers an FLF  $\Psi = \langle \psi_1, \dots, \psi_m \rangle$  iff for every network  $N_\Psi = \langle \psi^1 \dots \psi^n \rangle$  in  $\Psi$  there is a exactly one (possibly empty) network  $N_\Phi = \langle \varphi^1 \dots \varphi^m \rangle$  that *n*-covers it.

In order to back conjecture (6), which introduces *l*-coverage as a sufficient condition for entailment, we argue that we treat the FLF index essentially as a conjunction of independent clauses. FLF abstracts away from any non-commutative connective at LF, in particular from the implication. Although FLF-indexing itself is non-reversible, one observation is relevant. Conjunction reduces to implication if we neglect *ex falso* interpretation of implication in the combinatory semantics of natural languages. Seuren suggests in [14] that the cognitive-pragmatic construal of natural language does not leave space for zero-valued antecedents. Following his lead, we assume that phrasal semantics does not induce *ex falso* interpretation – in natural language, the universal quantifier has existential impact. Under this assumption, the ‘conjunctive’ logic of FLF as presented above, is more restrictive than the logic of LF, which stands for the relation between sentences. In that case, the conjecture (6) seems a fair and cautious approximation of entailment in natural language.

### 3 The Generator: From Flat Logical Form to Sentences

Entailment is a relation between natural language sentences. That relation can be judged by language users: natural language semantics is an empirical art when founded on entailment ([6]). At the end of the day, it is not the relation between formal representations but the informal relation between sentences that can and must be tested. To lead entailment and reasoning back to natural language processing, we should be able to generate from those logic representations that are exploited for reasoning and inference. We now present a non-deterministic procedure to generate Dutch sentences with a predefined, fully specified formal meaning. The procedure is grafted on the Delilah parser and generator (<http://www.berekendnederlands.nl>, [12]). The input to the procedure is an FLF index, as presented above. This index formula contains semantic information only. The output is a well-formed Dutch sentence with a full grammatical representation, providing again a formula in LF and its index FLF. The main characteristics of the procedure are:

- the input constraint is not biased towards the syntax or the lexicon;
- the generation procedure is non-deterministic, but finite;
- the result can be logically validated: input and output semantics are formulas in the same language

Here, we describe a method to relate FLF to the lexicon and the (categorical) grammar by exploiting the semantic nets induced by it, exemplified in (4). These networks are

shown to be able to steer lexical selection and grammatical unification. *L-coverage* (9) is applied to validate the result.

The Delilah system entertains a generator which is powered by a multimodal combinatory categorial grammar of Dutch, dubbed Minimal Categorical Grammar (MCG) in [8]. The combinatorics of MCG are controlled by a limited number of compositional modalities, not unlike the modalities proposed in [11] for Lambek categorial grammars and by [1] for Combinatory Categorical Grammar. Both in parsing and in generation, this grammar steers the unification of complex symbols. The unified graph is the main derivational result. An underspecified semantic representation emanates from this unification ([9]), which is fed into spell-out. The resulting full logical form result is the main outcome of both parsing and generation.

The generator is hypothesis-driven: it tries to construct a well-formed and meaningful phrase of a given category, with a complete parse in the form of a unified graph representing a complex symbol. The generation procedure is strictly meaning-driven: it operates without any structural preconditions, unlike the logical form-driven generators described in [4] and [17]. In these ‘realizers’, as in the famous Rosetta translation system ([13]), the input to the generation procedure contains essential pieces of structural information relevant to the output. Here, we propose a purely semantic way of constraining the realization, as our input constraint completely abstracts from syntax (see the discussion of the sentences (1)). It proceeds by selecting appropriate phrases from the lexicon after inspecting an agenda and by testing their unification. The generation succeeds if the hypothesis can be checked, no item is left on the agenda and some non-empty structure has been created.

Basically, the algorithm tries to find templates and to unify them according to an agenda which is set by an initial hypothesis and updated by applying combinatory categorial rules. The agenda consists of two parts: *given*, storing complex symbols already adopted, and *to find*, listing structures still to be checked. A successful unification of complex symbols according to the agenda is the proper result of the procedure.

An FLF offered to the generator, is ‘chunked’ into nets (cf. (4)). Below, is an example of the set of clauses in the FLF of the Dutch sentence *Elke vrouw probeerde te slapen* ‘Each woman tried to sleep’.

- (10) { state( $e1+\downarrow+\text{some}+[]$ , *woman*), th1( $e1+\downarrow+\text{some}+[]$ ,  $x+\downarrow+\text{every}+[]$ ),  
tense( $e2+\uparrow+\text{some}+[]$ , *past*), event( $e2+\uparrow+\text{some}+[]$ , *try*),  
th2( $e2+\uparrow+\text{some}+[]$ ,  $x+\uparrow+\text{every}+[]$ ), property( $y+\uparrow+\text{the}+[e2]$ ),  
th3( $e2+\uparrow+\text{some}+[]$ ,  $y+\uparrow+\text{the}+[e2]$ ), event( $e3+\uparrow+\text{some}+[y]$ , *sleep*),  
th4( $e3+\uparrow+\text{some}+[y]$ ,  $x+\uparrow+\text{every}+[]$ ), th5( $y+\uparrow+\text{the}+[e2]$ ,  $e3+\uparrow+\text{some}+[y]$ ) }
- (11) *e1-net*: <state( $e1+\downarrow+\text{some}+[]$ , *woman*), th1( $e1+\downarrow+\text{some}+[]$ ,  $x+\downarrow+\text{every}+[]$ )>  
*e2-net*: <event( $e2+\uparrow+\text{some}+[]$ , *try*), th2( $e2+\uparrow+\text{some}+[]$ ,  $x+\uparrow+\text{every}+[]$ ),  
th3( $e2+\uparrow+\text{some}+[]$ ,  $y+\uparrow+\text{the}+[e2]$ ), tense( $e2+\uparrow+\text{some}+[]$ , *past*)>  
*e3-net*: <event( $e3+\uparrow+\text{some}+[y]$ , *sleep*), th4( $e3+\uparrow+\text{some}+[y]$ ,  
 $x+\uparrow+\text{every}+[]$ ), th5( $y+\uparrow+\text{the}+[e2]$ ,  $e3+\uparrow+\text{some}+[y]$ )>  
*x-net*: < th1( $e1+\downarrow+\text{some}+[]$ ,  $x+\downarrow+\text{every}+[]$ ), th4( $e3+\uparrow+\text{some}+[y]$ ,  
 $x+\uparrow+\text{every}+[]$ ), th2( $e2+\uparrow+\text{some}+[]$ ,  $x+\uparrow+\text{every}+[]$ )>  
*y-net*: < property( $y+\uparrow+\text{the}+[e2]$ ), th3( $e2+\uparrow+\text{some}+[]$ ,  $y+\uparrow+\text{the}+[e2]$ )>

Each of the nets in (11) is taken to be a sequence of simultaneous conditions on the semantics of some lexical phrase. Together, the nets determine the lexical space for a generation process. Per net, all the candidates, in their lexical quality of complex symbols are selected. That being done, the generator tries to produce a grammatical construct in which each net is represented exactly once. The agenda for this process is derived from the contingent syntactic properties of the candidates.

The conceptual agenda raised by the FLF network strictly limits the freedom of the generator. Even when the available lexical space is extended by allowing purely functional additions, infinite looping is excluded under the cancellation agenda. On backtracking, the generator will produce all and only those sentences that dwell in the lexical space constructed by the input network and are reachable by the grammar.

Yet, this generation procedure from FLF is essentially non-deterministic in at least two senses:

- the (structure of the) FLF is not determining of the sentence, by definition of FLF;
- the output-FLF may not match the input-FLF according to a certain semantic standard.

FLF (like LF) underdetermines not just the syntax, *e.g.* the word order, of sentences generated in this way. Because of this ‘reverse underspecification’ – LF and FLF are not the outcome of a derivation but the spell-out of an underspecified unification result – the generation procedure cannot determine all the characteristics of the produced semantics in the logical space in advance or *on the fly*. There are two sources for this flaw:

- FLF may itself contain less specifications (aspect, mood, tense, ...) than any verbalization would introduce;
- inspection of the input-FLF cannot predict which logical dependencies between variables are blocked or enhanced by following a certain construction mode for the sentence.

The first aspect of generative underspecification is evident: one cannot be sure that an FLF contains all the information that a full sentence will produce by default. Natural language is meant to be more expressive than any logical representation. Complex symbols may introduce additional meanings to those mentioned in the conceptual agenda, *e.g.* by default specifications like tense on finite verbs. The concepts in the input are a subset of those in the output. Moreover, the input FLF, when constructed by reasoning, may not specify semantic dependencies that are inherent to sentential construal, like intensional inbedding.

The second incongruence between input and output FLF is due to the form-driven nature of sentence meaning. Whether or not a certain operator can have scope over another, depends partly, if not mainly, on its embedding. The generation process may isolate an operator on a strong or weak island, by choosing certain phrases or certain syntactic patterns for the respective nets (cf. [10]). For example, a quantifier embedded in a nominal construction (*to make the promise that ....*) has fewer scope options than a quantifier embedded in a non-nominal, but conceptually equivalent construction (*to promise that ...*). In the same vain, intensional domains are not

completely predictable from inspecting an FLF. Generally, weak and strong islands of any sort are induced by syntax, and the syntax is underspecified, by definition and inevitably. Consequently, the generation procedure cannot be enriched with an additional agenda controlling possible scopal dependencies. Scope can only be checked or compared *post hoc*.

Since FLF – in fact, every purely semantic logical form – contains too little information to fully determine the generation procedure, generating from logic is a non-deterministic trial, by necessity. The outcome of the process can or must be checked against the input constraint. It is important to realize, however, that the input constraint and the output FLF may differ in only a limited number of ways. For example, the output may contain concepts that are not present in the input, but only if these concepts are introduced by default, when applying certain complex symbols and if they passed the restrictions on unification imposed by the semantic networks. The output is far from being in free variation with the input.

As was argued above, it is unwise to check for strict identity or equivalence of input and output FLFs. Taking into account the considerations given above with respect to the 'inverse underspecification', we propose that the normal check would be as in

- (12) Accept S with OutputFLF as a generated translation of InputFLF into Dutch if OutputFLF *l-covers* InputFLF.

Informally, this means that the generated sentence is at least as specific as the input, or that a model for OutputFLF is also a model for InputFLF, but not necessarily the other way around. Again, it must be noted that the conceptual difference between InputFLF and OutputFLF will be very limited, given the restriction of the lexical resources to those induced by the semantic nets of InputFLF.

## 4 Conclusion

Accepting the post-derivational check (12), the original approximation (6) of natural language entailment in deep processing can be generalized to

- (13) For sentences S and T, S *entails* T if FLF(S) *l-covers* FLF F, T can be generated from F and FLF(T) *l-covers* F.

But as for results in natural language processing, there is always Gauß meeting with the diplomat Wilhelm von Humboldt in the early 19th century, according to Daniel Kehlman's *Die Vermessung der Welt*; Von Humboldt starts masochistically.

*...Er sei übrigens auch Forscher ! (...) Er untersuche alte Sprache.*

*Ach so, sagte Gauß.*

*Das, sagte der Diplomat, habe enttäuscht geklungen.*

*Sprachwissenschaft. Gauß wiegte den Kopf. Er wolle ja keinem zu nahe treten.*

*Nein, nein. Er solle es ruhig sagen.*

*Gauß zuckte die Achseln. Das sei etwas für Leute, welche die Pedanterie zur Mathematik hätten, nicht jedoch die Intelligenz. Leute die sich ihre eigene notdürftige Logik erfänden.*

*Der Diplomat schwieg.*

## References

1. Baldridge, J., Kruijff, G.-J.M.: Multi-modal Combinatory Categorical Grammar. In: Proceedings of the 10th Annual Meeting of the European Association for Computational Linguistics, pp. 211–218 (2003)
2. Bobrow, D., Condoravdi, C., Crouch, R., de Paiva, V., Karttunen, L., King, T., Nairn, L., Price, L., Zaenen, A.: Precision focused Textual inference. In: Proceedings of the workshop on textual entailment and Paraphrasing, pp. 16–21. ACL (2007)
3. Bos, J., Merkert, K.: When logical inference helps determining textual entailment (and when it doesn't). In: Proceedings 2<sup>nd</sup> Pascal RTE Challenge Workshop (2006)
4. Carroll, J., Copestake, A., Flickinger, D., Poznanski, V.: An Efficient Chart Generator for (semi-)Lexicalist Grammars. In: Proceedings of the 7th European Workshop on Natural Language Generation (EWNLG 1999), pp. 86–95 (1999)
5. Chambers, N., Cre, D., Grenager, T., Hall, D., Kiddon, C., MacCartney, B., de Marneffe, M., Ramage, D., Yeh, E., Manning, C.: Learning Alignments and Leveraging Natural Logic. In: Proceedings of the workshop on textual entailment and Paraphrasing, pp. 165–170. ACL (2007)
6. Chierchia, G., McConnel-Ginet, S.: Meaning and Grammar. MIT Press, Cambridge (2000)
7. Cooper, R., Crouch, D., van Eijck, J., Fox, C., van Genabith, J., Jaspers, J., Kamp, H., Milward, D., Pinkal, M., Poesio, M., Pulman, S.: Using the Framework. Technical Report Fracas Consortium, LRE 62-051 (1996)
8. Cremers, C.: On Parsing Coordination Categorially. Leiden University, HIL dissertations (1993)
9. Cremers, C., Reckman, H.: Exploiting logical forms. In: Verberne, S., Van Halteren, H., Coppen, P.-A. (eds.) Computational Linguistics in the Netherlands 2007. LOT, pp. 5–20 (2008)
10. Honcoop, M.: Dynamic excursions on weak islands. LOT (1998)
11. Moortgat, M.: Categorical Type Logics. In: Van Benthem, J., Ter Meulen, A. (eds.) Handbook of Logic and Language, pp. 93–177. Elsevier/Amsterdam, The MIT Press/Cambridge (1997)
12. Reckman, H.: Flat but not shallow. Towards flatter representations in deep semantic parsing for precise and feasible inferencing. LOT (2009)
13. Rosetta, M.T.: Rosetta. In: Compositional Translation. Kluwer, Dordrecht (1994)
14. Seuren, P.: The natural logic of language and cognition. *Pragmatics* 16(1) 16(1), 103–138 (2006)
15. Shieber, S.: The Problem of Logical-Form Equivalence. *Computational Linguistics* 19(1), 179–190 (1993)
16. Tatu, M., Boldovan, D.: COGEX at RTE3. In: Proceedings of the workshop on textual entailment and Paraphrasing, pp. 22–27. ACL (2007)
17. White, M., Baldridge, J.: Adapting Chart Realization to CCG. In: Proceedings Ninth European Workshop on Natural Language Generation, EACL (2003)

# An Analytic Tableau System for Natural Logic

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**Abstract.** In this paper we develop the beginnings of a tableau system for natural logic, the logic that is present in ordinary language and that is used in ordinary reasoning. The system is based on certain terms of the typed lambda calculus that can go proxy for linguistic forms and which we call Lambda Logical Forms. It is argued that proof-theoretic methods like the present one should complement the more traditional model-theoretic methods used in the computational study of natural language meaning.

## 1 Introduction

Logic has its roots in the study of valid argument, but while traditional logicians worked with natural language directly, modern approaches first translate natural arguments into an artificial language. The reason for this step is that some artificial languages now have very well developed inferential systems. There is no doubt that this is a great advantage in general, but for the study of natural reasoning it is a drawback that the original linguistic forms get lost in translation. An alternative approach would be to develop a general theory of the natural logic behind human reasoning and human information processing by studying formal logics that operate directly on linguistic representations. That this is possible we will try to make plausible in this paper. It will turn out that one level of representation, that of Logical Form, can meaningfully be identified with the language of an existing and well-understood logic, a restricted form of the theory of types. It is not difficult to devise inference systems for this language, and it is thus possible to study reasoning systems that are based directly on language.

We will define a tableau system and will place in focus tableau rules that are connected with certain properties of operators that seem important from a linguistic point of view. Our aim will not so much be to provide a proof system that is complete with respect to the semantics of our representations, but to provide rules that can be argued to be natural. The paper's purpose, therefore, is to contribute to the field of *natural logic*.<sup>1</sup>

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<sup>1</sup> Early contributions to natural logic are [16] and [22]. The research line we base ourselves upon is exemplified in [10][13][42][9][5][12][24][17][18].

## 2 Lambda Logical Forms

For our purpose it will be of help to have representations of natural language expressions that are adequate both from a linguistic and from a logical point of view. At first blush, this may seem problematic, as it may be felt that linguistics and logic require completely different and competing properties from the representations they use, but in fact the typed lambda calculus provides what we need, or at least a good approximation to it. In order to obtain a class of terms with linguistic relevance we will restrict attention to those (simply typed) lambda terms that are built up from variables and *non-logical* constants, with the help of application and lambda abstraction and will delimit this class further by the restriction that only variables of individual type are abstracted over. The resulting terms, which will be called *Lambda Logical Forms* (LLFs), are often very close to linguistic expressions, as the following examples illustrate.

- (1) a.  $((a \text{ woman})\text{walk})$   
 b.  $((\text{if}((a \text{ woman})\text{walk}))((no \text{ man})\text{talk}))$   
 c.  $(\text{mary}(\text{think}((\text{if}((a \text{ woman})\text{walk}))((no \text{ man})\text{talk}))))$   
 d.  $((a \text{ woman})(\lambda x(\text{mary}(\text{think}((\text{if}(\text{walk } x))((no \text{ man})\text{talk}))))))$   
 e.  $(\text{few man})\lambda x.(\text{most woman})\lambda y.\text{like } xy$

The terms in (1) were built up in the usual way, but no *logical* constants, such as  $=, \forall, \exists, \rightarrow, \wedge, \vee, \neg$  and the like, were used in their composition. The next section will make a connection between some of the non-logical constants used in (1) and logical ones, but this connection will take us from natural representations of linguistic expressions to rather artificial ones. Lambda terms containing no logical constants will therefore continue to have a special status.

Lambda Logical Forms come close to the Logical Forms that are studied in generative grammar. For example, in (13) trees such as the one in (2a) are found, strikingly similar to the  $\lambda$ -term in (2c).

- (2) a.  $[_S[_{DP} \text{ every linguist}][1[_S \text{ John}[_{VP} \text{ offended } t_1]]]]$   
 b.  $((\text{every linguist})(\lambda x_1(\text{john}(\text{offend } x_1))))$

## 3 A Natural Logic Tableau System

In this section we will discuss a series of rules for a tableau system directly based on LLFs. While tableau systems usually only have a handful of rules (roughly two for each logical operator under consideration), this system will be an exception. There will be many rules, many of them connected with special classes of expressions. Defining a system that comes even close to adequately describing what goes on in ordinary language will be a task far greater than what can be accomplished in a single paper and we must therefore contend ourselves with giving examples of rules that seem interesting. Further work should lead to less incomplete descriptions. Since the rules we consider typically are connected

to some algebraic property or other (such as monotonicity or anti-additivity—see below), it will also be necessary to specify to which class of expressions each rule applies. Describing exactly, for example, which expressions are monotone increasing in any given language requires a lot of careful linguistic work and for the moment we will be satisfied with providing examples (here: **some**, **some** N, **every** N, **many** N, and **most** N).

Familiarity with the method of tableaux will be assumed.

### 3.1 Tableau Entries

We will work with signed tableaux in which entries are formed by the rule that if  $A$  is an LLF of type  $\langle \vec{\alpha} \rangle$  and  $\vec{C}$  is a sequence of constants or LLFs of types  $\vec{\alpha}$ , then  $T\vec{C} : A$  and  $F\vec{C} : A$  are tableau entries.

An entry  $T\vec{C} : A$  ( $F\vec{C} : A$ ) intuitively states that  $A\vec{C}$  is true (false).

### 3.2 Closure

It will be assumed that the lexicon provides us with certain primitive entailment relations, such as **lark**  $\leq$  **bird** and **no**  $\leq$  **few**. A tableau branch is *closed* if it either contains both  $T\vec{C} : A$  and  $F\vec{C} : A$  or contains  $T\vec{C} : A$  and  $F\vec{C} : B$ , where  $A \leq B$  is lexical knowledge in this way.

A tableau is closed if all its branches are closed.

### 3.3 Rules Deriving from the Format

The format we have chosen itself validates some rules. First, we are only interested in LLFs up to  $\beta\eta$  equivalence and lambda conversions can be performed at will. Second, the  $X\vec{C} : A$  format (where  $X$  is  $T$  or  $F$ ) validates the following rules.

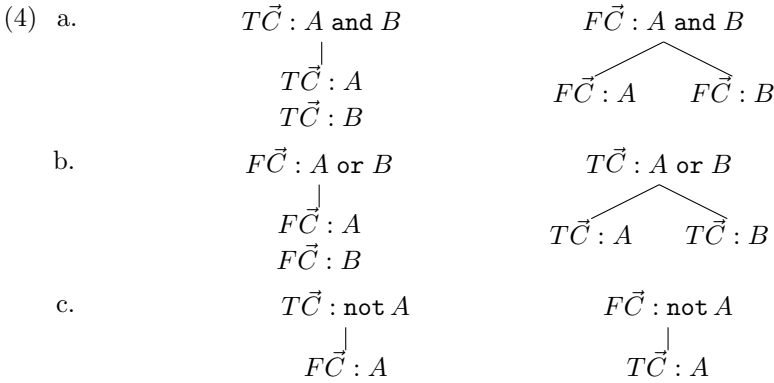
$$(3) \quad \begin{array}{ccc} X\vec{C} : AB & & XB\vec{C} : A \\ \downarrow & & \downarrow \\ XB\vec{C} : A & & X\vec{C} : AB \end{array}$$

So we can shift arguments to the front and shift them back again.

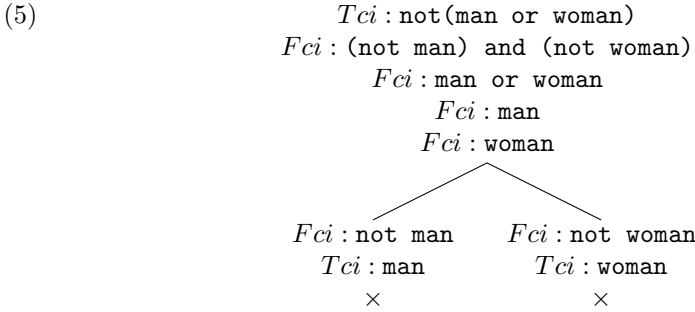
### 3.4 Boolean Rules

We can now give rules for the operators **and**, **or** and **not**, the first two of which we write between their arguments, that are much like the rules for  $\wedge$ ,  $\vee$  and  $\neg$  in signed tableau calculi. What is different here is that these rules are given for conjunction, disjunction and complementation in all categories, not just the category of sentences.

<sup>2</sup> Types will be relational, as in [20]. A relational type  $\langle \alpha_1 \dots \alpha_n \rangle$  is equivalent to the functional type  $\alpha_1 \rightarrow \dots \rightarrow \alpha_n \rightarrow t$  and  $\langle \rangle$  is equivalent to  $t$ .



Here is a tableau showing that  $\text{not}(\text{man or woman})$  entails  $(\text{not man})$  and  $(\text{not woman})$ .

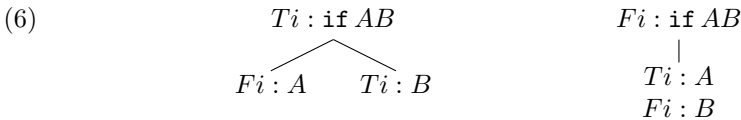


In order to refute the possibility that some object  $c$  and some world  $i$  satisfy  $\text{not}(\text{man or woman})$  but do not satisfy  $(\text{not man})$  and  $(\text{not woman})$  a tableau was developed which starts from the counterexample set

$$\{Tci : \text{not}(\text{man or woman}), Fci : (\text{not man}) \text{ and } (\text{not woman})\}.$$

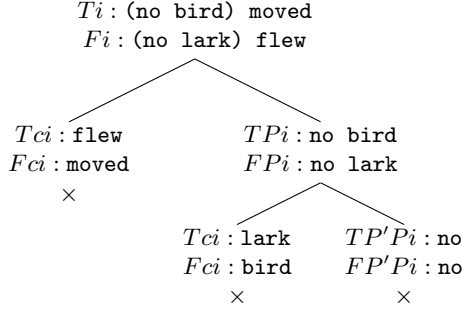
Since the tableau closes ( $\times$  signals branch closure) the possibility is indeed refuted.

While **and**, **or** and **not** seem to be operative in all categories, **if** is sentential. We formulate its rules as follows. Note that sentences still need a parameter (here:  $i$ ) since their type is  $\langle s \rangle$ , not just  $\langle \rangle$ .



### 3.5 Rules for Monotonic Operators

The rules we have discussed until now were either completely general or operated on specific words (constants), but it has been observed that natural reasoning

**Table 1.** Tableau for *no bird moved; therefore no lark flew*

hinges on properties that attach to certain *groups* of expressions. Let us write  $\subset_i$  for the relation that obtains between relations  $M$  and  $M'$  of the same type  $\langle \vec{\gamma}s \rangle$  if  $(\lambda \vec{x}. M \vec{x}i) \subset (\lambda \vec{x}. M' \vec{x}i)$ . A relation  $G$  of type  $\langle \langle \vec{\alpha}s \rangle \vec{\beta}s \rangle$  is called *upward monotone* if  $\forall XY \forall i (X \subset_i Y \rightarrow GX \subset_i GY)$  (where  $X$  and  $Y$  are of type  $\langle \vec{\alpha}s \rangle$ ). Examples of upward monotone expressions (already mentioned above) are **some**, **some** N, **every** N, **many** N, **most** N (where N varies over expressions of type  $\langle es \rangle$ ), but also **Mary**. Here is a tableau rule for upward monotone ( $\text{mon}\uparrow$ ) expressions.

$$\begin{array}{c}
(7) \quad T\vec{C}i : GA \quad \text{where } \vec{c} \text{ and } b \text{ are fresh, provided } G \text{ or } H \text{ is } \text{mon}\uparrow \\
F\vec{C}i : HB \\
\swarrow \quad \searrow \\
\begin{array}{cc}
T\vec{c}i : A & Tb\vec{C}i : G \\
F\vec{c}i : B & Fb\vec{C}i : H
\end{array}
\end{array}$$

And here is a dual rule for expressions that are downward monotone, i.e. that satisfy the property  $\forall XY \forall i (X \subset_i Y \rightarrow GY \subset_i GX)$ . Examples are **no**, **no** N, **every**, **few**, and **few** N.

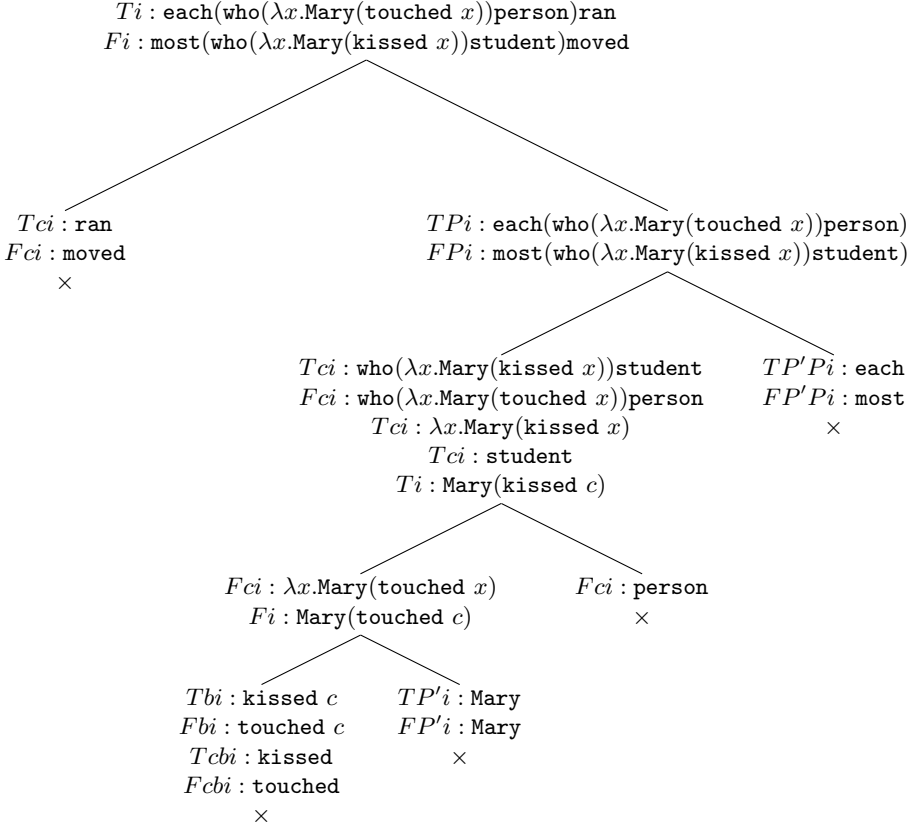
$$\begin{array}{c}
(8) \quad T\vec{C}i : GA \quad \text{where } \vec{c} \text{ and } b \text{ are fresh, provided } G \text{ or } H \text{ is } \text{mon}\downarrow \\
F\vec{C}i : HB \\
\swarrow \quad \searrow \\
\begin{array}{cc}
T\vec{c}i : B & Tb\vec{C}i : G \\
F\vec{c}i : A & Fb\vec{C}i : H
\end{array}
\end{array}$$

Using the second of these rules, the tableau in Table 1 shows, by way of example, that **no bird moved** entails **no lark flew**. Table 2 gives a more complex example, showing that *each person who Mary touched ran* entails *most students who Mary kissed moved*. Here the rules employed for **who** are essentially those for **and**.

### 3.6 Other Rules Connected to Algebraic Properties

Upward and downward monotonicity are not the only algebraic properties that seem to play a pivotal role in language. There is a literature starting with [25]

**Table 2.** Tableau for *each person who Mary touched ran; therefore most students who Mary kissed moved*



singling out *anti-additivity* as linguistically important. An operator  $A$  is anti-additive if it is downward monotone and satisfies the additional property that  $\forall XY((AX \cap AY) \subset A(X \cup Y))$ . A rule for anti-additive operators, examples of which are **no-one** and **without**, but also **not**, is easily given:

(9) If  $A$  is anti-additive:

$$\begin{array}{c}
 F\vec{D} : A(B \text{ or } C) \\
 \swarrow \quad \searrow \\
 F\vec{D} : AB \quad F\vec{D} : AC
 \end{array}$$

We can continue in this vein, isolating rules connected to semantic properties that have been shown to be linguistically important. For example, [8] mentions *splittingness*,  $\forall XY(A(X \cup Y) \subset (AX \cup AY))$ , and *having meet*,  $\forall XY((AX \cap AY) \subset A(X \cap Y))$ , which we can provide with rules as follows.

(10) If  $A$  has meet:

$$\begin{array}{c} F\vec{D} : A(B \text{ and } C) \\ \swarrow \quad \searrow \\ F\vec{D} : AB \quad F\vec{D} : AC \end{array}$$

(11) If  $A$  is splitting:

$$\begin{array}{c} T\vec{D} : A(B \text{ or } C) \\ \swarrow \quad \searrow \\ T\vec{D} : AB \quad T\vec{D} : AC \end{array}$$

no  $N$  and every  $N$  have meet, while some  $N$  is splitting.

### 3.7 Getting Rid of Boolean Operators

Many of the rules we have seen thus far allow one to get rid of Boolean operators, even if the operator in question is not the main operator in the LLF under consideration. Here are a few more. If a Boolean is the main connective in the functor of a functor-argument expression it is of course always possible to distribute it over the argument and Booleans can likewise be pulled out of lambda-abstractions.

$$(12) \quad \begin{array}{ccc} X\vec{C} : (A \text{ and } A')B & & X\vec{C} : (\lambda x.A \text{ and } B) \\ | & & | \\ X\vec{C} : AB \text{ and } A'B & & X\vec{C} : (\lambda x.A) \text{ and } (\lambda x.B) \end{array}$$

These rules were given for **and**, but similar rules for **or** and **not** are also obviously correct.

Other rules that help removing Booleans from argument positions are the following.

$$(13) \text{ If } A \text{ is } \text{mon}\uparrow: \quad \begin{array}{ccc} T\vec{C}i : A(B \text{ and } B') & & F\vec{C}i : A(B \text{ or } B') \\ | & & | \\ T\vec{C}i : AB & & F\vec{C}i : AB \\ T\vec{C}i : AB' & & F\vec{C}i : AB' \end{array}$$

$$(14) \text{ If } A \text{ is } \text{mon}\downarrow: \quad \begin{array}{ccc} T\vec{C}i : A(B \text{ or } B') & & F\vec{C}i : A(B \text{ and } B') \\ | & & | \\ T\vec{C}i : AB & & F\vec{C}i : AB \\ T\vec{C}i : AB' & & F\vec{C}i : AB' \end{array}$$

It is clear that not all cases are covered, but the rules allow us to get rid of **and** and **or** at least in *some* cases.

### 3.8 Rules for Determiners

Let us look at rules for determiners, terms of type  $\langle\langle es \rangle\langle es \rangle s\rangle$ . It has often been claimed that determiners in natural language all are *conservative*, i.e. have the property  $\forall XY (DXY \equiv DX(X \cap Y))$  ([2]). Leaving the question whether really *all* determiners satisfy this property aside, we can establish that for those which do we can use the following tableau rule.

(15) If  $D$  is conservative:

$$\begin{array}{c} Xi : DA(A \text{ and } B) \\ | \\ Xi : DAB \end{array}$$

This again is a rule that removes a Boolean operator from an argument position. Here is another. If determiners  $D$  and  $D'$  are *duals* (the pair **some** and **every** are prime examples), the following rule can be invoked. (We let  $\overline{T} = F$  and  $\overline{F} = T$ .)

(16) If  $D$  and  $D'$  are duals:

$$\begin{array}{c} Xi : DA(\text{not } B) \\ | \\ \overline{Xi} : D'AB \end{array}$$

The following rule applies to *contradictory* determiners, such as **some** and **no**.

(17) If  $D$  and  $D'$  are contradictories:

$$\begin{array}{c} Xi : DAB \\ | \\ \overline{Xi} : D'AB \end{array}$$

There must also be rules for the logical determiners **every** and **some**. Here are some.

$$\begin{array}{lll} (18) \text{ a.} & \begin{array}{c} Ti : \text{every } AB \\ \swarrow \quad \searrow \\ Fci : A \quad Tci : B \end{array} & \begin{array}{c} Fi : \text{some } AB \\ \swarrow \quad \searrow \\ Fci : A \quad Fci : B \end{array} & c \text{ must be } \textit{old}. \\ & \text{b.} & \begin{array}{c} Fi : \text{every } AB \\ | \\ Tci : A \\ Fci : B \end{array} & \begin{array}{c} Ti : \text{some } AB \\ | \\ Tci : A \\ Tci : B \end{array} & c \text{ must be } \textit{fresh}. \end{array}$$

We have now entered dangerous territory, as these are *complete* rules for **some** and **every** that will certainly lead to undecidability when adopted. It may be hypothesized that the human reasoner will prefer rules such as the ones discussed before, or the obvious rule for the symmetry of **some**:

$$\begin{array}{c} (19) \quad Xi : \text{some } AB \\ | \\ Xi : \text{some } BA \end{array}$$

How exactly the linguistic system avoids the ‘bleeding and feeding’ loops that can result from the availability of rules such as those in (18) seems an important question that may partly be open to empirical investigation. Logic may provide a space of possibilities here, but only experiment can show which possibilities were nature’s choice.

### 3.9 Further Rules

In a full paper we will add rules for the modal operators **may** and **must**, **think** and **know**. We will also consider rules that are connected to comparatives and other expressions.

## 4 Conclusion

One way to describe the semantics of ordinary language is by means of translation into a well-understood logical language. If the logical language comes with a model theory and a proof theory, the translation will then induce these on the fragment of language that is translated as well. A disadvantage of this procedure is that precise translation of expressions, taking heed of all their logical properties, often is difficult. Whole books have been devoted to the semantics of a few related words, but while this often was done with good reason and in some cases has led to enlightening results, describing language word by word hardly seems a good way to make progress. Tableau systems such as the one developed here provide an interesting alternative. They interface with the usual model theory, as developing a tableau can be viewed as a systematic attempt to find a model refuting the argument, but on the other hand they seem to give us a better chance in obtaining large coverage systems approximating natural logic. The format allows us to concentrate on rules that really seem linguistically important and squares well with using representations that are close to the Logical Forms in generative syntax. Tableau rules, moreover, do not only allow us to model the classical mode of reasoning, but are equally relevant for modelling alternative forms, such as abduction [17] or the reasoning discussed in Johnson-Laird [14,15], where a model of the premises is sought and the conclusion is then evaluated with respect to that model. In future work I will investigate such alternative forms of reasoning, using the natural language representations and the tableau systems presented here.

## References

1. Aliseda-Llera, A.: Seeking Explanations: Abduction in Logic, Philosophy of Science and Artificial Intelligence. PhD thesis, ILLC (1997)
2. van Benthem, J.F.A.K.: Questions about Quantifiers. *Journal of Symbolic Logic* 49, 447–478 (1984)
3. van Benthem, J.F.A.K.: *Essays in Logical Semantics*. Reidel, Dordrecht (1986)
4. van Benthem, J.F.A.K.: *Language in Action*. North-Holland, Amsterdam (1991)
5. Bernardi, R.: Reasoning with Polarity in Categorical Type Logic. PhD thesis, Utrecht University (2002)
6. Blackburn, P., Bos, J.: *Representation and Inference for Natural Language. A First Course in Computational Semantics*. CSLI (2005)
7. D’Agostino, M., Finger, M., Gabbay, D.: Cut-Based Abduction. *Logic Journal of the IGPL* 16(6), 537–560 (2008)
8. van der Does, J.: *Applied Quantifier Logics*. PhD thesis, University of Amsterdam (1992)
9. Dowty, D.: The Role of Negative Polarity and Concord Marking in Natural Language Reasoning. In: Harvey, M., Santelmann, L. (eds.) *Proceedings from SALT*, vol. IV, pp. 114–144. Cornell University, Ithaca (1994)
10. van Eijck, J.: Generalized Quantifiers and Traditional Logic. In: van Benthem, J., ter Meulen, A. (eds.) *Generalized Quantifiers in Natural Language*. Foris, Dordrecht (1985)

11. van Eijck, J.: Natural Logic for Natural Language. In: ten Cate, B., Zeevat, H. (eds.) *TbiLLC 2005. LNCS (LNAI)*, vol. 4363, pp. 216–230. Springer, Heidelberg (2007)
12. Fyodorov, F., Winter, Y., Francez, N.: Order-Based Inference in Natural Logic. *Logic Journal of the IGPL* 11(4), 385–416 (2003)
13. Heim, I., Kratzer, A.: *Semantics in Generative Grammar*. Blackwell, Oxford (1998)
14. Johnson-Laird, P.N.: *Mental Models: Towards a Cognitive Science of Language, Inference, and Consciousness*. Harvard University Press, Cambridge (1983)
15. Johnson-Laird, P.N.: *How We Reason*. Oxford University Press, Oxford (2006)
16. Lakoff, G.: Linguistics and Natural Logic. In: Davidson, D., Harman, G. (eds.) *Semantics of Natural Language*, pp. 545–665. Reidel, Dordrecht (1972)
17. MacCartney, B., Manning, C.: Natural Logic for Textual Inference. In: *ACL 2007 Workshop on Textual Entailment and Paraphrasing* (2007)
18. MacCartney, B., Manning, C.: An Extended Model of Natural Logic. In: Bunt, H., Petukhova, V., Wubben, S. (eds.) *Proceedings of the 8th IWCS*, Tilburg, pp. 140–156 (2009)
19. Montague, R.: The Proper Treatment of Quantification in Ordinary English. In: Hintikka, J., Moravcsik, J., Suppes, P. (eds.) *Approaches to Natural Language*, pp. 221–242. Reidel, Dordrecht (1973); Reprinted in [23]
20. Muskens, R.A.: *Meaning and Partiality*. CSLI, Stanford (1995)
21. Sánchez, V.: *Studies on Natural Logic and Categorical Grammar*. PhD thesis, University of Amsterdam (1991)
22. Sommers, F.: *The Logic of Natural Language*. The Clarendon Press, Oxford (1982)
23. Thomason, R. (ed.): *Formal Philosophy, Selected Papers of Richard Montague*. Yale University Press, New Haven (1974)
24. Zamansky, A., Francez, N., Winter, Y.: A ‘Natural Logic’ Inference System Using the Lambek Calculus. *Journal of Logic, Language and Information* 15, 273–295 (2006)
25. Zwarts, F.: Negatief-polaire Uitdrukkingen I. *Glot* 6, 35–132 (1981)

# The Data Complexity of the Syllogistic Fragments of English

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**Abstract.** Pratt and Third's syllogistic fragments of English can be used to capture, in addition to syllogistic reasoning, many other kinds of common sense reasoning, and, in particular (i) knowledge base consistency and (ii) knowledge base query answering, modulo their FO semantic representations. We show how difficult, in terms of semantic (computational) complexity and data complexity (i.e., computational complexity w.r.t. the number of instances declared in a knowledge base), such reasoning problems are. In doing so, we pinpoint also those fragments for which the reasoning problems are tractable (in **PTime**) or intractable (**NP-hard** or **coNP-hard**).

## 1 Introduction

Natural logic is concerned with capturing formal logic and common sense reasoning, by means of natural language insofar as we can ascribe to the latter a formal, compositional semantics. Montague, back in the 1970's [8] showed how to define a compositional, formal semantics for fragments of English by means of *compositional translations*  $\tau(\cdot)$  that recursively assign to each English syntactic constituent a HO or FO *meaning representation*<sup>1</sup>, and that gives rise to the notion of *semantic complexity* (see Pratt in [11,10]), viz., the computational properties of those meaning representations, which help in measuring the ease or the difficulty of the cognitive and/or computational processing involved in natural language understanding and reasoning [15,11,10,7]. Easy fragments are believed to exhibit "good" (tractable, i.e., in **PTime**) computational properties and difficult ones "bad" (intractable, i.e., **NP-hard** or **coNP-hard**) properties [15,11,10,7].

In this paper we study the *syllogistic fragments* (FOEs) of Pratt [11,10], who proposed them as a means of capturing in English syllogistic entailment and satisfiability. However, entailment and satisfiability are not the only relevant reasoning problems a fragment, let alone the FOEs, that is not "booleanly closed" (i.e., cannot express complete sets of boolean functions) captures. Instead, the common-sense reasoning problems we study in this paper are (i) identifying inconsistencies in known information and (ii) posing questions to known information. Such reasoning problems can be modelled in logic quite naturally via ontology and knowledge base reasoning tasks, resp., by the

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<sup>1</sup> HO can be conceived of as the extension of FO with the  $\lambda$ -abstraction,  $\lambda$ -application,  $\beta$ -normalization and, eventually, the types of the simply-typed  $\lambda$ -calculus.

**Table 1.** Coverage of the FOEs and of TSQs. See [11][10][16]

COP	Copula, common and proper nouns, negation, universal, existential quantifiers
COP+Rel	COP plus relative pronouns
COP+TV	COP plus transitive verbs
COP+TV+DTV	COP+TV plus ditransitive verbs
COP+Rel+TV	COP+Rel plus transitive verbs
COP+Rel+TV+DTV	COP+Rel+TV plus ditransitive verbs
TSQs	Copula, common and proper nouns, existential quantifiers, transitive verbs, noun and verb phrase coordination, relative pronouns, passives, query words

knowledge base satisfiability problem and the query answering problem. An ontology is a set  $\mathcal{O}$  of FO axioms and a knowledge base is a pair  $(\mathcal{O}, \mathcal{D})$ , where  $\mathcal{O}$  is an ontology and  $\mathcal{D}$  is a database containing data about a given domain of interest [14][4]. In this context it is assumed that the size of the data dominates that of the ontology and hence the main focus is in the *data complexity* of reasoning, i.e., in inferring computational properties w.r.t. the size of  $\mathcal{D}$  alone [17][4].

The FOEs capture problems (i) and (ii) when combined with suitable interrogative fragments such as *tree shaped questions* (TSQs) [16], an interrogative fragment with which a wide variety of information requests can be expressed. Since  $\tau(\cdot)$  allows us to assign to the FOEs knowledge bases as meaning representations, by combining the FOEs and TSQs and studying their data complexity, we can infer for which FOEs data complexity is tractable or intractable. Or, more precisely, which combinations of function words in the fragments give rise to such computational properties.

To infer data complexity bounds we adopt as main strategy *resolution-based saturation decision procedures* for fragments of FO as outlined by Joyner in [6]. We show that, in general, intractability w.r.t. data arises whenever fragments, alone or in combination with TSQs, are "booleanly closed". Otherwise, we obtain fragments where reasoning is tractable w.r.t. data.

## 2 The Syllogistic Fragments and Tree-Shaped Questions

The FOEs are defined incrementally. The idea is to start with a FOE, called COP, that covers: (i) copula ("is"), (ii) verb-phrase negation ("is not"), (ii) the determiners "some", "every" and "no", together with common and proper nouns. The fragment and the translation  $\tau(\cdot)$  are defined at the same time, by means of a semantically annotated context-free grammar. Standard HO meaning representations are used. Thereafter, by extending coverage to a new English construct, viz., transitive verbs (e.g., "likes"), ditransitive verbs (e.g., "gives"), relatives (e.g., "that") and anaphors (e.g., "him"), the other members of the family are defined. See Table 1. For the detailed definition of the fragments, we refer to [11][10]. See Table 2 for their meaning representations.

**Table 2.** The meaning representations generated by the FOEs and TSQs:  $\varphi_l$  (resp.  $\varphi_r$ ) stands for the meaning representation of the subject (resp. predicate) constituent of a main or subordinated sentence, whereas  $\varphi_n$  (resp.  $\varphi_{dn}$ ) denotes the meaning representation of a verb phrase containing a transitive (resp. ditransitive) verb;  $\psi(x, y)$  (resp.  $\chi(x, y, z)$ ) stands for some binary (resp. ternary) atom, while  $\pm$  means that a formula may or may not be negated

COP	$\varphi_l(x) \rightarrow A(x)$ $\varphi_r(x) \rightarrow \pm\varphi_l(x)$	$\forall x(\varphi_l(x) \Rightarrow \pm\varphi_r(x))$ $\exists x(\varphi_l(x) \wedge \varphi_r(x))$	No student failed. A student failed.
COP+TV	$\varphi_l(x) \rightarrow A(x)$ $\varphi_r(x) \rightarrow \pm\varphi_l(x) \mid \forall y(A(x) \Rightarrow \pm\psi(x, y))$ $\mid \exists y(A(x) \wedge \psi(x, y))$	$\forall x(\varphi_l(x) \Rightarrow \pm\varphi_r(x))$ $\exists x(\varphi_l(x) \wedge \varphi_r(x))$	No student failed. Some student follows every course.
COP+TV+DTV	$\varphi_l(x) \rightarrow A(x)$ $\varphi_n(x) \rightarrow \pm\varphi_l(x) \mid \forall y(A(x) \Rightarrow \pm\psi(x, y))$ $\mid \exists y(A(x) \wedge \psi(x, y))$ $\varphi_{dn}(x, y) \rightarrow \forall z(A(x) \Rightarrow \pm\chi(x, y, z))$ $\mid \exists z(A(x) \wedge \chi(x, y, z))$ $\varphi_r(x) \rightarrow \varphi_n(x) \mid \forall y(A(x) \Rightarrow \pm\varphi_{dn}(x, y))$ $\mid \exists y(A(x) \wedge \varphi_{dn}(x, y))$	$\forall x(\varphi_l(x) \Rightarrow \pm\varphi_r(x))$  $\exists x(\varphi_l(x) \wedge \varphi_r(x))$	Every student gives no credit to some student. A student borrowed a book from some library.
COP+Rel	$\varphi_l(x) \rightarrow A(x) \mid \pm\varphi_l(x) \wedge \pm\varphi_l(x)$ $\varphi_r(x) \rightarrow \varphi_l(x)$	$\forall x(\pm\varphi_l(x) \Rightarrow \pm\varphi_r(x))$ $\exists x(\pm\varphi_l(x) \wedge \pm\varphi_r(x))$	Every student who is not dum is smart.
COP+TV+Rel	$\varphi_l(x) \rightarrow A(x)$ $\varphi_r(x) \rightarrow \pm\varphi_l(x) \mid \forall y(A(x) \Rightarrow \pm\psi(x, y))$ $\mid \exists y(A(x) \wedge \psi(x, y))$	$\forall x(\varphi_l(x) \Rightarrow \pm\varphi_r(x))$ $\exists x(\varphi_l(x) \wedge \varphi_r(x))$	No student failed. Some student studies every course.
COP+Rel+TV+DTV	$\varphi_l(x) \rightarrow A(x) \mid \pm\varphi_r \wedge \pm\varphi_r$ $\varphi_n(x) \rightarrow \pm\varphi_l(x) \mid \forall y(A(x) \Rightarrow \pm\psi(x, y))$ $\mid \exists y(A(x) \wedge \psi(x, y))$ $\varphi_{dn}(x, y) \rightarrow \forall z(A(x) \Rightarrow \pm\chi(x, y, z))$ $\mid \exists z(A(x) \wedge \chi(x, y, z))$ $\varphi_r(x) \rightarrow \varphi_n(x) \mid \forall y(A(x) \Rightarrow \pm\varphi_{dn}(x, y))$ $\mid \exists y(A(x) \wedge \varphi_{dn}(x, y))$	$\forall x(\varphi_l(x) \Rightarrow \pm\varphi_r(x))$  $\exists x(\varphi_l(x) \wedge \varphi_r(x))$	Every helpful student gives some aid to some student. Some diligent student borrowed every book from every library.
TSQs	$\varphi_l(x) \rightarrow A(x) \mid \exists yR(x, y) \mid \varphi_r(x) \wedge \varphi_r(x)$ $\mid \exists y(R(x, y) \wedge \varphi_r(y))$ $\varphi_r(x) \rightarrow \varphi_l(x)$	$\varphi_l(x) \wedge \varphi_r(x)$	Which student who attends some course is diligent?

The information that we can express/store in the FOEs can be queried/accessed by TSQs<sup>2</sup>, which are built through query words (e.g., "who"), relatives, transitive verbs, copula, common nouns, the determiner "some", the pronoun "somebody", passives (e.g., "is loved by") and conjunction ("and"). See Table 1. For their formal definition we refer to [16]. See Table 2 for their meaning representations.

We intend to understand the computational properties of the FOEs *in the size of the data*. We consider sets  $\mathcal{S}$  of quantified and  $\mathcal{F}$  of ground sentences. We overload the notion of knowledge base to cover linguistic knowledge bases, i.e., pairs  $(\mathcal{S}, \mathcal{F})$ : since modulo  $\tau(\cdot)$ ,  $\mathcal{S}$  maps into ("expresses") an ontology  $\mathcal{O}$  and  $\mathcal{F}$  into ("expresses") a database  $\mathcal{D}$ , reasoning over  $(\mathcal{S}, \mathcal{F})$  reduces to reasoning over  $(\mathcal{O}, \mathcal{D})$ .

We study two decision problems. On the one hand, (i) knowledge base satisfiability (KB-SAT): **Given:**  $(\mathcal{S}, \mathcal{F})$ . **Check:** is  $\tau(\mathcal{S}) \cup \tau(\mathcal{F})$  satisfiable? And, on the other hand, (ii) query answering (KB-QA): **Given:**  $(\mathcal{S}, \mathcal{F})$ , a question  $Q$  and (possibly) a constant  $c$ . **Check:** does  $\tau(\mathcal{S}) \cup \tau(\mathcal{F}) \models \tau(Q)\{x \mapsto c\}$ ? Where  $\tau(Q)$  is a formula of (possibly) free variable  $x$ . By analogy to [17], we define the *data complexity* of KB-SAT and KB-QA as their computational complexity measured in the size of  $\mathcal{F}$  only ( $\mathcal{S}$  and possibly  $\mathcal{F}$  are considered constant). The *size*  $\#(\mathcal{F})$  of  $\mathcal{F}$  is defined as the number of proper names occurring in  $\mathcal{F}$ . This measure can be extended to  $\tau(\mathcal{F})$  by denoting with  $\#(\tau(\mathcal{F}))$  the number of individual constants in  $\tau(\mathcal{F})$ .

Note that as soon as we add relative pronouns, the FOEs become closed under negation (see Table 2). Note also that TSQs are strictly contained in the FOEs with transitive verbs and relatives (see Table 2). However, corpora analysis [2] show that while negation and universal quantification are rare in questions, relatives and existential quantifiers are common and that TSQs are a fragment that naturally expresses queries to knowledge bases.

*Example 1.* Suppose we want to reason about our (previous) knowledge regarding university students. We can capture such knowledge via a COP+TV knowledge base. Moreover, we can check whether such knowledge is consistent and, thus, meaningful, and pose to it TSQs:

$$\begin{array}{ll} \mathcal{S}_s := \begin{cases} \text{Every student attends some course.} & \forall x(\text{Student}(x) \Rightarrow \exists y(\text{attends}(x, y) \wedge \text{Course}(y))) \\ \text{Every bachelor student is a student.} & \forall x(\text{BachelorStudent}(x) \Rightarrow \text{Student}(x)) \end{cases} \\ \mathcal{F}_s := \begin{cases} \text{John is a bachelor student.} & \text{BachelorStudent}(\text{John}) \\ \vdots & \vdots \end{cases} \\ \mathcal{Q}_s := \begin{cases} \text{Which student attends some course?} & \text{Student}(x) \wedge \exists y(\text{attends}(x, y) \wedge \text{Course}(y)) \end{cases} \end{array}$$

Clearly, "John" is the answer to our question, and the knowledge base is consistent. But how difficult are these forms of reasoning over, ultimately,  $\tau(\mathcal{S}_s) \cup \tau(\mathcal{F}_s)$  and  $\tau(\mathcal{Q}_s)$  (viz., KB-SAT and KB-QA) in computational terms?

### 3 Resolution Saturations and Data Complexity

**Resolution decision procedures.** A *term*  $t$  is (i) a variable  $x$  or a constant  $c$  or (ii) an expression  $f(t_1, \dots, t_n)$  where  $f$  is a function symbol and  $t_1, \dots, t_n$  are terms. In the

<sup>2</sup> TSQs express a significant subset of database SQL SELECT-PROJECT-JOIN queries [1].

latter case, we speak about *function terms*. A *literal*  $L$  is a FO atom  $P(t_1, \dots, t_n)$ . By a *clause* we understand a disjunction  $L_1 \vee \dots \vee L_n \vee \overline{N}_{n+1} \vee \dots \vee \overline{N}_{n+m}$  of positive and negative literals. The *empty clause* or *falsum* is denoted  $\perp$ . By  $V(t)$ ,  $V(L)$  and  $V(C)$  we denote the sets of variables of, resp., term  $t$ , literal  $L$  and clause  $C$ . A term, literal, clause or set of clauses is said to be *ground* if it contains no free variables. A *substitution*  $\sigma$  is a function from variables to terms. It is called a *renaming* when it is a function from variables to variables. A *unifier* is a substitution  $\sigma$  s.t., given two terms  $t$  and  $t'$ ,  $t\sigma = t'\sigma$ . A *most general unifier* is a unifier  $\sigma$  s.t. for every other unifier  $\sigma'$  there exists a renaming  $\sigma''$  with  $\sigma' = \sigma\sigma''$ .

The *depth* of a term is defined by (i)  $d(x) := d(c) := 0$  and (ii)  $d(f(t_1, \dots, t_n)) := \max\{d(t_i) \mid i \in [1, n]\} + 1$ . The *depth*  $d(L)$  of a literal  $L$  or  $d(\Gamma)$  of a set of clauses  $\Gamma$  is the maximal depth of their terms. The *relative depth* of a variable  $x$  in a term is defined by (i)  $d(x, y) := d(x, c) := 0$  and (ii)  $d(x, f(t_1, \dots, t_n)) := \max\{d(x, t_i) \mid i \in [1, n]\} + 1$ . The *relative depth*  $d(x, L)$  of a variable  $x$  in a literal  $L$  is its maximal relative depth among  $L$ 's terms.

We consider the so-called *saturation-based* version of the resolution calculus in which we iteratively (monotonically w.r.t.  $\subseteq$ ) generate the set of all possible clauses derived from  $\Gamma$  using the rules

$$res \frac{C \vee \overline{L} \quad C \vee L'}{(C \vee C')\sigma} \quad \text{and} \quad fact \frac{C \vee L \vee L'}{(C \vee L)\sigma},$$

where  $\sigma$  is a most general unifier of  $L$  and  $L'$ , until either (i)  $\perp$  is derived or (ii) all possible clauses are generated (fixpoint computation). Formally, consider a function  $\rho(\cdot)$  over sets of clauses, defined in terms of *res* and *fact*. A *resolution calculus* is a function  $\mathcal{R}(\cdot)$  s.t.  $\mathcal{R}(\Gamma) := \Gamma \cup \rho(\Gamma)$ . A *derivation*  $\delta$  from  $\Gamma$  is defined by putting (i)  $\mathcal{R}^0(\Gamma) := \Gamma$  and  $\mathcal{R}^{i+1}(\Gamma) := \mathcal{R}(\mathcal{R}^i(\Gamma))$ , for  $i > 0$ . Thereafter the *saturation* of  $\Gamma$  is defined as  $\Gamma^\infty := \bigcup\{\mathcal{R}^i(\Gamma) \mid i \geq 0\}$ . The positive integer  $i$  is called the *depth* or *rank* of  $\delta$ . The set(s) of clauses derived at each rank  $i \geq 0$  of  $\delta$  is (are) called the *state(s)* of  $\delta$ . The *size* of  $\delta$  is defined as its total number of states. Resolution is sound and complete w.r.t. (un)satisfiability:  $\Gamma$  is unsatisfiable iff  $\perp \in \Gamma^\infty$ . Moreover, if  $\Gamma$  is satisfiable, we can build out of  $\Gamma^\infty$  a herbrand model of  $\Gamma$  [5].

Resolution saturations are not computable in general (they may not converge finitely). However, Joyner in [6] showed that finite convergence can be achieved provided that two conditions are met: (i) that the depth of literals does not grow beyond a certain bound  $d \geq 0$  and (ii) that the length of clauses (the number of disjunctions) does not grow beyond a bound  $l \geq 0$ . Several *refinements* can be used to ensure the existence of such bounds and a fortiori finite convergence for several fragments of FO.

To control depth, *acceptable orderings* (A-orderings), that is, well-founded and substitution-invariant partial orders on clause literals and sets thereof, can be used (which force resolution on literals that are maximal w.r.t. the ordering). The best known is the  $\prec_d$  ordering defined by

$$L \prec_d L' \text{ iff } d(L) < d(L'), V(L) \subseteq V(L') \\ \text{and, for all } x \in V(L), d(x, L) < d(x, L'),$$

a refinement sound and complete w.r.t. satisfiability. To control length the splitting rule

$$\text{split} \frac{\begin{array}{c} C \vee L \quad C \vee L' \\ \vdots \quad \vdots \\ C \vee L \vee L' \end{array} \quad \frac{C' \sigma}{C' \sigma} \quad C' \sigma}{C' \sigma} (V(L) \cap V(L') = \emptyset)$$

can be used (it is sound and complete w.r.t. satisfiability). These refinements are guaranteed to work the way we want them to in case they are applied to covering clauses. A literal  $L$  is said to be *covering* whenever (i)  $d(L) = 0$  or (ii) for every functional term  $t$  in  $L$ ,  $V(t) = V(L)$ . If all the literals of a clause  $C$  are covering, so is  $C$ . This property is not, however, closed under resolution or its refinements: applying them to covering clauses may result in non-covering clauses. To prevent this from happening, a further refinement is required: *monadization* [6]. Intuitively, what this does is to reduce the (un)satisfiability of non-covering clauses, satisfying some structural properties, into that of a set of covering clauses. The applicability of the refinements thus depends on the FO fragments such clauses are drawn from, but, whenever *all* are applicable, saturations finitely converge [5].

The different systems arising from the different combinations of rules, orderings and refinements are summarized below:

		<i>split</i>	<i>mon</i>	<i>split</i> <i>mon</i>
	$\mathcal{R}_{1,1}$	$\mathcal{R}_{1,2}$	$\mathcal{R}_{1,4}$	$\mathcal{R}_{1,5}$
$\prec_d$	$\mathcal{R}_{2,1}$	$\mathcal{R}_{2,2}$	$\mathcal{R}_{2,4}$	$\mathcal{R}_{2,5}$

In particular, the  $\mathcal{R}_{2,5}$  calculus decides the  $\mathcal{S}^+$  class of clauses [5]. The class  $\mathcal{S}^+$  is the class where every clause  $C$  satisfies: (i)  $V(C) = V(t)$ , for every functional term  $t$  in  $C$ , and (ii) either  $L$  has at most one variable or  $V(L) = V(C)$ , for every literal  $L$  in  $C$ . Note that saturations exhibit the shape of a tree (of branching factor 2) or of a sequence, depending on whether the calculi make use or not of the splitting rule.

**Data Complexity of KB-QA and KB-SAT.** In this section we study the data complexity of KB-SAT and KB-QA by applying resolution decision procedures to the FOEs. We apply data complexity arguments to sets  $\Sigma \cup \Delta$  of non-ground and ground clauses. This makes sense, because, modulo  $\tau(\cdot)$  and clausification, FOE sentences  $\mathcal{S}$  map to sets  $\Sigma$  of non-ground clauses, FOE facts  $\mathcal{F}$  map to sets  $\Delta$  of ground clauses, and, in general, knowledge bases  $(\mathcal{S}, \mathcal{F})$  to sets  $\Sigma \cup \Delta$  of clauses.

We do as follows. For the tractable FOEs we rely on the “separation” property of resolution saturations [5] (resolution of ground clauses can be delayed to the end). For the intractable, on the “monadic reducibility” property [10] that enforces a reduction to  $\mathcal{S}^+$  clauses for the fragments involved; this we combine with a data complexity of the  $\mathcal{S}^+$  class (and saturations).

- **Separation:**  $\perp \in (\Sigma \cup \Delta)^\infty$  iff there exists a set  $\Sigma' \subseteq \Sigma^\infty$  s.t. (i)  $d(\Sigma') \leq d(\Delta)$ , (ii)  $\perp \in (\Sigma' \cup \Delta)^\infty$  and (iii)  $\Sigma'$  is finite.

- **Monadic reducibility:** every set  $\Gamma$  of COP+TV+DTV+Rel classified meaning representations (or any fragment thereof) can be polynomially (in the size of  $\Gamma$ ) transformed into a set  $\Gamma_u$  of unary clauses s.t.  $\Gamma$  is satisfiable iff  $\Gamma_u$  is satisfiable.

**Lemma 1.** *Let  $(\mathbf{C}, \mathbf{F}, \mathbf{R})$  be a finite FO signature, where  $\mathbf{C}$  is a (finite) set of constants,  $\mathbf{F}$  a (finite) set of function symbols and  $\mathbf{R}$  a (finite) set of predicate symbols. Consider a clause set  $\Gamma$  over such a signature and suppose that there exist both a term depth bound  $d \geq 0$  and a clause length bound  $k \geq 0$ . Then, in the worst case,*

1. *the number of clauses derivable by the saturation is (i) exponential in the number of constants in  $\mathbf{C}$  if we use the splitting rule or (ii) polynomial in the number of constants in  $\mathbf{C}$  otherwise, and*
2. *the depth of the saturation is polynomial in the number of constants in  $\mathbf{C}$ .*

*Proof.* Assume that a depth bound  $d$  and a length bound  $l$  exist. Let  $c$  be the number of constant symbols in  $\mathbf{C}$ ,  $v$  the number of variables in  $\mathbf{V}$ ,  $f$  the number of function symbols in  $\mathbf{F}$ ,  $p$  the number of predicate symbols in  $\mathbf{R}$ ,  $ar_f$  the maximum arity of the function symbols, and  $ar_p$  the maximum arity of the predicate symbols. We can define the number  $te_i$  of terms of depth  $i \geq 0$  inductively by setting (i)  $te_0 := v + c$ , (ii)  $te_{i+1} := f \cdot te_n^{ar_f}$ . Thus, the number  $te$  of terms of depth  $\leq d$  is

$$te \leq \sum_{i=0}^d te_i = f^0 \cdot (v + c)^{ar_f^0} + \dots + f^d \cdot (v + c)^{ar_f^d} := p_{te}(c) \quad (1)$$

which defines a polynomial  $p_{te}(c)$ . This in its turn yields as upper bound to the number  $li$  of positive and negative literals

$$li \leq 2 \cdot p \cdot te^{ar_p} = 2 \cdot p \cdot p_{te}(c)^{ar_p} := p_{li}(c) \quad (2)$$

thus defining a polynomial  $p_{li}(c)$ . Finally, from  $li$  we derive an upper bound to the number  $cl$  of clauses of length  $\leq l$

$$cl \leq li^l = p_{li}(c)^l := p_{cl}(c) \quad (3)$$

which again defines a polynomial  $p_{cl}(c)$ . The splitting rule splits saturations into two, yielding a (saturation) tree of worst-case size  $\leq 2^{p_{cl}(c)}$ , largest (derived) state of size  $\leq p_{cl}(c)$  and that will converge after  $\leq p_{cl}(c)$  iterations.  $\square$

**Theorem 1.** *KB-SAT is in NP in data complexity for  $\mathcal{S}^+$ .*

*Proof.* Let  $\Sigma \cup \Delta$  be a set of  $\mathcal{S}^+$  clauses. Consider now a  $\mathcal{R}_{2,5}$ -saturation. Calculus  $\mathcal{R}_{2,5}$  decides  $\mathcal{S}^+$  and saturations finitely converge. Assume w.l.o.g. that  $\Sigma$  contains no constants and that  $\Delta$  is of depth  $d(\Delta) = 0$  and has  $c$  distinct constants (where  $c \geq 0$ ). By Lemma 1 we know that the saturation will be tree-shaped, of rank  $\leq p(c)$ , of size  $\leq 2^{p(c)}$  and of maximal state of size  $\leq p(c)$ .

Outline a non-deterministic algorithm for KB-SAT as follows. Start with  $\Sigma \cup \Delta$ . For each rank  $i \in [0, p(c)]$  of the saturation, guess/choose a state  $j \in [0, 2^i]$ . Notice that the algorithm will make polynomially many choices on  $c$ . Finally, check, in time polynomial in  $c$  whether  $\perp$  is in the resulting state, and, if no, compute, in time polynomial in  $c$ , a herbrand model of  $\Sigma \cup \Delta$ .  $\square$

**Theorem 2 (KB-SAT).** *The data complexity for KB-SAT is*

1. in **LSpace** for *COP*, *COP+TV*, *COP+TV+DTV* and *COP+Rel*, and
2. **NP**-complete for *COP+Rel+TV*, *COP+Rel+TV* and *COP+Rel+TV+DTV*.

*Proof.* (Sketch.) For *COP*, *COP+TV* and *COP+TV+DTV* we reason as follows. Let  $(\mathcal{S}, \mathcal{F})$  be a knowledge base and consider its meaning representations  $\tau(\mathcal{S})$  and  $\tau(\mathcal{F})$  (which can be computed in space logarithmic in  $\#(\mathcal{F})$ ). Computing their skolemization and clausification does not affect data complexity, since it is the identity for  $\tau(\mathcal{F})$ . By inspecting the resulting clauses we can observe that they are covering: using A-ordered resolution prevents clause depth from growing beyond a bound  $d$ . Furthermore, it can be proven that applying *res* and *fact*, does not increase clause length beyond a bound  $l$ , nor does it result in non-covering clauses. Thus, the A-ordered resolution calculi without splitting decide the satisfiability of  $\tau(\mathcal{S}) \cup \tau(\mathcal{F})$ . We also know by the "separation" property that we can "separate" data from facts provided  $\tau(\mathcal{S})$  is satisfiable.

Sketch a decision algorithm for KB-SAT as follows. Check whether  $\tau(\mathcal{S})$  is satisfiable, i.e., whether  $\perp \in \tau(\mathcal{S})^\infty$ , computation that does not depend on  $\#(\mathcal{F})$  (or  $\#(\tau(\mathcal{F}))$ ). If the answer is negative, return "no". If the answer is positive: (i) Compute a (finite) model/database  $\mathcal{D}_{\mathcal{F}}$  from  $\tau(\mathcal{F})$  (i.e., the herbrand model defined from  $\tau(\mathcal{F})$ ). (ii) Compute the FO formula  $\varphi_{\mathcal{S}} := \bigwedge \{C \mid C \text{ clause of } \tau(\mathcal{S})^\infty\}$ . Then,

$$\tau(\mathcal{S}) \cup \tau(\mathcal{F}) \text{ is satisfiable iff } \mathcal{D}_{\mathcal{F}} \models \varphi_{\mathcal{S}},$$

which outlines a reduction to relational database query answering, known to be in **LSpace** [1]. Membership in **LSpace** follows. The argument for *COP+Rel* is similar.

Membership in **NP** for *COP+Rel+TV* and *COP+Rel+TV+DTV* is derived as follows. Consider a knowledge base  $(\mathcal{S}, \mathcal{F})$ . Consider now the resulting meaning representations,  $\tau(\mathcal{S})$  and  $\tau(\mathcal{F})$ . Clausifying such meaning representations can be done in time constant in  $\#(\tau(\mathcal{F}))$ . By Pratt and Third's "monadic reducibility" property, we know that we can reduce, in time polynomial in  $\#(\tau(\mathcal{F}))$  their satisfiability to that of a set  $\tau(\mathcal{S})_u \cup \tau(\mathcal{F})_u$  of monadic clauses. By inspection, we can, moreover, observe that such classes belong to the  $\mathcal{S}^+$  class. We can now apply Lemma 1 whence it follows that KB-SAT is in **NP**.

Finally, **NP**-hardness for *COP+Rel+TV* and *COP+Rel+TV+DTV* can be inferred by a reduction from the **NP**-complete satisfiability problem for 2+2 clauses [12]. A 2+2 clause is a clause  $L_1 \vee L_2 \vee \bar{L}_3 \vee \bar{L}_4$  containing two positive literals and two negative literals.  $\square$

**Theorem 3 (KB-QA).** *If we consider TSQs, then the data complexity of KB-QA is*

1. in **LSpace** for *COP* and in **PTime** for *COP+TV*,
2. in **coNP** for *COP+TV+DTV*, and **coNP**-complete for *COP+Rel*, *COP+Rel+TV* and *COP+Rel+TV+DTV*.

*Proof.* (Sketch.) KB-QA for *COP* is in **LSpace** in data complexity, because it can be shown that its meaning representations are contained in the description logic *DL-Lite*,

for which such result holds [3]. Similarly, it can be shown that COP+TV KB-QA reduces to Datalog KB-QA. Furthermore, given a COP+TV knowledge base  $(\mathcal{S}, \mathcal{F})$  and a TSQ  $Q$ , such reduction proceeds in space logarithmic in  $\#(\mathcal{F})$ . It thus preserves data complexity. Since Datalog KB-QA is in **PTime** [1], the result follows.

The **coNP** upper bound for COP+Rel and COP+Rel+TV follows from the **coNP**-completeness for data complexity of KB-QA for the two-variable fragment of FO [9]. Regarding COP+TV+DTV and COP+Rel+TV+DTV, we observe that: (i) TSQs can be expressed quite easily by COP+Rel+TV+DTV, by extending this FOE with grammar rules accounting for wh- and y/n-questions. (ii) COP+Rel+TV+DTV is closed under negation. We can thus reduce KB-QA (again, by a reduction space logarithmic in the size of the data) to **coKB-SAT** (i.e., the complement of KB-SAT) and apply Theorem 2.

Finally, **coNP**-hardness derives from the fact that we can again reduce the satisfiability of 2+2 clauses to COP+Rel **coKB-QA** (i.e., the complement of KB-QA). This lower bound then propagates to COP+Rel+TV and COP+Rel+TV.  $\square$

## 4 Conclusions

We have studied the data complexity of Pratt's FOEs w.r.t. KB-SAT (viz., knowledge base satisfiability) and KB-QA (viz., answering TSQs over knowledge bases). In so doing, we have assessed their semantic complexity w.r.t. the common-sense problems of checking for the consistency of and posing questions to known information, which the aforementioned decision problems formalize.

Our results show that the data complexity of the fragments without relative clauses, namely, COP, COP+TV and COP+TV+DTV, are grosso modo, tractable (the upper bound for KB-QA for COP+TV+DTV is not tight), and that data complexity is grosso modo, intractable, when relatives are added (the upper bound for KB-SAT for COP+Rel is not tight either). The following table summarizes the computational properties associated to each such combination of function words:

<b>Tractable</b>	Negation in the predicates of declarations.	Relatives and conjunction everywhere but no negation in questions.
<b>Intractable</b>	Relatives and negation in the subject of declarations.	Relatives, conjunctions and transitive verbs in questions.

**Table 3.** Data complexity of KB-QA and KB-SAT for the FOEs and TSQs and complexity of satisfiability for the FOEs

	<b>KB-QA</b>	<b>KB-SAT</b>	<b>Satisfiability [11]</b>
COP	in <b>LSpace</b> [Th 3]	in <b>LSpace</b> [Th 2]	in <b>NLSpace</b>
COP+TV	in <b>PTime</b> [Th 3]	in <b>LSpace</b> [Th 2]	<b>NLSpace</b> -complete
COP+TV+DTV	in <b>coNP</b> [Th 3]	in <b>LSpace</b> [Th 2]	in <b>PTime</b>
COP+Rel	<b>coNP</b> -complete [9]	in <b>LSpace</b> [Th 2]	<b>NP</b> -complete
COP+Rel+TV	<b>coNP</b> -complete [9]	<b>NP</b> -complete [7, Th 2]	<b>ExpTime</b> -complete
COP+Rel+DTV	<b>coNP</b> -complete [Th 3]	<b>NP</b> -complete [Th 2]	<b>NExpTime</b> -complete
COP+Rel+DTV+TV	<b>coNP</b> -complete [Th 3]	<b>NP</b> -complete [Th 2]	<b>NExpTime</b> -complete

Intractability arises when the combination is "booleanly closed". Relatives express modulo  $\tau(\cdot)$  logical conjunction, hence, when combined with negation, yield intractability (we can encode propositional satisfiability). Note that the complexity of satisfiability (and entailment), and hence the *combined* complexity of KB-SAT and KB-QA, is significantly higher: it is, e.g., **ExpTime**-complete for COP+Rel+TV and **NExpTime**-complete for COP+Rel+TV+DTV (see Table 3).

As related work we must mention the results by Slavkovic [13] regarding approximate reasoning techniques for fragments that map into the two variable fragment of FO and Pratt's already cited results [11][10] regarding the (combined) computational complexity for satisfiability and entailment of the FOEs. In [15], on the other hand, the data complexity of model checking for several fragments is studied.

## References

1. Abiteboul, S., Hull, R., Vianu, V.: Foundations of Databases. Addison-Wesley, Reading (1995)
2. Bernardi, R., Bonin, F., Carbotta, D., Calvanese, D., Thorne, C.: English querying over ontologies: E-QuOnto. In: Proc. of the 10th Congress of the Italian Association for Artificial Intelligence AI\*IA 2007 (2007)
3. Bernardi, R., Calvanese, D., Thorne, C.: Expressing DL-Lite ontologies with controlled English. In: Proc. of the 20th Int. Workshop on Description Logics, DL 2007 (2007)
4. Calvanese, D., de Giacomo, G., Lembo, D., Lenzerini, M., Rosati, R.: Efficiently managing data intensive ontologies. In: Proc. of the 2nd Italian Semantic Web Workshop: Semantic Web Applications and Perspectives, SWAP 2005 (2005)
5. Fermüller, C.G., Leitsch, A., Hustadt, U., Tammet, T.: Resolution decision procedures. In: Robinson, A., Voronkov, A. (eds.) Handbook of Automated Reasoning, vol. 2, ch. 2, 1791–1849. Elsevier - The MIT Press (2001)
6. Joyner, W.H.: Resolution strategies as decision procedures. Journal of the ACM 23(3), 398–417 (1976)
7. Jurafsky, D., Martin, J.: Speech and Language Processing. Prentice-Hall, Englewood Cliffs (2000)
8. Montague, R.: Universal grammar. Theoria 36(3), 373–398 (1970)
9. Pratt, I.: Data complexity of the two-variable fragment with counting quantifiers. Information and Computation 207(8), 867–888 (2008)
10. Pratt, I., Third, A.: More fragments of language. Notre Dame Journal of Formal Logic 47(2), 151–177 (2006)
11. Pratt-Hartmann, I.: Computational complexity of controlled natural languages. In: Proc. of the Workshop in Controlled Languages CNL 2009 (2009)
12. Schaerf, A.: On the complexity of the instance checking problem in concept languages with existential quantification. Journal of Intelligent Information Systems 2(3), 265–278 (1993)
13. Slavkovic, M.: Deep analysis for an interactive question answering system. Master's thesis, Free University of Bozen-Bolzano (2007)
14. Staab, S., Studer, R. (eds.): Handbook on Ontologies. Int. Handbooks on Information Systems. Springer, Heidelberg (2004)
15. Szymanik, J.: The computational complexity of quantified reciprocals. In: Proc. of the 2007 Tbilisi Colloquium, TbiLLC 2007 (2007)
16. Thorne, C., Calvanese, D.: Tree shaped aggregate queries over ontologies. In: Proc. of the 8th Int. Conf. on Flexible Query Answering Systems FQAS 2009 (2009)
17. Vardi, M.: The complexity of relational query languages. In: Proc. of the 14th Annual ACM Symposium on Theory of Computing, STOC 1982 (1982)

# Extending Syllogistic Reasoning

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**Abstract.** In this paper syllogistic logic is extended first to full propositional logic, and then an interesting fragment of predicate logic that includes relations.

## 1 Introduction

Traditional logic, also known as term logic, is a loose term for the logical tradition that originated with Aristotle and survived until the advent of algebraic logic and modern predicate logic in the mid and late nineteenth century, respectively. Modern logicians used quite a number of arguments as to why traditional logic should be abandoned. First and foremost, the complaint is that traditional logic is *not rich enough* to account for mathematical reasoning, or to give a serious semantics of natural language. It only deals with part of monadic predicate logic, which doesn't say anything about multiple quantification. Russell (1900) blamed the traditional logical idea that every sentence is of subject-predicate form for giving sentences misleading logical forms. Due to the development of Montague grammar and especially Generalized Quantifier Theory in the 1960s-1980s the misleading form thesis of early proponents of modern logic is not a mainstream position anymore, and analyzing sentences in subject-predicate form is completely accepted again.

In this paper I will first quickly discuss traditional Aristotelian syllogistics, and how to extend it (also semantically) with negative and singular terms. Afterwards I will discuss how full propositional logic can be seen as an extension of Aristotelian syllogistics. Thus, in distinction with polish logicians like Lukasiewicz and others, I won't assume that to understand traditional logic we have to presuppose propositional logic<sup>†</sup> but instead formulate propositional logic by presupposing syllogistic reasoning. It is well-known that once we have full monadic predicate logic, we have full propositional logic as well. Indeed, syllogistics can be viewed as a fragment of propositional logic. In this paper I show how to complete this fragment. Afterwards I will follow (the main ideas, though not the details of) Sommers (1982) and his followers in showing how traditional logic can be extended so as to even account for inferences involving multiple quantification that almost all modern textbooks claim is beyond the reach of traditional logic: A woman is loved by every man, thus Every man loves a woman.

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<sup>†</sup> As explained by van Benthem et al. (to appear), Boole already suggested that propositional logic can be used for checking syllogistic validity.

## 2 From Syllogistics to Propositional Logic

Syllogisms are arguments in which a categorical sentence is derived as conclusion from two categorical sentences as premisses. As is well-known, a categorical sentence is always of one of four kinds: *a*-type ('All men are mortal'), *i*-type ('Some men are philosophers'), *e*-type ('No philosophers are rich'), or *o*-type ('Some men are not philosophers'). A rather standard proof theory **SYL** for syllogistic reasoning with negative terms (if  $P$  is a term,  $\overline{P}$  is a term as well) which only makes use of  $a$  and  $i$  propositions can make use of the fact whether a term occurs distributively, or monotone decreasingly/negatively within a sentence, or not. Denoting a distributed term by  $-$  and an undistributed term by  $+$ , the following follows at once:  $S^-aP^+$ ,  $S^+iP^+$ ,  $S^-eP^-$ , and  $S^+oP^-$ , which we might think of now as a *syntactic* characterisation. The proof system then consists of the following set of axioms and rules (with sentence-negation ' $\neg$ ' defined as follows:  $\neg(SaP) \stackrel{def}{=} SoP$ ,  $\neg(SiP) \stackrel{def}{=} SeP$ ,  $\neg(SeP) \stackrel{def}{=} SiP$ , and  $\neg(SoP) \stackrel{def}{=} SaP$ ):

- |   |                          |
|---|--------------------------|
| (1) $MaP, \Gamma(M)^+ \vdash \Gamma(P)$                                     | Dictum de Omni,          |
| where $\Gamma(M)^+$ is a sentence where $M$ occurs undistributed.           |                          |
| (2) $\vdash TaT$  | Law of identity          |
| (3) $\vdash T \equiv \overline{\overline{T}}$                               | Double negation          |
| (4) $Sa\overline{P} \vdash Pa\overline{S}$                                  | Contraposition           |
| (5) $\Gamma, \neg\phi \vdash \psi, \neg\psi \Rightarrow \Gamma \vdash \phi$ | Reductio per impossibile |
| (f) $\vdash \neg(Ta\overline{T})$ (i.e. $\vdash TiT$ )                      | Existential Import       |

We will now slightly extend syllogistic reasoning in some seemingly innocent ways. First, we add a distinguished 'transcendental' term ' $\top$ ' to our language, standing for something like 'entity'. Obviously, the sentence  $Sa\top$  should always come out true for each term  $S$ . To reflect this, we will add this sentence as an axiom to **SYL**. But adding  $\top$  as an arbitrary term to our language gives rise to a complication once we accept existential import for *all* terms, including negative ones: for negative term  $\overline{\top}$  existential import is unacceptable. One way to get rid of this problem is to restrict existential import to *positive* categorical terms only. Next, we add *singular*, or *individual* terms to our language. In contrast to in standard predicate logic, we will not assume that there is a type-difference between individual terms and standard predicates. Following Leibniz (1966b) and Sommers (1982), we will assume, instead, that for singular propositions,  $a$  and  $i$  propositions coincide, just like  $e$  and  $o$  propositions. Thus, 'Plato sleeps' is represented by a sentence like ' $PaS$ ', which is equivalent with ' $PiS$ '. Finally, we will add a rule (due to Sherperdson, 1956) saying what to do with empty terms. We will denote the system consisting of (1), (2), (3), (4), (5) together with the following four rules by **SYL**<sup>+</sup>.

<sup>2</sup> By this I really mean  $\vdash Ta\overline{\overline{T}}$  and  $\vdash \overline{\overline{TaT}}$ .

<sup>3</sup> By this I mean that *both*  $\psi$  and  $\neg\psi$  can be derived from  $\Gamma$  and  $\neg\phi$ .

<sup>4</sup> I will always assume that  $\phi_1, \phi_2 \vdash \psi$  iff  $\phi_2, \phi_1 \vdash \psi$ .

- (6)  $\vdash \neg(Ta\overline{T})$ , for all *positive* categorical terms  $T$
- (7)  $\vdash Sa\top$
- (8) for all singular terms  $I$  and terms  $P$ :  $IiP \dashv \vdash IaP$ .
- (9)  $\overline{Sa}S \vdash \overline{Sa}P$  (for any  $P$ )

To think of **propositional logic** in syllogistic terms, we will allow for 0-ary predicates as well. We will assume that if  $S$  and  $P$  are terms of the same arity,  $SaP$ ,  $SiP$  etc. are formulas of arity 0. Moreover, if  $S$  and  $P$  are 1-ary predicates, and  $\phi$  a 0-ary predicate, something like  $(Si\overline{P})a\phi$  will be 0-ary predicates as well. Starting with a non-empty domain  $D$ , we will (extensionally) interpret  $n$ -ary terms as subsets of  $D^n$  (thus  $D^0 = \{\langle \rangle\}$ ). If  $S$  and  $P$  are 0-ary or 1-ary terms, the categorical sentences are interpreted as follows:  $V_M(SaP) = \{\langle \rangle : V_M(S) \subseteq V_M(P)\}$ ,  $V_M(SiP) = \{\langle \rangle : V_M(S) \cap V_M(P) \neq \emptyset\}$ , and the  $e$  and  $o$ -propositions as negations of them. It is easy to see that all types of complex propositional formulas can be expressed in categorical terms ( $[\phi]a[\psi] \equiv \phi \rightarrow \psi$ ,  $[\phi]i[\psi] \equiv \phi \wedge \psi$ ,  $[\phi]e[\psi] \equiv \neg\phi$ , and  $[[\phi]e[\psi]]a[\psi] \equiv \phi \vee \psi$ ), and receive the correct interpretation.

Let us see now how things work from a proof-theoretic point of view. To implement the above suggestions, we will add to **SYL**<sup>+</sup> the following three ideas: (i) 0-ary terms don't allow for existential import, (ii)  $\top^0$  is a singular term, and (iii)  $P^0$  is equal to  $\top^0 i P^0$ . The first idea is implemented with the help of axiom (6) by stipulating that 0-ary terms are not categorical.

- (10) 0-ary terms are no categorical and  $\top^0$  is a singular term.
- (11)  $P^0 \dashv \vdash \top^0 i P^0$ .

We will denote the system **SYL**<sup>+</sup> together with (10) and (11) by **SYL**<sup>+</sup>**PL**. The claim of this section of the paper is that this system can indeed account for all inferences in propositional logic. It is almost immediately clear that Modus Ponens, Modus Tollens, the Hypothetical Syllogism, and the Disjunctive Syllogism can be thought of as ordinary valid syllogisms of the form Barbara, Camestres, Barbara, and Camestres, respectively. Also other 'monotonicity-inferences' follow immediately from the Dictum de Omni.

To show that **SYL**<sup>+</sup>**PL** is enough, we will show that also the following hold ' $p \vdash p \vee p$ ', ' $p \vee p \vdash p$ ', ' $p \vee q \vdash q \vee p$ ', and ' $p \rightarrow q \vdash (r \rightarrow p) \rightarrow (r \rightarrow q)$ '. The reason is that we can axiomatize propositional logic by these four rules, together with modus ponens (cf. Goodstein, 1963, chapter 4). We can conclude that propositional logic *follows* from syllogistic logic if we (i) make the natural assumption that propositions are 0-ary terms, (ii) assume that  $\top^0$  is a singular term, and (ii) treat singular terms as proposed by Leibniz.

It is important to realize that we represent  $p \vee q$  by  $\overline{p}aq$ . ' $p \vdash p \vee p$ ' immediately follows from the validity of  $\overline{p}a\top$ , the equivalence  $p \equiv \top ap$  and the Dictum. As for disjunctive elimination, note that because  $p \vee p \equiv \overline{p}ap$ , we can conclude by (9) to  $\overline{p}a\perp$ . Via contraposition and double negation we derive  $\top^0 ap$ . Because  $\top^0$  is a singular term (rule 10)) it follows by (8) that  $\top^0 i p$ , and thus via (11)

that  $p$ . So we have validated  $p \vee p \vdash p$ . It is easier to validate ' $p \vee q \vdash q \vee p$ ': it immediately follows by contraposition and double negation.

Notice that ' $p \rightarrow q \vdash (r \rightarrow p) \rightarrow (r \rightarrow q)$ ' follows from the Dictum if we could make use of the *deduction theorem*:  $\Gamma, P \vdash Q \Rightarrow \Gamma \vdash PaQ$  (for this to make sense,  $P$  and  $Q$  have to be 0-ary terms, obviously). But this deduction theorem follows from **SYL**<sup>+</sup>**PL**: Assume  $\Gamma, P \vdash Q$  and assume towards contradiction that  $\neg(PaQ)$ . This latter formula is equivalent to  $Pi\overline{Q}$ . We have seen above that  $\top iP$  can be derived from  $Pi\overline{Q}$  in **SYL**<sup>+</sup>. Because  $\neg(PaQ) \vdash P$ , it follows from the assumption  $\Gamma, P \vdash Q$  that  $\Gamma, \neg(PaQ) \vdash Q$ . From this we derive  $\top aQ$ , and together with the validity of  $Pa\top$  we derive via the Dictum that  $PaQ$ . Thus, from  $\Gamma$  and assuming  $\neg(PaQ)$  we derive a contradiction:  $\Gamma, \neg(PaQ) \vdash \neg(PaQ), PaQ$ . By the reductio-rule we conclude that  $\Gamma \vdash PaQ$ .

### 3 Relations

Traditional logicians were well aware of an important limitation of syllogistic reasoning. In fact, already Aristotle recognized that the so-called 'oblique' terms (i.e. ones expressed in a grammatical case other than the nominative) gives rise to inferences that cannot be expressed in the ordinary categorical syllogistic. An example used by Aristotle is 'Wisdom is knowledge, Of the good there is wisdom, thus, Of the good there is knowledge'. This is intuitively a valid inference, but it, or its re-wording, is not syllogistically valid: 'All wisdom is knowledge, Every good thing is object of some wisdom, thus, Every good thing is object to some knowledge'. The re-wording shows that we are dealing with a binary relation here: 'is object of'. Aristotle didn't know how to deal with such inferences, but he noted that *if* there is a syllogism containing oblique terms, there must be a corresponding syllogism in which the term is put back into the nominative case.

It is generally assumed that in traditional formal logic there is no scope for relations. Thus — or so the Frege-Russell argument goes — it can be used neither to formalize natural language, nor to formalize mathematics. What we need, — or so Frege and Russell argued — is a whole new logic. But the Frege-Russell argument is only partly valid: instead of inventing a whole new logic, we might as well just extend the traditional fragment. As far as semantics is concerned, it is well-known how to work with relations. The main challenge, however, is to embed relations into the traditional theory, and to extend the inference rules such that also proofs can be handled that crucially involve relations. As it turns out, part of this work has already been done by medieval logicians, and also by people like Leibniz and de Morgan when they were extending syllogistic reasoning such that it could account for inferences involving oblique terms, or relations.

We want to combine relations with monadic terms by means of the 'connectives'  $a, i, e$ , and  $o$  to generate new terms. This will just be a generalization of what we did before: When we combine a monadic term  $P$  with a monadic term  $S$  (and connective ' $a$ ', for instance), what results is a new 0-ary term like  $SaP$ . The generalization is now straightforward: if we combine an  $n$ -ary term/relation  $R$  with a monadic term  $S$  (and connective ' $a$ ', for instance), what results is a

new  $n-1$ -ary term  $(S^1aR^n)^{n-1}$ . The semantics should now determine what such new terms denote. The  $n-1$  ary term  $(S^1aR^n)^{n-1}$ , for instance, would denote  $\{\langle d_1, \dots, d_{n-1} \rangle : V_M(S) \subseteq \{d_n \in D : \langle d_1, \dots, d_n \rangle \in V_M(R^n)\}\}$  <sup>5</sup>

Now we can represent the natural reading of a sentence like ‘Every man loves a woman’ as  $Ma(WiL^2)$ . The meaning of this formula is calculated as follows:

$$V_M(Ma(WiL^2)) = \{\langle \rangle : I(M) \subseteq \{d \in D : \langle d \rangle \in V_M(WiL^2)\}, \\ \text{with } V_M(WiL^2) = \{d_1 : I(W) \cap \{d_2 \in D : \langle d_1, d_2 \rangle \in I(L^2)\} \neq \emptyset\}.$$

To represent the sentence ‘There is woman who is loved by every man’ we will follow medieval practice and make use of the *passive form* of ‘love’: *being loved by*. For every binary relation  $R$ , we represent its passive form by  $R^\cup$ , interpreted as indicated above:  $V_M(R^\cup) = \{\langle d_2, d_1 \rangle : \langle d_1, d_2 \rangle \in I_M(R)\}$  <sup>6</sup> Now we represent ‘There is woman who is loved by every man’ as follows:  $Wi(MaL^\cup)$ . This sentence is true iff:

$$V_M(Wi(MaL^\cup)) = \{\langle \rangle : I(W) \cap \{d \in D : \langle d \rangle \in V_M(MaL^\cup)\} \neq \emptyset\}, \\ \text{with } V_M(MaL^\cup) = \{\langle d_1 \rangle : I(W) \subseteq \{d_2 \in D : \langle d_1, d_2 \rangle \in V_M(L^\cup)\}\}.$$

Both truth conditions are intuitively correct, and correspond with those of the two first-order formulas  $\forall x[M(x) \rightarrow \exists y[W(y) \wedge L(x, y)]]$  and  $\exists y[W(y) \wedge \forall x[M(x) \rightarrow \wedge L(x, y)]]$ , respectively.

What we want to know, however, is how we can reason with sentences that involve relations. Let us first look at the re-wording of Aristotle’s example: ‘All wisdom is knowledge, Every good thing is object of some wisdom, thus, Every good thing is object of some knowledge’. If we translate this into our language this becomes  $WaK, Ga(WiR) \vdash Ga(KiR)$ , with ‘ $R$ ’ standing for ‘is object of’. But now observe that we immediately predict that this inference is valid by means of the Dictum de Omni, if we can assume that ‘ $W$ ’ occurs positively in ‘ $Ga(WiR)$ ’! We can mechanically determine that this is indeed the case. <sup>7</sup> First, we say that if a sentence occurs out of context, the sentence occurs positively. From this, we determine the positive and negative occurrences of other terms as follows:

$\overline{P}$  occurs positively in  $\Gamma$  iff  $P$  occurs negatively in  $\Gamma$ .

If  $(SaR)$  occurs positively in  $\Gamma$ , then  $S^-aR^+$ , otherwise  $S^+aR^-$ .

If  $(SiR)$  occurs positively in  $\Gamma$ , then  $S^+iR^+$ , otherwise  $S^-iR^-$ .

Thus, first we assume that ‘ $Ga(WiR)$ ’ occurs positively. From this it follows that the ‘ $WiR$ ’ occurs positively, from which it follows in turn that ‘ $W$ ’ occurs

<sup>5</sup> This by itself is not general enough. To express the mathematical property of *density*, for instance, we need to be able to combine a binary relation with a ternary relation.

<sup>6</sup> Of course, the active-passive transformation only works for binary relations. For more-ary relations it fails. Fortunately, we can do something similar here, making use of some functions introduced by Quine in his proof that variables are not essential for first-order predicate logic. We won’t go into this here.

<sup>7</sup> Cf. Sommers (1982) and van Benthem (1983).

positively. Assuming that ‘ $WaK$ ’ is true, the Dictum allows us to substitute  $K$  for  $W$  in  $Ga(WiR)$ , resulting in the desired conclusion:  $Ga(KiR)$ . Something very similar was done by medieval logicians (cf. Buridan, 1976).

As far as I know, this is how far medieval logicians went. But it is not far enough. Here is one classical example discussed by Leibniz (1966a): ‘Every thing which is a painting is an art (or shorter, painting is an art), thus everyone who learns a thing which is a painting learns a thing which is an art’ (or shorter: everyone who learns painting learns an art). Formally:  $PaA \vdash (PiL^2)a(AiL^2)$ . Semantically it is immediately clear that the conclusion follows from the premiss. But the challenge for traditional logic was to account for this inference in a proof-theoretic way. As Leibniz already observed, we can account for this inference in traditional logic if we add the extra (and tautological) premiss ‘Everybody who learns a thing which is a painting learns a thing which is a painting’, i.e.  $(PiL^2)a(PiL^2)$ . Now  $(PiL^2)a(AiL^2)$  follows from  $PaA$  and  $(PiL^2)a(PiL^2)$  by means of the *Dictum the Omni*, because by our above rules the second occurrence of ‘ $P$ ’ in  $(PiL^2)a(PiL^2)$  occurs in a monotone increasing position.<sup>8</sup>

To account for other inferences we need to assume more than just a tautological premiss. For instance, we cannot yet account for the inference from ‘There is a woman who is loved by every man’ represented by  $Wi(MaL^\cup)$  to ‘Every man loves a woman’ represented by  $Ma(WiL^2)$ . In standard predicate logic one can easily prove the equivalence of  $\forall x[M(x) \rightarrow \forall y[W(y) \rightarrow R(x, y)]]$  with  $\forall y[W(y) \rightarrow \forall x[M(x) \rightarrow R(x, y)]]$ . But in contrast to predicate logic, our system demands that the sequence of arguments of a relational term is in accordance with the scope order of the associated terms. Because of this, we have to use something like passive transformation to express ‘reverse scope’. Thus, to reason with relations, we have to say which rules passive transformation obeys. To do so, we will follow medieval logicians such as Buridan and enrich our system **SYL+PL** with the rule of *oblique conversion* (12), and the passive rule (13) (for binary relations  $R$ , and predicates  $S$  and  $O$ ), and the more general formulation of the Dictum in (1’):

- (12) Oblique Conversion:  $Sa(OaR) \equiv Oa(SaR^\cup)$   
 from ‘every man loves every woman’ we infer that ‘every woman is loved by every man’ and the converse of this.<sup>9</sup>
- (13) Double passive:  $R^{\cup\cup} \equiv R$
- (1’) **Dictum de Omni**:  $\Gamma(MaR)^+, \Theta(M)^+ \vdash \Gamma(\Theta(R))$

<sup>8</sup> In terms of our framework, Leibniz assumed that all terms being part of the predicate within sentences of the form  $SaP$  and  $SiP$  occur positively. But this is not necessarily the case once we allow for all types of complex terms:  $P$  doesn’t occur positively in  $Sa(PaR)$ , for instance. On Leibniz’s assumption, some invalid inferences can be derived (cf. Sanches, 1991). These invalid inferences are blocked by our more fine-grained calculation of monotonicity marking.

<sup>9</sup> From this rule we can derive that  $Si(OiR) \equiv Oi(SiR^\cup)$  is also valid. And that is correct: the sentence ‘A man loves a woman’ is truth-conditionally equivalent to ‘A woman is loved by a man’.

Let us see how we can account for the inference from  $Wi(MaL^U)$  to  $Ma(WiL)$ :

- |                                  |   |
|----------------------------------|---|
| 1. $Wi(MaL^\cup)$                | premiss   |
| 2. $(MaL^\cup)a(MaL^\cup)$       | a tautology (everyone loved by every man is loved by every man)                                       |
| 3. $Ma((MaL^\cup)aL^{\cup\cup})$ | from 2 and (12) ( $S = (MaL^\cup)$ and $S' = M$ )   |
| 4. $Ma((MaL^\cup)aL)$            | by 3 and (13), substitution of $L$ for $L^{\cup\cup}$   |
| 5. $Ma(WiL)$                     | by 1 and 4, by the Dictum de Omni (1') <span style="border: 1px solid red; padding: 0 2px;">10</span> |

Many other examples can be accounted for in this way as well.

## 4 A ‘natural’ Fragment of Predicate Logic

In this paper I have argued with Sommers (1982) and others that interesting parts of standard logic could have, and perhaps even have been, developed naturally out of traditional syllogistics. Singular propositions straightforwardly fit into the system, and the syllogistics can naturally be extended to account for propositional reasoning, and even for reasoning with relational terms. This is interesting, because syllogistic logic seems much closer to natural language than first-order predicate logic. In contrast to the latter, the former analyses sentences as being of subject-predicate structure. Indeed, the idea that natural language is misleading because it systematically analyses sentences in a wrong way is foreign to syllogistic logic. Syllogistic logic is also of great interest for empirical linguistics because it makes essential use of the *Dictum de Omni*, distributivity, or monotonicity. Though Geach (1962) famously argued that the traditional theory of distribution was hopelessly confused, developments in modern Generalized Quantifier Theory shows that the traditional notion of monotonicity is of great value for linguistic and logical purposes. Moreover, Geurts & van der Silk (2005) showed experimentally that inferences based on monotonicity are ‘easy’ in an interesting sense.

Though we used neither a distinguished relation of identity, nor make use of variables to allow for binding, we have seen that we could nevertheless adequately express many types of sentences for which these tools are normally used in predicate logic. This doesn't mean that our extended syllogistics is as expressive as standard first-order logic. What we cannot (yet) represent are sentences which *crucially* involve variables/pronouns and/or identity. Some examples for which these tools are crucial are the following: 'Every/some man loves himself', 'All parents love their children', 'Everybody loves somebody else', 'There is a unique king of France', and 'At least 3 men are sick'. As it turns out, we can extend our language with *numerical quantifiers* (cf. Murphee, 1997) and Quinean predicate functors (Quine, 1976) to solve these problems, but these extensions have their price. Pratt-Hartmann (2009) shows that syllogistic systems with numerical quantifiers cannot be axiomatized, and adding Quinean predicate functors forces the fragment to jump over the decidability border. In the formal system we have

<sup>10</sup> With  $\Gamma = Ma$ ,  $\Theta = Wi$ ,  $R = L$ , and  $M = MaL^\cup$ .

so far, the sequence of arguments of a relational term will always be in accordance with the scope order of the associated terms. Thinking of this system as a fragment of FOL means that this logic has a very interesting property. Following an earlier suggestion of Quine, Purdy (1998) shows that the limits of decidability are indeed close to the limits of what can be expressed in (our fragment of) traditional formal logic.<sup>11</sup> This suggests that the extended syllogistic system discussed in this paper — the part of logic where we don't essentially need variables or predicate functors — is indeed a very interesting part of logic. Indeed, a small contingent of modern logicians (e.g. Suppes, Sommers, van Benthem, Sanchez, Purdy, Pratt-Hartmann, Moss) seek to develop a system of *natural logic* which is very close to what we have done in this paper in that it crucially is essentially variable-free.

## References

1. van Benthem, J.: A linguistic turn: New directions in logic. In: Marcus, R., et al. (eds.) *Logic, Methodology and Philosophy of Science*, Salzburg, pp. 205–240 (1983)
2. van Benthem, J., et al.: *Logic in Action*, Amsterdam (to appear)
3. Burdidan, J.: *Tractatus de Consequentibus*. In: Hubien, H. (ed.) *Johannis Buridani tractatus de consequentiis*, critical edn. Publications universitaires, Louvain (1976)
4. Geach, P.: *Reference and Generality. An Examination of Some Medieval and Modern Theories*. Cornell University Press, Oxford (1962)
5. Geurts, B., van der Slik, F.: Monotonicity and processing load. *Journal of Semantics* 22, 97–117 (2005)
6. Goodstein, R.L.: *Boolean Algebra*. Pergamon Press, London (1963)
7. Leibniz, G.: A specimen of a demonstrated inference from the direct to the oblique. In: Parkinson (ed.) *Leibniz. Logical Papers*, pp. 88–89 (1966a)
8. Leibniz, G.: A paper on some logical difficulties. In: Parkinson (ed.) *Leibniz. Logical Papers*, pp. 115–121 (1966b)
9. Lukasiewicz, J.: *Aristotle's Syllogistic from the standpoint of Modern Formal Logic*. Garland Publishers, New York (1957)
10. Lyndon, R.C.: Properties preserved under homomorphism. *Pacific Journal of Mathematics* 9, 142–154 (1959)
11. McIntosh, C.: Appendix F. In: Sommers, F. (ed.) *The Logic of Natural Language*, pp. 387–425. Clarendon Press, Oxford (1982)
12. Mortimer, M.: On language with two variables. *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* 21, 135–140 (1975)
13. Murphee, W.A.: The numerical syllogism and existential presupposition. *Notre Dame Journal of Formal Logic* 38, 49–64 (1997)
14. Pratt-Hartmann, I.: No syllogisms for the numerical syllogistic. In: Grumberg, O., et al. (eds.) *Francez Festschrift. LNCS*, vol. 5553, pp. 192–203. Springer, Berlin (2009)

<sup>11</sup> As noted by Jan van Eijck (p.c.), the extensions of syllogistics explicitly discussed in this paper are more easily seen to be decidable: they are all in the two-variable fragment of first-order logic, which was proved to have the finite model property by Mortimer (1975), and hence has decidable satisfiability. However, an extension of the formal system with ternary relations along the same lines, for instance, is natural, and still proved to be decidable by the results of Purdy (1998).

15. Pratt-Hartmann, I., Moss, L.: Logics for the Relational Syllogistics. Review of Symbolic Logic (to appear)
16. Purdy, W.: Fluted formulas and the limits of decidability. *Journal of Symbolic Logic* 61, 608–620 (1996)
17. Quine, W.V.O.: Algebraic logic and predicate functors, pp. 283–307 (1976)
18. Russell, B.: A critical exposition of the Philosophy of Leibniz. Cambridge University Press, Oxford (1900)
19. Sanchez, V.: Studies on Natural Logic and Categorical Grammar, PhD thesis, Universiteit van Amsterdam (1991)
20. Sherpherdson, J.: On the interpretation of Aristotelian syllogistic. *Journal of Symbolic Logic* 21, 131–147 (1956)
21. Sommers, F.: The Logic of Natural Language. Clarendon Press, Oxford (1982)

# Internal and Interval Semantics for CP-Comparatives

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**Abstract.** The interval degree semantics for clausal (CP)-comparatives given in [5] is shown to be equivalent to a point degree semantics in which the CP-external degree relation is interpreted internal to the CP.

**Keywords:** semantics of comparatives, degrees, measure functions, interval semantics.

## 1 Semantics for DP-Comparatives

- (1) John is *taller /shorter /three cm more tall /less than three cm less short...* than Mary/ every girl/ at most three girl...

DP-comparatives are shown in (1): comparatives with a noun phrase complement (DP). In [3] I develop a theory which compositionally builds up the interpretations of the comparative expressions (italicized in (1)) as 2-place relations between degrees. A sketch of the semantic composition is provided in the appendix.

For measure  $H$  (height) and measure unit  $cm$  (centimeters), the domain of height-in- $cm$  degrees is the set  $\Delta_{H,cm}$  of all triples  $\langle r, cm, H \rangle$ , with  $r$  a real number. Relative to world  $w$ , the height-in- $cm$  measure function  $H_{cm,w}$  is a partial function from individuals to height-in- $cm$  degrees. For ease of presentation, I will assume throughout this paper a context which fixes the unit of measuring height as  $cm$ , even if the unit is not lexically present. The composition derives the following sample interpretations: (for degree  $\delta$ ,  $\delta^r$  is the first element of  $\delta$ , the real value):

<i>taller</i>	$\lambda\delta_2\lambda\delta_1 \in \Delta_{H,cm}: \delta_1^r > \delta_2^r$
<i>shorter</i>	$\lambda\delta_2\lambda\delta_1 \in \Delta_{H,cm}: \delta_1^r < \delta_2^r$
<i>more than three cm taller</i>	$\lambda\delta_2\lambda\delta_1 \in \Delta_{H,cm}: \delta_1^r > \delta_2^r + 3$
<i>less than three cm taller</i>	$\lambda\delta_2\lambda\delta_1 \in \Delta_{H,cm}: \delta_1^r < \delta_2^r + 3$
<i>more than three cm shorter</i>	$\lambda\delta_2\lambda\delta_1 \in \Delta_{H,cm}: \delta_1^r < \delta_2^r - 3$

The methodology of the theory is to keep the derivation at the level of degrees for as long as possible. Thus, the above degree relations only turn into relations to individuals when they combine with individual arguments, like the complements in the examples in (1). At that point, a type shifting operation of Composition with the Measure Function shifts them to relations to individuals:

Let  $P$  and  $R$  be 1-place and 2-place predicates of height-in-cm degrees.

$$P \circ H_{cm,w} = \lambda x. P(H_{cm,w}(x))$$

$$R \circ H_{cm,w} = \lambda y \lambda \delta_1. R(\delta_1, H_{cm,w}(y))$$

For example, to combine with complement *every girl*, *taller* shifts to:

$$taller \quad \lambda y \lambda \delta_1. \delta_1^r > H_{cm,w}(y)^r$$

Importantly, at the point that the comparative combines with its DP complement it is an extensional relation from degrees to individuals. We assume that all such relations fall under Montague's analysis in [4] for relations to individuals:

**Montague's Principle:** the interpretation of the DP-complement of a relation to individuals takes scope over that relation.

This gives us the following interpretation schema for DP-comparatives:

$$\alpha \text{ than DP} \quad \lambda \delta_1. \mathbf{DP}(\lambda y. \alpha(\delta_1, H_{cm,w}(y)))$$

With this, the analysis of DP comparatives is in essence an extension of that of Hoeksema's in [2]. Examples of predicted interpretations are given in (2):

- (2) a. taller than every girl  $\lambda \delta. \forall y [GIRL_w(y) \rightarrow \delta^r > H_{cm,w}(y)^r]$   
     any height above the height of the tallest girl  
     b. taller than some girl  $\lambda \delta. \exists y [GIRL_w(y) \wedge \delta^r > H_{cm,w}(y)^r]$   
     any height above the height of the shortest girl  
     c. less than 2 cm taller than Mary  $\lambda \delta. \delta^r < H_{cm,w}(Mary)^r + 2$   
     any height below Mary's height plus 2 cm  
     d. taller than exactly three girls  
     any height above height of the 3<sup>rd</sup> shortest girl and below that of any other girl.  
     e. more than 2 cm shorter than Mary  $\lambda \delta. \delta^r < H_{cm,w}(Mary)^r - 2$   
     any height below two cm below Mary's height

## 2 Semantics for CP-Comparatives

CP-comparatives are comparatives with a clausal complement, like in (3):

- (3) John is *taller* than Mary is *–*/ every girl is *–*/ at most three girls are *–*

The external comparative is the same as for DP comparatives. In this paper, I restrict my attention to CP comparatives of the form in (3):

$$\alpha \text{ than } [_{CP} DP \text{ is } [_{PRED-}]] \quad (\text{where } \alpha \text{ is the comparative relation})$$

I will start by making some uncontroversial assumptions about CP comparatives.

1. The CP complement is syntactically an operator-gap construction.
2. The operator-gap construction is semantically interpreted. This means that the gap involves a semantic variable that is abstracted over at the CP level.

3. The variable abstracted over is a degree variable.

4. The gap following the copula is a predicate gap.

(4) means that the gap will be interpreted as a predicate of individuals, to fit with the DP subject. Since the gap is based on a degree variable, it is natural to assume that it is in fact interpreted as a degree predicate, which shifts to a predicate of individuals though Composition with the Measure Function. What else is part of the interpretation of the gap depends on one's theory; this is indicated by relation variable **R** to be fixed by one's theory. This gives the following interpretation of the gap:

$$[\text{PRED} - ] \quad \lambda\delta_1. \mathbf{R}(\delta_1, \delta) \quad \text{with } \delta \text{ the variable bound at the CP level.}$$

We assume that  $\alpha$  denotes height-in-cm degree relation  **$\alpha$** , determining measure function  $H_{\text{cm},w}$ . We compose the predicate  $\lambda\delta_1. \mathbf{R}(\delta_1, \delta)$  with  $H_{\text{cm},w}$ , apply the DP subject, and abstract over degree variable  $\delta$  at the CP level. What else is part of the CP interpretation depends on one's theory, this is indicated by operation variable **M** to be fixed by one's theory. We have derived for the general case:

$$\begin{array}{ll} \alpha & \text{than } [\text{CP} \quad \text{DP is } [\text{PRED} - ] ] \\ \mathbf{\alpha} & - \quad \mathbf{M}(\lambda\delta. \text{DP}(\lambda y. \mathbf{R}(\delta, H_{\text{cm},w}(y)))) \end{array}$$

The advantage of this schema is that different theories fit into it. For instance, two influential approaches to comparatives are those by von Stechow and by Heim:

$$\begin{array}{ll} \text{Von Stechow [6]: } \mathbf{M} = t_{<} & \mathbf{R} = = \\ & \alpha \text{ than } [\text{CP} \quad \text{DP is } [\text{PRED} - ] ] \\ & \lambda\delta_1 \mathbf{\alpha}(\delta_1, t_{<}(\lambda\delta. \text{DP}(\lambda y. \delta^r = H_{\text{cm},w}(y)^r))) \\ \text{Heim [1]: } \mathbf{M} = t_{<} & \mathbf{R} = \lambda\delta_2 \lambda\delta_1. 0 < \delta_1^r \leq \delta_2^r \\ & \alpha \text{ than } [\text{CP} \quad \text{DP is } [\text{PRED} - ] ] \\ & \lambda\delta_1 \mathbf{\alpha}(\delta_1, t_{<}(\lambda\delta. \text{DP}(\lambda y. 0 < \delta^r \leq H_{\text{cm},w}(y)^r))) \end{array}$$

Sample predictions: von Stechow:

taller than some girl is – = any height above the height of the tallest girl

Heim: taller than every girl is – = any height above the height of the shortest girl

These approaches are criticized in [5] and in [3].

Central in the present paper is what I will call here the Internal Theory:

$$\begin{array}{ll} \text{The Internal Theory: } \mathbf{M} = \lambda P. P & \mathbf{R} = \mathbf{\alpha} \\ & \alpha \text{ than } [\text{CP} \quad \text{DP is } [\text{PRED} - ] ] \\ & \lambda\delta. \text{DP}(\lambda y. \mathbf{\alpha}(\delta, H_{\text{cm},w}(y))) \end{array}$$

In the Internal Theory, comparison relation  **$\alpha$**  is interpreted **inside** the CP at the position of the gap (and there is no **M**). Inspection of the semantics for DP comparatives discussed above should convince one that:

**Prediction of the Internal Theory: (i) and (ii) are equivalent:**

- (i)  $\alpha$  than DP
- (ii)  $\alpha$  than  $[\text{CP} \quad \text{DP is } - ]$

For discussion of the advantages and disadvantages of this theory, and a proposal for improving it, see [3].

### 3 Internal Semantics and Interval Semantics

So far, degrees have been point degrees: real numbers indexed for unit and measure, and the semantics has been a point degree semantics. [5] develops an interval semantics for CP-comparatives, based on *interval degrees*. I will call the proposal SW. I will show in this section that *as a semantics for CP-comparatives* SW is equivalent to the Internal Theory of CP-comparatives. This means that, despite claims to the contrary, the merits of this approach to CP-comparatives are not due to the interval semantics. On the positive side, the result will allow us to credit the Internal Theory of CP-comparatives to Schwarzschild and Wilkinson.

In the course of the following argument, I will simplify, modify, even at one point improve SW, keeping in mind that I am *only* concerned with the theory as an analysis of CP-comparatives. Also, I will explain the fine points of SW only as we go along.

I start with a first simplifying assumption. For SW, interval degrees are primitives, ordered in an interval structure. I will assume set theoretic interval structures, lifted from intervals as sets of points. A caveat: SW deviate from standard terminology in not requiring intervals to be convex: interval degrees are sets of points, not necessarily uninterrupted. This issue will be important later.

My second simplifying assumption concerns vagueness: I ignore it here. That is, changing from point degrees to interval degrees may be useful for vagueness: I am 1 meter 76 cm, according to a certain standard of precision. We can let 1.76 be an interval containing the point 1.76 and the points around it that form the margin of error. I do not object to such use of intervals, but will ignore it, because the problems concerning the semantics of quantificational DPs inside CP-comparatives are independent of vagueness. Thus, I will assume (with section 1) that the measure function assigns point degrees to individuals.

Thirdly, for ease of notation, I will ignore in this section the distinction between degree triples and their real value, writing  $\delta$  where I should write  $\delta^f$ .

With SW, we are concerned with the semantics of the following schema:

- (4)  $DP_1$  is  $\beta$ -taller than  $DP_2$  is –.  
 where  $\beta$  is: *at least two cm, at most two cm, exactly two cm,  $\emptyset$ , ....*

The semantics SW propose for (4) is (5) (based on their example (82), p. 23):

- (5)  $\exists j[ DP_1 \text{ is } j\text{-tall} \wedge DP_2 \text{ is } \max(\lambda i. \beta(j-i))\text{-tall} ]$

Here  $i$  and  $j$  are variables over interval degrees,  $j-i$ , the difference of  $j$  and  $i$ , is an interval degree, and  $\beta$  is a predicate of interval degrees. With this,  $\lambda i. \beta(j-i)$  is also a predicate of interval degrees, and  $\max(\lambda i. \beta(j-i))$  is again an interval degree.

Thus, *John is at least two cm taller than Mary* is true if for some interval degrees  $j$  and  $k$ , John is  $j$ -tall and Mary is  $k$ -tall, and  $k$  is interval degree  $\max(\lambda i. \textbf{at least two cm}(j-i))$ , whatever that is.

Schwarzschild and Wilkinson in [5] are not concerned with the external subject, only with quantificational DPs inside the CP-comparative. I will follow them, ignore the external subject and focus on the predicate:

(6)  $\beta$ -taller than DP is  $-$ .

$$\lambda x. \exists j [ x \text{ is } j\text{-tall} \wedge \text{DP is } \max(\lambda i. \beta(j-i))\text{-tall} ]$$

We look at the expression  $x$  is  $j$ -tall. I will write  $j\text{-tall}(x)$ .  $\dots\text{-tall}(\dots)$  is a relation between individuals and interval degrees; hence  $j\text{-tall}(\dots)$  is a predicate of individuals, and  $\dots\text{-tall}(x)$  a predicate of degrees. SW constrains  $\dots\text{-tall}(x)$  as follows:

Set of interval degrees  $I$  is a **proper filter** iff

1. if  $i \in I$  and  $i \subseteq j$  then  $j \in I$  (**persistence**)
2. if  $i \in I$  and  $j \in I$  then  $i \cap j \in I$  (**overlap**)
3.  $\emptyset \notin I$  (**properness**)

**Constraint:** for every individual  $x$ :  $\dots\text{-tall}(x)$  is a **proper filter**.

**Persistence** allows us to introduce a notion of height-ballpark. Suppose the heights of the girls vary from 1 meter 55 cm to 1 meter 72 cm. Then the smallest girl is  $[1.55, 1.55]\text{-tall}$  and the tallest girls is  $[1.72, 1.72]\text{-tall}$ . With persistence, each of the girls is  $[1.55, 1.72]\text{-tall}$ .

Thus the interval  $[1.55, 1.72]$  is the semantic ballpark within which we find the height of all the girls. The idea is: we compare John's height with that of the girls by comparing John's height with the ballpark interval for the girls.

**Overlap** and **properness** express degree-consistency. For instance, you cannot be both  $[1.72, 1.72]\text{-tall}$  and  $[1.74, 1.74]\text{-tall}$ , since then, by overlap, you should be  $[1.72, 1.72] \cap [1.74, 1.74]\text{-tall}$ , which is  $\emptyset\text{-tall}$ , and the latter is ruled out by properness.

Our assumption that for individual  $x$ ,  $H_{cm,w}(x)$  is a point allows a simplification: for individual  $x$  we just define  $\lambda i. i\text{-tall}(x)$  as the proper-filter **generated by**  $H_{cm,w}(x)$ :

**Ultrafilter:** For every individual  $x$ :  $\lambda i. i\text{-tall}(x) = \{i: H_{cm,w}(x) \in i\}$

With this, (6) becomes (7):

(7)  $\beta$ -taller than DP is  $-$ .

$$\lambda x. \exists j [ H_{cm,w}(x) \in j \wedge \text{DP is } \max(\lambda i. \beta(j-i))\text{-tall} ]$$

I will now argue that this account needs a correction. Look at (8):

(8) John is exactly two cm taller than Mary is  $-$ .

$$\exists j [ H_{cm,w}(\text{JOHN}) \in j \wedge \text{Mary is } \max(\lambda i. 2 \text{ cm}(j-i))\text{-tall} ]$$

What we need to understand at this point about the meaning of the second conjunct is that it associates with Mary a height-interval  $\max(\lambda i. 2 \text{ cm}(j-i))$ , the upper bound of which is exactly two cm below the lower bound of interval  $j$  and which has  $H_{cm,w}(\text{MARY})$  as upper bound (maximum). Let us set up the problem.

Assume that  $H_{cm,w}(\text{JOHN}) = 1.78$  and  $H_{cm,w}(\text{MARY}) = 1.72$ .

This means that  $[1.78, 1.78]\text{-tall}(\text{JOHN})$  and  $[1.72, 1.72]\text{-tall}(\text{MARY})$ .

Take the interval  $[1.74, 178]$ . Since  $[1.78, 1.78]$ -*tall*(JOHN), by persistence  $[1.74, 1.78]$ -*tall*(JOHN).

Given the meaning of  $\max(\lambda i. \mathbf{2\ cm}(j-i))$ -*tall*, it follow that:

$H_{\text{cm},w}(\text{JOHN}) \in [1.74, 178] \wedge \text{Mary is } \max(\lambda i. \mathbf{2\ cm}([1.74, 178]-i))$ -*tall* ]

Hence (9) is true:

(9)  $\exists j[ H_{\text{cm},w}(\text{JOHN}) \in j \wedge \text{Mary is } \max(\lambda i. \mathbf{2\ cm}(j-i))$ -*tall* ]

And so (8) is predicted to be true, incorrectly, because (8) is false in this context.

Clearly, the statement  $H_{\text{cm},w}(x) \in j$  in (8) is too weak: it needs to be replaced by a statement that makes  $H_{\text{cm},w}(x)$  the *lower bound* of the interval  $j$  in (8). In fact, since  $x$  is an individual, and we ignore vagueness, we solve the problem by requiring that  $j$  is the *point*  $H_{\text{cm},w}(x)$ . To simplify notation, I will set:  $[r,r] = r$ , for real number  $r$  (thus  $r$  itself counts as an interval) and we get:

(10)  $\beta$ -taller than DP is –.  
 $\lambda x. \text{DP is } \max(\lambda i. \beta(H_{\text{cm},w}(x)-i))$ -*tall* ]

(11) John is exactly two cm taller than Mary is –.  
 $\text{Mary is } \max(\lambda i. \mathbf{2\ cm}(H_{\text{cm},w}(\text{JOHN})-i))$ -*tall* ]

Given the description of the intended meaning of  $\max(\lambda i. \mathbf{2\ cm}(H_{\text{cm},w}(\text{JOHN})-i))$ -*tall*, this means that the interval with Mary's height as maximum is 2 cm below John's height, which is, of course, what we want.

We assume that (10) is derived through Composition with the Measure Function, and get (12), where  $\delta$  is a point degree:

(12)  $\beta$ -taller than DP is –.  
 $\lambda \delta. \text{DP is } \max(\lambda i. \beta(\delta-i))$ -*tall* ]

While it is clear from [5] what (12) means, it is not clear how (12) is derived, since [5] does not give an implementation of the grammar. This means that we can apply some charity: the obvious implementation is (13); (13) assigns to all examples discussed in [5] the same truth conditions as assigned in [5]; hence we take (12) to mean (13):

(13)  $\beta$ -taller than DP is –.  
 $\lambda \delta. \text{DP}(\lambda y. \max(\lambda i. \beta(\delta-i))$ -*tall*( $y$ ))

With the ultrafilter analysis of the degree predicates we have:

$\max(\lambda i. \beta(\delta-i))$ -*tall*( $y$ ) iff  $H_{\text{cm},w}(y) \in \max(\lambda i. \beta(\delta-i))$

Hence, (13) is equivalent to (14):

(14)  $\beta$ -taller than DP is –.  
 $\lambda \delta. \text{DP}(\lambda y. H_{\text{cm},w}(y) \in \max(\lambda i. \beta(\delta-i)))$

This we can rewrite as (15):

(15)  $\beta$ -taller than DP is –.

$$\lambda\delta.(\text{DP}(\lambda y. \mathbf{R}(\delta, H_{\text{cm},w}(y)) \\ \text{where } \mathbf{R} = \lambda\delta_2\lambda\delta_1 \in \Delta_{H,\text{cm}}: \delta_2 \in \max(\lambda i. \beta(\delta_1 - i)))$$

It should be clear at this point that if we can show that  $\mathbf{R}$  in (15) is the same relation as the Internal Theory uses, we have proved the equivalence between SW and the Internal theory. We will show this in two stages.

We start with SW's definitions of interval subtraction and max:

$$j - i = \begin{cases} (j \cup i)^{\text{cc}} - (j \cup i) & \text{if } i < j \quad (\text{where } X^{\text{cc}} \text{ is the convex closure of } X) \\ \emptyset & \text{otherwise} \end{cases}$$

The intuition is simple:  $j - i$  is the interval between the lower bound of  $j$  and the upper bound of  $i$ , if  $j > i$ , otherwise it is undefined.

$\max(\lambda i. \beta(j - i))$  is the unique interval  $k$  such that:

1. for every non-zero  $m \subseteq k$ :  $\beta(j - m)$
2. for every  $m \supseteq k$ : there is a  $p \subseteq m$ :  $\neg\beta(j - p)$

Instead of attempting to explain this notion, I will prove the following proposition, which is the first (and most important) step in the proof that  $\mathbf{R}$  in (15) is the relation of the Internal Theory:

Let  $\delta_1, \delta_2 \in \Delta_{\text{cm},w}$ .

**Proposition:**  $\delta_2 \in \max(\lambda i. \beta(\delta_1 - i))$  iff  $\beta(\delta_1 - \delta_2)$

**Proof:**

1. **If**  $\delta_2 \in \max(\lambda i. \beta(\delta_1 - i))$  **then**  $\beta(\delta_1 - \delta_2)$ .

Assume  $\delta_2 \in \max(\lambda i. \beta(\delta_1 - i))$ .

Then  $\delta_2 \subseteq \max(\lambda i. \beta(\delta_1 - i))$ . ( $\delta_2$  taken as a singleton interval)

The first clause of the definition of  $\max(\lambda i. \beta(\delta_1 - i))$  says that for all (non-empty) subintervals  $m$  of  $\max(\lambda i. \beta(\delta_1 - i))$ :  $\beta(\delta_1 - m)$  holds.

By the assumption, one of these is  $\delta_2$ , hence indeed:  $\beta(\delta_1 - \delta_2)$ .

2. **If**  $\beta(\delta_1 - \delta_2)$  **then**  $\delta_2 \in \max(\lambda i. \beta(\delta_1 - i))$ .

Assume  $\beta(\delta_1 - \delta_2)$ , and assume  $\delta_2 \notin \max(\lambda i. \beta(\delta_1 - i))$ .

Look at  $\max(\lambda i. \beta(\delta_1 - i)) \cup \delta_2$ .

$\max(\lambda i. \beta(\delta_1 - i)) \cup \delta_2 \supset \max(\lambda i. \beta(\delta_1 - i))$ .

(Note that at this point we use the fact that intervals are not necessarily convex, because this set counts as an interval, but is not necessarily convex.)

Let  $m \neq \emptyset$  and  $m \subseteq \max(\lambda i. \beta(\delta_1 - i)) \cup \delta_2$ .

-**Either**  $m \subseteq \max(\lambda i. \beta(\delta_1 - i))$ , and then  $\beta(\delta_1 - m)$ , by the first condition of the definition of max.

-**Or**  $m = \delta_2$  and, by the assumption,  $\beta(\delta_1 - m)$ .

-**Or**, for some non-empty  $k \subseteq \max(\lambda i. \beta(\delta_1 - i))$ :  $m = k \cup \delta_2$ .

In the latter case, we know that both  $\beta(\delta_1 - k)$  and  $\beta(\delta_1 - \delta_2)$ .

Now we look at  $k \cup \delta_2$ . The upperbound of this set is either the same as the upperbound of  $k$  or it is  $\delta_2$ . This means that:

$$\delta_1 - (k \cup \delta_2) = \delta_1 - k \quad \text{or} \quad \delta_1 - (k \cup \delta_2) = \delta_1 - \delta_2$$

In either case it follows that  $\beta(\delta_1 - (k \cup \delta_2))$ , hence also in this case  $\beta(\delta_1 - m)$ .

We see then that  $\max(\lambda i. \beta(\delta_1 - i)) \cup \delta_2 \supset \max(\lambda i. \beta(\delta_1 - i))$ .

But we have just shown that  $\max(\lambda i. \beta(\delta_1 - i)) \cup \delta_2$  doesn't have a non-zero subinterval  $m$  where  $\neg \beta(\delta_1 - m)$ . This contradicts the second clause of the definition of  $\max(\lambda i. \beta(\delta_1 - i))$ . We have derived a contradiction from the assumption that  $\delta_2 \notin \max(\lambda i. \beta(\delta_1 - i))$ . Hence  $\delta_2 \in \max(\lambda i. \beta(\delta_1 - i))$ .  $\square$

We are now concerned with comparative expression:  *$\beta$ -taller (than)*.

$$\begin{array}{ll} \delta_1 \text{ } \beta\text{-taller than } \delta_2: & \text{IT: } (\beta_{\text{IT}} + \text{more}) + \text{tall}(\delta_1, \delta_2) = \beta_{\text{IT}} \circ \neg_{\text{H}}(\delta_1, \delta_2) \\ & \text{SW: } \beta(\delta_1, \delta_2) \text{ where } \beta \text{ is an interval predicate.} \end{array}$$

[5] suggests the following semantics for differential interval predicates:

$$\begin{array}{ll} \emptyset(i) \text{ is true} & \text{iff the size of } i \text{ is bigger than 0 cm} \\ \text{at least two cm}(i) \text{ is true} & \text{iff the size of } i \text{ is at least 2 cm.} \\ \text{at most two cm}(i) \text{ is true} & \text{iff the size of } i \text{ is at most 2 cm.} \\ \text{exactly two cm}(i) \text{ is true} & \text{iff the size of } i \text{ is exactly 2 cm.} \end{array}$$

In [5] the notion *size of an interval* is a primitive notion. But obviously we want to assume at this point an adequacy constraint for points  $\delta_1, \delta_2$  where  $\delta_1 > \delta_2$ :

$$\text{Adequacy constraint: } \beta(\delta_1 - \delta_2) \text{ iff } \beta_{\text{IT}} \circ \neg_{\text{H}}(\delta_1, \delta_2)$$

The adequacy constraint makes a trivial connection between what SW does and what I do. For example, **at least two cm**( $\delta_1, \delta_2$ ) expresses for  $\delta_1 > \delta_2$  that the size of the interval  $(\delta_1, \delta_2)$  is at least 2 cm. The adequacy constraint tells us that this is the case exactly if and only if  $\delta_1$  and  $\delta_2$  are height-in-cm degrees such that  $\delta_1 > \delta_2 + 2$ .

As it happens, for the differentials that [5] uses, it is not a problem that SW's notion of interval subtraction is not classical subtraction. However, if SW were extended to other comparatives, like  *$\beta$  less tall than*, SW's non-standard notion would be problematic (as is briefly indicated in the appendix).

The proposition and the adequacy constraint together give us:

$$\text{Let } \delta_1, \delta_2 \in \Delta_{\text{cm}, w}.$$

$$\text{Corollary: } \delta_2 \in \max(\lambda i. \beta(\delta_1 - i)) \text{ iff } \beta_{\text{IT}} \circ \neg_{\text{H}}(\delta_1, \delta_2)$$

This means that (15) is equivalent to (16):

(16)  $\beta$ -taller than DP is  $\neg$ .

$$\begin{array}{l} \lambda \delta. (\text{DP}(\lambda y. \mathbf{R}(\delta, H_{\text{cm}, w}(y)) \\ \quad \text{where } \mathbf{R} = \beta_{\text{IT}} \circ \neg_{\text{H}} \end{array}$$

But, of course, as already mentioned,  $\beta_{\text{IT}} \circ \neg_{\text{H}}$  is the special case of external comparison relation  $\alpha$ : (16) is a special case of (17), the Internal Theory:

(17)  $\beta$ -taller than DP is  $-$ .

$\lambda\delta.(\text{DP}(\lambda y. \alpha(\delta, H_{\text{cm},w}(y)))$

where  $\alpha$  is the interpretation of the external comparative.

With this, we have shown, for the differentials discussed, SW to be equivalent to the Internal Theory of CP-comparatives.

## 4 Appendix: Compositional Derivation of Comparative Relations

We associate with an appropriate measure-unit pair  $\mathbf{M}, \mathbf{u}$  two scales:

**Basic scale for  $\mathbf{M}, \mathbf{u}$ :**  $\mathbf{s}_{\mathbf{M}, \mathbf{u}} = \langle \Delta_{\mathbf{M}, \mathbf{u}}, >_{\mathbf{M}}, t_{\mathbf{M}}, -_{\mathbf{M}}, \mathbf{M}_{\mathbf{u}} \rangle$  where measure domain  $\Delta_{\mathbf{M}, \mathbf{u}}$  and measure function  $\mathbf{M}_{\mathbf{u}}$  are as given in section 1, and  $>_{\mathbf{M}}, t_{>_{\mathbf{M}}}, -_{\mathbf{M}}$  are lifted from  $\mathbf{R}$ .

**Converse scale for  $\mathbf{M}, \mathbf{u}$ :**  $\mathbf{s}_{\mathbf{M}, \mathbf{u}}^c = \langle \Delta_{\mathbf{M}, \mathbf{u}}, >_{\mathbf{M}}^c, t_{\mathbf{M}}^c, -_{\mathbf{M}}^c, \mathbf{M}_{\mathbf{u}} \rangle$ , where  $\Delta_{\mathbf{M}, \mathbf{u}}$  and  $\mathbf{M}_{\mathbf{u}}$  are the same and  $\delta_1 >_{\mathbf{M}}^c \delta_2$  iff  $\delta_2 >_{\mathbf{M}} \delta_1$ ;  $t_{>_{\mathbf{M}}}^c = u_{>_{\mathbf{M}}}$ ;  $\delta_1 -_{\mathbf{M}}^c \delta_2 = \delta_2 -_{\mathbf{M}} \delta_1$

(Note: mnemonic superscripts:  $\delta^f, \delta^u, s^-$  refer to the  $r, u, -$  place of the relevant tuple.)

### Semantic derivation:

1. 1-place number predicate: *(somewhat)*:  $\lambda x. r > 0$   
 2-place number functions: *more*:  $-$ ; *less*:  $-^c$   
 A 1-place number predicate composes with a 2-place number function to give a 2-place number relation:  
 2-place number relations: *(somewhat) more than*:  $\lambda x. r >_{\mathbf{R}} 0^o - = >$   
*(somewhat) less than*:  $\lambda x. r >_{\mathbf{R}} 0^o -^c = <$
2. A 2-place number relation applies to a number to give a 1-place number predicate:  
 1-place number predicates: *more than three*:  $\lambda x. r > 3$ ;  
*less than three*:  $\lambda x. r < 3$
3. A number predicate combines with unit cm to give a cm-degree predicate:  
 1-place degree predicates: *more than three cm*:  $\lambda\delta. \delta^f > 3 \wedge \delta^u = \text{cm}$   
*less than three cm*:  $\lambda\delta. \delta^f < 3 \wedge \delta^u = \text{cm}$
4. Functions from scales to 2-place degree functions: *more*:  $\lambda s. s^-$   
*less*:  $\lambda s. (s^c)^-$

A 1-place predicate composes with a function from scales to degree relations to give a function from scales to degree relations:

*more than three cm more*:  $\lambda s. \lambda\delta_2 \lambda\delta_1. s^-(\delta_1, \delta_2)^f > 3 \wedge \delta_2^u = \text{cm}$

*more than three cm less*:  $\lambda s. \lambda\delta_2 \lambda\delta_1. (s^c)^-(\delta_1, \delta_2)^f > 3 \wedge \delta_2^u = \text{cm}$

5. Functions from units to scales: *tall*:  $\lambda u. s_{\mathbf{H}, \mathbf{u}}$   
*short*:  $\lambda u. s_{\mathbf{H}, \mathbf{u}}^c$

Apply step (4) to step (5), filling in cm for the unit in (5). This derives 2-place relations between height-in-cm-degrees.

Sample derivation:

*more than three cm + less* = *more than three cm less*

$\lambda\delta. \delta^f > 3 \wedge \delta^u = \text{cm} + \lambda s. (s^c)^- = \lambda s \lambda\delta_2 \lambda\delta_1. (s^c)^-(\delta_1, \delta_2)^f > 3 \wedge \delta^u = \text{cm}$

+ *tall* = *more than three cm less tall*

$$\begin{aligned}
+ \lambda u_{\text{S}_{H,u}} &= \lambda \delta_2 \lambda d_1 \in \Delta_{H,cm}: (\text{S}_{H,cm})^c (\delta_1, \delta_2)^r > 3 \\
(\text{S}_{H,cm})^c (\delta_1, \delta_2)^r &= -_H^c (\delta_1, \delta_2)^r = \delta_2^r - \delta_1^r \\
\text{So: more than three cm less tall } \lambda \delta_2 \lambda d_1 &\in \Delta_{H,cm}: (\delta_2^r - \delta_1^r) > 3 \\
(= \text{more than three cm shorter}) \lambda \delta_2 \lambda d_1 &\in \Delta_{H,cm}: \delta_1^r < \delta_2^r - 3
\end{aligned}$$

The computation shows the advantage of the scale being based on the full reals with normal subtraction (as opposed to subtraction on positive intervals in SW): even if the measure function doesn't assign negative heights, we want the equivalences that they allow, and use these in simplifying the meanings derived.

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## References

1. Heim, I.: Degree Operators and Scope. In: Féry, C., Sternefeld, W. (eds.) *Audiatur Vox Sapientiae*, A Festschrift for Arnim von Stechow, pp. 214–239. Akademie-Verlag, Berlin (2001)
2. Hoeksema, J.: Negative Polarity and the Comparative. *Natural Language and Linguistic Theory* 1(3), 403–434 (1983)
3. Landman, F.: An Almost (but not Quite) Naïve Semantics for Comparatives, Linguistics Department, Tel Aviv University, (2009) [tau.ac.il/~landman](http://tau.ac.il/~landman)
4. Montague, R.: The proper Treatment of Quantification in Ordinary English. In: Hintikka, J., Moravcsik, J., Suppes, P. (eds.) *Approaches to Natural Language*, pp. 221–242. Reidel, Dordrecht (1973)
5. Schwarzschild, R., Wilkinson, K.: Quantifiers in Comparatives: a Semantics of Degree based on Intervals. *Linguistics and Philosophy* 10(1), 1–41 (2002)
6. von Stechow, A.: Comparing Semantic Theories of Comparison. *Journal of Semantics* 3, 1–77 (1984)

# Temporal Propositions as Vague Predicates

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**Abstract.** The idea that temporal propositions are vague predicates is examined with attention to the nature of the objects over which the predicates range. These objects should not, it is argued, be identified once and for all with points or intervals in the real line (or any fixed linear order). Context has an important role to play not only in sidestepping the Sorites paradox (Gaifman [2002](#)) but also in shaping temporal moments/extent (Landman [1991](#)). The Russell-Wiener construction of time from events (Kamp [1979](#)) is related to a notion of context given by a string of observations, the vagueness in which is brought out by grounding the observations in the real line. With this notion of context, the context dependency functions in Gaifman [2002](#) are adapted to interpret temporal propositions.

## 1 Introduction

Fluents, as temporal propositions are commonly known in AI, have in recent years made headway in studies of events and temporality in natural language semantics (e.g. Steedman [2000](#), van Lambalgen and Hamm [2005](#)). The present paper concerns the bounded precision implicit in sentences such as (1).

- (1) Pat reached the summit of K2 at noon, and not a moment earlier.

Presumably, a moment in (1) is less than an hour but greater than a picosecond. Whether or not determining the exact size of a moment is necessary to interpret or generate (1), there are pitfalls well known to philosophers that lurk. One such danger is the Sorites paradox, which is commonly associated not so much with time as with vagueness. Focusing on time, Landman emphasizes the importance of context.

It is not the abstract underlying time structure that is semantically crucial, but the system of temporal measurements. We shouldn't ask just 'what is a moment of time', because that is a context dependent question. We can assume that context determines how precisely we are measuring time: it chooses in the hierarchy of temporal measurements one measurement that is taken as 'time as finely grained as this context requires it to be.' The elements of that measurement are then regarded as moments *in that context*. (Landman [1991](#), page 138)

Different notions of context are explored below. We start in section 2 with the use of context in Gaifman [2002](#) to sidestep the Sorites paradox before returning

in the succeeding sections to the special case of time. A basic aim is to critically examine the intuition that the temporal extent of an event is an interval — an intuition developed in Kamp [1979], Allen [1983] and Thomason [1989], among other works.

A concrete linguistic question concerning intervals is brought out by (2).

(2) John drank beer from nine to ten.

(2) is from Landman [1991], in page 137 of which we read

What should this mean? At some time in  $\llbracket \textit{from nine to ten} \rrbracket$  John drank beer? At all times in  $\llbracket \textit{from nine to ten} \rrbracket$  John drank beer? At most times in  $\llbracket \textit{from nine to ten} \rrbracket$  John drank beer? This is just not clear ...

Framed as a choice between different quantificational readings, the problem in understanding (2) becomes one of ambiguity rather than vagueness. (Multiple non-equivalent logical forms is commonly understood to indicate ambiguity, in contrast to a single logical form with various divergent interpretations, marking vagueness.) The approach pursued below puts vagueness ahead of ambiguity in asking not about quantificational force, but about the domain of quantification (arguably a more fundamental question). What is at issue is a notion of time that, as previously mentioned, depends on context. Ignoring tense and aspect (for the sake of simplicity), a minimal context where (2) is true is the cartoon strip (3) describing three temporal stretches in which John drinks beer, the first at nine, and the third at ten.

(3) 

nine, drink-beer(john)	drink-beer(john)	ten, drink-beer(john)
------------------------	------------------	-----------------------

Is (3) consistent with times between nine and ten at which John does not drink beer? Although (3) describes no such time, there remains the possibility of refining or interpreting (3) to grant John a break between beers. Motivating and fleshing out that possibility is a large part of what this paper is about.

## 2 Sorites and Appropriate Contexts

The tolerance of a unary predicate  $P$  to small changes is expressed in Gaifman [2002] through conditionals of the form (4).

(4)  $N_P(x, y) \rightarrow (P(x) \rightarrow P(y))$

$P$  is asserted in (4) to be tolerant insofar as  $P$  holds of  $y$  whenever  $P$  holds of an  $x$  that is  $N_P$ -near  $y$ . Repeatedly applying (4), we conclude  $P(z)$ , given any finite sequence  $y_1, \dots, y_n$  such that  $y_n = z$ ,  $P(y_1)$  and  $N_P(y_i, y_{i+1})$  for  $1 \leq i < n$ . A *Sorites chain* is a sequence  $y_1, \dots, y_n$  such that  $P$  holds of  $y_1$  but not  $y_n$ , even though  $N_P(y_i, y_{i+1})$  for  $1 \leq i < n$ . Gaifman's way out of the Sorites paradox is to interpret  $P$  against a *context dependency function*  $f$  mapping a finite set  $C$

(of objects in a first-order model) to a subset  $f(C)$  of  $C$ , understood to be the extension of  $P$  at “context”  $C$ . (In effect, the predication  $P(x)$  becomes  $P(x, C)$ , for some comparison class  $C$  that contains  $x$ .) The idea then is to pick out finite sets  $C$  that do *not* contain a Sorites chain. Such sets are called *feasible contexts*<sup>1</sup>. Formally, Gaifman sets up a *Contextual Logic* preserving classical logic in which tolerance conditionals (4) can be sharpened to (5), using a construct  $[C]$  to constrain the contexts relative to which  $P(x)$  and  $P(y)$  are interpreted.

$$(5) \quad [C] (N_P(x, y) \rightarrow (P(x) \rightarrow P(y)))$$

As contexts in Contextual Logic need not be feasible, (5) must be refined further to restrict  $C$  to feasible contexts

$$\text{feasible}(C) \rightarrow [C] (N_P(x, y) \rightarrow (P(x) \rightarrow P(y))).$$

The formal notation gets quite heavy, but the point is simple enough:

sentences and proofs have associated contexts. Those whose contexts are feasible form the feasible portion of the language; and it is within this portion that a tolerant predicate is meant to be used. The proof of the Sorites contradiction fails, because it requires an unfeasible context and in unfeasible contexts a tolerant predicate loses *[sic]* its tolerance; it has some sharp cutoff. Unfeasible contexts do not arise in practice. (Gaifman 2002, pages 23, 24)

The obvious question is why not build into Contextual Logic only contexts that *do* “arise in practice” — viz. the feasible ones? For tolerant predicates in general, such a restriction may, as Gaifman claims, well result in a “cumbersome system.” Fluents are, however, a very particular case of vague predicates, and surely practice is all that matters. That said, Contextual Logic leaves open the question of what the stuff of time is, demanding only that for every fluent  $P$ , finite sets  $C$  of times be picked out that validate (5), for a suitable interpretation of  $N_P$ . Such feasible contexts  $C$  avoid the sharp cutoffs characteristic of unfeasible contexts, and allow us to sidestep the difficulty of pinning down the precise moment of change by bounding granularity. Bounded granularity is crucial for making sense of talk about the first (or last) moment a fluent is true (or of claims that a fluent true at an interval is true at every non-null part of that interval).

### 3 Contexts for Temporal Extent

Where might a feasible context for a fluent come from? The present section traces context back to some (given) set  $E$  of events (of interest), making time just fine grained enough to compare events in  $E$ . The comparisons are, following Kamp 1979, given by binary relations on  $E$  of temporal overlap  $\bigcirc$  and complete

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<sup>1</sup> Unless I am mistaken, for  $C$  to be feasible, it suffices that there be *no* pairs  $a, b$  in  $C$  such that  $N_P(a, b)$  and  $P$  holds (at  $C$ ) of  $a$  but not  $b$ .

precedence  $\prec$  constituting an *event structure*  $\langle E, \bigcirc, \prec \rangle$  that satisfies (A<sub>1</sub>) to (A<sub>5</sub>)<sup>2</sup>

- (A<sub>1</sub>)  $e \bigcirc e$  (i.e.  $\bigcirc$  is reflexive)
- (A<sub>2</sub>)  $e \bigcirc e'$  implies  $e' \bigcirc e$
- (A<sub>3</sub>)  $e \prec e'$  implies not  $e \bigcirc e'$
- (A<sub>4</sub>)  $e \prec e' \bigcirc e'' \prec e'''$  implies  $e \prec e'''$
- (A<sub>5</sub>)  $e \prec e'$  or  $e \bigcirc e'$  or  $e' \prec e$

Before extracting temporal moments from  $\langle E, \bigcirc, \prec \rangle$ , it is useful for orientation to proceed in the opposite direction, forming an event structure from a linear order  $\langle T, < \rangle$  and a relation  $s \subseteq T \times E$  associating a time  $t \in T$  with an event  $e \in E$  according to the intuition that

$$s(t, e) \text{ says 'e s-occurs at t'.$$

It is natural to view  $s$  as a schedule, with temporal overlap  $ov(s)$  holding between events  $e$  and  $e'$  that  $s$ -occur at some time in common

$$e \text{ } ov(s) \text{ } e' \stackrel{\text{def}}{\iff} (\exists t) s(t, e) \text{ and } s(t, e')$$

and an event  $e$  completely preceding another  $e'$  if  $e$   $s$ -occurs only  $<$ -before  $e$

$$e <_s e' \stackrel{\text{def}}{\iff} (\forall t, t' \text{ such that } s(t, e) \text{ and } s(t', e')) t < t' .$$

**Proposition 1.**  $\langle E, ov(s), <_s \rangle$  is an event structure provided

- (i)  $<$  linearly orders  $T$
- (ii)  $(\forall e \in E)(\exists t \in T) s(t, e)$ , and
- (iii)  $s(t, e)$  whenever  $s(t_0, e)$  and  $s(t_1, e)$  for some  $t_0 < t$  and  $t_1 > t$ .

Let us call  $\langle s, T, < \rangle$  an *interval schedule* if it satisfies (i) – (iii).

Now for the Russell-Wiener construction in Kamp [1979] of time from an event structure  $\langle E, \bigcirc, \prec \rangle$ . We collect subsets of  $E$  any two in which  $\bigcirc$ -overlap in

$$O_{\bigcirc} \stackrel{\text{def}}{=} \{t \subseteq E \mid (\forall e, e' \in t) e \bigcirc e'\}$$

and equate temporal moments with  $\subseteq$ -maximal elements of  $O_{\bigcirc}$

$$T_{\bigcirc} \stackrel{\text{def}}{=} \{t \in O_{\bigcirc} \mid (\forall t' \in O_{\bigcirc}) t \subseteq t' \text{ implies } t = t'\} .$$

<sup>2</sup> Seven postulates are given in Kamp 1979, but two are superfluous. In Thomason [1984], event structures are called event orderings, with  $\bigcirc$  the complement of the union  $\prec \cup \succ$  of  $\prec$  and its converse  $\succ$ , and  $\prec$  an irreflexive interval ordering

$$\begin{array}{l} \text{not } e \prec e \\ e_1 \prec e_2 \text{ and } e'_1 \prec e'_2 \text{ imply } e_1 \prec e'_2 \text{ or } e'_1 \prec e_2 . \end{array}$$

We then lift  $\prec$  to  $T_\circ$  existentially

$$t \prec_\circ t' \stackrel{\text{def}}{\iff} (\exists e \in t)(\exists e' \in t') e \prec e'$$

and define  $\text{sched}_\circ \subseteq T_\circ \times E$  as the converse of membership

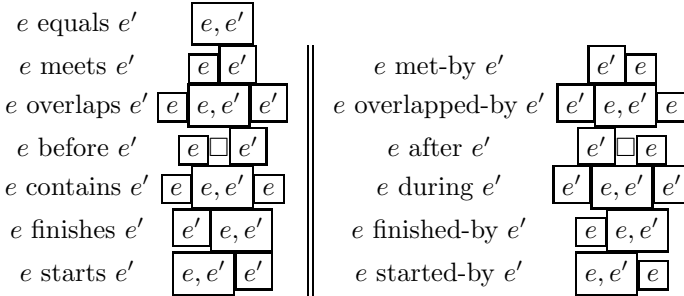
$$\text{sched}_\circ(t, e) \stackrel{\text{def}}{\iff} e \in t$$

for all  $t \in T_\circ$  and  $e \in E$ .

**Theorem 2.** (Russell, Wiener, Kamp)  $\langle \text{sched}_\circ, T_\circ, \prec_\circ \rangle$  is an interval schedule if  $\langle E, \circ, \prec \rangle$  is an event structure.

Applying the transformations in Theorem 2 and Proposition 1 in sequence to an event structure yields the same event structure, but applying the transformations in reverse to an interval schedule may result in a different (reduced) interval schedule. The notion of time obtained from events is fine enough just to determine overlap  $\circ$  and complete precedence  $\prec$  between events. But are there not other temporal relations to preserve?

Thirteen different relations between intervals are catalogued in Allen [1983], strung out below as sequences of snapshots (enclosed in boxes).



If a box must be  $\subseteq$ -maximal (as required by  $T_\circ$ ), only three of the thirteen strings above survive

$$\boxed{e} \boxed{e'} + \boxed{e'} \boxed{e} + \boxed{e, e'}.$$

To recover the ten other strings, it is useful to equip an event  $e$  with a pre-event  $\text{pre}(e)$  and a post-event  $\text{post}(e)$ , enriching a schedule  $s$  to

$$s^< \stackrel{\text{def}}{=} s \cup s^{\leq}_- \cup s^<_+$$

where

$$\begin{aligned} s^{\leq}_- &\stackrel{\text{def}}{=} \{ \langle t, \text{pre}(e) \rangle \mid (\exists t' > t) s(t', e) \text{ and } (\forall t' \leq t) \text{ not } s(t', e) \} \\ s^<_+ &\stackrel{\text{def}}{=} \{ \langle t, \text{post}(e) \rangle \mid (\exists t' < t) s(t', e) \text{ and } (\forall t' \geq t) \text{ not } s(t', e) \} \end{aligned}$$

so that, for example,  $\boxed{e} \sqsubset \boxed{e'}$  becomes  $\boxed{e, \text{pre}(e')} \boxed{\text{post}(e), \text{pre}(e')} \boxed{\text{post}(e), e'}$ .<sup>3</sup> It is easy to see that if  $\langle s, T, < \rangle$  is an interval schedule on  $E$ , then so is  $\langle s^<, T, < \rangle$  on the extended set

$$E_s^< \stackrel{\text{def}}{=} \{y \mid (\exists t) s^<(t, y)\}$$

of events, and moreover each of the thirteen Allen relations between  $e$  and  $e' \in E$  can be determined from the overlap relation  $ov(s^<)$  induced by  $s^< —$  e.g.

$$\begin{aligned} e \text{ before } e' &\text{ iff } \text{post}(e) \text{ } ov(s^<) \text{ pre}(e') \\ e \text{ meets } e' &\text{ iff } \text{post}(e) \text{ } ov(s^<) e' \text{ but neither} \\ &\quad e \text{ } ov(s^<) e' \text{ nor } \text{post}(e) \text{ } ov(s^<) \text{ pre}(e') . \end{aligned}$$

For the record,

**Proposition 3.** *For every interval schedule  $\langle s, T, < \rangle$ ,  $\langle E_s^<, ov(s^<), \prec \rangle$  is an event structure where  $\prec$  is the precedence  $<_{s^<}$  induced by  $<$  and  $s^<$ , and for  $\bigcirc \stackrel{\text{def}}{=} ov(s^<)$ ,*

$$T_{\bigcirc} = \{\{y \mid s^<(t, y)\} \mid t \in T\} .$$

According to the last equation,  $T_{\bigcirc}$  does *not* discard a time  $t \in T$  (as may be the case were  $\bigcirc = ov(s)$ ) but merely identifies it with  $t' \in T$  such that for all  $y$ ,

$$s^<(t, y) \quad \text{iff} \quad s^<(t', y) .$$

This equivalence is, in general, stronger than one with  $s$  in place of  $s^<$ . Note also that for  $T_{\bigcirc}$  to be finite, it suffices that  $E$  be finite (as  $T_{\bigcirc}$  consists of subsets of  $E_s^<$  for any schedule  $s$ , no matter how large  $T$  is). Even if  $E$  were not finite, we could apply the transformation described in Proposition 3 to the restriction

$$s \upharpoonright X \stackrel{\text{def}}{=} \{(t, e) \in s \mid e \in X\}$$

of  $s$  to a finite subset  $X$  of  $E$  for a finitary approximation that, as we shall see next, can be pictured as a string.

## 4 Contexts as Strings

In this section, we equate contexts with strings  $\in Pow(\Phi)^*$  over the alphabet  $Pow(\Phi)$  of subsets of some fixed set  $\Phi$ . Any relation  $s \subseteq T \times E$  over a finite linear order  $\langle T, < \rangle$  can be formulated as a string in  $Pow(\Phi)^*$  with  $\Phi = E \cup T$ . For example, the schedule  $\{(0, e), (1, e), (1, e'), (2, e), (2, e'), (3, e), (3, e'), (4, e')\}$  can be represented as (6), assuming the usual order on  $\{0, 1, 2, 3, 4\}$ .

<sup>3</sup> This construction marries and mangles ideas from Allen and Ferguson [1994] and Walker instants (Thomason [1984], van Lambalgen and Hamm [2005]). It goes without saying that for all  $e, e', e'' \in E$ , the set  $\{e, \text{pre}(e'), \text{post}(e'')\}$  has cardinality 3.

$$(6) \quad \boxed{0, e} \boxed{1, e, e'} \boxed{2, e, e'} \boxed{3, e, e'} \boxed{4, e'}$$

To restrict  $\Phi$  to some subset  $X$ , we project a string in  $Pow(\Phi)^*$  to one in  $Pow(X)^*$  by componentwise intersection  $r_X$  with  $X$

$$r_X(\alpha_1 \cdots \alpha_n) \stackrel{\text{def}}{=} (\alpha_1 \cap X) \cdots (\alpha_n \cap X)$$

so that for  $X = E \cup \{0, 4\}$ ,  $r_X$  maps (6) to

$$\boxed{0, e} \boxed{e, e'} \boxed{e, e'} \boxed{e, e'} \boxed{4, e'}$$

Another useful projection on  $Pow(\Phi)^*$  is *block compression*  $\pi$  reducing adjacent identical boxes  $\alpha\alpha^n$  in a string  $\sigma$  to one  $\alpha$

$$\pi(\sigma) \stackrel{\text{def}}{=} \begin{cases} \pi(\alpha\sigma') & \text{if } \sigma = \alpha\alpha\sigma' \\ \alpha\pi(\beta\sigma') & \text{if } \sigma = \alpha\beta\sigma' \text{ with } \alpha \neq \beta \\ \sigma & \text{otherwise} \end{cases}$$

so that, for example,

$$\pi(\boxed{0, e} \boxed{e, e'} \boxed{e, e'} \boxed{e, e'} \boxed{4, e'}) = \boxed{0, e} \boxed{e, e'} \boxed{4, e'}$$

Underlying the projection  $\pi$  is the slogan “no time without change” (Kamp and Reyle 1993, page 674), reducing any string

$$\boxed{\text{nine, drink-beer(john)}} \boxed{\text{drink-beer(john)}}^n \boxed{\text{ten, drink-beer(john)}}$$

of length  $n + 2$ , for  $n \geq 1$ , to (3).

$$(3) \quad \boxed{\text{nine, drink-beer(john)}} \boxed{\text{drink-beer(john)}} \boxed{\text{ten, drink-beer(john)}}$$

Next, to form the schedule induced by the set  $T_\circ$  in Proposition 3 as an inverse limit over finite subsets  $X$  of  $E$ , let the function  $\pi_X : Pow(E)^* \rightarrow Pow(X)^*$  restrict the events to  $X$  before applying  $\pi$

$$\pi_X(\sigma) \stackrel{\text{def}}{=} \pi(r_X(\sigma)) .$$

An instructive example is provided by the real line  $\langle \mathbb{R}, < \rangle$ ; for  $X = \{-1, 2, 7\}$ , the  $\pi_X$ -approximation of the schedule  $\{(r, r) \mid r \in \mathbb{R}\}$  is

$$\square \boxed{-1} \square \boxed{2} \square \boxed{7} \square$$

with empty boxes  $\square$  from pre- and post-events. In general, the  $\pi_X$ -approximation of  $s$  is the string corresponding to the result of the construction in Proposition 3 applied to the restriction  $s \upharpoonright X$  of  $s$  to  $X$ .

Why bother turning interval schedules into strings? One reason is that we can relax the interval requirement (condition (iii) in Proposition 1) on strings, as  $\pi$  is well-behaved without such an assumption (whereas Russell-Wiener-Kamp relies

on it). In particular, we can construe an element of  $\Phi$  as a point-wise fluent, and reconceive the string  $\boxed{pre(e)} \boxed{e} \boxed{post(e)}$  as  $\alpha(e) \boxed{oc(e)} \omega(e)$  where the negation  $\neg oc(e)$  of an occurrence of  $e$  is split between

$$\begin{aligned}\alpha(e) &\stackrel{\text{def}}{=} \boxed{\neg oc(e), \neg Past(oc(e)), Future(oc(e))} \\ \omega(e) &\stackrel{\text{def}}{=} \boxed{\neg oc(e), Past(oc(e)), \neg Future(oc(e))}\end{aligned}$$

not to mention

$$\begin{aligned}\text{hole}(e) &\stackrel{\text{def}}{=} \boxed{\neg oc(e), Past(oc(e)), Future(oc(e))} \\ \text{never}(e) &\stackrel{\text{def}}{=} \boxed{\neg oc(e), \neg Past(oc(e)), \neg Future(oc(e))} .\end{aligned}$$

For a model-theoretic interpretation based on observations that take time greater than 0, we shall use open intervals in the real line that have length greater than some fixed real number  $\epsilon > 0$ . Let

$$\mathcal{O}_\epsilon \stackrel{\text{def}}{=} \{(a, b) \mid a, b \in \mathbb{R}_{\pm\infty} \text{ and } b > a + \epsilon\}$$

where

$$(a, b) \stackrel{\text{def}}{=} \{r \in \mathbb{R} \mid a < r < b\} .$$

Complete precedence in  $\mathcal{O}_\epsilon$  is given by

$$o \prec_\epsilon o' \stackrel{\text{def}}{\iff} o, o' \in \mathcal{O}_\epsilon \text{ and } (\forall r \in o)(\forall r' \in o') r < r'$$

and abutment (or successors) by

$$o s_\epsilon o' \stackrel{\text{def}}{\iff} o \prec_\epsilon o' \text{ and not } (\exists o'' \prec_\epsilon o') o \prec_\epsilon o'' .$$

An  $\epsilon$ -chain  $\mathbf{c}$  is a sequence  $o_1 \dots o_n$  in  $\mathcal{O}_\epsilon^*$  such that

$$o_1 s_\epsilon o_2 s_\epsilon o_3 \dots s_\epsilon o_n .$$

A string  $\alpha_1 \dots \alpha_n \in Pow(\Phi)^*$  holds at an  $\epsilon$ -chain  $\mathbf{c} = o_1 \dots o_n$  if for  $1 \leq i \leq n$ ,

$$\mathbf{c}, o_i \models \varphi \text{ for every } \varphi \in \alpha_i$$

where the satisfaction relation  $\models$  is based on a function  $v : \Phi_0 \rightarrow Pow(\mathcal{O}_\epsilon)$ , kept fixed in the background, mapping an *atomic* fluent  $\varphi \in \Phi_0 \subseteq \Phi$  to a set  $v(\varphi)$  of open sets in  $\mathcal{O}_\epsilon$

$$\mathbf{c}, o_i \models \varphi \stackrel{\text{def}}{\iff} o_i \in v(\varphi) \quad \text{for } \varphi \in \Phi_0$$

extended in the classical manner — e.g.

$$\begin{aligned}\mathbf{c}, o_i \models \neg A &\stackrel{\text{def}}{\iff} \text{not } \mathbf{c}, o_i \models A \\ \mathbf{c}, o_i \models Past A &\stackrel{\text{def}}{\iff} (\exists j < i) \mathbf{c}, o_j \models A \\ \mathbf{c}, o_i \models Future A &\stackrel{\text{def}}{\iff} (\exists j > i) \mathbf{c}, o_j \models A .\end{aligned}$$

But what is gained by introducing a positive real number  $\epsilon > 0$ ? Very briefly,  $\epsilon$  gives us a handle on vagueness, if following Gaifman 2002 (where tolerance is distinguished from vagueness), we identify borderline cases via a modal logic with

possible worlds as representations of semantic views: ways of applying the vague predicates when “yes/no” decisions are required. (page 10)

In the present set-up, we can take a possible world to be an  $\epsilon$ -chain, over which an accessibility relation  $\sim_\epsilon$  is defined by putting

$$(a, b) \sim_\epsilon (a', b') \stackrel{\text{def}}{\iff} |a - a'| < \epsilon \text{ and } |b - b'| < \epsilon$$

for all  $(a, b), (a', b') \in \mathcal{O}_\epsilon$  and

$$o_1 \dots o_n \sim_\epsilon o'_1 \dots o'_m \stackrel{\text{def}}{\iff} n = m \text{ and } o_i \sim_\epsilon o'_i \text{ for } 1 \leq i \leq n$$

for all  $\epsilon$ -chains  $o_1 \dots o_n$  and  $o'_1 \dots o'_m$ . The idea is that an  $\epsilon$ -chain  $\mathbf{c}$  is a borderline of a string  $\sigma \in \text{Pow}(\Phi)^*$  if

$$(\exists \mathbf{c}_1 \sim_\epsilon \mathbf{c}) \sigma \text{ holds at } \mathbf{c}_1 \text{ and } (\exists \mathbf{c}_2 \sim_\epsilon \mathbf{c}) \sigma \text{ does not hold at } \mathbf{c}_2.$$

As required by the modal logic KTB in Gaifman 2002,  $\sim_\epsilon$  is reflexive and symmetric. Clearly,  $\sim_\epsilon$  is not transitive, and different strings may hold at  $\sim_\epsilon$ -related  $\epsilon$ -chains (making temporal relations possibly vary with the  $\epsilon$ -chain chosen).<sup>4</sup>

## 5 Conclusion

Three notions of context were considered above:

- feasible contexts in §2 for Sorites (Gaifman 2002) amounting to comparison classes
- selected events in §3 that induce temporal moments (applying the Russell-Wiener-Kamp construction on event structures with pre- and post-events)
- strings in §4 that generalize event occurrences to event types, and are interpretable as incomplete samples from open intervals in the real line  $\mathbb{R}$  of bounded granularity  $\epsilon > 0$ .

The incompleteness of the sequences of observations constituting strings gives rise to vagueness, with borderline cases analyzable in a modal logic, following Gaifman 2002. Focusing on the contexts that “arise in practice,” recall that Gaifman interprets a tolerant unary predicate  $P$  via a *context dependency function*  $f_P$  mapping a context  $C$  to the extension  $f_P(C) \subseteq C$  of  $P$  at  $C$ . Similarly, we might analyze a temporal proposition  $P$  as a function mapping  $C$  to the set  $f_P(C)$  of *parts of*  $C$  that *make*  $P$  *true* such that

$$P \text{ is true at } C \text{ iff } f_P(C) \neq \emptyset.$$

<sup>4</sup> Hence, (3) can be made consistent with John *not* drinking at 9:30. Also, note that the same conclusions can be drawn were  $\prec_\epsilon$  defined with  $r + \epsilon < r'$  in place of  $r < r'$ .

That is, if  $\sqsubseteq$  is the obvious part-of relation between strings with, for example,

$$\boxed{a,b}\boxed{c}\boxed{d} \sqsubseteq s \boxed{a,b,c}\boxed{a,c,d}\boxed{d} s' ,$$

and  $\mathcal{L}(P)$  is the set  $\bigcup_C f_P(C)$  of strings  $s$  that make  $P$  true, with for instance,

$$\mathcal{L}(\text{rain from dawn to dusk}) = \boxed{\text{dawn, rain}}\boxed{\text{rain}}^+ \boxed{\text{dusk, rain}}$$

(Fernando [2009a](#)), we can assert (7).

$$(7) \quad f_P(C) = \{s \sqsubseteq C \mid s \in \mathcal{L}(P)\}$$

The step up to a language  $\mathcal{L}(P) \subseteq \text{Pow}(\Phi)^*$  from a single string  $\sigma \in \text{Pow}(\Phi)^*$  accommodates the different ways a fluent  $P$  can be true, as well as multiple granularities. For instance, we might expect  $\mathcal{L}(P)$  to be closed under block compression  $\pi$  (from §4)

$$\sigma \in \mathcal{L}(P) \text{ implies } \pi(\sigma) \in \mathcal{L}(P) .$$

The proposed variant (7) of a context dependency function  $f_P$  enlists strings as situations to serve as indices  $C$  and elements  $s$  of denotations alike (Fernando [2009b](#)). The domain of  $f_P$  can be given by a system of interpretations in the real line as outlined in §4, although further work is required to clarify just what variations in the indices  $C$  such a system allows.

## References

- Allen, J.F.: Maintaining knowledge about temporal intervals. *Communications of the ACM* 26(11), 832–843 (1983)
- Allen, J.F., Ferguson, G.: Actions and events in interval temporal logic. *Journal of Logic and Computation* 4(5), 531–579 (1994)
- Fernando, T.: Situations in LTL as strings. *Information and Computation* 207(10), 980–999 (2009a)
- Fernando, T.: Situations as indices and as denotations. *Linguistics and Philosophy* 32(2), 185–206 (2009b)
- Gaifman, H.: Vagueness, tolerance and contextual logic, 41 pages (January 2002), <http://www.columbia.edu/~hg17/>, (August 2009)
- Kamp, H.: Events, instants and temporal reference. In: Bäuerle, R., Egli, U., von Stechow, A. (eds.) *Semantics from Different Points of View*, pp. 27–54. Springer, Berlin (1979)
- van Lambalgen, M., Hamm, F.: *The Proper Treatment of Events*. Blackwell, Malden (2005)
- Landman, F.: *Structures for Semantics*. Kluwer, Dordrecht (1991)
- Steedman, M.: *The Productions of Time*. (July 2000), Draft <ftp://ftp.cogsci.ed.ac.uk/pub/steedman/temporality/temporality.ps.gz>, Subsumes Temporality, van Benthem, J., ter Meulen, A. (eds.): *Handbook of Logic and Language*, pp. 895–935. Elsevier North Holland (1997)
- Thomason, S.K.: On constructing instants from events. *Journal of Philosophical Logic* 13, 85–96 (1984)
- Thomason, S.K.: Free constructions of time from events. *Journal of Philosophical Logic* 18, 43–67 (1989)

# Vagueness Is Rational under Uncertainty\*

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## 1 Introduction

We seek to show that some properties of vague scalar adjectives are consequences of rational communication. Theories of vagueness are usually directed at a cluster of traditional philosophical desiderata: how vagueness fits into a theory of truth (or metaphysics), the sorites, and related issues. These are important, and we will have more to say about some of them, but we also seek to refocus the analysis of vagueness, moving away from consideration of abstract philosophical problems, and towards consideration of the problems faced by ordinary language users. It is our contention that it is by considering the latter that the former can best be understood. Using standard mathematical tools, we suggest that, given a certain model of communication, what have typically been taken to be puzzles in the truth conditions of vague expressions, should just be expected consequences of communication under uncertainty.

The most basic criterion for any successful semantic theory is that it should establish when sentences are true, and when they are false. This is a tall order as regards sentences involving vague predicates. Some scholars settle for giving definite truth values for only a subset of conditions in the world, or for abandoning truth in favor of some notion of acceptability. We have no *a priori* argument against theories which attribute partial or fuzzy truth conditions to vague sentences<sup>1</sup> and no *a priori* argument against theories stated in terms of acceptability, but prefer to retain classical semantics. We will not claim that it is always possible to specify objective boolean truth values for every vague sentence even with complete knowledge of the utterance situation and speaker, but we will offer a theory which allows that every vague sentence has a boolean truth value, and we will offer a theory of how speakers can and must approximate it.

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\* We thank the anonymous reviewer and participants of the vagueness workshop at the 17th Amsterdam Colloquium and at the CAuLD workshop at INRIA. Earlier versions of this paper offered a less tenable notion of vagueness. The reactions and feedback we received have helped us revise our definition as well as bringing clarity to the future direction of this work.

<sup>1</sup> But we are claiming that vagueness is not a result of degrees of truth or likelihood. Vagueness rides atop distributions of certain facts. Contra fuzzy logic, an individual is either *tall* or *not tall*, rather than *tall* to degree  $x$  and *not tall* to degree  $1 - x$ .

## 2 A Free Variable Theory of Scalar Adjectives

Following Cresswell [1], we analyze the semantics of “Xena is tall” simply as saying that Xena’s height is greater than some standard of comparison or threshold degree  $d$ , where  $d$  is a free variable. Thus our analysis is semantically parallel to free variable accounts of e.g. modals [2] or quantifiers [3]. But there is a crucial difference. Consider the analysis of quantifiers: if “Everybody is happy” is analyzed as  $\forall x \in C, \text{happy}(x)$ , with the restrictor  $C$  being a free variable, then it is normally assumed that while the hearer may sometimes be uncertain as to the value of  $C$ , the speaker has a particular value in mind, and cannot be wrong about that value. However, in our free variable account of vagueness, there is no such asymmetry: when a speaker says “Xena is tall”, neither the speaker nor the hearer has privileged access to  $d$ . Indeed, even if both speaker and hearer are clear on issues such as the comparison class for which Xena’s tallness is being considered, and on normative issues such as the utility or aesthetics of various heights, they still cannot be certain as to the appropriate value of  $d$ . The best they can do, we will suggest, is form a probabilistic model of the value of  $d$ , based on their prior experience of how *tall* has been used.<sup>2</sup>

Probabilistic, statistical, and fuzzy theories of vagueness have previously been brushed aside and discounted because of the failure of rather naive straw-man statistical approaches to account for the properties of vague predicates. Arguments against probabilistic approaches to vagueness such as those of Fine, Klein, Parikh, and Kamp and Partee [4, 5, 6, 7] argue against the use for threshold values of point estimates such as averages, geometric means, or arbitrary probability densities.<sup>3</sup> And they’re right; such analyses are inadequate. There is another way though. Instead of merely using statistics to calculate a point estimate and then forgetting about the original statistics, we take the information conveyed by a vague sentence to be the statistical distribution.<sup>4</sup> A vague meaning provides a conditional distribution over facts, a probability function dependent on the value

<sup>2</sup> Accounts of the semantics of scalar predicates fall into two broad categories: those referring directly to a standard of comparison, which may be derived normatively, and those using a comparison class. Our main claims could be stated either way, but for concreteness we stick to an account based on an explicit degree. In terms of our statistical model, the comparison class would act as a prior in determining the distribution over values for the threshold, though we leave the details open. Likewise, we do not discuss how normative factors affect the threshold: it suffices for our main claims that while normative factors may constrain the threshold, they will typically not constrain the threshold to be a single, perfectly precise value.

<sup>3</sup> The arguments of [4, 5, 6, 7] fall short in a second way, by not distinguishing fuzzy truth-valued from probabilistic accounts. But many criticisms of fuzzy accounts do not extend to statistical approaches. For example, fuzzy accounts fail to explain the contradictoriness of “Xena is tall and she is not tall” (cf. fn. [1]), whereas a statistical account predicts this unproblematically, provided the same degree distribution is assumed for both uses of *tall*.

<sup>4</sup> Schmidt et al. [8] and Égré and Bonnay [9] have also recently advanced similar ideas. We share this point of view with those authors and contend that the time is ripe to revisit the probabilistic nature of vagueness.

of an unbound variable, a variable that is itself constrained by a pragmatically determined statistical distribution.

### 3 Communication under Uncertainty

We can imagine, contrary to fact, a system of linguistic communication in which all expressions are precise, in principle assessable by language users as definitely true or definitely false, i.e. there are no vague expressions. In fact, much of semantics as well as information theory is aimed at developing ways of guaranteeing precision. However, we are taking a somewhat different approach and asking whether there are situations guaranteeing a form of imprecision, vagueness. We apply standard results from information theory to obtain a model of communication under uncertainty in which the following hold: (i) communication under uncertainty is represented as a game against a malevolent nature who's goal is to disrupt communication, (ii) even in the face of a malevolent nature, non-trivial exchange of information is possible, (iii) the information conveyed by vague expressions is not truth values or the precise facts upon which a relation over degrees is determined, but a distribution over such facts, and (iv) vagueness is rational under uncertainty. The discussion in this section is presented informally by way of example, but the concepts and results have counterparts in the formal theory of information. Readers familiar with the basics of information theory should be able to make the relevant connections through the citations provided.

**Definition 1.** *Information channel:* An information channel [10] is a system of communication between a speaker and a hearer for conveying and interpreting observations about objects. The input to an information channel is a speaker's beliefs or observations and the output is the utterance heard.

Suppose a speaker observes and believes that Xena's height is defined by a degree of height (e.g. 175cm) and that it exceeds some standard. Both of which *are* objectively true or false. The speaker communicates the observation by uttering "Xena is tall", and in this case the information channel is the one where the speaker inputs observations about Xena's height relative to this standard and the hearer receives these via the utterance "Xena is tall".

Such a communicative setup can be represented statistically [10] as a vocabulary of observations and utterances along with probability distributions relating them. For a speaker or hearer to know how to use the channel, i.e. to know the communicative conventions, they must have knowledge of the space of observations and utterances and the probabilities governing them.

**Definition 2.** *Mutual information:* Mutual information [10] is a measure of overlap between the distributions of some observations.

E.g. if a speaker knows Xena's height as the uniform distribution over the interval (1.7m, 1.8m) but a hearer knows it as the uniform distribution over the interval (1.75m, 1.85m) then there is an overlap, a positive amount of mutual information, in the speaker and hearer's distributions. The mutual information is larger than it would have been if the hearer knew Xena's height to be in the interval (1.3m, 1.4m), but smaller than if they agreed exactly.

In order to be as precise as possible, rational speakers and hearers should strive to have channel distributions that overlap as much as possible [10]. Perfect overlap is unachievable under uncertainty, but maximizing mutual information is still the best option [11, 12, 13].

**Definition 3.** *Information channel game:* An information channel game [13] is a game between a speaker and a malevolent nature. In an information channel game, the speaker wants utterances to convey observations as precisely as possible, as measured by mutual information, but nature counters with noise making this difficult.

Suppose again that a speaker believes Xena’s true height is defined by a degree of height which exceeds some standard, and communicates this by uttering “Xena is tall”. Now suppose that every observation the speaker has ever made was interfered with via a series of funhouse mirrors created by a malevolent nature. Thus, when the speaker says “Xena is tall”, he is likely not only to be wrong as regards Xena’s height, but also to be misinformed as to what counts as *tall*. That is, his distribution over heights as well as the distribution over the standard of comparison have been distorted by the evil nature. The hearer will then recover something which is neither Xena’s true height nor what the speaker takes Xena’s height to be. For rational speakers and hearers to know how to use this channel is to have sufficient information to cope with the channel’s noisy distributions. Because the funhouse mirrors are not controlled by the speakers or hearers, though, the best that they can do is to jointly figure out the distributions of height and its standard that overlap as much as possible with the distributions that were distorted by nature, the true distributions plus some noise.

Notice that in the funhouse mirror game speakers and hearers do not play against each other. They play together, on the same team as it were, against the noise-introducing malevolent nature. They succeed when they maximize the overlap, the mutual information, of their communicative conventions, the channel distributions, under this uncertainty.

**Proposition 1.** *Communication under uncertainty can be modeled as an information channel game.*

Rational models of linguistic communication are often presented as Lewisian signaling games [14, 15] in which speaker and hearer play on separate teams with separate (though perhaps related) utilities. Such games<sup>5</sup> have proved very fruitful, but they bring with them the baggage of strong common knowledge and rationality (CKR) assumptions and also (in the most common applications) fail to recognize the importance of shared uncertainty in communication. Complete

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<sup>5</sup> This paper is not an argument against the inclusion of signaling games in the repertoire of game-theoretic semantics and pragmatics; rather, it’s an argument that we do have the option of accounting for vagueness in part outside of the signaling framework. We in fact believe that a proper account will draw on both ways of modeling linguistic behavior. E.g. De Jaegher and van Rooij [16] show, convincingly, examples in which signaling games are adequate for an account of vagueness.

CKR in an account of vagueness is a non-starting position, neither possible nor wanted because of unavoidable uncertainty and variability in communicative situations. What is wanted is a model of communication that can accommodate the uncertainty that speakers and hearers have. If (i) what you believe or even know is like what I believe or know up to a certain point and (ii) you believe this and can guess or estimate where we differ, then (iii) what really matters is whether you can communicate according to what you believe or know plus some guesses about how that might differ from what I believe or know. Insofar as what we are interested in is completely cooperative communication in which the speaker and hearer share common goals, we might say that rational communication under uncertainty is like talking to a noisy version of yourself.

The result is reminiscent of Davidson's notion of *charity* [17], which "counsels us quite generally to prefer theories of interpretation that minimize disagreement" [18, p. xix].<sup>6</sup> Davidson makes it clear that rational communication cannot and should not eliminate disagreement; it should offer a way to make disagreement possible and useful. Substituting CKR with charity, the problem rational speakers and hearers face is to do what's best with respect to certain beliefs plus some uncertain or noisy beliefs. The current paper can be seen as a way to refine Davidson's proposal, by introducing a method for working out what is *best* in a precise and motivated way. To wit, we treat disagreement between interlocutors as if it were introduced by a malevolent third party, and view communicative acts as part of an optimal strategy against that party.

For example, a rational speaker who tries to communicate an observation of Xena's height with an utterance of "Xena is tall", faces the problem that there is potential or actual variation in speaker and hearer beliefs about Xena, height, Xena's height, and standards of *tallness*. Letting these differences be our funhouse mirrors, the speaker that knows how to communicate over the channel can calculate the optimal channel distributions in the face of the uncertainty or noise created by the mirrors.

If our model is apropos, a welcome result is that even in the face of a malevolent nature, non-trivial exchange of information is possible. That is:

**Proposition 2.** *Communication under uncertainty is possible.*

It's not obvious it should be so, although many scholars implicitly assume the above without argument; in an information channel game, the possibility of communication under uncertainty can and must be demonstrated. Here is where standard mathematical results have a voice. In an information channel game with limited amounts of uncertainty, (i) a minimax solution exists, (ii) its value is a positive amount of mutual information, and (iii) when the noise or uncertainty becomes too large, as should be expected, communication will fail [12, 13].<sup>7</sup>

<sup>6</sup> Thanks to Nicholas Asher for pointing out the parallel.

<sup>7</sup> Implicit in all three points is an assumption that the communication channel has a limiting capacity. This capacity could be due to properties of the communication medium, restrictions on the range of sentences that can be used, or processing limitations of the interlocutors.

Suppose that the speaker observes Xena's height and says "Xena is tall", and the hearer knows that Xena is a warrior princess and has some guesses about the heights of warrior princesses and related standards of *tallness*. Then the hearer will come to know something about Xena's height even if there's disagreement about what counts as *tall* for a warrior princess. This follows from the existence of a minimax solution with positive mutual information for limited amounts of uncertainty. On the other hand, if the hearer does not know what sort of entity Xena is, or thinks warrior princesses are a kind of flower, things will not turn out so well. For in this case there will be a very large mismatch between the speaker and hearer's distributions of Xena's height and the relevant standard. In such circumstances it is quite possible that interpretation will fail, in the sense that the communicative act will not convey any information about Xena's height: chalk up a victory to malevolent nature. Fortunately, interlocutors commonly know enough about each other that they can make use of an information channel and succeed in meaningful information exchange.

There are two kinds of distributions that rational speakers and hearers must know to use an information channel: the prior probabilities and the conditional probabilities. In our example, the former are the distributions of Xena's height and *tallness* and the latter are distributions relating Xena's height to *tallness* and vice versa. The most important of these is the posterior distribution of Xena's height given an utterance of "Xena is tall". This allows a hearer to recover (estimates of) the speaker's observations.

**Proposition 3.** *The information conveyed<sup>8</sup> in communication under uncertainty is the posterior probability distribution of the input observations<sup>9</sup>*

<sup>8</sup> This is not a measure or quantity of information or information content such as entropy, self information, or mutual information. It is more akin to the notions of *semantic information* or *meaningful data* [see 19]. It is a way of drawing inferences or mapping from observations to unobserved but possible observations; not the facts but a tool that could be used to narrow in on them. As a tool or way of drawing inferences, this is also quite compatible with van Deemter's [20] proposal (also in this volume) that vagueness facilitates search.

<sup>9</sup> Lipman [21] suggests that propositions like that in Prop. 3 are inconsistent with communicative behavior involving vagueness. For Lipman, the crux of the problem is that probabilities in an optimal language should be precise. So if a speaker has subjective beliefs about an observation and wants to communicate these, the speaker should communicate his subjective beliefs, precisely, via the relevant distribution. On its face this is a reasonable argument, but only because Lipman does not consider one piece of the puzzle: the speaker's precise subjective beliefs about an observation need not be the precise subjective beliefs that the speaker believes he can communicate. *Ceteris paribus*, for arbitrarily precise subjective beliefs, we take Davidson's charity principle to imply that it is rational for the speaker to precisely communicate something less precise than those subjective beliefs, since attempting to communicate something more precise would neither minimize disagreement, nor account for the hearer's charitable tendency to minimize disagreement.

Typically in an information theoretic model the hearer decodes an utterance as follows: it is the most likely input observation given the output utterance. But this is problematic if a unique best observation does not exist, since selecting one reduces to choosing randomly from the posterior. As an alternative, we suggest not worrying about decoding at all. The information conveyed by communication under uncertainty is more generally the increased knowledge that a hearer has about the possible observations a speaker made. E.g. if a speaker says “Xena is tall”, what a hearer knows for sure is a distribution describing the speaker’s observation of Xena’s height. This even holds in the case when there is no way for the hearer to recover a unique best threshold of height! The information conveyed in the communication is not an exact observation of Xena’s height relative to the standard. Rather, the information conveyed is whatever knowledge would, in the best case, allow decoding, and, in the worst case, allow the hearer to draw conclusions about what possible observations a speaker could have made of Xena’s height. This takes us to propose a working definition of what it is for a claim involving a scalar predicate to be vague:

**Definition 4.** *Vague:* A scalar adjective is vague if it constrains some measure relative to a value which cannot be known in principle or in practice.

Suppose that the claim “Xena is tall” is made after an observation that Xena is 1.8m tall. Put in terms of information conveyed, the question is whether on the basis of hearing and accepting the claim “Xena is tall”, a hearer would be in a position to say with complete certainty whether the observation that Xena is 2m tall is correct or not. And the answer is no, because the information conveyed does not suggest a precise constraint but a statistical distribution for the standard of height, and there may be a non-zero probability that the standard is higher than 2m. Thus *tall* is vague. We may contrast the case of *tall* with the case of *taller than 1.8m*. To the extent that the degree *1.8m* is itself precise, the expression *taller than 1.8m* is not vague under our definition, since the measure (Xena’s height) is constrained with respect to a precisely known value.

We note here that the inability of the information conveyed by a scalar adjective to reveal the precise value by which its scale is constrained creates the appearance that facts underlying vague expressions are indeterminate, gappy, or simultaneously true and false. But this is only an illusion much like an observation made through a funhouse mirror. There is a fact to the matter of an observation that underlies a vague sentence, but it cannot be known in principle or practice on the evidence of the information conveyed.

At this point it may seem that this definition reiterates a common definition of vagueness vis-a-vis borderline cases. Crucially it does not. This is clearer if we recognize that communication games include not just the input of an observed height and standard or comparison class, but the context and goals of communication. If any of these, too, can only be known up to a noisy posterior distribution, then the cases in which the information conveyed is unreliable

for the determination of truth multiply. Vagueness, here, is uncertainty in the information conveyed and whether it can be used to determine the facts.

With the definition of vagueness in hand, we come to the central result:

**Proposition 4.** *Vagueness is rational under uncertainty.*

The rationality of vagueness follows directly from its definition and from constraints on maximizing mutual information with respect to uncertainty. If there is any uncertainty between a speaker and hearer then the resulting channel distributions, the best or rational way for the speaker and hearer to use the channel, have a margin of error or non-negative categorization error [10]. I.e. the information conveyed by an utterance of “Xena is tall” will be a non-trivial distribution over Xena’s height. Vagueness is immediate.

## 4 Conclusions

Theories generally agree that vague expressions are uncertain or imprecise [22, 23, 24]. Where we’ve gone further is by giving a reason for the imprecision and saying how rational speakers and hearers cope with it and why they are happy to accept it or even prefer it. And from that, we can say why the sorites and epistemic uncertainty are not surprising.

Consider the *tall* warrior princess Xena in a sorites sequence with Callisto, a hair’s breadth shorter: it seems Callisto must still be *tall*. Without reference to heights or standards, this case incorrectly seems to confirm a general inductive step. In our model, the distribution by which this inference is made treats Xena and Callisto as all but indistinguishable; indeed, we may even be psychologically unable to distinguish the information conveyed about adjacent individuals in a sorites sequence. So if Xena is *tall*, we correctly predict an inference to Callisto being *tall*, but we also predict that the inference should not continue much further. According to the statistical model we employ, each distribution along the scale is (slightly) different, and to the extent that speakers and hearers reason with such models, it is predicted that statements made about individuals far apart on the scale convey different information, i.e. different distributions over possible observations. The confidence as well as the uncertainty that speakers and hearers have does not remain constant moving along the scale. Thus if we were to allow Callisto to be 30,000 hair’s breadths shorter than Xena, then the inference from “Xena is tall” to “Callisto is tall” will not necessarily be made with the same confidence. This is, of course, precisely what we observe.

Now consider higher-order vagueness: the locations of the boundaries of a region where truth values are indeterminate may themselves fail to be precise. The awkward result is that imprecision remains as ever more regions of higher-order vagueness are added. The logical revisions in supervaluationist [4, 7] and dynamic accounts [25] try to eliminate imprecision rather than bringing it to the fore. Since we embrace imprecision within our model with statistical

uncertainty, we never make a decision about where precision stops and imprecision begins<sup>10</sup>

We claim that vague scalar adjectives fail to determine the standards of comparison upon which they supervene. This is similar to Williamson's [29, 30] epistemic view, which characterizes vagueness as a form of essential ignorance. When vague expressions are used, as speakers and hearers, we are only concerned with knowledge up to a margin of error; we are ignorant of the precise facts. Williamson's claims can be substantiated in a formal model such as ours<sup>11</sup>. The epistemic view, on its own, does not make it especially clear why ignorance bubbles up into communication (it's not obviously necessary; precise languages are staples in both philosophy and information theory) or why margins of error exist. In an information channel, though, we have ignorance because margins of error are unavoidable thus making ignorance rational under uncertainty. The ignorance in vagueness is strategic ignorance. Additionally, whereas Williamson [29, 30] gives a somewhat vague explanation for the widely accepted idea that vagueness supervenes on precision, we make it precise: vagueness is the product of limits on the information conveyed, information which supervenes on the precise facts in a statistical sense.

## References

- [1] Cresswell, M.J.: The semantics of degree. In: Partee, B. (ed.) *Montague Grammar*, pp. 261–292. Academic Press, New York (1976)
- [2] Kratzer, A.: The notional category of modality. In: *Words, Worlds, and Contexts: New Approaches in Word Semantics*, pp. 38–74. de Gruyter, Berlin (1981)
- [3] von Fintel, K.: *Restrictions on Quantifier Domains*. PhD thesis, University of Massachusetts at Amherst (May 1994)
- [4] Fine, K.: Vagueness, truth and logic. *Synthese* 30(3–4), 265–300 (1975)
- [5] Klein, E.: The semantics of positive and comparative adjectives. *Linguistics and Philosophy* 4, 1–45 (1980)

<sup>10</sup> It is common to attack vagueness with more sophisticated ways of comparing individuals to standards of comparison or comparison classes [3, 24]. These theories hint at the role of uncertainty and may provide an adequate model-theoretic semantics, but are silent regarding the source of the ordering relations. Are they the products of thought? Should we expect them to be viable strategies in communication? Our hope is that ideally such models could be motivated by looking at properties of rational communication.

Pragmatic accounts have also contributed to our understanding of vagueness: we are friends of the idea that vagueness is tied to the purposes and practices of communication and that the uncertainties present in the communicative situation impact what speakers and hearers are willing to accept or find useful [26, 27]. There have also been other models of vagueness as language games. As we do, these [6, 28] derive vagueness from assumptions about communication, but our model is unique in being an information channel game and in providing a criterion for vagueness along with a notion of the information conveyed by vague expressions.

<sup>11</sup> De Jaeger [23] also notes a deep connection between vague communication and epistemic uncertainty as suggested by a signaling game with correlated equilibria.

- [6] Parikh, R.: Vagueness and utility: The semantics of common nouns. *Linguistics and Philosophy* 17(6), 521–535 (1994)
- [7] Kamp, H., Partee, B.: Prototype theory and compositionality. *Cognition* 57, 129–191 (1995)
- [8] Schmidt, L.A., Goodman, N.D., Barner, D., Tenenbaum, J.B.: How tall is *Tall*? compositionality, statistics, and gradable adjectives. In: *Proceedings of the 31st Annual Conference of the Cognitive Science Society* (2009)
- [9] Égré, P., Bonnay, D.: Vagueness, uncertainty and degrees of clarity. *Synthese* (2009), doi:10.1007/s11229-009-9684-8
- [10] Cover, T.M., Thomas, J.A.: *Elements of Information Theory*. Wiley, Chichester (2006)
- [11] Verdu, S.: The exponential distribution in information theory. *Problems of Information Transmission* 32(1), 100–111 (1996)
- [12] Haussler, D.: A general minimax result for relative entropy. *IEEE Transactions on Information Theory* 43(4), 1276–1280 (1997)
- [13] Mathar, R., Schmeink, A.: Saddle-point properties and nash equilibria for channel games. *EURASIP Journal on Advances in Signal Processing* 2009(823513) (2009)
- [14] Lewis, D.: *Convention: A Philosophical Study*. Harvard University Press, Cambridge (1969)
- [15] Benz, A., Jäger, G., van Rooij, R.: An introduction to game theory for linguists. In: Benz, A., Jäger, G., van Rooij, R. (eds.) *Game Theory and Pragmatics*. Palgrave Macmillan, Basingstoke (2005)
- [16] De Jaegher, K., van Rooij, R.: Strategic vagueness, and appropriate contexts. ms. (2008)
- [17] Davidson, D.: On the very idea of a conceptual scheme. In: *Proceedings and Addresses of the American Philosophical Association*, vol. 47, pp. 5–20 (1974)
- [18] Davidson, D.: *Inquiries into Truth and Interpretation*. Oxford University Press, Oxford (2001)
- [19] Floridi, L.: Philosophical conceptions of information. In: Sommaruga, G. (ed.) *Formal Theories of Information*. LNCS, vol. 5363, pp. 13–53. Springer, Heidelberg (2009)
- [20] van Deemter, K.: Vagueness facilitates search. In: *Proceedings of the 17th Amsterdam Colloquium* (2009)
- [21] Lipman, B.L.: Why is language vague? ms. (December 2006)
- [22] Fara, D.G., Williamson, T.: Vagueness. In: Fara, D.G., Williamson, T. (eds.) *Vagueness (International Research Library of Philosophy)*, Ashgate (2002)
- [23] Kennedy, C.: Ambiguity and vagueness: An overview. In: Maienborn, C., von Stechow, P., Portner, P. (eds.) *The Handbook of Semantics* (to appear)
- [24] van Rooij, R.: Vagueness and linguistics. In: Ronzitti, G. (ed.) *The Vagueness Handbook* (2009) (to appear)
- [25] Barker, C.: The dynamics of vagueness. *Linguistics and Philosophy* 25, 1–36 (2002)
- [26] Lasnik, P.: Pragmatic halos. *Language* 75(3), 522–551 (1999)
- [27] Fara, D.G.: Shifting sands: An interest-relative theory of vagueness. *Philosophical Topics* 28(1), 45–81 (2000)
- [28] De Jaeger, K.: A game-theoretic rationale for vagueness. *Linguistics and Philosophy* 26(5), 637–659 (2004)
- [29] Williamson, T.: Vagueness and ignorance. In: *Proceedings of the Aristotelian Society*, pp. 145–162 (1992)
- [30] Williamson, T.: Imagination, stipulation and vagueness. *Philosophical Issues* 8(Truth), 215–228 (1997)

# Restricted Quantification over Tastes

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**Abstract.** This paper provides an analysis of statements with predicates of personal taste (*tasty*, *fun*, etc.). Rather than directly relativizing semantic interpretation to a judge (cf., Lasersohn, 2005), this paper aims to capture the phenomenon called ‘faultless disagreement’ (the fact that one can deny a speaker’s subjective utterance without challenging the speaker’s opinion) by means of pragmatic restrictions on quantification domains. Using vagueness models, a statement like *the cake is tasty* is analyzed as true in a partial context  $c$  iff it is true in the set of completions  $t$  consistent with  $c$  (Kamp, 1975), wherein *tasty* denotes different, contextually possible, taste measures (Kennedy, 1999). Phrases like *for me* restrict the set of completions to those with taste measures consistent with the speaker’s taste. Faultless disagreement naturally follows assuming speakers accommodate or reject implicit restrictions of this sort (Lewis, 1979).

**Keywords:** Taste; faultless disagreement; vagueness; context restriction.

## 1 Disagreements over Taste

One can deny an utterance like *the cake is tasty* or *running is fun* without challenging the speaker’s opinion. The speaker asserts ‘ $\phi$ ’, his addressee ‘ $\neg\phi$ ’, and still neither can be blamed for making a mistake. Lasersohn (2005); MacFarlane (2005), and Egan et al. (2005), among others, illustrate that such ‘faultless disagreements’ characterize, beside statements about taste, also aesthetic, moral and probability statements, future contingents, vagueness, and epistemic modals. These authors aim to account for faultless disagreements by relativizing semantic interpretation to a judge (or ‘context of assessment’). On this ‘relative truth’ approach, contexts specify in addition to a speaker, a world and a time of evaluation, also a judge. Due to variance in the judge-world-time parameters, the truth values of statements with, e.g., taste predicates, may vary between contexts (cf., (1a)), although their content remains the same (1c).

- (1)
- a.  $\llbracket \text{The cake is tasty} \rrbracket_{c,i,a} = 1$  iff  $\llbracket \text{The cake} \rrbracket_{c,i,a} \in \llbracket \text{tasty} \rrbracket_{c,i,a}$   
(where  $c$  is a context,  $i$  a world-time pair and  $a$  the judge).
  - b.  $\llbracket \text{The cake is tasty for Bill} \rrbracket_{c,i,a} = 1$  iff  $\llbracket \text{The cake is tasty} \rrbracket_{c,i,\text{Bill}} = 1$
  - c.  $\llbracket \text{The cake is tasty} \rrbracket_c = 1$  iff  $\llbracket \text{The cake is tasty} \rrbracket_c(W(c), A(c))$   
(where  $W(c)$  and  $A(c)$  are the world-time pair and judge in  $c$ .)

In this setting, faultless disagreement arise due to shifts in the contextual parameters, usually triggered by speakers’ tendency towards autocentric interpretations, e.g., while

a speaker may truthfully assert *the cake is tasty* based on herself as a judge, her addressee may truthfully assert *the cake is not tasty*, with himself as the judge. Still, no cake is both *tasty* and *not tasty* in any single context.

Dominant opponents of this approach argue against it based on theoretical and empirical considerations (for a detailed discussion see von Fintel and Gilles, 2008a). Importantly, if semantics is subjective to the extent suggested by relative truth theories, then why do speakers ever bother to deny others' utterances at all?

On the other side of the spectrum, theories claim for standard, 'impersonal' interpretations for taste predicates (Nouwen, 2009; Wolf, 2009), such as those in (2).

- (2)
- a.  $\llbracket \text{The cake is tasty} \rrbracket_i = 1$  iff  $\llbracket \text{the cake} \rrbracket_i \in \llbracket \text{tasty} \rrbracket_i$ .
  - b.  $\llbracket \text{Bill finds the cake tasty} \rrbracket_i = 1$  iff  $\llbracket \text{the cake} \rrbracket_i \in \llbracket \text{tasty} \rrbracket_i$ , for all indices  $i$  consistent with Bill's subjective experience.

Faultless disagreements are explained by virtue of the vagueness of taste predicates. We are inherently unable to reach full knowledge of the extension of expressions like *tasty*. Consequently, we can only base our claims on our own sensory experience (the gastronomic pleasure we feel while eating things), as well as on weak clues others give us about their experience. Thus, all edible things are *always* borderline of taste predicates (Nouwen, 2009).

Formally, vague predicates are often interpreted within contexts  $c$ , via a set of indices  $T_c$ , the worlds (Stalnaker 1978) or completions (van Fraassen, 1969; Kamp, 1975) consistent with the information in  $c$  (completions being 'classical' contexts, wherein every statement is either true or false). Truth of a statement  $S$  in  $c$  is defined based on these indices as follows (time indices are avoided, for simplicity sake):

- (3)
- $S$  is true in  $c$  iff  $S$  is true in every  $t \in T_c$ ;  $S$  is false in  $c$  iff  $S$  is false in every  $t \in T_c$ , and  $S$  is undetermined in  $c$ , otherwise.

The 'impersonal', vagueness based theory considers every taste statement always undetermined in actual contexts  $c$ . This has a variety of consequences. First and foremost, even when no attitude report or *for/ to* PP is present, there is still the intuition that taste predicates express someone's taste (after all, the most trustworthy source of beliefs about taste comes from our private sensory experience). Second, there is nothing scalar about taste predicates, as everything forms a borderline case. The problem is that if personal tastes do not tell us anything about the impersonal interpretation of *tasty*, the theory about it is impossible to refute. And if personal tastes do show anything, then, the large-scale taste differences between people prove false the idea of a single impersonal interpretation.

## 2 A New Proposal

In this paper, focusing on predicates of personal taste, I argue that relativity does not enter the semantics, except through independently-motivated, pragmatic mechanisms. While the approach developed in this paper is eventually vagueness-based, it also differs from, e.g., Nouwen (2009), in important respects. Part 2 elaborates on its

different components and how they bear on the personal and public aspects of taste predicates. Part 3 presents a variety of consequences. In addition, it grounds the discussion within a broader framework of linguistic analysis of different types of expressions with ‘personal’ ingredients.

## 2.1 The Personal Ingredient

First and foremost, this paper claims that there is no such thing as “one true answer” when it comes to taste. Do you, the readers, like salty French fries or sweet cream cakes, both or neither? Do you prefer Mozart to Bach, or the opposite? Let alone opera sopranos or rock stars. Different readers can definitely possess different tastes, and probably more often than not, they in fact do. Thus, a single unique interpretation of, e.g., *tasty* or *fun* need not necessarily exist in the actual world. In order to formally represent this idea, interpretation in contexts  $c$  can be modeled via the set  $T_c$  of completions  $t$  consistent with the information in  $c$ . These indices can be richer than worlds. For example, in Kamp (1975) completions determine cutoff points for vague adjectives, although the author holds that no such thing as a cutoff point exists in the actual world. On the present proposal, completions determine full fledged interpretations for, e.g., *tasty*. Each corresponds to the taste of one possible individual. Thus, different completions differ with respect to the interpretation of *tasty*, while none corresponds to the actual ‘objective’ (inter-personal) interpretation (as such probably does not exist).

What is the basic interpretation of *tasty* in each completion  $t$ ? Personal taste predicates  $P$  are typically gradable (cf., the felicity of *tastier*, *as tasty as*, *very / too / fairly tasty*, etc.), though the existence of different tastes imply different scales. Therefore, this paper associates taste predicates in each completion  $t$  with degree functions,  $f(P, t)$ , i.e. mappings of entities to degrees (cf., Kennedy, 1999). Thus, *tasty* holds true of an object  $x$  in  $t$  iff the value  $f(\text{tasty}, t)(x)$  exceeds the cutoff point (standard of membership) of *tasty* in  $t$ ,  $\text{standard}(\text{tasty}, t)$ , as illustrated in (4). *Tastier* holds true of an object pair  $\langle x, y \rangle$  in  $t$  iff the value  $f(\text{tasty}, t)(x)$  exceeds the value  $f(\text{tasty}, t)(y)$ , as illustrated in (5).<sup>1</sup>

- (4)  $\llbracket \text{The cake is tasty} \rrbracket_c = 1$  iff for all  $t \in T_c$ ,  $\llbracket \text{the cake} \rrbracket_t \in \llbracket \text{tasty} \rrbracket_t$   
 Iff for all  $t \in T_c$ ,  $f(\text{tasty}, t)(\llbracket \text{the cake} \rrbracket_t) > \text{standard}(\text{tasty}, t)$ .
- (5)  $\llbracket \text{The cake is tastier than the ice cream} \rrbracket_c = 1$   
 Iff for all  $t \in T_c$ ,  $\langle \llbracket \text{the cake} \rrbracket_t, \llbracket \text{the ice cream} \rrbracket_t \rangle \in \llbracket \text{tastier} \rrbracket_t$   
 Iff for all  $t \in T_c$ ,  $f(\text{tasty}, t)(\llbracket \text{the cake} \rrbracket_t) > f(\text{tasty}, t)(\llbracket \text{the ice cream} \rrbracket_t)$

Modifiers and subordinators such as *for X*, *to X*, *in X's opinion*, *I find X P* or *I consider X P*, are, therefore, explicit means of subjectively restricting contexts. For example, *the cake is tasty for me* is true in  $c$  iff *the cake is tasty* is true in every completion  $t$  of  $c$  in which the values of  $f(\text{tasty}, t)$  and  $\text{standard}(\text{tasty}, t)$  are consistent with my own taste: Entities’ values represent my opinion about their tastes, and so is the cutoff point between *tasty* and *not-tasty*.

<sup>1</sup> Nothing hinges on this type of an analysis of gradability. If an analysis without degrees can be made to work out, then probably it can also be made compatible with the present proposal.

- (6)  $\llbracket \text{The cake is tasty for Dan} \rrbracket_c = 1$  iff  
 For all  $t \in T_{\text{Dan}} \subseteq T_c$ ,  $f(\text{tasty}, t)(\llbracket \text{the cake} \rrbracket_t) > \text{standard}(\text{tasty}, t)$ .

As the information these constituents provide is presupposed, not asserted, it tends to be preserved under negation, e.g., *It is not tasty for Sam* doesn't normally mean it is tasty, but not for Sam.

## 2.2 The General (Public) Ingredient

Second, according to the present proposal, although tastes are personal, information about the interpretation of taste predicates *can* become publicly available.

Information about particular tastes can be gained through individual eating experiences, based on which generalizations on taste can be inferred. Thus, we can reasonably say that oil does not taste well, but chocolate does. Some people are less crazy than others about chocolate, but almost no one considers it disgusting. The opposite is the case with regard to oil. In fact, one's ability to draw generalizations about the taste of different groups of people plays an important role in one's social and cultural life. Conversations and disputes regarding utterances like, *the cake is tasty* are a main tool to this end. For example, it is publicly known that not everyone loves, say, avocado salads. Thus, before offering such a salad one is likely to consult with her partner about the matter. Similarly, before choosing a film to go to on a first date, one would usually try to find out whether her partner finds romantic comedies more *fun* than horror films, or vice versa, whether mainstream is fine, or independent cinema is preferred, and so on and so fourth. This is important because speakers order tastes in hierarchies. They argue for or against these orderings based on their cultural appropriateness or fitness to the spirit of the time and circumstances. Speakers do all that by uttering and disputing taste statements, thereby expressing or negotiating social dominance relations (thus, formally, completions are ordered by contextual relevance depending on how relevant and important the taste functions in them rank).

To recap, due to all that, if, for instance, one is to invite guests for dinner (including, say, one's bosses), one can fairly safely choose to serve pasta with a Bolognese sauce, rather than, say, an avocado with mint salad. But if some guests are vegetarian, then one may better go for a fungi cream sauce. And if some have children, getting the simplest sort of Ketchup, or even leaving some Pasta with no sauce at all, is probably a very good idea (adults will not like it, though). Of course, these and many others descriptions of facts about tastes are non-perfect generalizations. They admit exceptions. This brings us to the next point.

## 2.3 Restricted Generalizations: Synthesis of the Personal and General

Third, quantification in natural languages is by default restricted. Consider, for example, the quantifying expression *everything* in (7a). The domain of the universal quantifier denoted by *everything* clearly does not include every possible object, only *sites in Paris*, or maybe even only famous or adored sites.

- (7) I lived near the Seine, near Boulevard St. Germain and Rue St. Michel, near the market and the Pantheon, near everything (*Haaretz*, 04.01.2002).

Conventionally, contextual restrictions on quantifying expressions are represented by a context variable (say-,  $X_t$ ), whose value is a set of relevant individuals (cf., von Fintel 1994). Accordingly, the truth conditions of a statement with a quantifying expression, like (8a), require that every individual, which is a duck and is in the set of relevant individuals will be in the denotation of *lays whitish eggs*.

- (8)  $\forall t \in T_c: \llbracket \text{A duck lays whitish eggs} \rrbracket_t = 1$  iff  $\forall x \in (\llbracket \text{duck} \rrbracket_t \cap X_{\text{duck},t})$ :  
 $x \in \llbracket \text{lays whitish eggs} \rrbracket_t$  ( $X_{\text{duck},t}$  being the set of relevant entities in  $t$ ).<sup>2</sup>

We also find restrictions in theories of conditionals (Kratzer 1979, 1986; Kadmon and Landman 1993). The conditional in (9a) is restricted via  $X_{\text{John subscribes to a newspaper}}$  to be only about subscriptions to a newspaper that John can read. (9b) is restricted to eventualities (or completions) in which there is no oil in the tea. This is crucial, for instance, to account for the fact that, intuitively, (9b) fails to entail (10).

- (9) a. If John subscribes to a newspaper, he gets well informed.  
 b. If there is sugar in the tea, the tea tastes well.  
 (10) If there is sugar and oil in the tea, the tea tastes well.

Thus, grammar encompasses mechanisms of implicit domain restriction (Partee, 1989; Kratzer 1979, 1986; von Fintel 1994). They can be readily used also for our present purposes. Thus, we can say that some things (e.g., oil) are tasty according to no ‘relevant’ taster in a given context (restricting the tasters to be human, with a typical taste bud system, etc.) Other things (e.g., Pasta Bolognese) are tasty according to all tasters in a contextually given set (we do not always *have* to worry about, e.g., children or vegetarian, anymore). Thus, in each index, *tasty* measures the extent of gastronomic pleasure an eating event causes a certain type of subject. The subject in question, though, is most often not specified, allowing hearers to take into account all the contextually plausible and relevant tastes. Disputes (or agreements) about taste make sense precisely because taste statements are general, rather than personal.

To be sure, denotations of predicates are often interpreted restrictively even in the absence of a quantifying expression. Consider, for example, utterances of negated predicates like *not a bird* or *non-birds* in a context of a zoo. Hearers hardly ever interpret these predicates as referring to the garbage cans or to the cages or fences; nor do they interpret them as referring to visitors in the zoo, though all these constitute members of the given denotations. Moreover, hearers do not normally assume that sentences like *Tweety is a bird* are about ostriches. Atypical birds are, by default, considered irrelevant.<sup>3</sup> Finally, context restrictions are certainly at play when statements with vague predicates such as *the line is long* are evaluated. We can assert such statements only when we contextually restrict the set of possible cutoff points.

For the purposes of this paper (representing faultless disagreements), I make do by representing implicit context restrictions in taste statements via accommodation of,

<sup>2</sup> Notice that in this formulation  $X_t$  is given in each word/ completion separately, which allows us to represent vagueness with regard to the set of ‘relevant’ individuals (cf., Sassoon, 2009).

<sup>3</sup> For other examples and implications, see Sassoon (2009) and references therein.

e.g., an implicit *for* phrase or *find* subordinator's interpretation (Lewis, 1979), but for alternative proposals concerning the representation of implicit context restrictions see von Fintel (1994) and Stanley and Szabo (2000).

von Fintel and Gilles (2008) argue for a 'restricted quantification' analysis of faultless disagreement in statements with epistemic modality. Epistemic modals quantify over possibilities consistent with a relevant information state, where contexts of utterance decide whose information state that is. But given a context of utterance, there are multiple ways of drawing the boundaries of the group holding the relevant information. Thus, it is indeterminate just which set of possibilities is quantified over by bare epistemic modals.

Crucially, in von Fintel and Gilles (2008), the group whose knowledge is under discussion only enters into play indirectly (the set of possibilities forming the modal base is construed of worlds consistent with the information distributed over the group members). This analysis has an appealing advantage. Epistemic modals maintain the interpretation type other modals have (quantifiers over a modal base). For the very same reason, there is no a priori reason against taking the group of judges (the subjects whose taste is under discussion) to affect the truth conditions of taste statements only indirectly. The set of possibilities based on which a vague taste adjective is interpreted is construed based on the tastes of the group members. This analysis has an appealing advantage. Adjectives of personal taste maintain the interpretation type and characteristics of other vague adjectives. Let us, then, present the main consequences of such a proposal.<sup>4</sup>

### 3 The Consequences of Restricted Quantification over Tastes

First and foremost, the conjunction of two sentences such as those in (11a-b) is a contradiction *in the absence and only in the absence of an appropriate context restriction*. The contradictory interpretation illustrated in (12a-b) is that the cake is both above and below *tasty*'s standard in any completion *t* of *c* (i.e., according to any possible, conceivable, taste measure and cutoff point in *c*.)

- (11)      a. (Dan:) The cake is tasty                    .  
               b. (Sam:) No it's not.
- (12)      a.  $\forall t \in T_c, f(\text{tasty}, t)(\llbracket \text{The cake} \rrbracket_t) > \text{standard}(\text{tasty}, t)$ .  
               b.  $\forall t \in T_c, f(\text{tasty}, t)(\llbracket \text{The cake} \rrbracket_t) \leq \text{standard}(\text{tasty}, t)$ .

Second, context restriction (e.g., via accommodation) can turn the interpretation equivalent to that of sentences (13a-b) (Lewis, 1979). When discourse participants are cooperative, striving for a non-contradictory interpretation, they are likely to assume that context is thus restricted. The result is the non-contradictory interpretation in (14a-b).

<sup>4</sup> von Fintel and Gilles (2008) develop also a pragmatic theory of usage of epistemic modals. The present paper provides a theoretical setup, based on which this theory can be tested against data about taste statements. The discussion in the next section is preliminary, but nonetheless suggests that doing so can indeed be fruitful.

- (13) a. The cake is tasty (for Dan / in Dan's opinion).  
 b. No it's not (for Sam / in Sam's opinion).  
 (14) a.  $\forall t \in T_c$ , s.t. the values of  $f(\text{tasty}, t)$  and  $\text{standard}(\text{tasty}, t)$  represent Dan's taste in  $c$ ,  $f(\text{tasty}, t)(\llbracket \text{The cake} \rrbracket_t) > \text{standard}(\text{tasty}, t)$ .  
 b.  $\forall t \in T_c$ , s.t. the values of  $f(\text{tasty}, t)$  and  $\text{standard}(\text{tasty}, t)$  represent Sam's taste in  $c$ ,  $f(\text{tasty}, t)(\llbracket \text{The cake} \rrbracket_t) \leq \text{standard}(\text{tasty}, t)$ .

If Dan and Sam's tastes are different, they are adequately represented by different taste functions and/or cutoff points, namely by two different, non-overlapping sets of indices ( $T_{\text{Dan}} \subseteq T_c$  and  $T_{\text{Sam}} \subseteq T_c$ , but  $T_{\text{Dan}} \cap T_{\text{Sam}} = \emptyset$ ), differing along the interpretation of *tasty*. Thus, it may well be the case that in every  $t$  in  $T_{\text{Dan}}$ , the cake's degree of taste exceeds the standard, but in no  $t$  in  $T_{\text{Sam}}$ , the cake's degree does so. Still, the speakers may well agree that the indices in both these sets are consistent with the common ground, i.e., the interpretation of *tasty* in each represents a legitimate taste in  $c$ .

At any rate, such a polite discussion, with two purely personal interpretations, is characteristic of first dates or highly official meetings with very important personalities. Let us consider Dan and Sam after some period of happy relationships. Now they are less busy being polite (rendering utterances of the partner true), and more busy getting to know one another. Now Sam is more likely to respond to utterances such as *this c.d. is fun* or *this salad is tasty* with something like *C'mmon, that's teenage music!* or *No way! How can an Avocado-mint salad be tasty?*

Luckily, given the present analysis there is a natural sense in which two speakers in a dialogue such as (11a-b) may disagree. In uttering (11b) a bit less apologetically (or more assertively) than in a first date, Sam may imply that she prefers evaluating the asserted statement relative to a different set of taste functions and/or cutoff points, perhaps because she views them as superior (more plausible or relevant), and hence more appropriate in  $c$  (more likely to be considered part of  $T_c$ ). What is more, despite our pluralism with regard to taste, we all agree that, say, soap doesn't taste. Not everything goes. Thus, Sam's non-use of an explicit restrictor (e.g., *for Sam*) may even convey that in  $c$  we all agree that the cake doesn't taste (let us represent this proposition as  $\neg\phi$ ), i.e., that (11b) (namely,  $\forall t \in T_c, \llbracket \neg\phi \rrbracket_t = 1$ ) is true, while (13a) (namely,  $\forall t \in T_{\text{Dan}}, \llbracket \phi \rrbracket_t = 1$ ) is inappropriate, as  $T_{\text{Dan}} \cap T_c = \emptyset$ .

We have just illustrated a personal interpretation vs. public interpretation scenario. Alternatively, in a bad day, Sam may interpret Dan non-restrictively, as conveying (11a) (namely,  $\forall t \in T_c, \llbracket \phi \rrbracket_t = 1$ ), and reject that on the basis that  $c$  is more pluralistic. It is consistent with completions in which the cake isn't tasty. Sam can take her own taste to form evidence for that ( $\exists t \in T_{\text{Sam}} \subseteq T_c, \llbracket \neg\phi \rrbracket_t = 1$ ). This is a public vs. personal scenario. It is not cooperative in that Sam's response does not render Dan's utterance true (cf., von Fintel and Gilles, 2008). Dan could have been taken to base his generalization on his personal experience alone, but he was taken to provide a more general conjecture.

Thus, disagreements are taken to be faultless in a context  $c$  iff different context restrictions (and hence different completion sets) are used, whose legitimacy both sides appreciate. This condition is not always met, in particular not in personal vs. public scenarios characteristic of 'academic' discourse about art, food, and the like. In disagreements between students and their professor or between a listener and an expert

on a Radio program, the responders are taken to ‘know more’ about taste than the speakers, whose views might, therefore, be considered mistaken (if they are not assertive, their tastes might drop off the contextually reduced common grounds).

Conversely, the public vs. personal scenario is characteristic of, for instance, mothers trying to gently convince their children to eat healthy food. A mother may be in a position to assert a statement like *This cheese is very tasty* if, say, the children she knows find it tasty. She may assert the statement even if she herself does not find the cheese tasty, because adult tastes may simply be irrelevant.<sup>5</sup> If mother cannot really tell what her child’s taste is like, she can only convey that  $\forall t \in (\cup \{T_x : x \text{ is one of the other children mother knows}\}) \subseteq T_c$ ,  $\llbracket \text{This cheese is tasty} \rrbracket_t = 1$ . At the same time, mother may conjecture or ask about the interpretation based on (i) her child’s taste, or (ii) her child’s taste together with the taste of the other children she knows or additional children (von Fintel and Gilles, 2008). The child, who may well be convinced that a candy is tastier to eat, may answer *Yucky, this cheese is disgusting!* Being the speaker, a child, and one who is expected to eat the cheese, he can legitimately consider himself part of the group whose tastes are relevant. As such, his view about the cheese gives sufficient evidence against mother’s conjectures. The child can confirm or deny the strongest proposition she may have issued, whose truth value he presumes to know (von Fintel and Gilles, 2008 and references therein). Confirming her own proposition (as in *#ok, it is tasty*<sub>for every child you know</sub>) is not informative enough, certainly if he considers the cheese disgusting. One appropriate answer is then *No, this cheese is disgusting*, which can only be based on the child’s own taste ( $\forall t \in T_{me} \subseteq T_c$ ,  $\llbracket \text{this cheese is disgusting} \rrbracket_t = 1$ ). This interpretation is strong in that it entails that the cheese is not merely in the gap of *tasty* in *c*, but rather is in the negative denotation (which entails the falsity of (i), i.e. of “it is tasty for me”). Another appropriate answer is *No, this cheese is not tasty*, based on the taste of a group containing him and perhaps all other possible children. This answer conveys that  $\neg \forall t \in (\cup \{T_x : x \text{ is a child}\}) \subseteq T_c$ ,  $\llbracket \text{This cheese is tasty} \rrbracket_t = 1$ , which entails that (ii) (“it is tasty for every child you know plus me”) is not true. Denial makes sense because an interpretation based on a larger set of tastes (than those based on which mother made her assertion) is there to deny (von Fintel and Gilles, 2008).

In conclusion, disagreements have a substance because the interpretation of taste statements is mediated by an indeterminate parameter (e.g., whose taste is at stake), which tends to be interpreted as – restricted, but still – general, rather than personal. Generalizations can be disputed. Even speakers who only have a basis to assert an autocentric interpretation of a statement, conjecture about more general interpretations, checking whether their hearers are willing to accommodate their taste.

So far we have illustrated how disagreements on the application of taste predicates may arise due to inherent indeterminacy regarding the degree function they denote. This kind of disagreements is impossible if *tasty* in, say, (11), is substituted by a predicate such as *tall*. The latter’s degree function is fixed across contexts, or at least determines a context invariant ordering between entities’ (based on their heights).

<sup>5</sup> In fact von Fintel and Gilles (2008) illustrate this with epistemic modals, e.g., following a blood test that may rule out the possibility that John has cancer, an utterance of *I don’t know whether john might have cancer; I will ask the doctors* cannot mean “I don’t know whether, in view of what I know, John might have cancer”. The doctors’ knowledge is at stake.

Still, Sam and Dan can disagree about the comparison class. Sam may assert that *Joe is tall* based on Joe being taller than her (i.e. based on indices in which Joe and her form the comparison class), and Dan may disagree saying *no, Joe is not tall!*, based on Joe's being shorter than him (indices in which Joe and him form the given class).

Furthermore, again, a *for* phrase can provide a restriction to indices with a given comparison class, as in *Mary is tall for her age/ school*. In addition, in, for instance, *these cookies are tasty for cats*, the *for* phrase operates as a restrictor both at the level of the comparison class and at the level of the taste functions in question. Moreover, *for* phrases can restrict epistemic modal bases (as in *for all that I know,...*) Finally, the same data can be found in other languages, like Hebrew and Italian.<sup>6</sup> Altogether, the data support an analysis of *for* phrases as temporarily restricting evaluation contexts.

To sum up, the present approach captures different types of personal ingredients of interpretation and their relation to the common ground. It provides unified means to account for a variety of ingredients that grammar collapses together through the use of *for* phrases (e.g., personal degree functions, comparison classes, and epistemic modal bases). No new grammatical mechanisms are introduced. Independently-motivated, pragmatic mechanisms (accommodation of context restrictions) suffice. While the availability of faultless disagreements supports a classification of taste predicates as vague or indeterminate, it does not support their classification as true relative to a judge (Lasersohn, 2005). Nor does their availability depend on the way a personal ingredient may be realized (as indices, implicit indexicals, or non-indexical parameters). Rather, it depends on the existence of indeterminacy. If it is possible to interpret an ingredient affecting the truth conditions as general, the very attempt to generalize invokes disputes. If it is possible to withdraw (restrict the domain of application of the generalization) disagreement becomes faultless.

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## References

1. von Fintel, K.: Restrictions on Quantifier Domains. Ph.D. Dissertation, Uni. of Massachusetts, Amherst (1994)
2. von Fintel, K., Gillies, A.S.: Might Made Right. In: Egan, A., Weatherson, B. (eds.) *Epistemic modality*. Oxford University Press, Oxford (2008) (to appear)
3. von Fintel, K., Gillies, A.S.: CIA Leaks. *The Philosophical Review* 117(1), 77–98 (2008a)
4. Kadmon, N., Landman, F.: Any. *Linguistics and Philosophy* 16, 353–422 (1993)
5. Kamp, H.: Two theories about adjectives. In: Keenan, E.L. (ed.) *Formal Semantics of Natural Language*, pp. 123–155. Cambridge University Press, Cambridge (1975)

<sup>6</sup> In Hebrew we find: *Bill gavoha le-gilo* ('Bill is tall for his age'); *Ha-uga teima le-Bill/ le-xatulim* ('The cake is tasty for Bill/ for cats') and *Le-or ha-yeda shely, ...* ('For all that I know,...'). In Italian we find: *È grande per la sua età* ('(s/he) is big for his age'); *La torta è gustosa per me / per (i) gatti* ('The cake is tasty for me /for (the) cats'); and *Per tutto ciò che so, ...* ('For all that I know,...').

6. Kennedy, C.: Projecting the adjective: the syntax and semantics of gradability and comparison. Garland, NY (1997 UCSC PhD thesis) (1999)
7. Kratzer, A.: Conditional necessity and possibility. In: Bauerle, R., Egli, U., von Stechow, A. (eds.) *Semantics from different points of view*. Springer, Heidelberg (1979)
8. Kratzer, A.: Conditionals. In: Farley, A.M., Farley, P.T., McCullough, K.E. (eds.) *CLS 22 part 2: Papers from the parasession on pragmatics and grammatical theory*. CLS, University of Chicago, Chicago (1986)
9. Lasersohn, P.: Context dependence, disagreement, and predicates of personal taste. *Linguistics and Philosophy* 28, 643–686 (2005)
10. Lewis, D.: Scorekeeping in a Language Game. *Journal of Philosophical Logic* 8, 339–359 (1983); Lewis, D.K.: *Philosophical Papers* 1, pp. 233–249. Oxford University Press, Oxford (1979)
11. MacFarlane, J.: Making sense of relative truth. In: *Proceedings of the Aristotelian society*, pp. 321–339 (2005)
12. Nouwen, R.: Predicates of (Im)personal Taste. Manuscript (2007)
13. Partee, B.: Binding implicit variables in quantified contexts. In: *Papers from the 25th Regional Meeting, Part 1*. Chicago Linguistic Society, Chicago (1989)
14. Sassoon, W.: Galit: Semantics with clusters of properties. In: Kissine, M., De Brabanter, P. (eds.) *Utterance Interpretation and Cognitive models*, CRISPI, Emerald group publishing (2009)
15. Stanley, Szabo: *Mind and Language* vol. 15(2), pp. 219–261 (2000)
16. Wolf, L.: *Objective Judgment*. Manuscript. ILLC/University of Amsterdam (2009)

# Vagueness Facilitates Search

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**Abstract.** This paper addresses the question why language is vague. A novel answer to this question is proposed, which complements other answers suggested in the literature. It claims that vagueness can facilitate search, particularly in quasi-continuous domains (such as physical size, colour, or temperature), given that different speakers are likely to attach subtly different meanings to words (such as “tall”, “blue”, or “hot”) defined over such domains.

## 1 Introduction

Two questions dominate theoretical research on vagueness. The first is of a logical-semantic nature: *What formal models offer the best understanding of vagueness?* Many answers to this question have been proposed (e.g. [1], [2] for an overview), but none of these has found general acceptance so far. The second question is of a pragmatic nature and asks *Why is language vague?* This question has been asked forcefully by the economist Barton Lipman, who has shown that some seemingly plausible answers resist analysis in terms of classical Game Theory [3], [4]. While a number of tentative answers to this question have been suggested (for a survey, see [6], [7]), Lipman’s question is still partly unresolved, particularly with respect to situations where there is no conflict between the speaker and the hearer (cf. [8]).

The present paper will focus on the second question, and in doing so it will obtain some insights into the first question as well. We will elaborate on a novel answer to this question, which was sketched in broad outline in [6], [7], explaining the probabilistic basis of the argument, and discussing what we see as its merits more fully than before. In a nutshell, we argue primarily that vagueness can facilitate search. Additionally, we argue that Partial Logic is better placed to explain this phenomenon than Classical Logic, and that theories that give pride of place to *degrees* (including many-valued logics [9], but also two-valued theories that include degrees, e.g., [10]) are even better placed than Partial Logic to do this. We do not claim that facilitation of search is the only rationale for vagueness, or that degrees are necessary for explaining the benefits of vagueness: a non-quantitative model involving an ordinal scale might be equally suitable.

## 2 Informal Outline of the Argument

Let's call a domain *quasi-continuous* if it contains objects which resemble each other so much that they are indistinguishable. Domains do not have to be *mathematically* continuous to have this property: it suffices for them to contain objects that are similar enough in the relevant dimension (a person of 180.1cm and one of 180.2cm height, for example) that they cannot be told apart given the measurement tools at hand. Examples abound, including the heights of all the people you know, or all the colours that you have seen.

In a quasi-continuous domain, it is difficult for people to align the meanings of the predicates defined over them: there are bound to be people that one speaker calls 'tall' that another does not. The causes include physical and cultural differences between people. David Hilbert, for example, who focusses on colour terms, explains how the differences in people's eyes (e.g., in terms of the density of pigment layers on the lens and the retina, in terms of the sensitivity of the photo receptors) make it unavoidable that one normally sighted person can often distinguish between colour patches where another cannot [11]. The role of cultural issues was highlighted in [12], where it was shown that different weather forecasters use different criteria in their use of temporal phrases; according to some forecasters, for example, the start of the *evening* has something to do with dinner time, whereas for others, the time when the sun sets is more relevant, while yet others believe that the time on the clock is the only relevant consideration. Rohit Parikh has written insightfully about such matters, and we shall use and adapt one of his examples to present our own argument below.<sup>1</sup>

In Parikh's original story of Ann and Bob, Ann asks Bob to find her book on topology, adding that "it is blue" [13]. Ann and Bob use different concepts of 'blue', but if the overlap between them, as compared to their symmetric difference, is sufficiently large then Ann's utterance may still be very useful, because it may reduce the time that Bob should expect to take before finding the referent. All the same, the mismatch between speaker and hearer does cause Bob's search for the topology book to take more time than it would otherwise have done. This is particularly true because the book,  $b$ , may be an element of  $\|blue\|_{Ann} - \|blue\|_{Bob}$ . In this case, Bob must first search all of  $\|blue\|_{Bob}$ , then the ones he does not consider blue until he finds  $b$  there. His expected search effort can be equated to the cardinality of the set  $\|blue\|_{Bob}$  plus half that of the complement of this set. In this "unlucky" scenario, Ann's utterance has led Bob astray: without information about the colour of the book, he could have expected to examine only half the domain.

In Parikh's story, Ann and Bob both used a crisp (i.e., non-vague) concept 'blue'. In what follows, we will argue that it would be advantageous for Bob (and, by extension, for Ann, who wants the book to be found) if Bob was able to rise

<sup>1</sup> Differences between speakers are particularly difficult to accommodate in *epistemicist* (i.e., "vagueness as ignorance") approaches to vagueness, which often assume that there is always only one true answer to the question "Is this person tall". See [7], Chapter 7.

above thinking in terms of a simple dichotomy between blue and non-blue. Bob might argue, for example, that if the target book is not found among the ones he considers blue, then it is most likely to be one that he considers borderline blue; so after inspecting the books he considers blue, he would be wise to inspect these borderline cases. He might even think of the books as arranged in order of their degree of blueness, and start searching the ones that are most typically blue, followed by the ones that are just slightly less blue, and so on.

Colours are complex, multi-dimensional things. For simplicity, we shall focus on the one-dimensional word tall. Thus, of any two extensions that the word may be assigned in a given situation, one must always be a subset of the other ( $\|tall\|_{Ann} \subseteq \|tall\|_{Bob}$  or  $\|tall\|_{Bob} \subseteq \|tall\|_{Ann}$ , or both). More crucially, let us abandon the assumption that Ann and Bob must always think of the words in question as expressing a crisp dichotomy.<sup>2</sup> The story of the stolen diamond is set in Beijings Forbidden City, long ago:

A diamond has been stolen from the Emperor and, security being tight in the palace, the thief must have been one of the Emperors 1000 eunuchs. A witness sees a suspicious character sneaking away. He tries to catch him but fails, getting fatally injured in the process. The scoundrel escapes. With his last breath, the witness reports “The thief is tall!”, then gives up the ghost. How can the Emperor capitalize on these momentous last words? ([7], Chapter 9.)

Suppose the Emperor thinks of tall as a dichotomy, meaning taller than average, for instance. In this case, his men will gather all those eunuchs who are taller than average, perhaps about 500 of them. In the absence of any further clues, he should expect to search an average of as many as 250 tall people (i.e., half of the total number). Matters get worse if the witness has used a more relaxed notion of tall than the Emperor. If this mismatch arises, it is possible that the perpetrator will not be among the eunuchs whom the Emperor considers to be tall. Since the Emperor’s concept “tall” makes no distinctions between people who are not tall, the Emperors men can only search them in arbitrary order. In other words, he first searches 500 eunuchs in vain, then an expected  $0.5 * 500 = 250$ , totalling 750. Analogous to the previous section, the Emperor would have been better off without any description of the thieves height, in which case he should have expected to search  $0.5 * 1000 = 500$  eunuchs. The Emperor could have diminished the likelihood of a false start by counting more eunuchs as tall. But in doing so, he would have increased the search times that are necessary to inspect all the eunuchs he considers tall. The only way to avoid the possibility of a false start altogether is by counting *all* eunuchs as tall, which would rob the witness statement of its usefulness.

<sup>2</sup> Parikh hints briefly at a related possibility in a footnote, without discussing its implications: “It may be worth pointing out that probably Bob does have another larger set of Bluish(Bob) books which includes both Blue(Ann) and Blue(Bob). After looking through Blue(Bob), he will most likely look only through the remaining Bluish(Bob) books.” ([13], p. 533) See also our section 4, where expressions like “somewhat tall” are discussed.


If the Emperor thinks of tall as vague, however, then he might separate the eunuchs into three groups: the ones who are definitely tall, the ones who are definitely not tall, and the borderline cases characteristic of vague concepts. For concreteness, assume 100 eunuchs are definitely tall, 500 are definitely not tall, and 400 are doubtful. Surely, the eunuchs in the “definitely tall” category are more likely to be called tall than the ones in the “doubtful” category, while no one in the “definitely not tall” category could be called tall. To put some figures to it, let the chance of finding the thief in the group of 100 be 50% and the chance of finding him in the doubtful group of 400 likewise. Under this scenario, it pays off to search the “definitely tall” eunuchs first, as one may easily verify. In other words, the Emperor benefits from regarding tall as containing borderline cases (i.e., being vague). This thought experiment suggests that borderline cases, and hence vagueness, can facilitate search, because borderline cases allow us to distinguish more finely than would be possible if all our concepts were dichotomies. If your language contains only dichotomous concepts then separating the eunuchs into three different groups does not make sense: there are tall eunuchs, not-tall ones, and that’s it. But if you understand tall to have borderline cases then you can distinguish between the different people whom you do not consider tall, as well as between the ones you consider tall and all the others.

But if distinguishing between three different categories is better than distinguishing between just two, then it might be even better to distinguish even more finely. The Emperor can do even better than was suggested above if he uses a *ranking* strategy. Suppose he has the eunuchs arranged according to their heights. First the tallest eunuch is searched, then the tallest but one, and so on, until the diamond is found. This strategy is faster than each of the other ones if we assume that *the taller a person is, the more likely the witness is to have described him as tall*. Under this assumption, the same type of advantage obtains as in the previous case (where only borderline cases were acknowledged), but at a larger scale. – Note that we are ascribing a ranking strategy to the Emperor (i.e., the hearer) only. For all we know, the witness may be ignorant of the eunuchs heights while only possessing a rough impression of that of the thief. The Emperor and his men, by contrast, can rank the eunuchs at their ease.

This argument suggests an interesting possible *rationale* for understanding ‘tall’ as involving borderline cases or degrees, namely that this allows a more efficient search than would have been possible under a dichotomous understanding of these words. But borderline cases and degrees are the hallmark of vagueness. Consequently, if the argument is correct then we have found a so-far unnoticed *rationale* for vagueness: vagueness can facilitate search.

### 3 Towards a Formal Development of the Argument

We aim to show that, given a dichotomous model, it is normally possible to define a closely resembling vague model, which has a higher utility than the original crisp one, where utility is formalised by the amount of search that has to be undertaken by a hearer who uses the model in question. Let’s assume that

$\mathcal{A}$  is a standard two-valued model of the word ‘tall’ as defined on a domain  $D$  of people, where some people in  $D$  are tall (such people are in the extension  $\|tall\|$ ) and others are not (such people are in the extension  $\|\overline{tall}\|_{\mathcal{A}}$ ).  $\mathcal{B}$ , by contrast, has a truth-value gap: according to  $\mathcal{B}$ , there are not only tall and not-tall people, but borderline cases as well (such people are in the extension  $\|?tall?\|_{\mathcal{B}}$ ). As before, the search effort implied by a model  $\mathcal{X}$  (abbreviated  $s(\mathcal{X})$ ) will be formalised as the *expected* number of elements of  $D$  that the hearer will have to examine, under the assumption that she goes on searching until the intended referent (i.e., the man with the diamond in his pocket) is found. For simplicity, assume that the models  $\mathcal{A}$  and  $\mathcal{B}$  call exactly the same people tall, so both assign the same extension to  $\|tall\|$  .

### 3.1 The Advantage of Allowing Borderline Cases

Let us compare the models  $\mathcal{A}$  and  $\mathcal{B}$  above. Focussing on the witness’ reference to the thief ( $t$ ), there are three different types of situations. (In what follows,  $card(X)$  abbreviates “the cardinality of  $X$ ”).

**Type 1.**  $t \in \|tall\|$ . In this case,  $s(\mathcal{A})=s(\mathcal{B})$ , because the same sets are searched in both cases.

**Type 2.**  $t \in \|?tall?\|_{\mathcal{B}}$ . In this case,  $s(\mathcal{A})>s(\mathcal{B})$ , so  $\mathcal{B}$  leads to a lower search effort than  $\mathcal{A}$ . In other words, the model with borderline cases (i.e., model  $\mathcal{B}$ ) incurs an advantage over the one that does not (i.e.,  $\mathcal{A}$ ). The size of the advantage is  $1/2(card(\|\overline{tall}\|_{\mathcal{B}}))$ .

**Type 3.**  $t \in \|\overline{tall}\|_{\mathcal{B}}$ . In this case,  $s(\mathcal{B})>s(\mathcal{A})$ , in other words the model with borderline cases incurs a disadvantage. The size of the disadvantage is  $1/2(card(\|?tall?\|_{\mathcal{B}}))$ .

**Proofs.** of these claims use standard reasoning about probability. Consider Type 2, for example, where the thief  $t$  is borderline tall. Given our assumptions, this implies  $t \in \|\overline{tall}\|_{\mathcal{A}}$ . We can measure the hearer’s search effort implied by the model  $\mathcal{A}$  as  $s(\mathcal{A}) = card(\|tall\|) + 1/2(card(\|\overline{tall}\|_{\mathcal{A}}))$ . The search effort implied by  $\mathcal{B}$  is  $s(\mathcal{B}) = card(\|tall\|) + 1/2(card(\|?tall?\|_{\mathcal{B}}))$ , so  $s(\mathcal{A})>s(\mathcal{B})$  if  $card(\|\overline{tall}\|_{\mathcal{A}}) > card(\|?tall?\|_{\mathcal{B}})$ , which is true given that (as we assumed)  $\|\overline{tall}\|_{\mathcal{B}} \neq \emptyset$ . The size of the advantage is  $1/2(card(\|\overline{tall}\|_{\mathcal{A}}) - 1/2(card(\|?tall?\|_{\mathcal{B}})))$ , which equals  $1/2(card(\|\overline{tall}\|_{\mathcal{B}}))$ .

What we really like to know is the expected search effort *a priori*, when it is not known in which of the three Types of situations (listed above) we are (i.e., whether the thief is in  $\|tall\|$ , in  $\|?tall?\|_{\mathcal{B}}$ , or in  $\|\overline{tall}\|_{\mathcal{B}}$ ). Let “tall( $x$ )” (in double quotes) say that the witness calls  $x$  tall, then the following hypothesis,  $H$ , seems highly plausible:

**Hypothesis  $H$  :**

$$\forall xy((x \in \|?tall?\|_{\mathcal{B}} \wedge y \in \|\overline{tall}\|_{\mathcal{B}}) \rightarrow p(\text{“tall}(x)\text{”}) > p(\text{“tall}(y)\text{”})).$$

<sup>3</sup> Other assumptions can have similar consequences. See e.g. section 2, where we assumed that  $\|\overline{tall}\|_{\mathcal{A}} = \|\overline{tall}\|_{\mathcal{B}}$ .

(Note that  $H$  is not dependent on the size of  $\|?tall?\|_{\mathcal{B}}$  and  $\|\overline{tall}\|_{\mathcal{B}}$ .) In justification of  $H$ : a person in  $\|\overline{tall}\|_{\mathcal{B}}$  is considered clearly not-tall by the emperor so, although it cannot be ruled out that the witness described the same person as tall, such mismatches cannot occur too frequently amongst people who speak the same language, or else communication will break down. It would be far less unusual to see a person in the borderline area  $\|?tall?\|_{\mathcal{B}}$  being described as tall: intuitively speaking, this category exists precisely to take account of the fact that the individuals in it may be considered tall by some but not all speakers. Hypothesis  $H$  has important consequences because, given that only one individual is called “tall” and this individual is the (only) thief, it follows that

$$\forall xy((x \in \|?tall?\|_{\mathcal{B}} \wedge y \in \|\overline{tall}\|_{\mathcal{B}}) \rightarrow p(thief(x)) > p(thief(y))).$$

Consequently, it is advantageous to search (all of)  $\|?tall?\|_{\mathcal{B}}$  before  $\|\overline{tall}\|_{\mathcal{B}}$ . It follows that  $s(\mathcal{A}) > s(\mathcal{B})$ . In other words: given a dichotomous model, it is always possible to find a non-dichotomous model (i.e., with borderline cases) which agrees with it on all positive cases and which implies a smaller search effort on the part of the hearer.

### 3.2 The Advantage of Degrees and Ranking

To develop a formal take on what happens when a concept like “tall” is seen as having *degrees*, let us contemplate a degree model  $\mathcal{C}$ , alongside the dichotomous model  $\mathcal{A}$  and the three-valued model  $\mathcal{B}$ . Without loss of generality we can assume that  $\mathcal{C}$  assigns real-valued truth values in  $[0, 1]$  to each person in  $D$ . As is customary in Fuzzy Logic ([14], [9]), among other systems, let  $\mathcal{C}$  assign the value 0 to the shortest person and 1 to the tallest, while taller people are assigned values that are not lower than those assigned to shorter ones.

In the present context, the crucial advantage of degree models over 2- or 3-valued ones is that degree models tend to make finer distinctions. 2-valued models (i.e., dichotomous ones) are able to distinguish between two kinds of people (the tall ones and the not-tall ones), and 3-valued models (i.e., ones with a truth-value gap) are able to distinguish between three. Degree models have the capacity to distinguish between many more people – if need be, a mathematical continuum of them. Where this happens, the advantages are analogous to the previous subsection.

Suppose, for example, that the domain contains ten individuals:  $a1$ ,  $a2$ ,  $b1$ ,  $b2$ ,  $c1$ ,  $c2$ ,  $d1$ ,  $d2$ ,  $e1$ , and  $e2$ , where  $a1$  and  $a2$  have (approximately) the same height, so do  $b1$  and  $b2$ , and so on. (The number of members of each of the types a-e is immaterial: instead of all having two members, each of them could have any positive number of members.) Assume that the Emperor assigns “fuzzy” truth values as follows:

$$\begin{aligned} v(Tall(a1)) &= v(Tall(a2)) = 0.9, \\ v(Tall(b1)) &= v(Tall(b2)) = 0.7, \\ v(Tall(c1)) &= v(Tall(c2)) = 0.5, \\ v(Tall(d1)) &= v(Tall(d2)) = 0.3, \\ v(Tall(e1)) &= v(Tall(e2)) = 0.1. \end{aligned}$$

Recall that the witness described the thief as “tall”. It is not farfetched to think that  $a_1$  and  $a_2$  are more likely targets of this description than  $b_1$  and  $b_2$ , while these two are more likely targets than  $c_1$  and  $c_2$ , and so on. The Emperor should therefore start looking for the diamond in the pockets of the two tallest individuals, then in those of the two next tallest ones, and so on. The idea is the same as in the previous subsection, except with five rather than three levels of height: under the assumptions that were made, this search strategy is quicker than the previous two.

This example suggests that the key to the success of this strategy is the ability to *rank* the individuals in terms of their heights, assuming that this corresponds to a ranking of their likelihood of being called “tall”. Whenever this ability results in finer distinctions than 2- or 3-valued models, degree models lead to diminished search effort. To see how this works, it suffices to realise that the hypothesis  $H'$ , a minor variant of the earlier hypothesis  $H$ , is highly plausible:

**Hypothesis  $H'$  :**  $\forall xy(v(tall(x)) > v(tall(y)) \rightarrow p(\text{“tall}(x)\text{”}) > p(\text{“tall}(y)\text{”})).$

Given the correlation between  $v(tall(x))$  and the height of  $x$ , hypothesis  $H'$  says that taller individuals have a higher likelihood of being called tall than smaller ones. It follows from this hypothesis that it is advantageous to start searching the tallest individual (or individuals) in the domain, then the next tallest, and so on. If the domain contains individuals of four or more height levels (i.e., at least four different truth values of the form  $v(Tall(x))$ ) then this leads to an expected search time associated with the degree model  $\mathcal{C}$  which is smaller than that associated with model  $\mathcal{B}$ , which has a truth-value gap. So, given a three-valued model, it is always possible to find a degree model that respects the distinctions made by the three-valued model and that implies an even smaller search effort on the part of the hearer.

As it stands, hypothesis  $H'$  might be false. To see why, suppose the height of  $x$  is 210cm, while that of  $y$  is 190cm. It could be argued that, in this situation,  $y$  is *more* (instead of less) likely to be called tall than  $x$ , because  $x$  is quite untypical for someone designated as tall:  $x$  might be more likely to be called “extremely tall”, or even a “giant”. Wrinkles of this kind can only be ironed out by empirical research, which should tell us individuals of what height are most likely to be called “tall”, “extremely tall”, and so on. The outcome of these empirical investigations should then lead to a modified version of  $H'$ , which will tell the Emperor who to search first, based on the heights of the individuals in question. Like the original  $H'$ , the modified hypothesis would allow the Emperor to benefit from a degree model.

## 4 Discussion

We have argued that quasi-continuous domains make it difficult to align the meanings of the predicates defined over them: there are bound to be things that one person calls ‘large’ (or ‘blue’, or ‘warm’) that another person does not. Given such mismatches – which do not exist in standard game-theoretical analyses of

vagueness – we have argued that it is to the hearer’s advantage to distinguish shades of meaning in a way that is typical for vague concepts, namely using borderline cases or degrees. This argument suggests an answer to the *pragmatic* question that we asked in our Introduction which differs notably from the ones offered in the literature so far (see [6], [7]). To the extent that it supports degree-based models, ranging from Fuzzy Logic or probabilistic logic (e.g., [15], [16]) to Kennedy-style 2-valued semantics [10], our analysis also appears to shed light on the *logical-semantic* questions surrounding vagueness. – Let us discuss some possible objections against our argument.

**Objection 1.** It might be argued that the benefits that we ascribed to 3-valued and many-valued models can, in fact, also be obtained from 2-valued models. According to this view, the user of a 2-valued model is just as able to make fine distinctions as the user of any other kind of model. One can imagine a semantic and a syntactic version of this argument. The semantic version would argue that an intelligent user of a 2-valued model should be aware that *other* (2-valued) models may exist. She could argue, for example, that taller people are counted as tall by *more* models than less tall people. Clearly then, it pays to start searching amongst those people who are counted as tall by the largest set of models, that is, amongst the tallest people. The *syntactic* version of this argument would say that a person who is “quite” tall is a more likely to be called tall than someone is “somewhat” tall, who is more likely to be called tall than someone who is “a little bit on the tall side perhaps”, and so on; therefore, after unsuccessfully searching all the people who are downright tall, the hearer should direct her attention to the people who are quite tall, somewhat tall, and so on.

But all objections of this kind presuppose that Bob’s understanding of “large” goes beyond a simple dichotomous model. A language user who reasons as in the semantic version of this objection, for instance, knows that “tall” can have many different thresholds (corresponding with the different models), and reasons about these different thresholds. Essentially, this amounts to a *supervaluational* account of vagueness (e.g. [17]). The counterargument against the syntactic version of the objection is analogous: going beyond what the witness said, by exploring the extension of qualifications like “somewhat tall”, does not make sense unless one is aware that the word “tall” is used differently by different people. Once again, if the Emperor followed this strategy, we would be justified in ascribing to him an understanding of “tall” as a vague concept.

**Objection 2.** Why did the witness keep us guessing, by using a vague concept? Why did he NOT say “the thief is 185cm tall”, or something precise like that? – It is true that the utterance, “the thief is 185cm tall” might have been more helpful, but it is most naturally understood as vague too. To see this, note that the speaker may not know the exact height of the thief (nor his rank in terms of height), since he may have only a rough impression of his height, while he might know even less about other people’s heights. Consequently, the speaker is not in a position to pass on the exact height of the thief. For this reason, an utterance like “the thief is 185cm tall” would tend to be interpreted as true of

a person who is, for example, 184.4cm. At what height exactly the assessment starts being false would be difficult to say. Its meaning is perhaps best captured by a Gaussian function that asserts that 185cm is the most likely height, with other heights becoming less and less likely as they are further removed from 185cm. If such a vague estimate of the thief's height comes more naturally to human speakers than a precise assessment (e.g., "the thief's height is 185cm plus or minus 2cm") then Lipman's question can be repeated: why is this the case? Why, in other words, do statements in which speakers estimate heights tend to be vague? This new question can be answered in the same way as the question on which we focussed in this paper, by pointing out that a crisp concept like "height = 185cm plus or minus 2cm" would suffer from the same lack of flexibility as a crisp concept of "tall". Like before, vagueness allows speakers to deal flexibly with the differences among each other. Bob should start his search by focussing on individuals very close to 185cm, fanning out in both directions (i.e., below and above 185cm) until he has found the culprit.

Our account suggests a somewhat heretical view of *reference*. It is, of course, commonly understood that adjectives like "tall" are not intersective, but we can take this idea a step further. When a speaker refers to someone as "the tall man with the diamond in his pocket", one might believe that the hearer should consider the set of all men, intersect this with the set of individuals that have diamonds in their pockets, then intersects the result with the set of all people above a certain, contextually determined, height. The reason why this contextualised intersective interpretation will not work is *not* just that the hearer has incomplete information about the height standards employed by the speaker. The speaker did not necessarily employ any particular height standard; rather, she suggested that the best way to find the (unique) man with the diamond in his pocket is to start searching all the men in order of their height, because this is the quickest way to find the one with the diamond in his pocket.

**Objection 3.** It can be argued that a 3-valued model such as  $\mathcal{A}$  falls short of making "tall" a vague concept, given that its boundaries (i.e., between  $\|tall\|$ ,  $\|?tall?\|$ , and  $\|\overline{tall}\|$ ) are crisp instead of vague. One might even go further and argue that the same is true for the many-valued models discussed in section 3.2, since these, too, assign definite truth values to each statement of the form "Tall(x)". I would counter that, if these models are seen as failing to model genuine (i.e., higher-order) vagueness, then it is difficult to see what models *do* model genuine vaguenes. Certainly very few of the models on the theoretical market (see e.g. [1]) go further than many-valued models in acknowledging vagueness. Essentially, in this paper, I have taken the pragmatic question about vagueness to be "Why does language not make do with simple dichotomous concepts?"

**Objection 4.** Lipman, in [4], proves a game-theoretical theorem (framed within a standard model as proposed in Crawford and Sobel 1982) stating that, given a vague predicate  $P$ , there must always exist a non-vague predicate  $P'$  where the utility of  $P'$  is at least as great as that of  $P$ . It might be thought that this contradicts the main claim of the present paper, but this is not the case. To prove

his theorem, Lipman makes various assumptions which our analysis does not share. One of these assumptions is that a vague predicate is a *probability distribution* over functions that assign messages to heights. This is known as a mixed strategy, as opposed to a pure strategy, which is just a function from heights to messages. We have adopted a different attitude towards vagueness, without probability distributions. A second, and even more crucial assumption on which Lipman's theorem rests is that there are no mismatches between speaker and hearer. In particular, when a pure strategy is adopted by the speaker, he assumes that the hearer knows what this strategy is. Our own investigations, of course, start from a very different assumption, for which there exists ample empirical evidence (e.g. [11], [13], [12]) namely that mismatches between speakers' and hearers' understanding of concepts like 'tall' are unavoidable (i.e., perfect alignment would be a miracle).

## References

1. Keefe, R., Smith, P.: Vagueness: a Reader. MIT Press, Cambridge (1997)
2. van Rooij, R.: Vagueness and Linguistics. In: Handbook of Vagueness (to appear)
3. Lipman, B.: Economics and Language. In: The "Comments" section of [5] (2000)
4. Lipman, B.: Why is language vague? Working paper, Dept. of Economics, Boston University (Version of December 2006)
5. Rubinstein, A.: Economics and Language: Five Essays. Cambridge University Press, Cambridge (2000)
6. van Deemter, K.: Utility and Language Generation: The Case of Vagueness. *Journal of Philosophical Logic* 38(6), 607–632 (2009)
7. van Deemter, K.: Not Exactly: in Praise of Vagueness. Oxford University Press, (January 2010), <http://www.csd.abdn.ac.uk/~kvdeemte/NotExactly>
8. Crawford, V., Sobel, J.: Strategic information transmission. *Econometrica* 50, 1431–1451 (1981)
9. Bergmann, M.: An introduction to many-valued and fuzzy logic: semantics, algebras, and derivation systems. Cambridge University Press, Cambridge (2008)
10. Kennedy, C.: Vagueness and Grammar: The Semantics of Relative and Absolute Gradable Adjectives. *Linguistics and Philosophy* 30(1) (2007)
11. Hilbert, D.R.: Color and Color Perception: A Study in Anthropocentric Realism. In: CSLI Lecture Notes 9. CSLI Publications, Stanford (1987)
12. Reiter, E., Sripada, J., Hunter, J., Yu, J., Davy, I.: Choosing words in computer-generated weather forecasts. *Artificial Intelligence* 167, 137–196 (2005)
13. Parikh, R.: Vagueness and Utility: the semantics of common nouns. *Ling. and Phil.* 17, 521–535 (1994)
14. Zadeh, L.: Fuzzy Sets. *Information and Control* 8(3), 338–353 (1965)
15. Edgington, D.: Validity, uncertainty and vagueness. *Analysis* 52, 193–204 (1992)
16. Edgington, D.: Vagueness by degrees. In: [1] (1996)
17. Fine, K.: Vagueness, truth and logic. *Synthese* 30, 265–300 (1975)

# Meaning of ‘Now’ and Other Temporal Location Adverbs

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**Abstract.** This paper provides an analysis of the temporal location adverb *now*. The core data comes from free indirect discourse, where *now* often co-occurs with the past tense and has an affinity for stative sentences. Building on Kamp & Reyle’s (1993) analysis, I propose that *now* is a perspective setting anaphor: it requires an eventuality described by an aspectual phrase to hold throughout a salient event that serves as the ‘current perspective.’ The proposed meaning is compatible with both the past and present tenses and it has the same semantic type and uses the same ingredients as other temporal location adverbs.

**Keywords:** adverbs, aspect, tense, narrative progression, free indirect discourse, anaphora, indexicality, deixis.

## 1 Introduction

There is a particular use of *now* where it co-occurs with the past tense.<sup>1</sup> This usage is often found in *free indirect discourse* (FID) viz. (1), where it is possible to understand the described eventualities as happening from the point of view of a particular character, rather than the narrator ([1], [2]). This, however, is not a necessary condition for *now* to co-occur with the past tense. For example, the state of being unpleasant to look at in (2) *must be* interpreted from the point of view of the narrator.

He came to me and told me he had been dressing in my clothes whenever I wasn’t home for quite a few years, and *now he was ready to take the next step* and with the help of his doctor (that I didn’t even know about) he wanted to start the process of becoming female ([3]). (1)

Anna went to her plastic surgeon. She had won the beauty contest 30 years ago. *Now the old bag was a sight for sore eyes!* (Sam Cumming, p.c.) (2)

*Now* exhibits two key properties in discourses such as (1) and (2). The first is that *now* is used as an anaphor. This is especially clear when one compares (2) to its counterpart

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<sup>1</sup> Lee ([4]) showed that of the 100 randomly selected narrative discourses from the British National Corpus that contained *now*, 63 had the past tense.

without *now*. In such a case, the state of being unpleasant to look at would be understood to hold when Anna won the beauty contest. This would render the discourse infelicitous. With *now*, however, the discourse is felicitous because the described state is understood to hold throughout the event of going to the plastic surgeon. This particular event is chosen as an antecedent because *now* is an event seeking anaphor and the perfect clause in (2) makes the consequence of the winning event salient (rather than the winning event itself; see [5], [6]).<sup>2</sup> Further evidence comes from (3), which is infelicitous with *now* because the series of stative sentences don't provide an antecedent of the right type; cf. (4), which is like (3), except that a series of eventive sentences are inserted, rendering the discourse felicitous with *now*.

Samsa's room, a regular human bedroom, only rather too small, lay quiet between the four familiar walls. Above the table on which a collection of cloth samples was unpacked and spread out hung a picture. It showed a lady, with a fur cap on and a fur stole, sitting upright and holding out to the spectator a huge fur muff. Samsa {#was now/<sup>OK</sup>was} intrigued by this lady (modified from [8]). (3)

Samsa's room, a regular human bedroom, only rather too small, lay quiet between the four familiar walls. Above the table on which a collection of cloth samples was unpacked and spread out hung a picture. It showed a lady, with a fur cap on and a fur stole, sitting upright and holding out to the spectator a huge fur muff. *Suddenly, the lady dropped the muff and took off her cap.* Samsa could not believe his eyes. He {<sup>OK</sup>was now/<sup>OK</sup>was} intrigued by this lady. (4)

The other property of *now* is exemplified by discourses such as (5) and (6). Here we see that *now* is incompatible with eventive sentences ([7], pp. 595-596).<sup>3</sup> In this way, *now* differs from all other temporal location adverbs (cf. [12]), including the seemingly similar anaphor *at that point* in (7).

He came to me and told me he had been dressing in my clothes whenever I wasn't home for quite a few years, and now he {#took/<sup>OK</sup>was ready to take/<sup>OK</sup>was taking/<sup>OK</sup>had taken} the next step... (5)

Yesterday, Anna went to her plastic surgeon. She had won the beauty contest 30 years ago. Now she {#replaced/<sup>OK</sup>wanted to replace} her nose and upper lip with those of a donor. (6)

<sup>2</sup> Henriette de Swart (p.c.) offers an alternative analysis in which *now* locates a described eventuality onto the *main narrative timeline*—e.g. in (2), the described states are understood to hold at the time of the going-to-the-surgeon event and not the winning event because only the former event is on the *main narrative timeline*. I am open to such an analysis and note in passing that its preference to the one proposed here largely depends on the semantics of the pluperfect and an explicit theory of *narrative timelines*. The semantics of the pluperfect assumed here is motivated by flashback discourses discussed in [7] (pp. 593-611).

<sup>3</sup> Note that there is a reading of (5) and (6) in which the eventive predicates are acceptable. Such a reading, however, exemplifies a distinct *now*, which is not discussed in this paper (however, see [9]). The *now* considered here is truth-conditionally equivalent to *currently* and is morphologically distinguished from the other *now* in languages like Russian (cf. *sejčas* vs. *teper* discussed in [10]) and Korean (cf. *cikum* and *icey* discussed in [11]).

The first of Weiss' explicitly autobiographical novels, *Leavetaking*, describes his childhood and youth until 1940. {#Now/<sup>OK</sup>at that point} he essentially claimed his independence and set out to become an artist ([13]). (7)

Based on similar observations, Kamp and Reyle propose that *now* refers to a *temporal perspective point*—i.e. the speech time or a previously mentioned discourse event. The innovation of their analysis is that *now* is not treated as a *deictic expression*, i.e. one that *always* makes reference to the context of utterance (cf. [14]), viz. *will* or *I*, but rather as perspective setting anaphor whose value is always determined by the discourse context and is constrained by the tense. A problem with their analysis, however, is that they posit three past tenses even though the morphology indicates otherwise: (i) a past tense that only combines with stative sentences, (ii) a past tense that only combines with eventive sentences and (iii) a past tense that is required only in the presence of *now* ([7], pp. 601).

An alternative hypothesis is to say that in cases where *now* co-occurs with the past tense, there is an operator that 'shifts' *now*'s coordinates—e.g. in (2), an operator shifts the speech time coordinate to a past time, namely the time of Anna going to the plastic surgeon. In this way, we can maintain that *now* is a deictic expression; it refers to the (shifted) speech time.

One objection to this analysis comes from the seemingly ad-hoc motivation for using the shifting operator. Traditionally, such operators have been linked to propositional attitude verbs ([15]) and FID ([16], [17]), i.e. cases in which the 'perspective' shifts from the speaker (or narrator) to the attitude holder (or character in a novel). However, as we saw in (2), *now* can co-occur with the past tense in contexts where the described eventuality must be interpreted from the speaker's (or narrator's) point of view.

Another objection comes from the observation that shifting coordinates of *now* does not explain *now*'s reluctance to co-occur with eventive sentences viz. (5)–(7). This fact in particular, I believe, warrants a semantic reanalysis of *now*.

In this paper, I build on Kamp and Reyle's proposal that *now* is a perspective setting anaphor and propose a meaning that is (i) compatible with both the past and present tenses and (ii) has the same semantic type and uses the same ingredients as other temporal location adverbs. In particular, I propose that all temporal location adverbs have a temporal component and a discourse component. Depending on the nature of the adverb, one of these components typically plays a greater role in fixing the temporal location of an eventuality described by an aspectual phrase. In the case of *now*, however, both components play an instrumental role. They conspire to impose the following two requirements: (i) search for a topical event that serves as the 'current perspective' and (ii) describe what took place throughout this topical event. These two requirements capture *now*'s anaphoric nature and—given aspectual constraints on narrative progression discussed in the next section—they lead to a contradiction with eventive, but not stative verb phrases. All in all, the proposed analysis makes the correct predictions about the discourses considered in this section without "postulating apparently spurious ambiguities" ([7], pp. 599).

## 2 Background Assumptions

It is generally held that temporal anaphora in narrative discourse motivates the notion of a *reference time*—i.e. a placeholder for where the narrative has developed ([18],

[19], [20]). According to one influential analysis proposed by Bonnie Webber in [6], a reference time is either the time described by temporal location adverbs or the duration of the consequent state of a previously mentioned discourse event (cf. Partee's "time right after"). Moreover, following [18]–[20] Webber proposed that aspect constrains the temporal location of a described eventuality in the following way: Whereas events occur within a reference time, states hold throughout that time.

As an illustration of Webber's analysis, consider the Russian discourse below, in (8), which contains a sequence of sentences in the perfective aspect. With the adverb in (8b), this discourse entails that the dropping-off event preceded the money-giving event. However, without the adverb in (8b), the understood event ordering is reversed: the dropping-off event is understood to follow the money-giving event.

*Dudkin da-l nam ogromnuju summu deneg.*  
 Dudkin PFV.give-PST-3S us huge sum money.  
 'Dudkin gave us a large sum of money.'

(8a)

*Za nedljudo togo my zavez-l-i emu producty.*  
 From week to that we PFV.drop.off-PST-2P him products  
 'A week before that we had dropped off products at his place.'

(8b)

Applying Webber's analysis to (8), we would say that the dropping-off event is located within the reference time, namely a time that precedes the money-giving event by a week. When the adverb is not present, however, the reference time is supplied by the discourse context and refers to the consequent state of the giving event. Given that the dropping-off event is contained within this consequent state, it is correctly predicted that the dropping-off followed the money-giving.

In sum, Webber's analysis is elegant because it relates events to times specified by an adverb in the same way it relates events to times provided by the discourse context. Despite its elegance, however, I argued in [21] and [22] that Webber's analysis cannot account for the Russian imperfective aspect, which relates distinct event parts to the reference time. Which event part is at play depends on how the reference time is specified. For example, in (9b), the adverb specifies the reference time and the dropping-off event is understood to occur a week before the money-giving event as in (8b). This expected if we once again say that the dropping-off event is contained within the reference time. This relation, however, makes the wrong prediction about (9b) when there is no adverb and the reference time is therefore specified by the discourse context. In particular, it predicts that the dropping-off event follows the money-giving event, viz. (8b) without the adverb. However, the dropping-off event is still understood to precede the money-giving event, viz. (8b)/(9b) with the adverb.

*Dudkin da-l nam ogromnuju summu deneg.*  
 Dudkin PFV.give-PST-3S us huge sum money.  
 'Dudkin gave us a large sum of money.'

(9a)

*Za nedljudo togo my zavozi-l-i emu producty.*  
 From week to that we drop.off.IPF-PST-2P him products  
 'A week before that we had dropped off products at his place.'

(9b)

To account for the data above, I argued that it is necessary to split the notion of a *reference time* into two distinct parameters (cf. [7]; see also [23], [9] for independent evidence). I proposed a *birelational analysis* in which aspectual meaning involves both temporal information and information about discourse connectivity. In particular, aspect requires two inputs relative to which a described eventuality is located—(i) a *time* that is specified by a temporal location adverb (or some other grammatical expression) and (ii) a *state* that is specified by the discourse context. Events are required to be contained within the two inputs, while states hold throughout them.

Applying this analysis to the Russian imperfective, I proposed that a described event is required to be contained within a time input, while the consequent state of the described event is required to contain a state input. This allows us to say that when the adverb is present in (9b), the dropping-off event is contained within a time that precedes the money-giving event by a week; when the adverb is not present, the consequent state of the dropping-off event contains a state, namely the consequent state of the money-giving event. This correctly predicts that the dropping-off event preceded the money-giving event whether or not the adverb in (9b) is present.

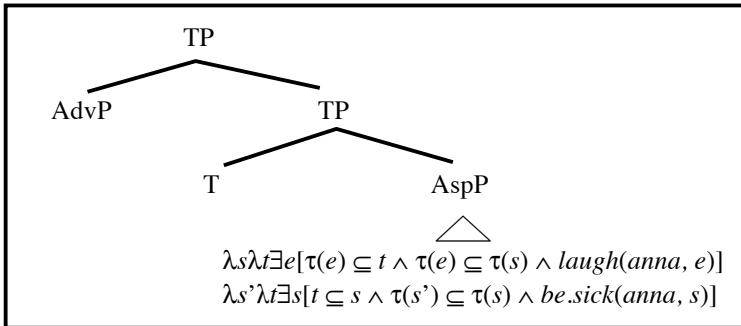
In what follows, I assume a birelational analysis of aspect without further comment. As shown in §4, this assumption is crucial to giving an adequate semantics for *now*. I end this section by providing birelational meanings of two English expressions: eventive and stative aspectual phrases in (10) and (11) respectively. In the next section, I provide details about how the inputs to these phrases are supplied.

$$\text{AspP [Anna laugh]} \rightsquigarrow \lambda s \lambda t \exists e [\tau(e) \subseteq t \wedge \tau(e) \subseteq \tau(s) \wedge \text{laugh}(\text{anna}, e)] \quad (10)$$

$$\text{AspP [Anna be.sick]} \rightsquigarrow \lambda s' \lambda t \exists s [t \subseteq \tau(s) \wedge \tau(s') \subseteq \tau(s) \wedge \text{be.sick}(\text{anna}, s)] \quad (11)$$

### 3 Meaning of Temporal Location Adverbs

I assume the syntactic architecture in Fig. 1, where aspectual phrases combine with a tense operator and the resulting denotation combines with an adverbial phrase.



**Fig. 1.** Assumed syntactic architecture

I treat tense operators as relations between the speech event  $e_0$  and a time argument  $t$ . For example, the past tense operator PST below, in (12), requires that a time argument precede the run time of the speech event and the present tense operator PRS in (13) requires that the two be identified.

$$T \text{ [PST]} \rightsquigarrow \lambda Q \lambda s \lambda t [t < \tau(e_0) \wedge Q(s, t)] \quad (12)$$

$$T \text{ [PRS]} \rightsquigarrow \lambda Q \lambda s \lambda t [t = \tau(e_0) \wedge Q(s, t)] \quad (13)$$

With regard to temporal location adverbs like *yesterday*, *the day before*, *at 5*, and *February 15, 1981*, I propose that they combine with TP and have two functions. They supply a time input—which serves as the *location time* for a described eventuality (cf. [7])—and specify its duration as well as its relation to a *perspectival event*—i.e. the speech event or a previously mentioned discourse event (cf. Kamp and Reyle's notion of a *temporal perspective point* in [7]).<sup>4</sup> Moreover, they supply a state input, which does not play a significant role in locating the described eventuality.

As an illustration of the analysis, consider the denotations of *yesterday* and *the day before* in (14) and (15):

$$\text{Adv [yesterday]} \rightsquigarrow \lambda Q \exists t \exists s [\text{day}(t) \wedge t <_{\text{day}} \tau(e_0) \wedge Q(s, t)] \quad (14)$$

$$\text{Adv [the day before}_n] \rightsquigarrow \lambda Q \exists t \exists s [\text{day}(t) \wedge t <_{\text{day}} \tau(e_n) \wedge Q(s, t)] \quad (15)$$

Both adverbs specify that the location time  $t$  is a 24-hour interval of time denoted by *day* that precedes the perspectival event by a day. However, *yesterday* is deictic, i.e. it requires that the perspectival event be the speech event  $e_0$ , whereas *the week before* requires that the perspectival event be a previously mentioned discourse event  $e_n$ . This explains why *yesterday* can be used discourse initially, but *the day before* cannot.

A question that arises is: Where does TP get its two inputs if there is no adverb present? Following work by Carlota Smith ([25], [26]), I assume that semantically, there is always an adverb present (even if it not there syntactically). In particular, I assume that episodic sentences in the past tense that do not have an overt adverb combine with a silent operator (cf. Bäuerle's silent 'once' in [27]). As illustrated in (16), the proposed operator supplies a state input  $s_n$  that requires a salient state antecedent that I will refer to as the *topic state*. Moreover, it supplies a time input which does not play a significant role in locating the described eventuality (cf. [7], pp. 528-529).

$$\text{Adv [<O}_n\text{>]} \rightsquigarrow \lambda Q \exists t [Q(s_n, t)] \quad (16)$$

The proposed semantics of this operator can be taken to correspond to the overt narrative marker *then*. This would explain, for example, why the sentences in (17) are infelicitous out-of-the-blue: there is no topic state provided by the discourse context.

$$\# \text{Avital came in} / \# \text{Then Avital came in.} \quad (17)$$

<sup>4</sup> On this analysis, the contribution of tense is superfluous in the presence of certain adverbs. Following [24], I take this to reflect a remarkable property of natural language that in the presence of an adverb like *a week ago*, the past tense must still be expressed.

Moreover, it would explain the understood event ordering in the discourses below in (18): *then* (in its covert or overt manifestation) requires that the sitting down event be contained within the topic state. Assuming this state is the consequent state of the coming in event, it is correctly predicted that the sitting follows the coming in.

- Yesterday, Avital came in. She sat down.  
 Yesterday, Avital came in. Then she sat down. (18)

In sum, temporal location adverbs supply both temporal information and information about discourse connectivity. In particular, they supply two inputs that are required by aspectual phrases—a *time* and a *state*. Adverbs like *yesterday* and *the day before* are similar insofar as the supplied *time* input plays a greater role in fixing the temporal location of the described eventuality; the two adverbs differ solely in whether the perspectival event is the speech event or a previously mentioned discourse event. On the other hand, the *state* input supplied by the adverb *then* (at least on its ‘narrative meaning’, cf. [28]) plays a greater role in fixing the temporal location of the described eventuality. This explains why we never see this adverb discourse initially. In the next section, I propose that both the *state* and *time* inputs supplied by *now* play a significant role in fixing the temporal location of the described eventuality.

## 4 Meaning of ‘now’

In the introduction, we saw that *now* has two key properties: it is an anaphor that seeks a salient event antecedent and it has an affinity for stative sentences. The nuts and bolts of my analysis are as follows. *Now* encodes the following directions: (i) search for a topical event that serves as the “current perspective” and (ii) describe what took place throughout this topical event. The latter direction is consistent with the aspectual requirements imposed on stative predicates, but not eventive ones, thereby explaining the contrast in (19). Moreover, the contrast in (20) is explained in the following way. Without *now*, the states of being old and sick are required to hold throughout the topic state, namely the consequent state of the winning event (cf. discussion of (18)). With *now*, however, the states of being old and sick are required to hold throughout a topical event that serves as the ‘current perspective.’ The topical event must be the event of going to the plastic surgeon assuming that the past perfective clause *had won the beauty contest* makes the consequent state of the winning event salient (and not the winning event itself), viz. fn.2.

- Yesterday, Anna went to her plastic surgeon. She had won the beauty contest  
 30 years ago. Now she {#replace/<sup>OK</sup>wanted to replace} her nose and upper lip. (19)

- Yesterday, Anna went to her plastic surgeon. She had won the beauty contest  
 30 years ago. {#She was old and sick/<sup>OK</sup>Now she was old and sick}. (20)

In order to make sense of this proposal within the theory outlined in the previous two sections, consider the meaning for *now* below, in (21):

$$\text{Adv} [\text{now}_n] \rightsquigarrow \lambda Q \exists t \exists s [t = \tau(e_n) \wedge \text{CONS}(e_n) = s \wedge Q(s, t)] \quad (21)$$

According to the formula above, *now* has the same semantic type and uses the same ingredients as other temporal location adverbs. In particular, it supplies a time input that is related to a salient event  $e_n$  that serves as the perspectival event. In this way, *now* is on a par with *the day before* viz. (15). It differs, however, in that it requires the supplied time input to be identified with (rather than precede) the run time of the perspectival event (viz. the relation  $t = \tau(e_n)$ ). As will be shown below, this difference is what makes *now* compatible with both the present and the past tense.

Like all other location adverbs, *now* also supplies a state input. However, unlike other adverbs, it relates this state to the perspectival event. In particular, it requires the supplied state input to be a consequent state of the perspectival event (viz. the relation  $\text{CONS}(e_n) = s$ ). In this way, both inputs supplied by *now* play a crucial role in fixing the temporal location of an eventuality described by AspP.<sup>5</sup> In particular, the relations  $t = \tau(e_n)$  and  $\text{CONS}(e_n) = s$  encoded by *now* entail that an eventuality described by AspP holds throughout the perspectival event as desired. Such is the case because—given the analysis sketched out in the previous section— $t$  and  $s$  are the inputs relative to which an eventuality described by AspP is located. In particular, events described by AspP are required to hold within these inputs, while states are required to hold throughout them. This leads to contradiction with eventive sentences, but not with stative sentences. That is, it follows from (21), (22) and (24) that the nose replacing event is contained within two non-overlapping eventualities—i.e. the perspectival event and its consequent state—thereby explaining the ungrammaticality of (19) with *replaced*. On the other hand, it follows from (21), (22) and (25) that a state of being sick held throughout the perspectival event as desired. In this way, we explain *now*'s affinity for stative sentences.

$$T \text{ [PST]} \rightsquigarrow \lambda Q \lambda s \lambda t [t < \tau(e_0) \wedge Q(s, t)] \quad (22)$$

$$T \text{ [PRS]} \rightsquigarrow \lambda Q \lambda s \lambda t [t = \tau(e_0) \wedge Q(s, t)] \quad (23)$$

$$\begin{array}{l} \text{AspP [she replace her nose]} \\ \lambda s \lambda t \exists e [\tau(e) \subseteq t \wedge \tau(e) \subseteq \tau(s) \wedge \text{replace.her.nose}(she, e)] \end{array} \quad (24)$$

$$\begin{array}{l} \text{AspP [Anna be sick]} \\ \lambda s' \lambda t \exists s [t \subseteq \tau(s) \wedge \tau(s') \subseteq \tau(s) \wedge \text{be.sick}(anna, s)] \end{array} \quad (25)$$

The proposed analysis not only accounts for *now* in contexts where it co-occurs with the past tense, but it also accounts for the seemingly deictic use of *now* in examples like (26). Here we see *now* appearing discourse initially and co-occurring with the present tense; it makes reference to the time at which it is uttered.

$$\text{Anna is sick now.} \quad (26)$$

This is just one way in which *now* is used in a discourse; its seemingly deictic behavior in (26) comes from the present tense, which identifies the run time of the speech event with the time introduced by *now*, which in turn is identified with the perspectival event. For this reason it follows from (21), (23) and (25) that the state of being sick described in (26) holds throughout the speech event as desired.

<sup>5</sup> Arguably the same can be said for the phrase *at the same time* and presumably others (cf. [9]).

## 5 Conclusion

I end this paper by mentioning two challenges that the proposed analysis faces. I begin with Hans Kamp’s example in (27), which entails that an earthquake is taking place at the speech event. What is interesting about this example is that there is no present tense in the sentence and the aforementioned entailment disappears without *now*.

I learned last week that there would now be an earthquake ([14], pp. 299). (27)

Given the analysis proposed here, one could say that the perspectival event in (27) must be the speech event because it is compatible with the semantics of *would* and there is no other possible antecedent; the learning event described by the matrix clause is presumably ruled out because *would* requires the earthquake to follow this event. In other words, the idea is that *now* is compatible with a *present* or a *past* perspectival event and—if no grammatical elements (viz. the present tense) indicate otherwise—independent rules of anaphora resolution determine which one is chosen.

Another challenge for the proposed analysis concerns the behavior of other temporal location adverbs that appear to ‘lose’ their deictic characteristics in FID analogous to *now*. For example, consider *tomorrow* in (28), where it does not refer to a day after the speech event (see [1] for examples involving other adverbs).

Tomorrow was Monday, Monday, the beginning of another school week! (28)  
(Lawrence, *Women in Love*, pp. 185; cited in [29])

Given the proposed analysis, there are two avenues to pursue: (i) like *now*, *tomorrow* is an anaphor or (ii) an FID operator is responsible for shifting *tomorrow*’s coordinates in (28). In its extreme, (i) leads to the perhaps undesirable claim that many (if not all) adverb expressions that are typically thought to be deictic are really anaphoric. The less radical view in (ii), on the other hand, suggests that an FID operator is also at play when *now* occurs in FID. If that’s right, then the effects of this operator are truth-conditionally undetectable given *now*’s proposed semantics.

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## References

1. Kamp, H., Rohrer, C.: Tense in texts. In: Bäuerle, R., et al. (eds.) *Meaning, Use and Interpretation of Language*, pp. 250–269. De Gruyter, Berlin (1983)
2. Doron, E.: Point of view as a factor of content. In: Moore, S.M., Wyner, A.Z. (eds.) *Proceedings of SALT*, vol. I, pp. 51–64. Cornell University Press, Ithaca (1991)

3. Moore, A., Knott, A.: I prefer my husband now he's a woman. In: Chesnutt, J. (ed.) *Woman's Day magazine*. Hachette Filipacchi Médias (2008)
4. Lee, E.: Discourse properties of 'now'. Presented at the 84th annual meeting of the Linguistic Society of America, Baltimore, Maryland, January 7-10 (2010)
5. Moens, M., Steedman, M.: Temporal Ontology and Temporal Reference. *Computational Linguistics* 14, 15–28 (1988)
6. Webber, B.: Tense as discourse anaphor. *Computational Linguistics* 14, 61–73 (1988)
7. Kamp, H., Reyle, U.: *From Discourse to Logic: Introduction to Model theoretic Semantics of Natural Language, Formal Logic and Discourse Representation Theory*. Kluwer, Dordrecht (1993)
8. Kafka, F.: *The Metamorphosis*. Waking Lion Press (2006)
9. Altshuler, D.: Temporal interpretation in narrative discourse and event internal reference. PhD dissertation, Rutgers (in prep.)
10. Mel'chuk, I.: Semanticheskie ètjudy i. 'sejchas' i 'teper' v russkom jazyke. *Russian Linguistics* 9, 257–279 (1985)
11. Lee, E., Choi, J.: Two nows in Korean. *Journal of Semantics* 26, 87–107 (2009)
12. Katz, G.: Event arguments, adverb selection, and the Stative Adverb Gap. In: Lang, E., Maienborn, C., Fabricius-Hansen, C. (eds.) *Modifying Adjuncts*, pp. 455–474. Mouton de Gruyter, Berlin (2003)
13. The Complete Review,  
<http://www.complete-review.com/reviews/weissp/fluchtp.htm>
14. Kamp, H.: Formal Properties of Now. *Theoria* 37, 227–273 (1971)
15. Schlenker, P.: Propositional attitudes and indexicality. PhD dissertation, MIT (1999)
16. Sharvit, Y.: The puzzle of free indirect discourse. *Linguistics & Philosophy* 31, 353–395 (2008)
17. Schlenker, P.: Context of thought and context of utterance (a note on Free Indirect Discourse and the Historical Present). *Mind & Language* 19, 279–304 (2004)
18. Kamp, H.: Events, instants and temporal reference. In: Bäuerle, R., Egli, U., von Stechow, A. (eds.) *Semantics from Different Points of View*, pp. 376–471. De Gruyter, Berlin (1979)
19. Hinrichs, E.: *Temporale Anaphora im Englischen*. Manuscript, University of Tübingen (1981)
20. Partee, B.: Nominal and Temporal Anaphora. *Linguistics and Philosophy* 7, 243–286 (1984)
21. Altshuler, D.: Aspect in English and Russian flashback discourses. *Oslo Studies in Language* 2 (to appear)
22. Altshuler, D.: A birelational analysis of the Russian imperfective. In: *Proceedings of Sinn und Bedeutung* 14 (to appear)
23. Nelken, R., Francez, R.: Splitting the Reference Time: The Analogy between Nominal and Temporal Anaphora Revisited. *Journal of Semantics* 14, 369–416 (1997)
24. Partee, B.: Some Structural Analogies between Tenses and pronouns in English. *Journal of Philosophy* 18, 601–609 (1973)
25. Smith, C.: The vagueness of sentences in isolation. In: Beach, W.E. (ed.) *Proceedings of the Chicago Linguistic Society*, pp. 568–577. University of Chicago Press, Chicago (1977)
26. Smith, C.: The syntax and interpretation of temporal expressions in English. *Linguistics and Philosophy* 2, 43–99 (1978)
27. Bäuerle, R.: *Temporale Deixis – Temporale Frage*. Tübingen, Narr (1979)
28. Shiffrin, D.: Anaphoric then: aspectual, textual and epistemic meaning. *Linguistics* 30, 753–792 (1992)
29. Banfield, A.: *Unspeakable Sentences*. Routledge & Kegan Paul, London (1982)

# Logical Consequence Inside Out<sup>\*</sup>

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Tarski's definition of logical consequence for an interpreted language rests on the distinction between *extra-logical symbols*, whose interpretation is allowed to vary across models, and *logical symbols*, aka logical constants, whose interpretation remains fixed. In this perspective, logicity come first, and consequence is a by-product of the division between logical and extra-logical symbols. The problem of finding a conceptually motivated account for this division is a long-standing issue in the philosophy of logic. Our aim here is to short-circuit this issue and lay the basis for a shift in perspective: let consequence come first, so that the demarcation of a set of constants can be viewed as the by-product of the analysis of a relation of logical consequence. The idea for *extracting* logical constants from a consequence relation is the following: they are the symbols which are essential to the validity of at least one inference, in the sense that replacing them or varying their interpretation would destroy the validity of that inference. Conversely, definitions of logical consequence can be construed as providing us with mappings from sets of symbols to consequence relations.<sup>1</sup> Extraction of constants is then expected to be an 'inverse' to generation of consequence relations.

In Sections [1](#) and [2](#), we introduce a substitutional framework for the abstract study of consequence relations. Extraction of constants is presented in Section [3](#). It is shown that extraction thus defined does not straightforwardly provide an inverse to generation of consequence relations. To circumvent this limitation, we consider in Section [4](#) 'richness' properties of languages that make things better. In Section [5](#), we prove that extraction and generation constitute a Galois connection, considering families of expansions of a language, instead of a single language at a time. This gives us the correspondence for non-substitutional Tarskian consequence as a limit case. Our final results thus bridge a gap between abstract consequence relations and logical consequence defined as truth-preservation.

## 1 Preliminaries

### 1.1 Languages

In the Bolzano setting, languages are *interpreted*; in particular every sentence is either true or false. We shall need very few assumptions about what sentences

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<sup>1</sup> A view of consequence advocated in van Benthem [\[2\]](#), who traces it back to Bolzano.

look like or how they are structured. For definiteness, a *language*  $L$  has a set  $Sent_L$  of *sentences*, which are finite strings of signs, some of which, called *symbols*, belong to a set  $Symb_L$ . Let  $u, v, u', \dots$  vary over  $Symb_L$ ,  $\varphi, \psi, \dots$  over  $Sent_L$ , and  $\Gamma, \Delta, \dots$  over finite subsets sets of  $Sent_L$ .  $V_\varphi$  is the set of symbols *occurring* in  $\varphi$ ; likewise  $V_\Gamma = \cup\{V_\varphi : \varphi \in \Gamma\}$ .  $Tr_L \subseteq Sent_L$  is the set of true sentences in  $L$ .

## 1.2 Replacement

We need a notion of ‘appropriate’ replacement of symbols by other symbols. To this end, think of  $Symb_L$  as partitioned into a set of *categories*. Then, a *replacement* is a partial function  $\rho$  from  $Symb_L$  to  $Symb_L$  such that for  $u \in \text{dom}(\rho)$ ,  $u$  and  $\rho(u)$  belong to the same category.  $\varphi[\rho]$  is the result of replacing each occurrence of  $u$  in  $\varphi$  by  $\rho(u)$ . It is convenient to assume that  $V_\varphi \subseteq \text{dom}(\rho)$  — in words,  $\rho$  is a replacement *for*  $\varphi$  — so that  $\rho$  is the identity on symbols that don’t get replaced. We may then assume that the following conditions hold<sup>2</sup>

- (1) a. If  $\rho$  is a replacement for  $\varphi$ ,  $\varphi[\rho] \in Sent$  and  $V_{\varphi[\rho]} = \text{range}(\rho \upharpoonright V_\varphi)$
- b.  $\varphi[id_{V_\varphi}] = \varphi$
- c. If  $\rho, \sigma$  agree on  $V_\varphi$ , then  $\varphi[\rho] = \varphi[\sigma]$ .
- d.  $\varphi[\rho][\sigma] = \varphi[\sigma\rho]$ , when  $\sigma$  is a replacement for  $\varphi[\rho]$

## 1.3 Consequence Relations

**Definition 1.** A relation  $R \subseteq \wp(Sent_L) \times Sent_L$  is

1. *reflexive* iff for all  $\varphi \in Sent_L$ ,  $\varphi R \varphi$ ;
2. *transitive* iff whenever  $\Delta R \varphi$  and  $\Gamma R \psi$  for all  $\psi \in \Delta$ , we have  $\Gamma R \varphi$ ;
3. *monotone* iff  $\Delta R \varphi$  and  $\Delta \subseteq \Gamma$  implies  $\Gamma R \varphi$ ;
4. *truth-preserving* iff whenever  $\Gamma R \varphi$  and (every sentence in)  $\Gamma$  is true,  $\varphi$  is also true.

**Definition 2.** A *consequence relation* in  $L$  is a reflexive, transitive, monotone, and truth-preserving relation between finite sets of  $L$ -sentences and  $L$ -sentences. Let  $\Rightarrow, \Rightarrow', \dots$  vary over the set  $CONS_L$  of consequence relations in  $L$ . Define:

- (2) a.  $\Gamma \Rightarrow^{max} \varphi$  iff it is not the case that  $\Gamma$  is true and  $\varphi$  is false.
- b.  $\Gamma \Rightarrow^{min} \varphi$  iff  $\varphi \in \Gamma$ .

$\Rightarrow^{max}$  is essentially material implication.

**Proposition 1.**  $\Rightarrow^{max}, \Rightarrow^{min} \in CONS_L$ , and  $(CONS_L, \subseteq)$  is partial order with  $\Rightarrow^{min}$  as its smallest and  $\Rightarrow^{max}$  as its largest element.

## 2 Bolzano Consequence

### 2.1 Definition of $\Rightarrow_-$

The following definition is familiar, except that (a) it is substitutional rather than model-theoretic; (b) it allows any set of symbols to be treated as logical.

<sup>2</sup> Essentially the conditions in Peter Aczel’s notion of a *replacement system* from [1].

**Definition 3.** For any  $X \subseteq \text{Symb}_L$ , define the relation  $\Rightarrow_X$  by

$\Gamma \Rightarrow_X \varphi$  iff for every replacement  $\rho$  (for  $\Gamma$  and  $\varphi$ ) which is the identity on  $X$ , if  $\Gamma[\rho]$  is true, so is  $\varphi[\rho]$ .

A relation of the form  $\Rightarrow_X$  is called a *Bolzano consequence* (relation);  $\text{BCONS}_L$  is the set of Bolzano consequences<sup>3</sup>

**Proposition 2.** (a)  $\text{BCONS}_L \subseteq \text{CONS}_L$

(b) In addition, Bolzano consequence is base monotone:

$$X \subseteq Y \text{ implies } \Rightarrow_X \subseteq \Rightarrow_Y$$

(c)  $(\text{BCONS}_L, \subseteq)$  is a partial order with  $\Rightarrow_\emptyset$  as its smallest and  $\Rightarrow_{\text{Symb}}$  as its largest element.

$(\text{BCONS}_L, \subseteq)$  is a sub-order of  $(\text{CONS}_L, \subseteq)$ , and  $\Rightarrow^{\text{max}} = \Rightarrow_{\text{Symb}}$ , although usually  $\Rightarrow^{\text{min}} \subsetneq \Rightarrow_\emptyset$ . The following two lemmas are trivial but fundamental:

**Lemma 1. (Replacement Lemma)** If  $\Gamma \Rightarrow_X \varphi$  and  $\rho$  doesn't move any symbols in  $X$ , then  $\Gamma[\rho] \Rightarrow_X \varphi[\rho]$ .

**Lemma 2. (Occurrence Lemma)**  $\Gamma \Rightarrow_X \varphi$  if and only if  $\Gamma \Rightarrow_{X \cap V_{\Gamma \cup \{\varphi\}}} \varphi$ .

## 2.2 Examples

**Propositional logic.** A standard language of *propositional logic* has symbols in a set  $X_0$  of connectives, say,  $X_0 = \{\neg, \wedge, \vee\}$ , and an infinite supply  $p_0, p_1, \dots$  of propositional letters. The usual definition of logical consequence,  $\models_{PL}$ , is model-theoretic, but we can ‘simulate’ it in the present substitutional setting, where  $p_0, p_1, \dots$  are sentences with fixed truth values. Assuming that the sequence of truth values of  $p_0, p_1, \dots$  is *not eventually constant*, one easily verifies that

$$(3) \quad \Gamma \models_{PL} \varphi \text{ iff } \Gamma \Rightarrow_{X_0} \varphi$$

**First-order logic.** For *first-order logic* the symbols are, say,  $X_1 = X_0 \cup \{\exists, \forall, =\}$ , and a supply of predicate symbols and individual constants. Now there is a difference between model-theoretic and substitutional definitions: in general we have

$$(4) \quad \models_{FO} \subsetneq \models_{FO_{\text{subst}}} \subsetneq \Rightarrow_{X_1}$$

where  $\models_{FO_{\text{subst}}}$ , i.e. the consequence relation you get with a standard *substitutional interpretation of the quantifiers*, as in [3]. So  $\models_{FO}$  is a consequence relation, but not a Bolzano consequence.

<sup>3</sup> We use ‘ $\Rightarrow_X$ ’ both as a relation symbol, which enables us to write  $\Gamma \Rightarrow_X \varphi$ , and as the value of the function  $\Rightarrow_\bullet: \text{Symb}_L \longrightarrow \text{BCONS}_L$  for the argument  $X$ .

### 2.3 Minimal Sets of Symbols

Different sets may generate the same Bolzano consequence, so one expects sets that are *minimal* in this respect to be particularly well behaved.

**Definition 4.**  $X$  is *minimal* iff for all  $u \in X$ ,  $\Rightarrow_X \neq \Rightarrow_{X-\{u\}}$ .

**Proposition 3.**  $X$  is *minimal* iff no proper subset of  $X$  generates the same consequence relation.

The next result shows that it is sufficient to look at consequence relations generated by minimal sets.

**Proposition 4.** Every  $X \subseteq \text{Symb}_L$  has a subset which is minimal among those generating  $\Rightarrow_X$ .

A stricter notion of minimality is the following:

**Definition 5.**  $X$  is *strongly minimal* iff for all  $u \in X$  there are  $\Gamma$ ,  $\varphi$ , and  $u'$  such that  $\Gamma \Rightarrow_X \varphi$ ,  $\Gamma[u/u']$  is true, but  $\varphi[u/u']$  is false.<sup>4</sup>

The following is practically immediate.

(5) If  $X$  is strongly minimal, it is minimal.

Let  $\text{MIN}_L$  ( $\text{SMIN}_L$ ) be the set of (strongly) minimal subsets of  $\text{Symb}_L$ . Strong minimality says that  $\Rightarrow_X \subseteq \Rightarrow_{X-\{u\}}$  fails in a particular way: a counterexample exists which involves replacing only  $u$ . One can show that, unless extra assumptions are made about the language (Section 4 below), not all Bolzano consequences are of the form  $\Rightarrow_X$  for strongly minimal  $X$ . But those of this form are particularly well behaved:

**Proposition 5.** If  $X$  is strongly minimal then, for all  $Y \subseteq \text{Symb}_L$ ,  $X \subseteq Y$  iff  $\Rightarrow_X \subseteq \Rightarrow_Y$ .

**Corollary 1.** The mapping  $\Rightarrow_{\_}$  is one-one on strongly minimal sets.

## 3 Extracting Constants from Consequence Relations

### 3.1 Defining Extraction

We now introduce an operation corresponding to the extraction of logical constants from a consequence relation. When a particular consequence relation is given, certain symbols are to be considered as logical constants because the consequence relation makes them play a special role with respect to validity. Our guiding intuition is that a symbol is constant if replacing it can destroy *at least one* inference.<sup>5</sup>

<sup>4</sup>  $u/u'$  is the replacement which maps  $u$  to  $u'$  but is the identity on all other symbols.

<sup>5</sup> This is a variation on a similar idea first introduced in [4], Ch. 9.

**Definition 6.** The function  $C_- : CONS_L \rightarrow \wp(Symb_L)$  is defined for  $\Rightarrow \in CONS_L$  and  $u \in Symb_L$  by  $u \in C_- \iff$  there are  $\Gamma, \varphi$  and  $u'$  such that  $\Gamma \Rightarrow \varphi$  but  $\Gamma[u/u'] \not\Rightarrow \varphi[u/u']$ .

Logical consequence can be construed as a function from sets of symbols to consequence relations. Extraction goes in the opposite direction. Moreover, the domains of both functions are naturally ordered by inclusion, so the situation is as shown in Figure 1. Proposition 2(b) said that  $\Rightarrow_-$  is an order-preserving mapping from  $(\wp(Symb_L), \subseteq)$  to  $(CONS_L, \subseteq)$ . We would like  $C_-$  to provide some sort of inverse order-preserving mapping. Before looking into this and other properties of  $C_-$ , let us see some examples of how  $C_-$  works.

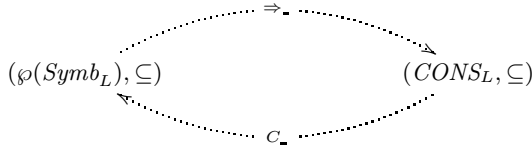


Fig. 1. Logical consequence and constant extraction

### 3.2 Examples

The function  $C_-$  might fail to yield the intended result because of its substitutional character. In particular, if a symbol  $u$  is unique in its category, there is no other symbol to replace  $u$  with, and trivially it will not count as a logical constant, no matter what inferential role it plays. This situation arises with negation, which is usually the only unary connective in the language. To sidestep this difficulty, let us assume, when considering propositional logic or first-order logic, that they come equipped with another unary connective, say  $\dagger$ , interpreted by the constant unary truth-function ‘equal to false’. Then we get:

**Proposition 6.**  $C_{\models_{PL}} is the standard set of logical constants of PL.$

Let us see why in two examples.  $p \models_{PL} p \vee q$  but  $p \not\models_{PL} p \wedge q$ . Replacing  $\vee$  by  $\wedge$  destroys the validity of the first inference, so  $\vee \in C_{\models_{PL}}$ . Similarly,  $\neg \neg p \models_{PL} p$  but  $\dagger \dagger p \not\models_{PL} p$ , therefore  $\neg \in C_{\models_{PL}}$ . Similarly for quantifiers in first-order logic.

**Proposition 7.**  $C_{\models_{FO}} = C_{\models_{FO_{subst}}}$  is the standard set of logical constants of first-order logic.

### 3.3 Facts about $C_-$ in the Bolzano Setting

As a direct consequence of the Replacement Lemma,  $C_-$  will never pick out non-logical constants when it is applied to a Bolzano consequence.

**Proposition 8.** For all  $X \in \wp(Symb_L)$ ,  $C_{\Rightarrow_X} \subseteq X$ .

Something stronger holds for strongly minimal  $X$ :

**Proposition 9.** *For all  $X \in \text{SMIN}_L$ ,  $C_{\Rightarrow_X} = X$ .*

From this and Proposition 5 we get:

**Proposition 10.**  *$\Rightarrow_-$  restricted to  $\text{SMIN}_L$  is an isomorphism with inverse  $C_-$ .*

This tells us that  $C_-$  plays its role as an order-preserving inverse mapping on some proper subset of  $\text{CONS}_L$ , namely the Bolzano consequences generated from strongly minimal sets of constants. These are of course severe limitations to the scope of the result, and the remainder of this paper will be devoted to providing an understanding of the global picture. But in the present framework,  $C_-$  is simply *not* an order-preserving inverse on all of  $\text{CONS}_L$ .

**Proposition 11.** *There are languages  $L$  and consequence relations  $\Rightarrow$  and  $\Rightarrow'$  in  $\text{CONS}_L$  such that:*

- (a)  $\Rightarrow \subseteq \Rightarrow'$  but  $C_{\Rightarrow} \not\subseteq C_{\Rightarrow'}$ ,
- (b)  $\Rightarrow \not\subseteq \Rightarrow_{C_{\Rightarrow}}$

The failure of (a) in particular is no surprise given that there are both a positive and a negative condition in the definition of  $C_-$ . The witness to a non-valid inference might disappear by shifting to a bigger consequence relation. More surprisingly, the situation is no better for Bolzano consequences.

**Proposition 12.** *There are languages  $L$  and sets  $X, Y \subseteq \text{Symb}_L$  such that:*

- (a)  $\Rightarrow_X \subseteq \Rightarrow_Y$  but  $C_{\Rightarrow_X} \not\subseteq C_{\Rightarrow_Y}$
- (b)  $\Rightarrow_X \not\subseteq \Rightarrow_{C_{\Rightarrow_X}}$

## 4 Extra Symbols

The following example vividly illustrates the importance of having extra symbols available in a language. Let  $\text{Symb}_L = \{a, b\}$ ,  $\text{Sent}_L = \{Rxy : x, y \in \text{Symb}_L\}$ , and  $\text{Tr}_L = \{Raa, Rbb, Rab\}$ . For example,

$$\Rightarrow_{\emptyset} Raa \not\models_{\emptyset} Rab, \quad \Rightarrow_{\{a\}} Rab$$

Here, you must move *two* symbols in order to turn a true sentence into a false one. As a result,  $C_-$  picks no constants at all: for all  $X \subseteq \text{Symb}_L$ ,  $C_{\Rightarrow_X} = \emptyset$ . However, expand  $L$  conservatively to  $L'$  by adding at least one new symbol  $c$ , while  $\text{Tr}_{L'} = \text{Tr}_L \cap \text{Sent}_{L'}$ . Then, regardless of the truth values of new sentences in  $L'$ :

$$(6) \quad \text{In } L', \text{ for all } X \subseteq \text{Symb}_L, C_{\Rightarrow_X} = X.$$

For example, we now have  $\Rightarrow_{\{a\}} Rab$  in  $L'$  (by conservativity), but  $\not\models_{\{a\}} Rcb$ , (by the replacement  $\rho(c) = b$ ,  $\rho(b) = a$ ). Thus, in  $L'$ ,  $a \in C_{\Rightarrow_{\{a\}}}$ .

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<sup>6</sup> Writing  $\Rightarrow_X \varphi$  for  $\emptyset \Rightarrow_X \varphi$ , i.e.  $\varphi$  is valid relative to the constants in  $X$ .

## 4.1 Richness and Abundance

One way of making extra symbols available is to simply assume that there are infinitely many symbols of each category in  $L$ . Call such languages *rich*. Note that nothing prevents most of these symbols from meaning the same.

**Proposition 13.** *If  $L$  is rich, then  $\Rightarrow_X \subseteq \Rightarrow_Y$  implies  $\Rightarrow_X = \Rightarrow_{X \cap Y}$ . In particular, for any  $X$ , the set  $\{Z : \Rightarrow_Z = \Rightarrow_X\}$  is closed under finite intersections, so if  $X$  is finite, this set has a smallest element.*

**Proposition 14.** *If  $L$  is rich, and  $X$  is minimal and finite, then  $C_{\Rightarrow_X} = X$ .*

Using a variant of the earlier example, one can show that the assumption of finiteness in this proposition is essential.

**Corollary 2.** *If  $L$  is rich, then for finite  $X$ ,  $\Rightarrow_X = \Rightarrow_{C_{\Rightarrow_X}}$ .*

**Corollary 3.** *If  $L$  is rich, then every finite  $X \subseteq \text{Symb}_L$  has a unique smallest subset, namely  $C_{\Rightarrow_X}$ , that generates  $\Rightarrow_X$ .*

Here is an even stronger requirement on  $L$ .  $u$  and  $u'$  (of the same category) are *synonymous*,

$$u \equiv_L u'$$

iff replacing (some) occurrences of  $u$  by  $u'$  or vice versa does not change the truth value of  $L$ -sentences. We say that  $L$  is *abundant* iff there are infinitely many synonyms of each symbol. So all the results above hold for abundant  $L$ , but in addition we have

**Proposition 15.** *If  $L$  is abundant, minimality and strong minimality coincide.*

Another variant of our example shows that this can fail when only richness is assumed.

**Corollary 4.** *If  $L$  is abundant, the results of Proposition 14 and Corollaries 2 and 3 hold for infinite  $X$  as well.*

## 4.2 Expansions

Richness and abundance may seem a bit extravagant assumptions. What one really needs, however, is the ability to *add* new symbols to  $L$ , in particular symbols with the same meaning as old ones, i.e. synonyms or *copies*. We now slightly revise our Bolzano set-up to make this possible.

Recall that an interpreted language  $L$ , as we defined it, comes with sets  $\text{Symb}_L$ ,  $\text{Sent}_L$ , and  $\text{Tr}_L$ ; in fact, we may set  $L = \langle \text{Symb}_L, \text{Sent}_L, \text{Tr}_L \rangle$ . We say that  $L'$  is an *expansion* of  $L$ ,  $L \leq L'$ , iff  $\text{Symb}_L \subseteq \text{Symb}_{L'}$ ,  $\text{Sent}_L = \{\varphi \in \text{Sent}_{L'} : V_\varphi \subseteq \text{Symb}_L\}$ , and  $\text{Tr}_L = \text{Tr}_{L'} \cap \text{Sent}_L$ .  $L'$  is an *expansion with copies*,  $L \leq_c L'$ , iff in addition every new symbol is synonymous, in  $L'$ , with some  $L$ -symbol. One readily shows that  $\leq$  is a partial order (reflexive, antisymmetric, and transitive), and  $\leq_c$  is a sub-order.

A partially ordered set  $Z$  is *directed* iff it is upward closed: if  $a, b \in Z$  there is  $c \in Z$  such that  $a \leq c$  and  $b \leq c$ . Now, our idea is to replace the fixed language  $L$  with a directed family  $\mathcal{L}$  of expansions of  $L$ . This requires a slight reformulation of what we have done so far. In what follows,  $\mathcal{L}$  is any directed family of expansions of  $L$ . To start with Bolzano consequence, suppose  $\Gamma \cup \{\varphi\} \subseteq \text{Sent}_L$  and  $X \subseteq \text{Symb}_L$ .

**Definition 7.**  $\Gamma \Rightarrow_{X,L} \varphi$  iff for every  $L' \in \mathcal{L}$  and every replacement  $\rho$  in  $L'$  (for  $\Gamma$  and  $\varphi$ ) which is the identity on  $X$ , if  $\Gamma[\rho] \subseteq \text{Tr}_{L'}$ , then  $\varphi[\rho] \in \text{Tr}_{L'}$ .

The family  $\mathcal{L}$  is suppressed in this notation, and has to be made clear in context. If  $\mathcal{L} = \{L\}$ , we have our previous notion of Bolzano consequence:  $\Rightarrow_{X,L} = \Rightarrow_X$ . Normally, the sentences we talk about will belong to several languages in  $\mathcal{L}$ . That this is not a problem follows from

**Lemma 3.** If  $\Gamma \cup \{\varphi\} \subseteq \text{Sent}_L$ ,  $X \subseteq \text{Symb}_L$ , and  $L' \in \mathcal{L}$ , then

$$\Gamma \Rightarrow_{X,L} \varphi \text{ iff } \Gamma \Rightarrow_{X,L'} \varphi,$$

where the right-hand side is relative to the subclass  $\mathcal{L}' = \{L'' \in \mathcal{L} : L' \leq L''\}$ .

In what follows, when  $\mathcal{L}$  is given and  $L' \in \mathcal{L}$ , we always understand  $\Rightarrow_{X,L'}$  to be relative to the corresponding subfamily generated by  $L'$ .

Next, we extend the notion of (strong) minimality to the new set-up:  $X$  is *minimal* iff for each  $u \in X$  there is  $L' \in \mathcal{L}$  such that  $\Rightarrow_{X,L'} \neq \Rightarrow_{X-\{u\},L'}$ , and analogously for strong minimality. Finally, we extend the definition of  $C_-$  to the case of consequence relations of the form  $\Rightarrow_{X,L}$ . Let  $u \in \text{Symb}_L$ .

**Definition 8.**  $u \in C_{\Rightarrow_{X,L}}$  iff there are  $L' \in \mathcal{L}$ ,  $\Gamma \cup \{\varphi\} \subseteq \text{Sent}_{L'}$ , and  $u' \in \text{Symb}_{L'}$  such that  $\Gamma \Rightarrow_{X,L'} \varphi$  but  $\Gamma[u/u'] \not\Rightarrow_{X,L'} \varphi[u/u']$ .

Call a family  $\mathcal{L}$  is *copy-closed* iff for every  $L' \in \mathcal{L}$  and every  $A \subseteq \text{Symb}_{L'}$ , there are  $L'' \in \mathcal{L}$  and  $B \subseteq \text{Symb}_{L''}$  such that  $B \cap \text{Symb}_{L'} = \emptyset$  and there is a copy  $b \in B$  of every  $a \in A$ . A simple case of a copy-closed family is  $\text{copies}(L) = \{L' : L \leq_c L'\}$ . This is essentially the interpreted language  $L$  with the possibility of adding (arbitrarily many) new names of already named things. Thus, it is a very mild extension of the original Bolzano set-up. Now all results assuming richness or abundance from the previous subsection hold automatically relative to copy-closed families, without any further assumptions:

**Proposition 16.** Relative to a copy-closed directed family of expansions of  $L$ :

- (a) Minimality and strong minimality coincide.
- (b) For all  $X \subseteq \text{Symb}_L$ ,  $\Rightarrow_{X,L} = \Rightarrow_{C_{\Rightarrow_{X,L}}}$ .
- (c) Each  $X \subseteq \text{Symb}_L$  has a unique smallest subset,  $C_{\Rightarrow_{X,L}}$ , generating  $\Rightarrow_{X,L}$ .

## 5 Galois Connections

### 5.1 General Consequence Relations

We generalized our initial framework by working with directed families of expansions of  $L$ . In terms of Figure [1](#), the sets of symbols we are interested in are

still subsets  $Symb_L$ . But such sets generate consequence relations for  $L$  as well as for the expansions of  $L$ . In particular, the extended Definition 8 of  $C_-$  appeals not only to the consequence relation of  $L$  but also to those of the expansions. As a result, we need to extend our notion of a consequence relation.

**Definition 9.** A *general consequence relation* for a family of languages  $\mathcal{L}$  is a family of consequence relations  $\Rightarrow = \{\Rightarrow_{L'}\}_{L' \in \mathcal{L}}$  such that for all  $L', L'' \in \mathcal{L}$  with  $L' \leq L''$ ,  $\Rightarrow_{L'} \subseteq \Rightarrow_{L''}$ .

We also need a generalized ordering  $\leq$  defined by  $\Rightarrow_X \leq \Rightarrow_Y$  iff for all  $L' \in \mathcal{L}$ ,  $\Rightarrow_{X, L'} \subseteq \Rightarrow_{Y, L'}$ .

For  $X \subseteq Symb_L$ , let  $\Rightarrow_X$  be the family  $\{\Rightarrow_{X, L'}\}_{L' \in \mathcal{L}}$ .  $\Rightarrow_X$  is a general consequence relation. Note that the extended Definition 8 of  $C_-$  applies not just to  $\Rightarrow_{X, L}$  but to any general consequence relation. Given a directed family  $\mathcal{L}$  of expansions of  $L$ , we let  $GCONS_L$  be the class of general consequence relations for  $\mathcal{L}$  and  $BGCONS_L$  the class of general consequence relations of the form  $\Rightarrow_X$ .

## 5.2 Galois Connection for Copies

What kind of correspondence do we get between  $\Rightarrow_X$  and  $C_-$  in this setting? We want something as close as possible to an isomorphism, with as few assumptions as possible on  $\mathcal{L}$ . A relevant notion of correspondence in that context is the notion of *Galois connection*. A Galois connection between two ordered sets  $A$  and  $B$  is a pair  $(f, g)$  of functions, with  $f : A \rightarrow B$  and  $g : B \rightarrow A$ , such that the following four conditions hold: (1)  $f$  is monotone, (2)  $g \circ f$  is decreasing, (3)  $g$  is monotone, (4)  $f \circ g$  is increasing.  $f$  is then an isomorphism with inverse  $g$  from  $g(B)$  to  $f(A)$ . Intuitively,  $f$  and  $g$ , even though they do not constitute a full-blown isomorphism, give rise to one between the sufficiently well-behaved subsets  $g(B)$  and  $f(A)$ .

**Proposition 17.** *Relative to a copy-closed family, the following hold:*

- (a)  $\Rightarrow_X \leq \Rightarrow_Y$  implies  $C_{\Rightarrow_X} \subseteq C_{\Rightarrow_Y}$
- (b)  $\Rightarrow_X \leq \Rightarrow_{C_{\Rightarrow_X}}$

Considering  $(\Rightarrow_-, C_-)$  as a tentative Galois connection between  $\wp(Symb_L)$  and  $GCONS_L$ , conditions (1) and (2) are always satisfied. (a) and (b) in Proposition 17 are (3) and (4), when the class of general consequence relations is restricted to  $BGCONS_L$ . We have shown:

**Theorem 1.**  $(\Rightarrow_-, C_-)$  is a Galois connection between  $\wp(Symb_L)$  and  $BGCONS_L$  for copy-closed families of languages.

The set  $C_{BGCONS_L}$  of sets of symbols which are the image of some  $\Rightarrow_X$  in  $BGCONS_L$  under  $C_-$  is of special interest, since it is the restriction of  $\wp(Symb_L)$  for which  $\Rightarrow_-$  is an isomorphism.

**Proposition 18.** *Relative to a copy-closed family of expansions of  $L$ ,  $C_{BGCONS_L}$  is the set of minimal sets in  $\wp(Symb_L)$ .*

On the left-hand side of our picture, the well-behaved objects in  $\wp(\text{Symb}_L)$  are the minimal sets. On the right-hand side, our Galois connection has the non-typical property that  $\Rightarrow_{\perp}$  is onto. If we were to extend the picture to include not only  $\text{BGCONS}_L$  but all of  $\text{GCONS}_L$ , we would get that the part of  $\text{GCONS}_L$  for which we have the isomorphism is precisely  $\text{BGCONS}_L$ , but the properties stated in Proposition 17 for relations in  $\text{BGCONS}_L$  do not hold for all relations in  $\text{GCONS}_L$ .

**Question:** Is there a natural class  $\text{GCONS}_L^* \supseteq \text{BGCONS}_L$  for which they hold?

### 5.3 Tarskian Consequence

$\text{copies}(L)$  is the simplest case of a copy-closed family of languages. A maximal case is the family of *all* expansions of a base-language  $L$ . Instead of just fixing a set of true sentences, consider interpreted languages  $L$  for which truth is defined as truth with respect to a fixed intended interpretation  $I$  of symbols of  $L$ . Let ' $J \models \varphi$ ' abbreviate ' $\varphi$  is true according to interpretation  $J$ '. *Tarskian consequence* (with a fixed domain of interpretation) can then be defined:

**Definition 10.**  $\Gamma \models_X \varphi$  iff for all  $J$  such that  $J \approx_X I$ , if  $J \models \Gamma$ , then  $J \models \varphi$ .

( $J \approx_X I$  means that  $J$  and  $I$  agree on  $X \subseteq \text{Symb}_L$ .) Tarskian consequence is equivalent to Bolzano consequence with respect to the family of *all* expansions:

**Proposition 19.**  $\Gamma \models_X \varphi$  iff for every  $L' \geq L$  and every replacement  $\rho$  that keeps all elements in  $X$  fixed, if  $I' \models \Gamma[\rho]$ , then  $I' \models \varphi[\rho]$ .

(Here  $I'$  is an interpretation extending  $I$  to the new symbols in  $\text{Symb}_{L'}$ .) Together with Proposition 17 and the fact that the family of all expansions is copy-closed, this implies that there is a Galois connection between  $\wp(\text{Symb}_L)$  and the class of Tarskian consequence relations.

## References

1. Aczel, P.: Replacement systems and the axiomatization of situation theory. In: Cooper, R., Mukai, K., Perry, J. (eds.) *Situation Theory and its Applications*, vol. 1, pp. 3–33. CSLI Publications, Stanford (1990)
2. van Benthem, J.: Is there still logic in Bolzano's key? In: Morscher, E. (ed.) *Bernard Bolzano's Leistungen in Logik. Mathematik und Physik Bd*, vol. 16, pp. 11–34. Academia Verlag, Sankt Augustin (2003)
3. Dunn, M., Belnap Jr., N.: The substitution interpretation of the quantifiers. *Noûs* 4, 177–185 (1968)
4. Peters, S., Westerståhl, D.: *Quantifiers in Language and Logic*. Oxford UP, Oxford (2006)

# Modified Numerals as Post-Suppositions

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**Abstract.** The paper provides a compositional account of cumulative readings with non-increasing modified numerals (aka van Benthem’s puzzle), e.g., *Exactly three boys saw exactly five movies*. The main proposal is that modified numerals make two kinds of semantic contributions. Their asserted/at-issue contribution is a maximization operator that introduces the maximal set of entities that satisfies their restrictor and nuclear scope. The second contribution is a post-supposition, i.e., a cardinality constraint that needs to be satisfied relative to the context that results after the at-issue meaning is evaluated. We take contexts to be sets of variable assignments relative to which quantificational expressions are interpreted and which are updated as a result of their interpretation.

## 1 Cumulativity and Modified Numerals

The goal of the paper is to provide a compositional account of cumulative readings with non-increasing modified numerals (aka van Benthem’s puzzle, [van Benthem 1986](#)), exemplified in [\(II\)](#) below. We discuss mainly *exactly n* numerals, but the same problem arises with other non-increasing numerals, e.g., *at most n*.

- (1) Exactly three<sup>x</sup> boys saw exactly five<sup>y</sup> movies.

The most familiar reading of sentence [\(II\)](#) is the surface-scope distributive one, namely: there are exactly three boys such that each of them saw exactly five movies (possibly different from boy to boy). We are not interested in this reading (although we discuss it briefly later on), but in the cumulative reading, namely: consider the maximal number of boys that saw a movie and the maximal number of movies seen by a boy; there are three such boys and five such movies. This reading of sentence [\(II\)](#) is true in Figure 2 below and false in Figure 1. Note that Figure 1 is exactly like Figure 2, with the addition of boy **b**<sub>1</sub>, movie **m**<sub>1</sub> and the arrow between them symbolizing the seeing relation at the very top of Figure 1.

Importantly, the cumulative reading is different from: the maximal number of boys that (between them) saw exactly five movies is three.[1](#) This is actually

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<sup>1</sup> As [Krifka \(1999\)](#), [Landman \(2000\)](#) and [Ferreira \(2007\)](#) observe. See [Robaldo \(2009\)](#) for a different take on the data.

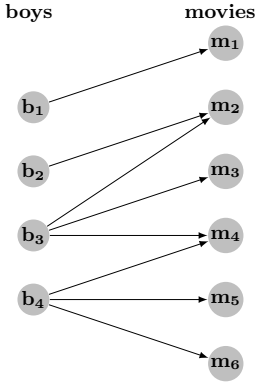


Fig. 1.

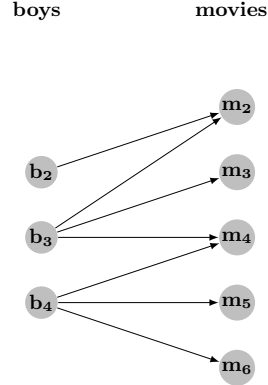


Fig. 2.

not a reading of sentence (11), although it bears some resemblance to its distributive reading. In fact, the situations in Figures 1 and 2 above distinguish between them: the cumulative reading is intuitively false in Figure 1 (4 boys and 6 movies) and true in Figure 2, while the other ‘reading’ just mentioned is true in both situations. The distinction between the cumulative reading and this other ‘reading’ is important for theoretical reasons: many formal systems derive something like it when they attempt to capture the cumulative reading.

Our proposal is that modified numerals make two kinds of contributions to the meaning of sentences like (11): (i) their asserted/at-issue contribution is a maximization operator that introduces the maximal set of entities that satisfies their restrictor and nuclear scope; (ii) they also contribute a post-supposition, i.e., a cardinality constraint (e.g., exactly five) that needs to be satisfied relative to the *context that results after* the at-issue meaning is evaluated. See Lauer (2009) for another use of this notion and Farkas (2002) and Constant (2006) for related post-assertion constraints on output contexts.

For our purposes, contexts are sets of total variable assignments relative to which quantificational expressions are interpreted and which are updated as a result of the interpretation of such expressions. That is, we work with a simplified version of Dynamic Plural Logic (DPIL, van den Berg 1996).

The main difference between the present account and Krifka (1999) is conceptual: we take modified numerals to constrain *quantificational* – not *focus* – alternatives, where a quantificational alternative is a set of assignments satisfying a quantificational expression. Thus, we reconceptualize DPIL as the logic of quantificational alternatives in natural language interpretation.

## 2 Modified Numerals as Post-Suppositions

We work with the usual models for classical first-order logic (FOL)  $\mathfrak{M} = \langle \mathfrak{D}, \mathfrak{I} \rangle$ :  $\mathfrak{D}$  is the domain of individuals and  $\mathfrak{I}$  is the basic interpretation function such

that  $\mathcal{J}(R) \subseteq \mathcal{D}^n$  for any  $n$ -ary relation  $R$ . An  $\mathcal{M}$ -assignment  $g$  is a total function from the set of variables  $\mathcal{V}$  to  $\mathcal{D}$ . The essence of quantification in FOL is point-wise/variablewise manipulation of variable assignments, abbreviated  $h[x]g$ . We generalize this to sets of assignments  $H[x]G$  cumulative-quantification style.

- (2)  $h[x]g := h$  differs from  $g$  at most with respect to the value assigned to  $x$
- (3)  $H[x]G := \begin{cases} \text{for all } h \in H, \text{ there is a } g \in G \text{ such that } h[x]g \\ \text{for all } g \in G, \text{ there is a } h \in H \text{ such that } h[x]g \end{cases}$

This is not a way of sneaking cumulativity into the system. Formally,  $H[x]G$  is a natural generalization of  $h[x]g$ : it is an equivalence relation over sets of total assignments, just as  $h[x]g$  is an equivalence relation over total assignments.

Atomic formulas are tests, i.e., they check that the input context  $G$  satisfies them and pass this context on. Cardinality constraints on the values of variables are also tests; the cardinality of the set of individuals  $X$  is symbolized as  $|X|$ . Dynamic conjunction and random assignment are interpreted DPL-style.

- (4)  $\llbracket R(x_1, \dots, x_n) \rrbracket^{(G, H)} = \mathbb{T}$  iff  $G = H$  and for all  $h \in H$ ,  $\langle h(x_1), \dots, h(x_n) \rangle \in \mathcal{J}(R)$
- (5)  $G(x) := \{g(x) : g \in G\}$
- (6)  $\llbracket x = n \rrbracket^{(G, H)} = \mathbb{T}$  iff  $G = H$  and  $|H(x)| = n$
- (7)  $\llbracket x \leq n \rrbracket^{(G, H)} = \mathbb{T}$  iff  $G = H$  and  $|H(x)| \leq n$
- (8)  $\llbracket x \geq n \rrbracket^{(G, H)} = \mathbb{T}$  iff  $G = H$  and  $|H(x)| \geq n$
- (9)  $\llbracket \phi \wedge \psi \rrbracket^{(G, H)} = \mathbb{T}$  iff there is a  $K$  s.t.  $\llbracket \phi \rrbracket^{(G, K)} = \mathbb{T}$  and  $\llbracket \psi \rrbracket^{(K, H)} = \mathbb{T}$
- (10)  $\llbracket [x] \rrbracket^{(G, H)} = \mathbb{T}$  iff  $H[x]G$

The translation of singular indefinite articles and bare numerals has the form given in (11) below. Square brackets  $[]$  indicate restrictor formulas and round brackets  $()$  indicate nuclear scope formulas. As (12) and (13) show,  $n$  is 1 for singular indefinite articles,  $n$  is 2 for the bare numeral *two* etc. This translation schema is just an abbreviation, provided in (14). Proper names are interpreted like indefinites; their restrictor requires the variable to have the same value as a non-logical constant, e.g., JASPER in (15) below (where JASPER denotes the individual Jasper). Pronouns are indexed with the variable introduced by their antecedent and their translation is that variable itself; we ignore differences between singular and plural pronouns. For example, (16) is translated as in (17).

- (11)  $\exists x[x = n \wedge \phi](\psi)$  intuitively:  $n$   $\phi$ -individuals are  $\psi$
- (12)  $A^x$  wolf came in.  $\rightsquigarrow \exists x[x = 1 \wedge \text{WOLF}(x)](\text{COME-IN}(x))$
- (13) Two<sup>x</sup> wolves came in.  $\rightsquigarrow \exists x[x = 2 \wedge \text{WOLF}(x)](\text{COME-IN}(x))$
- (14)  $\exists x[x = n \wedge \phi](\psi) := [x] \wedge x = n \wedge \phi \wedge \psi$
- (15)  $\exists x[x = \text{JASPER}](\phi) := [x] \wedge x = \text{JASPER} \wedge \phi$
- (16)  $A^x$  wolf came in. It<sub>x</sub> bit Jasper<sup>y</sup>.
- (17) a.  $\exists x[x = 1 \wedge \text{WOLF}(x)](\text{COME-IN}(x)) \wedge \exists y[y = \text{JASPER}](\text{BITE}(x, y))$   
b.  $[x] \wedge x = 1 \wedge \text{WOLF}(x) \wedge \text{COME-IN}(x) \wedge [y] \wedge y = \text{JASPER} \wedge \text{BITE}(x, y)$

The definition of truth below says that a formula  $\phi$  is true if there is at least one successful way to update the input context of evaluation  $G$  with  $\phi$ . Except

for the fact that we work with sets of assignments instead of single assignments, our interpretation function and truth definition are not different from the corresponding FOL or DRT/FCS notions (only their packaging is different).

- (18) Truth: a formula  $\phi$  is true relative to an input set of assignments  $G$  iff there is an output set of assignments  $H$  such that  $\llbracket \phi \rrbracket^{(G,H)} = \mathbb{T}$ .

## 2.1 Modified Numerals

We capture the meaning of modified numerals by means of a maximization operator **M** that enables us to introduce the set of all individuals satisfying their restrictor and nuclear scope. For example,  $\mathbf{M}([x] \wedge \text{BOY}(x))$  introduces the variable  $x$  and requires it to store *all* and *only* the individuals satisfying  $\text{BOY}(x)$ :  $[x] \wedge \text{BOY}(x)$  ensures that we store in  $x$  only individuals that satisfy  $\text{BOY}(x)$  and **M** ensures that we cannot store more individuals in  $x$  and still satisfy  $\text{BOY}(x)$ . We can now provide a preliminary translation for modified numerals, which has the form in (20). For example, *Exactly three<sup>x</sup> boys left* is translated as in (21): we store in  $x$  all the boys that left and test that there are 3 such entities.

- (19)  $\llbracket \mathbf{M}(\phi) \rrbracket^{(G,H)} = \mathbb{T}$  iff  
 $\llbracket \phi \rrbracket^{(G,H)} = \mathbb{T}$  and there is no  $H'$  s.t.  $H \subsetneq H'$  and  $\llbracket \phi \rrbracket^{(G,H')} = \mathbb{T}$   
 (20) *exactly n*  $\exists x = n[\phi]$  ( $\psi := \mathbf{M}([x] \wedge \phi \wedge \psi) \wedge x = n$ )  
 (21)  $\exists x = 3[\text{BOY}(x)]$  ( $\text{LEAVE}(x) := \mathbf{M}([x] \wedge \text{BOY}(x) \wedge \text{LEAVE}(x)) \wedge x = 3$ )

We can further elaborate on the above sentence with *They<sub>x</sub> were hungry* and derive the intuitively-correct truth conditions for the resulting discourse. But we derive incorrect truth conditions for sentence (II):

- (22) a.  $\exists x = 3[\text{BOY}(x)] (\exists y = 5[\text{MOVIE}(y)] (\text{SEE}(x, y)))$   
 b.  $\mathbf{M}([x] \wedge \text{BOY}(x) \wedge \mathbf{M}([y] \wedge \text{MOVIE}(y) \wedge \text{SEE}(x, y)) \wedge y = 5) \wedge x = 3$

We do not derive the cumulative reading, true only in Figure 2, but the ‘reading’ true in both Figure 1 and Figure 2: the maximal number of boys that (between them) saw exactly 5 movies is 3. What we want is a translation that places the cardinality requirement  $y = 5$  contributed by the direct object outside the scope of the operator  $\mathbf{M}([x] \wedge \dots)$  contributed by the subject, as shown in (23a) or, equivalently, (23b) below. These formulas capture the cumulative reading of (II): we introduce the maximal set  $x$  of boys that saw a movie and the maximal set  $y$  of movies seen by a boy and test that there are 5 such movies and 3 such boys.

- (23) a.  $\mathbf{M}([x] \wedge \text{BOY}(x) \wedge \mathbf{M}([y] \wedge \text{MOVIE}(y) \wedge \text{SEE}(x, y))) \wedge y = 5 \wedge x = 3$   
 b.  $\mathbf{M}([x] \wedge \text{BOY}(x) \wedge [y] \wedge \text{MOVIE}(y) \wedge \text{SEE}(x, y)) \wedge y = 5 \wedge x = 3$

## 2.2 Post-Suppositions

To compositionally derive such a representation, we will take cardinality requirements to be part of a dimension of meaning separate from the asserted/at-issue

meaning (but closely integrated with it). We take them to be *post-suppositions*, i.e., tests on output contexts, as opposed to presuppositions, which are tests on input contexts.

Post-suppositions are formulas introduced at certain points in the interpretation that are passed on from local context to local context and that need to be satisfied only globally, relative to the final output context. Our notion of context is now a set of assignments  $G$  indexed with a set of tests  $\zeta$ , represented as  $G[\zeta]$ . All the above formulas are interpreted in the same way, except that we accumulate post-suppositions as we incrementally update our input context. Thus, our interpretation function is of the form  $\llbracket \cdot \rrbracket^{\langle G[\zeta], H[\zeta'] \rangle}$ , where  $\zeta \subseteq \zeta'$ .

We mark a test  $\phi$  as a post-supposition by superscripting it, as shown in (24) below; semantically, we completely ignore the input set of assignments  $G$  and simply add  $\phi$  to the set of tests  $\zeta$ . Such superscripted tests are post-suppositional in the sense that they are required to be true relative to the final output context. This is formalized by the new definition of truth in (25) below, which treats the formulas  $\psi_1, \dots, \psi_m$  as post-suppositions because they are tests performed on the final output set of assignments  $H$  (again: contrast this with presuppositions).

$$(24) \quad \llbracket \phi \rrbracket^{\langle G[\zeta], H[\zeta'] \rangle} = \mathbb{T} \text{ iff } \phi \text{ is a test, } G = H \text{ and } \zeta' = \zeta \cup \{\phi\}$$

$$(25) \quad \text{Truth: a formula } \phi \text{ is true relative to an input context } G[\emptyset], \text{ where } \emptyset \text{ is the empty set of tests, iff there is an output set of assignments } H \text{ and a (possibly empty) set of tests } \{\psi_1, \dots, \psi_m\} \text{ s.t. } \llbracket \phi \rrbracket^{\langle G[\emptyset], H[\{\psi_1, \dots, \psi_m\}] \rangle} = \mathbb{T} \text{ and } \llbracket \psi_1 \wedge \dots \wedge \psi_m \rrbracket^{\langle H[\emptyset], H[\emptyset] \rangle} = \mathbb{T}.$$

Modified numerals are interpreted as before, except that the cardinality requirement is a post-supposition. Numeral modifiers *exactly*, *at most*, *at least* etc. are functions taking a bare numeral as their argument and introducing (i) a maximization operator  $\mathbf{M}$  scoping over the random assignment and the restrictor and nuclear scope formulas and (ii) a post-supposition consisting of the cardinality requirement ordinarily contributed by the bare numeral. The resulting translation of sentence (II) in (27) below derives the correct cumulative truth conditions. Note that the four formulas in (27a,b,c,d) are truth-conditionally equivalent.

$$(26) \quad \text{exactly } n \quad \exists^{x=n}[\phi] (\psi) := \mathbf{M}([x] \wedge \phi \wedge \psi) \wedge^{x=n}$$

$$(27) \quad \begin{aligned} \text{a. } & \exists^{x=3}[\text{BOY}(x)] (\exists^{y=5}[\text{MOVIE}(y)] (\text{SEE}(x, y))) \\ \text{b. } & \mathbf{M}([x] \wedge \text{BOY}(x) \wedge \mathbf{M}([y] \wedge \text{MOVIE}(y) \wedge \text{SEE}(x, y)) \wedge^{y=5}) \wedge^{x=3} \\ \text{c. } & \mathbf{M}([x] \wedge \text{BOY}(x) \wedge \mathbf{M}([y] \wedge \text{MOVIE}(y) \wedge \text{SEE}(x, y))) \wedge^{y=5} \wedge^{x=3} \\ \text{d. } & \mathbf{M}([x] \wedge \text{BOY}(x) \wedge [y] \wedge \text{MOVIE}(y) \wedge \text{SEE}(x, y)) \wedge y = 5 \wedge x = 3 \end{aligned}$$

Just as before, if we elaborate on (II) with *They<sub>x</sub> liked them<sub>y</sub>*, we derive the correct interpretation for the entire discourse: every one of the three boys liked every movie he saw (and not the movies some other boy saw).

The proposed analysis of modified numerals involves three crucial ingredients: (i) evaluation pluralities (sets of assignments), (ii) maximization operators over such pluralities and (iii) post-suppositions and their unusual scoping behavior. The following three sections provide independent evidence for each of them.

### 3 Universal Quantifiers

The fact that we use evaluation pluralities enables us to also account for cumulative readings of universals (Schein 1993, Kratzer 2000, Champollion 2009) <sup>2</sup>

Consider the sentence in (28) below (from Kratzer 2000). Its cumulative reading is: there are three editors such that each of them caught at least one mistake and every mistake was caught by at least one of the three editors. We translate distributive universal quantification as shown in (29): we introduce the set of all individuals  $x$  that satisfy the restrictor  $\phi$  by means of the maximization operator  $\mathbf{M}x$  and we check that *each* of these individuals also satisfies the nuclear scope  $\psi$  by means of the distributivity operator  $\delta$ .

- (28) Three <sup>$x$</sup>  copy editors (between them) caught every <sup>$y$</sup>  mistake in the manuscript.  
 (29)  $\forall x[\phi] \delta(\psi) := \mathbf{M}x(\phi) \wedge \delta(\psi)$

$\mathbf{M}x$  is the selective counterpart of the unselective, adverbial  $\mathbf{M}$ : unselective  $\mathbf{M}$  maximizes over sets of assignments  $H$ , while selective  $\mathbf{M}x$  maximizes over sets of individuals  $H(x)$  ('selective' and 'unselective' in the sense of Lewis 1975). Using unselective maximization for modified numerals is justified by the fact that their modifier can be non-adjacent/adverbial, as shown by the examples below.

- (30)  $\llbracket \mathbf{M}x(\phi) \rrbracket^{G[\zeta], H[\zeta']}] = \mathbb{T}$  iff  $\llbracket [x] \wedge \phi \rrbracket^{G[\zeta], H[\zeta']}] = \mathbb{T}$  and there is no  $H'$  s.t.  $H(x) \subsetneq H'(x)$  and  $\llbracket [x] \wedge \phi \rrbracket^{G[\zeta], H'[\zeta']}] = \mathbb{T}$ .  
 (31) Three boys saw five movies, exactly/precisely/at (the) most.  
 (32) The league limits teams to playing two games in a row – or, at the most, four games in five days, NBA spokesman Tim Frank says. <sup>3</sup>

The distributivity operator  $\delta$  in (33) (based on Brasoveanu 2008) says that we distributively update a set of assignments  $G$  with a formula  $\phi$  by updating each singleton set  $\{g\} \subseteq G$  with  $\phi$  and taking the union of the resulting output sets of assignments  $K$ . In addition,  $\delta$  discharges all post-suppositions contributed by the formula in its scope. Thus, just like presuppositions, post-suppositions are not always satisfied globally, but can be satisfied/discharged at intermediate points in the semantic composition, i.e., in more local output contexts.

- (33)  $\llbracket \delta(\phi) \rrbracket^{G[\zeta], H[\zeta']}] = \mathbb{T}$  iff  $\zeta = \zeta'$  and there exists a partial function  $\mathcal{F}$  from assignments  $g$  to sets of assignments  $K$ , i.e., of the form  $\mathcal{F}(g) = K$ , s.t.  
 (i)  $G = \mathbf{Dom}(\mathcal{F})$  and  $H = \bigcup \mathbf{Ran}(\mathcal{F})$  and (ii) for all  $g \in G$ , there is a (possibly empty) set of formulas  $\{\psi_1, \dots, \psi_m\}$  s.t.  
 $\llbracket \phi \rrbracket^{\{g\}[\zeta], \mathcal{F}(g)[\zeta \cup \{\psi_1, \dots, \psi_m\}]]} = \mathbb{T}$  and  $\llbracket \psi_1 \wedge \dots \wedge \psi_m \rrbracket^{\mathcal{F}(g)[\zeta], \mathcal{F}(g)[\zeta]} = \mathbb{T}$

The translation of sentence (28) above is provided below: we introduce a set  $x$  of three editors and the set  $y$  of all mistakes and check that, for every assignment  $h$  in the resulting output state  $H$ , the editor  $h(x)$  caught the mistake  $h(y)$ .

<sup>2</sup> I am indebted to Lucas Champollion for many helpful comments on this subsection.

<sup>3</sup> From the Corpus of Contemporary American English, [www.american corpus.org](http://www.american corpus.org)

- (34) a.  $\exists x[x = 3 \wedge \text{EDITOR}(x)] (\forall y[\text{MISTAKE}(y)] \delta(\text{CATCH}(x, y)))$   
 b.  $[x] \wedge x = 3 \wedge \text{EDITOR}(x) \wedge \mathbf{M}y(\text{MISTAKE}(y)) \wedge \delta(\text{CATCH}(x, y))$

The distributivity operator  $\delta$  is semantically vacuous in (34) – but not always. Consider the example below, from Kratzer (2000). This sentence does not have a cumulative reading to the effect that, between them, the editors caught a total of 500 mistakes. Its only reading is the distributive one: every editor is such that s/he caught 500 mistakes. We derive the distributive reading if the universal takes scope over the numeral *every*  $>>$  500. That is, cumulative readings are possible with universals only if they have narrow scope relative to the numerals they ‘cumulate’ with. As long as the non-surface scope 500  $>>$  *every* is blocked for sentence (35), we correctly derive the unavailability of the cumulative reading.

- (35) Every<sup>x</sup> copy editor caught 500<sup>y</sup> mistakes in the manuscript.  
 (36)  $\forall x[\text{EDITOR}(x)] \delta(\exists y[y = 500 \wedge \text{MISTAKE}(y)] (\text{CATCH}(x, y)))$

We also account for mixed cumulative-distributive sentences, e.g., *Three video games taught every quarterback two new plays* (Schein 1993): *every quarterback* is related cumulatively to *three video games* (a total of three video games taught all the quarterbacks), but distributes in the usual way over *two new plays* (every quarterback learned two possibly different plays). This automatically follows in our system if we preserve the surface-scope relations *three*  $>>$  *every*  $>>$  *two*.

Finally, we capture the distributive reading of (1) by means of the operator  $\delta$ : distributive modified numerals have a  $\delta$  operator over their nuclear scope.

- (37)  $\exists x^n[\phi] \delta(\psi) := \mathbf{M}([x] \wedge \phi \wedge \delta(\psi)) \wedge x^n$   
 (38)  $\exists x^=3[\text{BOY}(x)] \delta(\exists y^=5[\text{MOVIE}(y)] \delta(\text{SEE}(x, y)))$

## 4 Implicatures

Analyzing modified numerals by means of a maximization operator over evaluation pluralities enables us to account for the independent observation that modified numerals do not trigger scalar implicatures, unlike bare numerals/indefinites. This is because the operator  $\mathbf{M}$  contributed by modified numerals effectively eliminates referential uncertainty. In any given world, the variable introduced by a modified numeral can be associated with only one set of values: the set of all entities satisfying the restrictor and nuclear scope of the modified numeral. This is shown by the contrast below (from Umbach 2006).

- (39) {Two/#At least two} boys were selling coke. They were wearing black jackets. Perhaps there were others also selling coke, but I didn’t notice.

If there are more than two boys selling coke, the variable introduced by the bare numeral *two* can take different sets of two boys as values, i.e., the output contexts obtained after the update with a bare numeral may assign different sets of values to the variable contributed by the bare numeral. In contrast, the variable introduced by *at least two* has only one possible value: the set of all boys

selling coke. In any given world, all output contexts obtained after the update with a modified numeral assign the same value to the variable it contributes.

Thus, scalar implicatures are triggered by items that allow for referential indeterminacy/uncertainty. It is this semantic uncertainty that kicks off the pragmatic inferential process resulting in the addition of scalar implicatures.

But referential certainty is distinct from epistemic certainty. Suppose that our contexts are not simply sets of assignments  $G, H, \dots$ , but pairs of a world and a set of assignments  $\langle w, G \rangle, \langle w', H \rangle, \dots$  (in the spirit of Heim 1982). The *information state* at any point in discourse consists of all the pairs that are still live options, i.e., that are compatible with all the previous updates. Referential uncertainty is encoded by the second member of such pairs. Epistemic uncertainty is encoded by the first member of the pairs, i.e., by the set of worlds in the current information state – aka the current Context Set (Stalnaker 1978).

Modified numerals are referentially determined, but epistemically uncertain. If we fix the world, the variable contributed by the modified numeral has only one value, but this value may vary from world to world. Hence, we can use them (as opposed to their bare counterparts) only if we are epistemically uncertain about the cardinality of the maximal set of entities introduced by them.

This is the reason for the modal readings of indicative sentences with modified numerals (no need for insertion of covert modals, as in Nouwen 2009 and references therein). *Jasper invited maximally 50 people to his party* (from Nouwen 2009) is felicitous only if the speaker is uncertain with respect to the cardinality of the set of invited people (hence the ‘range of values’ interpretation). So, if the speaker does not know how many people Jasper invited, it is unacceptable to continue with: *43, to be precise*. The same pragmatic infelicity can arise intrasententially: *#A hexagon has at most/maximally/up to 11 sides* (Nouwen 2009) is infelicitous if we know what the word *hexagon* means.

Finally, given their epistemic uncertainty, modified numerals trigger epistemic implicatures of the kind proposed in Buring (2008) for *at least*.

## 5 Modals and Modified Numerals

This section provides independent evidence for the analysis of modified numerals in terms of post-suppositions. The unusual scoping behavior of post-suppositions and their interaction with distributivity enables us to capture the scopal interactions between modified numerals and modals. This is a novel result that solves an outstanding problem for the current analyses based on standard assumptions about the semantics of minimizers/maximizers and necessity/possibility modals (see Nouwen 2009 and references therein for more discussion). We provide the representations for two typical sentences (from Nouwen 2009) instantiating this problem and leave a more detailed discussion for another occasion.

Necessity modals are analyzed as distributive universal quantifiers in the modal domain; in (41) below,  $R$  is a contextually-provided accessibility relation and  $R_{w^*}(w)$  is interpreted as:  $w$  is an  $R$ -accessible world from the actual world  $w^*$ . The reading of sentence (40) we are after is: the minimum number of books that

Jasper is *allowed* to read is 10. The update in (42) captures this reading: each world  $w$  that is  $R$ -accessible from the actual world  $w^*$  is such that, if we store in  $y$  all the books Jasper read, the cardinality of the set of books is at least 10. That is: Jasper reads at least 10 books in every deontically-ideal world  $w$ .

- (40) Jasper <sup>$x$</sup>  should <sup>$w$</sup>  read at least ten <sup>$y$</sup>  books (to please his mother).
- (41)  $\mathbf{NEC}w(\phi) := \mathbf{M}([w] \wedge R_{w^*}(w)) \wedge \delta(\phi)$
- (42) a.  $\mathbf{NEC}w(\exists x[x = \text{JASPER}] (\exists y \geq_w^{10} [\text{BOOK}_w(y)] (\text{READ}_w(x, y))))$   
 b.  $\mathbf{M}([w] \wedge R_{w^*}(w)) \wedge \delta([x] \wedge x = \text{JASPER} \wedge \mathbf{M}([y] \wedge \text{BOOK}_w(y) \wedge \text{READ}_w(x, y)) \wedge y \geq_w^{10})$

We also account for maximal permissions like (43) below, interpreted as: the maximum number of people Jasper is allowed to invite is 10. We take possibility modals to be the counterparts of a modified numeral in the modal domain that contributes a non-singleton cardinality requirement. The maximization operator  $\mathbf{M}$  over worlds is independently justified by modal subordination (Roberts 1989), e.g., *A wolf might come in. It would eat Jasper first* is interpreted as: for *any* epistemic possibility of a wolf coming in, the wolf eats Jasper first. The update in (45) introduces all the worlds  $w$  that are  $R$ -accessible from the actual world  $w^*$  and such that Jasper invites some people in  $w$ . For each such world  $w$ ,  $y$  stores all the people invited by Jasper. Finally, we check that there is more than one world  $w$  and that the cardinality of the set  $y$  in *each* world  $w$  is at most 10.

- (43) Jasper <sup>$x$</sup>  is allowed <sup>$w$</sup>  to invite at most ten <sup>$y$</sup>  people.
- (44)  $\mathbf{POS}w(\phi) := \exists^{w>1} [R_{w^*}(w)] (\phi) = \mathbf{M}([w] \wedge R_{w^*}(w) \wedge \phi) \wedge^{w>1}$
- (45) a.  $\mathbf{POS}w(\exists x[x = \text{JASPER}] (\exists y \leq_w^{10} [\text{PERSON}_w(y)] (\text{INVITE}_w(x, y))))$   
 b.  $\mathbf{M}([w] \wedge R_{w^*}(w) \wedge [x] \wedge x = \text{JASPER} \wedge [y] \wedge \text{PERSON}_w(y) \wedge \text{INVITE}_w(x, y)) \wedge_{y \leq_w^{10} \wedge w>1}$

## 6 Conclusion

We introduced a framework that distinguishes evaluation plurality (sets of assignments) from domain plurality (non-atomic individuals). The maximization operator  $\mathbf{M}$  and the distributivity operator  $\delta$  are to evaluation pluralities what the familiar Link-style sum and distributivity operators are to domain pluralities. Cumulativity is just non-distributivity with respect to evaluation pluralities, while collectivity is just non-distributivity with respect to domain pluralities.

Modified numerals are maximal and introduce cardinality post-suppositions, which are constraints on output contexts – in contrast to presuppositions, which constrain input contexts. Just as presuppositions, post-suppositions can be satisfied/discharged non-globally, e.g., in the scope of distributivity operators. Post-suppositions are distinct from regular at-issue meaning with respect to their evaluation order: they can constrain the final, global output context. The exceptional scoping behavior of post-suppositions enables us to account for cumulative readings of non-increasing modified numerals and for their interaction with modal verbs. The referential maximality of modified numerals accounts for the fact that they do not trigger scalar implicatures, but only epistemic implicatures.

## References

- van Benthem, J.: *Essays in Logical Semantics*. Kluwer, Dordrecht (1986)
- van den Berg, M.: *Some Aspects of the Internal Structure of Discourse. The Dynamics of Nominal Anaphora*. PhD dissertation, Univ. of Amsterdam (1996)
- Brasoveanu, A.: Donkey Pluralities. *Linguistics and Philosophy* 31, 129–209 (2008)
- Büring, D.: The Least *at least* Can Do. In: Chang, C.B., Haynie, H.J. (eds.) *Proceedings of WCCFL 26, Cascadilla*, pp. 114–120 (2008)
- Champollion, L.: Cumulative Readings of *Every* Do Not Provide Evidence for Events and Thematic roles. In: Aloni, M., Bastiaanse, H., de Jager, T., Schulz, K. (eds.) *Amsterdam Colloquium 2009. LNCS (LNAI)*, vol. 6042, pp. 213–222. Springer, Heidelberg (2009)
- Chierchia, G., Fox, D., Spector, B.: The Grammatical View of Scalar Implicatures. In: Portner, P., et al. (eds.) *Handbook of Semantics*. de Gruyter, Berlin (to appear)
- Constant, N.: *English Rise-Fall-Rise: A study in the Semantics and Pragmatics of Intonation*. MA thesis, UC Santa Cruz (2006)
- Farkas, D.F.: Varieties of Indefinites. In: Jackson, B. (ed.) *Proceedings of SALT XII*, pp. 59–83. CLC (2009)
- Ferreira, M.: Scope Splitting and Cumulativity. In: Nouwen, R., Dotlačil, J. (eds.) *Proceedings of the ESSLI Workshop on Quantifier Modification* (2007)
- Geurts, B., Nouwen, R.: The Semantics of Scalar Modifiers. *Language* 83, 533–559 (2007)
- Heim, I.: *The Semantics of Definite and Indefinite Noun Phrases*, PhD dissertation, UMass (Published in 1988 by Garland) (1982)
- Kratzer, A.: The Event Argument and the Semantics of Verbs. UMass. ms (2000), <http://semanticsarchive.net>
- Krifka, M.: At least Some Determiners Aren't Determiners. In: Turner, K. (ed.) *The Semantics/Pragmatics Interface from Different Points of View*, pp. 257–291. Elsevier, Amsterdam (1999)
- Landman, F.: *Events and Plurality*. Kluwer, Dordrecht (2000)
- Lauer, S.: *Free relatives with -ever: Meaning and Use*. Stanford, ms (2009)
- Lewis, D.: Adverbs of Quantification. In: Keenan, E. (ed.) *Formal Semantics of Natural Language*, pp. 3–15. Cambridge University Press, Cambridge (1975)
- Nouwen, R.: Two Kinds of Modified Numerals. To appear in *Semantics and Pragmatics* (2009)
- Robaldo, L.: Distributivity, Collectivity and Cumulativity in terms of (In)dependence and Maximality. Univ. of Turin, ms (2009)
- Roberts, C.: Modal Subordination and Pronominal Anaphora in Discourse. *Linguistics and Philosophy* 12, 683–721 (1989)
- Schein, B.: *Plurals and Events*. MIT Press, Cambridge (1993)
- Schwarzschild, R.: *Pluralities*. Kluwer, Dordrecht (1996)
- Stalnaker, R.: Assertion. *Syntax and Semantics* 9, 315–332 (1978)
- Szabolcsi, A.: Strategies for Scope Taking. In: Szabolcsi, A. (ed.) *Ways of Scope Taking*, pp. 109–154. Kluwer, Dordrecht (1997)
- Umbach, C.: Why do modified numerals resist a referential interpretation? In: *Proceedings of SALT XV*, pp. 258–275. CLC (2006)

# Cumulative Readings of *Every* Do Not Provide Evidence for Events and Thematic Roles

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**Abstract.** An argument by Kratzer (2000) based on Schein (1986, 1993) does not conclusively show that events and thematic roles are necessary ingredients of the logical representation of natural language sentences. The argument claims that cumulative readings of *every* can be represented only with these ingredients. But scope-splitting accounts make it possible to represent cumulative readings of *every* in an eventless framework. Such accounts are motivated by obligatory reconstruction effects of *every* and by crosslinguistic considerations. Kratzer proposes that *agent* but not *theme* occurs in the logical representation of sentences because this allows her to model subject-object asymmetries in the distribution of cumulative *every*. But the reason for these asymmetries seems to be that *every* must be c-commanded by another quantifier in order to cumulate with it, no matter what its thematic role is. So the distribution of cumulative *every* does not provide support for Kratzer's proposal.

## 1 Introduction

The question whether events and thematic roles are part of the logical representation of natural language sentences has been debated for over forty years. Early formal semantic work, as well as some modern authors, simply represents the meaning of verbs with  $n$  syntactic arguments as  $n$ -ary relations. A transitive verb, for example, is assumed to denote a two-place relation. Against this, Davidson (1967) argued that verbs denote relations between events and their arguments, so that a transitive verb denotes a three-place relation. Once events have been introduced, it becomes possible to see verbs as predicates over events,

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**Table 1.** A summary of the positions in event semantics

Position	Verbal denotation	Example: Brutus stabbed Caesar
Traditional	$\lambda y \lambda x [\text{stab}(x, y)]$	$\text{stab}(b, c)$
Classical Davidsonian	$\lambda y \lambda x \lambda e [\text{stab}(e, x, y)]$	$\exists e [\text{stab}(e, b, c)]$
Neo-Davidsonian	$\lambda e [\text{stab}(e)]$	$\exists e [\text{stab}(e) \wedge \text{agent}(e, b) \wedge \text{theme}(e, c)]$
Asymmetric	$\lambda y \lambda e [\text{stab}(e, y)]$	$\exists e [\text{agent}(e, b) \wedge \text{stab}(e, c)]$

and to express the relationship between events and their arguments by separate predicates, i.e., thematic roles. This is the Neo-Davidsonian position (e.g. Parsons, 1990; Schein, 1993). Finally, Kratzer (2000) argues for an asymmetric position, according to which only agents are represented as thematic roles. The positions are illustrated in Table 1.

Over the course of the years, events and thematic roles have grown to be much more than mere notations.<sup>1</sup> For example, many theories that resort to the thematic role *agent* make specific claims about the semantic content of agenthood. But the choice between the representations in Table 1 has a more basic consequence. Because they use a larger number of relations, Neo-Davidsonian and asymmetric representations offer an additional degree of freedom: They make it possible to codify meanings in which one argument’s thematic role modifies a different event variable than the verb does. Such configurations are obviously impossible to write down without the help of events and thematic roles. Schein (1993) calls the property of such sentences *essential separation*.

The argument presented in Kratzer (2000), building on work by Schein (1986, 1993), holds that cumulative readings of *every* involve essential separation. While Schein’s entire argument is too intricate to be addressed adequately here, my goal is to refute the specific part that relates to the word *every*. I will show how these readings can, in fact, be adequately captured using an eventless representation that does not use explicit roles. The crux of the argument bears on how the meaning of *every* is adequately represented. There are many ways to adapt eventless frameworks to the task at hand; for example, see Brasoveanu (2009) for a dynamic framework. I will stay close to the framework used in Kratzer (2000) to make the comparison as easy as possible. I will focus on the parallels with existing approaches to quantification, rather than on technical aspects.

<sup>1</sup> In this paper, I talk of models and logical representation languages only for convenience. I don’t make any ontological claims about their existence. Readers who doubt that we should ascribe existence to models or logical representation languages in the first place should interpret the claims about whether events and thematic roles “exist” as claims about whether natural language is rich enough to express meanings which, if we choose to represent them formally, go beyond what can be expressed without using notational devices such as event variables and thematic relations.

Following Kratzer, I use the algebraic semantic framework of plurals introduced in Link (1983).<sup>2</sup> Since Schein not only argues for events and thematic roles but also, separately, against Link's framework, let me briefly justify my choice. As Schein points out, his two arguments are logically independent of each other, so his argument for events and roles can be recast in mereological terms, and this is in fact what Kratzer (2000) does.<sup>3</sup> I have two reasons for following her example. First, this makes it easier to compare my approach to Kratzer's. Second, I will argue that cumulative readings of *every* can be modeled using standard accounts of cumulative readings such as Krifka (1986) and Sternefeld (1998), and these accounts happen to be formulated in Link's framework. That said, choosing this framework is not essential for my purposes as long as the domain of individuals is grounded in *atoms* or individuals that have no parts. Under this standard assumption, join semilattices are isomorphic to an appropriate kind of set-theoretic lattice; see Schwarzschild (1996) for an example. So everything I say about individuals can be reformulated in a set-theoretic framework.

## 2 Schein's Argument as Presented in Kratzer

Schein's case for events is very intricate and relies on complicated sentences involving three or more quantifiers. In this short paper, I only consider the part of his argument which involves cumulative readings of *every*. This section summarizes Kratzer's exposition of this part. It is based on the following sentence:

- (1) Three copy editors caught every mistake in the manuscript.

Kratzer claims that (1) has a reading that can be paraphrased as "Three copy editors, between them, caught every mistake in the manuscript." In this reading, there are three copy editors, each of them caught at least one mistake, and every mistake was caught by at least one copy editor.<sup>4</sup> If the subject of (1) is understood distributively, neither the surface scope reading ("Each of three copy editors caught every mistake") nor the inverse scope reading ("Each mistake is

<sup>2</sup> In algebraic frameworks, the domains of individuals and, if present, of events are each partially ordered by a mereological *part-of* relation  $\sqsubseteq$ . On the basis of  $\sqsubseteq$ , an operation  $\oplus$  is defined that maps entities onto their *sum*, or least upper bound.  $\sqsubseteq$  orders the domains of individuals and events each into a complete join semilattice; in other words, the sum operation is defined for arbitrary nonempty subsets of these domains. Singular common nouns denote predicates over atomic individuals (individuals that have no parts); plural common nouns hold of sums. The pluralization operator, written  $*$ , closes predicates  $P$  under sum, i.e.  $*P$  is the smallest set such that (i) if  $P(X)$  then  $*P(X)$ ; (ii) if  $*P(X_1)$  and  $*P(X_2)$  then  $*P(X_1 \oplus X_2)$ . See Link (1998).

<sup>3</sup> For more on Schein's argument against sums, see Link (1998) and Schein (2006).

<sup>4</sup> Not all native speakers I consulted report that Kratzer's reading is in fact available from (1), though it seems present for everybody in the paraphrase that adds *between them*. In the following, I will grant that Kratzer's factual claim about (1) is correct. In any case, it is possible that her argument could also be based on that paraphrase, once the semantics of *between them* has been worked out.

such that it was caught by each of three copy editors”) is equal to Kratzer’s reading, because unlike it, they both entail that each mistake was caught by more than one copy editor. One possible analysis would be to claim that in Kratzer’s reading, the subject is understood collectively, so that any mistake that is caught by one of the editors counts as being caught by all three of them collectively (a “team credit” analysis). But, she argues, sentence (1) is true even if the editors worked independently of each other, which is incompatible with the usual understanding of the collectivity notion. In particular, (1) entails that every copy editor found at least one mistake, while collective readings do not always license this entailment. For additional arguments against a team-credit analysis, see Bayer (1997). For these reasons, I will not rely on team credit.

My strategy consists in analyzing Kratzer’s reading as a cumulative reading, the kind of reading which occurs in *600 Dutch firms own 5000 American computers* (Scha, 1981). It expresses that there are 600 firms and 5000 computers, each firm owns at least one computer, and each computer is owned by at least one firm. Following Krifka (1986) and Sternefeld (1998), this reading can be represented as follows, without events or thematic roles:

$$(2) \quad \exists X [600\text{-firms}(X) \wedge \exists Y [5000\text{-computers}(Y) \wedge \text{**own}(X, Y)]]$$

This representation makes use of the following ingredients and conventions. Uppercase letters are used for variables and constants that denote either atoms or sums, and lowercase letters for those that denote atoms. I use shorthands for the noun phrase denotations: for example, the predicate *600-firms* is true of any sum of firms whose cardinality is 600. The *cumulation* operator \*\*, a generalization of the pluralization operator from footnote 2, has been defined in various ways in the literature (see e.g. Beck and Sauerland, 2000). The definition I use is from Sternefeld (1998): Given a complete join semilattice  $\langle S, \sqsubseteq \rangle$  and a binary relation  $R \subseteq S \times S$ , \*\* $R$  is the smallest relation such that (i) if  $R(X, Y)$  then \*\* $R(X, Y)$ ; (ii) if \*\* $R(X_1, Y_1)$  and \*\* $R(X_2, Y_2)$  then \*\* $R(X_1 \oplus X_2, Y_1 \oplus Y_2)$ .

Cumulative readings express information about the cardinalities of the minimal witness sets associated with the quantifiers involved (Szabolcsi, 1997). Standard representations of *every* have problems with this kind of configuration (Roberts, 1987). For example, interpreting “every mistake” in situ as  $\lambda P. \forall x. \text{mistake}(x) \rightarrow P(x)$  leads to the interpretation in (3). But this is just the surface scope reading.

$$(3) \quad \exists Y [\text{three-copy-editors}(Y) \wedge \forall x [\text{mistake}(x) \rightarrow \text{**catch}(Y, x)]]$$

As Schein and Kratzer observe, if we adopt a Neo-Davidsonian position, the cumulative reading can nonetheless be represented adequately. Their idea is that once we have the agent role at our disposal, we can represent (1) roughly as “There is a sum  $E$  of mistake-catching events, whose agents sum up to three editors, and every mistake was caught in at least one of these events”, as in (4):

- (4)  $\exists E \exists X [\text{three-copy-editors}(X) \wedge \text{**agent}(E, X)$   
 $\wedge \forall y [\text{mistake}(y) \rightarrow \exists e [e \sqsubseteq E \wedge \text{catch}(e, y)]]$   
 $\wedge \exists Y [* \text{mistake}(Y) \wedge \text{**catch}(E, Y)]]$

Following Schein, Kratzer takes this fact to show that we need to have at least the relation *agent* at our disposal in our logical representation.

### 3 Modeling Cumulative *every* without Events

The part of Schein's argument that Kratzer presents and embraces is based on the assumption that the adequate translation of *every mistake* is in terms of a universal quantifier. The difficulty arises from the fact that the cumulative reading of (1) expresses something about the set or sum of all mistakes. But the universal quantifier does not give us a handle on this object, because it holds of any set that contains every mistake and possibly some non-mistakes.

The first step towards a solution was taken in Landman (2000), who shifts *every mistake* to a referential interpretation, one that denotes the sum or group of all mistakes, written  $\sigma x.\text{mistake}(x)$ .<sup>5</sup> On this view, *every mistake* is synonymous with *the mistakes*, if we disregard the fact that the latter sometimes allows nonmaximal interpretations (Krifka, 1996; Malamud, 2006).

At first sight, this suggestion faces an obvious problem: The distribution of *every mistake* is more restricted than the one of *the mistakes*. As is well known, *every* forces distributivity over its argument position (Kroch, 1974):

- (5) a. #Every soldier surrounded the castle. (*only distributive*)  
 b. The soldiers surrounded the castle. (*distributive or collective*)

This problem can be overcome by assuming that the restrictor of *every* is interpreted both in its base position as a restriction on the values of its argument position, and above a distributivity (\*) or cumulation (\*\*) operator, where it is the input to sum formation.<sup>6</sup>

Evidence that supports these assumptions comes from multiple sources. First, the assumption that it is not *every* itself but a separate operator that contributes distributivity is shared, in one way or another, by many authors for a variety of reasons: see e.g. Matthewson (2001), Sauerland (2003), Johnson (2007), and the papers in Szabolcsi (1997). This assumption finds crosslinguistic support in Chinese, where distributive readings require the presence of a VP modifier *dou*. For this reason, *dou* is often seen as an overt realization of the \* operator (Lee,

<sup>5</sup> Alternatively, the shift could be to a predicative interpretation, one that holds precisely of the sum of all mistakes. This solution is independently needed for variants *every other* and *almost every* which do not have a unique minimal witness. It could be exploited for explaining in terms of type mismatch why *every* is never interpreted in situ. For clarity of exposition, I stick to the referential interpretation of *every*.

<sup>6</sup> The granularity of *every* is determined by its complement and not by atomicity, as pointed out by Schwarzschild (1996), using examples like *Every three houses formed a block*. Here, quantification is over sums of three entities, not over atomic entities. So the level of granularity is sensitive to the restrictor of *every*.

[1986; Liu, 1990; Lin, 1998]). Whenever *meige*, the Chinese equivalent of *every*, is used, the VP modifier *dou* must be present; and when *meige* originates inside the VP, it must move out and take scope over *dou*, just as in the present account.<sup>7</sup>

Second, evidence that the complement of *every* is also interpreted in situ comes from obligatory reconstruction effects, i.e. cases in which a constituent behaves as if it was taking scope in two different places at once. Reconstruction effects attested specifically with *every* are well documented in various constructions. Examples are Condition C of binding theory ([Chomsky, 1993; Fox, 1999]) and antecedent-contained deletion ([Sauerland, 1998, 2004]).

Technically, the concept that restrictors of quantifiers are interpreted in several places can be expressed in any number of ways: syntactically, for example, by creating multiple copies of phrases ([Engdahl, 1986; Chomsky, 1993]) or multiply dominated phrases ([Johnson, 2007]); or semantically, by encapsulating the contribution of the restrictor into objects that the interpretation function makes accessible in several places, such as choice functions ([Sauerland, 2004]) or sets of assignments ([Brasoveanu, 2009]). Rather than comparing all these approaches, I simply choose the proposal with the lowest types and the least departure from ordinary syntactic assumptions, both for lack of space and because this makes the interaction with the cumulation operator easier to grasp. I adopt the proposal by [Fox (1999, 2002)], according to which in situ copies are interpreted by a special semantic rule, shown here in simplified form:

$$(6) \quad \text{Trace Conversion Rule: } [[(\text{Det})N]_i]^g = \iota y. [[N]^g(y) \wedge y = g(i)]$$

With Trace Conversion, the lower copy of a DP *every N* which bears the index *i* is interpreted as “the *N* which is *i*”. The contribution of the determiner in the lower copy is ignored. Similarly to what is assumed in event semantics ([Landman, 2000]), I assume that all quantifiers (even those in subject position) move before they are interpreted, so that trace conversion always applies. On *three copy editors*, the effect of trace conversion is vacuous, so I don’t show it.<sup>8</sup>

As an example, “Every dog barks” is interpreted as in (7). Here and below, the parts contributed by “every *N*” are underlined. The cumulative reading of (1) can be represented as in (8).<sup>9</sup>

$$(7) \quad \sigma x. \text{dog}(x) \in * \lambda X [\text{barks}(\iota x'. \text{dog}(x') \wedge x' = X)]$$

$$(8) \quad \exists X [\text{three-copy-editors}(X) \wedge \\ \langle X, \sigma y. \text{mistake}(y) \rangle \in ** \lambda X' \lambda Y [\text{catch}(X', \iota y'. \text{mistake}(y') \wedge y' = Y)]]$$

<sup>7</sup> Indeed, [Lin (1998)] makes a similar proposal for *meige* as Landman does for *every*.

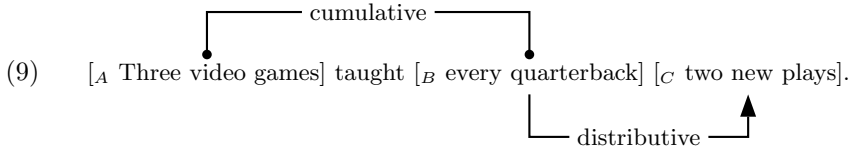
<sup>8</sup> Alternatively, one can assume in the style of [Matthewson (2001)] that *every N* is interpreted as a covert variant of the partitive construction *each of the Ns*, and furthermore that *the Ns* can raise out of that construction to take part in a cumulative relation. This way, subject quantifiers can be interpreted in situ.

<sup>9</sup> The \*\* operator makes sure that the cumulated relation applies to every member of the two sums. Here, it enforces that each of the three editors was involved in catching mistakes. This avoids the “leakage” problem faced by [Bayer (1997)]’s account.

This is provably equivalent to Kratzer's representation in (4), provided that  $catch(x, y)$  holds whenever  $\exists e [agent(e, x) \wedge catch(e, y)]$  and that (at least) the second argument of *catch* is always atomic.<sup>10</sup> Note that the requirement that  $y'$  range over singular mistakes effectively restricts  $Y$  to atomic values.

## 4 Mixed Cumulative-Distributive Readings

The sentences originally discussed by Schein (1993) are more complicated than Kratzer's in several respects. For one thing, they exhibit mixed distributive-cumulative configurations, such as in the following example:



The relevant reading of this sentence is the one in which there is a given set of three videos which between them were responsible for the fact that every quarterback learned two new plays. The solution from the previous section works here as well. We can represent the reading in an eventless framework as follows:

$$(10) \quad \begin{aligned} &\exists X [\text{three-video-games}(X) \\ &\quad \wedge \langle X, \sigma y. \text{quarterback}(y) \rangle \in ** \lambda X' \lambda Y [\exists Z \text{ two-new-plays}(Z) \\ &\quad \wedge *** \text{taught}(X', \iota y'. \text{quarterback}(y') \wedge y' = Y, Z)]] \end{aligned}$$

In this formula, the exhaustive component of “every quarterback” stands in a cumulative relation with “three video games”, while its distributive component makes sure that *teach* relates individual quarterbacks to sums of two plays each. \*\*\* is the ternary equivalent of \*\*. Two instances of cumulation are needed: the higher one to let *three video games* and *every quarterback* cumulate, and the lower one to reflect the lack of scopal dependency between any given set of two plays and the three video games. In other words, sentence (9) does not express for any set of two plays how many of the three video games taught that set.<sup>11</sup>

<sup>10</sup> This assumption is independently necessary to model the fact that if two mistakes A and B get caught, this always implies that A gets caught and B gets caught. It is necessary for the proof because Kratzer's representation in (4) does not otherwise exclude the technical possibility that the sum event E contains some catching events in which somebody catches a sum of mistakes which do not get caught individually.

<sup>11</sup> Schein (1993) also considers modified numerals, which are beyond the scope of this paper. Following Schein, replacing “two” by “exactly two” in (9) entails that each quarterback was taught exactly two new plays. This is admittedly hard to represent in terms of a ternary predicate *taught*, because for any given quarterback, the first argument of this predicate would have to be the sum of all video games that taught him plays. Even approaches such as Krifka (1999) or Brasoveanu (2009), which deal with cumulative readings of modified numerals, do not give us access to that sum.

## 5 Structural Asymmetries in Cumulative Readings

Recall that Kratzer's larger goal is to argue for a representation in which only the agent role, but not the theme role, is expressed as a separate relation. Kratzer is aware that the relevant reading of (1) can be described as a cumulative reading, but she prefers not to model it as such, observing that cumulative readings are less readily available with *every* in general. She illustrates this with the following:

- (11) a. Three copy editors caught every mistake. (= (1)) *Cumulative: OK*  
 b. Every copy editor caught 500 mistakes. *Cumulative: \**  
 c. 500 mistakes were caught by every copy editor. *Cumulative: \**

Cumulative readings are absent from both (11b) and (11c). In these examples, *every* is in agent position. Based on this, she generalizes that *every* can take part in cumulative readings only when it is not in agent position, cf. (11a). This is indeed predicted by the asymmetry in her representation.

The following minimal pair is a counterexample. For Bayer (1997), (12a) is “clearly bizarre”, since scripts cannot be written more than once. But he reports that (12b) has a reading where every screenwriter in Hollywood contributed to the writing of the movie. We can model this by assuming that *Gone with the Wind* denotes a sum entity. Then if we represent (12b) as a cumulative reading as in Sect. 3, it entails that every screenwriter wrote some part of this sum.

- (12) a. Every screenwriter in Hollywood wrote *Gone with the Wind*.  
 b. *Gone with the Wind* was written by every screenwriter in Hollywood.

Since *every* is in agent position in both cases, the asymmetry is unexpected on Kratzer's hypothesis. They should both be equally bizarre. This minimal pair suggests that what blocks the cumulative reading of certain *every*-phrases is not their thematic role. I propose instead that a noun phrase headed by *every* cannot cumulate with anything it c-commands. This constraint explains why *every* cannot cumulate in (11b) and (12a). It also predicts, contra Kratzer, that *every* as a passivized object noun phrase should be unable to cumulate even when it is not the agent. The following minimal pair confirms this prediction:

- (13) a. The Fijians and the Peruvians won every game. (Zweig, 2008)  
 b. Every game was won by the Fijians and the Peruvians.

According to Zweig, (13a) has a cumulative reading – either team won games and every game was won by one of the teams – but (13b) only has an odd distributive reading: each game was won by both teams at once. Kratzer cannot account for this contrast, since *every* has the same thematic role in both cases.

## 6 Conclusion

Cumulative readings of *every* do not pose a special problem for eventless representations, and they provide interesting evidence against its accepted translation as a generalized quantifier. The readings are not an argument that logical representations must contain events or thematic roles. The restriction on cumulative

readings of *every* is more accurately stated in terms of c-command than in terms of thematic roles, so it is not an argument for the asymmetric account in Kratzer (2000). Of course, this does not exclude the possibility that events and thematic roles might be present in the linguistic system for other reasons. The claim here is simply that cumulative readings of *every* do not bear on their status.

Further work is needed to explore the c-command constraint. Interestingly, the dynamic system in Brasoveanu (2009) derives this constraint, but only with the additional assumption that cumulative *every* cannot take inverse scope in the sentences in question. It remains to be seen whether this assumption can be given explanatory value, given that inverse scope of *every* is possible in general.

## References

- Bayer, S.L.: Confessions of a Lapsed Neo-Davidsonian: Events and Arguments in Compositional Semantics. Garland, New York (1997)
- Beck, S., Sauerland, U.: Cumulation is needed: A reply to Winter 2000. *Natural Language Semantics* 8(4), 349–371 (2000)
- Brasoveanu, A.: Modified numerals as post-suppositions. In: Aloni, M., Bastiaanse, H., de Jager, T., Schulz, K. (eds.) *Amsterdam Colloquium 2009*. LNCS (LNAI), vol. 6042, pp. 203–212. Springer, Heidelberg (2009)
- Chomsky, N.: A minimalist program for linguistic theory. In: Hale, K., Keyser, J. (eds.) *The View from Building 20. Essays in Linguistics in Honor of Sylvain Bromberger*, pp. 1–52. MIT Press, Cambridge (1993)
- Davidson, D.: The logical form of action sentences. In: Rescher, N. (ed.) *The logic of decision and action*, pp. 81–95. University of Pittsburgh Press, Pittsburgh (1967)
- Engdahl, E.: *Constituent Questions*. D. Reidel Publishing Company, Dordrecht (1986)
- Fox, D.: Reconstruction, binding theory, and the interpretation of chains. *Linguistic Inquiry* 30(2), 157–196 (1999)
- Fox, D.: Antecedent-contained deletion and the copy theory of movement. *Linguistic Inquiry* 33(1), 63–96 (2002)
- Groenendijk, J., Janssen, T., Stokhof, M. (eds.): *Truth, interpretation, information*. Foris, Dordrecht (1984)
- Johnson, K.: *Determiners*. Talk presented at On Linguistic Interfaces, Ulster (2007)
- Kratzer, A.: The event argument and the semantics of verbs. Manuscript, ch. 2. University of Massachusetts, Amherst (2000)
- Krifka, M.: Nominalreferenz und Zeitkonstitution. *Zur Semantik von Massentermen, Pluraltermen und Aspektklassen*. Fink, München (published 1986)
- Krifka, M.: Pragmatic strengthening in plural predications and donkey sentences. In: Galloway, T., Spence, J. (eds.) *Proceedings of SALT 6*, Ithaca. CLC Publications, Cornell University (1996)
- Krifka, M.: At least some determiners aren't determiners. In: Turner, K. (ed.) *The Semantics/Pragmatics Interface from Different Points of View*, pp. 257–291. Elsevier, Amsterdam (1999)
- Kroch, A.S.: The semantics of scope in English. PhD thesis, Massachusetts Institute of Technology, Cambridge, Mass. (1974)
- Landman, F.: *Events and plurality: The Jerusalem lectures*. Kluwer Academic Publishers, Dordrecht (2000)
- Lee, T.: *Studies on quantification in Chinese*. PhD thesis, University of California at Los Angeles (1986)

- Lin, J.-W.: Distributivity in Chinese and its implications. *Natural Language Semantics* 6, 201–243 (1998)
- Link, G.: The logical analysis of plurals and mass terms: A lattice-theoretical approach. In: Bäuerle, R., Schwarze, C., von Stechow, A. (eds.) *Meaning, use and interpretation of language*, pp. 303–323. de Gruyter, Berlin (1983)
- Link, G.: *Algebraic semantics in language and philosophy*. CSLI, Stanford (1998)
- Liu, F.-H.: *Scope Dependency in English and Chinese*. PhD thesis, University of California at Los Angeles (1990)
- Malamud, S.: (Non-)Maximality and distributivity: a decision theory approach. In: *Proceedings of the 16th Conference on Semantics and Linguistic Theory (SALT 16)*, Tokyo, Japan (2006)
- Matthewson, L.: Quantification and the nature of crosslinguistic variation. *Natural Language Semantics* 9, 145–189 (2001)
- Parsons, T.: *Events in the semantics of English*. MIT Press, Cambridge (1990)
- Roberts, C.: *Modal subordination, anaphora, and distributivity*. PhD thesis, University of Massachusetts, Amherst (1987)
- Sauerland, U.: *The meaning of chains*. PhD thesis, Massachusetts Institute of Technology, Cambridge, Massachusetts (1998)
- Sauerland, U.: A new semantics for number. In: *The Proceedings of SALT 13*, Ithaca, N.Y., pp. 258–275. Cornell University/CLC-Publications (2003)
- Sauerland, U.: The interpretation of traces. *Natural Language Semantics* 12, 63–127 (2004)
- Scha, R.: Distributive, collective and cumulative quantification. In: Groenendijk, J., Janssen, T., Stokhof, M. (eds.) *Formal methods in the study of language*, Mathematical Center Tracts, Amsterdam (1981); Reprinted in Groenendijk et al. (1984)
- Schein, B.: *Event logic and the interpretation of plurals*. PhD thesis, Massachusetts Institute of Technology, Cambridge, MA (1986)
- Schein, B.: *Plurals and events*. MIT Press, Cambridge (1993)
- Schein, B.: Plurals. In: Lepore, E., Smith, B.C. (eds.) *Oxford Handbook of Philosophy of Language*. Oxford University Press, Oxford (2006)
- Schwarzschild, R.: *Pluralities*. Kluwer, Dordrecht (1996)
- Sternefeld, W.: Reciprocity and cumulative predication. *Natural Language Semantics* 6, 303–337 (1998)
- Szabolcsi, A. (ed.): *Ways of scope taking*. Kluwer Academic Publishers, Dordrecht (1997)
- Zweig, E.: *Dependent plurals and plural meaning*. PhD thesis, NYU, New York, N.Y. (2008)

# Restricting and Embedding Imperatives

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**Abstract.** We use imperatives to refute a naïve analysis of update potentials (force-operators attaching to sentences), arguing for a dynamic analysis of imperative force as *restrictable*, *directed*, and *embeddable*. We propose a dynamic, non-modal analysis of conditional imperatives, as a counterpoint to static, modal analyses (e.g., Schwager [16]). Our analysis retains Kratzer’s [8] analysis of *if*-clauses as restrictors of some operator (with Schwager), but avoids typing it as a generalized quantifier over worlds (against her), instead as a dynamic force operator (cf. Portner [13, 14]; Potts [15]). Arguments for a restrictor treatment (but against a quantificational treatment) are mustered, and we propose a novel analysis of update on conditional imperatives (and an independently motivated revision of the standard ordering-semantics for root modals that makes use of it). Finally, we argue that imperative force is embeddable under an operation much like dynamic conjunction.

## 1 Plan

Sentences of the imperative clause-type (hereafter ‘imperatives’) are conventionally associated with a distinctive kind of force (what I will call ‘imperative force’) that is both *performative* and *directive* (see esp. Portner [13]). It is performative in the sense that the conventional discourse function of imperatives is not to describe facts about the world, but rather to introduce new facts (about obligations or commitments) into a discourse. It is directive in the sense that imperatives function primarily to shape the intentions (indirectly, by directly shaping things that, in turn, directly shape the intentions) of their addressees.

There is widespread agreement that a semantico-pragmatic analysis of imperatives should have something to say about this dimension (call it the ‘force dimension’) of the conventional meaning of imperatives.<sup>1</sup> What, *exactly*, needs to be said, beyond the fact that imperatives conventionally receive performative and directive interpretations, is often unclear. In this paper, I articulate

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<sup>1</sup> Some (e.g., Portner [14]: 366) have taken the stronger position that the unavailability of non-performative interpretations of imperatives means that the force dimension exhausts the dimensions of imperative meaning. This latter position is too strong. As I argue in [2], there are dimensions of imperative meaning (e.g., facts about their inferential and logical properties) that are paradigmatically static and do not emerge straightforwardly from an account of the force dimension.

substantive conditions of adequacy on an account of the force dimension of imperative meaning. My principal focus is on the performative effects of conditional imperatives (CIs; see [1]) and unconditional imperatives (UIs; see [2]).

(1) If the temperature drops, shut the window!  $\approx$  (if  $\phi$ )(! $\psi$ )

(2) Shut the window!  $\approx$  ! $\phi$

Schwager’s [16] account, which treats imperatives as a species of modal clause (hence, imperative operators as Kratzer-ian restrictable modal operators), is designed to handle CIs, but ultimately handles neither. Portner’s [14] account makes implicit use of directed speech-act (force) operators (à la Potts [15]), so that the force of an imperative is to add the content of the imperative (what’s commanded, i.e., the complement of the force operator) to the addressee’s To-Do List. It does well with UIs, but falters with CIs.

We strictly improve on these proposals by reconceptualizing force operators. The ordinary treatment (classic references are Stenius [17]; Lewis [11]) views speech-acts on the model of *propositional attitudes*: as an agent may believe  $\phi$ , she may assert  $\phi$ , command  $\phi$ , question whether  $\phi$ , etc.<sup>2</sup> Handling CIs requires a new approach: speech-acts are less like propositional attitudes, more like literal *actions* whose force (contextual effect) can be modulated, via linguistic and extra-linguistic mechanisms, and whose functional potential can be formally modeled in a familiar logic of programs. Conditional imperatives, we’ll see, illustrate a syntactic mechanism of force-modulation, which we model as force-restriction (in a sense to be precisified). (Making use of this analysis requires modifying the standard Kratzer [8] semantics for modals. There are, we’ll see, independent reasons for doing this.)

Our stance here resembles that of Krifka [9, 10], which emphasizes natural language devices (generally corresponding to regular operations on programs) for building complex speech-acts out of component speech-acts. The question naturally arises: which such operations are expressible in natural language? We end with some tentative remarks on this question.

## 2 Menu

The conventional discourse function of an imperative clause is, I will suppose, to introduce some sort of obligation or commitment on its addressee, via modification of parameters of the context to which the interpretation of obligation- or commitment-describing modalities is sensitive (cf. Han [5],

<sup>2</sup> See Krifka [10]. The traditional idea might be motivated by the idea that there is some sort of map from speech-acts onto propositional attitudes: every speech-act expresses some propositional attitude, and speech-act types are individuated by the sort of attitude they generally function to express—asserting that  $\phi$  expresses belief that  $\phi$ , questioning whether  $\phi$  expresses wondering whether  $\phi$ , etc.

Portner [13, 14]<sup>3</sup> Imperative force is performative because it generally yields a context in which certain obligation-descriptions are true (where previously they *were false*), directive because its target is the indirect regulation of the behavior of its addressee. Adequate accounts of imperative force will predict that CIs tend to introduce corresponding conditional obligations (COs), UIs tend to introduce corresponding unconditional obligations (UOs). Concretely, (1) and (2) should tend to make it the case that if the temperature drops, you must shut the window, and that you must shut the window, respectively. Should an account fail to predict this in a given context, there should be a plausible explanation (for instance, the prior context enforcing a conflicting obligation).

## 2.1 Modal Analyses

The paradigm modal analysis of imperatives is Schwager's [16]. Schwager assigns a CI (if  $\phi$ )(! $\psi$ ) the logical form  $O(\psi/\phi)$  (read: *if  $\phi$ ,  $\psi$  must be realized*); UIs are trivially restricted, so that  $!\phi := O(\phi/\top)$ . These LFs are interpreted via the standard Kratzer [8] ordering-restrictor semantics. On that semantics, if  $c$  is a context, then  $f_c$  is the *modal base* (a body of information),  $g_c$  the *ordering source* (for Schwager a set of contextually given preferences, generally supplied by the speaker). Both  $f_c$  and  $g_c$  map worlds to sets of propositions. The modal LFs are assigned the following truth-conditions<sup>4</sup>

**Definition 1.**  $\llbracket O(\psi/\phi) \rrbracket^{c,w} = 1 \Leftrightarrow \min(f_c(w) \cup \{\llbracket \phi \rrbracket^c\}, \preceq_{g_c(w)}) \subseteq \llbracket \psi \rrbracket^c$ , where:

1.  $\min(\Phi, \preceq_\Psi) := \{w \in \bigcap \Phi : \forall v \in \bigcap \Phi : v \preceq_\Psi w \Rightarrow w \preceq_\Psi v\}$
2.  $w \preceq_\Psi v \Leftrightarrow \{P \in \Psi : v \in P\} \subseteq \{P \in \Psi : w \in P\}$

The analysis thus assigns imperatives truth-conditions—the same as their modal LFs. As such, the analysis would appear to offer no account of imperative force—appear, indeed, to predict that imperative force is a subtype of assertoric force.

Schwager tries to avoid the worry by introducing contextual constraints on the felicitous utterance of an imperative. Imperative utterances are infelicitous at  $c$  unless the speaker of  $c$ :

1. Has exhaustive knowledge, à la Groenendijk and Stokhof [4], about  $f_c$  and  $g_c$ , so that he ‘utters a necessity proposition he cannot be mistaken about.’
2. Affirms the relevant preference for  $\phi$  a good ‘maxim for acting.’

When these conditions are met, an imperative utterance will generally receive the performative and directive interpretation adverted to above.

There are, however, deep problems. First, if the speaker of  $c$  isn't mistaken, then  $O(\psi/\phi)$  must *already* be true at  $c$ . Performative effect, which paradigmatically consists in updating  $c$  so that  $O(\psi/\phi)$  goes from false to true, is therefore

<sup>3</sup> Some non-imperative clauses (e.g., explicit performatives of the form *I hereby command you to...*) plausibly have conventionalized imperative force. While this complicates the posited link between clause-type and conventional force, it does not defeat it. Space prevents me from discussing such cases in any detail here, but see Asher and Lascarides [1] for relevant discussion.

<sup>4</sup> The Limit Assumption simplifies our discussion (with no worrying commitments).

erased. Second, affirmation that  $\phi$  is a good ‘maxim for acting’ is exactly the type of speech-act we should like to *analyze*. We would like to model *how* such affirmation is generally associated with the introduction of new obligations on the addressee. Saying that imperatives receive a performative and directive interpretation when certain presuppositions are met is no replacement for an account of what, precisely, such an interpretation consists in.

## 2.2 Dynamic Analyses

Portner [13, 14] (cf. Han [5]; Potts [15]) analyzes imperative performative effect as *addition to an addressee-indexed ordering source*, her ‘To-Do List’ (TDL). Imperative clauses are associated directly with a type of ‘sentential force,’ rather than indirectly (via analysis as a species of necessity modal with an exclusively performative interpretation). With  $[\cdot]$  a dynamic interpretation function, mapping formulas to update potentials,  $c$  a context,  $T_c$  a function from individuals to their TDLs,  $a_c$  the addressee, the idea is this:

**Definition 2.**  $c[!\phi] = c'$  is just like  $c$ , except  $\llbracket\phi\rrbracket^c$  is on  $T_{c'}(a_c)$

This analysis meets the criteria of adequacy on accounts of *unconditional* imperative force. Making use of the Kratzer semantics for modals, we can see that even where some of the  $T_c(a_c)$ -best worlds compatible with the  $c$ -relevant information do not satisfy  $\phi$ , it will tend to be the case that  $O(\phi/\top)$  is true at  $c'$ , since it will tend to be the case that all of the  $T_{c'}(a_c)$ -best worlds compatible with the same information do satisfy  $\phi$ , in virtue of the presence of  $\llbracket\phi\rrbracket^c$  on  $T_{c'}(a_c)$ . In cases where this does not reliably hold at  $c'$  (e.g., cases where updating  $c$  with  $!\phi$  introduces a logical incompatibility into the TDL), it’s not clear that we really do want to predict that new obligations are imposed. Such cases will tend to coincide with cases where the prior context enforces a conflicting obligation.

The analysis does not, however, meet the criteria of adequacy on accounts of conditional imperative force. In a footnote, Portner [13] moots an analysis in terms of conditional update: informally, he suggests,  $[(\text{if } \phi)(!\psi)]$  adds  $\llbracket\phi\rrbracket^c$  to  $a_c$ ’s TDL, once  $\phi$  is true. But this fails to explain the conventional discourse effect of CIs: the imposition of COs. Even when  $\phi$  is false at both  $c$  and the result of updating  $c$  with  $(\text{if } \phi)(!\psi)$ , this sort of update will typically introduce a CO of the form  $O(\psi/\phi)$ . Concretely: given an utterance of (11) at  $c$ , the associated CO (*if the temperature drops, you must shut the window*) will tend to be in force at the updated context, regardless of the truth of *the temperature drops* at either  $c$  or  $c$  updated with the CI.

A preliminary diagnosis of the problem: for CIs, we require an update on TDLs that is performed regardless of the antecedent’s truth then. The failure seems to stem from *deferring* update to the ordering source until the antecedent of the imperative is true. An immediate thought, then, is to treat conditional imperative force as a kind of unconditional imperative force:

**Definition 3.**  $c[(\text{if } \phi)(!\psi)] = c'$  is just like  $c$ , except  $\llbracket \phi \supset \psi \rrbracket^c$  is on  $T_{c'}(a_c)$

Call this the Wide-Scoping Proposal for CIs (WSPCI), so named because, according to the WSPCI,  $\llbracket (\text{if } \phi)(!\psi) \rrbracket = \llbracket !(\phi \supset \psi) \rrbracket$ <sup>5</sup>. The WSPCI runs into empirical problems. Consider the following case (from Kolodny and MacFarlane [7]): ten miners are trapped in a single shaft—A or B, although we do not know which—and threatened by rising waters. We can block one shaft or neither, but not both. If we block the shaft they are in, all are saved. If we guess wrong, all die. But, if we do nothing, water will distribute between the shafts and exactly one will die. Now consider the following set of imperatives.

- (3) If they're in A, block A!  $\approx$  (if  $\text{in\_A}$ )(!block\_A)
- (4) If they're in B, block B!  $\approx$  (if  $\text{in\_B}$ )(!block\_B)
- (5) Don't block either shaft!  $\approx$   $\neg(\text{block\_A} \vee \text{block\_B})$

The imperatives in (3–5) seem like sound advice. But if the WSPCI is right, they add the following to the addressee's TDL:  $\llbracket \text{in\_A} \supset \text{block\_A} \rrbracket^c$ ,  $\llbracket \text{in\_B} \supset \text{block\_B} \rrbracket^c$ , and  $\llbracket \neg(\text{block\_A} \vee \text{block\_B}) \rrbracket^c$ . The only way to satisfy all of these demands is to make sure the miners are in neither A nor B. But this is presupposed impossible at  $c$ . This does not square with intuitions: a speaker issuing these imperatives at  $c$  is not demanding the impossible<sup>6</sup>.

### 3 Restricting Force

To summarize, the dynamic account, as it stands, does well with UIs, but fails to predict the relevant phenomena for CIs. Why? There are only two sorts of update to perform on a TDL: deferred and non-deferred (immediate) addition. Deferred addition, we saw, cannot account for the conventional discourse effect of CIs. Immediate addition implies that there is some proposition that a CI adds to the addressee's TDL—that conditional commanding is a species of unconditional commanding. There are no obvious candidates for the identity of this proposition.

Each tack presupposes that imperative force involves a speaker demanding that *some proposition* be true (with the deferred update proposal making this demand contingent on some further condition). I see no way of predicting the relevant phenomena for CIs while preserving this assumption. So we will jettison it. The guiding idea here will be a familiar one: unconditional commanding is a species of conditional commanding. The former corresponds to a kind of unrestricted imperative force, the latter to a kind of restricted imperative force.

<sup>5</sup> Note: allowing ! to take widest scope in CIs lets us handle CIs with quantificational adverbials in consequent position. Consider the CI *if your boss comes in, never stare at him*. Schwager [16] assigns this sentence a wide-scope LF: the antecedent restricts the domain of the quantificational adverbial, and the necessity modal takes scope over the adverbial. Schwager takes this to be evidence for the modal analysis, but simply allowing ! to take widest scope (thus allowing the conditional antecedent to subsequently restrict the adverbial) lets us mimic her analysis.

<sup>6</sup> People do quibble with this judgment. Kolodny and MacFarlane [7] argue that they are mistaken. Space prevents me from rehearsing the arguments here.

### 3.1 First Pass

How to formalize this idea? The first thought is to type TDLs as Kratzer-ian conversational backgrounds: functions from worlds to a set of propositions. Doing this allows us to think of TDLs as something like a set of contingency plans: they furnish different practical ‘recommendations’ depending on the situation the agent finds herself in. Formally, we index TDLs to both agents *and* worlds, and treat  $(\text{if } \phi)(!\psi)$  as adding  $\llbracket \psi \rrbracket^c$  to  $a_c$ ’s TDL at the  $\phi$ -worlds (or some contextually selected subset thereof; cf. Mastop [12]: 103).

**Definition 4.**  $c[(\text{if } \phi)(!\psi)] = c'$  is like  $c$ , except  $\forall w \in \llbracket \phi \rrbracket^c : \llbracket \psi \rrbracket^c \in T_{c'}(a_c)(w)$

This is a natural and elegant extension of Kratzer’s restrictor analysis of conditional antecedents. Rather than restricting the domain of a generalized quantifier, however, CI antecedents function to restrict the *scope* of dynamic update. Update with UIs is thus understood in terms of update with CIs, rather than vice versa. UIs issue a demand on the addressee that holds in all possible contingencies, while genuine CIs issue a demand on the addressee that holds in some non-trivial restriction of the set of possible contingencies.

Elegant though it is, this proposal does no better at predicting the desired relationship between CIs and COs. We do get unconditional obligations (UOs) of the form  $O(\psi/\top)$  when evaluating these formulas at  $\phi$ -worlds. But we get nothing at  $\neg\phi$ -worlds. For the CI updates the addressee’s TDL only at the  $\phi$ -worlds, and does nothing otherwise. Thus, we have only a *metalinguistic* analogue of the desired prediction: given that  $(\text{if } \phi)(!\psi)$  is issued at  $c$ , if  $\phi$  is true at  $w$ , then typically  $\psi$  is required at  $w$  (i.e., typically,  $\llbracket O(\psi/\top) \rrbracket^{c',w} = 1$ , where  $c'$  is  $c$  updated with  $(\text{if } \phi)(!\psi)$ ). This isn’t good enough: we’d like to predict the *object-language* CO *if the temperature drops, you must shut the window* true at the updated context, regardless of whether *the temperature drops* is true then.<sup>7</sup>

### 3.2 Second Pass

Something is very intuitive about the contingency plan understanding of the TDL. The problem is that, on the standard Kratzer semantics for modals, the world of evaluation fixes the ordering source at a context: contingencies cease to be relevant (in the sense that they are ignored by the semantics) once the world of evaluation is fixed. Avoiding this, then, demands that TDLs be indexed to some semantic parameter other than the world of evaluation.

Our analysis indexes TDLs to *bodies of information* (modal bases, whether construed as sets of worlds or propositions), rather than worlds. On this picture, the contingencies relevant to planning are informational, rather than ‘factual,’ in character: the TDL furnishes different practical ‘recommendations’ for an agent depending on the information available to her at the context. Formally, we treat

<sup>7</sup> We could predict the right relationship between CIs and COs by rewriting the Kratzer semantics as a strict conditional semantics, so that  $\llbracket O(\phi/\psi) \rrbracket^{c,w} = 1$  iff  $\forall v \in \llbracket \phi \rrbracket^c : \llbracket O(\psi/\top) \rrbracket^{c,v} = 1$ . But this seems like an *ad hoc* revision of the semantics.

(if  $\phi$ )(! $\psi$ ) as adding  $\llbracket\psi\rrbracket^c$  to  $a_c$ 's TDL at every body of information  $\Phi \supseteq f_c(w) \cup \{\llbracket\phi\rrbracket^c\}$ , for each  $w$  (i.e., every  $\phi$ -containing expansion of the information at  $c$ ).

**Definition 5.**  $c[(\text{if } \phi)(!\psi)] = c'$  is like  $c$ , except:

$\forall w : \forall \Phi \supseteq f_c(w) \cup \{\llbracket\phi\rrbracket^c\} : \llbracket\psi\rrbracket^c \in T_{c'}(a_c)(\Phi)$

As before, a UI ! $\phi$  is a vacuously restricted CI:  $[\text{!}\phi] := [(\text{if } \top)(!\phi)]$ ; unconditional commanding is still a species of conditional commanding. The difference is that UIs add their consequents to every expansion of the information *sans phrase*.

This is the analysis of imperative force which I will be endorsing in this paper. It is a restrictor analysis of CI antecedents: CI antecedents restrict the set of contingencies to which a command pertains, thereby modulating the force of the associated speech-act. It represents an abandonment of the 'propositional attitude' model of speech acts described in this essay's introduction.

### 3.3 Information-Sensitive Ordering Semantics

Allowing the ordering source at a context  $c$  to be determined by the information at  $c$ , rather than the world of evaluation, does not, by itself, secure the desired relationship between CIs and COs. Getting this right requires modifying the semantics to *make use* of the information-sensitive ordering source<sup>8</sup>. The relevant change is having conditional antecedents function as both domain restrictors *and* ordering source shifters, so that the if-clause supplies the relevant contingency:

**Definition 6.**  $\llbracket O(\psi/\phi) \rrbracket^{c,w} = 1 \Leftrightarrow \min(f_c(w) \cup \{\llbracket\phi\rrbracket^c\}, \preceq_{T_c(a)(f_c(w) \cup \llbracket\phi\rrbracket^c)}) \subseteq \llbracket\psi\rrbracket^c$

Informally, the formula  $O(\psi/\phi)$  says the best-on-the supposition-that- $\phi$   $\phi$ -worlds are  $\psi$ -worlds. This secures the right result in the *if the temperature drops...* case. The relevant CI adds the proposition that the addressee shuts the window to her TDL at every body of information  $\Phi$  such that  $\Phi$  entails that the temperature is dropping. The information-sensitive semantics (ISS) evaluates the relevant CO by looking at the addressee's TDL with respect to such a body of information.

This is a major revision of the Kratzer [8] semantics, which allows contingency in ordering sources only via variation in the world coordinate, not via variation in the domain of quantification. So there is reason to worry that it is *ad hoc*. It can, in fact, be independently motivated. Consider, once again, Kolodny and MacFarlane's [7] miner case, and the obligation-descriptions in ([6][8]).

- (6) If they're in A, we gotta block A  $\approx O(\text{block\_A}/\text{in\_A})$
- (7) If they're in B, we gotta block B  $\approx O(\text{block\_B}/\text{in\_B})$
- (8) We may leave both shafts open  $\approx \neg O((\text{block\_A} \vee \text{block\_B})/\top)$

Given the case, informants reliably hear each of these obligation-descriptions as *true* (so, *a fortiori*, consistent). But, using the information-insensitive Kratzer semantics, whenever the modal base entails (i.e., it is known) that the miners are either all in A or all in B, these sentences are *provably inconsistent*.

<sup>8</sup> The issues here are discussed in more detail in my [3].

*Proof.* Suppose  $\llbracket \textcircled{6} \rrbracket^{c,w} = \llbracket \textcircled{7} \rrbracket^{c,w} = \llbracket \textcircled{8} \rrbracket^{c,w} = 1$  and  $\bigcap f_c(w) \subseteq \llbracket in\_A \vee in\_B \rrbracket^c$ .

1. Let  $g_c$  be an ordering source. Choose any  $v \in \min(f_c(w), \preceq_{g_c(w)})$ .
2. Since  $\bigcap f_c(w) \subseteq \llbracket in\_A \vee in\_B \rrbracket^c$ ,  $v \in \min(f_c(w) \cup \{\llbracket in\_A \rrbracket^c\}, \preceq_{g_c(w)})$  or  $v \in \min(f_c(w) \cup \{\llbracket in\_B \rrbracket^c\}, \preceq_{g_c(w)})$ <sup>9</sup>
3. By Kratzer's semantics (Def. [11](#)), since  $\llbracket \textcircled{6} \rrbracket^{c,w} = \llbracket \textcircled{7} \rrbracket^{c,w} = 1$ ,  $\min(f_c(w) \cup \{\llbracket in\_A \rrbracket^c\}, \preceq_{g_c(w)}) \subseteq \llbracket block\_A \rrbracket^c$  and  $\min(f_c(w) \cup \{\llbracket in\_B \rrbracket^c\}, \preceq_{g_c(w)}) \subseteq \llbracket block\_B \rrbracket^c$ .
4. So  $v \in \llbracket block\_A \rrbracket^c \cup \llbracket block\_B \rrbracket^c$ .
5. So  $\min(f_c(w), \preceq_{g_c(w)}) \subseteq \llbracket block\_A \rrbracket^c \cup \llbracket block\_B \rrbracket^c$ .
6. So, by Def. [11](#),  $\llbracket O((block\_A \vee block\_B)/\top) \rrbracket^{c,w} = 1$ . Contradiction.  $\square$

Space prevents me from discussing in detail the substance of the the proof or our proposal for COS. Briefly, the ISS blocks the proof by varying the ordering sources that are relevant for evaluating the CO-descriptions in [\(68\)](#): [\(6\)](#) uses an ordering source indexed to a body of information that entails that the miners are in shaft A, [\(7\)](#) uses an ordering source indexed to a body of information that entails that the miners are in shaft B, while [\(8\)](#) uses an ordering source indexed to a body of information that does not settle the miners' location<sup>10</sup>. The upshot: the ISS seems to be independently motivated, not *ad hoc*.

## 4 Sequencing

We may think of imperative force as a complex update on TDLs, constructed out of a set of basic updates on TDL components, together with a regular operation. In this case, the operation is  $;$  (SEQUENCING). Sequencing is function composition: if  $\alpha$  and  $\beta$  are context-change potentials, then  $\alpha; \beta = \lambda c. c\alpha\beta$ . A TDL is a set of contingency plans: a set of information-plan pairs. The basic updates are additions to contingency plans. Complex or composite update is understood as a *series* of additions to an addressee's contingency plans (which being a function of how the speaker modulates the force of her command). The update associated with a CI (if  $\phi$ )(! $\psi$ ) at  $c$  is a sequencing of the following basic update program:

$$\lambda \langle \Phi, \Psi \rangle. \begin{cases} \langle \Phi, \Psi \cup \{\llbracket \psi \rrbracket^c\} \rangle & \text{if } f_c(w) \cup \llbracket \phi \rrbracket^c \subseteq \Phi \\ \langle \Phi, \Psi \rangle & \text{otherwise} \end{cases}$$

<sup>9</sup> This step relies on a kind of monotonicity property of the Kratzer semantics: if  $u \in \min(\Phi, \preceq)$ , then for any  $\Psi$  such that  $\bigcap \Psi \subseteq \bigcap \Phi$  and  $u \in \bigcap \Psi$ ,  $u \in \min(\Psi, \preceq)$ .

<sup>10</sup> There is, I argue in my [3](#), a decision-theoretic motivation for allowing considerations of value (i.e., the elements of a deontic ordering source) to make a difference in determining what an agent should do *only when* they meet informational constraints: they must be actionable (roughly, knowably realizable) with respect to the relevant information (for relevant precedents, see Weirich [18](#), particularly his remarks on conditional utility, and Hawthorne and Stanley [6](#)). This both (i) motivates a semantics for deontic  $O$  on which its ordering source is information-sensitive, (ii) accounts for the variation in the ordering sources that blocks the proof.

There is, then, a sense in which utterances of CIs conventionally involve the performance of a composite speech-act: a ‘conjunction’ of instructions about updating individual contingency plans.<sup>11</sup> Standard treatments of force do not provide for complex updates built with regular operations: force is computed by applying a force-operator to a content, and doesn’t embed.

We also see speech-act sequencing, of a rather different sort, in the various ways a speaker may direct imperative force. So far we have (implicitly) construed imperatives as taking direction arguments: a context in which an imperative utterance occurs will tend to select someone *at whom* imperative force is targeted, i.e., an addressee. This orientation is, we see, sufficiently flexible to distinguish singular-addressee imperatives like (9) from group-addressee imperatives like (10). But it founders with plural-addressee imperatives like (11) and (12).

(9) Have the orchestra play Beethoven’s 5<sup>th</sup>  $\approx$  **!(make-play(5th)(orch))(a<sub>c</sub>)**

(10) Play Beethoven’s 5<sup>th</sup> (together)  $\approx$  **!(play(5th))(a<sub>c</sub>)** ( $a_c$  = the orchestra)

(11) Everyone play her part  $\approx$  ???

(12) (Conductor addressing orchestra members:) Play your part  $\approx$  ???

In (11) and (12) no single individual or group of individuals is targeted by the imperative. Rather, each individual in a set of addressees is targeted, separately. Their force is, for each addressee  $a$ , to instruct that  $a$  play  $a$ ’s part.

Plural addressee imperatives seem to involve some sort of quantification outscoping the imperative operator. So they demand a representation something like the following:  $\forall x!$ **play(the-part-of- $x$ )( $x$ )**.<sup>12</sup> In the absence of a vocative (as with, e.g., (12)), the default approach will be to bind free variables by  $\forall$ -closure. The intended interpretation of such formulas has them denoting sequences of updates: the result of sequencing the following set of updates, for all  $a \in \mathcal{A}_c$ , where  $\mathcal{A}_c$  is the set of addressees determined by  $c$ :

$$\{\beta \mid \exists a \in \mathcal{A}_c : \beta = [!(\text{play}(\text{the-part-of-}x)(x))][x/a]\}$$

In the general case, formulas of the form  $\forall x : (\text{if } \phi)(! \psi)$  are interpreted as in Def. 7. (We assume, implicitly, that quantification into an imperative operator must always bind a variable in the direction-argument-position.)

**Definition 7.**  $c[\forall x : (\text{if } \phi)(! \psi)] = c'$  is like  $c$ , except:

$$\forall a \in \mathcal{A}_c : \forall w : \forall \Phi \supseteq f_c(w) \cup \{[\phi]^c\} : [\psi]^{c[x/a]} \in T_{c'}(a)(\Phi)$$

Formulas of the form  $\forall x : (\text{if } \phi)(! \psi)$  can thus be viewed as expressing sequences of speech-acts along two dimensions. We have a ‘conjunction’ of instructions for the addressees, and each such instruction for a given addressee  $a$  is comprised of a ‘conjunction’ of instructions about updating  $a$ ’s contingency plans.

<sup>11</sup> Cf. the dynamic treatment of conjunction as function-composition:  $\sigma[\phi \wedge \psi] = \sigma[\phi][\psi]$ . On this view,  $[\phi \wedge \psi]$  is a composite speech-act—the result of sequencing  $[\phi]$  and  $[\psi]$ .

<sup>12</sup> Cf. Krifka [9, 10], who argues for representing pair-list readings of questions with universal quantification into questions.

The general orientation of this approach raises further questions. I can only gesture at their answers here. For instance: is embedding of speech-act-operators under an  $\exists$ -like operation (or of update potentials under a  $\vee$ -like operation) permitted (cf. Krifka [10], who suggests that it may be)? My tentative answer is: probably not. The purported evidence for the expressibility of such speech-acts in natural language is weak (see my [2]). There is, moreover, arguably no reasonable thing for such an operation to *mean*. Suppose basic update potentials are functions defined for contexts. Interpreting  $\vee$  in terms of  $\cup$  will tend to yield update potentials that are not functions from input contexts to output contexts, but rather *relations* between input contexts and several possible output contexts. Complex update potentials formed with such operations will tend, in other words, to be indeterministic programs. Indeterminism in update potentials is *prima facie* objectionable: basic conversational platitudes plausibly require that a cooperative speaker know how her utterance will update the context.<sup>13</sup>

## References

- [1] Asher, N., Lascarides, A.: Indirect Speech Acts. *Synthese* 128, 183–228 (2001)
- [2] Charlow, N.: Directives. Ms (2009a)
- [3] Charlow, N.: What we know and what to do. Ms (2009b)
- [4] Groenendijk, J., Stokhof, M.: Studies on the semantics of questions and the pragmatics of answers. Ph.D. Diss., ILLC (1984)
- [5] Han, C.: The structure and interpretation of imperatives: Mood and force in universal grammar. Ph.D. Diss. University of Pennsylvania (1998), <http://www.sfu.ca/~chunghye/papers/dissertation.pdf>
- [6] Hawthorne, J., Stanley, J.: Knowledge and action. *The Journal of Philosophy* 105, 571–590 (2008)
- [7] Kolodny, N., MacFarlane, J.: Ifs and oughts. Ms (2009), <http://johnmacfarlane.net/ifs-and-oughts.pdf>
- [8] Kratzer, A.: The notional category of modality. In: Eikmeyer, H., Rieser, H. (eds.) *Words, Worlds, and Contexts*, pp. 38–74. De Gruyter, Berlin (1981)
- [9] Krifka, M.: Quantifying into question acts. *Natural Language Semantics* 9, 1–40 (2001)
- [10] Krifka, M.: Semantics below and above speech acts. Talk delivered at Stanford (2004), <http://amor.rz.hu-berlin.de/~h2816i3x/Talks/StanfordLecture2004.pdf>
- [11] Lewis, D.: General semantics. *Synthese* 22, 18–67 (1970)
- [12] Mastop, R.: What can you do? Ph.D. Diss., ILLC (2005)
- [13] Portner, P.: The semantics of imperatives within a theory of clause types. In: Watanabe, K., Young, R. (eds.) *Proceedings of SALT 14*. CLC Publications (2004), <http://semanticsarchive.net/Archive/mJlZGQ4N/>
- [14] Portner, P.: Imperatives and modals. *Natural Language Semantics* 15, 351–383 (2007)

<sup>13</sup> There are possible interpretations for  $\vee$  that preserve determinism. For instance, ‘disjoined’ speech-acts might map a context into a *set* of alternative contexts (cf. Mastop [12]). But this gets formally unwieldy very quickly (see esp. Krifka [10]).

- [15] Potts, C.: Keeping world and will apart: A discourse-based semantics for imperatives. NYU Syntax/Semantics Lecture Series (2003), <http://people.umass.edu/potts/talks/potts-nyu-handout.pdf>
- [16] Schwager, M.: Conditionalized imperatives. In: Gibson, M., Howell, J. (eds.) Proceedings of SALT 16. CLC Publications (2006), <http://user.uni-frankfurt.de/~scheiner/papers/schwagerFEB07.pdf>
- [17] Stenius, E.: Mood and language game. *Synthese* 17, 254–274 (1967) doi:10.1007/BF00485030
- [18] Weirich, P.: Decision when desires are uncertain. In: Bradie, M., Sayre, K. (eds.) *Reason and Decision*, pp. 69–75. Bowling Green State UP (1982)

# A First-Order Inquisitive Semantics

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**Abstract.** This paper discusses the extension of propositional inquisitive semantics [Ciardelli and Roelofsen, 2009a, Groenendijk and Roelofsen, 2009] to the first order setting. We show that such an extension requires essential changes in some of the core notions of inquisitive semantics, and we propose and motivate a semantics which retains the essential features of the propositional system.

## 1 Introduction

The starting point of this paper is the propositional system of inquisitive semantics [Ciardelli, 2009, Ciardelli and Roelofsen, 2009b,a, Groenendijk and Roelofsen, 2009]. Whereas traditionally the meaning of a sentence is identified with its informative content, in inquisitive semantics –originally conceived by Groenendijk [2009a] and Mascarenhas [2009]– meaning is taken to encompass inquisitive content, consisting in the potential to raise issues.

More specifically, the main feature of this system is that a disjunction  $p \vee q$  is not only informative, but also inquisitive: it proposes two possibilities, as depicted in figure 1(b), and invites other participants to provide information in order to establish at least one of them.

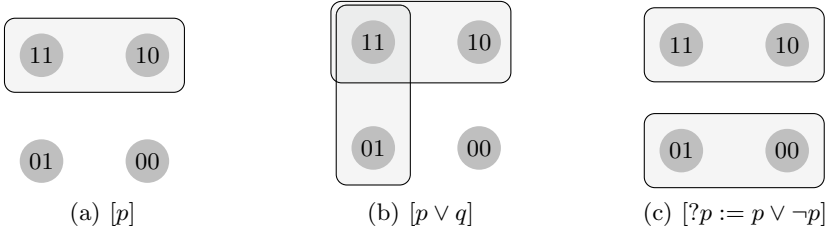
The main feature of a first-order extension can be expected to be that existential quantification also has inquisitive effects. A simplified version, assuming finite domains, was used in Balogh [2009] in an analysis of focus phenomena in natural language. However, as was shown in Ciardelli [2009], defining a first order system that can deal with infinite domains is not a trivial affair. While there I proposed to enrich the propositional system in order to make the predicate extension possible, what I outline here is a *conservative* extension of the original framework, which retains most of its essential features, in particular the decomposition of meanings into a purely informative and a purely inquisitive component.

## 2 Propositional Inquisitive Semantics

We start by recalling briefly the propositional implementation of inquisitive semantics. We assume a set  $\mathcal{P}$  of propositional letters. Our language will consist

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**Fig. 1.** Examples of propositional inquisitive meanings

of propositional formulas built up from letters in  $\mathcal{P}$  and  $\perp$  using the connectives  $\wedge, \vee$  and  $\rightarrow$ . We write  $\neg\varphi$  as an abbreviation for  $\varphi \rightarrow \perp$ .

Our semantics is based on *information states*, modeled as sets of valuations. Intuitively, a valuation describes a possible state of affairs, and a state  $s$  is interpreted as the information that the actual state of affairs is described by one of the valuations in  $s$ . In inquisitive semantics, information states are always used to represent the state of the common ground of a conversation, not the information state of any individual participant.

**Definition 1 (States).** A state is a set of valuations for  $\mathcal{P}$ . We denote by  $\omega$  the state of ignorance, i.e. the state containing all valuations. We use  $s, t, \dots$  as meta-variables ranging over states.

We get to inquisitive meanings passing through the definition of a relation called *support* between states and propositional formulas.

**Definition 2 (Support)**

$$\begin{aligned}
 s \models p &\iff \forall w \in s : w(p) = 1 \\
 s \models \perp &\iff s = \emptyset \\
 s \models \varphi \wedge \psi &\iff s \models \varphi \text{ and } s \models \psi \\
 s \models \varphi \vee \psi &\iff s \models \varphi \text{ or } s \models \psi \\
 s \models \varphi \rightarrow \psi &\iff \forall t \subseteq s : \text{if } t \models \varphi \text{ then } t \models \psi
 \end{aligned}$$

Support is used to define inquisitive meanings as follows.

**Definition 3 (Truth-sets, possibilities, meanings)**

1. The truth-set  $|\varphi|$  of  $\varphi$  is the set of valuations which make  $\varphi$  true.
2. A possibility for  $\varphi$  is a maximal state supporting  $\varphi$ .
3. The inquisitive meaning  $[\varphi]$  of  $\varphi$  is the set of possibilities for  $\varphi$ .

*Informativeness.* The meaning  $[\varphi]$  represents the proposal expressed by  $\varphi$ . One effect of the utterance of  $\varphi$  is to *inform* that the actual world lies in one of the specified possibilities, i.e. to propose to eliminate all indices which are not included in any element of  $[\varphi]$ : thus, the union  $\bigcup[\varphi]$  expresses the informative content of  $\varphi$ . A formula which proposes to eliminate indices is called *informative*. It is easy to see that the equality  $\bigcup[\varphi] = |\varphi|$  holds, insuring that inquisitive semantics preserves the classical treatment of information.

*Inquisitiveness.* What distinguishes inquisitive semantics from classical update semantics is that now the truth-set  $|\varphi|$  of a formula comes subdivided in a certain way, which specifies the possible resolutions of the issue raised by the formula. If resolving a formula  $\varphi$  requires more information than provided by  $\varphi$  itself, which happens iff  $|\varphi| \not\subseteq [\varphi]$ , then  $\varphi$  requests information from the other participants, and thus we say it is *inquisitive*. In the present system (but not in the unrestricted system mentioned below) a formula is inquisitive precisely in case it proposes more than one possibility.

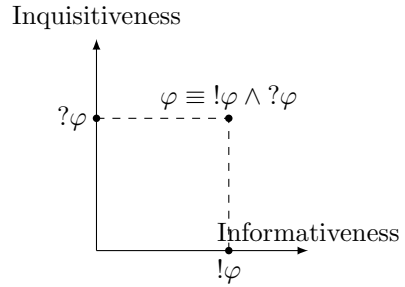
*Assertions and questions.* Notice that formulas which are neither informative nor inquisitive make the trivial proposal  $\{\omega\}$  (namely, they propose to stay in the given state). Thus, inquisitive meanings can be seen as consisting of an informative dimension and an inquisitive dimension. Purely informative (i.e., non-inquisitive) formulas are called *assertions*; purely inquisitive (i.e., non-informative) formulas are called *questions*. In other words, assertions are formulas which propose only one possibility (namely their truth-set), while questions are formulas whose possibilities cover the whole logical space  $\omega$ .

It is easy to see that disjunction is the only source of inquisitiveness in the language, in the sense that any disjunction-free formula is an assertion. Moreover, a negation is always an assertion: in particular, for any formula  $\varphi$ , its double negation  $\neg\neg\varphi$ , abbreviated by  $!\varphi$ , is an assertion expressing the informative content of  $\varphi$ .

An example of a question is the formula  $p \vee \neg p$  depicted in 1(c), which expresses the polar question ‘whether  $p$ ’. In general, the disjunction  $\varphi \vee \neg\varphi$  is a question which we abbreviate by  $?\varphi$ .

We say that two formulas  $\varphi$  and  $\psi$  are equivalent, in symbols  $\varphi \equiv \psi$ , in case they have the same meaning. The following proposition, stating that any formula is equivalent with the conjunction of an assertion with a question, simply reflects the fact that inquisitive meanings consist of an informative and an inquisitive component.

**Proposition 1 (Pure components decomposition).**  $\varphi \equiv !\varphi \wedge ?\varphi$



Obviously, the notions and the results discussed in this section may be relativized to arbitrary common grounds. For more details on the propositional system and its logic, the reader is referred to Groenendijk [2009b] and Ciardelli and Roelofsen [2009a].

### 3 The Maximality Problem

In this section I will discuss the main difficulty one encounters when trying to reproduce the above framework in a predicate setting; our analysis will lead to considerations which motivate the solution proposed in the next section.

Fix a first-order language  $\mathcal{L}$ . A *state* will now consist of a set of first-order models for the language  $\mathcal{L}$ : not to complicate things beyond necessity, we shall make the simplifying assumption that all models share the same domain and the same interpretation of constants and function symbols. Thus, let  $\mathbb{D}$  be a fixed structure consisting of a domain  $D$  and an interpretation of all (constants and) function symbols in  $\mathcal{L}$ ; a first-order model for  $\mathcal{L}$  based on the structure  $\mathbb{D}$  is called a  $\mathbb{D}$ -*model*.

**Definition 4 (States).** *A state is a set of  $\mathbb{D}$ -models.*

If  $g$  is an assignment into  $D$ , we denote by  $|\varphi|_g$  the state consisting of those models  $M$  such that  $M, g \models \varphi$  in the classical sense. The extension of the definition of support is unproblematic. Just like disjunction, an existential will only be supported in those states where a specific witness for the existential is known.

**Definition 5 (First-order support).** *Let  $s$  be a state and let  $g$  be an assignment into  $D$ .*

$$\begin{aligned} s, g \models \varphi & \iff \forall M \in s : M, g \models \varphi \text{ for } \varphi \text{ atomic} \\ \text{Boolean connectives} & \iff \text{as in the propositional case} \\ s, g \models \exists x \varphi & \iff s, g[x \mapsto d] \models \varphi \text{ for some } d \in D \\ s, g \models \forall x \varphi & \iff s, g[x \mapsto d] \models \varphi \text{ for all } d \in D \end{aligned}$$

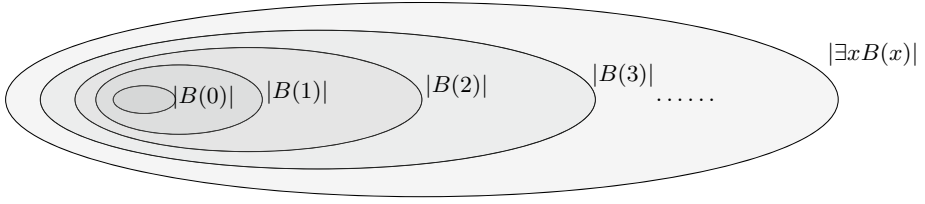
Based on support, we may define the informative content of a formula and prove that the treatment of information is classical. We may also define when a formula is inquisitive. However, there is a crucial thing that we *cannot* do: we cannot get a satisfactory notion of meaning by taking maximal supporting states, and indeed in any way which involves support alone. This is what the following examples show.

*Example 1.* Let our language consist of a binary function symbol  $+$  and a unary predicate symbol  $P$ ; let our domain be the set  $\mathbb{N}$  of natural numbers and let  $+$  be interpreted as addition. Moreover, let  $x \leq y$  abbreviate  $\exists z(x + z = y)$ .

Let  $B(x)$  denote the formula  $\forall y(P(y) \rightarrow y \leq x)$ . It is easy to check that a state  $s$  supports  $B(n)$  for a certain number  $n$  if and only if  $B(n)$  is true in all models in  $s$ , that is, if and only if  $n$  is an upper bound for  $P^M$  for any model  $M \in s$ , where  $P^M$  denotes the extension of the predicate  $P$  in  $M$ .

We claim that the formula  $\exists x B(x)$  –which expresses the existence of an upper bound for  $P$ – does not have any maximal supporting state. For, consider an arbitrary state  $s$  supporting  $\exists x B(x)$ : this means that there is a number  $n$  which is an upper bound for  $P^M$  for any  $M \in s$ .

Now let  $M^*$  be the model defined by  $P^{M^*} = \{n + 1\}$ .  $M^*$  does not belong to  $s$ , since we just said that the extension of  $P$  in any model in  $s$  is bounded



**Fig. 2.** The intended possibilities  $|B(n)|$  for the boundedness formula and its truth set  $|\exists x B(x)|$ , which is not itself a possibility

by  $n$ ; hence  $s \cup \{M^*\}$  is a proper superset of  $s$ . It is obvious that for any model  $M \in s \cup \{M^*\}$  we have  $P^M \subseteq \{0, \dots, n+1\}$  and thus  $M \models B(n+1)$ . Hence,  $s \cup \{M^*\} \models B(n+1)$  and therefore  $s \cup \{M^*\} \models \exists x B(x)$ . So,  $s \cup \{M^*\}$  is a proper extension of  $s$  which still supports  $\exists x B(x)$ .

This shows that any state that supports  $\exists x B(x)$  can be extended to a larger state which still supports the same formula, and therefore no state supporting  $\exists x B(x)$  can be maximal.

Let us meditate briefly on this example. What possibilities did *we* expect to come out of the boundedness example? Now,  $B(x)$  is simply supported whenever it is known to be true, so it has a classical behaviour. The existential quantifier in front of it, on the other hand, is designed to be satisfied only by the knowledge of a concrete bound, just like in the propositional case a disjunction (of assertions) is designed to be satisfied only by the knowledge of a disjunct.

Therefore, what we would expect from the boundedness formula is a hybrid behaviour: of course, it should inform that there is an upper bound to  $P$ ; but it should also raise the issue of *what* number is an upper bound of  $P$ . The possible resolutions<sup>1</sup> of this issue are  $B(0), B(1), B(2)$ , etc., so the possibilities for the formula should be  $|B(0)|, |B(1)|, |B(2)|$ , etc.

Now, the definition of possibilities through maximalization has the effect of selecting *alternative* ways to resolve the issue raised by a formula, i.e. ways which are incomparable relative to entailment. The problem is that obviously, if 0 is a bound for  $P$ , then so are 1, 2, etc.; if 1 is a bound, then so are 2, 3, etc. So, the ways in which the issue raised by the boundedness formula may be resolved cannot be regarded as *alternatives*. Still,  $B(0), B(1)$ , etc. are genuine solutions to the meaningful issue raised by the existential, and our semantics should be able to capture this.

This indicates that we need to come up with another way of associating a proposal to a formula; and if we are to be able to deal with the boundedness example, we need our notion to encompass proposals containing non-alternative possibilities. Notice that we cannot hope for a definition of such possibilities in terms of support: this is witnessed by the following example.

<sup>1</sup> For the precise definition of *resolutions* of a formula, the reader is referred to Ciardelli [2009].

*Example 2.* Consider the following variant of the boundedness formula:  $\exists x(x \neq 0 \wedge B(x))$ . Possibilities for this formula should correspond to the possible witnesses for the existential, and since 0 is *not* a witness, we expect  $|B(0)|$  *not* to be a possibility.

Thus, a system that represents the inquisitive behaviour of the existential quantifier in a satisfactory way should associate different possibilities to the formulas  $\exists x B(x)$  and  $\exists x(x \neq 0 \wedge B(x))$ . Capturing this distinction is quite important; for, intuitively, “Yes, zero!” would be a compliant response to “There exists an upper bound for  $P$ ”, but *not* to “There exists a positive upper bound to  $P$ ”, and being able to analyze compliance in dialogue is one of the principal aims of inquisitive semantics. However, the formulas  $\exists x B(x)$  and  $\exists x(x \neq 0 \wedge B(x))$  are equivalent in terms of support.

The point here is that, as argued in Ciardelli [2009], support describes the knowledge conditions in which the issue raised by a formula is resolved, but is not sufficiently fine-grained to determine what the resolutions of a formula are.

## 4 A First-Order Inquisitive Semantics

The discussion in the previous section indicates that we need to devise a non support-based notion of meaning which allows for non-alternative possibilities, i.e. possibilities which may be included in one another. In order to do so, we start from the observation that propositional inquisitive meanings may also be defined recursively, by means of an operator  $\text{MAX}$  which, given a set  $\Pi$  of states, returns the set  $\text{MAX}(\Pi)$  of maximal elements of  $\Pi$ .

### Definition 6

1.  $[p] = \{|p|\}$  if  $p \in \mathcal{P}$
2.  $[\perp] = \{\emptyset\}$
3.  $[\varphi \vee \psi] = \text{MAX}([\varphi] \cup [\psi])$
4.  $[\varphi \wedge \psi] = \text{MAX}\{s \cap t \mid s \in [\varphi] \text{ and } t \in [\psi]\}$
5.  $[\varphi \rightarrow \psi] = \text{MAX}\{\Pi_f \mid f : [\varphi] \rightarrow [\psi]\},$

where  $\Pi_f = \{w \in \omega \mid \text{for all } s \in [\varphi], \text{ if } w \in s \text{ then } w \in f(s)\}$

Restricting the clauses of this definition to indices belonging to a certain state  $s$  we obtain the proposal  $[\varphi]_s$  made by  $\varphi$  relative to the common ground  $s$ .

Now, the most obvious way to allow for non-maximal possibilities is to simply remove the operator  $\text{MAX}$  from the clauses. This strategy, pursued in my thesis [Ciardelli, 2009], changes the notion of meaning right from the propositional case.

In the resulting system, which we refer to as *unrestricted inquisitive semantics*, informativeness and inquisitiveness no longer exhaust the meaning of a formula. For, formulas such as  $p \vee \top$  are neither informative nor inquisitive, but they still

make a non-trivial proposal. Ciardelli et al. [2009] suggest that such formulas may be understood in terms of *attentive potential* and shows how the enriched notion of inquisitive meaning provides simple tools for an analysis of *might*. In this respect, the unrestricted system is a simple but powerful refinement of the standard system.

However, this solution has also drawbacks. For, in some cases the interpretation of possibilities included in maximal ones in terms of attentive potential does not seem convincing. For instance, consider a common ground  $s$  in which a concrete upper bound  $n$  for  $P$  is known, that is, such that  $s \models B(n)$ : intuitively, the boundedness formula should be redundant relative to such a common ground, that is, we should have  $[\exists x B(x)]_s = \{s\}$ . However, in the unrestricted system, the boundedness formula still proposes the range of possibilities  $B(0), \dots, B(n)$ , that is, we have  $[\exists x B(x)] = \{|B(0)| \cap s, \dots, |B(n)| \cap s, \emptyset\}$ .

The behaviour of the propositional connectives is sometimes also puzzling: for instance,  $(p \vee q) \wedge (p \vee q)$  also proposes the possibility that  $p \wedge q$  (but  $p \vee q$  does not), while the implication  $p \rightarrow ?p$  turns out equivalent with  $\neg p \vee \top$ .

My aim in the present paper is to outline a different road, to describe a way to extend propositional inquisitive semantics *as it is* to obtain a more “orthodox” predicate inquisitive semantics in which meaning still consists of informative and inquisitive potential.

**Definition 7.** *If  $\Pi$  is a set of states, say that an element  $s \in \Pi$  is optimally dominated in case there is a maximal state  $t \in \Pi$  with  $t \supsetneq s$ .*

In the unrestricted propositional semantics, due to the finitary character of propositional meanings, non-maximal possibilities are always properly included in some maximal one. Therefore, taking the maximal elements or filtering out optimally dominated ones are operations which yield the same result.

On the other hand, the example of the boundedness formula shows that the meanings we want to obtain in the first-order case may consist of an infinite chain of possibilities, none of which is maximal. Here, as we have seen, extracting maximal states in definition 6 leaves us with nothing at all; filtering out optimally dominated states, on the other hand, has no effect in this case and yields the intended meaning of the boundedness formula.

These observations lead to the idea of expanding definition 6 with the natural clauses for quantifiers (where the behaviour of  $\exists$  and  $\forall$  is analogous to that of  $\vee$  and  $\wedge$  respectively), while substituting the operator MAX with a more sensitive filter NOD which, given a set of states  $\Pi$ , returns the set of states in  $\Pi$  which are not optimally dominated. The result is the following definition.

**Definition 8 (First-order inquisitive meanings).** *The inquisitive meaning of a formula  $\varphi$  relative to an assignment  $g$  is defined inductively as follows.*

1.  $[\varphi]_g = \{[\varphi]_g\}$  if  $\varphi$  is atomic
2.  $[\perp]_g = \{\emptyset\}$
3.  $[\varphi \vee \psi]_g = \text{NOD}([\varphi]_g \cup [\psi]_g)$
4.  $[\varphi \wedge \psi]_g = \text{NOD}\{s \cap t \mid s \in [\varphi]_g \text{ and } t \in [\psi]_g\}$

5.  $[\varphi \rightarrow \psi]_g = \text{NOD}\{\Pi_f \mid f : [\varphi]_g \rightarrow [\psi]_g\}$
6.  $[\exists x \varphi]_g = \text{NOD}(\bigcup_{d \in D} [\varphi]_{g[x \mapsto d]})$
7.  $[\forall x \varphi]_g = \text{NOD}\{\bigcap_{d \in D} s_d \mid s_d \in [\varphi]_{g[x \mapsto d]}\}$

Again, the proposal  $[\varphi]_{s,g}$  made by  $\varphi$  relative to the common ground  $s$  and the assignment  $g$  is obtained by restricting the clauses to indices in  $s$ . Obviously, if  $\varphi$  is a sentence, the assignment  $g$  is irrelevant and we may therefore omit reference to it.

There is, however, a subtlety we must take into account. While in the propositional case a formula may propose the empty state only if it is inconsistent, with the given definition the empty state would pop up in totally unexpected circumstances, with unpleasant consequences in terms of entailment and equivalence; for instance, we would have  $[\exists x(x = 0 \wedge B(x))] = \{|B(0)|, \emptyset\} \neq \{|B(0)|\} = [B(0)]$ . To fix this problem, we modify slightly our definitions, stipulating that the empty state is optimally dominated in a set of states  $\Pi$  as soon as  $\Pi$  contains a non-empty possibility. For the rest, we can keep the definition of the system unchanged.

Notice that by definition of the operator NOD, we can never end up in an absurd situation like the one discussed in example 1, in which  $[\varphi] = \emptyset$  (in which, that is, a formula would propose *nothing*!) Moreover, it is easy to establish inductively the following fact, which shows that we have indeed defined a conservative extension of propositional inquisitive semantics.

**Proposition 2.** *If  $\varphi$  is a quantifier-free formula, then the meaning  $[\varphi]$  given by definition 8 coincides with the meaning of  $\varphi$  considered as a propositional formula, as given by definition 3.*

The system we defined can cope with the subtleties highlighted by example 2: formulas which are equivalent in terms of support may be assigned different meanings, and may even have *no* common possibility at all, thus differing dramatically in terms of the compliant responses they allow.

*Example 3.* In the context of example 1, let  $E(x) = \exists y(y + y = x)$  and  $O(x) = \neg E(x)$ ; clearly,  $E(x)$  and  $O(x)$  are assertions stating, respectively, that  $x$  is even and that  $x$  is odd. We have:

1.  $[\exists x B(x)] = \{|B(n)|, n \in \mathbb{N}\}$
2.  $[\exists x(x \neq 0 \wedge B(x))] = \{|B(n)|, n \neq 0\}$
3.  $[\exists x(E(x) \wedge B(x))] = \{|B(n)|, n \text{ even}\}$
4.  $[\exists x(O(x) \wedge B(x))] = \{|B(n)|, n \text{ odd}\}$

On the one hand, one knows an even upper bound for  $P$  iff one knows an odd upper bound, so the formulas  $\exists x(E(x) \wedge B(x))$  and  $\exists x(O(x) \wedge B(x))$  are resolved in exactly the same information states, which is what support captures. On the other hand, the sentences “there is an even upper bound to  $P$ ” and “there is an odd upper bound to  $P$ ” invite different responses, and the system rightly predicts this by assigning them distinct possibilities.

Moreover, unlike the unrestricted system, the proposed semantics correctly predicts that the boundedness formula is redundant in any information state in which an upper bound for  $P$  is known: if  $s \models B(x)$ , then  $[\exists x B(x)]_s = \{s\}$ .

Many features of the propositional system carry over to this first-order implementation. Crucially, meaning is still articulated in two components, informativeness and inquisitiveness. For, consider a  $\varphi$  which is neither informative nor inquisitive: since  $\varphi$  is not inquisitive,  $|\varphi| \in [\varphi]$ ; and since  $\varphi$  is not informative,  $|\varphi| = \omega$ ; finally, since the presence of the filter NOD explicitly rules out possibilities included in maximal ones,  $\omega$  must be the *unique* possibility for  $\varphi$ , that is,  $\varphi$  must be an inquisitive tautology.

Assertions and questions may be defined as usual, and it is still the case that for any formula  $\varphi$ ,  $!\varphi$  is an assertion,  $?\varphi$  is a question, and the decomposition  $\varphi \equiv !\varphi \wedge ?\varphi$  holds, where equivalence amounts to having the same meaning.

Obviously, the classical treatment of information is preserved, i.e. we have  $\bigcup[\varphi] = |\varphi|$ . Finally, the sources of inquisitiveness in the system are disjunction and the existential quantifier, in the sense that any formula not containing disjunction or the existential quantifier is an assertion.

## 5 Conclusions

In this paper we proposed a conservative extension of propositional inquisitive semantics to the first order setting, focussing on the essential changes that this move required. These were (i) to state the semantics in terms of a recursive specification of the possibilities for a sentence, rather than in terms of support; and (ii) to switch from the requirement of maximality to that of not being optimally dominated. These changes have no effect on the propositional case.

The proposed system was motivated here by the attempt to obtain correct predictions while retaining as much as possible of the propositional system: a very important thing which remains to be done is to provide a more conceptual justification for the given definitions.

Moreover, a task for future work is the investigation of both the logical features of the proposed semantics and its application to natural language, in particular to the semantics of interrogative sentences.

With regard to this latter aspect, notice that our logical semantics as such does not embody a specific theory on the semantic analysis of interrogatives. Instead, it offers a general logical framework in which also opposing empirical analyses may be formulated and studied. This is most obviously so for the Hamblin analysis of questions Hamblin [1973], which is covered by inquisitive existential quantification ( $\exists x Px$ ), and the partition approach of Groenendijk and Stokhof [1984], which is covered by universal quantification over polar questions ( $\forall x ?Px$ ). The treatment of *which*-questions in Velissaratou [2000], which analyzes such questions in terms of exhaustive answers, but not as partitions, may also be represented (by  $\forall x (Px \rightarrow ?Qx)$ ).

## References

- Balogh, K.: Theme with Variations. A Context-based Analysis of Focus. PhD thesis, ILLC, University of Amsterdam (2009)
- Ciardelli, I.: Inquisitive semantics and intermediate logics. Master Thesis, ILLC, University of Amsterdam (2009), <http://www.illc.uva.nl/inquisitive-semantics>
- Ciardelli, I., Roelofsen, F.: Generalized inquisitive semantics and logic. *Journal of Philosophical Logic* (2009a), (Forthcoming), [www.illc.uva.nl/inquisitive-semantics](http://www.illc.uva.nl/inquisitive-semantics)
- Ciardelli, I., Roelofsen, F.: Generalized Inquisitive Logic: Completeness via Intuitionistic Kripke Models. In: *Proceedings of Theoretical Aspects of Rationality and Knowledge XII* (2009b), [www.illc.uva.nl/inquisitive-semantics](http://www.illc.uva.nl/inquisitive-semantics)
- Ciardelli, I., Groenendijk, J., Roelofsen, F.: Attention! might in inquisitive semantics. In: *Proceedings of Semantics and Linguistic Theory XIX* (2009), [www.illc.uva.nl/inquisitive-semantics](http://www.illc.uva.nl/inquisitive-semantics)
- Groenendijk, J.: Inquisitive semantics: Two possibilities for disjunction. In: Bosch, P., Gabelaia, D., Lang, J. (eds.) *Seventh International Tbilisi Symposium on Language, Logic, and Computation*. Springer, Heidelberg (2009a)
- Groenendijk, J.: Inquisitive semantics: Questions, assertions, and hybrids. Amsterdam (2009b) (Manuscript), <http://www.illc.uva.nl/inquisitive-semantics>
- Groenendijk, J., Roelofsen, F.: Inquisitive semantics and pragmatics. In: Larrazabal, J.M., Zubeldia, L. (eds.) *Meaning, Content, and Argument: Proceedings of the ILCLI International Workshop on Semantics, Pragmatics, and Rhetoric* (2009), [www.illc.uva.nl/inquisitive-semantics](http://www.illc.uva.nl/inquisitive-semantics)
- Groenendijk, J., Stokhof, M.: *Studies on the Semantics of Questions and the Pragmatics of Answers*. PhD thesis, University of Amsterdam (1984)
- Hamblin, C.L.: Questions in Montague English. *Foundations of Language* 10, 41–53 (1973)
- Mascarenhas, S.: Inquisitive semantics and logic. Forthcoming Master Thesis, ILLC, University of Amsterdam (2009)
- Velissaratou, S.: Conditional questions and which-interrogatives. Master Thesis, ILLC, University of Amsterdam (2000)

# There Is Something about *Might*

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**Abstract.** In this paper we present an alternative interpretation of statements of epistemic possibility, which does not induce a consistency test on a common ground, as in (Veltman 1996), but which tests whether the possibility is supported by some update of the common ground, as in (Veltman 1984). The information space relative to which such claims are evaluated are taken to consist in the possible developments of a discourse in action. It is shown that this notion of *Might* not only behaves better logically and pragmatically speaking, but that it also allows for non-trivial attitude reports and questions about epistemic possibilities. These epistemic modal statements can also be understood to guide or focus the inquisitive actions of the discourse participants.

## 1 Epistemic Modalities

Epistemic modal operators like *Might* and *Must* in English, and semantically related verbs, adverbs and markers, express a kind of possibility or necessity relative to some body of knowledge, evidence, or other constraints. A sentence formalized as *Might*( $\phi$ ) (or:  $\Diamond\phi$ ) is used to express that  $\phi$  is not excluded relative to some source of evidence. In the standard semantic approach (Kratzer 1977) such a body of knowledge or evidence is conceived of as a set of possibilities (situations, worlds, ...), relative to which  $\Diamond\phi$  is true iff  $\phi$  is true with respect to some possibilities in  $K$ .

In the literature, this basic interpretation of the modalities has been challenged and modified in two respects. Firstly, epistemic modals are seen to be inherently contextual, or indexical. The relevant body of knowledge against which to evaluate epistemic modals has to be found relative to the discourse situation in which these modal sentences are uttered. Secondly, the relevant bodies of information have been argued to be those of the interlocutors in an actually unfolding discourse. Building on Stalnaker's idea of establishing common grounds, an utterance of  $\Diamond\phi$  has been taken to express consistency of  $\phi$  with the current information state of the interlocutors in a discourse. This idea has been formally developed in (Veltman 1996) and subsequent work.

Notoriously, such a consistency interpretation of  $\Diamond\phi$  can be deemed rather vacuous. While Veltman's update semantics is motivated in part by (Stalnaker 1978)'s idea that assertions, or utterances, are put to use to substantially contribute to a

common ground for the participants in a discourse, the epistemic test associated with  $\Diamond\phi$  appears to do nothing of the kind. In response to a claim that it might be the case that  $\phi$ , one can simply agree that it is consistent with the common ground that  $\phi$ , or just disagree that it is not. Upon the interpretation proposed, there is no other option available. Worse, assuming, as one would ideally do, that the common ground contains common knowledge, and that participants have the gift of introspection, a use of  $\Diamond\phi$  is utterly pointless, and would at best remedy possibly misconceptions of the common ground—while the remedies or required revisions typically remain beyond the scope of current systems of update or inquisitive semantics.

It has been suggested every here and there that epistemic modal statements additionally serve to “raise” possibilities, that they are used to bring us to “attend to” or “focus on” possibilities. (Hulstijn 1997; Groenendijk 2007; Yalcin 2008; Roussarie 2009; Brumwell 2009; Ciardelli, Groenendijk & Roelofsen 2009). However, it has so far remained unclear what exactly it means to raise a possibility, or for there to be one. As before, in response to a claim that  $\Diamond\phi$ , one might agree that, “Yes, there is the possibility that  $\phi$ .” or that “No, there is not.” but this will not all by itself serve to make  $\Diamond\phi$  any less pointless. Surely,  $\Diamond\phi$  can be taken to effectuate something like the presence or actuality of the possibility that  $\phi$  in the common ground. But what is the difference between a state of information with, or the same one without the possibility that  $\phi$ ? The only real, and vacuous, answer given is that the first does, and the second does not support that  $\Diamond\phi$ , or that the first does not, and the second does, require the hearer of the sentence to signal a possible inconsistency. (Ciardelli, Groenendijk & Roelofsen 2009) Nevertheless, it seems hardly anybody would deny that such possibility statements serve a non-trivial purpose, simply, because they have substance. In this paper I will polemically argue for this point by associating them with ordinary truth-conditions. As will come clear once we go along, nothing really hinges on the issue on whether to call these truth-conditions, or acceptability-conditions, or whatever conditions of your ilk.

The main idea pursued in this paper is that the epistemic *Might*-operator can be made more sense of if we revive an original interpretation of  $\Diamond$  as an ordinary modal operator defined over a space, not of simple possibilities, but of information states, as proposed in the so-called data semantics from (Veltman 1984; Landman 1986). Roughly,  $\Diamond\phi$  is taken to state that  $\phi$  holds in an update of the current information state. Like the  $\Diamond$ -operator from modal logic, which deems  $\Diamond\phi$  true in a situation (world, ...) iff there is an accessible situation (world, ...) in which  $\phi$  is true, epistemic *Might* ( $\Diamond\phi$ ) would be rendered true if there is an update, or extension, of the current information state in which  $\phi$  holds. As we will see below, this interpretation is practically sufficiently close to the interpretation of  $\Diamond\phi$  as a consistency test on information states; however, it also allows us to make more substantial sense of statements of epistemic possibility.

Veltman and Landman have originally focused on the logical aspects of their modal operators and related conditional sentences, but they have remained by and large silent about the set up of the space of information states in which the

modal operators get defined. There, it has been relatively classically assumed to be a fixed space, with a set of information states assumed given, together with a primitive and fixed extension or update relation. With all the work that has been done nowadays on the formal semantics and pragmatics of discourse, however, such spaces of information states and their updates have been and can be investigated and formalized in lots of further detail in the meantime. In this paper I want to show that indeed a neat formulation of  $\Diamond\phi$  can be given, drawing from the data semantics insights on *Might*, fleshing it out relative to a notion of a common ground, which is indexically linked to an actually occurring discourse. The space of updates or extensions of the relevant information states can be taken to consist in the future developments of the common ground in a discourse in action. And  $\Diamond\phi$  can be taken to state that  $\phi$  holds in a possible, maybe partial, resolution of the discourse.

## 2 Optimal Inquisitive Discourse

In order to implement the above ideas one can in principle take any classical or non-classical framework of interpretation, which deals with the raising and resolving of issues in discourse, like that of (Ginzburg1995; Roberts 1996; Hulstijn 1997; Groenendijk 2007; Ciardelli, Groenendijk & Roelofsen 2009), to name a few. For the present purposes it seems appropriate to build on my own (Dekker 2004; Dekker 2007), since the framework proposed there is framed in classical semantic and pragmatic terms, and arguably consistent with the others.

In (Dekker 2004; Dekker 2007) a notion of an optimal inquisitive discourse is defined that relates a set of agents whose epistemic states carry information and are troubled by questions. Let me first clarify what I mean with questions. There are questions which people have and questions people pose. Questions people have is what they wonder about, out of curiosity, but normally in relation to the Big Question, “What to do?” Questions people pose may and may not be questions people have, but normally they are, and they serve to make questions they have into issues which they share with others.

An simple way to model states with information and questions is given in (Groenendijk 2007) (originally from 1999) in which states are modeled by a symmetric and transitive relation on a set of possibilities. The idea is that possibilities that stand in that relation are considered possible ways the actual world or situation might be, and that the difference between connected possibilities is considered immaterial. Formally, a possibility  $i$  is considered to be a way the world might be in state  $\sigma$ , iff there is an  $i'$ , typically  $i$  itself, such that  $\langle i, i' \rangle \in \sigma$ . In such a case we say  $i \in D(\sigma)$ , with  $D(\sigma)$  representing the data in  $\sigma$ . If  $\langle i, j \rangle \in \sigma$ , it is considered no question whether the actual world is like  $i$  or  $j$ . However, if  $i, j \in D(\sigma)$ , and  $\langle i, j \rangle \notin \sigma$ , then the difference between the two does count. In that case the information state models the issue whether the actual world is an  $i$ - or a  $j$ -kind world. So  $I(\sigma) = D(\sigma)^2 \setminus \sigma$  can be taken to define the issues in  $\sigma$ . Some proposition or information state  $s$  answers a question state  $\sigma$ ,  $s \models \sigma$ , iff the information in  $s$  removes all issues in  $\sigma$ , i.e., iff  $(s^2 \cap I(\sigma)) = \emptyset$ .

The notion of an optimal inquisitive discourse in (Dekker 2004; Dekker 2007) is based on the simple assumption that agents involved in a communication aim to get their questions resolved in a reliable and respectable manner. In, indeed, the very simple cases, they have to do with the questions they have and with the information which is there, the joint information of the interlocuting participants. By the end of the day, the interlocutors want to get their questions resolved, so that they know what to do. Having no other information available than the information one has oneself, and what the others may provide, and, if necessary, the information from an oracle, the information which is exchanged and ends up in the common ground is ideally supported by the joint information of the interlocutors. Formally, a discourse situation involves a number of agents  $a_1, \dots, a_n \in A$ , each with their own (private) information and (private) questions, modeled by information states  $\sigma_1, \dots, \sigma_n$ , respectively. We also assume an oracle  $\mathcal{O} = \sigma_0$  to model the possibility of solicited and unsolicited information.

**Definition 1 (Optimal Inquiry).** *An inquisitive discourse  $\Phi$  among a set of agents  $a_1, \dots, a_n \in A$  with information states  $\sigma_1, \dots, \sigma_n$ , together with an oracle  $\mathcal{O} = \sigma_0$ , is optimal iff:*

- $\bigwedge_{1 \leq i \leq n} (D(\llbracket \Phi \rrbracket) \models \sigma_i)$  (relation)
- $D(\bigcap_{0 \leq i \leq n} (\sigma_i)) \subseteq D(\llbracket \Phi \rrbracket)$  (quality)
- $\Phi$  is minimal and well-behaved (quantity and manner)

(all relative to a standard intensional and indexical interpretation of  $\Phi$  in some pointed model  $\mathcal{M}$  where those agents have these information states).

Assuming  $\llbracket \Phi \rrbracket$ , the interpretation of the discourse  $\Phi$ , to convey information and raise issues, it can be rendered as an information state in its own right. The first requirement says that  $\Phi$  answers the questions of any participant. (This is the ‘ideal’ situation. If not all questions *can* be answered, we might say that an optimal discourse is one in which those are answered than can be answered.) In the second requirement  $\bigcap_{0 \leq i \leq n} (\sigma_i)$  presents the joint information and questions of the participants. The data provided by  $\Phi$  are required to be supported by the joint information of the participants. The minimality requirement obviously relates to Grice’s maxim of quantity and is motivated by the insight that the Big Question is never “What is the world exactly like?”, but, rather, “What to do?” with limited resources of information, reasoning, and time. A Gricean manner maxim is motivated by the observation that the exchange of information inherently involves engaging in a social practice.

The above definition indicates the way in which a discourse might ideally proceed. For instance: the participants each ask the questions they have, and the others give the required answers. We may first note a couple of intuitively obvious things, which some researchers think of theoretical significance: (i) an optimal inquisitive discourse can be impossible, since some questions simply can’t be answered with the tools and knowledge available; (ii) it need not be your intention to deliver or contribute to an optimal inquisitive discourse; (iii) you need not be rational. The above concept does not in any way exclude these

possibilities. Theoretically more interesting possibilities are those in which the idea or ideal of an optimal discourse is maintained, but in which non-standard methods have to be found to try and construe it.

It may be the case that the participants in a discourse fail the answer to the live questions, so that one may try to consult the oracle, but this one may fail the answer as well. The main goals can be achieved differently then, though. The required information may be there in a discourse situation, but distributed over the agents, or interlocutors.

An optimal inquiry, as defined, might run as follows.

- (1) *A*: Will Bernd be at the reception?
- (2) *B*: I don't know. He will be if he finished his grading.
- (3) *C*: Oh, but he just finished his grading.

This is an example where *B* provides unsolicited information, which nevertheless makes the exchange run smooth.

More interesting may be a case where it serves to ask a question one doesn't have, as has been elaborated in detail in (Dekker 2004; Dekker 2007). Someone may simply wonder whether or not to attend to the reception, the answer to which may depend on the configuration of lecturers attending it. Instead of spelling out the favorable and unfavorable configurations it may be worthwhile to simply ask which lecturers attend. A few sample answers of lecturers attending and those not attending may already suffice to get the original question answered. So-called conditional questions may also turn out to be very useful, potentially. I may ask "If Carla goes to the reception, will you go there as well?", and a positive reply to this question may sufficiently answer my own question in the sense that I then know I will not be going there. The moves in these examples are reasonable, because they *may* contribute to establishing an optimal exchange, even though they are not guaranteed to do so. The reason is that, while the global goal of an optimal exchange of information is clear, the agents have to act, and inquire, under uncertainty. It is against this general background that epistemic modality statements can be seen to make sense. Employing  $\diamond\phi$  one points at a possible resolution of the current discourse, and this may serve to point at a possibility which deserves further investigation. Of course, the ensuing investigation may always turn out negative after all.

### 3 Epistemic Modality in Discourse

The little discourse (1–3) above might have proceeded differently.

- (4) *A*: Will Bernd be at the reception?
- (5) *B*: He might have finished grading.
- (6) *A*: So, what?
- (7) *B*: If he has, he will definitely be there.

Upon this way of proceeding, the interlocutors have an incentive to go and find out whether John has indeed finished grading, that is, a new question has emerged from the possibility statement. Similarly, if I wonder whether or not to

go to the reception, and ask who will be there, the assertion that Bernd might be there would elicit a possibility that would directly decide my original question: if Bernd goes I wouldn't hesitate to go as well. Again it incites to investigate or query whether Bernd indeed will come. Finally, if we are looking for the bicycle keys, with the major issue being where the keys are, we are possibly facing a whole lot of questions, viz., for any possible location  $l$  the question whether the keys are at  $l$ . The statement that they might be in the basement would turn the main question into a more feasible one, viz, whether they are in the basement, and we may find reason to try and find evidence for that possibility, among the interlocutors, by consulting the oracle, or, what may amount to the same thing, go down the basement and look for the keys.

In each of the above cases, of course, there is no guarantee that the stated possibility will turn out true, or supported, and, hence, may help answer our question. Still, it does incite a specific investigative action, which may lead us to do at least something to achieve the required goal. By pointing at a possible resolution of the current discourse situation, one in which  $\phi$  holds, this automatically raises the question whether we can reach that state. This, naturally, provides the incentive to go and find out.

Before turning to the definition of the possibility statements themselves, we have to be more specific about possible resolutions of a discourse situation.

**Definition 2 (Possible Resolutions).** *A possible resolution of a discourse situation  $\mathcal{D}_i$  is a possible discourse situation  $\mathcal{D}_r$  that answers a reasonable update  $\mathcal{D}_{r-1}$  of  $\mathcal{D}_i$ .*

This definition is quite weak, because it allows for very partial resolutions of a discourse situation. (It also needs to be adjusted for the obvious that unreliable information (states) may have to be excluded.)

**Definition 3 (Epistemic Possibilities).**  $\mathcal{M}, v, g \models \Diamond\phi_i$  iff  $\mathcal{M}, \mathcal{D}_r, g \models \phi$  for a possible resolution  $\mathcal{D}_r$  of  $\mathcal{D}_{v,i}$ , where  $\mathcal{M}, \mathcal{D}_r, g \models \phi$  iff  $D(\mathcal{D}_r) \subseteq \llbracket \phi \rrbracket_{\mathcal{M}, g}$ .

The present definition of epistemic *Might* directly accounts for a number of typical features of its use. In the first place  $\Diamond\phi$  doesn't make sense in situations where  $\phi$  is an issue already, or where the issue whether  $\phi$  has been resolved. In the second place it is fully indexical. The truth of  $\Diamond\phi$  totally depends on the situation in the discourse where it is used, and on the information available there. In the third place it is non-persistent. Once new relevant information enters the common ground, the possibility that  $\phi$ , once acknowledged, may eventually have to be given up. By the same token, in the fourth, and final, place, the stated possibility or resolution should not be *any* theoretically possible update of the common ground: it should be a reasonably possible update, not one which is loaded with unsolicited details orthogonal to the issues which *are* raised in the common ground. The present definition thus suits some quite common opinions about epistemic *Might*.

By way of illustration, consider the following statement.

(8) Bernd might not go to the reception.

Out of the blue, this would appear to be a vacuous statement, to be rendered false indeed. However, in a context where one addresses Bernd's ex Denise, who wants to go to the reception, but who has plenty of reasons to not see Bernd, it makes sense. Suppose, that we are conversing about the reception and Ann knows that Pete goes to the reception, and Ben is sure that Pete will not go without his new friend Bernd. Denise makes the above statement. In the delicate circumstances, the statement seems true. However, in the same circumstances, delicate as they are, Ann and Ben may conjoin their information, and decide that, oops, Pete is going to the reception with Bernd. If Denise states that Bernd might not go to the reception, they will have to correct her. Notice that the sample statement changes from practically false, to deemed true, to eventually false again.

For a full account of possibility statements, and their use in discourse, we need of course to specify the notion of a reasonably possible update in much more detail. In part this will be framed against the background consisting of the interlocutors' understanding of an optimal inquisitive discourse, as defined above, but it will also have to take into account the actual discourse situation itself, the information the interlocutors have, about the (current stage of the) situation, and about each other's (lack of) information. We leave a specification of these details for the full version of the paper.

Although our understanding of epistemic modality is rather different, logically speaking, from Veltman's consistency *Might*, pragmatically speaking it makes quite similar predictions. For notice that, on the one hand, in run of the mill cases consistency of  $\phi$  with the common ground correlates with the theoretical possibility of an update with  $\phi$ . Moreover, on the second hand, the very fact that the update with  $\phi$  is suggested by any use of  $\Diamond\phi$  may automatically raise it as an issue in the current discourse, and, hence, as something true in a possible resolution of the ensuing discourse. Notice, though, that these systematic similarities are purely pragmatic, and, hence, defeasible.

For,  $\Diamond\phi$  can be rejected not just because of inconsistency of  $\phi$  with the common ground, but because an update with  $\phi$  is ruled out for other reasons. For instance, if  $\phi$  is refused as an issue. For instance, philosophically minded persons may at any moment bring up the possibility that there might be a cockroach in your coffee, that aliens from space may rule the world tomorrow, or that we are brains in a vat. Upon our understanding of might we need not believe these propositions to be false, in order to, still reject the accompanying statements of epistemic possibility. To accept these statements, it would normally require a reason to even consider the stated possibilities, while one may even also reject the possibility without further argument. Moreover,  $\Diamond\phi$  can be true and accepted even if  $\phi$  is inconsistent with our current implicit or explicit information. It may open up our eyes, for possibilities thoughtlessly excluded. Possibility statements may in principle announce or require an act of true belief *revision*. So while we may have not been truly believing that the keys are somewhere in the basement, but have been looking for them on the silent assumption that they are there, the announcement that we might have left them in the garage provides the incentive for another potentially very successful inquisitive action.

## 4 Questions and Beliefs about Modality

As defined, a possibility statement has truth-conditions, but its truth is very much context-dependent, unstable, and, hence, quite a bit negotiable. Nevertheless, with this little bit of truth-conditions  $\Diamond\phi$  may non-trivially figure in attitude reports and questions. As (Gillies & von Fintel 2008; Brumwell 2009; Roussarie 2009) have observed, the following sentences do not just report or question (in)consistencies, but true worries, beliefs and questions:

- (9) Timothy wonders whether he might go to the reception.
- (10) Sybille believes that he might stay home.
- (11) What do you think. Might Tim go somewhere else?

The present account can neatly account for this, but first observe that the interpretation of *Might* as just a consistency test appears to be quite inappropriate. When Tim is wondering whether he might go to reception, he is not just reflecting on his information. He is not inspecting his knowledge, with the question, “Well, is my information state consistent with this possibility?” Also, saying that Sybille believes that Ben might stay home does not just require that her information state be consistent with that possibility. The fact that her information does not exclude such a possibility is not sufficient for such an attribution to be true. (For, otherwise she could be attributed all kinds of epistemic possibilities about the whereabouts of my cousins whom she has never heard of.) Also, a question with *might* in it, as in (11) would really be no question. Assuming the common ground is public, we are all supposed to know whether it does or does not exclude the possibility that Tim goes somewhere else. Neither does it seem to ask for our beliefs about the common ground. (Like, “We are having a common ground together, but we don’t know what it is.”)

On the account presented in the previous section these statements gain full weight. Example (9) can be taken to state that Tim indeed wonders whether there is a reasonably possible update of his current state into one in which he comes—or if there is no such update. This does not require deciding yet, it is more like deciding if it is still conceivable to possibly decide positive. (Of course, if the outcome is negative, he would consistently decide he will not go, we hope.) Likewise, example (10) can be taken to state that Sybille believes that there is a reasonably possible update of her state to one in which Tim stays home. And finally, example (11) may be taken as a genuine question whether there is a reasonably possible update of the common ground in which  $\phi$  holds.

A formally elaborate account of these observations have been presented at the Colloquium. It requires not much more than an accessibility relation for epistemic agents, assigning them information states (relations) of the kind employed here, and a Hintikka-style analysis of beliefs. The only true extension is that discourse contributions must be understood entirely indexically. They relate to and contribute to any discourse situation in which they actually occur. For the rest, it is all standard truth conditions. Example (9), for instance, will be deemed true in a world  $v$  in a discourse  $\mathcal{D}_i$ , iff the information state

of Tim in  $v$  distinguishes worlds in which there is from those in which there is not a resolution of his current the discourse situation in which he goes to the reception.

## 5 Conclusion

In this paper I have presented a more or less classical interpretation of statements of epistemic possibility, according to which  $\Diamond\phi$  states that  $\phi$  holds in an update, or resolution, of the common ground. These statements have content, which make them suitable for use in non-trivial attitude reports and questions about epistemic possibilities. By staging and explaining *Might* utterances within a context of investigative discourse, these statements can be understood to guide and focus our inquisitive actions.

For substantial parts of the present proposal, intuitive motivation has been given. Modeling data or information in terms of non-excluded possibilities has been given the required philosophical motivation in the work of Frege, Wittgenstein, and Tarski. Modeling questions has been independently motivated using the tools and ideas of decision theory, as it has been most perspicuously formulated in the proposals from (van Rooy 2003). By understanding discourse acts as moves towards the goal of an optimal inquisitive discourse, we may now also gain understanding the use of possibilities ‘attended to’.

The perspective on the use of modality statements in discourse, which I have offered in this paper, may provide motivation for the primitive or stipulated concept of attending to possibilities, as in (Yalcin 2008; Roussarie 2009; Brumwell 2009; Ciardelli, Groenendijk & Roelofsen 2009). It seems notions of ‘congruence’, ‘answerhood’, or ‘compliance’, or other such pseudo-grammatical notions have no explanatory value when one tries to understand and explain what moves agents make in a communicative discourse, e.g., when people pose question they don’t have, or provide information unasked for. By viewing linguistic exchanges in the context of actual inquiries, for instance by means of a concept of an optimal inquisitive discourse, a lot of the data fall in place.

The present proposal seeks to understand the discourse contributions as more or less reasonable attempts to engage in the larger project of achieving an optimal inquisitive discourse. It is only relative to the wider goal of effective and reliable communication, of situated agents, that we can understand what the individual contributions can be taken to try or mean. In such a setting, it appears to be very reasonable indeed to sometimes raise questions and provide data which have been unsolicited, and, typically, to raise possibilities to attention, like we do with epistemic modality statements. A global perspective on discourse, and I think this is the one Grice originally must have had in mind, seems to automatically make sense of these contributions.

I would like to conclude the paper with a final observation, in line with the present discussion. First, maybe Goldbach’s second conjecture is true, while it is false to say that it might be true. We simply don’t know. Second, it is not so that we might be all wrong about everything. Surely, this is not to say that we are right about anything.

## References

- [Brumwell 2009] Brumwell, C.: A Dynamic Analysis of Epistemic Possibility. Master's thesis, ILLC, Universiteit van Amsterdam (2009)
- [Ciardelli, Groenendijk & Roelofsen 2009] Ciardelli, I., Groenendijk, J., Roelofsen, F.: *Might and Free Choice in Inquisitive Semantics*. In: Proceedings of SALT 18, Ohio State University, Ohio (2009)
- [Dekker 2004] Dekker, P.: Contexts for Questions. In: Hunyadi, L., Rákosi, G., Tóth, E. (eds.) *Proceedings of the Eighth Symposium of Logic and Language*, pp. 47–58. University of Debrecen, Debrecen (2004)
- [Dekker 2007] Dekker, P.: Optimal Inquisitive Discourse. In: Aloni, M., Butler, A., Dekker, P. (eds.) *Questions in Dynamic Semantics, CRiSPI 17*, pp. 83–101. Elsevier, Amsterdam (2007)
- [Gillies & von Fintel 2008] Gillies, A., von Fintel, K.: CIA Leaks. *The Philosophical Review* 117, 77–98 (2008)
- [Ginzburg1995] Ginzburg, J.: Resolving Questions, I & II. *Linguistics and Philosophy* 18(5,6), 459–527, 567–609 (1995)
- [Groenendijk 2007] Groenendijk, J.: The Logic of Interrogation. In: Aloni, M., Butler, A., Dekker, P. (eds.) *Questions in Dynamic Semantics. CRiSPI 17*, pp. 43–62. Elsevier, Amsterdam (2007)
- [Hulstijn 1997] Hulstijn, J.: Structured Information States. Raising and Resolving Issues. In: Benz, A., Jäger, G. (eds.) *Proceedings of MunDial 1997*, pp. 99–117. University of Munich (1997)
- [Kratzer 1977] Kratzer, A.: What *Must* and *Can* Must and Can Mean. *Linguistics and Philosophy* 1 (1977)
- [Landman 1986] Landman, F.: *Towards a Theory of Information*. Foris, Dordrecht (1986)
- [Roberts 1996] Roberts, C.: Information structure in discourse. In: Yoon, J.H., Kathol, A. (eds.) *Working Papers in Linguistics* 49, pp. 91–136. Ohio State University (1996)
- [van Rooy 2003] van Rooy, R.: Questioning to resolve decision problems. *Linguistics and Philosophy* 26, 727–763 (2003)
- [Roussarie 2009] Roussarie, L.: What might be known: epistemic modality and uncertain contexts. In: *Journées Sémantique and Modélisation (JSM 2009)*, Paris (2009)
- [Stalnaker 1978] Stalnaker, R.: Assertion. In: Cole, P. (ed.) *Syntax and Semantics 9 – Pragmatics*, pp. 315–332. Academic Press, New York (1978)
- [Veltman 1984] Veltman, F.: Data Semantics. In: Groenendijk, J., Janssen, T., Stokhof, M. (eds.) *Truth, Interpretation and Information*, pp. 43–63. Foris, Dordrecht (1984)
- [Veltman 1996] Veltman, F.: Defaults in Update Semantics. *Journal of Philosophical Logic* 25(3), 221–261 (1996)
- [Yalcin 2008] Yalcin, S.: *Modality and Inquiry*. Ph.D. thesis, Massachusetts Institute of Technology (2008)

# Incommensurability

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**Abstract.** This paper discusses subcomparatives with ‘incommensurable’ adjectives (e.g. *beautiful* and *intelligent*), which have received little attention in the literature so far. This is surprising, as the topic is of great importance for the current discussion with respect to the choice between a vague predicate analysis and degree-based approaches to gradability. This paper studies the properties of comparisons involving ‘incommensurable’ adjectives on the basis of a new collection of data. A confrontation of the data with both degree-based and non-degree-based theories offers evidence for the latter, and more in particular for a more constrained version of Klein’s analysis ([11],[12]) as presented in Doetjes, Constantinescu & Součková [6].

**Keywords:** subcomparatives, vague predicate analysis, degrees, comparison of deviation, relative comparison.

## 1 Introduction

In the literature, different judgments can be found for adjectival subcomparatives with so-called ‘incommensurable’ adjectives. Adjectival subcomparatives are comparatives which contain an overt adjective both in the main clause and in the *than*-clause. These two adjectives usually differ from one another. An example is given in (1):

(1) The table is longer than the desk is wide.

The adjectives *long* and *wide* correspond to the dimensions of length and width respectively, and these dimensions can be measured by the same measurement system. According to Kennedy [9], the number of adjectives that may occur in subcomparatives is limited by the fact that adjectives in these structures need to be commensurable. In case they are incommensurable, like the adjectives in (2), the sentence is not felicitous. Thus he concludes that incommensurability constitutes an argument against the vague predicate analysis of adjectives as developed by Klein ([11],[12]) and recently defended by Van Rooij [16].

(2) #My copy of *The brothers Karamazov* is heavier than my copy of *The Idiot* is old.

Even though this example is convincing, and indeed seems to be rather odd, Bartsch & Vennemann ([1]:91) discuss another case of a comparative with supposedly incommensurable adjectives, and claim that the sentence is fine:

(3) Marilyn is more beautiful than she is intelligent.

As Bartsch & Vennemann indicate, the important reading is not the metalinguistic one (where *more* could be replaced by *rather*), but the reading in which a comparison is made between Marilyn's beauty and her intelligence. Surprisingly, the relevant reading of (3) is mostly ignored in the literature (with the exception of [1], [8], [2], [3]), and if addressed, relatively few examples are taken into account. Some of Bale's [2] examples are given in (4):

- (4) a. Seymour is as intelligent as Esme is beautiful.
- b. If Esme chooses to marry funny but poor Ben over rich but boring Steve, [...] Ben must be funnier than Steve is rich.
- c. Although Seymour was both happy and angry, he was still happier than he was angry.

The properties of subcomparatives with so-called incommensurable adjectives are important for the way gradability is represented. The only way to handle this type of phenomenon in a degree-based approach is to assume some sort of a mapping mechanism that turns the incommensurable degrees, that is, degrees on different scales, into objects that may be compared (cf. [1], [8], [2], [3]). On the other hand, sentences such as (3) and (4) can be seen as an argument in favor of the vague predicate analysis, as the vague predicate analysis does not introduce degrees on different scales, and as such no mapping mechanism is required.

The first part of this paper examines a collection of examples of subcomparatives containing 'incommensurable' adjectives in English and Dutch. I will argue that sentences such as (3) and (4) have to be seen as a subcase of what I will call Relative Comparison or **RC** (following [6]), and that Comparison of Deviation ([9],[10]) should also be seen as an instance of RC. I will also discuss conditions on RC that can account for the contrast between (2) on the one hand, and well-formed cases of RC on the other. In the second part of the paper, the data will be confronted to both the vague predicate analysis and theories of gradability that make use of degrees. I will argue that RC should be seen as evidence for a constrained version of the vague predicate analysis, as proposed by Doetjes, Constantinescu & Součková [6].

## 2 What Is Relative Comparison?

The sentences in (3) and (4) raise two different questions. In the first place, one needs to know whether the phenomenon exemplified in (3) and (4) is limited to subcomparatives with incommensurable adjectives, or whether there are also sentences with commensurable adjectives that exhibit similar behavior and should be analyzed in the same way. In the second place, given the contrast between the judgments given in the literature for (2) and (3)-(4), one wants to know under what conditions this type of sentences can be used.

Before addressing this second question, I will first discuss some properties of these structures. More in particular, I will argue that RC is a rather broad phenomenon, which covers all subcomparatives with a relative interpretation, excluding only subcomparatives with an absolute interpretation such as (1) above, in which the absolute width of the table is compared to the absolute length of the desk (cf. [6]).

According to Bale [2], relative comparison (in his terms ‘indirect comparison’) is not restricted to subcomparatives with incommensurable adjectives. It also occurs in elliptical comparatives with two different norms, as in (5):

- (5) Ella is heavier for a baby than Denis is for a three year old.

As Bale notes, an ordinary degree-based theory would expect this type of sentences to be impossible, as (5) certainly does not compare the weight of the baby to the weight of the three year old in absolute terms. What is compared here is the relative weight of the baby (as compared to other babies) and the relative weight of the three year old (as compared to other three year olds).

The sentence in (5) is actually very similar to subcomparatives with two polar opposites and a comparison of deviation interpretation, as in (6a) below. Kennedy ([9],[10]), who discusses this type of sentences in detail, argues that direct comparison of the degrees corresponding to polar opposites is not possible, as these form different objects. In his theory, degrees introduced by positive adjectives constitutes positive extents (ranging from zero to some point on the scale), while degrees introduced by negative adjectives correspond to negative extents (ranging from some point on a scale to infinity). As a result, the positive adjective *tall* conveys information about the height an object has, while the negative adjective *short* conveys information about ‘the height an object does not have’ ([9]:193). As comparison of two degrees is based on the inclusion relation, this way of modeling positive and negative degrees excludes comparison of a positive and a negative degree even if they are defined as degrees on the same scale. However, in this scenario there is a way out. Comparatives and equatives with two polar opposites that make use of the same scale may be interpreted as instances of Comparison of Deviation (COD). Kennedy derives the example in (6a) as in (6b), which results in a comparison of the two differential extents, measuring the difference between the actual degree and the standard. The ZERO function maps the two differential extents onto two extents that both start at the zero point of the same scale, and as such can be compared.

- (6) a. The Cubs are as old as the White Sox are young.  
 b.  $\text{ZERO}(\text{old}(\text{Cubs}) - d_{s,\text{oldness}}) \geq \text{ZERO}(\text{young}(\text{White Sox}) - d_{s,\text{youthness}})$

Kennedy’s analysis predicts that COD is restricted to adjectives that project degrees on the same scale (antonyms and dimensional adjectives that are compatible with the same measurement system, such as *long* and *tall*). As such, he excludes the possibility of COD in subcomparatives with incommensurable adjectives (cf. (2)). The analysis of sentences such as (6a) as involving a comparison of the deviation from the standard implies that the degrees are at least equivalent to the standard value, and as such these sentences entail that the positive form of the adjectives holds of the subject (cf. [5] for the meaning of the positive).

For Bartsch & Vennemann [1] there is no fundamental difference between sentences such as (6) (Kennedy’s COD) and cases such as (2) and (3) (comparisons with incommensurable adjectives). In both cases, the comparison concerns the difference between the actual degree and the norm, and as such they are all analyzed in terms of a comparison of the deviations from the respective standards introduced by the two adjectives. In the case of dimensional adjectives, this comparison makes use of a conventional measure. In the case of sentences such as (3), they propose that

specific and average beauty/intelligence values can be assigned numbers on a single scale (specific and average BQs and IQs, as they call them). As such, the differences between the specific and the average values may be compared. Hamann, Nerbonne & Pietsch [8] arrive at a similar result by forcing the standard values corresponding to the two adjectives to be mapped onto the same point of the derived scale.

However, Bale [3] argues that (2) and (3) cannot be analyzed in a similar way to COD, as this type of sentence does not entail the positive, as illustrated in (7).

(7) Unfortunately, Mary is more intelligent than Medusa is beautiful.

Bale concludes that the neutralization effect found in normal comparatives (cf. [4]) is also present in this type of comparison and that the analysis of (2) and (3) should not introduce the standard. This implies that there are two different phenomena: COD on the one hand, which receives an analysis along the lines of (6a) and as such is restricted to polar opposites, and relative comparison (his indirect comparison) on the other, which accounts for cases with ‘incommensurable’ adjectives.

According to Bale, the difference in interpretation between COD and relative comparison is correlated with the use of the analytic versus the synthetic form of the comparative. However, there are two facts that complicate the picture. In the first place, the use of an analytic comparative as opposed to a synthetic form may introduce an evaluative interpretation of the adjective (see [6], [15]), and as such the lack of a neutral interpretation may well be directly due to the use of the analytic comparative, which would make it independent from the use of a subcomparative with two polar opposites and from the analysis in (6a). On the other hand, it is questionable whether the neutralization effect Bale talks about is always available, even for sentences with a synthetic comparative. In this regard it is interesting to look at COD sentences in German, where only the synthetic form is used. Yet, these sentences have a comparison of deviation type of interpretation, which corresponds to a non-neutral interpretation [4]. The sentence in (8) entails that the positive form of the adjectives holds of the subjects, contrary to what is found in (7).

(8) ?Hans is kleiner als Eva groß ist  
 ‘Hans is shorter than Eva is tall’.

At this point the picture seems to be rather complicated: on the one hand, people do not agree on whether we are dealing with one phenomenon or with two. On the other hand, even though neutralization effects can be found, as shown by Bale’s data, they do not always occur. Obviously one could say that (7) is a case of relative comparison while (8) is a case of COD, but this does not explain why this would be so. More in particular, there does not seem to be any reason not to apply Bale’s analysis to cases such as (8), and this raises the question why the effect found in (7) is necessarily absent in (8).

When looking at the sentences Bale uses in order to show that the neutral interpretation exists, it turns out that they have two things in common. In the first place, they contain positive adjectives (*beautiful, intelligent, pretty*), and in the second place, they all have a strong ironical flavor. This is also clear in the example in (9), which is an attested Dutch sentence (internet).

- (9) Gelukkig was [de hond] veel slimmer dan hij mooi was.  
 'Luckily the dog was much smarter than he was good-looking.'

The sentence in (9) does not only lack the presupposition that the dog is good-looking, it strongly suggests that the dog is ugly, and in this respect the interpretation differs from a neutral or non-evaluative interpretation. As such the sentence can be seen as a case of what Leech [13] calls 'criticism under the guise of praise': even though the dog is claimed to be pretty, the person who uses this sentence wants to convey that the dog is ugly. Given that praise usually involves positive adjectives, one expects this effect to arise only when positive adjectives are used. Interestingly, when one tries to formulate a negative counterpart of (9), one does not succeed. There is no way to interpret (10) without presupposing that the dog is ugly.

- (10) Jammer genoeg was [de hond] veel dommer dan hij lelijk was  
 'Unfortunately the dog was much more stupid than he was ugly'.

The effect in (9) might be analyzed as resulting from an 'ironic standard': the normal standard corresponding to the adjective *mooi* has been replaced by an ironic standard, which stretches up the domain and as such permits to even include the ugly dog in the set of good-looking individuals. Obviously this effect is by no means limited to comparatives (cf. *Mooie hond heb je daar!* 'That's a pretty dog you've got there!', which may be used ironically as well).

Given that it is possible to force a neutral reading of the first adjective, as in (7), it should be possible to stretch up the domain of both adjectives. A closer look at the data shows that this seems to be the default case. Evidence for this comes from the fact that it is very hard to get the ironic reading of the sentence in (9) when the first adjective *slim* 'smart' is replaced by its negative counterpart *dom* 'stupid', as in (11a). Moreover, in equatives with two positive adjectives, it is not possible to interpret only one of the two adjectives ironically. In (11b), either both adjectives are ironical, or both are not. If one assumes that the ironic reading forces an 'ironic standard' for both adjectives, these restrictions can be understood.

- (11) a. Jammer genoeg was [de hond] veel dommer dan hij mooi was  
 'Unfortunately the dog was much more stupid than he was good-looking'.  
 b. De hond was even slim als hij mooi was  
 'The dog was as smart as it was good-looking'.

An analysis of cases such as (7) and (9) in terms of an ironic 'standard' has an important advantage for the interpretation of the data. It makes it possible to assume that even (7) and (9) involve a comparison of deviation in the sense that they presuppose the positive. However, in this case, this positive has an ironic interpretation. The apparent lack of this type of effect in the traditional COD environments, and in particular in (8), follows from the fact that the domain of both adjectives needs to be stretched up, while this is only possible when a positive adjective is used. As such the effect is not expected to occur in sentences such as (8), which contain both a positive and a negative adjective.

A further argument for treating RC and the traditional COD cases with polar opposites as one single phenomenon has been offered in [6]. In this paper, it is claimed that in both types of sentences a similar interpretation is obtained, which is

not the interpretation in (6b) above. Standard cases of COD are argued to not involve a comparison of differential extents in an absolute sense, as predicted by Kennedy's analysis, but rather in a relative sense: if the two standards introduced by the two adjectives are clearly different, the same deviation (in absolute terms) counts as a smaller deviation from the higher standard than from the lower one. This can be illustrated by the example in (12a), which is arguably true under the COD interpretation in (12b) [10]:

- (12) a. The Sears Tower is as tall as the San Francisco Bay Bridge is long.  
 b. The degree to which the Sears Tower exceeds a standard of tallness (for buildings) is at least as great as the degree to which the San Francisco Bay Bridge exceeds a standard of length (for bridges).

If one compares the differential extents, one cannot do so in an absolute way, given that the total length of Sears Tower (527 meters) might well be less than the difference between the length of San Francisco Bay Bridge (5,920 meters) and the standard length for bridges. Such a scenario could still make the sentence true, as long as the two differential extents are comparable to one another in a relative way, given the size of the standard. The deviations are rather measured as a percentage of the standard than as an absolute value.

To conclude the first part of the section, there are good reasons to treat the original COD cases (involving polar oppositions) and subcomparatives with incommensurable adjectives as manifestations of one single phenomenon. In the first place, these sentences presuppose the positive (even though this fact may be obscured by the effect of irony). Moreover, all of these cases involve a relative, strongly context dependent interpretation, which makes them very different from subcomparatives that involve an absolute comparison such as the one in (1).

The next question to address is what constraints are placed on relative comparison. As observed at the beginning of this paper, not all combinations of adjectives seem to lead to a felicitous result (cf. (2)). However, a well chosen example in the right type of context can be fully felicitous and there is no reason to assume that the structure as such is ungrammatical. In the remainder of this section, I will argue that RC requires the two adjectives to be semantically or contextually associated to one another.

A closer look at the difference between the infelicitous example in (2) and for instance the fully felicitous example in (9) is that it is not easy to find a connection or relation between the two adjectives used in (2) (*heavy* and *old*). On the other hand, the two adjectives used in (9) (*mooi* 'pretty, good-looking' and *slim* 'smart') are conventionally associated to one another ('the looks and the brains'). It is not by accident that both Bartsch & Vennemann and Bale use many examples with similar adjectives (see (3) and (4a)); this type is also very easy to find on the internet.

When looking at other examples and at the contexts in which they are used, one can find further evidence for the idea that there needs to be some kind of association between the two properties in order for the sentence to be felicitous. This association can be of various kinds. In the original COD-cases, for instance, antonymy or 'near' antonymy as in (12) and (13), seems to play a role in licensing the use of the RC structure (note that in (13) the analytic form of the comparative is used).

- (13) Do you see a rectangle, that is taller than it is narrow? That's what I see and that's what other people see when you are wearing a long dress.

In many cases context plays a crucial role. The examples in (14) illustrate some felicitous uses of the adjective *zwaar* 'heavy' which contrast with its infelicitous use in (2). In (14a) (source: internet), the adjectives (*zwaar* 'heavy' in the main clause and *sappig en aromatisch* 'juicy and aromatic' in the *than*-clause) are all used to characterize peaches as being delicious. In (14b) (source: internet), the adjectives *smakelijk* 'tasty' and *zwaar* 'heavy' are both typical characterizations of a meal.

- (14) a. [We hebben de] laatste Hongaarse perzikken (*sic*) gekocht, 4 in 800 gram (en even sappig en aromatisch als ze *zwaar* zijn).  
'We bought the last the last Hungarian peaches, 4 in 800 grams (and as juicy and aromatic as they are heavy).'
- b. Gelukkig was het [eten] even smakelijk als het *zwaar* was.  
'Luckily the meal was as tasty as it was heavy.'

The example in (14b) is interesting for another reason as well. It falls in a class of examples in which the two adjectives give a positive and a negative qualification, and as such suggest that the positive property compensates for the negative one. Some more examples of this kind are given in (15) (source: internet):

- (15) a. Exercise is far more invigorating than it is tiring.  
b. [Een tweeling hebben] is minder *zwaar* dan het leuk is.  
'Having twins is less difficult than it is fun.'

Besides these cases and the ones with antonyms, the two adjectives usually have the same connotation and polarity. In many cases, both properties could explain a certain situation, and the sentence evaluates the contribution of each property (see also Bale's example in (4b)):

- (16) "Als de graaf een schelm en een schurk is, zou hij het Geschrift nooit aan zijn neef gegeven hebben!" zei Frans. "Of hij moet nog dommer zijn dan hij schurkachtig is."  
(Tonke Dragt, *De zevensprong*)  
"If the count is a scoundrel and a villain [as you were saying], he would never give the Manuscript to his nephew!" said Frans. "Or he has to be even more stupid than he is villainous."

Finally, there are some rare cases in which the adjective in the *than*-clause expresses a particularly salient property of its subject, as in (17), taken from a poem by Gaston Bursens. The use of the RC structure insists on the silence of the willows by comparing it to a contextually salient property that is known to hold to a high degree.

- (17) De wilgen zijn nog stiller dan ze krom zijn  
'The willows are even more silent than they are bent'.

To conclude, subcomparatives with so called incommensurable adjectives fall into a much larger class of subcomparatives with a non-absolute interpretation, which also includes the traditional cases of comparison of deviation. In some cases the non-neutral reading these sentences have, may be obscured by a stylistic use of the structure, involving an ironic interpretation of the adjectives. These comparisons may contain all sorts of adjectives, but in order to have a felicitous result, the two adjectives need to be associated to one another. As the examples above show, there are various ways in which this association can be obtained.

### 3 Theoretical Consequences

At this point it is clear that subcomparatives with ‘incommensurable’ adjectives are not excluded and thus a complete theory of comparatives has to be able to derive them. As indicated in the introduction, these sentences cannot be handled by a standard degree-based approach, because these adjectives do not project comparable degrees on a single scale. Various authors [1], [8], [2], [3]) solve this problem by mapping the degrees to different objects that can be compared, as indicated above. In the vague predicate analysis such a complication is not necessary in order to deal with this type of comparatives, which may be seen as an argument in favor of this approach. In what follows I will discuss a number of possible accounts in the light of the empirical generalizations made in the previous section.

Bartsch and Vennemann [1] argue in their account of sentences such as (2) that these have to be treated on a par with COD cases. For them, a sentence such as (2) involves a scale on which specific and average values for beauty and intelligence (“specific and average BQ’s and IQ’s as they call them) can be assigned in such a way that these numerical values can be compared. The interpretation of the sentence amounts to a comparison of the deviations between the specific and the average values for beauty and intelligence respectively. For COD sentences involving dimensional adjectives, they make the same assumption. However, in this case the grammar can make use of measures, such as feet or centimeters. Interestingly, this is the point at which their proposal makes a false prediction. As shown above, COD makes a comparison between the relative lengths of two differential extents and not between their absolute lengths. This is not expected in their proposal, as their analysis of the incommensurable cases is modeled on the existence of measurement systems.

Bale [2],[3] offers a detailed analysis of the mapping between degrees on ordinary scales to degrees on a universal scale. In his view, the only difference between cases such as (1) (absolute comparison, where two measures are compared in an absolute way) and (3) (relative comparison) is that the domain of individuals that has to be taken into account for sentences such as (1) contains measures (which he considers to be a special type of individuals). Given that sentences such as (1) normally have neutral interpretations (see [7] for discussion), Bale predicts RC sentences to have a neutral interpretation as well. As such he fails to account for the limited nature and the ironic effect of this type of interpretation that has been illustrated in (9)-(11) above. A further problem of the type of mapping Bale proposes (which I will not describe in detail here for reasons of space) is that he predicts a fine-grainedness that is not justified by the data. Bale reconstructs the precise position of every degree on the universal scale from the relative position the individual occupies on the primary scale with respect to other values on that scale. He assumes that a value on the primary scale is mapped onto a fraction on the universal scale. This fraction corresponds to the position of the value (where the lowest value equals one and the highest value equals the total number of values) divided by the total number of values on the scale. This is problematic in two respects. On the one hand, RC does not require the amount of information about the domain that Bale’s system needs, and on the other hand, the meaning of these sentences is not as clear-cut as he predicts. RC is a coarse-grained phenomenon. Take the interpretation of the equative in (24a). The sentence implies that the peaches are both very juicy and aromatic and very heavy,

and the use of a comparative rather than an equative would only be possible if for instance the peaches were extremely juicy and aromatic while being only slightly bigger than average. In this respect, a less constrained mapping, as proposed by Hamann, Nerbonne & Pietsch [8] is to be preferred. However, as shown below, the coarse-grained nature of RC follows directly from an account of comparatives that takes the vague predicate analysis as a starting point.

In approaches to comparatives based on the vague predicate analysis, the meaning of RC cases (that is, including the original cases of COD) involves the use of degree functions such as *quite*, *very* and *extremely* (cf. [12]:130, [14]). A sentence such as (4c) would be analyzed as in (18), where *d* could be *quite*, *very* or *extremely*.

(18)  $\exists d[(d(\text{funny}))(\text{Ben}) \wedge \neg(d(\text{rich}))(\text{Steve})]$

This captures the coarse-grained nature of RC, as these modifiers are vague themselves and only allow for a rough division of the domain. This is an advantage over degree-based approaches, as these necessarily involve a mapping, and this mapping may be done in a very precise way. Also, the fact that we are (necessarily) dealing with a rough type of comparison seems to be at least part of the reason why the two adjectives in this structure need to be associated to one another. Consider again the equative in (24a). The only information the comparative conveys is that the peaches are very juicy and aromatic. The fact that they are heavy is already present in the context. However, by using the RC structure, the fact that all these properties add to the satisfaction of the person who bought the peaches is focused on. A further advantage of this type of approach is that it predicts the relative interpretation of COD-cases involving a dimensional adjective to be the only possible one, as the interpretation of expressions such as *very* and *extremely* varies with the standard. Finally, given that *John is quite/ very/ extremely tall* cannot be followed by the sequence *#but he is not tall*, the use of these modifiers makes it in principle possible to derive the COD-type interpretation of RC.

This last point is more complex, however. When looking in more detail at the standard formalization of the comparative under the vague predicate analysis, as formulated by Klein, it turns out that this approach does not account for the COD-type of interpretation, nor for the data in (9)–(11). The formalization in (19) only implies that the dog should be smart, not that it should be good-looking.

(19)  $\exists d[(d(\text{smart}))(\text{the dog}) \wedge \neg(d(\text{good-looking}))(\text{the dog})]$

This problem is solved in a more constrained version of Klein's analysis as proposed in [6] and [7], which restate Klein's analysis in terms of a comparison between degree functions. The *than*-clause introduces the maximally informative degree function  $\delta$  that, if applied to the adjective in the *than*-clause results in a set including the subject of this adjective (this is the formalization used in [7]). The analysis of (4b) is given in (20), where  $\delta_1 >_A \delta_2$  iff  $\delta_1(A) \subset \delta_2(A)$ .

(20)  $\exists \delta_1[(\delta_1(\text{funny}))(\text{Ben}) \ \& \ \delta_1 >_A \text{MAX}_{\text{rich}}(\lambda \delta_2(\delta_2(\text{rich}))(\text{Steve}))]$

In this analysis, the functions that can be used must be inherently ordered with respect to one another (which is a consequence of Klein's Consistency Principle), and it is assumed that *quite*, *very* and *extremely* fulfill this requirement as well. As such, the sentence in (4b) states that if Steve is quite rich, Ben has to be very funny, or,

alternatively, if Steve is very rich, then Ben has to be extremely funny. This seems to be exactly what the sentence means. The analysis differs from Klein's original formalization by putting a much stronger constraint on the semantic contribution of the *than*-clause. As for the example in (9), the *than*-clause introduces the maximally informative degree function which, when applied to *mooi* 'good-looking', results in a set containing the dog. As a consequence, the ironic reading of the sentence can be attributed to a stylistic effect that stretches up the domain for *mooi* as to even include the ugly dog.

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## References

1. Bartsch, R., Vennemann, T.: *Semantic Structures*. Athenäum, Frankfurt (1972)
2. Bale, A.: *The universal scale and the semantics of comparison*. Dissertation. McGill University (2006)
3. Bale, A.: A universal scale of comparison. *Linguistics & Philosophy* 31, 1–55 (2008)
4. Bierwisch, M.: The semantics of gradation. In: Bierwisch, M., Lang, E. (eds.) *Dimensional Adjectives*, pp. 71–262. Springer, Berlin (1989)
5. Cresswell, M.: The semantics of degree. In: Partee, B. (ed.) *Montague grammar*, pp. 261–292. Academic Press, New York (1976)
6. Doetjes, J., Constantinescu, C., Součková, K.: A neo-Kleinian approach to comparatives. In: Ito, S., Cormany, E. (eds.) *Proceedings of SALT XIX* (to appear)
7. Doetjes, J.: Cross polar (a)nomalies without degrees. In: Cornilescu, A., Avram, L. (eds.) *Proceedings of the International conference of the English Department at the University of Bucharest* (2009) (to appear)
8. Hamann, C., Nerbonne, J., Pietsch, R.: On the Semantics of Comparison. *Linguistische Berichte* 67, 1–23 (1980)
9. Kennedy, C.: *Projecting the adjective. The syntax and semantics of gradability and comparison*. Garland, New York (1999)
10. Kennedy, C.: Polar opposition and the ontology of 'degrees'. *Linguistics & Philosophy* 24, 33–70 (2001)
11. Klein, E.: A semantics for positive and comparative adjectives. *Linguistics & Philosophy* 4, 1–46 (1980)
12. Klein, E.: The interpretation of adjectival comparatives. *Journal of Linguistics* 18, 113–136 (1982)
13. Leech, G.: *A linguistic guide of English poetry*. Longman, Harlow (1969)
14. McConnell-Ginet, S.: *Comparative constructions in English*. Dissertation, University of Rochester (1973)
15. Rett, J.: Antonymy and evaluativity. In: Gibson, M., Friedman, T. (eds.) *Proceedings of SALT XVII*. CLC Publications (2008)
16. van Rooij, R.: Vagueness and Linguistics. In: Ronzitti, G. (ed.) *The Vagueness Handbook* (to appear)

# Distributivity in Reciprocal Sentences

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**Abstract.** In virtually every semantic account of reciprocity it is assumed that reciprocal sentences are distributive. However, it turns out that the distributivity must be of very local nature since it shows no effect on the predicate or other arguments in reciprocal sentences. I present a semantic analysis of reciprocals that treats reciprocal sentences as distributive but captures the local nature of distributivity.

## 1 Introduction

Two meaning components are present in reciprocals. First, reciprocals express anaphoricity to a plural argument. Second, they specify that the causal relation holds between distinct parts of the plural argument. I call the first meaning component of reciprocals *anaphoric condition*, and the second component *distinctness condition*. In (1) the anaphoric condition ensures that the object has the same reference as the subject. The distinctness condition specifies how the relation of hating is satisfied. More concretely, (1) is only true if Morris hated Philip and Philip hated Morris.

- (1) Morris and Philip hated each other.

In order to capture the distinctness condition of reciprocals we have to interpret the relation in reciprocal sentences distributively. That is, the relation *hate* in (1) does not hold of the plurality Morris and Philip itself, rather, it holds of distinct individuals forming this plurality. To account for the distinctness condition we thus need some way of ensuring distributive quantification in reciprocal sentences.

In this paper I am going to argue that the distributive quantification necessary for capturing the distinctness condition of reciprocals must have a very limited scope. In fact, it should scope only over the reciprocal itself, and exclude other arguments, as well as the verb. The observation is not new. It has already been made in Williams' response to Heim et al. (1991a). However, Williams himself notes this as a problem but does not propose a semantic analysis. Subsequent analyses of *each other* either ignored this problem, admitted that their account cannot deal with it or claimed that the problem is not real. I am going to argue against the last solution and propose a semantic analysis of reciprocals with a limited scope of distributivity. The analysis is possible if we combine the theory of reciprocity with Landman's analysis of distributivity limited to thematic roles (Landman, 2000).

The paper is organized as follows. In the next section I list three arguments that point to the very limited nature of distributive quantification in reciprocal sentences. In Section 3 I show that parallel arguments exist in case of cumulative quantification, which led Landman (2000) to postulate a novel type of distributivity. Building on his idea (albeit not his actual implementation) I show how the same approach can account for the behavior of reciprocals. Section 4 is the conclusion.

## 2 Data

At least three arguments point to the conclusion that distributivity in reciprocal sentences is very limited in its scope.

The first argument comes from reciprocal sentences of type DP-V-each other-(P) DP, that is, where another argument is present. Consider (2a). As noted in Williams (1991), Moltmann (1992), to get the interpretation ‘each child giving a different present’, the plural DP is preferred over the singular one, cf. the difference between (2a) and (2b) in this interpretation.

- (2) a. Two children gave each other a Christmas present.  
       ?? under the reading ‘each child giving a different present’
- b. Two children gave each other Christmas presents.  
       OK under the reading ‘each child giving a different present’

There are two strategies how one builds distributivity into reciprocal sentences. One option (Dalrymple et al. 1998, Sabato and Winter 2005 among others) is to build distributivity into the meaning of reciprocals. The second option makes use of distributivity postulated independently of reciprocals (Heim et al. 1991b, Beck 2001, among others). I focus here on the first option and come to the second option at the end of this section.

In the first approach one assumes that reciprocals scope over relations and require, in its basic reading, that the extension of relation includes all pairs of non-identical individuals. In example (2a), the relation is ‘ $\lambda x \lambda y. x$  gave  $y$  a Christmas present’. Since  $x$  and  $y$  are distinct individuals (2a) can mean that the first child gave one present to the second child and the second child gave another present to the first child. Thus, we derive as the default reading the reading which is dispreferred in (2a). Obviously, the problem would disappear if we ensured that a *Christmas present* is outside the scope of the reciprocal, hence *each other* would not distribute over it. However, it is unclear why indefinites should by default scope over reciprocals given that normally the inverse scope is a dispreferred option.

This is Williams’ and Moltmann’s argument why distributivity should be very local or absent in reciprocal sentences. Dalrymple et al. (1998) and Beck (2001) respond to this by claiming that the reading marked as ‘??’ in (2a) is possible so there is nothing bad after all if we derive it. I think that this cannot be the end of story, though. Williams’ point was not that the reading (2a) is impossible, only that it is marked, roughly equally as the distributive reading of (3) is marked.

- (3) Two children gave Mary a Christmas present.

The distributive reading of (3) improves if we substitute the indefinite subject with a DP whose head is a universal quantifier *both* or *all*, and the same holds for (2a). This intuition has also been confirmed in a questionnaire studies, see Dotlačil (2009). Suppose we derive the marked status of the distributive reading in (3) by assuming that numeral noun phrases do not distribute, unlike quantifiers. This solution would fail to extend to (2a) since here there is an independent source of distributivity, namely, the reciprocal itself, which gives rise to the dis-preferred reading by default. Thus, contrary to Dalrymple et al. (1998) and Beck (2001) I believe that even if one accepts (2a) under the relevant reading, one still needs to explain why the reading is somewhat marked, in a parallel fashion to (3), and accounts in which reciprocals freely distributes over *a Christmas present* lack the explanation. Notice that if *each other* distributed only very locally, we might be able to say that the marked status of (2a) has the same reason as the marked status of (3).

The second argument for the very local scope of distributivity comes from a cumulative quantification, studied by Scha (1981) and Krifka (1989), among many others. Its connection to reciprocity has been discussed in Sauerland (1998) and Sternefeld (1998). The problem can be shown on (2b) but since this involves complications due to the presence of bare plurals, I use two different examples. The first one is a variation on (2b), the second one is from the Corpus of Contemporary American English.

- (4) a. Two children gave five presents to each other (in total).  
 b. Critics and defenders of the Catholic Church have been aligned against each other in two conflicting camps.

A possible reading of (4a) is that two children gave each other some presents, such that in total five presents were given. (4b) can mean that critics have been aligned against defenders in one camp and defenders were aligned against critics in another camp so in total there were two competing camps. Consider (4b) in more detail. If the reciprocal scopes above *two conflicting camps* we get the reading that critics were aligned against defenders in two conflicting camps and defenders were aligned against critics in two (possibly different) camps. If *two conflicting camps* scopes above the reciprocal, we get the reading that there are two conflicting camps and critics were aligned against defenders in these two camps, and so were defenders. Neither of the readings is correct. In a nutshell the problem is as follows. (4a) and (4b) are cases of a cumulative quantification. Normally, we can derive the cumulative reading if we assume that none of the arguments distributes over the others and all arguments are interpreted in their thematic positions (in line of Krifka 1989 and others since). However, this is incompatible with the account of *each other* which requires distributivity. The problem could be avoided if we had a system where *each other* does require distributivity, but distributivity is only very local, not affecting the interpretation of other arguments.

The third argument comes from the fact that reciprocal sentences can combine with collective predicates. This is shown in (5a), from Dimitriadis (2000), and (5b), from the Corpus of Contemporary American English.

- (5) a. Bill and Peter, together, carried the piano across each others lawns.  
 b. Cooper and friends gather at each other's homes to perform tunes and ballads.

The problem is that in Dalrymple et al. (1998) and others, (5a) ends up meaning that Bill together carried the piano and so did Peter, which is nonsense. However, (5a) and (5b) can be interpreted. The problem would again be avoided if we ensured that the distributivity associated with *each other* does not scope over the adverb *together* in (5a) or the collective verb *gather* in (5b).

These are problems for Dalrymple et al. (1998) and Sabato and Winter (2005) but they are similarly problematic for accounts in which reciprocals make use of independently postulated distributivity. Consider (6).

- (6) Morris and Philip hated each other.

There is a long tradition of analyzing referring expressions (like coordinations of proper names in (6)) as possibly distributing over the predicate. Various alternative analyses of how to achieve this exist. Regardless of the option we choose we build the distinctness condition of *each other* upon the capability of the subject to distribute over the predicate (Heim et al. 1991b, Beck 2001, among others). In particular, we might interpret *each other* as follows (see Beck 2001):

- (7)  $\llbracket \text{each other} \rrbracket = \text{the other one(s) among } x \text{ different from } y$

Now, we let  $x$  to be bound by the plural argument, anteceding the reciprocal, and  $y$  to be bound by the distributive quantifier. In (6) we thus derive the reading which could be (somewhat clumsily) paraphrased as 'each of Morris and Philip hated the other one among Morris and Philip different from himself', that is, Morris hated Philip and Philip Morris. Since it is necessary that the antecedent of reciprocals distribute, we run again into the problem why (2a) is degraded under the indicated interpretation. We also cannot explain why (5a) and (5b) are possible. Finally, (4a) and (4b) are problematic. Since the subject has to distribute in these readings, we derive that, for instance, (4b) is interpreted as 'critics were aligned in two competing camps, and so were defenders' which is not the correct interpretation.<sup>1</sup>

To sum up, three arguments point to the conclusion that distributivity, necessary to capture the distinctness condition of reciprocals, applies only very locally. In the next section, I propose an analysis of these data.

<sup>1</sup> The last problem can be avoided but we have to assume an operator that applies to syntactically derived relations and cumulates on their both arguments (see Beck and Sauerland 2000 and literature therein). This analysis has been assumed in Sternefeld (1998) and Sauerland (1998). I am assuming that this operation is not possible. Even if we allow it the analysis still faces the two other problems.

### 3 Distributivity and Reciprocals

#### 3.1 Background Assumptions

I assume that the interpretive model includes  $D_e$ , the domain of individuals, and  $D_v$ , the domain of events. Both  $D_e$  and  $D_v$  are structures ordered by ‘sum’,  $\oplus$  in such a way that  $\langle D_e, \oplus \rangle$  is isomorphic to  $\langle \wp(D_e) - \{\emptyset\}, \cup \rangle$ , similarly for  $D_v$ . For more details, see Landman (1991). I furthermore assume that sentences are interpreted in neo-Davidsonian fashion. Verbs are predicates of events, and arguments are introduced through separate thematic roles. For example, ((8a)) is interpreted as (8b).

- (8) a. Burt and Greg kissed Clara and Lisa  
 b.  $(\exists e)(\text{*kiss}(e) \wedge \text{*Ag}(\text{BURT} \oplus \text{GREG}, e) \wedge \text{*Th}(\text{CLARA} \oplus \text{LISA}, e))$

Notice that, as is standard in event semantics (see Krifka 1989, Landman 2000, Kratzer 2003, among others), predicates and thematic roles are pluralized by \*. \* is defined below. It should be straightforward to see how we could extend \* to cumulate on relations of higher arity than 2.

- (9) a.  $\text{*}Px = 1$  iff  $Px = 1$  or  $x_1 \oplus x_2 = x$  and  $\text{*}Px_1$  and  $\text{*}Px_2$   
 b.  $\text{*}R(x, y) = 1$  iff  $R(x, y) = 1$  or  $x_1 \oplus x_2 = x$  and  $y_1 \oplus y_2 = y$  and  $\text{*}R(x_1, y_1)$  and  $\text{*}R(x_2, y_2)$

Thus, the event  $e$  is possibly a plural event that has subevents in which parts of the plurality Burt  $\oplus$  Greg kissed parts of the plurality Clara  $\oplus$  Lisa. This would be true, if, for example,  $e$  consisted of subevents  $e_1$  and  $e_2$ , where Burt kissed Clara in  $e_1$  and Greg kissed Lisa in  $e_2$ . This is the so-called cumulative reading.

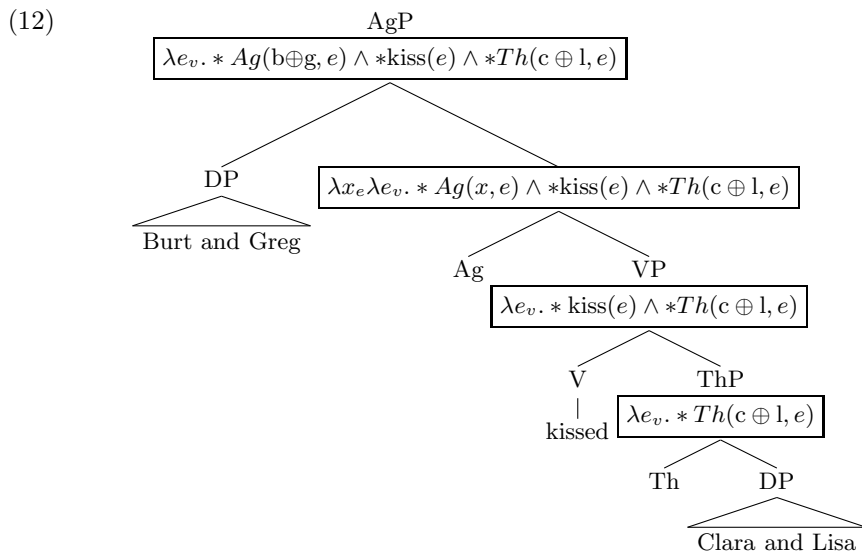
To arrive at (8b) compositionally, I make the following assumptions. First, thematic roles are introduced separately in the syntax. Since each thematic role is of type  $\langle e, \langle v, t \rangle \rangle$  to combine them together we either need to assume some lift operator which lifts one thematic role so it can apply to the other or we can assume a special mode of composition, event identification (Kratzer, 2003). I am going to assume the latter here, for any two arbitrary types that end in  $\langle v, t \rangle$ :

- (10) Event Identification (ei):  
 a.  $Y_{\langle v, t \rangle} \text{ ei } Z_{\langle v, t \rangle} = \lambda e. Y(e) \wedge Z(e)$   
 b.  $Y_{\langle \sigma_1, \sigma_2 \rangle} \text{ ei } Z_{\langle v, t \rangle} = \lambda P_{\sigma_1}. Y(P) \text{ ei } Z$  if  $\sigma_2$  ends in  $\langle v, t \rangle$   
 c.  $Y_{\langle \sigma_1, \sigma_2 \rangle} \text{ ei } Z_{\langle \sigma_3, \sigma_4 \rangle} = \lambda P_{\sigma_1} \lambda Q_{\sigma_3}. Y(P) \text{ ei } Z(Q)$  if  $\sigma_{2,4}$  end in  $\langle v, t \rangle$

Finally, we want generalized quantifiers to be interpretable in their thematic positions (see Krifka 1989). For that we assume LIFT:

- (11) LIFT:  $\lambda R_{\langle e, \langle v, t \rangle \rangle} \lambda Q_{\langle \langle e, t \rangle, t \rangle} \lambda e. Q(\lambda x. R(x, e))$

The syntactic structure (12) shows how we derive (8b) in a stepwise fashion using the assumptions we made so far.



In the next section, I want to discuss more complicated cases in which cumulative readings intertwine with distributive readings, which will form the key insight for understanding what is going on in reciprocal sentences.

### 3.2 Distributivity in Cumulative Readings

Consider the following sentence, from Landman (2000).

- (13) Three boys gave six girls two flowers.

Example (13) can be true if there are three boys and six girls and each boy gave flowers to some of the six girls and each girl received flowers from some of the three boys, and each boy gave out two flowers. For instance, one boy gave one flower to girl<sub>1</sub> and one flower to girl<sub>2</sub>, the second boy gave one flower to girl<sub>3</sub> and one flower to girl<sub>4</sub> and the third boy gave one flower to girl<sub>5</sub> and girl<sub>6</sub>. The problem with this reading is that *three boys* distributes over *two flowers* (so, each boy gives out two flowers in total) but three boys and six girls are interpreted cumulatively, that is, neither of these arguments distributes over the other argument. For more discussion and more examples showing the same point, see Roberts (1990), Schein (1993), Landman (2000) and Kratzer (2003). To account for this reading, we need to allow the subject to distribute only very locally, over the theme argument, and excluding the goal argument. The analysis of this is proposed in Landman (2000). However, I am going to differ from his approach because it is not clear how it could be extended to reciprocals.<sup>2</sup>

<sup>2</sup> Champollion (2010) offers an account that can deal with examples like (13) in an eventless framework. I have to leave it open whether his analysis could be extended to reciprocals.

The basic idea is that we let some thematic roles be related not to the event  $e$  but to some subevent  $e'$ . Thus, we assume a null operator which optionally applies to a thematic role and requires it to relate to  $e'$ , a subevent of  $e$ :

- (14) The operator making a thematic role related to the subevent  $e'$   
 $\lambda R_{\langle e, \langle v, t \rangle \rangle} \lambda x \lambda e' \lambda e. R(x, e') \wedge e' \leq e$

We can then distribute only over  $e'$  and exclude distribution over the whole event  $e$ . In (13) we let the agent and theme be related to  $e'$ , the subevent of  $e$ :

- (15)  $\lambda x \lambda e' \lambda e. \exists y, z \left( 2 \text{ flowers}(y) \wedge 6 \text{ girls}(z) \wedge *Ag(x, e') \wedge *Th(y, e') \right)$   
 $\wedge e' \leq e \wedge *give(e) \wedge *Go(z, e)$

To let the agent distribute over the theme argument it suffices to allow cumulation of (=the application of  $*$  to) the first two arguments of this function. I notate the distributive operator which enables this as  $\mathcal{D}$ .  $\mathcal{D}$  is defined as:

- (16)  $\mathcal{D}(Q_{\langle e, \langle v, \langle v, t \rangle \rangle \rangle}) = \lambda x \lambda e. *(\lambda y \lambda e'. Q(y, e')(e))(x, e)$

We cumulate on the first two arguments of  $Q$ . Thus,  $x$  and  $e$  can be split into parts, for instance,  $x_1, x_2$  and  $e_1, e_2$  and  $x_1, e_1$  satisfies  $\lambda y \lambda e'. Q(y)(e')(e)$ , and the same holds for  $x_2, e_2$ . To see what  $\mathcal{D}$  is doing consider the example above.  $\mathcal{D}$  applies to (15) which derives the following:

- (17)  $\lambda x \lambda e. * \left( \lambda y \lambda e'. \exists z, z' (2 \text{ flowers}(z) \wedge 6 \text{ girls}(z') \wedge *Ag(y, e') \wedge *Th(z, e') \wedge e' \leq e \wedge *give(e) \wedge *Go(z', e)) \right) (x, e)$

If *three boys* applies to (17) (by LIFT) we derive that *three boys* and  $e$  can be split into parts, and we can pair up the parts of the nominal argument with the parts of the event such that each pair satisfies the following function:

- (18)  $\lambda y \lambda e'. \exists z, z' \left( 2 \text{ flowers}(z) \wedge 6 \text{ girls}(z') \wedge *Ag(y, e') \wedge *Th(z, e') \wedge e' \leq e \wedge *give(e) \wedge *Go(z', e) \right)$

This is true if, for instance, there are three subevents of  $e$  and every boy is the agent argument of one of the subevents and for each of the subevents there are two flowers that are the theme argument. Notice that even though the goal argument is in the syntactic scope of  $\mathcal{D}$ , it does not covary with the agent. Only the arguments that are related to the subevent  $e'$  show covariation. Thus, this system is useful for dealing with cases in which one argument syntactically scopes over another argument but does not induce covariation over it. For more discussion and details on the compositional analysis, see Dotlačil (2009).

### 3.3 Reciprocal Sentences

We have seen that distributivity in reciprocal sentences should be limited in scope. The same strategy which allows us to combine distributive and cumulative readings in one clause could be used for reciprocals. Consider the sentence *Morris and Philip hated each other*. We let the agent and theme be related to the subevent  $e'$ . The two thematic roles combine and give us (19). (19) is parallel to the previous cases where thematic roles were related to subevents, the only difference is that now we abstract over the theme argument.

$$(19) \quad \lambda x \lambda y \lambda e' \lambda e. * Ag(x, e') \wedge * Th(y, e') \wedge * hate(e)$$

We need to let *each other* apply to this function and express that it holds for distinct parts of a plural argument. We assume the following interpretation:

$$(20) \quad \llbracket \text{each other} \rrbracket = \lambda Q \lambda x \lambda e. * (\lambda y \lambda z \lambda e'. Q(y, z, e', e) \wedge \wedge \text{distinct}(y, z))(x, x, e)$$

In standard accounts like Dalrymple et al. (1998), reciprocals take a relation as its argument and require that  $y, z$ , parts of the plural argument  $x$ , which apply to the relation are distinct. In the account here *each other* applies to  $Q$ , the relation of arity 4: this relates two individual arguments and two events.  $Q$  can be built up by letting thematic roles relate to the subevent  $e'$  and abstracting over the object argument of the relation. Thus, *each other* can apply to (19). Notice that I leave it open how the distinctness itself should be understood. For the purposes of this paper, assume that it is equivalent to non-overlap.

Letting *each other* apply to (19) and to the subject *Morris and Philip* we get (21). This formula is true if the plurality Morris and Philip can be split into parts and one part (say, Morris) hates the other, distinct, part (say, Philip) in some subevent of  $e$ , and the other way round, that is, Philip hates Morris in some other subevent of  $e$ . This is what we want.

$$(21) \quad \lambda e. * \left( \lambda x \lambda y \lambda e'. * Ag(x, e') \wedge * Th(y, e') \wedge \right) (\text{m} \oplus \text{p}, \text{m} \oplus \text{p}, e)$$

Consider now (22), repeated from above. As we have discussed in Section 2 the interpretation in which the indefinite is distributed over is marked.

$$(22) \quad \text{Two children gave each other a Christmas present.} \\ ?? \text{ under the reading 'each child giving a different present'}$$

We let the agent and goal be related to subevents, which gives us:

$$(23) \quad \lambda x \lambda y \lambda e' \lambda e. \exists z \left( \text{present}(z) \wedge * Ag(x, e') \wedge * Go(y, e') \right) \wedge e' \leq e * \text{give}(e) \wedge * Th(z, e)$$

If we let *each other* apply to (23) and to the subject (notated as “2 c.”) we get:

$$(24) \quad \lambda e. * \left( \lambda x \lambda y \lambda e'. \exists z (\text{present}(z) \wedge * Ag(x, e') \wedge * Go(y, e')) \right) (2 \text{ c.}, 2 \text{ c.}, e)$$

Even though *a present* is in scope of  $*$  and thus, one might think, could be interpreted as varying with respect to each child, it does not. The reason is that unlike the agent and goal argument, the theme argument is related to the event  $e$ . Therefore, (24) is true if one child gave another child a present, and the other child gave the first child a present, and in total *one* present was exchanged. Thus, unlike every single analysis of reciprocals I know of (with the exception of Moltmann 1992) we do not derive the distributive reading as the default one. We can still derive the distributive reading if we assume that the theme argument is also related to the subevent  $e'$ . However, notice that this requires an extra operation, namely modification of the theme thematic role. It

is likely that this extra operation makes the particular reading less likely. As we have seen above, it is also dispreferred to interpret *two children gave Mary a Christmas present* with Christmas presents varying for each kid. Here again, the dispreferred interpretation only follows if we let the theme argument be related to the subevent and the subject distribute over it. If these optional operations are dispreferred in this case we expect them to be dispreferred in (22) as well. Thus, unlike previous accounts, we correctly capture the parallelism between reciprocal and non-reciprocal sentences concerning distribution over indefinites.

We can also derive the reading of *Two children gave five presents to each other (in total)* in which two children gave each other some presents, such that in total five presents were given. This reading is in fact captured in the representation (24), the only difference is that ‘a present’ is substituted by ‘five presents’. Finally, let me come back to reciprocal sentences with collective predicates, like (25a), repeated from above, which gets the interpretation (25b):

- (25) a. Cooper and friends gather at each other’s homes.  
 b.  $\lambda e.*\left(\lambda y\lambda z\lambda e'.*Ag(y,e')\wedge*Th(\text{house of } z,e')\right)(C\oplus_{\text{fr.}},C\oplus_{\text{fr.}},e)$   
 $\wedge e'\leq e\wedge\text{gather}(e)\wedge\text{distinct}(y,z)$

(25b) is true if, for instance, each friend is the agent of gathering at his friends’ homes. One might find this a non-sensical interpretation since it seems strange that a single person could be the agent of gathering. However, it has been argued in the work of Dowty and Brisson (see Brisson 2003 and references therein) that the agent of collective predicates like *gather* needs to satisfy only some general requirements that gathering might impose (getting to some particular place, for instance) and does not need to “undergo gathering” himself. This enables (25b) to have a possible interpretation. We furthermore expect that collective predicates which do not have such unspecific requirements on their agents should not combine with reciprocals. One test to distinguish the two types of collective predicates is using the quantifier headed by *all*. While *gather* is compatible with this quantifier, other collective predicates like *outnumber* are not. It turns out that collective predicates of the latter type cannot appear in reciprocal sentences either. For example, *The boys in our class outnumber each other’s families* is uninterpretable. We expect this since reciprocals should only combine with collective predicates whose agents can be atomic individuals.

## 4 Conclusion

In order to accommodate the distinctness condition of *each other* we need to assume that reciprocal sentences include some sort of distributivity. I have shown that this distributivity is very local and has no effect on the predicate or other arguments in reciprocal sentences. This can be accommodated by using a very local version of distributivity which operates only between thematic roles hosting the reciprocal and its antecedent. The analysis gives an independent support to distributivity which does not scope over the whole clause but only over selected arguments.

## References

- Beck, S.: Reciprocals are definites. *Natural Language Semantics* 9, 69–138 (2001)
- Sigrid, B., Sauerland, U.: Cumulation is needed: A reply to Winter. *Natural Language Semantics* 8, 349–371 (2000)
- Brisson, C.: Plurals, *all*, and the nonuniformity of collective predication. *Linguistics and Philosophy* 26, 129–184 (2003)
- Champollion, L.: Cumulative readings of every do not provide evidence for events and thematic roles. In: Aloni, M., Bastiaanse, H., de Jager, T., Schulz, K. (eds.) *Amsterdam Colloquium 2009*. LNCS (LNAI), vol. 6042, pp. 213–222. Springer, Heidelberg (2010)
- Dalrymple, M., Kanazawa, M., Kim, Y., Mchombo, S., Peters, S.: Reciprocal expressions and the concept of reciprocity. *Linguistics and Philosophy* 21, 159–210 (1998)
- Dimitriadis, A.: Beyond identity: problems in pronominal and reciprocal anaphora. Doctoral Dissertation, University of Pennsylvania (2000)
- Dotlačil, J.: Anaphora and distributivity. A study of *same*, *different*, reciprocals and *others*. Doctoral Dissertation, Utrecht Institute of Linguistics OTS, Utrecht, The Netherlands (2009)
- Heim, I., Lasnik, H., May, R.: Reciprocity and plurality. *Linguistic Inquiry* 22, 63–192 (1991a)
- Heim, I., Lasnik, H., May, R.: Reply: on reciprocal scope. *Linguistic Inquiry* 22, 173–192 (1991b)
- Kratzer, A.: The event argument and the semantics of verbs. Ms. Four chapters (2003), <http://semanticsarchive.net>
- Namfred, K.: Nominal reference, temporal constitution, and quantification in event semantics. In: Bartsch, R., van Bentham, J., van Emde Boas, P. (eds.) *Semantics and contextual expressions*. Foris, Dordrecht (1989)
- Landman, F.: *Structures for semantics*. Kluwer, Dordrecht (1991)
- Landman, F.: *Events and Plurality: The Jerusalem Lectures*. Kluwer, Dordrecht (2000)
- Moltmann, F.: Reciprocals and *same/different*: Towards a semantic analysis. *Linguistics and Philosophy* 15, 411–462 (1992)
- Roberts, C.: *Modal subordination, anaphora, and distributivity*. Garland Publishing, New York (1990)
- Sabaro, S., Winter, Y.: From semantic restrictions to reciprocal meanings. In: *Proceedings of FG-MOL* (2005)
- Sauerland, U.: Plurals, derived predicates, and reciprocals. In: *The interpretative tract: MIT working papers in linguistics*, Cambridge, Massachusetts (1998)
- Scha, R.: Distributive, collective, and cumulative quantification. In: Janssen, T., Stokhof, M. (eds.) *Formal methods in the study of language*, pp. 483–512. Mathematical Centre Tracts, Amsterdam (1981)
- Schein, B.: *Plurals and events*. MIT Press, Cambridge (1993)
- Sternefeld, W.: Reciprocity and cumulative predication. *Natural Language Semantics* 6, 303–337 (1998)
- Williams, E.: Reciprocal scope. *Linguistic Inquiry* 22, 159–173 (1991)

# A Logic for Easy Linking Semantics

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**Abstract.** I will define a logic with case-indexed variables and partial variable assignments for a lean syntax-semantics interface. Specifically, the syntax-semantics mapping does not require obligatory quantifier raising (as Heim+Kratzer, 1998) and does not require a fixed underlying order of arguments of the verb. The latter feature will facilitate semantic research on free word order languages and semantic research on languages where no specific syntactic claims about word order are as yet available.

## 1 Linking: Troubles and a Vision

Several problems have been identified in the past that complicate the mapping between syntax and semantics.

**Type mismatch problem:** Verbs denote relations between entities, whereas names, indexicals or definite NPs refer to entities. Quantificational DPs denote quantifiers over entities. In that case, a type mismatch between verb argument and DP denotation has to be resolved. While some theories endorse the assumption that verbs denote relations between generalized quantifiers, most people prefer to retain the original logical type of verbs. Heim + Kratzer (1998) develop the by now standard way to resolve the type mismatch between verb and quantifiers where quantifier raising, coindexing and the interpretation of traces as variables settles matters of scope, but also enables semantic composition of verb projection and quantificational DP. Hence, the type mismatch problem is considered as solved by many semanticists. However, the semantic composition of even a simple sentence like *John likes most Fellini movies* requires quantifier raising, interpreted traces, coindexing, and lambda abstraction.

**Order codes argument structure:** Standard semantic treatments of English and other languages assume a fixed (underlying) order of arguments of the verb. Word order, rather than case marking, is the factor that ensures that each DP or PP instantiates the correct argument place of the verb. According to this analysis, free word order languages should not exist. If a language is suspected to be of that type (see Haug, 2009 on Ancient Greek), or if a language is not as yet sufficiently well understood to make claims about word order, semantic analysis requires to stipulate a basic order of verbal arguments. To overcome such arbitrary stipulations, Montagovian semantics with interpreted case markers could be an attractive generalization of the standard framework.

**The tacit argument problem:** Many analyses propose that the verb has arguments that are not instantiated by overt phrases in the sentence, e.g. the

temporal argument (e.g. Stechow et al. 2009). In order to instantiate this argument, one has to assume that there is a tacit temporal PRO as a dummy syntactic object that figures in QR. PRO leaves a trace which can instantiate the temporal argument of verbs. In non-embedded sentences, von Stechow has to assume that PRO (not being a quantifier itself) passes its index to an independent lambda operator and gets deleted. While technically feasible, the process is an artifact of a specific kind of theory without offering further insights.

**The event problem:** In a standard Davidsonian analysis, event modifiers can apply to the event argument of the verb at many levels in syntax. In the standard fixed word order paradigm, we have to make a claim whether the event argument should be the first, or the second, etc. or the last argument of the verb. There is no agreed answer to this question and authors tend to avoid any principled position. We could claim that the event is an early argument of the verb such that, for instance, *love* denotes  $\lambda e \lambda y \lambda x \text{LOVE}(x, y, e)$ .  $\lambda e$  gets instantiated by the trace  $x_e$  of an uninterpretable dummy E-PRO. E-PRO is co-indexed with  $x_e$  and has to be raised to all positions immediately below an event modifier MOD. In that position, it has to pass its index to an independent lambda operator that makes  $x_e$  accessible. After combination of MOD and verb projection, another trace of PRO instantiates the event argument of the verb, thereby making the argument inert until needed the next time. (Note: if there is more than one event modifier in a sentence, we will need a chain of traces of PRO). We could alternatively claim that the event is a late argument of the verb, and our example verb *love* denotes  $\lambda y \lambda x \lambda e \text{LOVE}(x, y, e)$ . If an event modifier wants to combine with the verb before the verb has met all its DP arguments, the modifier has to use some standard procedure to instantiate the innermost argument of an n-place relation and to reopen all other arguments after modification. Such modes of combination can be defined but again carry the flavor of repairing theory-internal problems without offering further insights.

It should be pointed out that Kratzer (2002/unpublished) might offer a solution, based on plural events. I will not recapitulate her analysis here, suffice it to say that event semantics loses much of its original appeal: making semantic representations elegant and perspicuous, and not redundant and unperspicuous.

In this paper, I will define Linking Logic, a type logic on finite variable assignments, and Easy Linking Logic which endorses variables that are indexed with abstract case labels. This will allow us to design Easy Linking Semantics, a format for semantic analysis and composition that is independent of any specific grammatical framework and yet draws on earlier Montagovian semantics in a maximally conservative manner.

## 2 Linking Logic

I will define a type logic which operates on partial variable assignments.<sup>1</sup> All terms  $t$  and formula  $\phi$  are interpreted relative to models  $M$  and variable assignments  $g$ .

<sup>1</sup> An extended version of the paper also includes predicate logic on partial variable assignments, which might offer an easier way into the format.

Unlike normal logics, however, the interpretation will only be defined for variable assignment functions which have the free variables of  $t$  or  $\phi$  as their domain. No formula can be evaluated relative to an assignment which is too "rich". As a consequence, variable binding will not always lead to interpretable formula. E.g.  $\exists x\phi$  will only be interpretable if  $x$  occurs free in  $\phi$ . These properties are not desired or desirable in logics for mathematics and philosophy in general, perhaps. However, they reflect deep insights about natural language interpretation. For example, the ban on vacuous quantification, as a principle at LF, is hardwired in the logical backbone.

Logical types are recursively defined on basis of atomic types  $e, s, t$  in the usual manner.

**A type logical syntax:** A type logic language  $L$  on basis of these types consists of a set of constants for each type  $\tau$ , and a set of variables for each type  $\tau$ . The function  $fr$  will map any term to the set of free variables that occur in that term.

- For each type  $\tau$ , any constant  $c$  of type  $\tau$  is a term of type  $\tau$ . The set of free variables  $fr(c) := \emptyset$ .
- For each type  $\tau$ , any variable  $v_i$  of type  $\tau$  is a term of type  $\tau$ . The set of free variables  $fr(v_i) := \{v_i\}$ .
- If  $A$  is a term of type  $\langle\sigma, \tau\rangle$  and  $B$  is a term of type  $\sigma$ , then  $A(B)$  is a term of type  $\tau$ . The set of free variables  $fr(A(B)) := fr(A) \cup fr(B)$ .
- Logical connectives on type  $t$ : If  $\phi$  and  $\psi$  are of type  $t$ , then  $\phi \wedge \psi$ ,  $\phi \vee \psi$ ,  $\phi \rightarrow \psi$  and  $\neg\phi$  are terms of type  $t$ . The free variables are defined as follows:  $fr(\neg\phi) := fr(\phi)$ , and  $fr(\phi \wedge \psi) = fr(\phi \vee \psi) = fr(\phi \rightarrow \psi) = fr(\phi) \cup fr(\psi)$ .
- If  $\phi$  is a term of type  $\tau$ , and if  $fr(\phi)$  contains variable  $v_i$  of type  $\sigma$  then  $\lambda v_i.\phi$  is a term of type  $\langle\sigma, \tau\rangle$ . The set of free variables  $fr(\lambda v_i.\phi) := fr(\phi) - \{v_i\}$ .

The present system does not introduce syncategorematic quantification as an operation on type  $t$  terms. Determiners can denote generalized quantifiers as usual. In the following, I will use the notation  $g|_A$  for the partial function  $g*$  which arises by restricting  $g$  to domain  $A$ . Hence,  $g|_{fr(\phi)}$  stands for  $g$ , restricted to the free variables in term  $\phi$ .

**Interpretation:** Let  $D_e, D_s$  be domains of entities and worlds, and let  $D_t := \{0, 1\}$  as usual. Let  $D_{\langle\sigma, \tau\rangle} := \{f | f : D_\sigma \rightarrow D_\tau\}$ , and use  $\mathbf{D}$  to refer to this hierarchy of sets. Let moreover  $I$  be a function which maps all constants of type  $\tau$  into  $D_\tau$ . The type logical language  $L$  is interpreted relative to the model  $M = \langle \mathbf{D}, \mathbf{I} \rangle$  and partial variable assignments  $g$  from  $\mathbf{Var}$  into  $\mathbf{D}$  where  $\|\phi\|$  will only be defined for assignments  $g$  such that  $dom(g) = fr(\phi)$ .  $\emptyset$  is used for the empty variable assignment.

- Let  $c$  be a constant of type  $\tau$ .  $\|c\|^{M, \emptyset} := I(c)$ .
- Let  $v_i$  be a variable of type  $\tau$ . Let  $g$  be an assignment which is defined on  $fr(v_i) := \{v_i\}$ . Then  $\|v_i\|^{M, g} := g(v_i)$ .
- Let  $A$  be term of type  $\langle\sigma, \tau\rangle$  and  $B$  a term of type  $\sigma$ . Let  $g$  be a variable assignment with  $dom(g) = fr(A(B)) = fr(A) \cup fr(B)$ . Then  $\|A(B)\|^{M, g} :$

$= ||A||^{M,g_1} (||B||^{M,g_2})$  where  $g_1 := g$  restricted to  $fr(A)$  and  $g_2 = g$  restricted to  $fr(B)$ .

- Logical connectives on type  $t$ : Let  $\phi$  and  $\psi$  be of type  $t$ . Let moreover  $g$  be any assignment with  $dom(g) = fr(\phi) \cup fr(\psi)$ .

$$||\phi \wedge \psi||^{M,g} = 1 \text{ iff } ||\phi||^{M,g_1} = 1 \text{ and } ||\psi||^{M,g_2} = 1.$$

$$||\phi \vee \psi||^{M,g} = 1 \text{ iff } ||\phi||^{M,g_1} = 1 \text{ or } ||\psi||^{M,g_2} = 1.$$

$$||\phi \rightarrow \psi||^{M,g} = 1 \text{ iff } ||\phi||^{M,g_1} = 0 \text{ or } ||\psi||^{M,g_2} = 1.$$

$$||\neg\phi||^{M,g_1} = 1 \text{ iff } ||\phi||^{M,g_1} = 0$$

In all cases,  $g_1 := g|_{fr(\phi)}$  and  $g_2 := g|_{fr(\psi)}$ .

- If  $\phi$  is a term of type  $\tau$ , and if  $fr(\phi)$  contains variable  $v_i$  of type  $\sigma$  then  $\lambda v_i.\phi$  is a term of type  $\langle\sigma, \tau\rangle$ . Let  $g$  be an assignment with  $dom(g) = fr(\phi) - \{v_i\}$ . Then  $||\lambda v_i.\phi||^{M,g} :=$  the function which maps all  $m \in D_\sigma$  to  $||\phi||^{M,g'}$  where  $g' := g \cup \{v_i, m\}$ .

Any term in  $L$  can exclusively be interpreted with respect to variable assignments that run exactly on the free variables of the term. While this may look like a restriction at first sight, the system covers all and exactly the functions served by variable assignments elsewhere. The mayor difference between sparse assignment logics and classical logics arises already in the definitions of well-formed terms. Whereas classical logics allow for vacuous binding, the use of  $\lambda$ -abstraction is restricted to terms where the bound variable actually occurs free in the term. Let  $\phi$  be a term of type  $t$  and let the variable  $v_i$  be in  $fr(\phi)$ . We will use the following abbreviations:

$$\exists v_i \phi := \neg(\lambda v_i.\phi = \lambda v.v(v = v))$$

$$\forall v_i \phi := \lambda v_i.\phi = \lambda v.v = v$$

Quantifiers inherit the ban on vacuous binding from  $\lambda$ -abstraction. Apart from that, they have the usual truth conditions. Let us check this for the existential quantifier  $\exists v_i \phi$ . We know that  $v_i \in fr(\phi)$  and  $fr(\exists v_i \phi) = fr(\phi) - \{v_i\}$ . Given a model  $M$  and assignment  $g$  which is defined on  $fr(\phi) - \{v_i\}$ ,  $||\exists v_i \phi||^{M,g} = 1$  iff there is an extension  $g* = g \cup \{v_i, m\}$  such that  $||\phi||^{M,g*} = 1$ . Note that  $\phi$  is defined for assignment  $g*$  because we assumed that  $v_i$  is free in  $\phi$ .

Another operator that will be used later is the subset relation  $\subset$  of type  $\langle\langle e, t \rangle, \langle\langle e, t \rangle, t \rangle\rangle$ . If  $A, B$  are terms of type  $\langle e, t \rangle$ , then  $||A \subset B||^{M,g}$  is defined for all  $g$  with  $dom(g) = fr(A) \cup fr(B)$ .

$||A \subset B||^{M,g} = 1$  iff  $||A||^{M,g|_{fr(A)}}$  is the characteristic function of a set  $A'$  in  $M$ ,  $||B||^{M,g|_{fr(B)}}$  is the characteristic function of a set  $B'$  in  $M$  and  $A' \subset B'$ .

Let me illustrate that bound variables do not have any influence on the meaning of terms. Consider the terms  $\lambda v_2.MAN(v_2)$  and  $\lambda v_g.WALK(v_g)$ .

$$||\lambda v_2.MAN(v_2) \subset \lambda v_g.WALK(v_g)||^{M,g} = 1$$

$$\text{iff } ||\lambda v_2.MAN(v_2)||^{M,g} \subset ||\lambda v_g.WALK(v_g)||^{M,g},$$

that is iff the set  $MAN$  with the characteristic function  $||\lambda v_2.MAN(v_2)||^{M,g}$  is a subset of the set  $WALK$  with the characteristic function  $||\lambda v_g.WALK(v_g)||^{M,g}$ .

Although the computation of the two latter characteristic functions operates via  $v_2$  and  $v_g$ , the same functions would result if we execute the computation via

any other variable. Generally, bound variables can be renamed like in classical logics (i.e. taking care that the new variable isn't one bound by an operator inside the scope of the original binding operator). We can hence freely use renaming of variables, for instance in order to graphically distinguish saturated arguments from open arguments of the verb.

### 3 Easy Linking Semantics

In what follows, I will use an Easy Linking Logic  $L_{link}$  which deviates from the systems above in its variables of type  $e$ . Apart from ordinary variables, we will use variables with abstract case labels like *nom*, *acc*, *dat*, *gen*. These include labels for prepositional cases like *by*, *for*, *to*, *with*. We will also assume that if the same preposition can be used with different thematic roles, and combines with the same verb twice, it will count as two different labels. Hence, *with*<sub>1</sub> in *with great care* counts as a different abstract prepositional case than the *with*<sub>2</sub> in *with a hammer* in the following sentence.

- (1) *With great care, Joan opened the box with a hammer.*

Finally, I propose to use the labels  $t$ ,  $pl$ ,  $e$  for times, place and events. Hence,  $\mathbf{Var} = \{v_{nom}, v_{acc}, v_{dat}, \dots, e, t, pl, v_1, v_2, v_3, \dots\}$ . The exact choice of labels can be adapted if necessary. Likewise, we can assume that the linking logic  $L_{link}$  has more abstract case indices than we actually want to use of some specific semantic analysis. As before, formulae in  $L_{link}$  will be interpreted in suitable models  $M$  relative to finite assignments  $g$ .

What is the meaning of a verb in Easy Linking Semantics? I assume that the "conceptual" content of verbs in English should be captured in a variable-independent way as an  $n$ -place relation between objects, events, and worlds as usual. Hence, we will use conceptual denotations of verbs like the following:

$$\begin{aligned} [[stab]]^c &= ||\lambda x \lambda y \lambda e \lambda w. STAB(x, y, e, w)||^M \\ [[buy]]^c &= ||\lambda x \lambda y \lambda z \lambda e \lambda w. BUY(x, y, z, e, w)||^M \\ [[sell]]^c &= ||\lambda x \lambda y \lambda z \lambda e \lambda w. SELL(x, y, z, e, w)||^M \\ [[kiss]]^c &= ||\lambda x \lambda y \lambda e \lambda w. KISS(x, y, e, w)||^M \\ [[rain]]^c &= ||\lambda e \lambda w. RAIN(e, w)||^M \end{aligned}$$

These denotations can be viewed as conceptual values of English as well as German, Dutch, Russian or Japanese verbs, and they are not committed to any syntax-semantics interface. For the sake of illustration, I decided to use the Davidsonian format with an event argument for the verb. This is *not* what Beaver & Condoravdi propose, but Easy Linking Semantics is particularly attractive if you want to use events.

When verbs enter into the composition of a sentence, they change to their linking semantics. Each verbal argument is instantiated with a variable which carries the abstract case label that corresponds to the phrase that realizes this argument in sentences. Event and world argument will likewise be instantiated

by specific event- and world variables. The following examples illustrate the step. I use  $[[\dots]]$  for the linking semantics of words in English, whereas  $||\dots||$  evaluates terms in  $L_{link}$  in a model  $M$ .

$$\begin{aligned} [[stab]] &\longrightarrow ||STAB(v_{nom}, v_{acc}, e, w)||^M \\ [[buy]] &\longrightarrow ||BUY(v_{nom}, v_{acc}, v_{from}, e, w)||^M \\ [[sell]] &\longrightarrow ||SELL(v_{nom}, v_{acc}, v_{to}, e, w)||^M \\ [[kiss]] &\longrightarrow ||KISS(v_{nom}, v_{acc}, e, w)||^M \\ [[rain]] &\longrightarrow ||RAIN(e, w)||^M \end{aligned}$$

These  $L_{link}$  terms each denote a set of partial assignments from variables with case labels into the model domain  $M$ . In using variables, I make the syntax look as similar to traditional logic as possible. In using variables with case indices, I endorse Beaver & Condoravdi's proposal that linking should be part of the semantic value of verbs rather than part of a trace mechanism at the syntax-semantics interface.

### 3.1 Saturation of Arguments

We will assume that DPs carry their abstract case as a syntactic feature. These cases will enter the semantic composition; hence the denotation of  $DP_{case}$  is a tuple which consists of generalized quantifier (the same as in classical semantics) and its case label. In a sentence like the following, the subject DP *Ann* hence is interpreted as  $\langle \lambda P.P(ANN), nom \rangle$ .

(2) *Ann coughed*

Generally, a DP combines with a sister constituent XP as follows:

$$\begin{aligned} [[DP_{case} \text{ XP}]] &= \langle [[DP]], case \rangle \oplus [[XP]] \\ &= [[DP]](\lambda v_{case}.\psi) \end{aligned}$$

where  $\psi$  is an  $L_{link}$  term that codes the denotation of XP:  $[[XP]] = ||\psi||^M$ . Note that this definition does not depend on any specific term that is used to represent the meaning of XP. It can be shown that for any two terms  $\Psi_1, \Psi_2$  which both code the meaning of XP, the result of the above lambda-abstraction is identical for both terms. The crucial insight is that all ways to code the meaning of XP must coincide in their free variables, and these always have to contain  $v_{case}$ . Equivalent codings will then yield the same logical object for the same variable assignments; which is all that is needed to ensure identical results of lambda-abstraction over  $v_{case}$ . Hence, the result of semantic composition is well-defined. Let me show an example.

$$\begin{aligned} [[Ann]] &= \langle ||\lambda P.P(ANN)||^M, nom \rangle \\ [[coughed]] &= ||COUGH(v_{nom}, e, w)||^M \\ [[Ann \text{ coughed}]] &= ||\lambda P.P(ANN)||^M (||\lambda v_{nom}.COUGH(v_{nom}, e, w)||^M) \\ &= ||\lambda v_{nom}.COUGH(v_{nom}, e, w)(ANN)||^M \\ &= ||COUGH(ANN, e, w)||^M \end{aligned}$$

The next example shows object quantification. The procedure is very similar to a Heim-Kratzer treatment, though without any need to raise the object DP.

(3) *Ann read every book.*

$$\begin{aligned}
[[read]] &= ||READ(v_{nom}, v_{acc}, e, t)||^M \\
[[every\ book]] &= \langle ||\lambda Q_{<e,t>}\forall x(BOOK(x) \rightarrow Q(x))||^M, acc \rangle \\
[[VP]] &= [[read\ every\ book]] \\
&= ||\lambda Q_{<e,t>}\forall x(BOOK(x) \rightarrow Q(x))||^M (||\lambda v_{acc}.READ(v_{nom}, v_{acc}, e, t)||^M) \\
&= ||\forall x(BOOK(x) \rightarrow \lambda v_{acc}.READ(v_{nom}, v_{acc}, e, t)(x))||^M \\
&= ||\forall x(BOOK(x) \rightarrow READ(v_{nom}, x, e, t))||^M \\
[[S]] &= [[Ann\ read\ every\ book.]] \\
&= \langle ||\lambda P.P(ANN)||^M, nom \rangle \oplus ||\forall x(BOOK(x) \rightarrow READ(v_{nom}, x, e, t))||^M \\
&= ||\lambda P.P(ANN)||^M (||\lambda v_{nom}.\forall x(BOOK(x) \rightarrow READ(v_{nom}, x, e, t))||^M) \\
&= ||\lambda v_{nom}.\forall x(BOOK(x) \rightarrow READ(v_{nom}, x, e, t)(ANN))||^M \\
&= ||\forall x(BOOK(x) \rightarrow READ(ANN, x, e, t))||^M
\end{aligned}$$

The derivation of subject quantifiers and two quantificational DPs can combine in one sentence. The order of application will determine scope relations; I leave it to the reader to compute more examples<sup>2</sup>

The world variable is generally bound as in Fintel & Heim (2007). Actually, their use of partial assignments as part of their metalanguage is the same as our use of partial assignments as part of the underlying logic. Unlike the world index, the event parameter can take low scope and will be bound by an existential closure operator ECL for the variable  $e$  at any point. Let  $\Phi$  be some  $L_{link}$  term that represents the meaning of XP where  $e$  occurs free in  $\Phi$ .  $[[ECL\ XP]] = ||\lambda e.\Phi \neq \emptyset||^M = ||\exists e\Phi||^M$ . As before, existential closure is only defined if  $e$  occurs free in  $\Phi$ , and yields the same result for all equivalent terms that could represent the meaning of XP.

Unlike DP arguments, the Davidsonian event variable is often used in order to collect several event modifications before it undergoes existential closure. This can be implemented in the present system by assuming that event modifiers leave the event argument as an open variable.

(4) *Ann read every book carefully*

$$\begin{aligned}
[[read]] &= ||READ(v_{nom}, v_{acc}, e, t)||^M \\
[[carefully]] &= \langle ||\lambda P(CAREFUL(e) \wedge P(e))||^M, e \rangle \\
[[read\ carefully]] &= ||\lambda P(CAREFUL(e) \wedge P(e))||^M (||\lambda e.READ(v_{nom}, v_{acc}, e, t)||^M) \\
&= ||(CAREFUL(e) \wedge \lambda e.READ(v_{nom}, v_{acc}, e, t)(e))||^M \\
&= ||(CAREFUL(e) \wedge READ(v_{nom}, v_{acc}, e, t))||^M \\
[[ECL\ read\ carefully]] &= ||\exists e(CAREFUL(e) \wedge READ(v_{nom}, v_{acc}, e, t))||^M \\
[[every\ book]] &= \langle ||\lambda Q_{<e,t>}\forall x(BOOK(x) \rightarrow Q(x))||^M, acc \rangle \\
[[ [ECL\ read\ carefully] every\ book]] &= ||\lambda Q_{<e,t>}\forall x(BOOK(x) \rightarrow Q(x))||^M \\
&\quad (||\lambda v_{acc}.\exists e(CAREFUL(e) \wedge READ(v_{nom}, v_{acc}, e, t))||^M) \\
&= ||\forall x(BOOK(x) \rightarrow \exists e(CAREFUL(e) \wedge READ(v_{nom}, x, e, t))||^M
\end{aligned}$$

<sup>2</sup> Longer version with more examples available at Semantics Archive.

$$\begin{aligned}
& [[Ann \text{ read every book carefully}]] \\
& = \langle ||\lambda P.P(ANN)||^M, nom \rangle \\
& \quad \oplus ||\forall x(BOOK(x) \rightarrow \exists e(CAREFUL(e) \wedge READ(v_{nom}, x, e, t)))||^M \\
& = ||\forall x(BOOK(x) \rightarrow \exists e(CAREFUL(e) \wedge READ(ANN, x, e, t)))||^M
\end{aligned}$$

Alternatively, we can apply ECL after combining verb and object DP and get the following.

$$||\exists e(\forall x(BOOK(x) \rightarrow CAREFUL(e) \wedge READ(ANN, x, e, t)))||^M$$

Finally, the following example can be treated similarly if we replace ECL by the event quantifier *twice*.

(5) *Ann twice read every book carefully.*

The quantifier *twice* contributes  $\langle ||\lambda P \exists e_1 \exists e_2 (e_1 \neq e_2 \wedge P(e_1) \wedge P(e_2))||^M, e \rangle$ . Combination with any XP proceeds by lambda-abstraction over the event argument in the semantics of XP, and functional application. We can derive the following two readings.

$$\begin{aligned}
& ||\exists e_1 \exists e_2 (e_1 \neq e_2 \wedge \forall x(BOOK(x) \rightarrow CAREFUL(e_1) \wedge READ(ANN, x, e_1, t)) \\
& \quad \wedge \forall x(BOOK(x) \rightarrow CAREFUL(e_2) \wedge READ(ANN, x, e_2, t)))||^M \\
& ||\forall x(BOOK(x) \rightarrow \exists e_1 \exists e_2 (e_1 \neq e_2 \wedge CAREFUL(e_1) \wedge READ(ANN, x, e_1, t)) \\
& \quad \wedge CAREFUL(e_2) \wedge READ(ANN, x, e_2, t)))||^M
\end{aligned}$$

The linking mechanism rests on the idea that clauses are closed domains in which every argument of the verb occurs only once. In this preliminary version, I will leave it open whether we will combine Easy Linking Semantics with indices in those cases where parts of a clause undergo long distance movement (or scope). Likewise, I will not detail the analysis of passives here. Passivation requires a different instantiation in linking semantics value of the verb which reflects the shifted grammatical roles. So far, I have demonstrated how Easy Linking Semantics can implement quantification, argument saturation and argument modification without binding the argument. Note that Beaver & Condoravdi propose a particular way of shifting the value of the time arguments, which is effected by temporal modifiers. I will not take a stand as to whether this is the best way of treating temporal modification, but it can be implemented in Easy Linking Semantics.

### 3.2 Functional Shifting of Arguments

Beaver & Condoravdi use operators that shift the value of variable assignments as part of their overall tense semantics. Let us see how values of the time argument of verbs can be shifted by means of a simple function, e.g. the function which maps a time point  $\tau$  to  $\tau + 1$ . I will generally use  $t$  for the time argument (variable) and greek letters for time points. For the sake of simplicity, I will omit the Davidsonian event argument in the present section. This is not to

say that the technique is restricted to non-Davidsonian semantics. Consider the following formula in  $L_{link}$  which states that Ann coughed in  $w$  at  $t$ .

$$||COUGH(ANN, w, t)||^{M, g}$$

The formula is defined for all  $g$  with the domain  $\{t, w\}$  on times and worlds in  $M$  which are such that their extension to  $v_{nom}$  which map  $v_{nom}$  to  $ANN$  is in  $[[cough]]$ . Assume that we want to modify this formula in a way that ensures that Ann coughed at the time point that follows on  $g(t)$ . If you need a linguistic counterpart of this modification, you could imagine that it is contributed by *one moment later*. We can achieve this modification by lambda-abstraction over  $t$  and applying the resulting function to  $(t + 1)$ . The computation proceeds as follows:

1.  $||COUGH(ANN, w, t)||^{M, g} = 1$  iff  $\text{dom}(g) = \{t, w\}$  and all extensions of  $g$  to  $v_{nom}$  which map  $v_{nom}$  to  $ANN$  are in the denotation of *cough*.
2.  $||\lambda t. COUGH(ANN, w, t)||^{M, g'}$  is defined for all assignments where  $\text{dom}(g') = \{w\}$ . It denotes that function  $\mathbf{F}$  from time points  $\tau$  to  $\{0, 1\}$  which maps  $\tau$  to 1 iff the extension  $g'' := g' + \langle t, \tau \rangle$  is such that  $||COUGH(ANN, w, t)||^{M, g''} = 1$ .

We will now apply this function to the term  $t + 1$ .

1.  $||\lambda t. COUGH(ANN, w, t)(t + 1)||^{M, g}$  is defined for our old assignments  $g$  with  $\text{dom}(g) = \{t, w\}$ . (Note that  $t$  is again free in the new formula, because it was free in the argument term.)
2. According to our definition of functional application in EasyLL, we get  $||\lambda t. COUGH(ANN, w, t)(t + 1)||^{M, g} = ||\lambda t. COUGH(ANN, w, t)||^{M, g_1} ||(t + 1)||^{M, g_2}$  where  $g_1 = g|_{\{w\}}$  and  $g_2 = g|_{\{t\}}$ . This latter combination is equal to:
3.  $\mathbf{F}(g_2(t) + 1)$ , where  $\mathbf{F}$  is  $||\lambda t. COUGH(ANN, w, t)||^{M, g_1}$ . Given that  $g_2(t) = g(t)$ , this application yields *true* exactly if  $ANN$  coughs at time  $g(t) + 1$ . The application yields *false* else.

Generalizing this mechanism, we can apply a functional shift to the time argument  $t$  in a given formula. Any modifier that involves time will first effect lambda abstraction over the time variable. The resulting term is applied to a term  $F(t)$ . The time argument remains open; the formula is still defined for partial variable assignments  $g$  which have the time variable  $t$  in their domain.

Functional shifts can be combined. We could decide to apply a function  $G(t) := 2t$  in addition to  $F(t) = t + 1$  (whatever sense this may make on times). The order of semantic application determines the order in which  $\mathbf{F}$  and  $\mathbf{G}$  operate on the tense argument. Remember that, in the following formulae,  $\lambda t$  binds only the open variable  $t$  in  $\phi$ .

$$||\lambda t. \phi(t)(F(t))||^M = ||\phi(t + 1)||^M$$

$$||\lambda t. \phi(F(t))(G(t))||^M = ||\lambda t. \phi(t + 1)(G(t))||^M = ||\phi(2t + 1)||^M$$

$$||\lambda t. \phi(t)(G(t))||^M = ||\phi(2t)||^M$$

$$||\lambda t. \phi(G(t))(F(t))||^M = ||\lambda t. \phi(2t)(F(t))||^M = ||\phi(2(t + 1))||^M = ||\phi(2t + 2)||^M$$

Beaver & Condoravdi (2007) use functional composition in order to model stacked temporal modifiers of the kind *in the morning on Saturday for three weeks in 2008*. They exploit the fact that the syntactic order of temporal modifiers determines the order of application in the semantic representation. In their framework, certain ungrammatical orders of modifiers can be explained by the fact that the respective composition of functions is undefined or yields empty results.

## 4 Summary

The present paper spells out a type logic on partial variable assignments which allows for full control over the open variables of each term, and with variables which are indexed with abstract case labels. This type logic can serve as the backbone of semantic analysis, offering a convenient way to activate and inactivate parameters in the semantic computation. I proposed a specific example of Easy Linking Semantics to illustrate the potential of the linking mechanism. It allows to define semantic combination of argument and operator in much the same way as the QR-based mechanism proposed in Heim & Kratzer (1998), but without quantifier raising at LF. This is particularly advantageous for verb arguments which do not meet their modifying or saturating phrase at a fixed place in the sentence. Such verb arguments include the time argument, space argument, but also the event argument (Parsons, 1990). Easy Linking Semantics is an attractive alternative framework in modeling the semantics of free word order languages, as well as the semantic component in grammars that do not make use of movement operations. Easy Linking Semantics, finally, is closely related to Linking Semantics as in Beaver & Condoravdi (2007). It offers a near-type logic way to refer to denotations in their linking structures and can be generalized to accommodate their event-free semantic fragment of English.

## References

- Beaver, D., Condoravdi, C.: On the Logic of Verbal Modification. In: Aloni, M., Dekker, P., Roelofson, F. (eds.) *Proceedings of the Amsterdam Colloq. 2007*, pp. 6–12 (2007)
- Davidson, D.: *The Logical Form of Action Sentences*. In: *Essays on Actions and Events*, pp. 105–122. Clarendon Press, Oxford (1980)
- von Fintel, K., Heim, I.: *Intensional Semantics* (2007), <http://semantics.uchicago.edu/kennedy/classes/s08/semantics2/vonfintel+heim07.pdf>
- Hang, D., Eckhoff, H., Majer, M., Welo, E.: Breaking down and putting back together again. *Analysis and Synthesis of New Testament Greek*. *J. of Greek Linguistics* 9(1), 56–92 (2009)
- Heim, I., Kratzer, A.: *Semantics in Generative Grammar*. Blackwell, Malden (1998)
- Kratzer, A.: 2002/in prog. *The Event Argument of the Verb*. Ms., Semantics Archive (2002)
- Parsons, T.: *Events in the semantics of English*. MIT Press, Boston (1990)
- von Stechow, A., Grønn, A.: *The (Non-)Interpretation of Subordinate Tense*. Manuscript presented at Oslo University, Göttingen University (2009)

# Rivalry between French *-age* and *-ée*: The Role of Grammatical Aspect in Nominalization<sup>\*</sup>

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**Abstract.** This paper will provide an account for the existence of pairs of deverbal nominals with *-age* and *-ée* giving rise to event readings. We first study the argument structure of the bases and of the derived nominals, and establish the general tendencies. We further examine the Aktionsart of the nominalizations and of the verbal bases. We conclude that these levels of investigation are not sufficient to determine the proper contribution of the two nominalization patterns and further demonstrate that the relevant contribution they make is at the level of grammatical aspect. We therefore propose that *-age* introduces the imperfective viewpoint, whereas *-ée* introduces the perfective viewpoint.

**Keywords:** nominalizations, event and argument structure, grammatical aspect.

## 1 Introduction

In this paper, we will study French deverbal nouns with *-age* and *-ée* which are derived from the same verbal base —a case of nominalization rivalry ignored in the literature. Based on a corpus of event nominal pairs derived from 29 verbal bases (which we selected from the TLFi dictionary and completed with web occurrences), we will provide an account of the existence of such pairs in the language. Two questions immediately arise in light of such cases:

- Is there any linguistic reason for the existence of these pairs?
- Do these nominalizations have a distinctive contribution?

Looking at the interplay between event structure, Aktionsart and grammatical aspect, we will try to sketch an answer to these general questions, and propose that the nominalizations under consideration contribute different grammatical aspect values.

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## 2 Argument Structure of the Verbal Bases

### 2.1 All Verbal Bases Selected

We begin by examining two existing hypotheses: (a) the suffix *-age* selects transitive verbal bases (Dubois-Charlier (1999)), and (b) only unaccusative verbs allow *-ée* nominalization (Ruwet (1988)). Examination of the argument structure type of the bases leads us to conclude that there is no clear specialization of the two nominalizations: both can combine with transitive, unaccusative and unergative bases (cf. Legendre (1989) for unaccusativity tests in French). However, some trends and regularities are visible. The transitive base is the primary type selected by both processes that construct N-*age* and N-*ée* pairs:

- (1) couler du bronze 'to cast bronze', couler une cloche, 'to cast a bell'  
→ le coulage / la coulée du bronze/ d'une cloche 'the casting of bronze/a bell'

However, nominalizations with both *-age* and *-ée* also select unergative bases (2) and unaccusatives (3).

- (2) CHEVAUCHER 'aller à cheval' 'to ride' → la chevauchée hebdomadaire 'the weekly ride' / le chevauchage sous un soleil éclatant 'the riding under a blazing sun'  
(3) ARRIVER 'to arrive' → l'arrivage / l'arrivée de la marchandise, 'the arriving'/ the arrival of the merchandise

### 2.2 General Preferences

When selected by only one of the two nominalizations, there is a general preference for certain bases: *-age* shows a tendency to select transitive bases (4) while unaccusatives are selected by *-ée* (5).

- (4) tourner le film 'to shoot the film'  
→ le tournage du film / \*la tournée du film 'the shooting of the film'  
(5) le fascisme monte en Europe 'fascism grows in Europe'  
→ la montée du fascisme/\*le montage du fascisme 'the growth of fascism'

On the one hand, the data confirm the arguments of Martin (2008) (that nominalization with *-age* is not limited to transitive bases) and of Legendre (1989) (that *-ée* nouns are not a valid test for unaccusativity). On the other hand, this result determines the argument structure of the verbal bases selected by the two nominalizations.

### 2.3 Proposal: Highlighting of Causation

#### 2.3.1 Transitive Bases

Nominalization with *-age* highlights the proto-agent property (cf. Dowty (1991)) of the external argument of the verb (cf. Kelling (2001) and Martin (2008) for an earlier analysis). Our analysis is supported by the different meanings associated with N-*age* and N-*ée* derived from the same transitive verb base (6a) and by neologisms (6b). Nominalization with *-age* underlines the causative sense while with *-ée* it highlights the resultative sense.

- (6) a. Le montage des briques / la montée des briques 'the lifting of bricks' (cause/result)  
 b. @...avec Sarkozy, on est entré dans l'ère de l'effrayage !  
 with Sarkozy, we entered the age of scaring  
 (built on EFFRAIER 'scare' transitive-causative : x CAUSE y is scared)

### 2.3.2 Unaccusative Bases

#### (i) Nominalization with *-age* seems to introduce a semantic participant into the event structure of the base verb which has the proto-agent property external causation.

For deverbal nouns built from some unaccusative bases, such as ARRIVAGE 'arrival', POUSSAGE 'growth', nominalization with *-age* seems to introduce causation thus allowing a verbal paraphrase with *faire* 'make'.

- (7) a. l'arrivage des légumes 'the "arriving" = arrival of the vegetables'  
 = 'faire arriver les légumes' 'make the vegetables arrive'  
 b. le poussage des poils sur le torse 'the growth of hair on the chest'  
 = 'faire pousser les poils à l'aide d'une lotion' 'make the hair grow using a lotion'  
 c. le levage de la pâte = 'faire lever la pâte'  
 'the rising of the dough' = 'make the dough rise'

This is also true for other deverbal nouns with *-age* that have no morphological counterpart with *-ée*, like ATERRISSAGE, which is derived from an unaccusative verb that has no transitive counterpart in French (unlike in English and German).

- (8) a. l'avion a atterri 'the plane landed'  
 b. \*le pilote a atterri l'avion 'the pilot landed the plane'  
 c. l'atterrissage de l'avion 'the landing of the plane'

#### (ii) Exceptions

But this pattern is not systematic. An *-age* nominal is ungrammatical when the unaccusative V selects an internal argument that cannot be affected by (agentive or instrumental) causation.

- (9) a. la coulée / \*le coulage / de lave 'the flow / of lava'  
 b. la couchée / le couchage des réfugiés – la couchée / \*le couchage du soleil  
 'the going-to-bed of refugees / the setting of the sun'

The contrasts in (9a-b) are explained by the fact that it is not possible to cause the sunset, or to take into account an external cause (other than natural) for the flowing of lava. Conversely, the examples in (10) are acceptable because it is possible to have an external initiator of the situation expressed by the verb COULER 'flow', and therefore the property 'causally affected' of the proto-patient is present:

- (10) le coulage / la coulée d'eau 'the flowing of water'

We can therefore conclude that in the case of unaccusative verbs that select an internal argument which cannot be affected by causation, the internal argument cannot figure as a participant (y) in the complex event structure in (11), following Levin and Rappaport (1993) and subsequent work.

- (11) [ x CAUSE [BECOME y <STATE>]]

In addition, it also allows us to refine the 'agentivity' property of *-age*, proposed in Kelling (2001) and Martin (2008).

### 2.3.3 Refinement of Our Proposal

Martin (2008) proposed to extend the 'agentivity' property characterizing *-age* deverbals on transitive bases to account for two unaccusative verbs, ARRIVER 'arrive' and POUSSER 'grow', giving rise to *-age* nouns. However, she neither mentions the conditions in which this property is neutralized, nor whether these unaccusative verbs are the only ones that may involve "agentivity" when nominalized by *-age*.

Our study reveals several points.

(i) This 'agentivity' property cannot be extended to all the unaccusatives (even those without a transitive counterpart), as in (12).

- (12) COULER<sub>[unacc]</sub> 'flow' → coulée / \*coulage de lave 'flow / \*flowing of lava'

(ii) An unaccusative verb can be nominalized by *-age* and yet not involve agentivity (13).

- (13) PASSER 'pass' → le passage de l'ouragan 'the passing of the hurricane'

(iii) Unergatives (as in 14a) and some transitive verbs in the corpus (14b) are not causative, even if they allow nominalization by *-age*.

- (14) a. SAUTER 'jump' → sautage 'jumping' (trampoline)  
b. remonter l'escalier 'to climb back upstairs' → le remontage d'escalier 'the climbing back upstairs'

Causation is therefore highlighted by *-age* nominalization in a very particular way. We propose that causation is not directly introduced by *-age* (since certain *-age* nominalizations of unaccusative bases are not causative) but only highlighted when the verb inherently possesses this property. In other words the internal argument must have a proto-P property "be causally affected", which must be specified in the lexical entry of the verb. The proto-P property on the internal argument implies a proto-A property: "x causally affects y". It is conceivable, according to our study, that this lexical property of the verb is only activated through morphological derivation.

## 3 Aspectual Properties

Since the rivalry between nominalizations with *-age* and *-ée* does not seem to be constrained by the argument structure of the verbal base, we continue our investigations by examining the lexical-aspectual properties of the verbs.

### 3.1 Aspectual Properties of the Verbal Bases

Our corpus analysis shows that *-age* and *-ée* nominalizations are not sensitive to the lexical-aspectual class of their verbal bases, since they can select bases from all the aspectual classes except for pure states: activities (15), accomplishments (16), and achievements (17).

- (15) POUSSER 'push' ACT → deux heures de poussage / de poussée (naissance)  
two hours of pushing / of push (delivery)

- (16) PESER (tr) 'weigh' ACC → pesage / pesée de l'enfant 'the weighing of the baby'

- (17) ARRIVER 'arrive' ACH → l'arrivage du navire / l'arrivée du navire  
'the "arriving" / the arrival of the ship'

### 3.2 Aspectual Inheritance vs. Aspectual Shift

The application of the set of tests for French nominalizations elaborated by Haas *et al.* (2008) to the *-age* and *-ée* pairs allows us to conclude that the two constructions have different lexical-aspectual values, which they generally inherit from the verbal bases, but which can also be the result of an aspectual shift induced by nominalization.

#### 3.2.1 Aspectual Inheritance

Activity verbs can give rise to activity nominals with *-age* and *-ée*, as shown by the fact that these nominals reject the structure *un N de x-temps* 'a N of x-time' in (18a), excluded for ACT nominals (Haas *et al.* (2008)). Accomplishments give rise to Durative Culminative Occurrences (DCO, following terminology and tests from Haas *et al.* 2008). This is indicated in (18b) by the fact that the corresponding nominals appear in 'x time of N'. There are also achievements that give rise to Punctual Occurrences (PO, 18c). Contrary to ACT nominals, DCO and PO nominals appear in the subject of a *eu lieu* 'happened'. DCO nominals, but not PO nominals, can be the subject of a *duré* 'lasted' and appear in *en cours de* 'in the process of' N.

- (18) a. V ACT → N ACT  
CRIER (unerg.) 'shout' :  
Il a crié pendant une heure / #en une heure  
He shouted for an hour / #in an hour  
→ une heure de criage / # un criage d'une heure  
an hour of shouting / #a shouting of an hour  
b. V ACC → N DCO  
plumer un volatile → pendant le plumage des oies / entre deux plumées d'oies  
'to pluck a bird' → 'during the plucking of geese / between two pluckings of geese'  
c. V ACH → N PO  
ARRIVER 'arrive' (unacc.) :  
le train est arrivé à 20h00 → l'arrivée du train à 20h00  
the train arrived at 8p.m → the arrival of the train at 8p.m.

#### 3.2.2 Aspectual Shift

Haas *et al.* (2008) added a new category of deverbal nouns: Durative non-Culminative Occurrences (DnCO). The DnCO MANIFESTATION 'demonstration' is derived from an activity verb MANIFESTER 'demonstrate' but successfully passes the test 'subject of a *eu lieu* 'happened' (which excludes activity nominals). DnCOs differ from other Occurrences (DCO, PO) in not being culminative; that is, if the process denoted by the noun is interrupted, we can nonetheless assert that the denoted event took place (*e.g.*, the manifestation has been interrupted → they manifested, vs. the delivery has been interrupted → # she gave birth).

Consequently, there are cases in which the aspectual value of the base is shifted in the nominalization process. Such cases include (i) activity bases which derive DnCOs (19a), as shown by their ability to appear with *pendant* 'during'; (ii) achievement bases giving rise to DCO (instead of PO), which can take *en cours de* 'in the process of' in (19b).

- (19) a. V ACT  $\rightarrow$  N DnCO ( for *-ée* ):  
 chevaucher pendant deux heures (activity) 'to ride for two hours'  
 $\rightarrow$  le jour de la chevauchée (DnCO) 'the day of the ride'  
 b. V ACH  $\rightarrow$  N ODC (for *-age*)  
 ARRIVER 'arrive' (ACH)  
 $\rightarrow$  5173 tonnes (de céréales) étaient en cours d'arrivée par camions  
 '5173 tones (of cereals) were in the process of arriving by trucks'

These results show that the two nominalizations *-age* and *-ée* are not tied to specific lexical-aspectual values. However, in the case of *-age*, we can remark that the shift goes in the direction of durativity (as in 19b), whereas in the case of *-ée*, the shift is associated with terminativity (19a). Nonetheless, the Aktionsart of these nominals seems to be insufficient in distinguishing their properties. In the following section, we will show that the distinguishing factor is in fact their contribution on the level of grammatical aspect (viewpoint – Smith (1991)).

### 3.3 Grammatical Aspect in Nominalizations

Given the existence of these pairs of nouns, it is reasonable to hypothesize that the two nominalizations correspond to different ways of conceptualizing events: focusing on the event as a whole (closed) in the case of *-ée*, or, in the case of *-age*, focusing on the ongoing process or on an internal phase of the event denoted by the verbal base. Thus, *-age* introduces the imperfective aspect, while *-ée* introduces the perfective aspect. The difference should therefore be situated on the level of grammatical aspect (viewpoint), which usually shows up in the verbal domain in French. In this light, we propose the following account of the pairs:

**(20) Proposal: With the same verbal base (tr., unacc. and unerg.) *-age* and *-ée* contribute grammatical aspect introducing an imperfective vs. perfective value**

#### 3.3.1 Series of Arguments Supporting This Semantic Difference

The first argument is provided by the semantic difference between the two nominalizations, which is highlighted by the following distributional tests.

(i) Event nominals with *-ée*, but not with *-age*, can appear with the preposition APRÈS 'after', which requires a perfective event as its complement, exactly as in the case of (finite and non-finite) complement clauses.

- (21) a. ??après l'arrivage de la marchandise / après l'arrivée de la marchandise'  
 'after the arriving of the merchandise / after the arrival of the merchandise'  
 b. après être arrivée, la marchandise a été vendue  
 'after being arrived, the merchandise has been sold'
- (22) a. ??après le pesage du bébé / après la pesée du bébé 'after the weighing of the baby'  
 b. après avoir pesé le bébé 'after having weighed the baby'

(ii) Event nominals with *-age*, but not with *-ée*, can appear as object of INTERROMPRE 'interrupt' (23), or as subject of PROGRESSER 'progress' (24).

- (23) L'arrivage / ??l'arrivée des ouvriers a été interrompu(e) par un convoi de police  
 'the arriving / the arrival of the workers has been interrupted by a police crew'

- (24) Le perçage / ??la percée du tunnel a progressé.  
'the drill-age / the "drill-ée" of the tunnel progressed'
- (iii) The two nominalizations have different meanings (namely 'process in development' with *-age* and 'whole process' with *-ée*), when they appear as objects of FILMER 'to film' (25) or SURVEILLER 'supervize' (26).
- (25) a. J'ai filmé le pesage du bébé  
I filmed the weighing of the baby ( the ongoing event/ a phase of the event)  
b. J'ai filmé la pesée du bébé (la globalité de l'événement : début, milieu, fin)  
I filmed the weighing of the baby (the whole event: start, development, end)
- (26) a. J'ai surveillé l'arrivée des marchandises  
'I supervised the arriving of goods' ('supervise the ongoing event')  
b. #J'ai surveillé l'arrivée des marchandises (épier, guetter, attendre)  
'I supervised the arrival of goods' ('look for, wait for the arrival')

(iv) pluractionality of *-age* as manifestation of its imperfectivity value

Another argument for imperfectivity in the case of *-age* nominals is their pluractional meaning. Recall that in the literature on pluractionality, pluractional markers are defined as imperfective (iterative or habitual) aspectual operators (*cf.*, Van Geenhoven (2004)). Nominalizations with *-age* involve a pluractional meaning which conflicts, in the case of achievement verbal bases, with the cardinality of the internal argument, thus explaining the contrasts in (27) and (28).

- (27) \*l'arrivée d'un légume / OK de légumes, de la marchandise  
'the arriving of a vegetable / of vegetables, of the merchandise'
- (28) \*le tuage d'une mouche / OK de mouches 'the killing of a fly / of flies'

Similar tests have been used crosslinguistically in the domain of verbal aspect for West Greenlandic in Van Geenhoven (2004) and aspectual periphrases with *andar* in Spanish by Laca (2006). Pluractionality has been also documented for Romanian Supine nominalizations by Iordăchioaia & Soare (2008), Alexiadou & al (2008). In (29), the supine derived from 'kill' is ruled out when combined with a singular argument:

- (29) ucisul \*unui jurnalist / jurnaliștilor de către mafia politică  
'the killing \*of a journalist / of journalists by the political mafia' [Romanian]

### 3.3.2 Extension to Nominalizations with *-age/-ment*

Our proposal, according to which *-age/-ée* introduce an opposition at the level of grammatical aspect, allows us to reconsider the treatment of nominalization with *-age/-ment* put forward in Martin (2008). Martin (2008) explains the contrast in (30b) through the fact that *a pedestrian* is not an incremental Theme.

- (30) a. Pierre a écrasé une banane/ un piéton  
'Peter crushed a banana / ran over a pedestrian'  
b. l'écrasage d'une banane / # l'écrasage d'un piéton  
'the crushing of a banana' / "the running over of a pedestrian"  
c. l'écrasement d'un piéton "the running over of a pedestrian"

If our proposal for *-age/-ée* pairs can be extended to *-age/-ment*, more precisely, if the nominalization with *-ment* can be considered as highlighting the global event, then the

contrast in (30b) - (30c) is predicted<sup>1</sup>. In pairs, *-age* nominals denote an ongoing event, so in (30b), *écrasage* cannot take *a pedestrian* as an argument, because *run over a pedestrian* denotes a punctual event (an achievement), and cannot be conceptualized in its development, but only as a global (closed) situation.

## 4 Consequences and Refinements of the Analysis

### 4.1 Selectional Restrictions on the Nominalization of Transitive-Unaccusative Verbs

Our proposal is further confirmed by selectional restrictions on these nominalizations in the case of transitive-unaccusative verbs (see also Martin 2008 for *-age/-ment*). As shown in (31), *-age* selects the transitive base whereas the unaccusative base is selected by *-ée*.

- (31) a. Marie a percé son abcès > le perçage de l'abcès.  
'Mary burst her abscess / the bursting of the abscess'  
b. Son abcès a percé > la percée de l'abcès /vs. #le perçage de l'abcès  
'Her abscess burst / the bursting of the abscess'

(i) Proposal: Given that *-age* conceptualizes the situation type denoted as ongoing, then it is expected that *-age* selects the event structure involving the initiator (or the volitional causer) of the ongoing event: the complex one (transitive pattern) where figures the initiator, *x*, whereas *-ée* will select the simple one (unaccusative pattern):

- (32) PERCER 'to burst'  
a. [*x* CAUSE [BECOME *y* <BURST>]] for (31a) → PERÇAGE  
b. [BECOME *y* <BURST>] for (31b) → PERCÉE

(ii) Account of these selectional restrictions for *-age* vs. *-ment* deverbals of transitive-unaccusative verbs by Martin (2008). According to Martin (2008) [Property 1], for GONFLER 'inflate, blow' (transitive-unaccusative verb), *-age* deverbals are built on the long eventive chain of the verb (the transitive pattern): *gonflage du ballon par Pierre*, while *-ment* deverbals are built on the short one (the unaccusative pattern): *gonflement du ballon* 'the inflation of the balloon'. This distribution is correct, but, as noted by Martin (2008) herself, *-ment* deverbals can also be built on the long eventive chain of the alternating verbs (*gonflement du ballon par Pierre* 'the inflation of the balloon by Pierre'). This casts doubts on the exploitation of the notion of length of the eventive chain for explaining the selectional restrictions.

### 4.2 Transitive-Unaccusative and Transitive Bases Selected by Both *-age/-ée*

If *-age* and *-ée* respectively introduce imperfective and perfective grammatical aspect, the selectional restrictions in the case of transitive-unaccusative verbs follow

<sup>1</sup> These examples would also involve, for *-age/-ment* pairs, an interplay between the Aktionsart of the verb and the grammatical aspect of the nominalization, which may *a priori* not hold for *-age/-ée*.

naturally : *-age* is predicted to select only the complex event structure because it contains the initiator of the denoted situation type (33a) – transitive pattern ; whereas *-ée* will select the simple event structure ((33b) – unaccusative pattern)<sup>2</sup> :

- (33) a. [x CAUSE [BECOME y <STATE>]]                      b. [BECOME y <STATE>]]  
 ..... // ..... *-age*                      // ..... *-ée*

Because *-ée* presents the situation as closed, we predict that *-ée* can also select a complex event structure including the initiator (33a) in the case of transitive – unaccusative verbs, then also accounting for *le gonflement du ballon par Pierre* ‘the inflation of the balloon by Pierre’ exactly as for transitive base verbs of our corpus selected by both nominalizations.

- (34) rentrer les vaches ‘to bring in the cows’: [x CAUSE [BECOME y <PLACE>]]
- (35) a. La rentrée des vaches ‘the bringing in of cows’  
 b. @ *j’ai effectué* la rentrée des bêtes ‘I did the bringing in of the animals’  
 c. [x CAUSE [BECOME y <PLACE>]]  
 // ..... *-ée*
- (36) a. Le rentrage des vaches ‘the bringing in of cows’  
 b. @ opération rentrage des vaches *avec une voisine qui n’y connaît rien*  
 ‘the operation of bringing in the cows with a neighbour who knows nothing about’  
 c. [x CAUSE [BECOME y <PLACE>]]  
 ..... // ..... *-age*

For transitive-unaccusative verbs, *-ée* can also select the complex event structure, but *-age* can only select the complex one, because of their respective grammatical aspect values.

### 4.3 Nominalization of Unaccusative Verbs without Transitive Counterparts

Our proposal makes the following prediction: because these unaccusative verbs have a simple event structure [without an external initiator (x)] they will only be selected by *-ée* (37). The prediction is borne out: (38c) vs. (38b) :

- (37) PERCER<sub>2</sub> émerger ‘to emerge’: [BECOME y <emerged>]
- (38) a. @les fleurs ont percé / l’entreprise a rapidement percé (PERCER<sub>2</sub> émerger)  
 ‘the flowers “broke through” / ‘the enterprise broke through’  
 b. # le perçage des fleurs/ # le perçage de l’entreprise  
 ‘the “breaking-through” of the flowers’ / ‘the breaking through of the enterprise’  
 c. @la percée des fleurs / la percée de l’entreprise  
 ‘the “break-through” of the flowers’ / ‘the breaking through of the enterprise’

Our proposal then covers the distribution of patterns that Martin (2008) treats in terms of length of the eventive chain, but it goes further: (i) by proposing a principled reason for this distribution : because N *-age* denotes an ongoing event (so a portion of it) – imperfective view point – it highlights the initiator of the situation denoted by the verb, involved in the ongoing event; (ii) by accounting for the fact that the complex event structure is not only combinable with *-age*, but also with *-ée* (and also

<sup>2</sup> In this event structure, the slashes note the portion of the event highlighted by each nominalization.

with *-ment* cf. examples of Martin (2008)). The same proposal (i.e. (20)) allows us to account for the selectional restrictions in the case of transitive-unaccusative verbs (*-age* selects the transitive one, *-ée* selects the unaccusative one) but also to predict the nominalization of 'pure' unaccusative verbs.

## 5 Conclusion

By comparing the properties of *-age* and *-ée* in pairs, we have shown that these nominalizations encode different values of grammatical aspect, e.g. imperfective vs. perfective, in other words that there is a complementary distribution.

The originality of the proposal consists in the existence, in the nominal domain, of a level of grammatical aspect, normally reserved to the verbal domain in French. We support our proposal by specific tests devoted to nominalizations.

- Consequently, the various properties associated with *-age* nominals in the literature (e.g., agentivity, incrementality, length of the eventive chain) follow from our general proposal that *-age* and *-ée* convey different grammatical aspectual values.
- The other properties we studied also fall under this proposal, namely the fact that nominalizations with *-age* and *-ée* can select all types of bases, but *-age* exhibit preference for transitive bases, whereas *-ée* for unaccusative ones. Correlatively, *-age* nominals built on unaccusative bases reveal a part of the argument structure of the verb, which is invisible to syntax. Moreover, in spite of the general inheritance of the lexical aspectual value of the base verb in the deverbal nouns, there is also aspectual shift, reflecting durativity in the case of *-age* and terminativity in the case of *-ée*.
- The next step of our research will be to test the hypothesis of the existence of grammatical aspect to the whole *-age* / *-ée* corpus. We hypothesize that generally the grammatical aspect oppositions are encoded in pairs (*-age/-ée*, *-age/-ment*, etc) but they can be lexicalized by a single nominalization rule, when pairs are not available.

## References

1. Alexiadou, A., Iordăchioaia, G., Soare, E.: Nominal/Verbal Parallelisms and Number/Aspect Interactions in the Syntax of Nominalizations. Submitted to Journal of Linguistics (2008)
2. Dowty, D.: Thematic Proto-roles and Argument Selection. *Language* 67(3), 547–619 (1991)
3. Dubois, J., Dubois-Charlier, F.: La dérivation suffixale en français, Paris, Nathan (1999)
4. Kelling, C.: French Psych Verbs and Derived Nouns. In: Butt, M., King, T.H. (eds.) *Nominals. Inside and out*. CSLI, Stanford (2003)
5. Haas, P., Huyges, R., Marin, R.: Du verbe au nom: calques et décalcages aspectuels, Actes du Congrès Mondial de Linguistique Française (2008)
6. Laca, B.: Indefinites, Quantifiers and Pluractionals: What Scope Effects Tell us about Event Pluralities. In: Tasmowski, L., Vogelee, S. (eds.) *Non-definiteness and Plurality*. John Benjamins, Amsterdam (2006)

7. Levin, B., Rappaport, M.: Levels of Lexical Representation. In: Pustejovsky, J. (ed.) *Semantics and Lexicon*. Kluwer Academic Publishers, Dordrecht (1993)
8. Martin, F.: The Semantics of Eventive Suffixes in French. In: Schäfer, F. (ed.) 'SinSpec', Working Papers of the SFB 732, vol. 1. University of Stuttgart, Stuttgart (2008)
9. Ruwet, N.: Les verbes météorologiques et l'hypothèse inaccusative. In: Benveniste, C.B., Chervel, A., Gross, M. (eds.) *Mélanges à la mémoire de Jean Stéfanini* (1988)
10. Smith, C.: *The Parameter of Aspect*. Kluwer Academic Press, Dordrecht (1991)
11. Van Geenhoven, V.: For-adverbials, Frequentative Aspect, and Pluractionality. *Natural Language Semantics* 12, 135–190 (2004)

# Free Choice from Iterated Best Response

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**Abstract.** This paper summarizes the essence of a recent game theoretic explanation of free choice readings of disjunctions under existential modals ([8]). It introduces principles of game model construction to represent the context of utterance, and it spells out the basic mechanism of iterated best response reasoning in signaling games.

**Keywords:** conversational implicatures, game theoretic pragmatics, free choice disjunction, iterated best response.

## 1 Free Choice Disjunctions and Game Theory

Contrary to their logical semantics, disjunctions under modal operators as in (1a) may receive free-choice readings (FC-readings) as in (1b) ([12]).

- (1) a. You may take an apple or a pear.  $\Diamond(A \vee B)$
- b. You may take an apple and you may take a pear.  $\Diamond A \wedge \Diamond B$

This inference is not guaranteed by the standard logical semantics which treats disjunction as truth-functional connective and the modal as an existential quantifier over accessible worlds. Of course, different semantics of disjunctions or modals are conceivable and have been proposed by, for instance, [12], [18] or [1]. But, all else being equal, a *pragmatic* solution that retains the logical semantics and treats FC-readings as Gricean inferences seems preferable (cf. the arguments in [16]).

Unfortunately, a naïve approach to Gricean scalar reasoning does not suffice. If we assume that the set of expression alternatives with which to compare an utterance of (1a) contains the simple expressions in (2), we run into a problem.

- (2) a. You may take an apple.  $\Diamond A$
- b. You may take a pear.  $\Diamond B$

Standard scalar reasoning tells us that all semantically stronger alternatives are to be inferred *not* to be true. This yields that  $\Box \neg A$  and that  $\Box \neg B$  are true, which contradicts (1a) itself.

This particular problem has a simple solution. [14] observe that the FC-reading follows from naïve scalar reasoning based on the alternatives in (2) if we use the already exhaustified readings of the alternatives as in (3).

- (3) a. You may take an apple, but no pear.  $\Diamond A \wedge \neg \Diamond B$   
 b. You may take a pear, but no apple.  $\Diamond B \wedge \neg \Diamond A$

Truth of (1a) together with the falsity of both sentences in (3) entails the FC-reading in (1b).

There is clearly a certain intuitive appeal to this idea: when reasoning about expression alternatives it is likely that potential pragmatic enrichments of these may at times be taken into account as well. But when and how exactly? Standard theories of scalar reasoning do not integrate such nested pragmatic reasoning. This has been taken as support for theories of local implicature computation in the syntax where exhaustivity operators can apply, if necessary, several times (57). But the proof that such nested or iterated reasoning is very much compatible with a systematic, global, and entirely Gricean approach amenable to intuitions about economic language use is still up in the air.

Enter game theory. Recent research in game theoretic pragmatics has produced a number of related models of agents' step-by-step pragmatic reasoning about each others' hypothetical behavior (17, 210). This is opposed to the more classical equilibrium-based solution concepts which merely focus on stable outcomes of, mostly, repeated play or evolutionary dynamics. The main argument of this paper is that such step-by-step reasoning, which is independently motivated, explains free-choice readings along the lines sketched above: early steps of such reasoning establish the exhaustive readings of alternative forms, while later steps of the same kind of global reasoning can pick on previously established readings.

In order to introduce and motivate this game theoretical approach, two sets of arguments are necessary.<sup>1</sup> Firstly, we need to settle on what kind of *game model* is required in order to represent conversational moves and their interpretation. This is to be addressed in section 2. Secondly, we need to spell out a *solution concept* by means of which pragmatic language use can be explained in the chosen game models. This is the topic of section 3. Finally, section 4 reviews briefly how this approach generalizes.

## 2 Interpretation Games as Context Models

It is standard in game-theoretic pragmatics to assume that an informative assertion and its uptake can reasonably be modelled as a *signaling game*. More specifically then, the pragmatic interpretation of assertions can be modelled by a particular kind of signaling game, which I will call *interpretation game*. These latter games function as representations of the context of utterance (as conceived by the receiver) and are constructed from a given target expression whose interpretation we are interested in, together with its natural Neo-Gricean alternatives and their logical semantics. Let me introduce both signaling games and interpretation games one after the other.

<sup>1</sup> These arguments can only be given in their bare essentials here (see [8] for the full story).

*Signaling Games.* A signaling game is a simple dynamic game with imperfect information between a sender and a receiver. The sender has some private information about the state of the world  $t$  which the receiver lacks. The sender chooses a message  $m$  from a given set of alternatives, all of which we assume to have a semantic meaning commonly known between players. The receiver observes the sent message  $m$  and chooses an action  $a$  based on this observation. An *outcome* of playing a signaling game for one round is given by the triple  $t$ ,  $m$  and  $a$ . Each player has his own preferences over such outcomes.

More formally speaking, a signaling game (with meaningful signals) is a tuple

$$\langle \{S, R\}, T, \text{Pr}, M, [\![\cdot]\!] , A, U_S, U_R \rangle$$

where sender  $S$  and receiver  $R$  are the players of the game;  $T$  is a set of states of the world;  $\text{Pr} \in \Delta(T)$  is a probability distribution over  $T$ , which represents the receiver's uncertainty which state in  $T$  is actual;<sup>2</sup>  $M$  is a set of messages that the sender can send;  $[\![\cdot]\!] : M \rightarrow \mathcal{P}(T) \setminus \emptyset$  is a denotation function that gives the predefined semantic meaning of a message as the set of all states where that message is true;  $A$  is the set of response actions available to the receiver; and  $U_{S,R} : T \times M \times A \rightarrow \mathbb{R}$  are utility functions for both sender and receiver.

*Interpretation Games.* For models of natural language interpretation a special class of signaling games is of particular relevance. To explain pragmatic inferences like implicatures we should look at *interpretation games*. I assume here that these games can be constructed generically from a set of alternatives to the to-be-interpreted expression, together with their logical semantics. Here are the assumptions and the construction steps.

Firstly, the set of receiver actions is equated with the set of states  $A = T$  and the receiver's utilities model his interest in getting to know the true state of affairs, i.e., getting the right *interpretation* of the observed message:

$$U_R(t, m, a) = \begin{cases} 1 & \text{if } t = a \\ 0 & \text{otherwise.} \end{cases}$$

Moreover, in the vein of [9], we assume that conversation is a *cooperative* effort—at least on the level of such generic context models—so that the sender shares the receiver's interest in correct interpretation.<sup>3</sup>

$$U_S(t, m, a) = U_R(t, m, a).$$

The set  $T$  of state distinctions is to be derived from the set  $M$  of messages given by some (normal, natural, Neo-Gricean) set of alternative forms to the

<sup>2</sup> As for notation,  $\Delta(X)$  is the set of all probability distributions over set  $X$ ,  $Y^X$  is the set of all functions from  $X$  to  $Y$ ,  $X : Y \rightarrow Z$  is alternative notion for  $X \in Z^Y$ , and  $\mathcal{P}(X)$  is the power set of  $X$ .

<sup>3</sup> Notice that this implicitly also commits us to the assumption that all messages are equally costly, or, if you wish, costless.

target sentence whose implicatures we are interested in. Clearly, not every possible way the world could be can be distinguished with any set  $M$ . So we should restrict ourselves to only those states that can feasibly be expressed with the linguistic means at hand. What are those distinctions? Suppose  $M$  contains only logically independent alternatives. In that case, we could in principle distinguish  $2^M$  possible states of the world, according to whether some subset of messages  $X \subseteq M$  is such that all messages in  $X$  are true, while all messages in its complement are false. (This is what happens in propositional logic, when we individuate possible worlds by all different valuations for a set of proposition letters.) But for normal pragmatic applications the expressions in  $M$  will not all be logically independent. So in that case we should look at states which can be *consistently* described by a set of messages  $X \subseteq M$  all being true while all expressions in its complement are false. Moreover, since at least the target message may be assumed true for pragmatic interpretation, we should define the set of states of the interpretation game as given by the set of all subsets  $X \subseteq M$  containing the target message such that the formula

$$\bigwedge X \wedge \neg \bigvee M \setminus X$$

is consistent. With this, also the semantic denotation function  $\llbracket \cdot \rrbracket$  is then straightforwardly defined as:

$$\llbracket m \rrbracket = \{t \in T \mid m \in t\}.$$

Finally, since we are dealing with general models of utterance interpretation, we should not assume that the receiver has biased beliefs about which specific state obtains. This simply means that in interpretation games  $\text{Pr}(\cdot)$  is a flat probability distribution.

*Example.* To give a concrete example, here is how to construct an interpretation game for the target expression in (1a). Everything falls into place once a set of alternatives is fixed. To keep the exposition extremely simple, let us first only look at the set of messages in (4). (See section 4 for more discussion.)

- |     |                                     |                          |
|-----|-------------------------------------|--------------------------|
| (4) | a. You may take an apple or a pear. | $m_{\Diamond(A \vee B)}$ |
|     | b. You may take an apple.           | $m_{\Diamond A}$         |
|     | c. You may take a pear.             | $m_{\Diamond B}$         |

Based on these alternatives, there are three states we need to distinguish:

$$\begin{aligned} t_A &= \{m_{\Diamond A}, m_{\Diamond(A \vee B)}\} \\ t_B &= \{m_{\Diamond B}, m_{\Diamond(A \vee B)}\} \\ t_{AB} &= \{m_{\Diamond A}, m_{\Diamond B}, m_{\Diamond(A \vee B)}\}. \end{aligned}$$

Here,  $t_A$  is a state where the hearer may take an apple but no pear, and  $t_{AB}$  is a state where the hearer may take both an apple and a pear. These states yield the interpretation game in figure 1. Notice that we consider only these states,

	$\Pr(t)$	$a_A$	$a_B$	$a_{AB}$	$m_{\Diamond A}$	$m_{\Diamond B}$	$m_{\Diamond(A \vee B)}$
$t_A$	$\frac{1}{3}$	1,1	0,0	0,0	✓	—	✓
$t_B$	$\frac{1}{3}$	0,0	1,1	0,0	—	✓	✓
$t_{AB}$	$\frac{1}{3}$	0,0	0,0	1,1	✓	✓	✓

**Fig. 1.** Interpretation game constructed from (1a) and (4)

because these are the only distinctions we can make between worlds where the target message (1a) is true that can be expressed based on consistent valuations of all alternatives. Certainly, in the present case, this is nearly excessively simple, but it is not trivial and, most importantly, there is still room for pragmatic interpretation: there are still many ways in which sender and receiver could coordinate on language use in this game. What is needed is a solution concept that singles out uniquely the player behavior that explains the free choice inference.

### 3 Iterated Best Response Reasoning

Generally, behavior of players is represented in terms of *strategies*. A *pure sender strategy*  $s \in S = M^T$  is a function from states to messages and a *pure receiver strategy*  $r \in R = A^M$  is a function from messages to actions. A *pure strategy profile*  $\langle s, r \rangle$  is then a characterization of the players' *joint behavior* in a given signaling game. For instance, the tuple:

$$s = \left\{ \begin{array}{l} t_A \mapsto m_{\Diamond A} \\ t_B \mapsto m_{\Diamond B} \\ t_{AB} \mapsto m_{\Diamond(A \vee B)} \end{array} \right\} \quad r = \left\{ \begin{array}{l} m_{\Diamond A} \mapsto t_A \\ m_{\Diamond B} \mapsto t_B \\ m_{\Diamond(A \vee B)} \mapsto t_{AB} \end{array} \right\} \quad (1)$$

is a strategy profile for the game in figure 1. And a special one, indeed. It corresponds to the intuitive way of using the corresponding natural language expressions: the interpretation of  $m_{\Diamond A}$ , for instance, is the exhaustive reading that only  $A$ , but not  $B$  is allowed; and the interpretation of  $m_{\Diamond(A \vee B)}$  is the free choice inference that both taking  $A$  and taking  $B$  are allowed. This is therefore what a solution concept is required to predict in order to explain FC-readings based on the game in figure 1.

But the strategy profile in (1) is not the only one there is. Also, the rather unintuitive pooling strategy profile

$$s = \left\{ \begin{array}{l} t_A \mapsto m_{\Diamond(A \vee B)} \\ t_B \mapsto m_{\Diamond(A \vee B)} \\ t_{AB} \mapsto m_{\Diamond(A \vee B)} \end{array} \right\} \quad r = \left\{ \begin{array}{l} m_{\Diamond A} \mapsto t_{AB} \\ m_{\Diamond B} \mapsto t_{AB} \\ m_{\Diamond(A \vee B)} \mapsto t_{AB} \end{array} \right\} \quad (2)$$

is conceivable. What is worse, both strategy profiles describe an equilibrium state: given the behavior of the opponent neither player has an incentive to deviate. But, clearly, to explain the FC-reading, the profile in (1) should be selected, while the profile in (2) should be ruled out. In other words, we need a mechanism with which to select one equilibrium and rule out others.

*IBR Models.* One way of looking at an *iterated best response model* (IBR model) is exactly that: a plausible mechanism with which reasoners (or a population) may arrive at an equilibrium state (rather than another). An IBR model assumes that agents reason about each other's behavior in a step-by-step fashion. The model is anchored in naïve behavior of level-0 players that do not take opponent behavior into account, but that may be sensitive to other non-strategic, psychological factors, such as, in our case, the semantic meaning of messages. Players of level- $(k + 1)$  assume that their opponent shows level- $k$  behavior and play a best response to this belief<sup>4</sup>.

Here is a straightforward IBR sequence as a solution concept for signaling games. Naïve players of level-0 are defined as playing some arbitrary strategy that conforms to semantic meaning. For the sender, this yields:

$$S_0 = \{s \in S \mid \forall t \in T : t \in \llbracket s(t) \rrbracket\}.$$

Level-0 senders are characterized by the set of all pure strategies that send only true messages. For interpretation games, naïve receiver types receive a similarly straightforward characterization:

$$R_0 = \{r \in R \mid \forall m \in M : r(m) \in \llbracket m \rrbracket\}.$$

Level-0 receivers are characterized by the set of all pure strategies that interpret messages as true.

In order to define level- $(k+1)$  types, it is necessary to define the notion of a *best response* to a belief in level- $k$  behavior. There are several possibilities of defining beliefs in level- $k$  behavior<sup>5</sup>. The most convenient approach is to assume that agents have *unbiased beliefs* about opponent behavior. Unbiased beliefs in level- $k$  behavior do not favor any one possible level- $k$  behavior, if there are several, over any other, and can therefore be equated simply with a flat probability distribution over the set of level- $k$  strategies.

Turning first to higher-level sender types, let us write  $R_k(m, a)$  for the probability that a level- $k$  receiver who is believed to play a random strategy in  $R_k$  will play  $a$  after observing  $m$ . Then level- $(k + 1)$  senders are defined by

$$S_{k+1} = \left\{ s \in S \mid s(t) \in \arg \max_{m \in M} \sum_{a \in A} R_k(m, a) \times U_S(t, m, a) \right\}$$

as the set of all best responses to that unbiased belief.

For higher-level receiver types the same standard definition applies once we have characterized the receiver's *posterior beliefs*, i.e., beliefs the receiver holds about the state of the world after he observed a message. These need to be

<sup>4</sup> Models of this kind are good predictors of laboratory data on human reasoning (see, for instance [3]), but also solve conceptual issues with equilibrium solution concepts (see [6]). Both of these aspects make IBR models fit for use in linguistic applications.

<sup>5</sup> This is the crucial difference between various IBR models such as given by [4], [11] and [8], for instance.

derived, again in entirely standard fashion, from the receiver's prior beliefs  $\Pr(\cdot)$  and his beliefs in sender behavior as given by  $S_k$ . Let  $S_k(t, m)$  be the probability that a level- $k$  sender who is believed to play a random strategy in  $S_k$  will send  $m$  in state  $t$ . A level- $(k+1)$  receiver has posterior beliefs  $\mu_{k+1} \in (\Delta(T))^M$  calculated by Bayesian conditionalization, as usual:

$$\mu_{k+1}(t|m) = \frac{\Pr(t) \times S_k(t, m)}{\sum_{t' \in T} \Pr(t') \times S_k(t', m)}.$$

Level- $(k+1)$  receivers are then defined as best responding to this posterior belief:

$$R_{k+1} = \left\{ r \in R \mid r(m) \in \arg \max_{a \in A} \sum_{t \in T} \mu_{k+1}(t|m) \times U_R(t, m, a) \right\}.$$

This last definition is incomplete. Bayesian conditionalization is only defined for messages that are not *surprise messages*. A surprise message for a level- $(k+1)$  receiver is a message that is not used by any strategy in  $S_k$  in any state. A lot can be said about the proper interpretation of surprise messages (see the discussion in [11, 8, 15]). This is the place where different *belief revision strategies* of the receiver could be implemented, if needed or wanted. For the purposes of this paper it is sufficient to assume that whatever else the receiver may come to believe if he observes a surprise message, he will stick to the belief that it is true. So, if for some message  $m$  we have  $S_k(t, m) = 0$  for all  $t$ , then define  $\mu_{k+1}(t|m) = \Pr(t \mid \llbracket m \rrbracket)$ .

*Example.* The simple IBR model sketched here does what we want it to: it uniquely singles out the intuitive equilibrium state in equation (II) for the game in figure I. To see how this works, and to see where IBR may rationalize the use of exhausted alternatives in Gricean reasoning, let us calculate the sequence of reasoning starting with  $R_0$  for the simple game in figure I (the case starting with  $S_0$  is parallel).<sup>6</sup>

$$\begin{aligned} R_0 &= \left\{ \begin{array}{ll} m_{\Diamond A} & \mapsto t_A, t_{AB} \\ m_{\Diamond B} & \mapsto t_B, t_{AB} \\ m_{\Diamond(A \vee B)} & \mapsto t_A, t_B, t_{AB} \end{array} \right\} & S_1 &= \left\{ \begin{array}{ll} t_A & \mapsto m_{\Diamond A} \\ t_B & \mapsto m_{\Diamond B} \\ t_{AB} & \mapsto m_{\Diamond A}, m_{\Diamond B} \end{array} \right\} \\ R_2 &= \left\{ \begin{array}{ll} m_{\Diamond A} & \mapsto t_A \\ m_{\Diamond B} & \mapsto t_B \\ m_{\Diamond(A \vee B)} & \mapsto t_A, t_B, t_{AB} \end{array} \right\} & S_3 &= \left\{ \begin{array}{ll} t_A & \mapsto m_{\Diamond A} \\ t_B & \mapsto m_{\Diamond B} \\ t_{AB} & \mapsto m_{\Diamond A \vee B} \end{array} \right\} \\ R_4 &= \left\{ \begin{array}{ll} m_{\Diamond A} & \mapsto t_A \\ m_{\Diamond B} & \mapsto t_B \\ m_{\Diamond(A \vee B)} & \mapsto t_{AB} \end{array} \right\}. \end{aligned}$$

Naïve receiver behavior only takes semantic meaning into account and this is what  $S_1$  plays a best response to. Given  $S_1$ , message  $m_{\Diamond A}$  is interpreted exhaustively by  $R_2$ , as meaning “you may do  $A$ , but not  $B$ ” (and similarly for  $m_{\Diamond B}$ ),

<sup>6</sup> Sets of pure strategies  $Z \subseteq X^Y$  are represented by listing for each  $x \in X$  the set of all  $y \in Y$  such that for some strategy  $z \in Z$  we have  $z(x) = y$ .

while message  $m_{\Diamond(A \vee B)}$  is a surprise message, and will be interpreted merely as true. This makes  $m_{\Diamond(A \vee B)}$  the only rational choice for  $S_3$  to send in  $t_{AB}$ , so that in one more round of iteration we reach a fixed point equilibrium state in which  $R_4$  assigns to  $m_{\Diamond(A \vee B)}$  the FC-reading that he may do  $A$  and that he may do  $B$ . In sum, the FC-reading of  $m_{\Diamond(A \vee B)}$  is derived in two steps of receiver reasoning by first establishing an exhaustive interpretation of the alternatives, and then reasoning with this exhaustive interpretation to arrive at the FC-reading.

## 4 IBR Reasoning: The Bigger Picture

The previous two sections have tried to give, as short and yet accessible as possible, the main mechanism of IBR reasoning and the demonstration that IBR reasoning can account for FC-readings of disjunctions. Many assumptions of this approach could not have possibly been spelled out sufficiently, and so the impression may arise that IBR reasoning, as outlined here, is really only arbitrarily designed to deal with a small problem of linguistic interest. This is, decidedly, not so. There are good and independent motivations for both game model construction and solution concept, and both in tandem do good explanatory work, both conceptually and empirically (see [21][8]).

Moreover, it should be stressed that the IBR approach also handles more complex cases than the easy example discussed above, of course. Most importantly, it predicts well also when other scalar contrasts, such as given by (5a) or (5b), are taken into account as well.

- |     |                                      |                            |
|-----|--------------------------------------|----------------------------|
| (5) | a. You must take an apple or a pear. | $m_{\Box(A \vee B)}$       |
|     | b. You may take an apple and a pear. | $m_{\Diamond(A \wedge B)}$ |

Including more alternative messages results in bigger context models that include more state distinctions. But still IBR reasoning gives intuitive results. For instance, [8] spells out the IBR reasoning based on a set of alternatives that includes (4) and the conjunctive alternative in (5b). Doing so, we derive that (1a) is taken to implicate that  $\Diamond(A \wedge B)$  is false. This is as it should be: in a context where the conjunctive alternative (5b) is salient, this inference should be predicted, but for the FC-reading alone the simple alternatives as in (4) should suffice. Similar considerations apply to the stronger modal alternative in (5a).

Generalizing the result further, it is possible to show that for any  $n$ -place case of the form  $\Diamond(A_1 \vee \dots \vee A_n)$  we derive the inference that  $\Diamond A_i$  under IBR logic. The argument that establishes this result is a so-called *unravelling argument* which I can only sketch here: in the first step (of receiver reasoning) all “singleton” messages of the form  $\Diamond A_i$  are associated with their exhaustive readings; in the second step all two-place disjunctions  $\Diamond(A_i \vee A_j)$ ,  $i \neq j$ , are associated with states in which exactly two actions are allowed one of which must be  $A_i$  or  $A_j$ .<sup>7</sup>

<sup>7</sup> In order to make this inference more specific, as it clearly should be, a slightly more careful setup of the reasoning sequence is necessary than given here. But this is a technical problem that does not disturb the conceptual point that is of relevance.

continuing in this way, after  $n$  rounds of reasoning the form  $\Diamond(A_1 \vee \dots \vee A_n)$  gets the right interpretation that all actions  $A_i$  are allowed.

Interestingly, IBR does *not need* to assume conjunctive alternatives even for the general  $n$ -place case, while [14]’s approach *has to*<sup>8</sup>. To see this, look at the three-placed case  $\Diamond(A \vee B \vee C)$  with only alternatives  $\Diamond A$ ,  $\Diamond B$  and  $\Diamond C$ . The exhaustive readings of these are given in (6).

- (6) a.  $\Diamond A \wedge \neg \Diamond B \wedge \neg \Diamond C$
- b.  $\Diamond B \wedge \neg \Diamond A \wedge \neg \Diamond C$
- c.  $\Diamond C \wedge \neg \Diamond A \wedge \neg \Diamond B$

But truth of  $\Diamond(A \vee B \vee C)$  together with the falsity of all sentences in (6) does not yield the FC-reading that any of  $A$ ,  $B$  or  $C$  are allowed. To establish the FC-reading, we also need the alternatives  $\Diamond(A \wedge B)$ ,  $\Diamond(A \wedge C)$  and  $\Diamond(B \wedge C)$  with their exhaustive readings in (7).

- (7) a.  $\Diamond(A \wedge B) \wedge \neg \Diamond C$
- b.  $\Diamond(A \wedge C) \wedge \neg \Diamond B$
- c.  $\Diamond(B \wedge C) \wedge \neg \Diamond A$

If we then want to account for the presence of the FC-reading in the absence of the scalar inference that  $\Diamond(A \wedge B \wedge C)$  is false, we need to assume that all alternatives with two-placed conjunctions are given, but *not* the three-placed conjunctive alternative. This is not impossible, but also not very plausible.

Finally, let me also mention for the sake of completeness that the IBR approach also deals with free choice readings of disjunctions under universal modals in the exact same fashion as outlined here. A parallel account also deals with the structurally similar inference called *simplification of disjunctive antecedents* as exemplified in (8).

- (8) a. If you take an apple or a pear, that’s okay.
- b. If you take an apple, that’s okay. And if you take a pear, that’s also okay.

The IBR model is also capable of dealing with epistemic ignorance readings such as forced by (9).

- (9) You may take an apple or a pear, but I don’t know which.

To capture these, however, the game models have to be adapted to include also possible sender uncertainty (see [8] for details).

<sup>8</sup> And with it, in slightly amended form, the syntactic account of [7].

## References

1. Asher, N., Bonevac, D.: Free choice permission as strong permission. *Synthese* 145(3), 303–323 (2005)
2. Benz, A., van Rooij, R.: Optimal assertions and what they implicate. *Topoi* 26, 63–78 (2007)
3. Camerer, C.F.: *Behavioral Game Theory: Experiments in Strategic Interaction*. Princeton University Press, Princeton (2003)
4. Camerer, C.F., Ho, T.-H., Chong, J.-K.: A cognitive hierarchy model of games. *The Quarterly Journal of Economics* 119(3), 861–898 (2004)
5. Chierchia, G.: Scalar implicatures, polarity phenomena and the syntax/pragmatics interface. In: Belletti, A. (ed.) *Structures and Beyond*, pp. 39–103. Oxford University Press, Oxford (2004)
6. Crawford, V.P.: Lying for strategic advantage: Rational and boundedly rational misrepresentation of intentions. *American Economic Review* 93(1), 133–149 (2003)
7. Fox, D.: Free choice and the theory of scalar implicatures. In: Sauerland, U., Staveva, P. (eds.) *Presupposition and Implicature in Compositional Semantics*, pp. 71–120. Palgrave MacMillan, Hampshire (2007)
8. Franke, M.: *Signal to Act: Game Theory in Pragmatics*. PhD thesis, Universiteit van Amsterdam (2009)
9. Grice, P.H.: *Studies in the Ways of Words*. Harvard University Press, Cambridge (1989)
10. Jäger, G.: Game dynamics connects semantics and pragmatics. In: Pietarinen, A.-V. (ed.) *Game Theory and Linguistic Meaning*, pp. 89–102. Elsevier, Amsterdam (2007)
11. Jäger, G., Ebert, C.: Pragmatic rationalizability. In: Riester, A., Solstad, T. (eds.) *Proceedings of Sinn und Bedeutung*, vol. 13, pp. 1–15 (2009)
12. Kamp, H.: Free choice permission. *Proceedings of the Aristotelian Society* 74, 57–74 (1973)
13. Kamp, H.: Semantics versus pragmatics. In: Guenther, F., Schmidt, S.J. (eds.) *Formal Semantics and Pragmatics for Natural Languages*, pp. 255–287. Reidel, Dordrecht (1978)
14. Kratzer, A., Shimoyama, J.: Indeterminate pronouns: The view from Japanese. In: Otsu, Y. (ed.) *Proceeding of the 3rd Tokyo Conference on Psycholinguistics*, pp. 1–25 (2002)
15. Mühlenbernd, R.: *Kommunikationsmodell für den Entwicklungsprozess von Implikaturen*. Master's thesis, University of Bielefeld (2009)
16. Schulz, K.: A pragmatic solution for the paradox of free choice permission. *Synthese* 147, 343–377 (2005)
17. Stalnaker, R.: Saying and meaning, cheap talk and credibility. In: Benz, A., Jäger, G., van Rooij, R. (eds.) *Game Theory and Pragmatics*, pp. 83–100. Palgrave MacMillan, Hampshire (2006)
18. Zimmermann, T.E.: Free choice disjunction and epistemic possibility. *Natural Language Semantics* 8, 255–290 (2000)

# A Formal Semantics for Iconic Spatial Gestures

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**Abstract.** In this paper I describe a formal semantics for iconic spatial gestures. My claim is that the meaning of iconic gestures can be captured with an appropriate mathematical theory of space and the familiar notion of intersecting modification. I support this claim with the analysis of some examples extracted from an annotated corpus of natural human-human interaction.

**Keywords:** gesture semantics, non-verbal communication, formal semantics, applied spatial logic.

## 1 Introduction

The study of gestural behaviour in human communication has recently seen a rapid development, partially increased by the possibility of incorporating this knowledge in the design of embodied artificial agents for human-machine interfaces. However, to this date, the number of attempts to specify a formal framework for the analysis of gesture has been limited, and to our knowledge the only extensive attempt in this direction is the one by Lascarides and Stone [5]. In this paper, I address the same question of Lascarides and Stone, namely what the criteria that determine the semantic “well-formedness” of a gesture are, but we take a different approach. Rather than considering gestures a discourse-bound phenomenon, I assume that they contribute to communication at the meaning level. I will employ a montagovian perspective and show how we can account for their contribution to meaning formation in a way not dissimilar to verbal language. My proposal is complementary to the one of Lascarides and Stone, providing a more precise description of the mechanism of gesture meaning determination, which is left mainly unspecified in their account.

To keep things manageable, I restrict my attention to those gestures categorized in the literature as *iconic*. These gestures do not have a conventionalized meaning, but their interpretation is possible in conjunction with the interpretation of the accompanying verbal sentence. They *iconically* represent spatial or physical properties of the entities or events under discussion, in the sense that

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their formal appearance is determined by the spatial properties of the individuals/events under discussion. Another property that distinguishes these gestures from other typologies is the fact that they are completely independent of the lexical items they accompany. Their distribution is not tied to specific lexical items and similarly the lexical items they accompany are not dependent on the gestures, ruling out any deictic dimension of the gestures.

The semantics I propose is based on the notion of *iconic equivalence* and of *intersecting modification*. The former concept corresponds roughly to the relation holding between two spaces that are indistinguishable. My claim is that these two concepts are sufficient to explain a wide range of cases of gesture and speech interaction.

The paper is structured as follows: in Sect. 2 I will introduce first informally and then more precisely what I propose to be the meaning of iconic gestures; in Sect. 3 I will then outline a theory of space that capture most of the spatial information expressed in gestures and conclude in Sect. 4 by illustrating the semantics on the base of two examples extracted from an annotated corpus of spontaneous gestures.

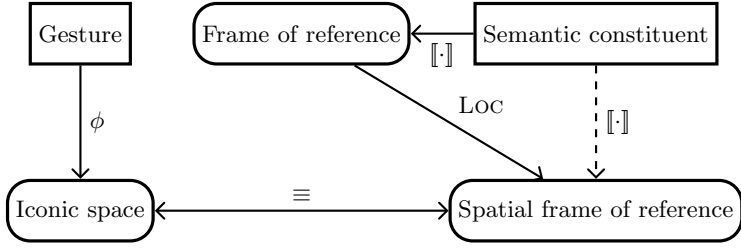
## 2 Semantics

### 2.1 Informal Introduction

The meaning of purely iconic gestures can be analyzed in terms of two simple concepts: *iconic equivalence* and *intersectivity*. Iconic equivalence is the relation holding between two spaces that are indistinguishable when *observed* at a specific *resolution*. With resolution I mean a mathematical language that describe certain properties of a space and an associated notion of equivalence between spaces. The notion of equivalence determines the descriptive limits of the language, or equivalently the ability of the language of identifying differences in two spaces. An observation becomes then a description of a space in the mathematical language in question. For instance we can observe a space using Euclidean geometry and consider it iconically equivalent to another space if the two spaces are congruent up to rigid transformations. If we observe the same space using the language of topology we would consider it iconically equivalent to another space if there is an homeomorphism between the spaces.

The second component at the heart of the analysis of iconic gestures meaning is intersectivity. My claim is that iconic gestures can be analyzed as modifiers of the interpretation of the fragment of verbal language they accompany that contribute additional constraints to the interpretation. The constraints are expressed in terms of iconic equivalence between the space shaped by the gesture and the space occupied by the referents introduced by verbal language. The assumption is of course that a gesture combines only with semantically well-typed expressions, to which I will refer as *semantic constituents*.

The process of interpretation of a fragment of natural language accompanied by a gesture can then be visualized as in Fig. 1. The gesture (considered as a physical act) is interpreted as describing a spatial configuration, called an *iconic*



**Fig. 1.** Combined interpretation of speech and gesture

*space*. This space is generated from the kinetic representation of the gesture by a procedure  $\phi$ . The exact nature of this procedure is beyond the scope of this paper as it depends mainly on contextual and pragmatic factors. The semantic constituent (a string of words) is interpreted through a standard arbitrary interpretation function that associates with each word an element of a montagovian *frame of reference*. Additionally the words of the verbal language are given an interpretation also in a *spatial frame of reference*. This frame is an abstract representation of the physical space in which the individuals of the discourse universe exist. The two frames are connected by a family of mappings LOC that assign to the objects of the montagovian frame the space they occupy.

## 2.2 Formal Semantics

As already stated, we interpret natural language expressions and gestures with respect to two types of ontologies, or frames of reference. The first frame of reference is a classical montagovian individual-based ontology  $\mathcal{F}$ . This frame is defined inductively as follows:

1.  $D_e \in \mathcal{F}$ , where  $D_e$  is a primitive set of individuals,
2.  $D_t \in \mathcal{F}$ , where  $D_t = \{1, 0\}$ ,
3. if  $\Gamma \in \mathcal{F}$  and  $\Delta \in \mathcal{F}$  then  $\Gamma^\Delta \in \mathcal{F}$ , where  $\Gamma^\Delta$  is the set of all functions from  $\Gamma$  to  $\Delta$ .

As it is the case in many semantic analyses of natural language I will assume that the domain  $D_e$  presents an internal structure that identifies sub-kinds of individuals, in particular I assume a distinction between singular and plural individuals.

The second frame of reference is a spatial ontology called  $\mathcal{S}$ . The frame  $\mathcal{S}$  is defined inductively as follows:

1.  $D_r \in \mathcal{S}$ , where  $D_r$  is a primitive set of regions of a space<sup>1</sup> equipped with some additional structure that characterizes this collection as a space (e.g.

<sup>1</sup> Equivalently we could use a point-based geometry. I choose here to use a region-based geometry because the logical language I propose to describe iconic spaces uses regions as primitive objects.

a relation of inclusion among regions together with the property of being an open region to consider the set a mereotopology)

2.  $D_t \in \mathcal{S}$ ,
3. if  $\Gamma \in \mathcal{F}$  and  $\Delta \in \mathcal{F}$  then  $\Gamma^\Delta \in \mathcal{F}$ .

It is important to point out that in the definition of  $D_r$  the notion of space is used in a flexible way. In most cases  $D_r$  can be considered a physical space in the classical sense but, as we will see later, sometimes we need to extend this notion to include the additional dimension of time, when for example we are interpreting gestures involving actions or events.

In what follows, we will assume the usual convention of saying that elements of  $D_e$ ,  $D_t$ , and  $D_r$  have respectively type  $e$ ,  $t$  and  $r$  and that elements of any domain  $\Gamma^\Delta$  have type  $\delta\gamma$ .

The two frames are connected by a family LOC of (possibly partial) injective mappings from elements of  $\mathcal{F}$  to  $\mathcal{S}$ . The elements of LOC are indexed by their domain, so for instance we will write for the member of LOC that has  $D_e$  as its domain  $loc_e$ . This implies that for each element of  $\mathcal{F}$  we will allow only one mapping. We restrict the possible members of LOC with the following conditions:

1. for all  $x \in D_e$ ,  $loc_e(x) = r$ , where  $r$  is an arbitrary element of  $D_r$ <sup>2</sup>,
2. for all  $x \in D_t$ ,  $loc_t(x) = x$ ,
3. for all  $f \in \Gamma^\Delta$ ,  $loc_{\delta\gamma}(f) = f'$ , such that  $\forall x \in \Delta. f'(loc_\delta(x)) = loc_\gamma(f(x))$ .

In this way the structure of the frame  $\mathcal{F}$  is reflected in  $\mathcal{S}$  through LOC, which is a homomorphism from  $\mathcal{F}$  to  $\mathcal{S}$ . Also the types of  $\mathcal{F}$  are reflected in the types of  $\mathcal{S}$ . These conditions have also the pleasant property of allowing us to define the family LOC by simply defining  $loc_e$ .

The meaning of an iconic gesture can then be expressed as a function that intersects an element of a domain in  $\mathcal{F}$  with an element of the corresponding domain in  $\mathcal{S}$  under the LOC mappings. We split the denotation of the gestures in two objects: a first object that inhabits a domain in  $\mathcal{S}$  and that expresses the condition of iconic equivalence between the iconic space and the reference space, and a second object expressed in term of a combinator that intersects the gesture with the accompanying semantic constituent bridging in this way the interpretation of the two modes of communication.

The denotation of an iconic gestures is expressed as the characteristic function of a set of  $n$ -tuples, with  $n \geq 1$ , of regions such that the restriction of the space at the base of  $\mathcal{S}$  to an element of this is set is iconically equivalent to the iconic space described by the gesture. Let  $\rho(S, X)$  be the function that restrict the space  $S$  to its sub-region  $X$ , let  $\equiv$  be the iconic equivalence relation and let  $\gamma$  be the iconic space associated with a gesture, we say that the denotation of a gesture  $g$  is the following function of type  $r^n t$  (where with  $\tau^n \sigma$  we mean a function with  $n \geq 1$  abstractions of type  $\tau$ ):

$$[g] = \lambda r_1 \dots \lambda r_n. \rho \left( D_r, \bigcup_{i=1}^n r_i \right) \equiv \gamma . \quad (1)$$

<sup>2</sup> If we choose to work with a point-based geometry then  $loc_e$  maps individuals to *sets* of points.

The combinator on the other hand acts as a glue between the interpretation of the semantic constituent and the interpretation of the gesture. We define two combinators, the first one  $C_P$  intersecting gestures of type  $r^nt$  with constituents of type  $e^nt$  (predicates) and the second one  $C_M$  intersecting gestures of type  $r^nt$  with constituents of type  $(e^nt)e^nt$  (predicate modifiers). The combinators also ensure that the entities depicted in the gesture co-refer with the entities introduced by natural language

$$C_P = \lambda G. \lambda P. \lambda x_1 \dots \lambda x_n. P \ x_1 \dots x_n \wedge G \ loc_e(x_1) \dots loc_e(x_n) \ . \quad (2)$$

$$C_M = \lambda G. \lambda M. \lambda P. \lambda x_1 \dots \lambda x_n. M \ P \ x_1 \dots x_n \wedge G \ loc_e(x_1) \dots loc_e(x_n) \ . \quad (3)$$

The application of  $C_P$  or  $C_M$  to a gesture results in an intersecting modifier in the sense of [3]. We can in fact prove the following two propositions:

**Proposition 1.** *Let  $G$  be the denotation of a gesture of type  $r^nt$ , then for every function  $P$  of type  $e^nt$  we have that  $C_P \ G \ P = P \sqcap_{e^nt} C_P \ G \ 1_{e^nt}$ , where  $\sqcap_{e^nt}$  is the meet operation for objects of type  $e^nt$  and  $1_{e^nt}$  is the unit of  $\sqcap_{e^nt}$ .*

**Proposition 2.** *Let  $G$  be the denotation of a gesture of type  $r^nt$ , then for every function  $M$  of type  $(e^nt)e^nt$  we have that  $C_M \ G \ M = M \sqcap_{(e^nt)e^nt} C_M \ G \ 1_{(e^nt)et}$ .*

The fact that we require our combinators to correspond to the intersection (under the LOC mappings) of the meaning of the gesture and of the semantic constituent rules out the possibility of having combinators that combine iconic gestures with higher order constituent like generalized quantifiers. This restriction seems to be supported empirically by the fact that we were not capable of finding iconic gestures accompanying higher order quantifiers in a survey of a section of the Speech and Gesture Alignment (SAGA) corpus [2] developed by the University of Bielefeld [3].

### 3 A Logic for Iconic Spaces

In this short paper I will only sketch the spatial language that captures the spatial properties usually expressed with gestures. The language has been designed on the base of the analysis of the SAGA corpus. However it is probably impossible to give a general account of the spatial properties that we observe expressed in gestures, and for this reason the language has been designed to be flexible and allow the construction of different *spatial theories* for different applications. The language is inspired by various logical languages proposed in the literature, in

<sup>3</sup> A possible counterexample could be for example the arc-like gesture that commonly accompany a generalized quantifier like **everyone** or **everything**. However this gesture does not seem to qualify as an iconic one, given that its distribution is quite constrained to the lexical item it accompanies and moreover it is unclear which type of spatial information it is expressing.

particular the seminal analysis of Euclidean geometry by Tarski [8] and the logical interpretation of Mathematical Morphology, an image processing technique, proposed by Aiello and Ottens [11].

The language is a first order language whose intended domain is the set of sub-regions of an euclidean vector space and a set of scalars. The non-logical primitives of the language are the inclusion relation ( $\subseteq$ ) among regions, a distinguished region  $\mathbf{n}$  corresponding to the points close to the origin (including the origin and) two binary operations  $\oplus$  and  $\odot$ . The first operation  $\oplus$  is defined with respect to two regions and corresponds to a generalized vector sum, known as Minkowski sum. It is defined as follows:

$$A \oplus B = \{a + b \mid a \in A, b \in B\} . \quad (4)$$

The second operation is defined between a scalar and a vector and is defined as follows:

$$s \odot A = \{sa \mid a \in A\} . \quad (5)$$

The resulting language is capable of expressing a wide range of spatial properties. It can express mereotopological properties (inclusion, partial overlap, tangential contact, etc.). The language can express the relative position of two regions (in a categorical way) by simply adding to it a number of properly defined distinguished primitive regions. It can also express relative size and with the introduction of appropriate primitives more refined comparative relations like “taller than” or “larger than”. Another type of spatial feature that the language can express and that we can observe often expressed in gestures is the orientation of the main axis of a region. More in general the language is capable of expressing many size and position independent spatial properties through the use of classes of prototypes expressed as primitive regions that are scaled and translated and then used to probe the space.

To express the notion of *iconic equivalence* I will adopt a weaker version of the standard relation of *elementary equivalence* between models. I will consider two models iconically equivalent if they satisfy the same *iconic theory*. An iconic theory is simply a conjunction of atomic formulae and negations of atomic formulae. In what follows I will assume that the iconic theory has been built by the following procedure. Given a space with  $n$  distinguished regions (for instance the regions described by a gesture), we assign to each region a constant  $\mathbf{r}_i$  with  $1 \leq i \leq n$  and we call the set of all region constants  $R$ . Let  $D_r$  be the set of regions in the space, and  $\nu$  the interpretation function that maps every  $\mathbf{r}_i$  to the corresponding region of space, then for every  $k$ -ary predicate  $P$  we take the Cartesian product  $R^k$  and build the following conjunction:

$$\bigwedge_{t \in R^k} \begin{cases} P(t) & \text{if } S, \nu \models P(t) \\ \neg P(t) & \text{otherwise.} \end{cases} \quad (6)$$

The iconic theory is obtained by conjoining the resulting formulae.

Consequently the denotation of a gesture can be reformulated to incorporate this specific instance of iconic equivalence:

$$\llbracket g \rrbracket = \lambda r_1 \dots r_n. \rho(D_r, \bigcup_{i=1}^n r_i), \nu [\mathbf{r}_i \mapsto r_i] \models \Theta(\gamma) \ , \quad (7)$$

where  $\Theta$  is the procedure described above for some fixed set of predicates.

## 4 Examples

I now analyze two examples extracted from the SAGA corpus. Beside illustrating the proposed semantics, the examples are meant to show the deep interaction between natural language semantics and gesture semantics. For this reason I selected two slightly involved cases that challenge our proposal in different ways. I will only outline the analysis of these examples: in particular I will only give an informal description of the iconic spaces associated with the gestures, as a complete formal characterization of these space would require the introduction of the complete spatial logic just sketched in Sect. 3.

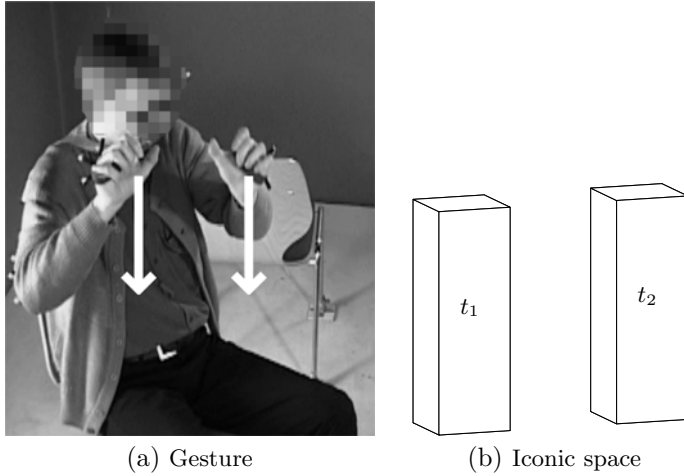
### 4.1 Interaction between Gestures and Plurals

The first example involves the interaction between plurality in natural language semantics and gestures. The example is taken from a dialogue between *Router* and *Follower*, the first describing the visible landmarks as seen during a bus ride. In the fragment we are interested in *Router* is describing a church with two towers. The speaker utters the sentence *die [...] hat zwei Türme*<sup>4</sup> (“that [...] has two towers”) with an accompanying iconic gesture roughly synchronized with the noun-phrase *zwei Türme*. The gesture is depicted in Fig. 2 together with the associated iconic space.

As a first step we need to define the semantics the constituent *zwei Türme*. To give a proper treatment of the plural *Türme* I assume the fairly standard extension of the montagovian frame  $\mathcal{F}$  discussed in Sec. 2 consisting in the introduction of *sum individuals* (see [6]). The sum individuals are members of the type  $e_+$  and we can know their cardinality with the function  $|\cdot|$  and extract from them the individuals that compose them with a number of projection functions. I also assume a standard interpretation of a numeral like *zwei* as a function of type  $(e_+t)e_+t$  that restrict a set of sum individuals to the subset composed by the elements with the correct cardinality (see [4]). The denotation of *zwei Türme* corresponds then to the set of sum individuals that have cardinality equal to 2 and that are the sum of individuals that are towers.

The proposed semantics seem inadequate to analyze this example because the number entities introduced in the verbal language does not match the number of regions depicted by the gesture (1 vs 2). However the gesture is combined in this

<sup>4</sup> The speaker is also introducing other architectonic features of the church before introducing the two towers.



**Fig. 2.** Gesture accompanying the utterance *die [...] hat zwei Türme* and its associated iconic space

case with a constituent referring to a plural individual and thus we can simply refine our semantics to take into account the refined individuals ontology. We extend the definition of LOC in such a way that the spatial projection of a sum individual is the tuple of the spatial projections of its composing atoms. So we say that for all  $x \in D_{e+}$ ,  $loc_{e+}(x) = \langle r_1, \dots, r_n \rangle$ , where  $n = |x|$ ,  $x$  is the result of summing  $x_1, \dots, x_n$  and for  $1 \leq i \leq n$  we have that  $loc_e(x_i) = r_i$ . We also need to introduce a combinator of type  $(r^nt)(e_+t)e_+t$  to intersect the interpretation of a gesture with a plural predicate:

$$C_{P+} = \lambda G. \lambda P. \lambda x. P \ x \wedge G \ \pi_1(loc_{e+}(x)) \dots \pi_n(loc_{e+}(x)) \ . \quad (8)$$

The resulting interpretation for the noun-phrase accompanied by the gesture is the following:

$$\lambda x. |x| = 2 \wedge \mathbf{towers} \ x \wedge \rho(D_r, r_1 \cup r_2), \nu [\mathbf{r_1} \mapsto r_1, \mathbf{r_2} \mapsto r_2] \models \Theta(\gamma) \ , \quad (9)$$

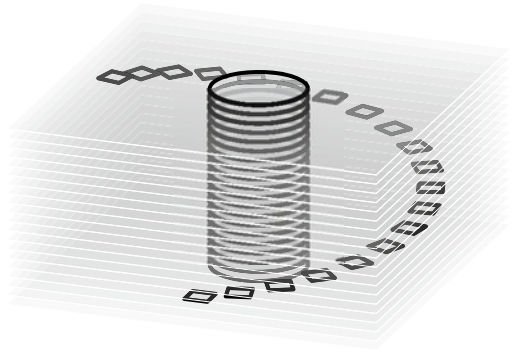
where the theory  $\Theta(\gamma)$  could describe for instance a space with two disconnected, vertical regions, possibly with a certain shape (e.g. a prism-like shape rather than a cylindrical one).

## 4.2 Gestures in the Space-Time

Quite often gestures accompany description of actions, for example by exemplifying the trajectory of a movement. The following example is aimed at showing how we can treat time in iconic gestures. My claim is that for the purposes of determining the meaning of a gesture depicting an action or an event we can consider time as an additional dimension in our spatial ontology. A realistic



(a) Gesture



(b) Iconic space

**Fig. 3.** Gesture accompanying *du fährst um den Teich herum* and corresponding iconic space

spatio-temporal ontology would also require additional restrictions that rule out impossible situations like objects that move with infinite velocity or that cease to exist for a certain period of time, but for the goal of demonstrating how the semantics can cope with time related issues the simple addition of time as an unrestricted dimension will suffice.

The example is taken from the same portion of the SAGA corpus. In this case *Router* explains how the bus ride goes around a pond. *Router* utters the sentence *du fährst um den Teich herum* (“you drive around the pond”) accompanied by the gesture presented in Fig. 3. We represent the iconic space as a three dimensional space in which the vertical dimension represents time. The time dimension is “sliced” into instants to show that each instant is in itself a two dimensional space. The cylindrical region in the middle represents the constant position of the pond while the arch formed of squares represents the different positions occupied by the bus at different instants.

The analysis of this example is in all ways similar to the analysis of the previous one. In this case I assume that the gesture combines with the predicate *fährst ... herum* extended by the locative preposition *um*.<sup>5</sup> The meaning of the gestures is represented as the characteristic function of a set of pairs of regions such that one represents a static circular bi-dimensional object and the other an object moving in time with an arc-like trajectory. The two regions moreover are located in the space in such a way that the circular one is roughly at the center of the trajectory followed by the other region. The set of regions satisfying these constraints is then intersected with the set of pairs of individuals corresponding

<sup>5</sup> Nam in [7] shows how locative prepositions can be equivalently analyzed as operators that generate an intersecting predicate modifier when combined with a noun-phrase or as predicate extensors, i.e. functions that take a predicate of arity  $n$  and return as a result a predicate of arity  $n + 1$ .

to the denotation of the preposition *um* applied to the predicate *fährst ... herum*, i.e. the set of pairs of individuals such that the first one drives around the second one. In this way the referents introduced by the pronoun *du* and by the definite description *den Teich* are shared by the verb and the gesture resulting in the intuitive meaning that we would associate with this speech and gesture exchange.

## 5 Conclusion

I presented a formal semantics for iconic gestures capable of capturing what is conceivably the meaning of iconic gestures. At the moment of writing I have implemented this semantics in a speech and gesture generation prototype that can produce simple descriptions of static and dynamic space configurations that are then rendered using an animated conversational agent. I have also started testing experimentally the assumption that gesture meaning is combined with the propositional meaning of verbal language. At the same time I am also extending the semantics to treat different types of gestures in order to provide a more uniform perspective on the way verbal language is augmented by non-verbal means.

## References

1. Aiello, M., Ottens, B.: The Mathematical Morpho-Logical View on Reasoning about Space. In: International Joint Conference on Artificial Intelligence, pp. 205–211. Morgan Kaufmann Publishers Inc., Hyderabad (2007)
2. Bergmann, K., Hahn, F., Kopp, S., Lücking, A., Mehler, A., Rieser, H.: The Bielefeld Speech and Gesture Alignment Corpus (SAGA) and How It Was Put to Use (in preparation)
3. Keenan, E.L., Faltz, L.M.: Boolean Semantics for Natural Language. D. Reidel Publishing Company, Boston (1985)
4. Geurts, B.: Take Five. In: Vogeeler, S., Tasmowski, L. (eds.) *Non-Definiteness and Plurality*, pp. 311–329. John Benjamins, Amsterdam (2006)
5. Lascarides, A., Stone, M.: A Formal Semantic Analysis of Gesture. *Journal of Semantics* 26, 393–449 (2009)
6. Link, G.: The Logical Analysis of Plural and Mass Nouns: A Lattice Theoretic Approach. In: Bäuerle, R., Schwarze, C., von Stechow, A. (eds.) *Meaning, Use and Interpretation of Language*, pp. 302–323. De Gruyter, Berlin (1983)
7. Nam, S.: The Semantics of Locative PPs in English. PhD Dissertation, UCLA (1995)
8. Tarski, A.: What is Elementary Geometry? In: Henkin, L., Suppes, P., Tarski, A. (eds.) *The Axiomatic Method, with Special Reference to Geometry and Physics*, pp. 16–29, North-Holland, Amsterdam (1959)

# On the Scopal Interaction of Negation and Deontic Modals

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**Abstract.** In this paper we argue that the different scopal relations that deontic modal auxiliaries cross-linguistically exhibit can be explained by assuming that (i) polarity effects arise in the domain of universal deontic modals and therefore not in the domain of existential deontic modals; and (ii) that all deontic modals must be interpreted VP in situ if their polarity requirements allow for that.

**Keywords:** Negation, Deontic Modality, Negative Polarity Items, Positive Polarity Items, Negative Quantifiers.

## 1 Introduction

### 1.1 The Data

Universal deontic modals come about in different kinds: English deontic *must*, *ought* and *should* scope over negation. On the other hand, *have to*, *need to* and *need* (without *to*) scope under negation. *Need* is a clear Negative Polarity Item (NPI) and may thus not appear in non-negative sentences.

- |     |    |                            |               |
|-----|----|----------------------------|---------------|
| (1) | a. | John mustn't leave         | $\Box > \neg$ |
|     | b. | John oughtn't to leave     | $\Box > \neg$ |
|     | c. | John shouldn't leave       | $\Box > \neg$ |
| (2) | a. | John doesn't have to leave | $\neg > \Box$ |
|     | b. | John doesn't need to leave | $\neg > \Box$ |
|     | c. | John need*(n't) leave      | $\neg > \Box$ |

Unlike universal deontic modals, existential deontic modals may only appear under the scope of negation, as is shown below for *may* and *can*:

- |     |    |                    |                   |
|-----|----|--------------------|-------------------|
| (3) | a. | John cannot leave  | $\neg > \Diamond$ |
|     | b. | John may not leave | $\neg > \Diamond$ |

This pattern is not unique for English. In fact, to the best of our knowledge, this pattern (universal deontic modals can either scope over or under negation; existential ones can only scope under negation), applies to all languages that exhibit universal and existential modals. Spanish *deber* and *tener* for instance, behave on a par with English *must* and *have to*, in the sense that *deber* outscopes negation, whereas *tener* does not. Given that the Spanish negative marker *no* is always attached to the left of the finite verb, this shows even more that the observed pattern must reduce to properties of the modal verbs rather than their structural position with respect to negation at surface structure.

- |     |    |                         |               |
|-----|----|-------------------------|---------------|
| (4) | a. | Juan no debe salir      | $\Box > \neg$ |
|     | b. | Juan no tiene que salir | $\neg > \Box$ |

In German, things are slightly different: *sollen* ('should') behaves like English *should* and outscopes negation; *brauchen* ('need to') is an NPI comparable to English *need*; and *müssen* ('must'), like English *have to*, scopes under negation. There is no modal verb with the meaning of English *must/have* that outscopes negation. Existential deontic modals (e.g. *dürfen* ('may')), finally, always scope under negation

- |     |    |                                  |                   |
|-----|----|----------------------------------|-------------------|
| (5) | a. | Hans soll nicht abfahren         | $\Box > \neg$     |
|     | b. | Hans braucht *(nicht) abzufahren | $\neg > \Box$     |
|     | c. | Hans muss nicht abfahren         | $\neg > \Box$     |
|     | d. | Hans darf nicht abfahren         | $\neg > \Diamond$ |

Although the cross-linguistic overview is far from complete, the picture that emerges is that languages are uniform in their scope-internal relation between existential deontic modals and negation, but that languages allow different scopal relations between negation and universal deontic modals depending on which modal element (verb/adjective) is taken.

## 1.2 Questions

The pattern above obviously calls for an explanation and therefore the two following questions need to be addressed: (i) what determines the scopal properties of universal deontic modals with respect to negation; and (ii) why do existential deontic modals always appear under the scope of negation?

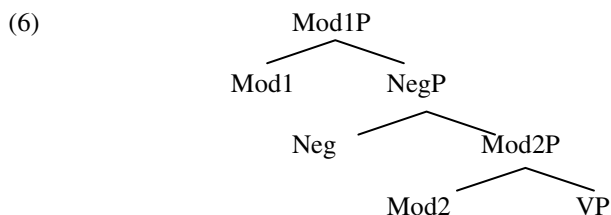
In the rest of this paper we will address these questions and argue that the scopal behaviour of deontic modals follows from independently motivated assumptions concerning (i) the status of polarity items and (ii) the possible positions of interpretation of lexical elements in the tree.

## 2 Previous Proposals

The scopal relations between modals and negation has been observed and studied by a number of scholars, most notably [1], [2], [3], [4], [5] and [6]. In this section we will discuss and evaluate two proposals, which are quite similar in nature.

## 2.1 Cormack and Smith (2002)

According to Cormack and Smith [4], there are two positions for modals, Modal1 and Modal2, and (sentential) negation scopes in between them.



Cormack and Smith adopt the following assumptions: (i) the scopal order between modal types is derived by semantic / conceptual necessity (though their formulation of this is not quite clear), i.e. the fact that epistemic modals scope over deontic does not follow from any syntactic principle; (ii) it is a property of syntax that there are two possible positions for modals, one above and one below negation (the position that the negative marker occupies); and (iii) which specific modals go in Modal1 and which in Modal2 is lexically specified and therefore idiosyncratic in nature.

- (7) John doesn't have to leave  
 [John [<sub>NegP</sub> doesn't [<sub>Mod2P</sub> have to leave]]]  $\neg > \square$
- (8) John mustn't leave  
 [John [<sub>Mod1P</sub> must [<sub>NegP</sub> n't [<sub>VP</sub> leave]]]  $\square > \neg$
- (9) ... dass Hans nicht abfahren muss  
 [<sub>CP</sub> dass Hans [<sub>NegP</sub> nicht [<sub>Mod2P</sub> [<sub>VP</sub> abfahren] muss]]]  $\neg > \square$

However, this analysis faces several problems. Although the assumption that the epistemic > deontic ordering is semantically / conceptually necessary, the necessity of the split between Modal1 and Modal 2 is less plausible. First in many languages there is no syntactic evidence for two different positions. This is illustrated for Spanish below. (Note that this may not be derived from movement of the negative marker *no*, as generally the surface position of the negative marker *no* always corresponds to its LF position.)

- (10) a. Juan no debe salir  $\square > \neg$   
 b. Juan no tiene que salir  $\neg > \square$

Secondly, it remains unclear why only deontic universal modals allow for a lexical split. Why couldn't deontic existentials be analysed as Modal1? Cormack and Smith argue that children start out with a learning algorithm that takes all (deontic) universals to be Modal1 and all existentials to be Modal2 and that children may reanalyse some Modal1's as Modal2's if the language input forces them to so (e.g. *need* is reanalysed from Modal 1 to Modal2). But why couldn't a Modal2 be reanalysed as a Modal1?

## 2.2 Butler (2003)

Butler's analysis [5] is similar in spirit to [4]. He also derives the scopal properties from a universal syntactic template. For that he distinguishes between different functional projections for epistemic and root modals as well as different functional projections for existential and universal modals. Butler's analysis follows Cinque's/Rizzi's cartographic approach in the sense that all scopal properties reflect a universal basic structure. For negation and modality that is:

- (11) EpistNecP > (NegP) > EpistPosP > (strong) subject > RootNecP > NegP > RootPosP > vP

Under Butler's proposal it follows immediately that all epistemic deontic modals take scope under negation, whereas a deontic universal like *must* outscopes negation. However, it becomes unclear now why some deontic universals may not outscope negation, such as English *have to* or German *müssen*. Although Butler only briefly addresses this question, the only way to deal with such examples is to posit that the negative marker in those cases is in the higher NegP. However, such a solution introduces new problems as well. First, it becomes unclear again why other modals, such as *must*, may not be outscoped by such a high negation and secondly, it predicts that in all cases where negation outscopes *have to* (or any other deontic modal that scopes under negation), it also outscopes the subject. However, this prediction is too strong as it incorrectly rules out cases such as (12):

- (12) Many people don't have to work  
'There are many people who are not required to work'

Finally it should be noted that this solution reduces the syntactic approach that Butler proposes into a lexically idiosyncratic approach as well: it needs somehow to be lexically encoded which position negation occupies when combined with which deontic universal. It is however unclear what kind of a mechanism could be responsible for that.

## 3 Analysis

In order to overcome the problems that approaches that are built on syntactic templates face, we argue instead that the scopal behaviour of deontic modals results from their lexical semantic properties, *in casu* their polarity properties. In accordance with two additional assumptions concerning the locus of interpretation of negative and deontic modal elements, we argue that all discussed facts follow directly.

### 3.1 Neutral and Polar Modals

As discussed before, the domain of (universal) deontic modals is one where NPI specifications hold.

- (13) a. Sue need \*(not) leave.  $\neg > \square$   
b. Je hoeft dat \*(niet) te doen Dutch  $\neg > \square$

- c. Du brauchst dass \*(nicht) zu tun                      German  $\neg > \square$   
 You need.NPI that (NEG) to do  
 ‘You don’t need to do that’

Since NPIs surface in the domain of deontic modality, we should also expect there to be Positive Polarity Items (PPIs), as any domain that has one of these classes also exhibits the other class (quantifiers over individuals, adverbs, etc.).

PPIs have the ‘boring property that they cannot scope under negation’ [7: 409], which means that they can occur either in positive sentences or in negative sentences when outscoping the negation: this is indeed what is attested with deontic modals *must*, *should* and *ought*. Adopting that these modals are PPIs, the scopal properties of English are already captured as these elements necessarily scope over negation.

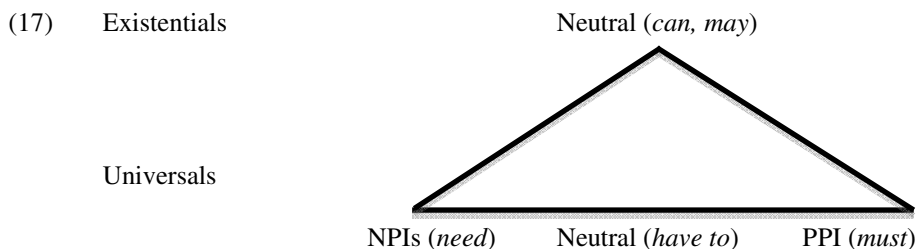
- (14) a. John must/should/ought (to) leave  
 b. John mustn’t/shouldn’t/oughtn’t (to) leave                       $\square > \neg$

Homer (2009), who has independently and for different reasons reached the conclusion that *must* is a PPI provides a series of additional arguments, adopting diagnostics provided by [7], which show that *must* indeed is a PPI [8: 30-32], since it obeys the Intermediate Scope Constraint.

- (15) a. Fun mustn’t be expensive                       $* \neg > \square$   
 b. Fun mustn’t always be expensive                       $\neg > \forall > \square$   
 (16) a. He mustn’t rake the leaves (too)                       $* \neg > \square$   
 b. I’m not sure he mustn’t rake the leaves too                       $\neg > \neg > \square$

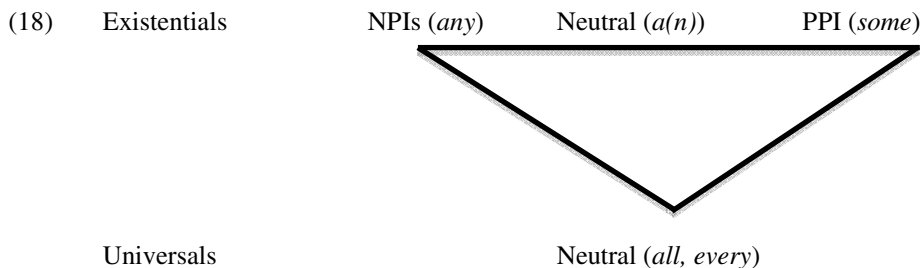
Finally, it should be noted that not all deontic modals are polarity items. English *have to* or German *müssen* can occur in positive sentences (hence they are not NPIs) and they appear under the scope of negation in negative sentences (hence they are not PPIs). This class of modals are referred to as ‘neutral deontic modals’.

At the same time no NPIs surface in the domain of deontic existential modals. On the basis of the same type of reasoning we applied above, no PPI deontic existential modal is expected to surface either, a prediction that to the best of our knowledge is borne out. The landscape of deontic modals thus looks as follows:



The obvious question is why deontic modals exhibit this ‘triangle of polarity’. Without further speculating, the fact that a triangle emerges in (17) is not surprising given the shape of landscape of polarity items that are attested in the domain of

quantificational DPs. There we find series of indefinites/existentials that are PPIs or NPIs, but no universal quantifier NPIs/PPIs have ever been attested (pace [9] who argues that Greek *n*-words are NPIs, but see [10], [11] for a discussion of a series of problems this analysis faces):



So, although it is unclear what constitutes the better-known pattern in (18), it is likely to assume that what is the cause of the shape in (18), also underlies (17). We therefore tentatively conclude that the fact that universal deontics come about in NPIs, PPIs and neutral elements, bans existential deontics from doing so as well.

### 3.2 Deontic Modals and Negation

However, this specification of deontic modals in terms of their polarity properties does not suffice to account for the scopal behaviour that deontic modals exhibit. It only explains the fixed scopal properties of NPI/PPI modals with respect to negation, but not the scopal relations between neutral deontic modals and negation. I.e., why does *have to* always scope under negation (and is that really always the case)?

Let us make the following two assumptions: (i) negation never lowers at LF: it is interpreted in its surface position and may only raise to a higher position at LF if it moves along with another, independently, raising element; (ii) deontic modals are base-generated VP-in situ. The first assumption is uncontroversial; the second, however, is not.

Received wisdom has it that in English these (and other) modals are base-generated in  $I^0$  (Dutch and German modals e.g. are generally assumed to be base-generated inside VP). If so, then there is no position for them to reconstruct to under negation. But is received wisdom correct in this case? The argument for generation in  $I^0$  stems from the fact these modals always *appear* in  $I^0$ . Such modals are taken to differ in two ways from regular verbs: they only come in tensed forms *and* they are generated in  $I^0$ . However, only the first of these characterizations is needed, as it by itself derives the second one. We know that these deontic modal auxiliaries are moving verbs since they can make it up to  $C^0$ :

(19)    Can/may/must he leave?

If these modals are movers, and if they are always tensed, then it follows that if they are generated in a VP, they will always move to at least  $I^0$ . In short, this view is as consistent with the facts as the generation-in- $I^0$  view is, and, as we will see, it is superior to the latter in getting the facts with one fewer special assumption about modals.

The only difference between deontic modals being base-generated in  $I^0$  and being base-generated inside VP is that in the latter case, these modals are taken to be lexical verbs and therefore they must be interpreted in their base position as well.

On the basis of these assumptions all facts follow naturally. Let's discuss first the examples in (1)-(3), repeated as (20)-(22) below:

- (20) a. John mustn't leave  $\square > \neg$   
 b. John oughtn't to leave  $\square > \neg$   
 c. John shouldn't leave  $\square > \neg$

*Must*, *ought* and *should* are base-generated VP in situ, and thus in a position lower than negation. However, since they are PPIs, their appearance under negation would make the sentences crash at LF and therefore, as a last resort option, these modals are interpreted in a higher head position to which they have moved in order to check their tense features and where they outscope negation.

- (21) a. John doesn't have to leave  $\neg > \square$   
 b. John doesn't need to leave  $\neg > \square$   
 c. John need\*(n't) leave  $\neg > \square$

In (21) the same story applies, except for the facts that these modals, being neutral or even NPIs, do not render the sentence ungrammatical if they are interpreted in their base position, which is lower than negation. Therefore there is no proper trigger that could force them to raise across negation and the only reading these sentences receive is one where negation outscoops the modal.

- (22) a. John cannot leave  $\neg > \diamond$   
 b. John may not leave  $\neg > \diamond$

Since there are no polar deontic existential modals all deontic existentials are neutral and remain to be interpreted at their base position, just like the cases in (21).

The Spanish facts are also covered, as the PPI modal *deber* will be forced to raise to a higher position at LF, whereas no such trigger exists for 'tener', which will therefore remain in its surface position at LF.

- (23) a. Juan no debe salir  $\square > \neg$   
 b. Juan no tiene que salir  $\neg > \square$

Now, let's consider the German cases (24). Note that German exhibits V2 in main clauses. However, V2 does not change the position where lexical verbs are interpreted in general. In this sense, V2 is to be considered a PF phenomenon. At LF, lexical verbs are still present at their base position. *Sollen* is a PPI and thus raises across negation at LF. *Brauchen* on the other hand is an NPI and will thus remain in situ (there is no trigger for raising; in fact the presence of such a trigger would violate its NPI licensing conditions). *Müssen* is neutral and won't raise at LF either. *Dürfen*, finally, is an existential and therefore neutral as well: hence  $\neg > \square$ .

- (24) a. Hans soll nicht abfahren  $\Box > \neg$   
b. Hans braucht \*(nicht) abzufahren  $\neg > \Box$   
c. Hans muss nicht abfahren  $\neg > \Box$   
d. Hans darf nicht abfahren  $\neg > \Diamond$

3.3 Deontic Modals and Negative DPs

Another puzzle concerning the interaction between (deontic) modals and negation concerns the ambiguity of neutral modals with respect to Negative DPs, as has been observed by [6]:

(25) [6: 11]

Type of Modal wrt sentential negation	Interpretive possibilities of (Negative component of) NegDP
Mod>Neg	Mod > Subject <sub>Neg</sub>
	Mod > Object <sub>Neg</sub>
Neg>Mod	Subject <sub>Neg</sub> > Mod
	Object <sub>Neg</sub> > Mod
	Mod > Object <sub>Neg</sub>

While neutral and NPI modals behave similarly w.r.t. sentential negation, they behave differently with negation inside NegDPs. Iatridou & Sichel show that neutral modals scope under a NegDP in subject position but are ambiguous with respect to a NegDP in object position:

- (26) a. Nobody has to/needs drive.  $\neg > \Box$   
b. He has to/needs to do no homework tonight. (pref.)  $\neg > \Box$   
c. In order to see how other people live, he  
has to/needs to get no new toys for a while.  $\Box > \neg$

However, an NPI modal will scope under negation no matter where that negation is. English NPI *need* is not sufficiently part of colloquial English for reliable judgments, but for German neutral DM *müssen* versus NPI *brauchen*, the facts are very clear: while *müssen* behaves like English *have to/need to* in (26), *brauchen* is fine in (27)a-b; in (27)c the intended reading cannot be yielded with *brauchen*:

- (27) a. Keiner muss/braucht (zu) fahren  $\neg > \Box$   
Noone muss/braucht (to) leave  
(28) b. Er muss/braucht keine hausarbeiten (zu) machen  $\neg > \Box$   
He muss/braucht no homework do  
c. Um zu sehen, wie andere leben, muss/\*braucht er eine  
Zeitlang keine neuen Geschenke (zu) bekommen  $\Box > \neg$   
In order to see how other people live, he muss/\*bracht to get  
no new toys for a while

These facts immediately follow from the presented analysis that takes modals such as English *have to* and German *brauchen/muessen* to be interpreted in their base position. Since objects are in the complement of the modal verb, they allow for an interpretation where the neutral modal outscopes them, but as these Negative DPs are

able to undergo quantifier movement, the negation is able to outscope the modals as well. Subject Negative DPs, on the other hand, already at surface structure outscope the neutral modal, which therefore can never be put in a position where it outscores the negation. Note that since NPI modals must be under the scope of negation, in these cases the narrow scope reading of the object is never available.

## 4 Conclusion and Discussion

In the beginning of this paper we addressed two questions: (i) what determines the scopal properties of universal deontic modals with respect to negation; and (ii) why do existential deontic modals always appear under the scope of negation?

In this paper we argue that once it is adopted that (i) modals that always outscope negation are PPIs, (ii) only deontic universal modals exhibit polarity effects (there are no PPI/NPI deontic existentials), (iii) deontic modals are lexical verbs (sometimes in disguise), and (iv) negation does not lower at LF, all known facts concerning the scopal behaviour of deontic modals with respect to negation naturally follow.

However, at least two questions remain open. First, why is it the case that only deontic modals (and, similarly, existential/indefinite quantificational DPs) exhibit polarity effects? In other words, why are the triangles in (17) and (18) triangles?

Second, under this analysis it is assumed that that Negative DPs may undergo (some kind of) quantifier raising. It is a known fact, however, that Negative DPs do not outscope higher quantifiers (i.e., yield to reverse readings). Take (29) from [12].

- (29) Everybody touched no desert  $\forall > \neg\exists; *\neg\exists > \forall$

However, we assume that what (29) shows is that the relative scopal ordering of two quantifiers remains frozen. It does not show that *no desert* is forbidden to (overtly) raise across the subject, as long as the subject raises again across the object again. So (29) itself is not a valid counterargument against a QR analysis of negative DPs. The more general question as to what blocks the inverse reading in (29) remains an open question though, but it should be noted that mechanisms that rule out reverse readings in cases of multiple QR are needed independently. Take for instance the German scrambling data in (30)–(32), taken from [13: 37], which show that if two DPs have been moved, their scopal order remains identical to their surface orderings:

- (30) ... dass fast jeder Mann mindestens eine Frau kennt  $\forall > \neg\exists; *\neg\exists > \forall$   
 ... that nearly each man at-least one woman knows  
 ‘... that nearly every man knows at least one woman’
- (31) ... dass [mindestens eine Frau], fast jeder  $t_i$  Mann kennt  $\forall > \neg\exists; \neg\exists > \forall$   
 ... that at-least one woman nearly each man knows  
 ‘... that nearly every man knows at least one woman’
- (32) ... dass fast jedem Kind mindestens ein Buch nur Hans vorlas  $\forall > \exists$   
 ... that nearly each child at-least one book only hans read  $*\exists > \forall$   
 ‘... that only hans ready at least one book to nearly every child’
- (33) [ $\beta$  ... [ $\alpha$  ... [ $t_b$  ...]]]  $\alpha > \beta; \beta > \forall$
- (34) [ $\alpha$  ... [ $\beta$  ... [ $t_\alpha$  ... [ $t_b$  ...]]]]  $\alpha > \beta; *\beta > \forall$

## References

1. Picallo, M.: Modal verbs in Catalan. *Natural Language and Linguistic Theory* 8, 285–312 (1990)
2. De Haan, F.: The interaction of negation and modality: A typological study. *Outstanding dissertations in linguistics*. Garland Publishing, New York (1994)
3. Palmer, F.: *Mood and Modality*. Cambridge University Press, Cambridge (2001)
4. Cormack, A., Smith, N.: Modals and negation in English. In: Barbiers, S., Beukema, F., Van der Wurff, W. (eds.) *Modality and its interaction with the verbal system*, pp. 133–163. John Benjamins, Amsterdam (2002)
5. Butler, J.: A Minimalist Treatment of Modality. *Lingua* 113, 967–996 (2003)
6. Iatridou, S., Sichel, I.: Negative DPs and Scope Diminishment: Some basic patterns. In: Schardl, A., Walkow, M., Abdurrahman, M. (eds.) *NELS 38: Proceedings of the 38th Annual Meeting of the North East Linguistics Society*. GLSA, Amherst (2009)
7. Szabolcsi, A.: Positive Polarity – Negative Polarity. *Natural Language and Linguistic Theory* 22, 409–452 (2004)
8. Homer, V.: Epistemic Modals: High, ma non troppo. Paper presented at NELS 40, MIT, Cambridge (2009)
9. Giannakidou, A.: Negative...Concord. *Natural Language and Linguistic Theory* 18, 457–523 (2000)
10. Watanabe, A.: The Genesis of Negative Concord: Syntax and Morphology of Negative Doubling. *Natural Language and Linguistic Theory* 35, 559–612 (2004)
11. Zeijlstra, H.: Negative Concord is syntactic agreement. *LingBuzz* 000645 (2009)
12. Von Stechow, K., Iatridou, S.: Epistemic containment. *Linguistic Inquiry* 34, 173–198 (2003)
13. Fanselow, G.: Features,  $\theta$ -Roles and Free Constituent Order. *Linguistic Inquiry* 32, 405–437 (2001)

# Projective Meaning and Attachment

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**Abstract.** This paper examines the possibility of providing a unified account of the projection properties of presuppositions, conventional and conversational implicatures. I discuss the solution offered in (Roberts et al. 2009) and show that the central notion we need to cover the spectrum of observations is that of *attachment*.

## 1 Introduction

The most basic observations about presuppositions concern what is called their *projection* behaviour. Roughly speaking, a presupposition can be characterised as an implication which is able to project. A sentence *S* presupposes a proposition  $\phi$  whenever *S* implies  $\phi$  and certain ‘suitably modified’ versions of *S* also imply  $\phi$  (projection). The ‘suitably modified’ qualification encompasses negation, interrogation and a variety of embeddings. For instance, *Mary knows that Paul cheated on the exam* and its modified versions (*Mary does not know / Does Mary know that Paul cheated on the exam*) all imply that Paul cheated.

Projection is not automatic. It depends on context and the properties of embedding, as studied in the vast literature on presupposition projection. A less well-known property concerns the limitations on *attachment*. Ducrot (1972) had noted that it is difficult to attach a discourse constituent to a presupposition. For instance, the salient reading of (1) is that Paul does not cheat (asserted content) because he was behind in his work. The probably more natural interpretation that Paul was in the habit of cheating (presupposed content) because he was always behind cannot be construed.

- (1) Paul has stopped cheating on exams because he was always behind in his work

The question naturally arises whether these two properties can be unified in some way and perhaps ultimately viewed as two sides of the same coin. In the next section, I describe in more detail the symmetry between projection and attachment constraints. In section 3, I present the approach of Roberts et al. (2009), based on the notion of *Question Under Discussion* (QUD) and highlight the possibility of deriving attachment constraints from it. In section 3.2, I show that attachment is not limited to some carefully chosen examples but has a clear experimental reflection. Finally, in section 4, I discuss the QUD-based approach and show that attachment is a fundamental notion to analyse the interaction between discourse and projection.

## 2 Extending the Symmetry between Projection and Attachment

There is little doubt that presuppositions tend to project and do not provide a natural attachment site. Roberts et al. (2009) suggest that projection extends to conventional implicatures (CIs). For instance, they borrow from Chierchia and McConnell-Ginet the observation that non-restrictive relative clauses project. An example parallel to (1) is (2). According to Potts (2005), non-restrictive trigger a CI and, indeed, (2b) entails that Paul cheated.

- (2) a. Paul, who has cheated on the exam, might be dismissed
- b. Do you think that Paul, who has cheated on the exam, might be dismissed?

Roberts et al. recall Simons' (2005) observation that certain conversational implicatures (cis) project. Consider (3) (Simons' example 27). Answer B1 makes sense only if one assumes some sort of negative connection between rain and going on a picnic. This connection is preserved in B2 variants.

- (3) A – Are we going on a picnic?
- B1 –It's raining
- B2 –It's not raining / Is it raining?

Attachment limitations have also been investigated with a similar result. Ducrot's (1972) *loi d'enchaînement* ('linking law') targets presuppositions. In a nutshell, the linking law forbids any attachment to a presupposition, whether by way of a subordinating or coordinating conjunction, except for *et* ('and') and *si* ('if'), or by way of a 'logical relation'. In (Jayez 2005, Jayez and Tovenà 2008), it is claimed that CIs are subject to the same limitations. For instance, in (4), the preferred interpretation is that John being unable to register for the next term is the cause of his failure. The more natural interpretation that it is bad luck for him since he cannot register would involve recruiting the CI trigger *unfortunately* for the attachment (see Potts 2005 for evaluative adverbs and Jayez and Rossari 2004 for parentheticals).

- (4) Unfortunately, Paul has failed his exam because he cannot register for the next term

Finally, it has been noted in various works that CIs cannot provide natural targets for refutation, see (Jayez and Rossari 2004, Potts 2005). E.g. the refutations in (5) target only the asserted proposition that Paul has failed his exam, leaving aside the evaluative CI trigger *unexpectedly*.

- (5) A – Paul has unexpectedly failed his exam
- B – You lie / You are wrong / Impossible / Quite the contrary

In a refutation, the attempt by an addressee to attach a new constituent to a presupposition or to an implicature is bound to be perceived as artificial. In this respect, refutations belong to the category of attachments. As noted by David Beaver (p.c.), a similar observation and conclusion holds for *confirmations* ('You are right', 'Quite so', etc.).

It is of course tempting to hypothesize that there is a common source behind the projection and the attachment observations, and that presuppositions, CIs and cis can be grouped into a natural class, whose members differ essentially by specific lexical profiles.

### 3 Accounting for the Symmetry: The QUD Approach

#### 3.1 Basics

Recently, Roberts et al. (2009) have proposed that presuppositions, CIs and cis, which they group under the generic term of *not-at-issue content* (henceforth non-AI content), after Potts' term for CIs, share indeed a central property: they do not necessarily address the Question Under Discussion (QUD). Assuming that each discourse is organised around at least one common topic (the QUD), they offer the following principle.

(6) **QUD principle.** All and only the non-AI content may project.

Two important points are to be mentioned at this stage. First, if we decide to see presuppositions and implicatures as members of a common family, it is no longer possible to attribute their common behaviour to properties that do not hold for the whole class. So, anaphoric or dynamic theories of presuppositions, whatever their merits, are not plausible candidates for unifying presuppositions, CIs and cis since, for instance, they do not make room for CIs (Potts 2005). Roberts et al. make the same point for common ground theories of presuppositions.

Second, if the QUD theory is correct, it should allow one to derive the attachment properties. Roberts et al. include the refutation test among those properties that characterise the projecting elements but they do not tackle the general question of attachments. Generalising from Potts, I assume that the semantic and pragmatic contribution of a discourse constituent can be seen as a  $n$ -tuple  $\langle q, a_1 \dots a_n \rangle$ , where the first element (AI content) addresses the QUD and the other ones are presupposed or implicated material. Appropriate functions can extract the relevant material. If  $C$  is a constituent,  $AI(C)$  extracts the AI content of  $C$ ,  $pres(C)$  the presuppositions, etc. Consider now a pair of adjacent constituents  $(C_1, C_2)$  in a monologue, typically two successive sentences or clauses that convey a proposition. By using  $C_1$ , the speaker signals that she contributes to the QUD with  $AI(C_1)$ . If the next constituent is connected to an element of  $C_1$  different from  $AI(C_1)$ , the speaker abandons the QUD. In most contexts, this is an odd move because the speaker just addressed the QUD *via*  $AI(C_1)$ , hence the impression of a non sequitur. In dialogues, the situation is a little different since we cannot, in general, assign to participants a unique discourse strategy. It may be the case that participants disagree on certain issues. This accounts for the fact, noted by Jayez and Rossari and von Stechow (2004), that it is perfectly possible to interrupt the discourse trajectory ascribed to a participant, for instance by questioning a presupposition or a CI she endorses, in general at the cost of a metalinguistic effect.

- (7) A – Unfortunately, Paul has failed his exam  
B – (Well, I wouldn’t call that ‘unfortunate’ / It’s not really unfortunate, you know,) he’s so lazy. He got what he deserves

In monologues, the price to pay for abandoning the QUD is higher since the speaker is supposed to have a coherent strategy. This is not quite impossible, however. A speaker may signal explicitly that she is abandoning the QUD with a special discourse marker such as *by the way*. In that case, the speaker may sound uncooperative, especially if she abruptly shifts the topic in the middle of a serious discussion, but she is not incoherent since she makes clear that she is not currently following a plan to tackle the QUD (8).

- (8) Paul stopped smoking. By the way, Mary never took to smoking.

3.2 Simple Experimental Evidence

One might argue that the QUD hypothesis, in its current stage, is only a clever guess. However, preliminary experimental evidence is clearly consonant with the hypothesis. If the QUD approach is right, competent speakers should process more easily an attachment to the AI content than to the non-AI content. In order to evaluate this prediction, I carried out a simple categorisation experiment.

46 French students were asked to classify 40 French two-sentence pairs, including 8 fillers, as either *banale* (ordinary) or *bizarre* (weird). They were all native speakers, with an age range of 17-27 and an age mean of 20.1. The test was administered collectively (all the subjects rated the pairs together). Subjects had to read and rate pairs following the order on the test sheet and were not allowed to correct a previous choice. They were asked to run through the pair list as fast as possible. In each pair, the sentences were related by a consequence discourse marker (*donc* or *alors* ≈ ‘therefore’, ‘so’) or by a causal/justification subordinating conjunction (*parce que* ≈ ‘because’ or *puisque* ≈ ‘since’). The pairs exploited either an AI or a non-AI linking and featured a presupposition or conventional implicature trigger. The following table shows the translations of four pairs, with the expected answer in the last column. The general idea was to have a markedly selective preference for attaching either to the AI or to the non-AI content in each case. For instance, in the fourth example, assuming an analysis of *almost* like in Jayez & Tovenà (2008), attachment to the AI content (Laetitia was near to desperation) would not make much sense whereas attachment to the non-AI content (Laetitia was not (entirely) desperate) is plausible.

trigger	connection mode	text	expected answer
<i>know</i>	AI	Serge knows that there is a strike because he listened to the radio	OK
<i>know</i>	non AI	Martine knows that it’s raining, so people have umbrellas	weird
<i>almost</i>	AI	Mary is almost late, so she hurries up	OK
<i>almost</i>	non AI	Laetitia was almost desperate because there were some positive points	weird

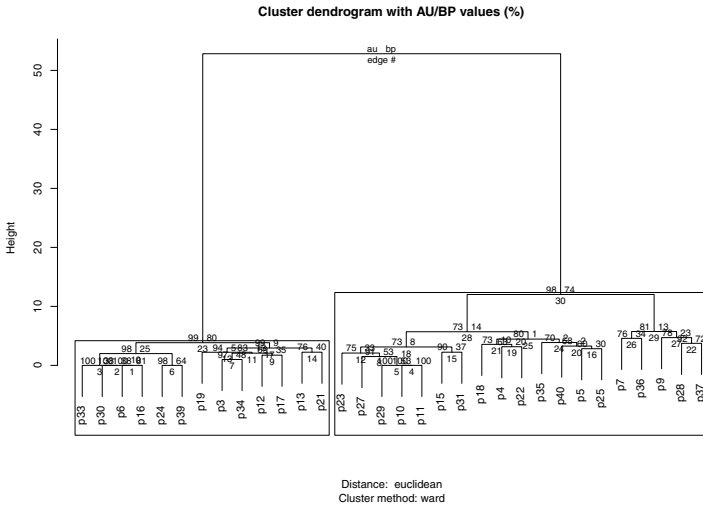
A simple comparison of means suggests that there is a sharp contrast between the AI and non-AI conditions: 88% of OK answers for the first, 18% (presuppositions) and 12% (CIs) for the second. The results can be analysed in several ways, which I illustrate briefly. The most obvious possibility is to examine the differences between individual sentence pairs. I used the Mac Nemar test for paired samples to compare the proportions of binary responses of the subject group in two different conditions, two types of sentence pairs in our case. The sentence pairs (excluding fillers) were classified into different categories, according to their connection mode (AI or non-AI) and the presupposition or CI triggers they contained. They were compared pairwise and the 300 resulting tests were themselves classified into different categories according to (i) which mode of connection (AI, non-AI with presupposition, non-AI with CI) each pair exhibited and (ii) whether the trigger was identical in the two pairs. When the trigger is the same, there are 5 pairs (out of 6) that show a significant difference in the AI vs. presupposition attachment conditions, and 7 pairs (out of 10) that show a significant difference for CIs. A second technique is to cluster the responses in order to see whether and how they pattern together. Figure 2 shows the result of an application of the R *pvclust* package for hierarchical clustering.<sup>1</sup> The rectangles represent the best guesses of the algorithm (with an estimated probability of error inferior to 0.05). The left rectangle contains only AI attachment pairs, the right rectangle contains three AI attachment pairs which are rejected (p9) or not significantly accepted by the subjects (p28 and p36) and all non-AI attachment pairs. This suggests again that the perception of AI and non-AI attachments tends to diverge. Finally, one can fit a series of linear mixed models using the *lme4* R package.<sup>2</sup> Subjects are considered as groups and the reference level of the attachment type factor is rotated to highlight the different hierarchies between levels (AI, non-AI+presupposition, non-AI+CI). The results are shown in figure 1. They are all significant ( $P < 0.05$ ). In a nutshell, table 1 says that the probability of getting an ‘OK’ (vs. ‘weird’) answer from subjects decreases as the proportion of presuppositional or CI attachments increases and increases with attachments to the AI content. Presuppositions partially compensate for CIs, as shown by the figures where CI constitutes the reference level (line 3). In other terms, they have a lesser negative influence than CIs. However, this might reflect a bias in the data rather than a general tendency and more work is needed to evaluate the import of the present difference.

Reference level			
AI	intercept = 2.1265	pres. = -3.6645	CI = -4.1617
pres.	intercept = -1.5379	AI = 3.6644	CI = -0.4973
CI	intercept = -2.0352	AI = 4.1617	pres. = 0.4973

Fig. 1. Results of mixed modelling

<sup>1</sup> See <http://www.is.titech.ac.jp/~shimo/prog/pvclust/>

<sup>2</sup> <http://cran.r-project.org/web/packages/lme4/index.html>



**Fig. 2.** Probabilistic clustering

Although preliminary and imperfect in some respects, this procedure delivers a result clearly consonant with the QUD approach and also with the extended version of Ducrot’s *loi d’enchaînement*.

## 4 The Attachment Approach

In spite of its attractiveness, the QUD approach faces some problems and I will defend the view that the notion of attachment (i) is a better candidate to address them and (ii) provides a basis for explaining why, in most examples, projective meaning does not address the QUD, as made clear by Roberts et al.

## 4.1 The Attachment Principle

In some cases, the non-AI content *does* address the QUD. In (9), B uses the double fact that Paul has been smoking and that he does not smoke as an argument in favour of her conclusion that Paul has a strong will.

- (9) A – Does Paul have a strong will?  
B – Generally speaking, yes. He has stopped smoking, for instance

A variation on this type of configuration is illustrated by (10). The preferred interpretation of B's answer is that Paul answers to mails, but not very quickly. Thus, the proposition that Paul answers to mails survives the negation and projects. However, it is difficult to say that it does not address the QUD, at least not if we consider what is relevant to the topic made salient by A's question.

- (10) A – Is Paul a good partner?  
B – He does not answer to mails very quickly

Note that with (10) or with (9) we do not base our understanding *only* on a general or circumstantial rule like [addiction  $\leadsto$  no strong will]. It is necessary to make the *fact* that Paul answers to mails or that he has smoked enter the picture, in order to draw from B's answers various inferences relevant to the QUD. So, the situation cannot be reduced to the type of examples studied by Simons that we mentioned in section 2.

(10) and (9) illustrate the possibility that information pieces which address the QUD nonetheless project. Crucially, in both cases, one observes attachment limitations. E.g. it is impossible to interpret (11a) and (11b) as meaning that Paul answers to mails because he is professional and that he smoked because he liked smoking.

- (11) a. He does not answer to mails very quickly because he is very professional  
 b. He didn't stop smoking, for instance, because he liked that

Such observations have two consequences. First, they show that material usually considered as implicated or presupposed can address the QUD *and* be projected. Second, if attachment limitations were a reflection of not addressing the QUD, as I have proposed, they should disappear. In view of these problems, I introduce a new, different principle for dealing with projective meaning. The intuition behind it is that the only strongly preferred attachment in linguistic communication is to AI content.

- (12) **Attachment Principle.** In linguistic communication, the AI content provides the preferred site for attachment.

In other terms, one cannot 'ignore' the AI content. It is possible to attach to presupposed or implicated material only if there is a simultaneous attachment to the AI content, as in (9). Three questions remain open. What is AI content? Can we motivate the attachment principle? How come that QUD addressing generally patterns with attachment?

## 4.2 One How and Two Whys

How do we discern attachable content? Every linguistic piece of communication can come with conventionally attachable and non-attachable content. The linguistic marking of AI content vs. presuppositions or CIs provides a typical case. I leave open the possibility that a linguistic item contains no conventionally attachable content, as might be the case for interjections (Wharton 2003).

Conventionally attachable content contains those elements which contribute to 'what is said' in the Gricean sense, that is, all the non-presupposed and non-implied propositions resulting from exploiting the linguistic code *and* assigning values to those indexical arguments that occur in the predicates of such propositions. This amounts to saying that the conventionally attachable content comprises entailments and certain *explicatures*<sup>3</sup> (Sperber and Wilson 1986). For instance, in (13a) the attachable content includes all the entailments of the proposition that it

<sup>3</sup> Standard explicatures result from interpreting pronouns and providing spatio-temporal coordinates.

is raining at  $t$ , where  $t$  is the value assigned to the time indexical associated with the sentence tense. In contrast, whereas the existence of a consequence relation between the rain and staying at home in (13b) is also considered as an explication in some recent approaches (see Ariel 2008 for a survey), it is not integrated into the attachable content under the present analysis, since non-metalinguistic attachment to the consequence relation is odd. Answer A1 is perfectly standard. Admittedly, it exploits an inferential connection ( $\text{rain} \leadsto \text{staying at home}$ ) which is not part of the AI content, but it *also* targets directly the AI content of the second clause. In this respect, the AI content is not ignored and the attachment principle is not violated. In contrast, answer A2 is clearly off the track because it is intended to falsify the non-AI content associated with *so* (see Jayez 2004 for a discussion of the (non-) AI content of discourse markers).

- (13) a. It is raining  
 b. It is raining, so I prefer to stay at home  
 A1 – Rain should not prevent you from going out  
 A2 – ??It's false, there is no relation between the two things

In cases such as (10), mentioned in (Schlenker 2008), there is (at least) a strong preference for the adjunct to bear the presuppositional non-AI content. Whether this is the result of a conventional preference or a pragmatic inference *via* (an equivalent of) the Quality maxim is open to discussion. In any case, it is not necessary in the proposed approach to postulate that AI content, which is conventionally attachable, must be itself strictly conventional, since nothing essential depends on this assumption.

Why is there an attachment principle at all? Could not we be 'free' to attach, in the limits of inferential plausibility? Consider an example like *I have to pick up my sister at the airport* and suppose that it can be represented as a set of Boolean combinations of literals  $\{L_1 = \text{pick-up}(x) \ \& \ x = y, L_2 = \text{sister}(y)\}$ , where the second element stands for the presupposition. These two elements are 'unrelated', in a sense I am going to clarify. First, the fact that I have a sister does not make it intuitively more plausible that I must pick her up at the airport. Second, the fact that I have to pick up my sister at the airport entails that I have a sister. However, in using the sentence, I am not asserting that I have a sister but that I must fetch at the airport the person who is described by the presupposition. This assertion can come out false in several ways, including those cases in which I have no such obligation and those in which the person I must fetch at the airport is not correctly described by the presupposition, as, for instance, when I must pick up my father or my aunt. This granted, can we imagine a proposition that would be related to the two pieces of information that I have a sister (presupposition) and that I must pick up at the airport a person who is correctly described by the presupposition? It is very unlikely that we can spontaneously produce such a proposition. The task is inferentially too complex.

In most situations, AI and non-AI contents are unrelated. There is no self-evident proposition that would be a common consequence of both contents or would entail them jointly. More generally, given the contribution of a discourse constituent,  $\langle L_1, \dots, L_n \rangle$ , there is no guarantee that  $L_1 \dots L_n$  can be jointly

connected to a common proposition through some discourse relation. If attachment were unconstrained, the general independence of the contribution members would make the construction of an interpretation in discourse even more difficult than it is. For instance, assuming a simple two-sentence dialogue of the form (A:S1= $\langle L_1, L_2 \rangle$ –B:S2= $\langle L'_1, L'_2 \rangle$ ), B would have to eliminate one of  $L_1, L_2$  to determine which part of the contribution is intended by A to require a continuation. More generally, the relation of this problem to the notion of entropy is hard to miss. If we represent a conversational move as a tuple of information pieces  $cm = \langle a_1 \dots a_n \rangle$ , every participant different from the speaker has to choose an intended attachment site among the components of the  $cm$  vector and to provide an appropriate reaction. We can assimilate such reactions to messages. By her reaction, the addressee gives an indication as to the attachment site she has chosen, thus sending a message which consists of a partial ordering of  $cm$ , according to which elements of  $cm$  the addressee's reaction applies more naturally to. In addition, the addressee creates a new conversational move,  $cm'$  to which other participants can react. A series of conversational moves can be seen as a sequence  $(cm_1^x, \langle o^y(cm_1^x, cm_2^y), cm_2^y \rangle, \dots, \langle o^z(cm_{n-1}^u, cm_n^z), cm_n^z \rangle)$ , where  $x, y, u, z$  etc., are discourse participants and  $o(cm_1, cm_2)$  is the ordering on  $cm_1$  induced by the explicit response  $cm_2$ . Consider the probability of an ordering at each stage; if there is no guidance as to preferred attachments, every information piece of the last move can provide an equally plausible attachment site and Shannon entropy is maximum.<sup>4</sup> After the addressee's move, the probability of an attachment by the addressee is estimated with respect to the move itself, which may lead to inferential overload as the discourse grows. Given the cognitive difficulty of dealing with massive equiprobability or inference, a plausible conjecture is that languages have developed conventionalised preferences for attachment in order to streamline discourse management.

Finally, why is the QUD approach empirically robust? The elements that are marked for attachment are preferably interpreted as addressing the QUD because the constraints on attachment help keeping the thread in discourse evolution. Accordingly, when an element is marked for attachment, it is very likely that it contributes to the discourse topic at the current point. Elements that are not so marked can project, since they are subtracted from the current discussion thread. As we saw in the previous section, this does not prevent something from addressing the QUD and projecting, if this element is not conventionally marked as attachable.

## 5 Conclusion

The upshot of the previous discussion is that an element can address the QUD and nonetheless project. This is so because projection is (negatively) associated with conventionalised attachment preferences, that do not noticeably vary with the context. Several important issues are still pending. I will mention three of them. First, additional experimental work is necessary to construct models of

<sup>4</sup> Under the present setting, Shannon entropy is  $-\sum_{a \in cm} p(a) \log p(a)$  for a move  $cm$ ,  $p(a)$  being the probability that the site chosen for attachment is  $a$ .

cognitive processing for non-AI content. In particular, recent work on anticipatory effects (Chambers and San Juan 2008) might complicate the debate over the role of common ground and, more generally, the dynamic character of presuppositions, questioned in various approaches (Abbott, Schlenker). Second, the status of non-conventional elements, so called ‘conversational implicatures’, is unclear. Since they do not necessarily correspond to a segment of linguistic code, their integration into a layered conventional system, as is proposed here, has to be reconsidered. Last, it must be determined whether all discourse relations are equally sensitive to attachment or whether there is a significant variation and why, see (Winterstein 2009) on *too* for an illustration of this problem.

## References

- Ariel, M.: *Pragmatics and Grammar*. Cambridge University Press, Cambridge (2008)
- Chambers, C.G., San Juan, V.: *Perception and Presupposition in Real-Time Language Comprehension: Insights from Anticipatory Processing*. *Cognition* 108, 26–50 (2008)
- Ducrot, O.: *Dire et ne pas Dire*. Hermann, Paris (1972)
- von Fintel, K.: *Would you Believe it? The King of France is Back! Presuppositions and Truth-Value Intuitions*. In: Reimer, M., Bezuidenhout, A. (eds.) *Descriptions and Beyond*, pp. 315–341. Oxford University Press, Oxford (2004)
- Jayez, J.: *Presuppositions and Pedigrees for Discourse Markers*. In: Bonami, O., Patricia Cabrero Hoffherr, P. (eds.) *Empirical Issues in Syntax and Semantics*, vol. 5, pp. 89–110 (2004)
- Jayez, J.: *How Many are ‘several’? Argumentation, Meaning and Layers*. *Belgian Journal of Linguistics* 19, 187–209 (2005)
- Jayez, J., Rossari, C.: *Parentheticals as Conventional Implicatures*. In: Corblin, F., de Swart, H. (eds.) *Handbook of French Semantics*, pp. 211–229. CSLI Publications, Stanford (2004)
- Jayez, J., Toven, L.: *Presque and Almost: How Argumentation Derives from Comparative Meaning*. In: Bonami, O., Cabredo Hofherr, P. (eds.) *Empirical Issues in Syntax and Semantics*, vol. 7, pp. 217–240 (2008)
- Potts, C.: *The Logic of Conventional Implicatures*. Oxford University Press, Oxford (2005)
- Roberts, C., Simons, M., Beaver, D., Tonhauser, J.: *Presuppositions, Conventional Implicatures and beyond: A Unified Account of Projection*. In: *Proceedings of the ESSLLI 2009 Workshop New Directions in the Theory of Presupposition* (2009)
- Schlenker, P.: *Presupposition Projection: The New Debate*. In: Friedman, T., Ito, S. (eds.) *Proceedings of SALT XVIII*, pp. 655–693 (2008)
- Simons, M.: *Presupposition and Relevance*. In: Szabo, Z.G. (ed.) *Semantics vs. Pragmatics*, pp. 329–355. Oxford University Press, Oxford (2005)
- Sperber, D., Wilson, D.: *Relevance. Communication and Cognition*. Blackwell, Oxford (1986)
- Wharton, T.: *Interjections, Language, and the ‘Showing/Saying’ Continuum*. *Pragmatics and Cognition* 11, 39–91 (2003)
- Winterstein, G.: *The Meaning of Too: Presupposition, Argumentation and Optionality* (2009), <http://www.linguist.univ-paris-diderot.fr/~gwinterstein/docs/WintersteinG-MeaningToo-Tbilisi.pdf>

# Adverbs of Comment and Disagreement

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**Abstract.** Adverbs of comment (AOCs) such as *sadly* raise a question of subjective meaning, much like predicates of personal taste (*fun*, *tasty*), namely, to whom the speaker attributes the emotion or evaluation when there is no overt *for*-PP. I extend Lasersohn's (2005) and Stephenson's (2007) judge analysis for predicates of personal taste to AOCs. My proposal is that disagreement on one and the same proposition  $sad(p, a)$  expressed by these adverbs only arises when the hearer correctly resolves the argument of judge  $a$  (a constant) despite its absence in overt syntax, i.e.  $sad(p, a)$  vs.  $\neg sad(p, a)$ . Otherwise, only mis- or incomprehension occurs where the speaker and the hearer actually express two different propositions on the same issue, i.e.  $sad(p, a)$  vs.  $\neg sad(p, b)$ .

**Keywords:** incompleteness, subjective meaning, disagreement, judge, mis/incomprehension.

## 1 Introduction

Adverbs of comment (henceforth, AOCs) such as *sadly* or *fortunately* raise a question of subjective meaning, much like predicates of personal taste (Lasersohn 2005) and epistemic modality (Stephenson 2007), namely, to whom the speaker attributes the emotion/evaluation when she uses an AOC, like *sadly* in e.g. (3-a).

- (1) a. Roller coasters are fun.  
b. Roller coasters are fun [for kids].
- (2) a. The computer might be at risk.  
b. In some world compatible with what [the technician] knows in the actual world, the computer is at risk.
- (3) a. Sadly, the Pink Panther is just one of those jokes that gets lost in translation.  
b. Sadly [for Steve Martin], the Pink Panther is just one of those jokes that gets lost in translation.

In (3-b), the speaker makes it linguistically explicit that the state of affairs at issue is sad for Steve Martin, while it is left open in (3-a) for whom it is so.

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This kind of subjective meaning arises, as shown in (1)-(3), due to a hidden argument at the logical form, and it disappears once the argument is made explicit. Predicates of personal taste and AOCs are more similar to each other than (cf. (1)-(2)) to epistemic modal verbs (cf. (3)) in that syntactic evidence for this argument is in both cases provided by a *for*-PP.

Following Bach (1999) and Potts (2005), I assume that a sentence with AOCs such as (3-b) is double-propositional, one proposition  $p$  expressed by the sentence without the parenthetic *sadly for Steve Martin* and the other one  $sad(p, a)$ ,  $a$  being Steve Martin in this case. According to Potts (2005), the first proposition is at-issue while the second one is a conventional implicature (CI). In comparison to (3-b), the second proposition in (3-a) is incomplete. Because of this, disagreement on the second proposition (i.e.  $sad(p, a)$ ) in (4) demonstrates two different cases: in one case that I call real disagreement, A and B express two propositions differing in polarity, i.e.  $sad(p, a)$  and  $\neg sad(p, a)$ . In the other case that I call mis- or incomprehension, they express two propositions that differ both in polarity and in the hidden argument or "judge" (Lasersohn 2005), i.e.  $sad(p, a)$  and  $\neg sad(p, b)$ .

- (4) A: Sadly, the Pink Panther is just one of those jokes that gets lost in translation.  
 B: Ok, but this is not sad.

The paper is organized as follows. In Section 2, I elaborate on the idea of incomplete propositions in a sentence with AOCs but with no overt judge argument in syntax. In Section 3, I discuss predicates of personal taste in terms of subjective meaning. Section 4 provides a formal analysis for AOCs following Lasersohn (2005) and Stephenson (2007). The last section contains a concluding remark.

## 2 Incomplete Propositions

I assume that sentences sometimes do not express a complete proposition (Bach 1997, 2008, *contra* Cappelen and Lepore 2005). Incomplete propositions arise often due to a syntactically silent but semantically obligatory argument. Take the famous sentence about meteorological conditions for example, (5-b) is propositionally complete but (5-a) is not. This means, I hold that (5-a) does not express one proposition but is used to express different propositions, depending on what the hidden argument - the location - is. Usually, the context of utterance makes the location explicit for such sentences. In comparison, (5-b) expresses one unambiguous proposition that it is raining in Amsterdam.

- (5) a. It is raining.  
 b. It is raining in Amsterdam.

The role of the person(s) to a certain emotional state is not that different from that of the location to a certain meteorological condition. In the latter case, it is commonly assumed that a time argument and a place argument are needed

for the sentence to make sense, while in the former case (putting tense aside), we have a judge instead of a place argument. In a nutshell, I assume the incompleteness of *It is raining* and of *Sadly, p* is due to a missing place/judge argument, which are needed to fill the necessary referential information to give (by the speaker) or get (by the hearer) a complete proposition. Sentences such as (3-a) express two propositions (Bach 1999, Potts 2005), one main, complete proposition that *the Pink Panther is just one of those jokes that gets lost in translation* and the other, secondary, incomplete proposition that *this is sad*. Nothing can be sad if no person is subject to this emotion. The exact group that the speaker has in mind can be made linguistically explicit as in (3-b).

Jackendoff (1972) proposes that such adverbs as *sadly* predicate over a sentence and a second argument SPEAKER. In the literature they are sometimes called speaker-oriented adverbs. With reference to the overt argument by the *for*-PP in (3-b), one can argue against the speaker-orientation of AOCs. Rather, they should be treated as two-place predicates (Liu 2009) taking a judge (in the Lasersohnian sense) as the second argument so that the evaluation can be attributed to the speaker, the addressees, the subject of the sentence, etc. Although this argument can be syntactically silent, a felicitous use of AOCs presupposes the existence of the judge. For example, in a war situation where the speaker informs his own party about the serious casualties of the opposite party, the literal use of *unfortunately* or *tragically* will be outrageous. This means that the argument of judge should be in the semantics of a sentence with AOCs, or in other words, with no judge, no complete proposition is expressed. The same holds for their adjective equivalents of e.g. *It is sad (for Steve Martin) that ...*. With no PP, the sentence does not express a complete (CI) proposition, i.e. is not truth-evaluable.

### 3 Predicates of Personal Taste

#### 3.1 Subjectivity vs. Objectivity

The subjective meaning due to the covert PP in the case of AOCs resembles that of predicates of personal taste, e.g. *fun*, *tasty*. For the latter, Lasersohn (2005) initiates a novel analysis with the judge parameter in which he claims that sentences like (6-a) are not truth-evaluable until the intended judge is resolved.

- (6) a. The chili is tasty.  
       b. A: Is the chili tasty?  
           B: Yes, it is. / No, it isn't.

At least two cases need be distinguished for (6-b): in one, A just saw B taste the chili and thus means to ask whether the chili is tasty for B and B is supposed

<sup>1</sup> To keep things brief, the adjective counterparts of AOCs differ in that AOCs contribute a CI content to the sentence meaning (Potts 2005), while with evaluative adjectives, the propositional content (as that by AOCs) is an at-issue content (see Bonami and Godard 2008 for more detailed comparisons).

to answer whether the chili is tasty for her. Another case is for example where A has assigned B to find out whether the new chili product of their company is tasty for the customers. In this case, A means to ask whether the chili is tasty for the customers and B is supposed to give an answer - probably based on sampling statistics or sales simply. In the second case, the question awaits an objective answer, while in the first case it is about the subjective meaning. Sam's answer in Stephenson's (2007) example (7-a) is odd only when the predicate is used as one of *personal* taste, i.e. subjectively. If Sam answers without having tried but based on the nice looks of the cake, the dialogue in (7-a) is fine.

- (7) a. Mary: How's the cake?  
       Sam: It's tasty.  
       Sue: No, it isn't, it tastes terrible.  
       Sam: # Oh, then I guess I was wrong.  
       b. Mary: Is Petra a doctor?  
       Sam: Yes, she is.  
       Sue: No, she isn't. She is a teacher.  
       Sam: Oh, then I guess I was wrong.

This point is crucial for the issue of subjective meaning with predicates of personal taste and AOCs (and possibly also epistemic modals), namely, there might be no single truth about whether 'roller coasters are fun' or whether 'sadly, Michael Jackson died' (the death of Michael Jackson is sad), as opposed to something like whether *Petra is a doctor* (Stephenson 2007). Or in other words, whether Petra is a doctor can be objectively true or false, while a statement with predicates of personal taste (in the subjective meaning) is true as long as the speaker speaks truthfully. But such a true proposition is just part of *the* truth about whether the chili is tasty when more than one judge is involved.

In the absolute sense, the truth about whether something is fun/tasty consists of a set of (true) propositions, each of which takes a member of the relevant domain as the judge. In this use, when there are more judges involved, there is no single truth (or to put more accurately, the truth is not single-propositional) about whether something is tasty/fun. In a dialogue where two persons Jane and Ben are asked to answer the questions, compare the different semantics of the two questions (as denoting sets of all mutually exclusive, possible answers, in the Hamblin-style semantics): *tc* stands for *the chili*.

- (8) a.  $||is\ petra\ a\ doctor|| = \{doctor(petra), -doctor(petra)\}$   
       b.  $||is\ the\ chili\ tasty|| = \{tasty(tc, jane) \wedge tasty(tc, ben), tasty(tc, jane) \wedge \neg tasty(tc, ben), \neg tasty(tc, jane) \wedge tasty(tc, ben), \neg tasty(tc, jane) \wedge \neg tasty(tc, ben)\}$

Issues are more complicated when collective taste or emotion (where inter-subjectivity instead of subjectivity is involved) rather than personal taste or emotion counts. I leave this for future work.

### 3.2 Disagreement

In (9), what Ben does is comment on whether the chili is tasty (an incomplete proposition, Bach 2008) by his own judge (for a complete proposition). Although it seems that Jane and Ben disagree on the same proposition, they actually express different propositions on the same issue (with the autocentric perspective), that *tasty(tc, jane)* and  $\neg$ *tasty(tc, ben)*. When the domain of the intended judge is made linguistically explicit such as in (10), Ben can no longer felicitously disagree by simply taking a different judge.<sup>2</sup>

- (9) Kim: Is the chili tasty?  
 Jane: The chili is tasty.  
 Ben: No, the chili is not tasty.
- (10) Kim: Is the chili tasty for Peter?  
 Jane: This chili is tasty for Peter.  
 Ben: No, this chili is not tasty for Peter. / # No, this chili is not tasty for Mark.

In the following, I will introduce the previous analyses (Lasersohn 2005, Stephenson 2007) for predicates of personal taste.

### 3.3 Previous Analyses

Kaplan (1989) proposes a two-step derivation for demonstratives, i.e. *character* as a function from context to *content* (proposition) and *content* as a function from world-time pairs  $\langle w, t \rangle$  to truth values  $\{0, 1\}$ . Following this, Lasersohn (2005) argues that a sentence with predicates of personal taste has a stable *content* but that the truth value of this *content* is relativized to individuals. He therefore introduces a new judge index, the value of which is provided “in the derivation of truth values from content, not in the derivation of content from character” (Lasersohn 2005: p.643), that is, by the pragmatic context. To

<sup>2</sup> The same observation holds in connection with the domain restriction of quantifiers (von Stechow and Gillies 2008). Take *only* as an example, the truth value of the sentence obtains only if the domain restriction gets resolved. Accordingly, if the domain restriction of quantification is made linguistically explicit, no disagreement by taking a different domain restriction is felicitous, as shown in (ii).

- (i) A: Only Peter came to the party.  
 B: Really? I heard Sue was there too.  
 A: Yeah, but she was supposed to be there helping me.
- (ii) A: Among the people I invited, namely, Peter, Ben and Jane, only Peter came to the party.  
 B: # Really? I heard Sue was there too.

Neutral and nonneutral modals e.g. in (2), where “the kind of modality is linguistically specified in the former, but provided by the non-linguistic context in the latter” (Kratzer 1991: p.640), should demonstrate similar effects of disagreement.

sum up, Lasersohn assumes that the *content* of (11) is semantically complete, i.e. it expresses a complete proposition, but its truth value is relativized to a world-time-judge triple  $\langle w, t, j \rangle$ .

(11) The chili is tasty.

(12) Predicates of personal taste (Lasersohn 2005):

$$\|tasty\|^{c;w,t,j} = [\lambda x_e. x \text{ tastes good to } j \text{ in } w \text{ at } t]$$

$$\|tasty \text{ for } DP\|^{c;w,t,j} = \|tasty\|^{c;w,t,\|DP\|^{c;w,t,j}}$$

Revising Lasersohn (2005), Stephenson (2007) takes *tasty* or *fun* as two-place predicates, taking PRO or a *for*-PP as the second argument, i.e. the judge.

(13) Predicates of personal taste (Stephenson 2007):

$$\|taste\|^{c;w,t,j} = [\lambda y_e. [\lambda x_e. x \text{ tastes good to } y \text{ in } w \text{ at } t]]$$

$$\|PRO_J\|^{c;w,t,j} = j$$

- (14) a.  $\|The \text{ chili is tasty } PRO_J\|^{c;w,t,j} = 1$  iff the chili tastes good to  $j$  in  $w$  at  $t$   
 b.  $\|The \text{ chili is tasty } PRO_{John}\|^{c;w,t,j} = 1$  iff the chili tastes good to John in  $w$  at  $t$   
 c.  $\|The \text{ chili is tasty for John}\|^{c;w,t,j} = 1$  iff the chili tastes good to John in  $w$  at  $t$

As (14) shows, the sentence *The chili is tasty* is judge-dependent only in (14-a), where neither linguistic context, i.e. by use of the *for*-PP in (14-c) nor pragmatic context in (14-b) provides the value of the judge argument.

## 4 AOCs

Concerning AOCs, direct disagreement by beginning with *No* is not possible due to the relation between the at-issue content and the CI content, i.e.  $\forall p(sad(p) \rightarrow p)$ : disagreement on the CI content presupposes the agreement on the at-issue content, but disagreement on the at-issue content invalidates the issue of the CI content (Liu 2009).

(15) A: Sadly, the Pink Panther is just one of those jokes that gets lost in translation.

B: Ok, but this is not sad. (# No, this is not sad.)

AOCs differ from predicates of personal taste or adjectives of comment when they are embedded. As Lasersohn (2005) and Stephenson (2007) point out, when predicates of personal taste are embedded, for example in (16-a), the sentence gets a salient reading that Mary is the judge. The same is true with adjectives of comment shown in (16-b), whereas it is not so with AOCs.

- (16) a. Mary<sub>*j*</sub> thinks that the chili is tasty (for  $j$ ).  
 b. Mary<sub>*j*</sub> thinks that it is sad (for  $j$ ) that the Pink Panther is ...

- c. Mary thinks that sadly (for *j*), the Pink Panther is ...

According to Potts (2005), AOCs are of CI type  $\langle e^a, \langle \langle s^a, t^a \rangle, \langle s^a, t^c \rangle \rangle \rangle$  taking an individual of type  $\langle e^a \rangle$  and a proposition of type  $\langle s^a, t^a \rangle$  as the two arguments, yielding a proposition of type  $\langle s^a, t^c \rangle$ . In comparison to the at-issue content, this CI content is not necessarily part of Mary's beliefs.

Due to the *for*-PP, (3-a) and (3-b) have different effects in terms of disagreement. In (15), by a positive answer, B agrees with A on the at-issue content of A's utterance and disagrees with A on the CI content. However, the disagreement of B can be attributed to two reasons:

- **mis- or incomprehension:** possibly because the context is not informative enough or because B somehow fails to comprehend even when it is. In this case, the issue is *for whom it is sad*, and A and B disagree in the sense that they take different judges: *it is sad* for one and for the other *it is not sad*. They can either take an autocentric (*sadly*  $PRO_{[+speaker]}$ ) or an exocentric (*sadly*  $PRO_{[-speaker]}$ ) perspective.
- **real disagreement:** this presupposes that B understands exactly what A means but disagrees with her, as if the argument of judge (e.g. *for Steve Martin*) were overt. In this case, the issue is *whether this is sad for Steve Martin*. More likely they (or at least one of them) take an exocentric perspective.

Only in the latter case is the disagreement on one and the same complete proposition. In (17), B cannot simply take another judge, different from Steve Martin, whether explicitly or implicitly. The disagreement has to be on the same proposition, that *this is sad for Steve Martin*.

- (17) A: Sadly for Steve Martin, the Pink Panther ...  
 B: Ok, but this is not sad for Steve Martin.

The same distinction holds also with agreement: in (18), it is still not clear that A and B express the same CI proposition. Rather, they could fairly well take two different judges for whom this is sad and thus express two propositions.

- (18) A: Sadly, the Pink Panther is ...  
 B: Ok, this is indeed sad.

#### 4.1 Judge Shifting

With two different judges, the same speaker can express propositional fragments differing in polarity, but this is not possible if the judge argument is silent. In other words, with an explicit judge argument, a new judge can be introduced to make up a new proposition of the same or opposite polarity. With implicit judge, judge-shifting is ruled out. (19) provides evidence that with implicit judges, judge-shifting is only possible with context-shifting, for instance by speaker change or by change of the same speaker's mental state e.g. in (20).

- (19) a. Sadly for Steve Martin, the Pink Panther is just one of those jokes that gets lost in translation. But this is not sad for anybody else.

- b. Sadly, the Pink Panther is just one of those jokes that gets lost in translation. #But this is not sad.
- (20) Sadly, (or maybe not sadly) the Pink Panther is just one of those jokes that gets lost in translation.

(21-a) provides an example where two propositions are expressed by two AOCs taking the same propositional argument, and (21-b) gives the same, but with their adjective counterparts as commentaries.

- (21) a. Fortunately for them, unfortunately for us, it was a good choice.  
 b. It's fortunate for us, but unfortunate for the auto industry as a whole.

## 4.2 Judge as a Constant

Lasersohn (2005) points out that the interpretation with existential closure on judges for predicates of personal taste  $\exists x.the\ chili\ is\ tasty\ for\ x$  is too weak. So is it also with the judge for AOCs. There are two ways to formulate that the existence of judge is presupposed, either  $\exists x(sad(p, x))$  or  $sad(p, a)$ . If we take the former as the interpretation, the disagreement can go two ways (22): B1 is too strong and B2 is too weak. This provides extra evidence for the constant interpretation of the judge for AOCs.

- (22) A:  $\exists x(sad(p, x))$   
 B1:  $\neg\exists x(sad(p, x))$   
 B2:  $\exists x\neg(sad(p, x))$

To sum it up, the disagreement on one and the same proposition only arises when the hearer correctly resolves the argument of judge despite its absence in overt syntax, i.e.  $sad(p, a)$  vs.  $\neg sad(p, a)$ . Otherwise, only mis- or incomprehension occurs where the speaker and the hearer actually express two different propositions on the same issue, i.e.  $sad(p, a)$  vs.  $\neg sad(p, b)$ .

- (23) A:  $sad(p, a)$   
 B1:  $\neg sad(p, b)$  -mis- or incomprehension  
 B2:  $\neg sad(p, a)$  -real disagreement

## 4.3 Analysis

Stephenson (2007: p.500) claims that “epistemic modals are inherently judge-dependent, whereas predicates of personal taste become judge-dependent only if they take  $PRO_J$  as an argument”. In the following, I extend her analysis for predicates of personal taste to AOCs, as the subjective meaning results from the absence of an overt *for*-PP in both cases. The interpretation for *sadly* is provided below: the same analysis applies to their (evaluative) adjective counterparts.

(24) AOCs:

$$\begin{aligned} \|\text{sadly}\|^{c:w,t,j} &= [\lambda x_e. [\lambda p_{<s,t>.} p \text{ is sad for } x \text{ in } w \text{ at } t]] \\ \|\text{sadly for } DP\|^{c:w,t,j} &= \|\text{sadly}\|^{c:w,t,j} (\|DP\|^{c:w,t,j}) \end{aligned}$$

Predicates of personal taste, epistemic modals and AOCs co-occur with one another. If we believe Stephenson (and I do) that epistemic modals are inherently judge-dependent, we can leave them aside first and concentrate on (25-a) to see how the two judges interact: *PP* stands for *Pink Panther* in (26).

(25) Sadly, the Pink Panther is boring.

- a. Sadly  $\text{PRO}_{j_2}$ , the Pink Panther is boring  $\text{PRO}_{j_1}$ .
- b. Sadly  $\text{pro}_{\text{Steve Martin}}$ , the Pink Panther is boring  $\text{PRO}_{\text{the reviewer}}$ .
- c. Sadly for Steve Martin, the Pink Panther is boring for the reviewer.

(26)  $\|\text{boring}\|^{c:w,t,j} = [[\lambda y_e. [\lambda x_e. x \text{ is boring for } y \text{ in } w \text{ at } t]]$   
 $\|\text{boring } \text{PRO}_{j_1}\|^{c:w,t,j} = [\lambda x_e. x \text{ is boring for } j_1 \text{ in } w \text{ at } t]$   
 $\|[\text{the } PP \text{ is boring}]\|^{c:w,t,j} = [\text{the } PP \text{ is boring for } j_1 \text{ in } w \text{ at } t]$   
 $\|\text{sadly}\|^{c:w,t,j} = [\lambda x_e. [\lambda p_{<s,t>.} p \text{ is sad for } x \text{ in } w \text{ at } t]]$   
 $\|[\text{sadly } \text{PRO}_{j_2}]\|^{c:w,t,j} = [\lambda p_{<s,t>.} p \text{ is sad for } j_2 \text{ in } w \text{ at } t]$   
 (25-a)  $\|^{c:w,t,j} = 1$  iff [the PP is boring for  $j_1$  in  $w$  at  $t$ ] and [it is sad for  $j_2$  that [the PP is boring for  $j_1$  in  $w$  at  $t$ ] in  $w$  at  $t$ ]

## 5 Conclusion

In the foregoing, I extended the judge parameter (relativism: Lasersohn 2005) to analyze AOCs, but the basic assumption I adopted is rather the radical invariantism (Bach 2008). The former holds that the stable content (proposition) is truth-evaluable relative to the judge and the latter holds that the content (proposition) is not complete and thus not truth-evaluable due to the missing argument. As is shown below, Bach's derivation seems more economical where the argument of judge is resolved like other indexicality. But this is just a conceptual note: for the interpretation of AOCs, the two approaches both work, no matter you take the judge as an argument or a parameter.

<sup>3</sup> Interestingly, for sentences with two explicit judge arguments e.g. (i), the quantifier in the at-issue content scopes over the quantifier in the CI content.

- (i) Sadly for everyone, the Pink Panther is boring for someone.
  - a. Reading a ( $\exists > \forall$ ):  $\exists x[\text{boring}(PP, x) \wedge \forall y(\text{person}(y) \rightarrow \text{sad}(\text{boring}(PP, x), y))]$
  - b. Reading b ( $\forall > \exists$ ): unavailable  
 $\forall y[\text{person}(y) \rightarrow \exists x[\text{sad}(\text{boring}(PP, x), y)]]$

(ii) Mary thinks that sadly for everyone, the Pink Panther is boring for someone.

Is the CI content here more at-issue than is assumed in Potts (2005)? In (ii), the CI content seems to be part of Mary's beliefs, different from (16-c). Or does *everyone* simply denote plural individuals? I don't have an answer yet.

- (27) a. Lasersohn's relativism: *character*  $\Rightarrow$  *resolve indexicality*  $\Rightarrow$  *content*  
 $\Rightarrow$  *judge – dependent truth value*  
 b. Bach's radical invariantism: *character (incomplete proposition)*  $\Rightarrow$   
*resolve indexicality (including judge)*  $\Rightarrow$  *content (complete*  
*proposition) / truth value*

To conclude, AOCs such as *sadly, tragically, unfortunately* are another example to be taken into consideration in the general phenomenon of subjective meaning, along with predicates of personal taste (and also quantifier domain restrictions / epistemic modality). A sentence with an AOC but with no overt argument of judge is propositionally incomplete and thus semantically vague. In this case, disagreement either involves the hearer's mis- or incomprehension of the intended judge by the speaker and her adoption of a different judge or her real disagreement on the same proposition with the intended judge correctly resolved.

## References

- Bach, K.: The Semantics-Pragmatics Distinction: What it is and Why it matters. *Linguistische Berichte. Special Issue on Pragmatics* 8, 33–50 (1997)
- Bach, K.: The Myth of Conventional Implicature. *Linguistics and Philosophy* 22, 327–366 (1999)
- Bach, K.: Perspectives on Possibilities: Contextualism, Relativism, or What? In: Egan, A., Weatherson, B. (eds.) *Draft for Epistemic Modality*, Oxford University Press, Oxford (2008)
- Bonami, O., Godard, D.: Lexical semantics and pragmatics of evaluative adverbs. In: McNally, L., Kennedy, C. (eds.) *Adverbs and Adjectives: Syntax, Semantics, and Discourse*, pp. 274–304. Oxford University Press, Oxford (2008)
- Cappelen, H., Lepore, E.: *Insensitive Semantics*. Blackwell, Oxford (2005)
- von Fintel, K., Gillies, A.S.: 'Might' Made Right. In: Egan, A., Weatherson, B. (eds.) *To appear in a volume on Epistemic Modality*, Oxford University Press. Oxford (2008)
- Jackendoff, R.S.: *Semantic Interpretation in Generative Grammar*. The MIT Press, Cambridge (1972)
- Kaplan, D.: Demonstratives. In: Almog, J., et al. (eds.) *Themes from Kaplan*, pp. 481–563. Oxford University Press, Oxford (1989)
- Kratzer, A.: Modality. In: von Stechow, A., Wunderlich, D. (eds.) *Semantik. Ein internationales Handbuch der zeitgenössischen Forschung*, pp. 639–650. de Gruyter, Berlin (1991)
- Lasersohn, P.: Context dependence, disagreement, and predicates of personal taste. *Linguistics and Philosophy* 28, 643–686 (2005)
- Liu, M.: Speaker-oriented Adverbs of the German *-weise* Sort. In: *Proceedings of Sinn und Bedeutung*, vol. 13 (2009)
- Potts, C.: *The Logic of Conventional Implicatures*. Oxford Studies in Theoretical Linguistics. Oxford University Press, Oxford (2005)
- Stephenson, T.: Judge Dependence, Epistemic Modals, and Predicates of Personal Taste. *Linguistics and Philosophy* 30, 487–525 (2007)

# Two Puzzles about Requirements

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**Abstract.** I discuss the semantics of statements of minimum and maximum requirement. I show that, on standard assumptions, such statements receive a non-sensical interpretation. I solve this puzzle by pointing out certain semantic features shared by all end-point markers akin to *minimum* and *maximum*.

## 1 First Puzzle: Minimum Requirements

My friend and me are having a bet in which I claim to be able to score at least 300 points in the game of scrabble we are about to start. The following would be an accurate paraphrase for the bet in question.

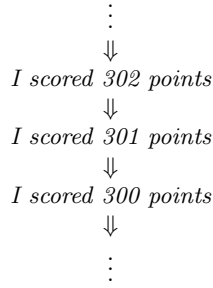
- (1) The minimum number of points I need to score to win the bet is 300.

That is, the bet involves a *minimum requirement*: If I score 300, I win. If I score more, I also win. But if I score less, then I lose. To be sure, at first sight it seems obvious why (1) is interpreted as such, for I do *need* to score 300, and 300 is the *minimum* score that makes me win the bet. Yet, when we make things precise, and given common assumptions on the semantics of modal auxiliaries, it turns out that it is rather mysterious why (1) means what it means.

The common assumptions I am alluding to are, first of all, that *need* is a universal quantifier over possible worlds and, second, that the *to*-phrase in (1) restricts quantification over possible worlds (as in von Stechow and Iatridou's [2]). In other words, "*to p, need to q*" is true if and only if all the *p*-worlds are *q*-worlds; i.e. if *p* entails *q*. At first sight, this view appears to make good predictions. For example, a case like (2) is now interpreted as saying that you went to the Twijnstraat in all the worlds where you got good cheese.

- (2) To get good cheese, you need to go to the Twijnstraat.

However, when we apply the above assumptions to (1), the outcome is very puzzling. Note first the following: in the scenario I sketched about the scrabble bet, there are no worlds in which I win the bet while scoring fewer than 300 points. Furthermore, the worlds where I do win come in many variations: in some (but not all) of them I score 300 points, in some (but not all) my score is 301, in some (but not all) it is 302, 310, or even 550. The problem is that the most obvious referent for *the minimum number of points I need to score* is the smallest number *x* such that *I scored x points* is true in all relevant worlds. However, for no value for *x* is this open sentence true in all bet-winning worlds.



**Fig. 1.** The *at least* interpretation for numerals

In other words, one would expect the definite description *the minimum number of points I need to score* to fail to refer in the described situation.

It might appear that there is an obvious solution. If we assume that numerals are interpreted as lower-bounded only (usually, this is dubbed the *at least* interpretation for numerals), then we get an entailment scale as in figure 1. Given this scale, it is now true that in all bet-winning worlds I scored 300 points, for the worlds where my exact score was higher will be worlds in which I scored at least 300 points. Unfortunately, if we assume that such an entailment scale is appropriate then, by entailment, it is also true that in all bet-winning worlds I scored 200 points. In fact, it is entailed that I score a single point in every bet-winning world. Consequently, *the minimum number of points I need to score to win the bet* is now predicted to be 1, not 300. Yet, (3), obviously, seems an unacceptable way of paraphrasing the bet between my friend and me.

(3) The minimum number of points I need to score to win the bet is 1.

In sum, independent of how we interpret numerals, it appears far from straightforward how to come to a compositional interpretation of (1).

## 2 Second Puzzle: Maximum Requirements

If we assume the entailment scale in figure 1, then there is a further puzzle. The proposition *I scored 1 point* is true in all bet-winning worlds, and so is the proposition *I scored 300 points*. The proposition *I scored 301 points*, however, is the first proposition in the scale that is not true in all bet-winning worlds. (It's only true in some.) This makes 300 the highest value for  $x$  such that *I scored  $x$  points* is true in all victorious worlds and so we predict that instead of (1), the correct way of paraphrasing the bet is, in fact, (4). Clearly, this is an unwelcome prediction, for the scenario I sketched was a prime example of what we call *minimum requirements*, not of *maximum* ones.

(4) The maximum number of points I need to score to win the bet is 300.

In the given scenario there was no upper limit on my score. That is, the bet is only about me scoring 300 or more. No matter how much higher than 300 I score, I will keep on winning. In certain other scenarios maximal requirements do make sense, however. Take the following example.

- (5) The maximum number of sets needed to decide a men's tennis match is 5.

In men's tennis, the first player to win three sets wins, hence (5). At the same time, this means that there can be no men's tennis match which lasts for fewer than 3 sets. In other words:

- (6) The minimum number of sets needed to decide a men's tennis match is 3.

Once again, we derive the wrong interpretation for such examples under the standard assumptions discussed above. Let us first assume that numerals create entailment scales, as in figure 1. That is, if 4 sets were played in a match, then this entails that 3 sets were played, as well as that 2 sets were played, etc. The minimum number  $n$  of sets such that  $n$  sets are played in every world in which the match is decided is now 1. The maximum number  $n$  of sets such that  $n$  sets are played in every match-deciding world is 3, since all such worlds contain (at least) 3 sets, while only some contain a 4th or a 5th one.

Things are no better if we interpret numerals as doubly-bound. In that case the definite descriptions in (5) and (6) will fail to refer, for there exists no  $n$  such that in all possible ways in which a tennis match could be decided the match counted exactly  $n$  sets.

### 3 Modal Force

The two puzzles above point out a problem with the interaction between universal modals and scalar operations. Abstracting away from the scenarios above, the puzzles can be generalised as follows. Let  $\mathcal{P}$  be a scale of propositions, as in (7).

- (7)  $p_1 < p_2 < p_3 < \dots < p_i < p_{i+1} < \dots$   $\mathcal{P}$

Following standard assumptions, a proposition  $p_k$  is a *minimum requirement* for  $q$  if it is the case that  $p_k$  is entailed by  $q$ , while for no  $l < k$ ,  $p_l$  is entailed by  $q$ . If (7) is an entailment scale, however, then for any  $p_k$  it holds that  $p_k \models p_{k-1}$  and, in fact,  $p_k \models p_1$ . So, if  $q$  entails  $p_k$ , then it also entails  $p_1$ . In other words, the minimum requirement for  $q$  could only ever be  $p_1$ , which renders the notion of minimum requirement quite useless. If, however,  $\mathcal{P}$  is not an entailment scale, then it is easy to imagine that  $p_k$  is entailed by  $q$  while this is not the case for any proposition that is lower on the scale. Unfortunately, this is not how we normally understand the notion of *minimum requirement*. In fact, part of the problem with our assumed definition of minimum requirement is that if  $p_k$  is a

minimum requirement for  $q$ , then  $q$  need not entail  $p_k$ . For instance, if my bet is that I will score 300 or more in the game of scrabble, then me winning the bet does not entail me scoring exactly 300 points<sup>1</sup>.

There is a surprising way out: minimal and maximal requirements are not about necessity, but rather about possibility. Assume  $\mathcal{P}$  to be a non-entailment scale:  $p_k$  is a minimal (maximal) requirement for  $q$  iff  $p_k$  is compatible with  $q$  and there is no  $l < k$  ( $l > k$ ) such that  $p_l$  is compatible with  $q$ . In the scrabble bet scenario, the proposition that I scored exactly 300 points is the lowest ranked proposition such that there *exists* a bet-winning world in which that proposition is true. Furthermore, 3 is the smallest number  $n$  such that there is a world in which the tennis match is decided in such a way that the number of played sets is exactly  $n$ . Also, 5 is the highest number  $n$  such that there is a world in which the match is decided such that the number of played sets is exactly  $n$ . Thus, under the assumption that the modal force of *need* is existential, the analysis of examples like (1), (5) and (6) appears straightforward.

There is however clear evidence that the modal force of *need* and its kin is not existential. For instance, if it were, we would predict (8) to be true. The intuition, however, is that it is false.

(8) To decide a men's tennis match, you have to play exactly 3 sets.

Below, I will argue that although the semantics of *need* and its kin involves universal quantification, operators like *minimum* and *maximum* are special in that they tend to force a weaker interpretation of their complement.

Before I provide the details of this analysis and give motivation that favours it, I will first briefly say a bit more about the data. Central to the puzzles that I presented above is a rather specific family of noun phrases, namely definite descriptions that contain a minimality or maximality operator and a necessity modal. It is important to note that the specific modal or the specific scalar operator does not matter. First of all, statements of minimal or maximal requirement

<sup>1</sup> Connected to this discussion is the following. Von Fintel and Iatridou [2] observe that their theory predicts that (i-a) entails (i-b).

- (i) a. To get good cheese, you have to go to the Twijnstraat.
- b.  $\Rightarrow$  To get good cheese, you have to breathe.

If you go to the Twijnstraat in all worlds in which you get good cheese, then since you breathe in all the worlds in which you go to the Twijnstraat, it follows that you breathe in all worlds in which you get good cheese. Von Fintel and Iatridou judge (i-b) *true, yet unhelpful* in the context of (i-a), an intuition I agree with. This means that even though the data in (i) might be slightly unintuitive at first sight, von Fintel and Iatridou's theory appears on the right track in predicting that there is an entailment here. However, the truth of the following example, suggested to me by David Beaver, is a further prediction of the theory, but it is not clear that it is a welcome one.

- (ii) To climb Everest you need 3 to equal 2+1.

are not limited to the modal *to need*. *Must*, *require*, *should* and *have to* allow for similar constructions, as in the following naturally occurring examples, and pose exactly the same puzzles.

- (9) Determine the smallest number of digits that must be removed from *x* so that the remaining digits can be rearranged to form a palindrome.
- (10) REM level is the minimum number of BYTES you require to continue.
- (11) What is the minimum number of karanga I should know before I can say that I can karanga?
- (12) We are usually interested in knowing the smallest number of colors that have to be used to color a graph.
- (13) What is the shortest distance one needs to travel to visit all 30 teams in 28 major league cities?

Note too that expressing minimal (or maximal) requirements is not limited to the expressions *minimum* and *maximum* (and their derivations). The phenomenon extends to using superlatives (like *smallest* in (9) and (12), or *shortest* in (13)).

## 4 Towards an Analysis: Restricting the Modal Base

Above, I argued that despite the puzzles central to this work, modals like *need* are best analysed as universal quantifiers. They just appear to have existential modal force whenever they interact with expressions of minimality/maximality. This is in a rather specific sense reminiscent of the discussion of modals in Lillooet Salish by Rullmann et al. [7], for they argue that in that language modals have *variable quantificational force*, with universal force being the default [2]. Their analysis is based on an idea originating from Klinedinst [4]. Klinedinst proposes that modals are distributivity operators with respect to a plurality of worlds. According to this proposal modal force is not a matter of existential versus universal quantification over possible worlds, but rather a matter of whether the modal base is definite (necessity) or indefinite (possibility) [3]. Put differently, whereas necessity modals quantify distributively over the whole modal base, possibility modals quantify distributively over just a subset of modal base worlds.

<sup>2</sup> A different possible route to follow would have been to connect the puzzles of minimum and maximum requirement to Schwager's observations in [8]. She discusses the fact that necessity modals are sometimes compatible with anti-exhaustifiers like *for example*, resulting in a weakening of the modal force. I leave it to further research to investigate whether Schwager's observations are in any deeper way linked to the puzzles discussed above.

<sup>3</sup> I should note, however, that the semantics of minimum/maximum requirement does not turn out to be solely due to the interaction between minimality/maximality operators and *definiteness*. There is no clear parallel between minimum requirements and cases where minimality scopes over a definite noun phrase. For instance, it appears impossible to interpret (i) as saying that the man who married the youngest woman married a woman of 20 years old.

Rullmann et al. implement Klinedinst's idea by providing the general scheme for the interpretation of modal expressions in (14).

$$(14) \quad \llbracket \text{MODAL}(f)(w)(\varphi) \rrbracket = \forall w' \in f(\text{modal\_base}(w)) : w' \in \varphi$$

where  $f$  maps a set of worlds to one of its subsets

If  $f$  is the identity function, then the scheme represents an operator which universally quantifies over the worlds in the modal base, which is equivalent to the standard interpretation of necessity modals. If  $f$  is existentially closed, thus allowing quantification over a proper subset of the modal base, then the scheme represents a possibility modal. Rullmann et al. suggest that in Lillooet salish,  $f$  is not specified for modal expressions, which is why these expressions have variable modal force. In English, expressions like *need* fix  $f$  to be the identity function.

Obviously, a view on modals along these lines opens up the theoretical possibility for variable quantificational force for English models too. Say, we interpret *need* as follows, where the underlined conjunct  $\exists f[f = \lambda W.W]$  is interpreted with widest possible scope (akin a presupposition). (I'm assuming a dynamic interpretation of  $\exists$ ).

$$(15) \quad \llbracket \text{need} \rrbracket = \lambda p. \lambda w. \exists f[f = \lambda W.W] \ \& \ \forall w' \in f(\text{modal\_base}(w)) : w' \in p$$

The choice of the identity function for determining the domain of quantification can be overruled by existential quantification over  $f$  with narrower scope. As I will suggest below, scalar operators like *minimum*/*maximum* introduce existential quantifiers in their scope. It is this fact that weakens the modal force of necessity modals in the scope of such operators. For instance, example (16) is interpreted as (17), where the first conjunct is rendered vacuous by the local re-quantification over  $f$ .

$$(16) \quad \text{The minimum number of points I need to score is 300.}$$

$$(17) \quad \exists f[f = \lambda W.W] \ \& \ \min(\lambda n. \exists f[\forall w' \in f(\lambda w. \text{I win the bet in } w) : \text{I score } n \text{ points in } w']) = 300$$

This says that there exists a subset of worlds in which I win the bet such that I score exactly 300 points in every world in that subset and that there is no subset of the set of bet-winning worlds such that for each world in that set I score  $n < 300$  points.

The mechanism with which we derive (17) (i.e. existential closure in the scope of the minimality operator) might seem far-fetched, but as I will show next, it turns out that there is a general case to be made that scalar end-point markers, like superlatives and expressions like *minimum* and *maximum*, introduce existential structures in their scope. To be able to show this I will need modified numerals to enter into the discussion.

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- (i) (Five older men get married in the same year, all to significantly younger spouses.) The youngest woman the men married is 20 years old.

## 5 Modified Numerals

So far, I have been assuming that, in statements of minimum or maximum requirement, the scope of *minimum* and *maximum* is wider than that of the modal. So, I have been analysing (1) as (18-a), rather than (18-b)<sup>4</sup>

- (1) The minimum number of points I need to score to win the bet is 300.
- (18) a.  $\min_n(\Box[\text{I score } n \text{ points}])=300$   
 b.  $\Box[\min_n(\text{I score } n \text{ points}) = 300]$

Note that an analysis along the lines of (18-b) does not solve our puzzles. On an ‘exactly’ reading for *n points*, there is just the single value for *n* which makes *I score n points* true in a specific world. The use of *minimally* would then be vacuous. Moreover, (18-b) is true if and only if I win the bet providing I score exactly 300 points, no more, no fewer. This is not a minimum requirement. Worse still, on an analysis along the lines of (18-b), we would expect that there is no difference between (1) and (19).

- (19) The maximum number of points I need to score to win the bet is 300.

On an at least perspective for *n points*, (18-b) will be a contradiction. Since for any *n*, *I score n points* entails that I scored a single point, (18-b) ends up stating that  $\Box[1 = 300]$ .

Interestingly, there is a variation on (18-b) that yields the correct truth-conditions without the need for a change in modal force for *need*.

- (20)  $\Box[l_n(\text{I score } n \text{ points}) \geq 300]$

This analysis is not as far fetched as it might seem at first sight. As a numeral modifier, *minimally* shares its semantics with *at least*. In other words, the proper treatment of (1) could be thought to be whatever works for (21) or (22).

- (21) To win the bet, I need to score minimally 300 points.  
 (22) To win the bet, I need to score at least 300 points.

<sup>4</sup> It is difficult to extend the above puzzles of minimal and maximal requirement to cases of epistemic modality. This might actually be expected if the analysis of a wide scope minimality operator is on the right track, given the generalisation that epistemic modals tend to take wide scope (cf. [11]).

Consider the following example. Say, you have seen me put 10 marbles in a box, but you do not know how many marbles there were in the box to begin with. Structurally, your knowledge state now resembles that of a minimal requirement scenario: in all compatible worlds, there are (at least) 10 marbles in the box, while in no compatible worlds there are fewer than 10 marbles in the box. Yet, in contrast to the examples given above, we cannot express this knowledge state as (i).

- (i) ??The minimum number of marbles that must be in the box is 10.

Unfortunately, there are reasons to believe that (20) is too simplistic as an analysis for (21) or (22). As Geurts and Nouwen [3] argue in detail, *at least* does not correspond to the  $\geq$ -relation. Moreover, I recently showed that both *minimally* and *at least* are part of a class of numeral modifiers that is incompatible with specific amounts [6]. That is, whereas (23) is felicitous and true, (24) is unacceptable.

(23) A hexagon has more than 2 sides.

(24) #A hexagon has { at least / minimally } 2 sides.

If *minimally* corresponded to  $\geq$ , then it is a mystery why (24) is infelicitous, for it ends up stating that  $6 \geq 2$ , which is simply a true proposition. Note also, that one cannot argue that (24) is true but underinformative and therefore infelicitous, for the same argument would wrongly predict (23) to be infelicitous, since this states the true and underinformative  $6 > 2$ . The conclusion I draw from this in [6] is that, unlike the comparative *more than*, modifiers like *minimally* are not operators that compare two values. For our present purposes this means that (20) will not do as an analysis of (21) or (22), and that it therefore seems unlikely that the solution to the minimum requirement puzzle lies in interpreting *minimum* as a (reconstructed) narrow scope  $\geq$ -relation.

A further property of numeral modifiers like *minimally* is that they trigger readings of speaker uncertainty (cf. [3], [5], [6]). For instance, (25) is interpreted as being about the minimum number of people John *might* have invited (according to the speaker).

(25) John invited { minimally / at least } 30 people to his party.  
(#To be precise, he invited 43.)

Such speaker uncertainty readings carry over to adjectives like *minimum*.<sup>5</sup>

(26) The { minimum / smallest } number of people John invited to the party is 30.  
(#To be precise, it's 43.)

Apart from understanding (26) as a case of speaker uncertainty, one might also understand it as saying that 30 is the smallest number of people that John, *at some time in the past*, invited to the party. Crucially, all available readings somehow involve existential quantification.

The point I want to make is that there is a general puzzle underlying the interaction of universal modals and scalar operators, be they adjectives like *minimum*, *smallest*, *highest* etc. or numeral modifiers like *minimally* and *at least*.<sup>6</sup>

<sup>5</sup> I am grateful to an anonymous Amsterdam Colloquium reviewer for urging me to attend to the relevance of such data.

<sup>6</sup> In fact, an anonymous reviewer suggests that the data extends to cases where *minimum* is used as a noun, as in (i).

(i) I need a minimum of 300 points to win the bet.

What such expressions appear to have in common is that they operate on existential structures. To be more precise, operators like superlatives and *minimum/maximum* select the top-most or bottom-most element from a scale. In examples like (25), no scale is available: the number of people invited by John is a definite value (a singleton set):

- (27)  $\lambda n$ .John invited (exactly)  $n$  people

Applying *minimally* to (27) makes no sense, since this set contains just the single value. In order to make the application of the end-point operator meaningful, a scale needs to be created. This can be done by assuming that there is a silent existential quantifier somewhere in the embedded structure. For instance, the following would result in a scale for which the application of an end-point operator does make sense.

- (28)  $\lambda n$ .John may have invited (exactly)  $n$  people

Note that we can observe something similar with superlatives. The amount of prize money won by Federer is in principle a definite amount. However, in the scope of a superlative, we need to divide this amount up into parts.

- (29) The highest amount of prize money that Federer won so far is \$1 million.  
That was for Wimbledon 2009.

*the amount of prize money that Federer won in some tournament*

- (30) The highest amount of prize money that Federer won so far is \$8.3 million. That was over 2006.

*the amount of prize money that Federer won for some year*

When we interpret (29) or (30) we need to assume that the speaker had some silent indefinite in mind in the scope of the superlative. Without such an assumption, interpretation will fail, for no scale will have been created.

What the puzzles of minimum and maximum requirement have in common with examples like (29)/(30) is then the presence of an operator that requires a certain scale structure. When the overt material does not lead to an interpretation in which the operator is applied to a scale, the insertion of some existential quantificational structure can be used to ‘save’ interpretation. In the case of minimum and maximum requirements, this last-resort strategy involves local existential closure over the function selecting the relevant part of the modal base.

## 6 To Conclude

The main goal of this paper is to point out that the semantics of minimum (and maximum) requirements is not trivial. The analysis I have sketched is open for improvement, but, importantly, I hope to have shown that there is a connection between the puzzles I discussed here and the recent literature on modified numerals.

I have remained silent about a further possible connection. As I mentioned in the previous section, an operator like *minimum* has a kinship to the relation  $\geq$ , which in turn is linked to disjunction:  $x \geq y \Leftrightarrow x = y \vee x > y$ . *To do minimally p* means *to do p* or *to do more than p*. With this in mind, the puzzles I have discussed above are yet further additions to the list of puzzles that involve modality and disjunctive structures, most notably the issue of free choice possibility. However, I will need to leave a detailed comparison to free choice for further research. The analysis I proposed is based on a general observation I made concerning scalar end-point markers, namely that since their complement needs to denote a scale, their presence sometimes leads to the insertion of a covert existential quantifier to facilitate interpretation. Although the analysis incorporates elements familiar from the free choice literature (such as Klinedinst's view on modality), the main mechanism seems to me to be based on an independent observation.

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## References

1. von Fintel, K., Iatridou, S.: Epistemic containment. *Linguistic Inquiry* 34(2), 173–198 (2003)
2. von Fintel, K., Iatridou, S.: What to do if you want to go to harlem: Anankastic conditionals and related matters. Ms. MIT (2005), <http://mit.edu/fintel/www/harlem-rutgers.pdf>
3. Geurts, B., Nouwen, R., et al.: The semantics of scalar modifiers. *Language* 83(3), 533–559 (2007)
4. Klinedinst, N.: Plurality and Possibility. PhD thesis, University of California Los Angeles (2007)
5. Krifka, M.: More on the difference between more than two and at least three. Paper presented at University of California, Santa Cruz (2007)
6. Nouwen, R.: Two kinds of modified numerals. *Semantics and Pragmatics* 3(3), 1–41 (2010)
7. Rullmann, H., Matthewson, L., Davis, H.: Modals as distributive indefinites. *Natural Language Semantics* 16, 317–357 (2008)
8. Schwager, M.: Exhaustive imperatives. In: Dekker, P., Franke, M. (eds.) *Proceedings of the 15th Amsterdam Colloquium*. Universiteit van Amsterdam (2005)

# Two Sources of *Again*-Ambiguities: Evidence from Degree-Achievement Predicates<sup>\*</sup>

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**Abstract.** This paper provides evidence that *again*-ambiguities derive from two distinct sources, with the precise nature of a particular ambiguity being dependent on the particular type of predicate (Result-State or Degree-Achievement) present in the sentence. Previous research has focused primarily on sentences containing Result-State predicates (e.g. *to open*) rather than Degree Achievements (e.g. *to widen*), and has located the source of the ambiguity in the scope that *again* takes with respect to BECOME in a syntactically decomposed predicate. I argue that entailment facts preclude such an analysis from applying to sentences containing Degree Achievements and *again*. Instead, I propose that Degree Achievement predicates should be decomposed into comparative structures, and that the ambiguity in such sentences arises from the scope *again* takes with respect to a comparative Degree Phrase, rather than a BECOME operator.

## 1 Introduction

The proposal that certain morphologically simple words should be realized as multiple syntactic objects in order to explain paraphrasability and to capture certain entailment patterns originated in the late 1960s and early 1970s with the Generative Semantics (GS) movement; since Dowty [3], an analysis of this type has often been referred to as a ‘lexical decomposition’ account. Evidence brought forth for a decomposition analysis came in part from purported ambiguities found in sentences containing (i) an adverbial such as *again*, and (ii) an achievement-type verb. That is, it was claimed that there are two readings available for a sentence such as (1).

(1) The door opened again.

In one reading of this sentence, termed the *repetitive* reading, the door is understood to have opened previously; in the other reading, termed the *non-repetitive* or *restitutive* reading, the door is understood to have merely been in an open state before (though it need not ever have been opened before).

According to a GS-style analysis, the ambiguity found in (1) is said to result from the scope of *again* with respect to elements in a decomposed predicate (see [1], [3],

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[8], [9]). A sentence like *The door opened* is said to be decomposable into two propositional levels: the level of the small clause, and the level of BECOME plus the small clause. This leaves two possible attachment sites for *again*, shown below, which correspond to the two readings for (1).

- (2)     a. [again [BECOME [the door open]]                    *repetitive*
- b. [BECOME [again [the door open]]                *non-repetitive*

Intuitively, a repetitive reading includes a non-repetitive one; if the door was previously opened, it follows that the door was previously open. Evidence that there are two distinct readings comes from the fact that when *again* is preposed, as in (3), only a repetitive reading is available.

- (3)     Again, the door opened.

This entailment between readings will turn out to be crucial in the discussion that follows. As it turns out, a BECOME-*again* analysis of an *again*-ambiguity always predicts such an entailment to hold between readings. Thus, such an analysis is problematic when we consider sentences containing Degree Achievement (DA) predicates and *again*; such sentences do demonstrate an ambiguity, but it is one in which neither reading entails the other. Examples of DA predicates include many deadjectival verbs, such as *widen*, *narrow*, *lengthen*, *shorten*, as well as predicates such as *grow* and *shrink*.

Consider the sentence below, which contains the DA predicate *widen*.

- (4)     The river widened again.

Like (1), the sentence in (4) has both a repetitive and a non-repetitive reading. The repetitive reading is true only if the river widened previously. The non-repetitive reading of (4) (called the *counter-directional* reading by von Stechow [9]) is true only if the river narrowed previously. Crucially, neither reading entails the other. The sentences in (5) highlight both of these readings.

- (5)     a. The river widened two months ago, and this month it widened again. (*rep.*)
- b. The river narrowed last month, but this month it widened again. (*non-rep.*)

To demonstrate more precisely the nature of the two readings, consider the following set of situations.

Table 1.

	Sit. 1	Sit. 2	Sit. 3
<b>April 1<sup>st</sup></b>	12m	10m	10m
<b>May 1<sup>st</sup></b>	12m	11m	12m
<b>June 1<sup>st</sup></b>	10m	11m	10m
<b>July 1<sup>st</sup></b>	12m	12m	12m

In situation 1, the river narrows between May 1<sup>st</sup> and June 1<sup>st</sup>, and widens between June 1<sup>st</sup> and July 1<sup>st</sup>; in such a situation the non-repetitive, but not the repetitive, reading is true. In situation 2, the river widens between May 1<sup>st</sup> and June 1<sup>st</sup>, keeps a constant width for the month of June, and then widens between June 1<sup>st</sup> and July 1<sup>st</sup>; in such a situation, only the repetitive reading is true. We thus see that the two readings have distinct truth-conditions. Note that we can, however, have a situation in which both readings are true; situation 3 is such a case.

In general, we find a similar pattern of non-entailing readings for all sentences containing an atelic DA predicate and *again*; for more discussion on telicity and DA predicates, see [6], [7]. As §2 will demonstrate, the lack of entailment between readings in sentences like (4) shows clearly that the source of the ambiguity for such sentences cannot be explained in terms of the relative scope of *again* and a BECOME operator. In §3 it will be argued that the correct decomposition of DA predicates does not contain a BECOME operator, but instead contains a comparative structure. The ambiguity found in (4) will then be accounted for in terms of the scope *again* takes with respect to the comparative Degree Phrase in the decomposed predicate.

## 2 BECOME and *again*

In what follows, a semantics relativized to time intervals is assumed [2], [3].

- (6) A *time interval* is a subset  $i$  of a dense linear order  $T$  of moments  $t_n$  such that  $\forall t_1, t_3 \in i$  where  $t_1 < t_3$ , if  $t_1 < t_2 < t_3$ , then  $t_2 \in i$  (from Bennett & Partee [2])

Only closed time intervals are assumed below; note that it is possible for an interval to contain only one moment. Intervals are ordered as follows:

- (7)  $i < j$  iff for all  $t$  in  $i$  and all  $t'$  in  $j$ ,  $t < t'$  (from Bennett & Partee [2])

The BECOME-*again* analysis of *again*-ambiguities requires that predicates like the verb *open* be decomposed into BECOME and a small-clause containing a stative predicate, which denotes a stative property of intervals. Stative properties are defined as follows.

- (8)  $P$  is a *stative property of intervals* only if  
 i.  $P(i)$  can be true of a single-moment interval  
 ii.  $P(i)$  is true of an interval  $i$  containing  $n > 1$  moments only if  $\forall i' \subseteq i$ ,  $\phi(i')$  is true (from Dowty [3])

The following is an example of a stative property:

- (9)  $[\lambda i: \parallel [\text{the door is open}_{\text{ADJ}}] \parallel^i] = [\lambda i: \forall t \in i, \text{the door is open at } t]$

Also assumed here is the standard meaning for BECOME from Dowty [3].

- (10)  $\parallel \text{BECOME} \parallel^{\text{g.i}}(P)$  is defined only if  $\exists i' \mid P(i') = 1$  where defined,  
 $\parallel \text{BECOME} \parallel^{\text{g.i}}(P) = 1$  iff  $P(\text{beg}(i)) = 0$  &  $P(\text{end}(i)) = 1$

Finally, the denotation assumed for *again* is based on von Stechow [9]. *Again* introduces presuppositional content in the form of a definedness condition.

- (11)  $\|again\|^{g,i}(P)$  is defined only if  
 (i)  $P(i)$  is defined  
 (ii)  $\exists g, h . g < h \ \& \ end(h) \leq beg(i) \mid P(g) = 1 \ \& \ P(h) = 0$

where defined:

$$\|again\|^{g,i}(P) = 1 \text{ iff } P(i) = 1$$

The definition given above for *again* differs from the standard one in that it allows  $end(h) \leq beg(i)$ , rather than requiring  $h < i$ . More will be said on this below.

A simple example demonstrates how *again* introduces presuppositional content into the truth-conditions of a sentence.

- (12)  $\| [again \ [the \ door \ is \ open]] \|^{g,i}$  is defined only if  
 $\exists g, h . g < h \ \& \ end(h) \leq beg(i) \ \& \$   
 $\forall t \in g, \text{ the door is open at } t$   
 $\exists t \in h, \text{ the door is not open at } t$

where defined, is true iff

$$\forall t \in i, \text{ the door is open at } t$$

Under these assumptions, the sentence *The door is open again* asserts that the door is open, and presupposes both that it was open and then closed prior to its current state of being open.

It can be demonstrated that the BECOME-*again* analysis predicts an entailment between readings, no matter what stative-property-denoting small clause is in the scope of BECOME. The claim that we prove is the following.

- (13)  $\forall S, \forall i$ , if  $[\lambda i : \|S\|^i]$  = a stative property, then  
 $\| [again \ [BECOME \ [S]]] \|^{g,i} = 1 \rightarrow \| [BECOME \ [again \ [S]]] \|^{g,i} = 1$

We can prove this claim by showing that any interval index for which the repetitive reading of an arbitrary *again*-sentence is true is also one for which the non-repetitive reading of that sentence is true. The following is a proof of (13).

Let  $a$  be an arb. chosen interval and  $P$  an arb. chosen stative predicate such that  $AGAIN(BEC(P))(a) = 1$ . Given this assumption, we know that intervals must exist that satisfy the definedness condition introduced by *again*. Thus, we know  $\exists b, c$  such that

- (i)  $c < b \ \& \ end(b) \leq beg(a)$   
 (ii)  $\neg P(beg(c)) \ \& \ P(end(c))$  (i.e.  $BEC(P)(c)$ )  
 (iii)  $P(beg(b)) \vee \neg P(end(b))$  (i.e.  $\neg BEC(P)(b)$ )  
 (iv)  $\neg P(beg(a)) \ \& \ P(end(a))$  (i.e.  $BEC(P)(a)$ )

From the existence of the intervals  $a, b$  and  $c$  we can then infer the existence of intervals  $d$  and  $e$  that satisfy the non-repetitive presupposition (note that it is important for

$P$  to be a stative predicate, since we rely on the fact that it can be true of single moment intervals when defining the intervals  $d$  and  $e$  below):

- (v) Let  $d = \text{end}(c)$ , and let  $e = \text{beg}(a)$ . Then,
- (vi)  $d < e \ \& \ \text{end}(e) \leq \text{beg}(a)$
- (vii)  $P(d) \ \& \ \neg P(e)$  (from ii, iv, v)

From steps (iv), (vi) and (vii) it follows that  $\text{BEC}(\text{AGAIN}(P))(a) = 1$ . The above proof thus shows that any interval index for which the repetitive reading of an *again*-sentence is true is also one for which the non-repetitive reading is true. This is the case regardless of what stative predicate is in the scope of BECOME; hence, we can say that the fact that a repetitive reading entails a non-repetitive one is a direct consequence of the BECOME-*again* analysis.

The revision to *again* mentioned above is what allows for the proof to go through. However, it is important to stress that the main argument does not crucially depend on this revision. First of all, the revision does not change the truth-conditions of *again*-sentences in any noticeable way. Second, if we adopt the standard definition of *again* rather than the revised one, the repetitive reading of (1) will not logically entail the non-repetitive reading, but it will still *practically* entail it. The repetitive reading of (1) asserts that the door became open; thus, for the reading to be true the door must thus be closed at the beginning of the topic interval. The repetitive reading presupposes (i) that the door became open before the topic interval, and also (ii) that between these two openings it did not become open. However, the negation of  $\text{BEC}(P)(i)$  is  $P(\text{beg}(i)) \vee \neg P(\text{end}(i))$ ; it thus does not follow from the fact that something did *not* become open that thing became *not* open. With both versions of *again*, the repetitive reading is predicted to be true in a situation where the door did not actually close until the very beginning of the topic interval, i.e. for a situation in which the door was only fully closed for a single moment. Thus, taking the standard definition of *again* rather than the revised one, the entailment will fail only in a situation in which the door is closed for precisely one moment; in such a case the repetitive reading, but not the non-repetitive one, will hold. Since such situations do not play any role in what follows, the revised version of *again* will be adopted for the remainder of the discussion.

### 3 Degree Achievements and *again*

As we saw above, there is no entailment between the two readings of *The river widened again*; the BECOME-*again* analysis thus cannot apply to this sentence. Von Stechow [9] assumes that the decomposition of a sentence like (4) does contain a BECOME operator, along with a comparative structure. While he derives the correct presupposition for the non-repetitive reading (i.e. a reading which only presupposes a previous narrowing), he derives the incorrect presupposition for the repetitive reading; his analysis predicts that the repetitive reading of a sentence like (4) can only be uttered truthfully in a situation that includes both a previous widening and a narrowing. His account thus predicts that (4) cannot be uttered truthfully in a situation like situation 2 in table 1; it also predicts that a sentence like (4) demonstrates the same kind of entailment as (1). Both of these results are intuitively incorrect.

The account argued for here follows von Stechow [9] in assuming that DA predicates are decomposed into comparative structures, but holds that this decomposition does not contain BECOME at all. The proposed structure is shown in (14).

- (14) The river widened.  
at END [the river is [more than [at BEG it is *wh* wide]] wide]

The assumptions regarding comparatives adopted here are based on Heim [5], with a maximality semantics for *more/-er* and an ‘at least’ semantics for gradable adjectives.

- (15)  $\| \text{more} \|^{g,i} = [\lambda f_{\langle d \rangle} \lambda g_{\langle d \rangle} : \max\{d \mid g(d) = 1\} > \max\{d \mid f(d) = 1\}]$   
(to be slightly amended below)

- (16)  $\| \text{wide} \|^{g,i} = [\lambda d. \lambda x. \forall t \in i, x \text{ is at least } d \text{ wide at } t]$

The structure in (14) also contains two sentential operators BEG and END, which shift the interval of evaluation to, respectively, the initial and final moment of the index interval.

- (17) a.  $\| \text{at BEG} \|^{g,i}(P) = 1$  iff  $P(\text{beg}(i)) = 1$   
b.  $\| \text{at END} \|^{g,i}(P) = 1$  iff  $P(\text{end}(i)) = 1$

The structure in (14) is uninterpretable as is, since  $\| \text{more} \|$  requires two predicates of degrees as input. However, following Heim [5], if we assume that a comparative DegP – like an object quantifier – raises for interpretation, the structure becomes interpretable (also assuming null-operator movement in the *than*-clause). The interpretable structure is shown in (18), along with the derived truth-conditions.

- (18) more than [*wh* 2 at BEG it is  $d_2$  wide] [1 at END the river is  $d_1$  wide]

$$\| (18) \|^{g,i} = 1 \text{ iff } \max\{d \mid \text{river is } d\text{-wide at } \text{end}(i)\} > \max\{d \mid \text{river is } d\text{-wide at } \text{beg}(i)\}$$

Given this analysis, the sentence *The river widened* can be paraphrased as ‘the river is wider at the end of the interval than at the beginning of the interval’.

Heim [5] proposes that certain ambiguities can be explained by allowing a comparative DegP to scope above or below certain elements; the elements she considers are the intensional verbs *require* and *allow*. The ambiguity displayed in a sentence like (4) can be explained in a similar fashion, with *again* being the relevant element which DegP can scope over. The pre-LF structure for (4) is shown below.

- (19) *The river widened again*  
before LF movement:  
again [at END the river is [more than at BEG it is *wh* wide] wide]

The DegP in (19), like that in (14), must move for interpretation. However, there are now two possible movement sites for DegP to move to: above *again*, or below. If

DegP moves below *again*, the repetitive reading of (4) is derived; if it moves above *again*, the non-repetitive reading is derived. The repetitive reading is shown below.

- (20) *repetitive reading*  
again [more than [wh 2 at BEG it is  $d_2$  wide] [1 at END the river is  $d_1$  wide]]

$\| (20) \|^{g,i}$  is defined only if:

$\exists g, h . g < h \ \& \ end(h) \leq beg(i) \ \& \\
max\{d : \text{river is } d\text{-wide at } end(g)\} > max\{d : \text{river is } d\text{-wide at } beg(g)\} \\
max\{d : \text{river is } d\text{-wide at } end(h)\} \leq max\{d : \text{river is } d\text{-wide at } beg(h)\}$

Where defined, is true iff

$max\{d : \text{river is } d\text{-wide at } end(i)\} > max\{d : \text{river is } d\text{-wide at } beg(i)\}$

The truth-conditions derived for (20) assert that the river widened over the topic interval  $i$ , and presuppose only that the river also widened at some time  $g$  prior to  $i$ . The presupposition is silent as to whether the river narrowed or stayed at the same width during the interval  $h$  between  $g$  and  $i$ . This is the desired result for the repetitive reading, as it allows the sentence to be true in both situation 2 and situation 3 in table 1.

We turn now to the non-repetitive reading of (4), where the DegP moves above *again*.

- (21) *non-repetitive reading*  
more than [wh 2 at BEG it is  $d_2$  wide] [1 again at END the river is  $d_1$  wide]]

Roughly, this reading can be paraphrased ‘at the end of  $i$  the river is again wider than its width at the beginning of  $i$ ’. Notice that, in the non-repetitive LF, *again* scopes over a clause containing an unbound variable of degrees, i.e. over the trace left by DegP movement; *again* thus introduces its definedness condition over the clause in the DegP only. Assuming predicate abstraction limits input degrees to ones that satisfy the presupposition (see Heim & Kratzer [4] p.125), the denotation for the lambda-abstracted function is as follows.

- (22)  $\| 1 \text{ again [at END [river is } d_1 \text{ wide]]} \|^{g,i}$  is defined only for degrees  $d$  such that  
 $\exists g, h . g < h \ \& \ end(h) \leq beg(i) \ \& \\
(i) \text{ the river is } d\text{-wide at } end(g) \\
(ii) \text{ the river is not } d\text{-wide at } end(h).$

where defined, is true of a degree  $d$  only if  
the river is  $d$ -wide at  $end(i)$

This function will only have a non-empty domain if the river narrowed sometime prior to the beginning of the topic interval  $i$ , as can be deduced from conditions (i) and (ii) in (22). To see examples how this follows, consider again the following situations.

Table 2.

	Sit. 1	Sit. 2	Sit. 3	Sit. 4
<b>April 1<sup>st</sup></b>	12m	10m	10m	12m
<b>May 1<sup>st</sup></b>	12m	11m	12m	12m
<b>June 1<sup>st</sup></b>	10m	11m	10m	10m
<b>July 1<sup>st</sup></b>	12m	12m	12m	10m

Let  $g$  be the interval between April 1<sup>st</sup> and May 1<sup>st</sup>,  $h$  be the interval between May 1<sup>st</sup> and June 1<sup>st</sup>, and  $i$  be the interval between June 1<sup>st</sup> and July 1<sup>st</sup>. In situations 1, 3 and 4, the function in (22) will be defined for all degrees in the half-open interval (10m-12m]; in situation 2 it will not be defined for any degrees. In situations 1 and 3, the function will be true of all degrees for which it is defined. In situation 4, it will be not be true of any degrees for which it is defined.

The situations in which the domain of the function in (22) is non-empty (situations 1, 3 and 4) thus match those situations in which the presupposition of the non-repetitive reading is intuitively satisfied. In order to derive the correct presupposition for the entire sentence (i.e. in order to have the presupposition in the DegP project), we need to assume that the comparative morpheme has a definedness condition which requires that its two input <dt> functions are also defined. This condition is shown below.

(23)  $\ll more \ll (f)(g)$  is defined iff  $\exists d \mid f(d)$  is defined &  $\exists d \mid g(d)$  is defined

Note that this condition seems to be independently needed, as comparative sentences appear in general to allow for presupposition projection in both the matrix and the DegP clause. For example,

(24) My boat is longer than your boat.  
           *presupposes*  
       I have a boat & you have a boat

Assuming the above definedness condition for *more*, the truth-conditions for the non-repetitive reading come out as follows:

(25)  $\ll (21) \ll^{g,i}$   
       is defined only if:  
        $\exists d, \exists g, h . g < h \ \& \ end(h) \leq beg(i) \ \&$   
       the river is  $d$ -wide at  $end(g)$  &  
       the river is not  $d$ -wide at  $end(h)$

Where defined, is true iff

$max\{d : \text{river is } d\text{-wide at } end(i)\} > max\{d : \text{river is } d\text{-wide at } beg(i)\}$

These truth-conditions contain the presupposition only that the river narrowed sometime before the beginning of  $i$ . As such, the sentence is predicted to be true in situations 1

and 3, which correctly matches speaker intuitions. The DegP scope account thus correctly derives a repetitive and a non-repetitive reading for (4), neither of which entails the other.

## 4 Conclusion

The DegP scope account presented above derives the correct truth-conditions for both readings of the sentence in (4), which can be seen as a general case of a sentence containing a Degree Achievement predicate and *again*. A number of conclusions follow from the above discussion. First of all, it is clear that not all *again*-ambiguities can be explained by the BECOME-*again* scope analysis, since not all ambiguities demonstrate the entailment between readings that such an analysis predicts.

Second, the *again*-ambiguity found in sentences with DA predicates like *widen* can be explained in terms of the position a comparative DegP takes with respect to *again*, if we assume that DA predicates are decomposed into the comparative structures proposed in §3. This account follows Heim [5], where it is proposed that DegP can scope above certain elements. If the current proposal is on the right track, *again* should be added to this list of elements.

Finally, the fact that DA predicates give rise to a different type of *again*-ambiguity than result-state predicates provides strong evidence that the two types of predicates have different internal structure. In particular, the specific ambiguity found in sentences with DA predicates demonstrates that such predicates cannot contain a BECOME operator.

While the above discussion has shown it to be quite plausible that *again*-ambiguities have different sources in different sentences, it is left to future work to determine whether a more general account of *again*-ambiguities can be provided which can apply to all of the various cases.

## References

1. Beck, S., Johnson, K.: Double Objects Again. *Linguistic Inquiry* 35, 97–124 (2004)
2. Bennett, M., Partee, B.: Toward the Logic of Tense and Aspect in English. Technical report, Indiana University Linguistics Club, Bloomington, Indiana (1978)
3. Dowty, D.: Word Meaning and Montague Grammar. Reidel, Dordrecht (1979)
4. Heim, I., Kratzer, A.: Semantics in Generative Grammar. Blackwell, Malden (1998)
5. Heim, I.: Degree Operators and Scope. *Semantics and Linguistic Theory* 10, 40–64 (2000)
6. Kearns, K.: Telic Senses of Deadjectival Verbs. *Lingua* 117, 26–66 (2007)
7. Kennedy, C., Levin, B.: Measure of Change: The Adjectival Core of Degree Achievements. In: McNally, L., Kennedy, C. (eds.) *Adjectives and Adverbs: Syntax, Semantics and Discourse*, pp. 156–182. Oxford University Press, Oxford (2008)
8. McCawley, J.D.: Syntactic and Logical Arguments for Semantic Structures. In: Fujimura, O. (ed.) *Three Dimensions in Linguistic Theory*, pp. 259–376. TEC Corp., Tokyo (1973)
9. Stechow, A.: The Different Readings of Wieder ‘Again’: A Structural Account. *Journal of Semantics* 13, 87–138 (1996)

# Equatives, Measure Phrases and NPIs

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**Abstract.** Standard semantic accounts of the equative ascribe it an ‘at least’ meaning, deriving an ‘exactly’ reading when necessary via scalar implicature. I argue for a particular formulation of this scalar implicature account which considers that (i) equatives license NPIs in their internal arguments, and (ii) equatives whose internal arguments are measure phrases (MPs) are, in contrast to clausal equatives, ambiguous between ‘at most’ and ‘exactly’ interpretations. The analysis employs particular assumptions about MPs, scalar implicature and the notion of set complementation to enable ‘at least’ readings to be sensitive to the direction of a scale, thereby becoming ‘at most’ readings in certain constructions.

## 1 Introduction

I begin by discussing some data that present a challenge to standard accounts of the equative: MP equatives, whose internal arguments are measure phrases (MPs). Standard equatives are ambiguous between an ‘at least’ and ‘exactly’ interpretation; for many speakers of English, MP equatives are instead ambiguous between an ‘at most’ and ‘exactly’ interpretation. Section 2 briefly presents standard accounts of the semantics of the equative and then discusses the fact that NPIs are licensed in the internal argument of equatives. This observation plays a significant role in the analysis, which is presented at the end of Section 2. Section 3 expands the analysis to a treatment of equative modifiers and discusses extensions of the analysis to the semantics of the comparative.

### 1.1 Equatives and MPs

It’s been observed that equatives are ambiguous. These two possible meanings are reflected in the two felicitous responses to (A) in (1). In (B), John’s being taller than Sue is incompatible with (A) (on the ‘exactly’ reading); in (B’), John’s being taller than Sue is compatible with (A) (on the ‘at least’ reading).

- (1) (A) John is as tall as Sue is.

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- (B) No, he's taller than Sue is.    (B') Yes, in fact he's taller than Sue is.

Equatives with measure phrases or numerals in their internal argument ('MP equatives') present a challenge to this account. (I follow [Pancheva \(2006\)](#) in using the term 'clausal' to describe comparatives and equatives which have a potential clausal source and 'phrasal' to describe those with no potential clausal source, like MP equatives and e.g. *John is as tall as himself* <sup>\*(is)</sup>. Following [Hankamer \(1973\)](#), I will use the terms **target** and **correlate** to refer to the subordinate and matrix material in comparatives, respectively.)

- (2) a. (I think) John biked as far as 500 miles yesterday.  
b. The DOW dropped as much as 150 points yesterday.  
c. Sometimes Mars is as far as 235 million miles from Earth.  
d. The waves reached as high as 6ft before nightfall.  
e. GM plans on laying off as many as 5,000 employees.

The first thing to note about MP equatives is that their distribution is restricted relative to clausal equatives. Roughly speaking, they are licensed when the value of the correlate is indeterminate.<sup>1</sup> This is manifested in different ways: the measure could be the imprecise either due to speaker ignorance (2a) or because the measure need not be precise in the context. Alternatively, the correlate could denote a range, either of times (2c), individuals (2d) or worlds (2e). This restriction on the distribution of MP equatives seems related to their being more marked than their (roughly synonymous) MP construction counterparts like *John biked 500 miles yesterday*.

Nouwen (2008, *to appear*), discussing what he calls ‘Class B’ comparative quantifiers (*at most 6ft*, *up to 6ft*), provides a different perspective on this indeterminacy requirement: he suggests that such constructions involve quantification over ranges (epistemic ranges in the case of examples like (2a) and (2b)).

When clausal equatives involve ranges, as in (3), their ‘at least’ interpretation is manifested as follows.

- (3)  $\frac{\text{The boys are as tall as Sue (is).}}{\text{correlate}} \quad \frac{}{\text{target}}$

In a situation in which the boys' heights range from 4ft to 6ft, (3) is true if Sue is 4ft tall or shorter (and false if Sue is 4½ft tall). This means that, for clausal equatives with range-denoting correlates, the heights of the boys denoted by the

<sup>1</sup> There are a few instances of MP equatives being used in the absence of indeterminacy, however. An article on Vitamin D states “A study from the University of California, San Diego recommended doses as high as 2,000 international units a day – far more than the 200 to 600 governments recommend.” (<http://www.theglobeandmail.com/news/national/d-is-the-new-c-sunshine-vitamin-is-suddenly-hot/article1423352>) In this case, the MP equative does not seem to be indeterminate, and is used in a context in which the recommendation was exactly 2,000 units. These MP equatives therefore seem to be more like DP equatives, which I discuss briefly in Section 3.

correlate must be at least as high as the value denoted by the target. Clausal equatives therefore receive an **at least** interpretation.

When MP equatives have range-denoting correlates, they are interpreted differently. One article reports “The European Union will impose import duties as high as 20 percent on some leather shoes from China and Vietnam” and goes on to explain that “it will impose rising tariffs over six months, to a maximum of almost 20 percent of their value”<sup>2</sup> This example demonstrates that, in MP equatives, when the correlate expresses a range the maximum value of the correlate range must be at most as high as the value denoted by the target. MP equatives therefore can receive an **at most** interpretation.

This seems to be the case for MP equatives generally, like those in (2). (2e), for instance, is reported by speakers of the relevant dialect to be true if GM plans on laying off 4,500 employees, but not if GM plans on laying off 5,500 employees. (In other words: (2e) is true if, given all of the possible worlds consistent with GM’s plans, the world in which GM lays off the highest number of employees (i.e. the maximum in the range of ordered worlds) is a world in which they hire 5,000 employees or fewer than 5,000 employees.).

In this paper, I argue that there is a natural account for the apparent fact that MP equatives receive an interpretation opposite that of clausal equatives. It draws on the observations that MPs are themselves scalar, that the target of equatives is downward-entailing, and that scalar implicatures are interpreted differently in downward-entailing contexts. Before presenting the analysis in Section 2, I’ll discuss some background assumptions.

## 1.2 Background Assumptions

I follow many others in assuming that gradable adjectives denote relations between individuals and degrees.

$$(4) \quad \llbracket \text{tall} \rrbracket = \lambda x \lambda d. \text{tall}(x, d)$$

I additionally assume that positive and negative antonyms (like *tall* and *short*) differ in their ordering, which is observable in their behavior in comparatives (Seuren, 1984; von Stechow, 1984, a.o.). Positive antonym scales are downward-monotonic, with open lower bounds of zero and closed upper bounds (5a). Negative antonym scales like *short* are upward-monotonic, with closed lower bounds and closed upper bounds of infinity (5b).

(5) Context: John is 5ft tall.

$$\text{a. } \lambda d. \text{tall}(\text{john}, d) = (0, 5] \quad \text{b. } \lambda d. \text{short}(\text{john}, d) = [5, \infty]$$

Following Bresnan (1973), I assume that comparatives and equatives with overt tense morphology are clauses that have undergone elision along the lines of (6).

$$(6) \quad \text{John is taller than } [\text{CP OP}_d \text{ Sue is } \cancel{d} \text{tall}]$$

<sup>2</sup> Sentences from “EU to impose 20% duties on shoes from China”,  
[http://www.chinadaily.com.cn/english/doc/2006-02/21/content\\_522184.htm](http://www.chinadaily.com.cn/english/doc/2006-02/21/content_522184.htm)

I adopt the ‘A-not-A’ account of the comparative in (7) (McConnell-Ginet, 1973; Kamp, 1975; Hoeksema, 1983; Seuren, 1984; Schwarzschild, 2008, a.o.). An important consideration of this theory is the observation that NPIs are licensed in the targets of (clausal) comparatives (8).

$$(7) \quad \lambda D' \lambda D \exists d [D(d) \wedge \neg D'(d)]$$

- (8) a. He would rather lose his honor than so much as a dime.  
b. She is happier now than ever before.

This is predicted by the A-not-A analysis given the assumption that the downward-entailingness of degree quantifiers is calculated in terms of sets of degrees, rather than sets of individuals (cf. Seuren, 1984; von Stechow, 1984; Hoeksema, 1983, 1984; Heim, 2003).

- (9) Context: Mary is 6ft tall, John is 5ft tall, Sue is 4ft tall.  
a. Mary is taller than John.  $\rightarrow$  Mary is taller than Sue.  
b. Mary is taller than Sue.  $\nrightarrow$  Mary is taller than John.

I have had to omit the discussion of this calculation and its consequences due to space restrictions, but I hope to return to address the topic in future papers.

To sum up: NPIs appear to be licensed in the targets of comparatives, and entailment patterns between supersets and subsets of degrees (9) confirm that the targets of comparatives are downward-entailing. This fact is captured by the ‘A-not-A’ analysis in (7) because it characterizes the target of comparatives as downward-entailing. Finally, I would like to point out Hoeksema’s (1983) observation that the definition in (8b) is equivalent to the one in (10) that invokes set complements (written as  $\overline{D}$ ).

$$(10) \quad \llbracket \text{-er} \rrbracket = \lambda D' \lambda D \lambda d. d \in D \wedge d \in \overline{D'}$$

(10) additionally differs from (8) in not existentially binding the differential degree  $d$ . This allows for further modification by e.g. *much* and *3 inches* in *John is much/3 inches taller than Sue*. I assume that, in the absence of a differential modifier, the differential argument  $d$  is bound via existential closure.

## 2 Equatives

Previous analyses of the equative have exploited the fact that to be exactly as tall as Sue is to be at least as tall as Sue, and therefore that the ‘exactly’ reading entails the ‘at least’ reading (but not vice-versa). The equative has for this reason been considered to be another type of scalar-implicature phenomenon (on a Horn scale with the comparative), assigning the weak ‘at least’ reading to the semantics of the equative and deriving the former from the latter via scalar implicature where context allows (Horn, 1972; Klein, 1980; Chierchia, 2004). This suggests an analysis in which the maximum degree denoted by the correlate (John’s height in (1)) must be greater than or equal to the maximum degree denoted by the target (Sue’s height in (1)).

$$(11) \quad \llbracket \text{as} \rrbracket = \lambda D' \lambda D. \text{MAX}(D) \geq \text{MAX}(D')$$

This definition will erroneously predict that MP equatives, too, have an ‘at least’ interpretation, not an ‘at most’ interpretation. (This is true regardless of whether we interpret the MP as denoting a single degree  $n$  or an upward-monotonic scale with a lower bound of  $n$ , which I argue for in the next section.)

I present an alternative analysis below, in Section 2.2. Before doing so, I introduce some independent observations and assumptions which motivate it.

## 2.1 MPs and Scalar Implicatures

The analysis below relies on the observation that MPs (and numerals) are themselves scalar, thus accounting for their different interpretation in equatives. The traditional SI account of sentences like *John has 3 children* assigns the numeral an ‘at least’ semantics ( $\geq 3$ ), deriving the ‘exactly’ interpretation via scalar implicature, where appropriate (contra Geurts, 2006). This means that the denotation of an MP target (in a positive-antonym equative, like those in (2)) is an upward-monotonic set of degrees, with a lower bound of  $d$  (for a  $d$ -denoting numeral) and an upper bound of  $\infty$ .

In a context in which Sue is 5ft tall, the target of the equative *John is as tall as Sue (is)* denotes the degrees to which Sue is tall (12a), which is downward-monotonic. The target of the equative *John could be as tall as 5ft*, on the other hand, denotes the degrees greater than or equal to 5ft (12b) (an upward-monotonic scale).

$$(12) \quad \begin{array}{ll} \text{a.} & \llbracket \text{Op}_d \text{ Sue is } d \text{ tall} \rrbracket = \lambda d. \text{tall}(\text{sue}, d) = (0, 5] \\ \text{b.} & \llbracket 5\text{ft} \rrbracket = \lambda d. d \geq 5\text{ft} = [5, \infty] \end{array}$$

This particular characterization of MPs is significant in light of independent observations tying it to SIs in downward-entailing contexts. Chierchia (2004) argues that SIs (a) can be calculated sub-sententially, and (b) are calculated differently in downward-entailing contexts. I’ll illustrate this point as Chierchia does, independently of equatives and MPs. *Or* is typically characterized as scalar, ambiguous between a weak reading ( $A$  or  $B$  or both) and a strong reading ( $A$  or  $B$  but not both). The strong reading then comes about, where pragmatically possible, as the result of scalar implicature (13a). In downward-entailing environments, though, this SI is affectively cancelled; (15b) cannot be used to negate the claim that Sue didn’t meet both Hugo and Theo (and is therefore incompatible with Sue having met both). Chierchia’s explanation is that SIs are calculated in terms of informativity, and what counts as the most informative in upward-entailing contexts is actually the least informative in downward-entailing contexts (and vice-versa).

$$(13) \quad \begin{array}{ll} \text{a.} & \text{Sue met Hugo or Theo.} \\ \text{b.} & \text{Sue didn’t meet Hugo or Theo.} \end{array}$$

Importantly, it looks as though the targets of equatives are downward-entailing environments as well; they license NPIs.

- (14) a. He would just as much lose his honor as he would a dime.  
 b. She is as happy now as ever before.

Extending Chierchia's observation to equatives therefore has the consequence that targets of MP equatives are always interpreted weakly. The consequence of this is that, while the targets of (positive-antonym) clausal equatives denote downward-monotonic degree sets, the targets of (positive-antonym) MP equatives denote upward-monotonic degree sets.

## 2.2 A More Sensitive Semantics

The crux of the analysis that follows is a reformulation of the equative morpheme which takes into account that equative targets are downward-entailing and that the difference between the targets of clausal equatives and the targets of MP equatives is one of monotonicity.

The definition of the equative morpheme below draws on the set-complement reformulation of the comparative in (10).<sup>3</sup>

- (15)  $\llbracket \text{as} \rrbracket = \lambda D' \lambda D [\text{MAX}(D) \in \widehat{D'}]$ , where  
 $\widehat{D} =_{\text{def}}$  the smallest  $D'$  such that  $\overline{D} \subseteq D'$  and  $D'$  is a closed set.

This definition invokes the notion of a 'closure of the complement', the smallest superset of the complement with closed bounds.<sup>4</sup> It is downward-entailing in its target ( $D'$ ), correctly predicting the licensing of NPIs.

- (16) Context: Mary is 6ft tall, John is 5ft tall, Sue is 4ft tall.  
 Mary is as tall as John.  $\rightarrow$  Mary is as tall as Sue. *is true iff*  
 $\text{MAX}((0,6]) \in \widehat{(0,5]} \rightarrow \text{MAX}((0,6]) \in \widehat{(0,4]} \quad \text{is true iff}$   
 $6 \in [5, \infty] \rightarrow 6 \in [4, \infty] \checkmark$

Positive-antonym MP equatives differ from positive-antonym clausal equatives in that their target is upward-monotonic. The definition in (15) allows the 'greater than' relation we associate with the 'at least' reading of the equative to be sensitive to the ordering on the target scale; it affectively employs a different relation ('at least', 'at most') based on the direction of the target scale.

- (17) John is as tall as Sue. (John's height = 5ft; Sue's height = 5ft; **true**)  
 $\text{MAX}((0,5]) \in \widehat{(0,5]} \rightsquigarrow 5 \in [5, \infty] \checkmark$   
 (18) John is as tall as Sue. (John's height = 6ft; Sue's height = 5ft; **true**)  
 $\text{MAX}((0,6]) \in \widehat{(0,5]} \rightsquigarrow 6 \in [5, \infty] \checkmark$

<sup>3</sup> The definition in (15) is a simplified version of  $= \lambda D' \lambda D \lambda d [d = \text{MAX}(D) \wedge d \in \widehat{D'}]$ , which is required for an account of modified equatives (see §4).

<sup>4</sup> Direct application of (15) will result in some scales having a closed lower bound of zero. This is formally unattractive but actually harmless, assuming that it is infelicitous to predicate a gradable property of an individual if that individual doesn't exhibit that property at all (cf. #*That couch is intelligent*). We could alternatively reformulate the definition of a closure of a complement to omit this possibility.

- (19) John is as tall as Sue. (John's height = 5ft; Sue's height = 6ft; **false**)  
 $\text{MAX}([0, 5]) \in \widehat{[0, 6]} \rightsquigarrow 5 \in [6, \infty] \text{ ✗}$
- (20) The waves reached as high as 6ft. (waves' height = 6ft; **true**)  
 $\text{MAX}([0, 6]) \in \widehat{[6, \infty]} \rightsquigarrow 6 \in [0, 6] \checkmark$
- (21) The waves reached as high as 6ft. (waves' height = 5ft; **true**)  
 $\text{MAX}([0, 5]) \in \widehat{[6, \infty]} \rightsquigarrow 5 \in [0, 6] \checkmark$
- (22) The waves reached as high as 6ft. (waves' height = 7ft; **false**)  
 $\text{MAX}([0, 6]) \in \widehat{[6, \infty]} \rightsquigarrow 7 \in [0, 6] \text{ ✗}$

(15) works just as well for negative-antonym equatives, whose clausal arguments are upward-monotonic (see (5b)). I assume a definition of the maximality operator in which it is sensitive to the direction of the scale (see Rett, 2008).

- (23) John is as short as Sue. (John's height = 5ft, Sue's height = 5ft; **true**)  
 $\text{MAX}([5, \infty]) \in \widehat{[5, \infty]} \rightsquigarrow 5 \in [0, 5] \checkmark$
- (24) John is as short as Sue. (John's height = 4ft, Sue's height = 5ft; **true**)  
 $\text{MAX}([4, \infty]) \in \widehat{[5, \infty]} \rightsquigarrow 4 \in [0, 5] \checkmark$
- (25) John is as short as Sue. (John's height = 5ft, Sue's height = 4ft; **false**)  
 $\text{MAX}([5, \infty]) \in \widehat{[4, \infty]} \rightsquigarrow 5 \in [0, 4] \text{ ✗}$

To extend the analysis to negative-antonym MP equatives (like *The temperature dropped as low as 2° Kelvin*), we must recall that the target also involves a negative antonym (e.g. 2° low, rather than 2° high). This is consistent with Bresnan's (and Kennedy's (1999)) assumptions about the syntax of comparatives and equatives ((26), cf. (6)).

- (26) John has fewer children than Sue.  
 -er ([Op'<sub>d</sub> Sue has *d'*-few children]) ([Op<sub>d</sub> John has *d*-few children])

MP targets of negative-antonym equatives are thus in fact downward-monotonic, which results in the correct truth conditions.

- (27) The temperature dropped as low as 2° Kelvin. (highest temp = 2°; **true**)  
 $\text{MAX}([2, \infty]) \in \widehat{[0, 2]} \rightsquigarrow 2 \in [2, \infty] \checkmark$
- (28) The temperature dropped as low as 2° Kelvin. (highest temp = 3°; **true**)  
 $\text{MAX}([3, \infty]) \in \widehat{[0, 2]} \rightsquigarrow 3 \in [2, \infty] \checkmark$
- (29) The temperature dropped as low as 2° Kelvin. (highest temp = 1°; **false**)  
 $\text{MAX}([1, \infty]) \in \widehat{[0, 2]} \rightsquigarrow 1 \in [2, \infty] \text{ ✗}$

### 3 Expansions and Extensions

*More about MP equatives.* It should be obvious that there is more empirical ground to cover before this analysis can be complete. Footnote 1 discusses MP

equatives that don't appear to involve ranges; these equatives seem to unambiguously have an 'exactly' interpretation, and be evaluative (in the sense of Rettl, 2008). Furthermore, there seems to be a fair amount of speaker variation with respect to grammaticality judgments and interpretation of MP equatives; it'd be nice to know more about the source of this variation. Finally, few languages allow MP equatives; it'd be nice to have a better understanding of the relevant cross-linguistic variation. I am in the process of conducting a cross-linguistic survey on the interpretations of equatives in an attempt to address these issues.

*Equative modifiers.* Importantly, this analysis calls for a semantics of superlative modifiers like *at least* and *at most* that are not sensitive to the direction of the scale: *at least* can modify MP equatives, forcing them to have an 'at least' interpretation (30a), and *at most* can modify clausal equatives, forcing them to have an 'at most' interpretation (30b).<sup>5</sup>

- (30) a. John biked at least as far as 500 miles yesterday.  
b. John is at most as tall as Sue (is).

I argue that such an analysis requires the assumption that pragmatic strengthening is applied to equatives before the equatives are modified. The modifiers therefore take strengthened, 'exactly' equative meanings as their arguments, and add a restricting clause based on an objective scale direction ( $\leq$  or  $\geq$ ).

*MP comparatives.* The assumptions made above about the denotation of MPs in DE contexts doesn't extend straightforwardly to comparatives given the definition in (10). In particular, feeding an upward-monotonic denotation of MPs into (10) erroneously predicts that all MP comparatives are true.

- (31) John is taller than 5ft. (John's height = 4ft; false)  
 $\exists d[d \in (0,4) \wedge d \in [5, \infty]] \rightsquigarrow \exists d[d \in (0,4) \wedge d \in (0,5)] \checkmark$

It seems that the incorrect truth conditions in (31) underscore the argument in Pancheva (2006) that comparative subordinators are meaningful and can differ semantically. Some languages employ different comparative subordinators for MP targets than they do for clausal targets (cf. Spanish *de lo que* DP versus *de* MP). One possible way of adopting Pancheva's analysis while holding fixed this particular characterization of MPs as denoting their weak meaning in DE contexts is to argue that the comparative morpheme *-er* is a simple quantifier over degrees, while clausal *than* is a function from a set to its complement (thus resulting in the NPI data above), and MP *than* is an identity function over degree sets.

- (32) a.  $\llbracket \text{-er} \rrbracket = \lambda D' \lambda D \lambda d. d \in D \wedge d \in D'$   
b.  $\llbracket \text{than}_{\text{clausal}} \rrbracket = \lambda D \lambda d. d \notin D$       b.  $\llbracket \text{than}_{\text{MP}} \rrbracket = \lambda D \lambda d. D(d)$

<sup>5</sup> 'At least' and 'at most' interpretations also differ in whether they compare the maximum degree of the minimal individual in the correlate range or the maximum degree of the maximal individual in the correlate range. I'm assuming for the present paper that this is a pragmatic effect, but there might be more to the difference.

Slavic languages provide independent evidence that MP targets of comparatives are treated differently from clausal targets of comparatives. ((33) is Pancheva's example from Russian, in which clausal comparatives are formed with the *wh*-phrase *čem*, and phrasal comparatives are formed with a covert subordinator.)

- (33) a. ??Ivan rostom bol'she čem dva metra.  
           Ivan in-height more what two meters  
       b. Ivan rostom bol'she dvux metrov.  
           Ivan in-height more [two meters]<sub>GEN</sub>  
           'Ivan measures in height more than two meters.'

This discussion of MPs in comparative and equative targets helps provide an explanation for why languages employ two different subordinators for clausal comparatives and MP comparatives: the two types of targets denote different types of scales (in relation to the scale denoted by the correlate). It's also compatible with the possibility that some languages disallow MP equatives entirely.

*DP equatives.* Some phrasal equatives have DP rather than MP targets.

- (34) a. John can reach as high as the ceiling (\*is).  
       b. This rubber band can stretch as wide as a house (\*is).

It appears as though these equatives, too, must be indeterminate, or a range of some sort ((35a), but this requirement comes in the absence of any obvious unmarked counterparts ((35b), cf. MP constructions).

- (35) a. ??John reached as high as the ceiling.  
       b. ??John can reach the ceiling's height.

It's not clear to me which of the three readings ('at least', 'at most', 'exactly') DP equatives have. ((34a), for instance, seems both compatible with John being capable of reaching lower than the ceiling's height and with John being capable of reaching higher than the ceiling. I suspect that the meaning of these DPs rely heavily on the contextual salience of the DP, not just the measure denoted by the DP. This point is made especially clear by DP equatives like *This train will take you as far as Berkeley*, which is intuitively false if the train will take you somewhere equidistant to Berkeley (but not to Berkeley itself).

*Conclusion.* Clausal equatives have a weak 'at least' interpretation while MP equatives have a weak 'at most' interpretation. I argue that these phenomena can be assimilated in a neo-Gricean SI framework if we characterize the weak meaning of the equative in a way that is sensitive to the scalar ordering of its internal argument. The account relies on independent observations that numerals (and therefore MPs) are themselves scalar, and that scalar implicature is calculated sub-sententially and differently in downward-entailing contexts (Chierchia, 2004). It has potential implications for studies of the comparative, as well as modifiers derived from degree quantifiers with MP targets which have independently been observed to display similar characteristics (Nouwen *to appear*).

## References

- Bresnan, J.: Syntax of comparative clause construction in English. *Linguistic Inquiry* 4, 275–344 (1973)
- Cartwright, H.: Amounts and measures of amounts. *No<sup>us</sup>* 9, 143–164 (1975)
- Chierchia, G.: Scalar implicatures, polarity phenomena and the syntax/pragmatic interface. In: Beletti (ed.) *Structures and Beyond*, Oxford (2004)
- Geurts, B.: Take five. In: Vogeleer, S., Tasmowski, L. (eds.) *Non-definiteness and plurality*, Benjamins, pp. 311–329 (2006)
- Hankamer, J.: Why there are two thans in English. *Chicago Linguistics Society* 9, 179–191 (1973)
- Heim, I.: On quantifiers and NPIs in comparative clauses. Ms., MIT, Cambridge (2003)
- Hoeksema, J.: Negative polarity and the comparative. *Natural language and linguistic theory* 1, 403–434 (1983)
- Hoeksema, J.: To be continued: the story of the comparative. *Journal of Semantics* 3, 93–107 (1984)
- Horn, L.: On the Semantic Properties of the Logical Operators in English. PhD Thesis, University of California, Los Angeles (1972)
- Kamp, H.: Two theories of adjectives. In: Keenan, E. (ed.) *Formal Semantics of Natural Language*, pp. 123–155. Cambridge University Press, Cambridge (1975)
- Kennedy, C.: *Projecting the Adjective*. Garland Press, New York (1999)
- Klein, E.: A semantics for positive and comparative adjectives. *Linguistics and Philosophy* 4, 1–45 (1980)
- McConnell-Ginet, S.: *Comparative Constructions in English: A Syntactic and Semantic Analysis*. PhD Thesis, University of Rochester (1973)
- Nerbonne, J.: Nominalized comparatives and generalized quantifiers. *Journal of Logic, Language and Information* 4, 273–300 (1995)
- Nouwen, R.: Two kinds of modified numerals. *Semantics and Pragmatics* (to appear)
- Pancheva, R.: Phrasal and clausal comparatives in Slavic. In: Lavine, J., Franks, S., Tasseva-Kurkchieva, M., Filip, H. (eds.) *Formal Approaches to Slavic Linguistics 14: The Princeton Meeting*, pp. 236–257 (2006)
- Rett, J.: Antonymy and evaluativity. In: Gibson, M., Friedman, T. (eds.) *Proceedings of SALT XVII*. CLC Publications (2008)
- Schwarzschild, R.: The grammar of measurement. In: Jackson, B. (ed.) *Proceedings of SALT XII* (2002)
- Schwarzschild, R.: The role of dimensions in the syntax of noun phrases. *Syntax* 9, 67–110 (2006)
- Schwarzschild, R.: The semantics of the comparative and other degree constructions. *Language and Linguistic Compass* 2(2), 308–331 (2008)
- Seuren, P.: The comparative revisited. *Journal of Semantics* 3, 109–141 (1984)
- von Stechow, A.: Comparing semantic theories of comparison. *Journal of Semantics* 3, 1–77 (1984)

# Squiggly Issues: Alternative Sets, Complex DPs, and Intensionality

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**Abstract.** In this paper, we investigate a number of long-standing issues in connection with (i) focus interpretation and its interrelation with complex definite descriptions, and (ii) the intensional properties of sentences with focus constituents. We revitalize the use of Root’s (1992)  $\sim$  operator, clarify its definition as an anaphoric operator, discuss the principles that govern its placement in logical forms and show how it can be successfully employed to replace the notion of Krifka’s (2006) focus phrases. Finally, we argue that a proper view of the intensional dimension of retrieving the antecedent sets required by the operator can account for problems relating to the intensionality of sentences with focus sensitive operators that are discussed by Beaver & Clark (2008).

**Keywords:** anaphora, Alternative Semantics, DP, DRT, focus interpretation, focus phrase, intensionality.

## 1 Introduction: Focus Semantic Values and Context Sets

According to Root (1985, 1992, 1996), focusing – the semantic reflex of an F feature assigned to some constituent X in logical form – leads to the creation of a focus semantic value  $\llbracket X \rrbracket^f$ . The FSV is simply the domain of objects of the same semantic type as the ordinary semantic value  $\llbracket X \rrbracket^o$  relative to some model. For instance, the FSV of the word  $[\text{THEodore}]_F$  is the domain of individuals  $D_e$ .

Note that, other than in the case of mathematical models, natural discourse does not enable us to exhaustively list all entities that belong to  $D_e$  since we are not omniscient. All we know is that if  $d$  is an individual then it is a member of  $D_e$ . We shall therefore consider focus semantic values to be (anonymous) characterizations rather than extensionally determined sets.

It is well-known since Root (1992) that FSVs are not as such suited to function with conventionally focus-sensitive particles<sup>1</sup>; first, they need to undergo contextual restriction. Consider the sequence in (1).

- (1) a. We have invited all siblings of your mom but, I noticed, we have really neglected your father’s relatives.  
b. So far, we have only invited  $[\text{uncle THEodore}]_F$ .

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<sup>1</sup> Beaver & Clark (2008: Chap. 3) distinguish conventional, free, and quasi-sensitivity.

$$(2) \quad \forall x[x \in C \wedge \text{invite}(\mathbf{we}, x) \rightarrow x = \mathbf{t}]$$

Using a standard semantics for *only* yields (2) as the reading for (1b). We obtain the wrong result if the quantificational domain  $C$  for *only* is set to  $D_e$  since this set also comprises Mom's invited siblings, which are then ruled out by (2). This goes against what is said in (1a). Therefore, in order to get the proper meaning for (1b),  $C$  must be restricted to a contextually available set, in this case "your father's relatives".

For this and a number of other focus-related purposes, Root (1992, 1996) defines, in addition to the focus feature  $F$ , a *focus interpretation operator*  $\sim$  (informally known as "squiggle operator"), which can in principle attach to arbitrary constituents. If  $X$  is some constituent,  $\llbracket X \rrbracket^o$  is the ordinary meaning of  $X$  and  $\llbracket X \rrbracket^f$  is the FSV, then  $\sim X$  triggers a presupposition such that a *context set*  $C$  containing a contrastive item  $y$  must be identified, with the properties given in (3).<sup>2</sup>

$$(3) \quad (i) C \subseteq \llbracket X \rrbracket^f \quad (ii) y \in C \quad (iii) y \neq \llbracket X \rrbracket^o$$

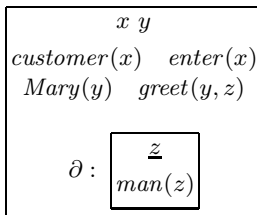


Fig. 1(a).

Preliminary DRS for (4)

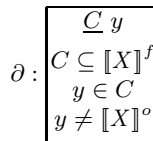


Fig. 1(b).

Presupposition triggered by  $\sim X$

In the following we would like to scrutinize the anaphoric nature of  $\sim$ . For this purpose we provide a translation of the constraints in (3) into DRT, which is geared to the treatment of presupposition and anaphora in the framework of van der Sandt (1992), Geurts (1999) and Kamp (2001). Definite descriptions like that in the second sentence of (4) are represented as in Fig. 1(a), where the "anaphoric" variable  $z$  is waiting to get bound to the previously mentioned customer  $x$ . In this vein, we formulate the  $\sim$  conditions from (3) as in Fig. 1(b).

$$(4) \quad \text{A customer entered. Mary greeted } \boxed{\text{the man}}.$$

<sup>2</sup> We ignore a fourth condition according to which  $\llbracket X \rrbracket^o \in C$ , since we think it is generally superfluous and sometimes even out of place. While it is unproblematic that the retrieved set  $C$  sometimes contains  $\llbracket X \rrbracket^o$  there are cases in which imposing this as a *constraint* is implausible; for instance, overtly contrastive focus.

<sup>3</sup> We ignore tense.

## 2 Squiggle Placement

A representation like the one in Fig. 1b – in particular the treatment of  $C$  as an anaphoric variable – clearly shows that the semantic type which the variables adopt is dependent on the attachment site of the squiggle operator. If  $\sim$  attaches to a DP then  $C$  must be a set of individuals. If it attaches to a VP then  $C$  is a set of properties or, preferably, a set of events or states. Seen in this light, it is surprising that Rooth (1992: 89) chooses to attach the  $\sim$  in (5) at VP level.

(5) Mary only  $\sim$ [ $_{VP}$  introduced  $BILL_F$  to Sue].

Rooth assumes that *only* is syntactically adjoined to VP and that it quantifies over the set provided by a variable  $C$  which gets instantiated by means of  $\sim$ . The squiggle operator, in its designated location, triggers the presupposition in Fig. 2a,b.

$$\partial : \begin{array}{c} \underline{C} \ P \\ C \subseteq \{\lambda x.introd(x, z, s) \mid z \in D_e\} \\ P \in C \quad P \neq \lambda x.introd(x, \mathbf{b}, s) \end{array}$$

**Fig. 2(a).**

Presupposition triggered by  $\sim$ [ $_{VP} \dots$ ]

$$\partial : \begin{array}{c} \underline{C} \ e' \\ C \subseteq \{e \mid introd(e) \wedge GO(e, s)\} \\ e' \in C \quad TH(e') \neq \mathbf{b} \end{array}$$

**Fig. 2(b).**

Same issue, using event semantics

We provide two variants of this presupposition. Figure 2(a) is immediately derived from Rooth's original account, Fig. 2(b) is a reformulation in Neo-Davidsonian semantics, which uses discourse referents for events rather than properties (as it is common practice in DRT)<sup>4</sup>. The meaning of (5) is correctly represented as (6a)<sup>5</sup> or (6b).

- (6) a.  $\forall P[P \in C \wedge P(\mathbf{m}) \rightarrow P = \lambda x.introd(x, \mathbf{b}, s)]$   
 b.  $\forall e[e \in C \wedge AG(e, \mathbf{m}) \rightarrow TH(e, \mathbf{b})]$

The question is whether it is plausible to assume that the instantiation of  $C$  is due to anaphoric retrieval as suggested by the definitions in Fig. 2a,b. Consider the discourse in (7).

- (7) a. At the party, there were Alex, Bill, and Carl, none of whom Sue had met before.  
 b. Mary only introduced  $BILL_F$  to Sue.

There are no introduction *events* in the discourse context given by (7a). It seems therefore wrong to assume that (7b) involves anaphoric retrieval of a set of VP-meanings of the form [introduced  $z$  to Sue]. On the other hand, it is highly likely

<sup>4</sup> Cf. Bonomi and Casalegno (1993), Beaver and Clark (2008) for an elegant treatment of focus in event semantics.

<sup>5</sup> Here, we ignore intensionality.

that retrieval is of a set of alternatives to *Bill*. But in that case it is more intuitive for  $\sim$  to attach to  $[BILL_F]$  as shown in (8).

(8) Mary only introduced  $\sim_{[DP\ BILL_F]}$  to Sue.

The problem is how to bring this insight in line with the semantics in (6a), which was found to be essentially correct. First of all, since  $C$  is now the set of individuals  $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$  rather than a set of predicates, it can no longer be used in formula (6a) as before. What we want instead is (9).

(9)  $\forall P[P \in \llbracket \text{introd. } \sim[BILL_F] \text{ to Sue} \rrbracket^\sim \wedge P(\mathbf{m}) \rightarrow P = \lambda x.\text{introd}(x, \mathbf{b}, \mathbf{s})];$   
 where  $\llbracket \text{introd. } \sim[BILL_F] \text{ to Sue} \rrbracket^\sim = \{\lambda x.\text{introd}(x, z, \mathbf{s}) \mid z \in C\}$

$\llbracket \cdot \rrbracket^\sim$  is, like Rooth's  $\llbracket \cdot \rrbracket^o$  and  $\llbracket \cdot \rrbracket^f$ , a mapping from well-formed expressions to semantic values (associated with some model).  $\llbracket \cdot \rrbracket^\sim$  differs from  $\llbracket \cdot \rrbracket^f$  in that it is defined only for constituents which contain an occurrence of the “context resolution” marked  $\sim$ . There is a switch from  $\llbracket \cdot \rrbracket^f$  to  $\llbracket \cdot \rrbracket^\sim$  when  $\sim$  is encountered, indicating that the operation it triggers subjects  $\llbracket \cdot \rrbracket^f$  to the relevant contextual restriction. Thus the constituent  $[BILL_F]$  of (8) has the focus semantic value  $\llbracket BILL_F \rrbracket^f = D_e$ , but the  $\sim$ -marked constituent  $\sim[BILL_F]$  has instead a value  $\llbracket \sim[BILL_F] \rrbracket^\sim$  (a contextually determined subset of  $\llbracket BILL_F \rrbracket^f$ ). We call  $\llbracket \sim[BILL_F] \rrbracket^\sim$  no longer FSV but *context set*. A comparison between Rooth's and our account is shown in Table 1.

**Table 1.** Reversed order of compositional Alternative Semantics and context resolution

Rooth (1992)	Our Account
$\llbracket BILL_F \rrbracket^f = D_e$	$\llbracket BILL_F \rrbracket^f = D_e$
<b>Foc.int.</b> $\rightarrow$	$\llbracket \sim[BILL_F] \rrbracket^\sim = \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$
$\llbracket \text{introd. } BILL_F \rrbracket^f$ $= \{\lambda y \lambda x.\text{introd}(x, z, y) \mid z \in D_e\}$	$\llbracket \text{introd. } \sim[BILL_F] \rrbracket^\sim$ $= \{\lambda y \lambda x.\text{introd}(x, z, y) \mid z \in \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}\}$
$\llbracket \text{introd. } BILL_F \text{ to Sue} \rrbracket^f$ $= \{\lambda x.\text{introd}(x, z, \mathbf{s}) \mid z \in D_e\}$	$\llbracket \text{introd. } \sim[BILL_F] \text{ to Sue} \rrbracket^\sim$ $= \{\lambda x.\text{introd}(x, z, \mathbf{s}) \mid z \in \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}\}$
$\llbracket \sim[\text{introd. } BILL_F \text{ to Sue}] \rrbracket^\sim$ $= \{\lambda x.\text{introd}(x, z, \mathbf{s}) \mid z \in \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}\}$	$\leftarrow$ (flawed) <b>Foc.int.</b>

### 3 Benefits of Our Account

In (8)  $\sim$  is adjoined to the focus constituent itself. But we do not propose that this is always so. Our interpretation of the  $\sim$  operator allows us, for instance, to handle the issue of *focus phrases* (Drubig, 1994, Krifka, 2006). Sentence (10) demonstrates what Krifka calls “the problem of the only child”.

(10) Sam only talked to  $[BILL's_F\ \text{mother}]_{FP}$ .

Drubig and Krifka noticed the problem that (10) presents for a Structured Meanings account which would analyse the sentence as involving *only*-quantification over *Bill* and the other members of his alternative set. If the set contains a sibling

of *Bill* then *Sam* must both have talked to their mother and, at the same time, not have talked to her, and the sentence would come out as a contradiction, although intuitively it isn't. Krifka (2006) solved the problem by postulating that *only* instead associates with *focus phrases (FP)*, cf. (10), which means that quantification is about referentially distinct alternatives to *Bill's mother* rather than alternatives to *Bill*.

By applying our strictly anaphoric definition of the squiggle operator we automatically get the correct semantics for (10).  $\sim$  is attached to  $[_{DP} \text{ BILL's}_F \text{ mother}]$ , giving rise to the presupposition in Fig. 3.

$$\partial : \begin{array}{c} \mathcal{C} y \\ C \subseteq [\text{BILL's}_F \text{ mother}]^f \\ y \in C \quad y \neq [\text{BILL's}_F \text{ mother}]^o \end{array}$$

Fig. 3.  $\sim[\text{BILL's}_F \text{ mother}]$

- (11) a.  $[\text{BILL's}_F \text{ mother}]^o = \iota x. \text{mother\_of}(x, \mathbf{b})$   
 b.  $[\text{BILL's}_F \text{ mother}]^f = \{d \mid \exists x. \text{mother\_of}(d, x)\}$

The ordinary value occurring in Fig. 3 is simply Bill's mother – representable as the  $\iota$ -expression in (11a). (Here we ignore the presupposition that is arguably generated by the definite description.) The focus semantic value is the anonymous set given in (11b), the set of all mothers of individuals in  $D_e$ , regardless of who or how numerous they are. During the process of anaphoric retrieval this set undergoes restriction, and  $C$  is resolved to whatever mothers play a role in the given context. Compare, for instance, sentence (12).

- (12) At the party there were Alex, Bill, Carl and Daniel, and also Bill's mother and Carl's mother. I only knew  $\sim[\text{BILL's}_F \text{ mother}]$ .

The second sentence of (12) is naturally interpreted as saying that the speaker knew Bill's mother but not Carl's mother, leaving it open whether he also knew the unmentioned mothers of Alex and Daniel. This interpretation can be obtained when  $\sim$  is attached to  $[\text{BILL's}_F \text{ mother}]$ , but not when it is attached to  $[\text{BILL's}_F]$ . Note also that the semantics correctly predicts that the other persons mentioned, who are not mothers, do not become elements of  $C$ .

A further benefit of the way we propose to use  $\sim$  arises in connection with an example discussed in von Hensinger (2007). He notices a problem with complex definite descriptions like the one occurring in (13a), which involves adjectival modification.

- (13) a. John only talked to  $[\text{the GERman}_F \text{ professor}]$ .  
 b.  $\{[\text{the German professor}], [\text{the French professor}], [\text{the English professor}], \dots\}$

Something is wrong if (13a) is analyzed under the assumption that determining the truth conditions of the sentence involves computing denotations of expressions of the form [the A professor]<sup>6</sup>, in other words a set like (13b). For it might well be that on the occasion that (13a) speaks of there were besides the one German professor several French professors and therefore the expression *[[the French professor]]* would fail to properly refer. Still, if the only professor that John talked to was the only German professor there, then (13a) is a perfectly good way of saying that John only talked with this one professor. The problem recurs every time we want to determine the alternative set of a DP. Consider (14a) as a more complex example. Again, (14b) does not seem to be an appropriate alternative set, because it involves uncontrollable presupposition-triggering expressions itself.

- (14) a. John only caught [the monkey which threw a toMAtO<sub>F</sub> at Lisa].  
 b.  $\{ \llbracket \text{the monkey which threw a tomato at Lisa} \rrbracket, \\ \llbracket \text{the monkey which threw a cucumber at Lisa} \rrbracket, \\ \llbracket \text{the monkey which threw a carrot at Lisa} \rrbracket, \dots, ?? \}$

The general solution we offer for cases like these will be demonstrated for (13a). The FSV of the phrase [the GERman<sub>F</sub> professor] is determined by a purely mechanical process as the set characterized by (15a), which does not run into the problems that (13b) caused. The set can even be further simplified to (15b).

- (15) a.  $\{d \mid \exists P[P(d) \wedge \text{professor}(d)]\}$     b.  $\{d \mid \text{professor}(d)\}$

The  $\sim$  is then adjoined to [<sub>DP</sub> the GERman<sub>F</sub> professor], which simply defines the task of retrieving from the context a set of professors, e.g.  $\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}$ , who are of course distinct from each other and whose nationality doesn't play any role.<sup>7</sup>

## 4 Intensionality

Discussions of the intensional aspects of information structure are not very common, but an exception is Beaver & Clark (2008: 95ff.) (in the following: B & C),

<sup>6</sup> A is some other nation-denoting adjective.

<sup>7</sup> Since a number of people have questioned the compositionality of this proposal, here comes the derivation:  $\llbracket \text{professor} \rrbracket^f = \{\lambda x. \text{professor}(x)\}$ ;  
 $\llbracket \text{GERman}_F \rrbracket^f = \{\lambda Q \lambda x. [P(x) \wedge Q(x)] \mid P \in D_{\langle e, t \rangle}\}$  (set of intersective modifiers);  
 $\llbracket \text{GERman}_F \text{ professor} \rrbracket^f = \{\lambda x. [P(x) \wedge \text{professor}(x)] \mid P \in D_{\langle e, t \rangle}\}$

As the alternative meaning  $\llbracket \text{the} \rrbracket^f$  we assume – similar to a proposal made in von Stechow (2007: Sect. 3.3) –  $[\lambda P. \cup \mathcal{P}]_{\langle \langle \langle e, t \rangle, t \rangle, \langle e, t \rangle \rangle}$  (involving a typeshift from a set of properties to a set of individuals). The resultant set of professors (15) is then – other than on von Stechow's account – restricted and contextually identified by means of  $\sim$ . If this seems a bit curious at first, recall that the *ordinary meaning* of the definite determiner  $\llbracket \text{the} \rrbracket^o$  consists of precisely the same two aspects: a typeshifter  $[\lambda P. x]_{\langle \langle e, t \rangle, t \rangle}$  and a presuppositional condition (sometimes written as  $\iota$ -operator) to identify the free variable  $x$  as the unique individual with property  $P$  (here: *German professor*) in the relevant context.

which contains a detailed discussion of the sentence in (16a) (the F-marking is theirs, a translation to our account is (16b)).

- (16) a. Sandy only met [the PREsident]<sub>F</sub>.  
 b. Sandy only met  $\sim$ [the PREsident<sub>F</sub>].

B & C argue roughly as follows. An extensional evaluation of (16) involves a set *A* of alternatives for the denotation (= the *extensional value*) of *the president*. *A* is a set of ordinary individuals (of which the actual president is one) that enters into the determination of the extensional value of the sentence (its actual truth value), like the actual president himself does. If instead we want to obtain the *intensional* value of the sentence (i.e. the proposition it expresses), then we must start with the intensions of its smallest constituents and compute the intensions of the complex constituents from the intensions of their components, in the manner familiar from Montague Grammar, arriving eventually at the intension of the sentence as a whole. In this way we obtain as intension for *the president* an individual concept *pr* (a function from possible worlds to individuals; for each possible world *w*, *pr*(*w*) is the president in *w*). B & C's next assumption is that if the semantic value of *the president* is an individual concept, then the members of the *alternative set* invoked by the F-marking of this phrase must consist of individual concepts as well. But if that is what we want to assume about the alternative set *A*, we have to be a very careful. For one thing we cannot assume *A* to be the set of *all* individual concepts. For if there is at least one world *w*<sub>1</sub> other than the actual world *w*<sub>0</sub>, and there are at least two individuals, then there will be different individual concepts that both assign the actual president **a** to the actual world but differ in what they assign to *w*<sub>1</sub>: *c*<sub>1</sub> = {⟨*w*<sub>0</sub>, **a**⟩, ⟨*w*<sub>1</sub>, **a**⟩} and *c*<sub>2</sub> = {⟨*w*<sub>0</sub>, **a**⟩, ⟨*w*<sub>1</sub>, **b**⟩}. And then the usual semantics for *only* will yield a contradiction for a sentence like (16). Furthermore, even when we accept that in general the alternative set is contextually restricted, it isn't immediately clear how this kind of conflict can be avoided. B & C discuss a number of options. But as we see it, the problem that these options are trying to deal with need not arise in the first place. The solution we suggest starts from the observation that all compositional steps in the computation of the truth value of sentences like (17) (in any possible world *w*) are extensional. In this regard (16) is no different than e.g. (17).

- (17) Sandy met the president.

The intension of such a “purely extensional” sentence  $\phi$  can be obtained by simple “abstraction with respect to possible worlds”. (In an intensional model  $M = \langle W, M \rangle$ , where *W* is a set of possible worlds and *M* a function which assigns each *w* ∈ *W* an extensional model *M*(*w*), the intension  $\llbracket \phi \rrbracket_M$  of  $\phi$  in *M* can be obtained as  $\lambda w. \llbracket \phi \rrbracket_{M,w}$ , where  $\llbracket \phi \rrbracket_{M,w}$  is the truth value of  $\phi$  in *M*(*w*).)

Our second assumption is that retrieval of alternative sets is in actual fact always retrieval of a set description – or, if you prefer, of a predicate. Intuitively, interpreting the focus of (18b) triggers retrieval of the predicate “(*member of*) *the president's family*” (in the following: *MPF*).

- (18) a. Sandy wanted to meet the president's family.  
 b. But she only met  $\sim$ [the PRESident<sub>F</sub>].

To capture the intensional dimension in our representations there are various ways in which one could proceed. For present purposes the simplest solution is to adopt a DRT-based version of the Type2 logic of [Gallin (1975)], in which possible worlds are represented explicitly by variables or, for us, discourse referents, cf. [Roberts (1989)], [Brasoveanu (2007)], [Bittner (to appear)]. This entails that ordinary predicates get an additional argument position that is to be filled by a possible world. (Some modifications are needed for the algorithm that constructs (preliminary) DRSs of this new form from sentences, but this is not a serious problem.) In this formalism the representation for [17] can be given the representation in Fig. 4<sup>8</sup>

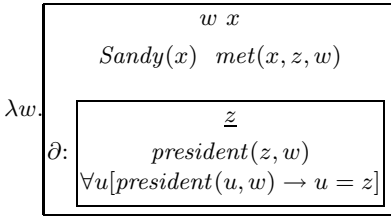


Fig. 4. DRS for [17]

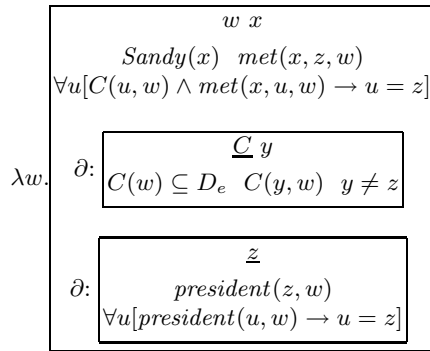


Fig. 5. DRS for [18b]

A representation of this kind should be evaluated with respect to contexts  $c$  which specify (among other things) a set  $W_c$  of worlds. For instance,  $W_c$  could be the set of worlds that are assumed to be compatible with the belief of some given person  $A$ . The embedded presupposition in Fig. 4 imposes on  $c$  the constraint that every world in  $W_c$  must satisfy it. If the presupposition is not satisfied, then the context  $c$  should, if possible, be accommodated so that the constraint is satisfied by all its worlds after all.

Given that the context  $c$  satisfies the presupposition, the non-presuppositional part of Fig. 4 modifies  $c$  to an output context  $c'$  whose world set  $W_{c'}$  consists of those worlds of  $W_c$  in which the non-presuppositional part of Fig. 4 is true.

A representation of [18b] which captures intensionality along the same lines must include a presuppositional requirement for a description of the alternative

<sup>8</sup> In the following, we explicitly indicate the uniqueness condition of the definite, which is treated implicitly in [van der Sandt (1992)] and in Fig. 1a, where it is assumed that contextual *identification* via anaphora – if successful – is necessarily unique. However, when as is the case in [17] the presupposition has to be justified in the encyclopaedic context, its representation is arguably defective without the uniqueness condition.

set for the focused constituent  $[\sim [the PRESident_F]]^\sim$ . We represent this presupposition by means of a predicate discourse referent  $C$ , which has besides its ordinary argument slot also a slot for a possible world. One such representation is given in Fig. 5. In the context provided by (18a)  $C$  can be resolved to the predicate “member of the president’s family”. This turns Fig. 5 into the representation in Fig. 6.

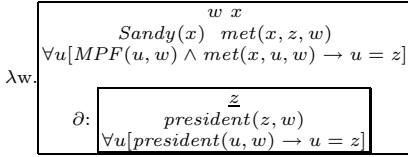


Fig. 6. Doubly *de dicto*

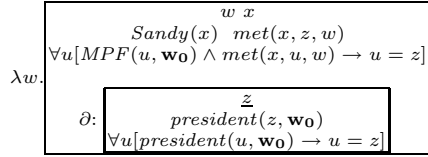


Fig. 7. Doubly *de re*

Figures 5 and 6 represent only one interpretation of (18b). We call this interpretation the “doubly *de dicto*” interpretation, since both the DP *the president* and the alternative set description *MPF* are evaluated at the different evaluation worlds  $w$ , at which the non-presuppositional content is then evaluated for truth or falsity. Figure 6 would be a natural interpretation in a situation where Sandy set out on her quest for satisfaction of the desire described in (18a) with a purely descriptive conception of the president and knows no more about his family than that he has one. But this is not the only interpretation of (18b). For one thing, as often observed, the DP *the president* can not only be interpreted *de dicto* but also *de re*. We can represent this interpretation by replacing the relevant occurrences of the discourse referent  $w$  by the discourse referent  $w_0$ , an “indexical” discourse referent that always stands for the actual world. But not only *the president* can be given either a *de dicto* or a *de re* interpretation, the same holds for the description *MPF*. Figure 7 represents the “doubly *de re*” interpretation of (18b).

Certain “mixed” interpretations seem possible also. For instance, Sandy may have directly referential knowledge of the president (say, by having seen him on TV), but know little about his family. In that case an interpretation might seem reasonable in which DP *the president* is interpreted *de re* but the alternative set description *MPF* *de dicto*. On the other hand, the combination of a *de dicto* interpretation of *the president* in combination with a *de re* interpretation of *MPF* makes little intuitive sense. As things stand, however, we do not know what general principles if any limit the number of interpretational options for such sentences. We leave this as a question for further investigation.

## References

- Beaver, D., Clark, B.: Sense and Sensitivity. How Focus Determines Meaning. Wiley-Blackwell, Chichester (2008)
- Bittner, M.: Tense, Mood and Centering. In: Musan, R., Rathert, M. (eds.) Tense across Languages, Niemeyer, Tübingen (to appear)

- Bonomi, A., Casalegno, P.: Only: Association with Focus in Event Semantics. *Natural Language Semantics* 2(1), 1–45 (1993)
- Brasoveanu, A.: Structured Nominal and Modal Reference. PhD thesis, Rutgers University, New Brunswick (2007)
- Drubig, H.B.: Island Constraints and the Syntactic Nature of Focus and Association with Focus. In: *Arbeitspapiere des SFB 340*, vol. 51 (1994)
- Gallin, D.: *Intensional and Higher-Order Modal Logic*. North-Holland, Amsterdam (1975)
- Geurts, B.: *Presuppositions and Pronouns*. Elsevier, Oxford (1999)
- von Heusinger, K.: Alternative Semantics for Definite NPs. In: Schwabe, K., Winkler, S. (eds.) *Information Structure and the Architecture of Grammar. A Typological Perspective*, Benjamins, Amsterdam, pp. 485–508 (2007)
- Kamp, H.: The Importance of Presupposition. In: Rohrer, C., Roßdeutscher, A. (eds.) *Linguistic Form and its Computation*. CSLI, Stanford (2001)
- Krifka, M.: Association with Focus Phrases. In: Molnár, V., Winkler, S. (eds.) *The Architecture of Focus*, pp. 105–136. Mouton de Gruyter, Berlin (2006)
- Roberts, C.: Modal Subordination and Pronominal Anaphora in Discourse. *Linguistics and Philosophy* 12(6), 683–721 (1989)
- Rooth, M.: Association with Focus. PhD thesis, University of Massachusetts, Amherst (1985)
- Rooth, M.: A Theory of Focus Interpretation. *Natural Language Semantics* 1(1), 75–116 (1992)
- Rooth, M.: Focus. In: Lappin, S. (ed.) *The Handbook of Contemporary Semantic Theory*, pp. 271–297. Blackwell, Oxford (1996)
- van der Sandt, R.: Presupposition Projection as Anaphora Resolution. *Journal of Semantics* 9, 333–377 (1992)

# Disjunctive Questions, Intonation, and Highlighting<sup>★</sup>

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**Abstract.** This paper examines how intonation affects the interpretation of disjunctive questions. The semantic effect of a question is taken to be three-fold. First, it raises an issue. In the tradition of inquisitive semantics, we model this by assuming that a question proposes several possible updates of the common ground (several *possibilities* for short) and invites other participants to help establish at least one of these updates. But apart from raising an issue, a question may also *highlight* and/or *suggest* certain possibilities, and intonation determines to a large extent which possibilities are highlighted/suggested.

## 1 Preliminaries: Basic Assumptions and Data

**Syntactic structure.** Syntactically, we distinguish between two kinds of disjunctive interrogatives. On the one hand there are those that consist of a single interrogative clause containing a disjunction. On the other hand there are those that consist of two interrogative clauses, conjoined by disjunction. We will refer to the former as *narrow-scope* disjunctive interrogatives, and to the latter as *wide-scope* disjunctive interrogatives. Some examples are given in (1) and (2) below.

- (1) Narrow-scope disjunctive interrogatives:
  - a. Does Ann or Bill play the piano?
  - b. Does Ann love Bill or Chris?
- (2) Wide-scope disjunctive interrogatives:
  - a. Does Ann play the piano, or does Bill play the piano?
  - b. Does Ann play the piano, or Bill?

We will assume that (2b) has exactly the same underlying syntactic structure as (2a); only some material is left unpronounced.

**Intonation patterns.** Disjunctive questions can be pronounced in different ways, and their interpretation is partly determined by the choice of intonation pattern. We concentrate on two prosodic features that seem to have significant semantic impact. First, in the case of a narrow-scope disjunctive interrogative it is important whether the disjunction is pronounced ‘as a block’ or whether each of the disjuncts is given separate emphasis. Second, in case the disjuncts are given separate emphasis, it is important

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<sup>★</sup> This paper has benefited enormously from discussions with Maria Aloni, Ivano Ciardelli, Jeroen Groenendijk, and Kathryn Pruitt. Due to space limitations, the mode of presentation here is rather dense. For a slightly more slow-paced and elaborate version of the paper, see [1].

<sup>1</sup> The semantic significance of these prosodic features has been established experimentally [2].

whether there is a rising or a falling pitch contour on the second disjunct. The different intonation patterns are given in (3) and (4), where underlining is used to represent emphasis, and  $\uparrow$  and  $\downarrow$  indicate rising and falling pitch.<sup>2</sup>

(3) Intonation patterns for narrow-scope disjunctive interrogatives:

- a. *Block intonation*: Does Ann-or-Bill $\uparrow$  play the piano?
- b. *Open intonation*: Does Ann $\uparrow$  or Bill $\uparrow$  play the piano?
- c. *Closed intonation*: Does Ann $\uparrow$  or Bill $\downarrow$  play the piano?

(4) Intonation patterns for wide-scope disjunctive interrogatives:

- a. *Open intonation*: Does Ann $\uparrow$  play the piano, or Bill $\uparrow$ ?
- b. *Closed intonation*: Does Ann $\uparrow$  play the piano, or Bill $\downarrow$ ?

**Focus and closure.** We take it that emphasis in the acoustic signal is a reflex of a *focus* feature in the logical form, and that the rising-and-falling pitch contour in (3c) and (4b) correlates with a *closure* feature in the logical form. It seems that this closure feature affects the pronunciation of the entire sentence (not just of, say, the contrastive elements in both disjuncts). Therefore, we assume that it is adjoined to the sentence as a whole. The ensuing logical forms are listed in the table below. Focus features, closure features, and interrogative complementizers are denoted by F, C, and Q, respectively.

Pattern	Acoustic signal	Logical form
Narrow		
- block	Does <u>Ann-or-Bill</u> $\uparrow$ play?	[Q-does [Ann or Bill] <sub>F</sub> play]
- open	Does <u>Ann</u> $\uparrow$ or <u>Bill</u> $\uparrow$ play?	[Q-does [Ann] <sub>F</sub> or [Bill] <sub>F</sub> play]
- closed	Does <u>Ann</u> $\uparrow$ or <u>Bill</u> $\downarrow$ play?	[Q-does [Ann] <sub>F</sub> or [Bill] <sub>F</sub> play] <sub>C</sub>
Wide		
- open	Does <u>Ann</u> $\uparrow$ play, or <u>Bill</u> $\uparrow$ ?	[[Q-does [Ann] <sub>F</sub> play] or [Q-does [Bill] <sub>F</sub> play]]
- closed	Does <u>Ann</u> $\uparrow$ play, or <u>Bill</u> $\downarrow$ ?	[[Q-does [Ann] <sub>F</sub> play] or [Q-does [Bill] <sub>F</sub> play]] <sub>C</sub>

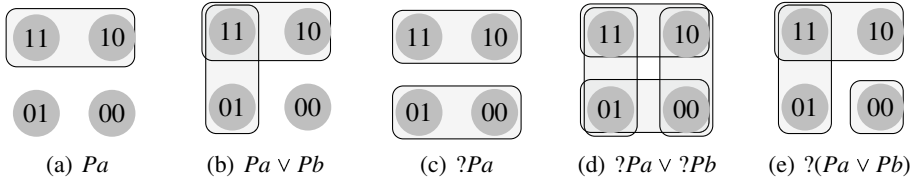
**Basic data.** Our theory should capture, at the very least, the effects of intonation on answerhood conditions. The basic empirical observations are summed up in (5), (6), and (7) below (wide-scope disjunctive interrogatives are not explicitly listed here; they behave exactly like their narrow-scope counterparts in the relevant respects). Notice that open intonation behaves in some ways like block intonation, but in others more like closed intonation: it licenses a *no* answer, but it does not license a *yes* answer. To the best of our knowledge, this observation has not been taken into account before.

- |  |  |  |
|--|--|--|
| (5) Does <u>Ann-or-Bill</u> $\uparrow$ play? | (6) Does <u>Ann</u> $\uparrow$ or <u>Bill</u> $\uparrow$ play? | (7) Does <u>Ann</u> $\uparrow$ or <u>Bill</u> $\downarrow$ play? |
| a. No. $\Rightarrow$ neither                 | a. No. $\Rightarrow$ neither                                   | a. #No.  |
| b. Yes. $\Rightarrow$ at least one           | b. #Yes. $\Rightarrow$ yes what?!                              | b. #Yes.   |
| c. (Yes,) Ann does.                          | c. Ann does.   | c. Ann does.   |
| d. (Yes,) Bill does.                         | d. Bill does.  | d. Bill does.  |

A further observation that should be accounted for is that disjunctive interrogatives with closure intonation convey that the speaker expects that exactly one of the disjuncts is

<sup>2</sup> Most previous work on disjunctive questions (3, 4, 5, 6) does not take the open intonation pattern into account. Bartels (7) does observe the pattern but does not account for its semantics.

d.  $\llbracket \text{love} \rrbracket := \{\lambda y. \lambda x. \lambda w. \text{love}_w(x, y)\}$



**Fig. 1.** Some propositions visualized

**Disjunction.** Disjunction introduces alternatives. The denotation set of a phrase ‘ $\alpha$  or  $\beta$ ’, where  $\alpha$  and  $\beta$  are two expressions of some type  $\tau$ , is the union of the denotation set of  $\alpha$  and the denotation set of  $\beta$ :

(12) For any type  $\tau$ , if  $\llbracket \alpha \rrbracket, \llbracket \beta \rrbracket \subseteq D_\tau$ , then  $\llbracket \alpha \text{ or } \beta \rrbracket := \llbracket \alpha \rrbracket \cup \llbracket \beta \rrbracket$

For example:

(13) a.  $\llbracket \text{Ann or Bill} \rrbracket = \left\{ \begin{array}{l} \text{Ann,} \\ \text{Bill} \end{array} \right\}$     b.  $\llbracket \text{Ann or Bill plays} \rrbracket = \left\{ \begin{array}{l} \lambda w. \text{play}_w(\text{Ann}), \\ \lambda w. \text{play}_w(\text{Bill}) \end{array} \right\}$

Notice that the denotation set of a complete sentence, such as ‘Ann or Bill plays’ is a set of objects in  $D_{(st)}$ . Such objects are functions from indices to truth values, or equivalently, sets of indices. In inquisitive semantics, sets of indices are referred to as *possibilities*, and a set of possibilities is called a *proposition*. So complete sentences express propositions.

**Visualization.** Propositions can be visualized in a helpful way. For instance, the sentence ‘Ann plays’ expresses the proposition  $\{\lambda w. \text{play}_w(\text{Ann})\}$ , which contains a single possibility consisting of all indices in which Ann plays. This proposition is depicted in figure [I\(a\)](#), where 11 is the index in which both Ann and Bill play, 10 the index in which only Ann plays, etcetera. Figure [I\(b\)](#) depicts the proposition expressed by ‘Ann or Bill plays’. As we saw in (13b), this proposition consists of two possibilities: the possibility that Ann plays, and the possibility that Bill plays.

**Excluded possibilities.** Recall that the possibilities for a sentence  $\alpha$  embody the ways in which  $\alpha$  proposes to update the common ground. If some index  $i$  is not included in any possibility for  $\alpha$ , then we say that  $i$  is *excluded* by  $\alpha$ . For in this case,  $i$  will be eliminated from the common ground by any of the updates proposed by  $\alpha$ . If  $\alpha$  excludes any indices, then we refer to the set of all such indices as the *possibility excluded* by  $\alpha$ . If  $\alpha$  does not exclude any indices, then we say that it does not exclude any possibility. We use  $\llbracket \alpha \rrbracket$  to denote the set of possibilities excluded by  $\alpha$ .

**Interrogative clauses.** The interrogative complementizer,  $Q$ , always operates on an expression  $\alpha$  of type  $(st)$ , and the resulting clause  $[Q \alpha]$  is always again of type  $(st)$ . So even though there is a shift in syntactic category, there is no shift in semantic type. The proposition expressed by  $[Q \alpha]$  consists of the possibilities for  $\alpha$  itself, plus the possibility that  $\alpha$  excludes.

(14)  $\llbracket [Q \alpha] \rrbracket := \llbracket \alpha \rrbracket \cup \llbracket \alpha \rrbracket$

For example, the proposition expressed by the simple polar interrogative ‘Does Ann play?’ consists of two possibilities: the possibility that Ann plays, and the possibility

that she does not play. These possibilities embody two possible updates of the common ground, and the responder is invited to provide information such that either one of these updates can be established.

$$(15) \quad \llbracket \text{Q-does Ann play} \rrbracket \\ = \llbracket \text{Ann plays} \rrbracket \cup \llbracket \neg \text{Ann plays} \rrbracket = \left\{ \lambda w. \text{play}_w(\text{Ann}), \right. \\ \left. \lambda w. \neg \text{play}_w(\text{Ann}) \right\} \Rightarrow \text{see figure 1(c)}$$

**Disjunctive interrogatives.** Given these assumptions, the propositions expressed by wide- and narrow-scope disjunctive interrogatives are the following:

$$(16) \quad \text{Wide-scope disjunctive interrogative: Does Ann play or does Bill play?} \\ \llbracket \text{Q-does Ann play or Q-does Bill play} \rrbracket \\ = \llbracket \text{Q-does Ann play} \rrbracket \cup \llbracket \text{Q-does Bill play} \rrbracket \\ = \left\{ \lambda w. \text{play}_w(\text{Ann}), \right. \\ \left. \lambda w. \neg \text{play}_w(\text{Ann}) \right\} \cup \left\{ \lambda w. \text{play}_w(\text{Bill}), \right. \\ \left. \lambda w. \neg \text{play}_w(\text{Bill}) \right\} \Rightarrow \text{see figure 1(d)}$$

$$(17) \quad \text{Narrow-scope disjunctive interrogative: Does Ann or Bill play?} \\ \llbracket \text{Q-does Ann or Bill play} \rrbracket \\ = \llbracket \text{Ann or Bill plays} \rrbracket \cup \llbracket \neg \text{Ann or Bill plays} \rrbracket \\ = \left\{ \lambda w. \text{play}_w(\text{Ann}), \right. \\ \left. \lambda w. \text{play}_w(\text{Bill}) \right\} \cup \left\{ \lambda w. \neg \text{play}_w(\text{Ann}) \wedge \neg \text{play}_w(\text{Bill}) \right\} \Rightarrow \text{see figure 1(e)}$$

So much for the compositional treatment of our basic fragment in inquisitive semantics. Notice that this treatment does not yet say anything about the licensing and interpretation of yes/no answers, or about the ‘exactly one implication’ of disjunctive interrogatives with closure intonation. The following sections propose an extension of the system that will allow us to capture these phenomena.

### 3 Focus and Highlighting

The general idea that we will pursue in this section is that a sentence, besides proposing one or more possible updates, may also *highlight* certain possibilities, and that *focus* plays an important role in determining the possibilities that a sentence highlights.

We think that highlighting is of particular relevance for the licensing and interpretation of yes/no answers. More specifically, we hypothesize that a *yes* answer to a question  $\alpha$  presupposes that  $\alpha$  highlighted exactly one possibility, and if this presupposition is met, *yes* confirms that highlighted possibility. A *no* answer on the other hand, if felicitous, simply rejects all the possibilities highlighted by  $\alpha$  (for now we will assume that a *no* answer is always felicitous; a felicity condition will be specified in section 4).

**Initial motivation: opposing polar questions.** Initial motivation for this idea comes from an old puzzle concerning polar questions, exemplified by the contrast between (18a) and (18b):

- (18) a. Is the door open?                      b. Is the door closed?

According to inquisitive semantics, as it has been developed so far, (18a) and (18b) are equivalent: they both express a proposition consisting of two possibilities, the possibility that the door is open, and the possibility that the door is closed. However, there is a clear empirical difference between the two: in reply to (18a), *yes* means that the door is open, while in reply to (18b), it means that the door is closed.<sup>3</sup>

This difference is captured straightforwardly if we assume that (18a) highlights the possibility that the door is open, that (18b) highlights the possibility that the door is closed, and that the interpretation of *yes* and *no* is as hypothesized above. Our aim is to give a similar explanation of the licensing and interpretation of yes/no answers in response to disjunctive questions. In order to do so, we must first specify how the possibilities highlighted by a given sentence are compositionally determined, and in particular how focus affects this process.

**Proposing and highlighting.** We will henceforth assume that the semantic value of a sentence  $\alpha$  consists of two components,  $\llbracket \alpha \rrbracket_P$  and  $\llbracket \alpha \rrbracket_H$ . Both  $\llbracket \alpha \rrbracket_P$  and  $\llbracket \alpha \rrbracket_H$  are sets of possibilities;  $\llbracket \alpha \rrbracket_P$  embodies the proposal that  $\alpha$  expresses, and  $\llbracket \alpha \rrbracket_H$  consists of the possibilities that  $\alpha$  highlights. The semantic value of subsentential expressions will also consist of these two components. For any expression  $\alpha$ , sentential or subsentential, we will refer to  $\llbracket \alpha \rrbracket_P$  as its P-set, and to  $\llbracket \alpha \rrbracket_H$  as its H-set. Both P-sets and H-sets are composed by means of pointwise function application.

What we used to call the denotation set of an expression, then, is now called its P-set. As far as names, verbs, and disjunction are concerned, H-sets are defined just as P-sets. However, as soon as interrogative complementizers enter the derivation, P-sets and H-sets start to diverge. Recall that the proposal expressed by  $[Q \alpha]$  consists of the possibilities for  $\alpha$  itself, plus the possibility that  $\alpha$  excludes:

- (19)  $\llbracket [Q \alpha] \rrbracket_P := \llbracket \alpha \rrbracket_P \cup \llbracket \alpha \rrbracket$

We will assume that  $[Q \alpha]$  simply highlights the possibilities that  $\alpha$  itself highlights, not the possibility that  $\alpha$  excludes:

- (20)  $\llbracket [Q \alpha] \rrbracket_H := \llbracket \alpha \rrbracket_H$

These assumptions suffice to capture the contrast between opposing polar questions:

- |  |  |
|--|--|
| <p>(21) [Q-is the door open]</p> <p>Proposes: open/closed</p> <p>Highlights: open</p> <p><i>yes</i> <math>\Rightarrow</math> the door is open</p> <p><i>no</i> <math>\Rightarrow</math> the door is closed</p> | <p>(22) [Q-is the door closed]</p> <p>Proposes: open/closed</p> <p>Highlights: closed</p> <p><i>yes</i> <math>\Rightarrow</math> the door is closed</p> <p><i>no</i> <math>\Rightarrow</math> the door is open</p> |
|--|--|

**Highlighting and focus.** We will assume that focus affects the computation of H-sets. To see why, consider the two focus structures that give rise to block intonation and open intonation, respectively:

<sup>3</sup> This is sometimes taken to be a general argument against ‘proposition set’ approaches to questions—which include, besides inquisitive semantics, classical theories such as [14], [17], and [18]—and in favor of alternatives such as the ‘structured meaning’ approach or the ‘orthoalgebraic’ approach [19/20]. Here, we choose not to pursue a full-fledged alternative to the proposition set approach, but rather to extend it in a suitable way.

- (23) a. Does [Ann or Bill]<sub>F</sub> play the piano?  $\Rightarrow$  block intonation  
 b. Does [Ann]<sub>F</sub> or [Bill]<sub>F</sub> play the piano?  $\Rightarrow$  open intonation

Recall that (23a) licenses both *yes* and *no* as an answer, while (23b) only licenses *no*. Our hypothesis about the interpretation of *yes* and *no* captures this contrast if we assume that (23a) highlights a single possibility (the possibility that Ann or Bill plays), while (23b) highlights two possibilities (the possibility that Ann plays, and the possibility that Bill plays). But this can only be if focus affects the computation of H-sets. For, apart from their focus structures, (23a) and (23b) are perfectly identical.

The intuitive idea that we will pursue is that ‘focus makes H-sets collapse’. Let us first make this more precise for the case where  $\alpha$  is a complete sentence, of type  $(st)$ :

- (24) If  $\alpha$  is of type  $(st)$ , then:

$$\llbracket \alpha_F \rrbracket_H := \{ \bigcup_{\pi \in \llbracket \alpha \rrbracket_H} \pi \}$$

If  $\alpha$  is of type  $(st)$ , then every element of  $\llbracket \alpha \rrbracket_H$  is a possibility  $\pi$ , a set of indices. The focus feature collapses all these possibilities into one big possibility,  $\bigcup_{\pi \in \llbracket \alpha \rrbracket_H} \pi$ . This, then, is the unique possibility in  $\llbracket \alpha_F \rrbracket_H$  [\[4\]](#)

If  $\alpha$  is a sub-sentential expression, of some type  $\sigma$  different from  $(st)$ , then the elements of  $\llbracket \alpha \rrbracket_H$  are not full-fledged possibilities, so we cannot simply take their union. However, following [\[21\]](#), we can take their ‘generalized union’:

- (25) If  $\alpha$  is of some type  $\sigma$ , different from  $(st)$ , then:

$$\llbracket \alpha_F \rrbracket_H := \{ \lambda z. \bigcup_{y \in \llbracket \alpha \rrbracket_H} z(y) \} \quad \text{where } z \text{ is a variable of type } (\sigma(st))$$

For our examples, the relevant case is the one where  $\alpha$  is of type  $e$ . In this particular case, we have:

- (26)  $\llbracket \alpha_F \rrbracket_H := \{ \lambda P. \bigcup_{y \in \llbracket \alpha \rrbracket_H} P(y) \}$  where  $P$  is a variable of type  $(e(st))$

Let us first consider what this means for some disjunctive declaratives with different focus structures:

- (27)  $\llbracket [\text{Ann}]_F \text{ or } [\text{Bill}]_F \text{ plays} \rrbracket_H = \left\{ \begin{array}{l} \lambda w. \text{play}_w(\text{Ann}), \\ \lambda w. \text{play}_w(\text{Bill}) \end{array} \right\}$

- (28)  $\llbracket [\text{Ann or Bill}]_F \text{ plays} \rrbracket_H = \{ \lambda w. \text{play}_w(\text{Ann}) \cup \lambda w. \text{play}_w(\text{Bill}) \}$

With narrow focus on each individual disjunct, ‘Ann or Bill plays’ highlights two possibilities. But, as desired, focus on the whole disjunctive subject NP collapses these two possibilities into one. Now let us turn to disjunctive interrogatives. First consider the narrow-scope variant. Recall that, by definition, an interrogative clause  $[Q \alpha]$  highlights the same possibilities as  $\alpha$  itself. So we have:

- (29)  $\llbracket Q\text{-does } [\text{Ann}]_F \text{ or } [\text{Bill}]_F \text{ play} \rrbracket_H = \left\{ \begin{array}{l} \lambda w. \text{play}_w(\text{Ann}), \\ \lambda w. \text{play}_w(\text{Bill}) \end{array} \right\}$

- (30)  $\llbracket Q\text{-does } [\text{Ann or Bill}]_F \text{ play} \rrbracket_H = \{ \lambda w. \text{play}_w(\text{Ann}) \cup \lambda w. \text{play}_w(\text{Bill}) \}$

Thus, it is predicted that the question ‘Does Ann or Bill play?’ only highlights two distinct possibilities if it has narrow focus on ‘Ann’ and on ‘Bill’. Wide-scope disjunctive interrogatives on the other hand, always highlight two distinct possibilities:

<sup>4</sup> Notice that this is reminiscent of what is called *non-inquisitive closure* in inquisitive semantics [\[10\]](#), and what is called *existential closure* in alternative semantics [\[15\]](#).

$$(31) \quad \llbracket \text{Q-does [Ann]}_F \text{ play or Q-does [Bill]}_F \text{ play} \rrbracket_H = \left\{ \begin{array}{l} \lambda w. \text{play}_w(\text{Ann}), \\ \lambda w. \text{play}_w(\text{Bill}) \end{array} \right\}$$

The analysis so far yields a number of satisfactory predictions:

- (32) Does [Ann or Bill]<sub>F</sub> play?
- a. Highlights the possibility that Ann or Bill plays.
  - b. *yes*  $\Rightarrow$  at least one of them plays
  - c. *no*  $\Rightarrow$  neither Ann nor Bill plays
- (33) Does [Ann]<sub>F</sub> or [Bill]<sub>F</sub> play?
- a. Highlights the possibility that Ann plays and the possibility that Bill plays.
  - b. *yes*  $\Rightarrow$  presupposition failure (the question highlights more than one possibility)
  - c. *no*  $\Rightarrow$  neither Ann nor Bill plays
- (34) Does [Ann]<sub>F</sub> play or does [Bill]<sub>F</sub> play?
- a. Highlights the possibility that Ann plays and the possibility that Bill plays.
  - b. *yes*  $\Rightarrow$  presupposition failure (the question highlights more than one possibility)
  - c. *no*  $\Rightarrow$  neither Ann nor Bill plays

We seem to have obtained a better understanding of the basic difference between block intonation and open intonation. Now let us consider the effect of closure.

## 4 Closure and Suggestions

Our basic intuition is that closure suggests that *exactly one of the highlighted possibilities can be realized*. (Recall that possibilities embody possible updates of the common ground; as such it makes sense to speak of them as ‘being realized’.) To see what this amounts to, consider our running examples (35a) and (35b):

- (35) a. Does Ann<sub>↑</sub> or Bill<sub>↓</sub> play?                      b. Does Ann<sub>↑</sub> play, or Bill<sub>↓</sub>?

These questions both highlight two possibilities: the possibility that Ann plays, and the possibility that Bill plays. To suggest that exactly one of these possibilities can be realized is to suggest that exactly one of Ann and Bill plays the piano.

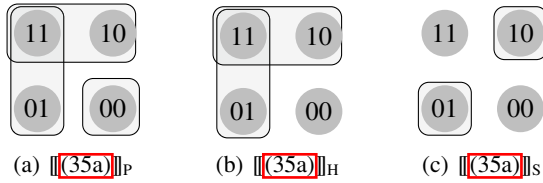
There are several ways to formalize this intuition. We will assume here that the meaning of a sentence  $\alpha$  does not just consist of  $\llbracket \alpha \rrbracket_P$  and  $\llbracket \alpha \rrbracket_H$ , but has a third component,  $\llbracket \alpha \rrbracket_S$ , which is the set of possibilities/updates that  $\alpha$  *suggests*. We will refer to  $\llbracket \alpha \rrbracket_S$  as the S-set of  $\alpha$ .

We will assume that the S-set of expressions that do not bear a closure-feature is always empty, and define the semantic contribution of the closure-feature as follows:

- (36) *The effect of closure:*

$$\llbracket \alpha_C \rrbracket_P := \llbracket \alpha \rrbracket_P \quad \llbracket \alpha_C \rrbracket_H := \llbracket \alpha \rrbracket_H \quad \llbracket \alpha_C \rrbracket_S := \mathcal{EX}(\llbracket \alpha \rrbracket_H)$$

The definition of  $\llbracket \alpha_C \rrbracket_S$  makes use of the *exclusive strengthening* operator  $\mathcal{EX}$ . For any set of possibilities  $\Pi$ , and for any possibility  $\pi \in \Pi$ , the exclusive strengthening of  $\pi$  relative to  $\Pi$  is defined as:



**Fig. 2.** Exclusive strengthening illustrated

$$(37) \quad \mathcal{EX}(\pi, \Pi) := \pi - \bigcup \{ \rho \mid \rho \in \Pi \text{ and } \pi \not\subseteq \rho \}$$

and the exclusive strengthening of  $\Pi$  itself is defined as:

$$(38) \quad \mathcal{EX}(\Pi) := \{ \mathcal{EX}(\pi, \Pi) \mid \pi \in \Pi \}$$

The effect of exclusive strengthening is illustrated for example (35a) in figure 2. Recall that (35a) proposes three possibilities, as depicted in figure 2(a), and highlights two possibilities, as depicted in figure 2(b). Applying  $\mathcal{EX}$  to these two highlighted possibilities removes the overlap between them, resulting in the two possibilities in figure 2(c).<sup>5</sup>

**Accepting and canceling suggestions.** Suggestions can either be accepted or canceled by a responder. We will assume that acceptance is the default. That is, if a suggestion is not explicitly contradicted, then all conversational participants assume that it is commonly accepted, and the suggested information is added to the common ground. Thus, if you ask (35a) or (35b), and I reply: ‘Ann does’, then I tacitly accept your suggestion. As a result, the common ground will not only be updated with the information that Ann plays, but also with the information that Bill does not play (see also [13/23]).

**Licensing *no*.** At the beginning of section 3 we hypothesized that *no*, in response to a question  $\alpha$ , simply denies all the possibilities that  $\alpha$  highlights. We left the felicity condition on the use of *no* unspecified at that point. Now that suggestions have entered the picture, we are ready to make this felicity condition explicit.

Recall the contrast between disjunctive declaratives and interrogatives:

- (39) Ann↑ or Bill↓ plays the piano.      (40) Does Ann↑ or Bill↓ play the piano?
- a. No, neither of them does.      a. #No, neither of them does.
- b. Actually, neither of them does.      b. Actually, neither of them does.

The declarative licenses a *no* response; the interrogative does not. The declarative really *asserts* that at least one of Ann and Bill plays the piano (in the sense that it excludes—technically speaking—the possibility that neither Ann nor Bill plays), whereas the interrogative merely *suggests* that at least one of Ann and Bill plays. Thus, this example illustrates that *no* can be used to deny an assertion, but not to cancel a suggestion. As

<sup>5</sup> It should perhaps be emphasized that closure is *not* interpreted here as signaling *exhaustivity* (as in [22]). That is, it does not imply that ‘nobody else plays the piano’ or something of that kind. And this is for a good reason: disjunctive interrogatives with closure intonation generally do not exhibit any exhaustivity effects. Therefore, closure intonation and exhaustivity effects should be seen as (at least partly) independent phenomena.

illustrated in (40b), cancellation of a suggestion requires a ‘weaker’ disagreement particle such as *actually* or *in fact* (see [10,23] for similar observations).

Thus, *no*, in response to a question  $\alpha$ , denies the possibilities that  $\alpha$  highlights, but is felicitous only if denying these possibilities does not cancel the suggestion that  $\alpha$  expresses. This accounts for the contrast between (39) and (40), and also for the licensing and interpretation of *no* in response to disjunctive interrogatives with block or open intonation. And that brings us full circle. All the empirical observations that we started out with have been accounted for.

We believe that the analysis proposed here may shed light on a wider range of phenomena than the ones explicitly discussed here. For instance, it leads to a simple account of ignorance implicatures, and of the exclusive component of disjunctive declaratives. It may also illuminate the cross-linguistic manifestation of disjunctive interrogatives. For more discussion, we refer to the extended version of this paper [1].

## References

1. Roelofsen, F., van Gool, S.: Disjunctive questions, intonation, and highlighting. Technical report, ILLC, Univ. of Amsterdam (2009)
2. Pruitt, K.: Perceptual relevance of prosodic features in non-*wh*-questions with disjunction. University of Massachusetts Amherst (2007) (manuscript)
3. Aloni, M., van Rooij, R.: The dynamics of questions and focus. In: Jackson, B. (ed.) Proceedings of SALT 12, Cornell University. CLC Publications (2002)
4. Han, C., Romero, M.: The syntax of whether/Q... or questions: Ellipsis combined with movement. *Natural Language & Linguistic Theory* 22(3), 527–564 (2004)
5. Beck, S., Kim, S.: Intervention effects in alternative questions. *The Journal of Comparative Germanic Linguistics* 9, 165–208 (2006)
6. Biezma, M.: Alternative vs polar questions: the cornering effect. In: Ito, S., Cormany, E. (eds.) Proceedings of Semantics and Linguistic Theory, SALT XIX (2009)
7. Bartels, C.: The intonation of English statements and questions: a compositional interpretation. Routledge, New York (1999)
8. Groenendijk, J.: Inquisitive semantics: Two possibilities for disjunction. In: Bosch, P., Gabelaia, D., Lang, J. (eds.) Seventh International Tbilisi Symposium on Language, Logic, and Computation. Springer, Heidelberg (2009)
9. Mascarenhas, S.: Inquisitive semantics and logic. MSc Thesis, Univ. of Amsterdam (2009)
10. Groenendijk, J., Roelofsen, F.: Inquisitive semantics and pragmatics. Presented at the Workshop on Language, Communication, and Rational Agency at Stanford (2009), [www.illc.uva.nl/inquisitive-semantics](http://www.illc.uva.nl/inquisitive-semantics)
11. Ciardelli, I., Roelofsen, F.: Inquisitive logic. To appear in the *Journal of Philosophical Logic* (2009), [www.illc.uva.nl/inquisitive-semantics](http://www.illc.uva.nl/inquisitive-semantics)
12. Ciardelli, I.: Inquisitive semantics and intermediate logics. MSc Thesis, Univ. of Amsterdam (2009)
13. Balogh, K.: Theme with variations: a context-based analysis of focus. PhD thesis, University of Amsterdam (2009)
14. Hamblin, C.L.: Questions in Montague English. *Foundations of Language* 10, 41–53 (1973)
15. Kratzer, A., Shimoyama, J.: Indeterminate pronouns: The view from Japanese. In: Otsu, Y. (ed.) The Proceedings of the Third Tokyo Conference on Psycholinguistics, pp. 1–25 (2002)
16. Alonso-Ovalle, L.: Disjunction in Alternative Semantics. PhD thesis, University of Massachusetts, Amherst (2006)

17. Karttunen, L.: Syntax and semantics of questions. *Linguistics and Philosophy* 1, 3–44 (1977)
18. Groenendijk, J., Stokhof, M.: *Studies on the Semantics of Questions and the Pragmatics of Answers*. PhD thesis, University of Amsterdam (1984)
19. Krifka, M.: For a structured meaning account of questions and answers. *Audiatur Vox Sapientia. A Festschrift for Arnim von Stechow*, 287–319 (2001)
20. Blutner, R.: Questions and answers in an orthoalgebraic approach. University of Amsterdam (2009) (manuscript), <http://www.blutner.de>
21. Partee, B.H., Rooth, M.: Generalized conjunction and type ambiguity. In: von Stechow, A. (ed.) *Meaning, Use, and Interpretation*. de Gruyter, Berlin (1982)
22. Zimmermann, E.: Free choice disjunction and epistemic possibility. *Natural Language Semantics* 8, 255–290 (2000)
23. Groenendijk, J.: *Inquisitive semantics and dialogue pragmatics*. Rutgers lecture notes (2008), [www.illc.uva.nl/inquisitive-semantics](http://www.illc.uva.nl/inquisitive-semantics)

# The Semantics of Count Nouns

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**Abstract.** We offer an account of the semantics of count nouns based on the observation that for some count nouns, the set of atoms in the denotation of the singular predicate is contextually determined. The denotation of singular count nouns is derived relative to a context  $k$ , where  $k$  is a set of entities which count as atoms in a particular context. An operation  $\text{COUNT}_k$  applies to the mass noun denotation and derives the count meaning: a set of ordered pairs  $\langle d, k \rangle$  where  $d$  is a member of  $N \cap k$  and  $k$  is the context relative to which  $d$  counts as one. Count nouns and mass nouns are thus typically distinct and the grammatical differences between them follow from this. We distinguish between naturally atomic predicates, which denote sets of inherently individuable entities or Boolean algebras generated from such sets, and semantically atomic predicates, which denote sets which are atomic relative to a particular context  $k$ . This distinction is orthogonal to the mass count distinction.

**Keywords:** mass/count distinction, atomicity, counting, measuring, homogeneity, nominal interpretations, semantics of number.

## 1 Introduction

This paper proposes a semantics for count nouns which makes explicit the grammatical basis of counting. We assume the semantics for mass nouns proposed in Chierchia[1], according to which mass nouns denote atomic Boolean algebras generated by the complete join operation from a possibly vague set of atoms. However, we differ from Chierchia in our analysis of count nouns. Chierchia argues that the atomic elements in mass denotations cannot be grammatically accessed because a mass noun is lexically plural, i.e. the mass lexical item denotes a Boolean algebra. In contrast, singular count nouns denote a unique set of salient atoms, which as a consequence are grammatically accessible. Plural count nouns denote the closure of the singular denotation under the complete join operation; thus plural count nouns and mass nouns denote the same kind of entities (although plural count nouns do not include the singular entities in their denotations). The grammatical difference between mass nouns and plural count nouns is only whether the set of atoms from which the Boolean algebra is generated is or is not lexically accessible. Lexical accessibility is determined by the pragmatic accessibility of a salient, stable set of atoms (Chierchia[2]). We argue in this paper that Chierchia's account of count nouns is inadequate and that the salience or non-vagueness of a presupposed atomic set cannot be at the basis of count noun semantics. There are two reasons for this: (i) the existence of

mass predicates such as *furniture* which denote sets generated from a set of non-vague, salient atoms, and (ii) the existence of context dependent count nouns such as *wall* and *hedge*.

### 1.1 Mass Nouns May Denote Sets of Salient Atoms

As Chierchia[1],[2] and Gillon[3] have pointed out, some mass nouns, like *furniture*, denote Boolean algebras generated from sets of inherently individuable atoms. Barner and Snedeker[4] show that these mass nouns, in contrast to mass nouns like *mud* but like count nouns, allow quantity judgments in terms of number rather than overall volume. Thus *Who has more furniture?* will be answered by comparing numbers of pieces of furniture, while *Who has more sand/mud?* will be answered by comparing overall quantities of mud or sand, no matter how many individual piles or heaps or units the stuff is arranged in. Rothstein[5] and Schwarzschild[6] both show that predicates like *furniture* (which Rothstein calls ‘naturally atomic’) make the atomic entities salient for distributive adjectives such as *big*. Pires de Oliveira and Rothstein[7] show that naturally atomic predicates may be antecedents for reciprocals in Brazilian Portuguese, although this is impossible in English.

### 1.2 Singular Count Noun Denotations May Be Contextually Determined

There are a significant number of count nouns which are not associated with a unique set of salient atoms: instead the set of atoms in the denotation of these count nouns may be variable and highly context dependent. Krifka[8] shows that nouns such as *sequence* and *twig* are non-quantized, and Mittwoch[9] shows that this is true also of mathematical terms such as *plane* and *line*. Rothstein[10] shows that this generalises to classes of singular count nouns denoting sets of entities with context dependent physical boundaries. These include nouns such as *fence*, *wall*, *hedge* and *lawn*, where the boundaries of the atomic entities are defined by Cartesian coordinates, and classificatory nominals such as *bouquet/bunch*. For example, if a square of land is fenced or walled in on four sides, with the fence or wall on each side built by a different person, we can talk of one (atomic) fence/wall enclosing the field, or we can talk of the field being enclosed by four fences or walls, each one built by a different person. The atomic (and countable) units thus depend on the contextually relevant choice of what counts as one. Similarly, flowers are often sold in bunches, but I may decide that a ‘predesignated’ bunch of flowers is not big enough for my purposes and buy two bunches which I then put together and deliver as a single bunch. Many other such examples can be constructed. Thus, count noun meanings must involve sets of context dependent atoms. Crucially, fences, walls and bunches in these contexts can be counted, as in *four fences/three walls/two bunches of flowers*, whereas furniture cannot be counted (*\*three furnitures*), even though *furniture* may be naturally associated with a uniquely determined set of salient atomic entities. This indicates that the counting operation can apply to count nouns because the association with the set of contextually relevant atoms is grammatically encoded.

## 2 Count Noun Denotations

We now look at how to encode the contextual dependence of count nouns. Nominals are interpreted with respect to a complete atomic Boolean algebra  $M$ .  $\sqcup_M$ , the sum operation on  $M$  is the complete Boolean join operation (i.e. for every  $X \subseteq M$ :  $\sqcup_M X \in M$ ). With Chierchia, we assume that the set of atoms  $A$  of  $M$  is not fully specified, vague. We assume that mass and count nouns are derived from an abstract root noun, with its denotation in  $M$ . This gives the following noun denotations:

**Root nouns:**  $N_{\text{root}} \subseteq M$ : Root nouns denote  $N_{\text{root}}$ , a Boolean algebra, the closure of a set of atoms in  $M$  under the sum operation  $\sqcup_M$ . Their type is  $\langle d, t \rangle$  ( $d$  the type of individuals).

**Mass nouns:**  $N_{\text{mass}} = N_{\text{root}}$ : Mass nouns just are root nouns, of type  $\langle d, t \rangle$ .

**Singular count nouns:** A singular count noun denotes  $N_k$ , a set of ordered pairs, derived from  $N_{\text{root}}$  via an operation  $\text{COUNT}_k$ , to be specified below.  $N_k$  is of type  $\langle d \times k, t \rangle$  i.e.  $N_k \subseteq M \times \{k\}$ . The first projection of an element of  $N_k$  is an entity in  $N_{\text{root}}$ , and the second projection is  $k$ , where  $k$  indicates the context relative to which the singular count noun denotation has been derived. Context dependency is captured via the dependency of the  $\text{COUNT}_k$  operation on the choice of context  $k$ .

**Plural count nouns:** In a default context  $k$ ,  $\text{PL}(N_k) \subseteq M \times \{k\}$ , where the first projection is the closure of  $N_{\text{root}} \cap k$  under sum, and the second projection is  $k$ .

We now show in more detail how these types are derived. The denotation of a root noun,  $N_{\text{root}}$ , is the Boolean algebra generated under  $\sqcup_M$  from a set of atoms  $A_N \subseteq A$  (so root noun denotation  $N_{\text{root}}$  has the same 0 as  $M$ , its atoms are  $A_N$ , and its 1 is  $\sqcup_M(A_N)$ ). Mass nouns have the denotation of root nouns, so  $N_{\text{mass}} = N_{\text{root}}$ . (Note that the choice of this particular theory of mass nouns is not essential to what follows. We assume it for simplicity.) For mass nouns like *furniture*, the atoms in the denotation of the nominal will be the salient indivisible entities, while for mass nouns like *mud* the atoms will be an underdetermined vague set of minimal mud parts.

Singular count nouns denote sets of countable atoms. Counting is the operation of putting entities which are predesignated as atoms, i.e. entities that count as 1, in one-to-one correspondence with the natural numbers. We have seen that what counts as ‘one entity’ is contextually determined. Since countability is a grammatical property, we claim that the grammatical encoding of the context is what makes a noun count.

We propose that singular count nouns are interpreted relative to a *counting context* (for simplicity *context*)  $k$ . A *context*  $k$  is a set of objects from  $M$ ,  $k \subseteq M$ ,  $K$  is the set of all contexts. The set of count atoms determined by context  $k$  is the set  $A_k = \{ \langle d, k \rangle : d \in k \}$ .  $A_k$  is going to be the set of atoms of the count structure  $B_k$  to be determined below. The objects in  $k$  are not mutually disjoint with respect to the order in  $M$ , since we may want, in a single context, my hands and each of my fingers to count as atoms, i.e. to be members of the same contextual set of atoms  $k$ . Thus it may be the case that for two entities  $lt$  and  $lh$  (my left thumb and my left hand),  $lt \sqsubseteq_M lh$ , but nevertheless  $lt, lh \in k$ . In that case  $\langle lt, k \rangle, \langle lh, k \rangle \in A_k$ . So both my left thumb and my left hand

are atoms which count as one entity in context  $k$ . Given this we cannot lift the order on the count Boolean domain from the mass domain.

We want the count domain  $B_k$  to be a complete atomic Boolean algebra generated by the set of atoms  $A_k$ . Up to isomorphism, there is only one such structure,  $B_k$ .

**Definition of  $B_k$ :**  $B_k$  is the unique complete atomic Boolean algebra (up to isomorphism) with set of atoms  $A_k$ . We let  $\sqcup_k$  stand for the corresponding complete join operation on  $B_k$ .

However, we would like to lift this order from the mass domain as much as we can. If  $k' \subseteq k$  and  $k'$  is a set of mutually non-overlapping objects in  $M$ , there is no problem in lifting part-of relations of the sums of  $k'$ -objects from the mass domain. ( $k'$  is a set of mutually non-overlapping objects in  $M$  iff for all  $d, d' \in k'$ :  $d \sqcap_M d' = 0$ ;  $\sqcap_M$  is the standard meet operation on  $M$ ). Thus we impose the following constraint on  $B_k$ :

**Constraint on  $B_k$ :** For any set  $k' \subseteq k$  such that the elements of  $k'$  are mutually  $M$ -disjoint, the Boolean substructure  $B_{k'}$  of  $B_k$  is given by:  $B_{k'} = \{<\sqcup_M X, k'>: X \subseteq k'\}$  with the order lifted from  $\sqcup_M$ .

The plurality order is not lifted from the mass domain for objects that overlap. i.e. the sum of my hands and my fingers is a sum of twelve atoms, hence not lifted from the mass domain (*atom*, here is a metalanguage predicate).

**(Singular) count predicates**, in particular count nouns, denote subsets of  $A_k$ . Count nouns are derived as follows. All nouns are associated with a root noun meaning,  $N_{\text{root}}$ , a Boolean algebra generated under  $t_M$  from a set of  $M$ -atoms. As noted above,  $N_{\text{mass}} = N_{\text{root}} \subseteq M$ . A singular count noun denotation  $N_k$  is derived by an operation  $\text{COUNT}_k$  which applies to  $N_{\text{root}}$  and picks out the set of ordered pairs  $\{<d, k>: d \in N \cap k\}$ . These are the entities which in the given context  $k$  count as atoms, and thus can be counted. The parameter  $k$  is a parameter manipulated in context. Thus, in the course of discourse we have as many relevant  $k$ s around as is contextually plausible. We can think of these contexts as contextually defined perspectives on a situation or model, and the set of contextually relevant contexts is rich enough to allow different numbers of  $N$  entities in a situation, depending on the choice of  $k$ , i.e. the choice of counting perspective that is chosen. The  $\text{COUNT}_k$  operation is defined in (1):

$$\text{For any } X \subseteq M: \text{COUNT}_k(X) = \{<d, k>: d \in X \cap k\} \quad (1)$$

Thus the interpretation of a count noun in context  $k$  is:  $N_k = \text{COUNT}_k(N_{\text{root}})$ .

The denotation of a singular count noun is thus a set of ordered pairs whose first projection is an entity in  $N_{\text{root}} \cap k$ , and whose second projection is context  $k$ . these sets are *semantically atomic sets*, since the criterion for what counts as an atom is semantically encoded by the specification of the context  $k$ . The  $N_{\text{root}} \cap k$ , or  $N_{\text{root},k}$ , is the set of semantic atoms in  $N_{\text{root}}$  relative to  $k$ . This is the set of atomic  $N$ -entities used to evaluate the truth of an assertion involving a count noun  $N_k$  in context  $k$ .

The atoms in  $k$  are not constrained by a non-overlap condition, since we want to allow examples such as *I can move my hand and my five fingers* and *It took 2500 bricks to build this wall*, which make reference to atomic elements and their atomic parts. Non-overlap comes in as a constraint on default contextual interpretations:

**Constraint on count predicates:** In a default context  $k$ , the interpretation of singular count predicate  $P$  is a set of mutually non-overlapping atoms in  $k$  (where  $\langle a, k \rangle$  and  $\langle a', k \rangle$  don't overlap iff  $a \sqcap_M a' = 0$ ).

This guarantees that when we count entities in the denotation of  $N_k$  we will be counting contextually discrete, non-overlapping entities. This allows us to capture the generalisation that counting, which presumes non-overlap of atomic entities, is restricted to lexical predicates and does not usually occur across conjunctions. Thus, *I moved my hand and its five fingers*, does not imply a felicitous use of *Hence I moved six body parts*, since the predicate *body part* in a default context will be interpreted as denoting a set of non-overlapping objects.

**Plural count nouns** are derived from singular count noun meanings, using the standard plural operation, defined in the current count structures, and thus adapted to the meaning of the count noun. The plural operation gives the closure of  $N_{\text{root},k}$  under the sum operation, while keeping track of the context. Link's plural operation '\*' (Link 1983) is as follows:

$$*A = \{d: \exists Y \subseteq A: d = \sqcup Y\} \quad (2)$$

We adapt this for application to singular count nouns in the following way. For a relation  $N_k$ , we define the  $n$ -th projection of  $N_k$  as follows:

$$\pi_1(N_k) = \{d: \langle d, k \rangle \in N_k\} = N_{\text{root},k} \quad (3)$$

$$\pi_2(N_k) = k \quad (4)$$

For convenience we also define  $\pi_n$  directly for pairs:

$$\begin{aligned} \pi_1(\langle d, k \rangle) &= d \\ \pi_2(\langle d, k \rangle) &= k \end{aligned} \quad (5)$$

Note that for any  $\langle d, k \rangle \in N_k$ ,  $\pi_2(\langle d, k \rangle) = \pi_2(N_k) = k$ .

With this we lift the \*-operation to the present count structures:

$$\text{In default context } k: PL(N_k) = *N_k = \{\langle d, k \rangle: d \in \pi_1(N_k)\} \quad (6)$$

(In non-default contexts, we don't lift plurality from the mass domain. Thus in a non-default context  $k$ :  $*\pi_1(N_k) = \{d: \exists Y \subseteq A_k: d = \sqcup_k Y\}$  )

The denotation of the plural count noun depends on the contextually determined denotation of the singular  $N_k$ . However, the non-overlap condition in the constraint on count predicates guarantees that in default contexts, the order of the plural count noun denotation is lifted directly from  $M$ . The plural noun denotes a set of ordered pairs

where the first element is in the closure of  $N_{\text{root},k}$  under sum and the second element is the context  $k$ .  $N_{\text{root},k}$  may vary depending on choice of  $k$ , and the denotation of the plural set will similarly vary, depending on  $N_{\text{root},k}$ . Crucially, the information about the context determining the set of atoms is preserved in the plural denotation. Since there is no guarantee that, even with a predicate like *hair*,  $\text{HAIR}_{\text{root},k}$  and the set of atoms in  $\text{HAIR}_{\text{root}}$  is the same set, there is also no guarantee that  $\text{HAIR}_{\text{root}}$  and  $*\text{HAIR}_k$  is the same set. So though *the hair* and *the hairs* may well refer to the same real-world entity, there is no guarantee that they do so. Note that, since  $k$  itself is not constrained by a non-overlap condition, the plural domain may contain elements not lifted from  $M$ . These will not be in the denotations of lexical predicates, but they will be in the denotations of other expressions built up in the grammar like the conjunctive definite *my hand and its five fingers*. However, as noted above, in a normal context, although *my hand and its five fingers* denotes a sum of 6 atoms, we do not normally count across conjunctions. Thus, *I moved my hand and its five fingers*, does not imply a felicitous use of *Hence I moved six body parts*. Counting across conjunctions is usually permissible only when the conjoined nouns can be reanalyzed as a complex lexical predicate, e.g. *three boys and girls* has a felicitous interpretation as *three children*. (For discussion of non-default contexts, situations where overlapping entities are felicitously counted, and the interpretation of bunch predicates such as *deck of cards*, see Rothstein, to appear.)

### 3 Implementing the Analysis

Mass nouns and count nouns are of different types: mass nouns denote subsets of  $M$ , and are of type  $\langle d, t \rangle$ ; count nouns denote subsets of  $M \times K$  and are of type  $\langle \langle d \times k \rangle, t \rangle$ . In this section we explore how this works compositionally:

#### 3.1 Operations Which Are Not Sensitive to the Count/Mass Distinction

**Adjectival modification:** Some grammatical operations apply equally well to both types, for example adjectival modification as in *an expensive chair*, *expensive furniture*. We treat *expensive* as denoting a property of individuals, which in its attributive reading shifts to the predicate modifier type  $\langle \langle d, t \rangle, \langle d, t \rangle \rangle$ . As a predicate modifier, *expensive* applies to mass nominal expressions of type  $\langle d, t \rangle$ , denoting the function  $\lambda P \lambda x. P(x) \wedge \text{EXPENSIVE}(x)$ . ( $P$  is a variable over expressions of type  $\langle d, t \rangle$ .) We assume a count modifier  $\text{EXPENSIVE}_{\text{count}}$  modifying expressions of type  $\langle d \times k, t \rangle$ , which is defined in terms of  $\text{EXPENSIVE}$ , using the  $\pi_n$  function defined in (5). ( $P$  is a predicate variable type  $\langle d \times k, t \rangle$ , and  $x$  is a variable of type  $d \times k$ ).  $\text{EXPENSIVE}_{\text{count}}$  denotes the function  $\lambda P \lambda x. P(x) \wedge \text{EXPENSIVE}(\pi_1(x))$ .

**Conjunction:** Conjunction of count and mass nouns such as *tables and other furniture* must be at the type of mass noun. This is shown in the partitive constructions in (7), where a conjunction of a mass DP and a count DP can occur in the partitive only with mass determiners:

\*Three/\*many of [the tables and the furniture] arrived damaged. (7)

Some/much of the tables and the furniture arrived damaged. (8)

We assume that *and* conjoins arguments at the same type. In cases of type mismatch, the count noun lowers to a mass reading via the  $\pi_n$  function.

$$\begin{aligned} \llbracket \text{tables and (other) furniture} \rrbracket &= \text{AND}(\pi_1(\llbracket \text{tables} \rrbracket), \llbracket \text{furniture} \rrbracket) \\ &= \text{AND}(*\text{TABLES}_{\text{root}\leq k}, \text{FURNITURE}) \end{aligned} \quad (9)$$

### 3.2 Operations Which Distinguish between Mass and Count Nouns

**Grammatical counting:** Grammatical counting, i.e. modification by numerical expressions is sensitive to the count/mass distinction. We count entities which are atomic in a particular context, and thus the cardinality function, used in the interpretation of numeral modifiers, must be interpreted relative to the context used to derive the noun and encoded in the count noun meaning. This means that number expressions must be sensitive to the typal difference between mass and count nouns, and select only expressions of type  $\langle d \times k, t \rangle$ . A numerical expression such as *three* denote a function from  $\langle d \times k, t \rangle$  into  $\langle d \times k, t \rangle$ , and thus cannot apply to mass nouns. *Three* applies to a set of ordered pairs  $N_k$  and gives the subset of  $N_k$ , such that all members of  $\pi_1(N_k)$  are plural entities with three parts, each of which is an entity in  $k$ . The index on the cardinality function,  $\pi_2(\mathcal{P})$ , shows that the interpretation of the cardinality function is dependent on the  $k$  variable relative to which  $N_k$  is derived.

$$\llbracket \text{three} \rrbracket (N_k) = \lambda \mathcal{P} \lambda x. \mathcal{P}(x) \wedge |\pi_1(x)|_{\pi_2(\mathcal{P})} = 3 \quad (10)$$

*Three* denotes a function which applies to a count predicate  $N_k$  and gives the subset of ordered pairs in  $N_k$ , where the first projection of each ordered pair has three parts which count as atoms in  $k$ .

**Determiner selection:** Determiners are sensitive to the typal difference between  $\langle d, t \rangle$  and  $\langle d \times k, t \rangle$ , as in *every chair* vs *\*every furniture*.

**Partitive constructions:** Partitive constructions such as *three of the chairs*/*\*three of the furniture* are sensitive to the properties of the nominal head of the partitive complement. The determiner heading a partitive shows the same selectional restrictions with respect to the nominal head of its complement as determiners usually show within a DP: *three of the chairs*/*three chairs*; *\*three of the furniture*/*\*three furniture(s)*; *\*much of the chairs*/*\*much chairs*; *much of the furniture*/*much furniture*; We thus need to recover the predicate expression from the DP. This is possible because the operations which construct the DP keep track of the original context  $k$  at all stages. The partitive operates on a definite complement, which is defined using Link's [11] operation,  $\llbracket \text{the} \rrbracket (X) = \sigma(X) = \sqcup X$  if  $\sqcup X \in X$ , otherwise undefined:

$$\begin{aligned} \text{For mass nouns: } \text{the } N_{\text{mass}} &= \sigma N_{\text{mass}} = \sqcup_M N_{\text{mass}}, \text{ if defined.} \\ \text{For count nouns: } \text{the } N_k &= \sigma N_k = \sqcup_k(N_k) = \langle \sqcup_M \pi_1(N_k), k \rangle \text{ if defined} \end{aligned} \quad (11)$$

We lift the part-of relation on ordered pairs in  $M \times \{k\}$  from  $M$ :  $\langle x_1, k \rangle \sqsubseteq_k \langle x_2, k \rangle$  iff  $x_1 \sqsubseteq_M x_2$ .

The partitive operation follows the following definition schema: it operates on a definite complement and gives the set of parts:  $\text{PARTITIVE}(\sigma N) = \{x: x \sqsubseteq (\sigma N)\}$

For a mass predicate,  $\text{PARTITIVE}(\sigma N_{\text{mass}}) = \{x: x \sqsubseteq_M \sqcup_M(N_{\text{mass}})\}$ , i.e.  $N_{\text{mass}}$ .

For a count predicate in context  $k$ ,  $\text{PARTITIVE}(\sigma N_k)$  is again lifted from  $M$ :  $\text{PARTITIVE}(\sigma N_k) = \{\langle x, k \rangle: \langle x, k \rangle \sqsubseteq_k \langle \sqcup_M(\pi_1(N_k)), k \rangle\}$

Crucially, since we kept track of the context  $k$  during all the operations involving the composition of the embedded DP, the operation giving the set of parts of  $\sqcup_k N_k$  will still have access to the original context  $k$ . Determiners in partitives apply to the result of applying  $\text{PARTITIVE}$  to the DP meaning exactly as they would apply to NP within the DP. Since *three* makes use of the parameterised cardinality function which makes reference to  $k$ , it can apply to  $\text{PARTITIVE}(\text{the chairs})$  or  $\text{PARTITIVE}(\text{the pieces of furniture})$  which denote sets of type  $\langle d \times k, t \rangle$ , but not to  $\text{PARTITIVE}(\text{the furniture})$ , which denotes a set of type  $\langle d, t \rangle$ . *Some* applies equally well to both types.

**Reciprocal resolution:** Reciprocal resolution is sensitive to the mass/count distinction. A reciprocal cannot take a mass noun as antecedent, although the mass noun is lexically plural. We assume that reciprocals (in English) are constrained to take as antecedents plural entities in  $M \times K$ . We use this constraint to explain Gillon's[3] observation that *The curtains and the carpets resemble each other* (the 'count' reciprocal) is ambiguous between the collective reading in which the sum of curtains resembles the sum of carpets and vice versa and the distributive reading in which each member of the set  $\text{CURTAINS}_k \cup \text{CARPETS}_k$  resembles all the other members of the set. The mass counterpart, *the curtaining and the carpeting resemble each other*, has only the first, collective, reading. Space constraints prevent giving a full analysis here, but the outline of the explanation is as follows.

On the distributive reading of the count reciprocal, the conjoined DP *the curtains and the carpets* denotes the sum of the maximal plurality of curtains and the maximal plurality of carpets, and the interpretation of the reciprocal requires every two atomic parts of this sum, i.e. atomic individuals in the denotation of  $\sigma \text{CURTAINS}_k \cup \sigma \text{CARPETS}_k$ , to resemble each other. On the second reading of the sentence, the curtains as a group, or singular collectivity, resemble the carpets as a group, or singular collectivity, and vice versa. On this reading the plurality of curtains,  $\sigma \text{CURTAINS}_k$  and the plurality of carpets,  $\sigma \text{CARPETS}_k$  are treated as collections and are raised to the group-atoms  $\text{GR}(\sigma \text{CURTAINS}_k)$  and  $\text{GR}(\sigma \text{CARPETS}_k)$  (see Landman[12] for details of a raising to group atoms operation). We assume that raising to group atoms is relative to a context  $k$ , and that the group atoms are indexed for the context in which they are atomic. Group atoms thus have their denotations in  $M \times K$ , and pluralities of group atoms such as *the curtains and the carpets* can be antecedents for reciprocals, giving the collective interpretation. When the antecedent of the reciprocal is *the curtaining and the carpeting*, the distributive reading is not available. This is because *curtaining* and *carpeting* are nominals of type  $\langle d, t \rangle$ , and distribution down to the atomic parts of

the nominal is distribution down to entities of type  $\langle d, t \rangle$  and not to entities of type  $\langle \langle d \times k \rangle, t \rangle$ . However, the group reading is available, since  $\sigma$ CURTAINING and  $\sigma$ CARPETING can be raised to the atomic collections  $GR(\sigma$ CURTAINING) and  $GR(\sigma$ CARPETING) respectively, and the conjunction will then denote a plurality of atoms in the count domain, i.e. of type  $\langle \langle d \times k \rangle, t \rangle$ .

#### 4 Formal Atomicity, Natural Atomicity and Semantic Atomicity

Our account assumes a single domain  $M$ , and analyses the count/mass distinction as a typal distinction between mass nouns, which denote subsets of the domain  $M$ , and (singular) count nouns which denote sets of indexed entities in  $M \times K$ , where the index indicates the context in which they count as one. Count noun meanings are derived from mass noun (or root noun) meanings by a lexical operation  $COUNT_k$ , which picks out those entities in  $N_{root}$  which count as atoms in the context  $k$  and indexes them as such. While similar to Krifka[13,14], insofar as both this and Krifka's account derive count nouns from mass nouns by a lexical operation, the theories are very different conceptually and formally. Krifka proposes analysing count nouns as extensive measure functions of type  $\langle n, \langle d, t \rangle \rangle$  which apply to a number to give a measure predicate. *Cattle* is a mass predicate of type  $\langle d, t \rangle$ , denoting  $\lambda x. CATTLE(x)$ . It is similar in meaning to the root noun *COW*, and we can assume for our purposes that they are synonymous. The count noun meaning *COW'* is derived from *COW* (or *CATTLE*) and is the function  $\lambda n \lambda x. COW(x) \wedge NATURAL\ UNIT(x) = n$ . This applies to a number and yields a count expression, indistinguishable in type from the mass predicate, but with a different meaning. In the absence of an explicit number word, the predicate is reduced from  $\langle n, \langle d, t \rangle \rangle$  to  $\langle d, t \rangle$  via existential quantification over the  $n$  argument. Crucially, for Krifka, the typal difference between mass and count terms is neutralised before the higher nodes of the  $N$  tree are constructed. In the theory presented here, the typal difference persists up to the DP level and allows grammatical operations such as partitive construction and reciprocal resolution to exploit the typal contrast. This technical difference reflects a deeper conceptual difference. Krifka analyses count nouns as extensive measure functions directly analogous to expressions such as *kilo* and *litre*. We start here from the premise that measuring and counting are very different operations: measuring assigns to a quantity a value on a dimensional scale, while counting puts contextually determined atomic entities in one-to-one correspondence with the natural numbers. I pursue this contrast between measuring and counting in ongoing research[15].

The theory developed here allows us to distinguish three kinds of atomicity:

- (i) Formally atomic sets are sets of atoms which generate atomic Boolean algebras under the complete join operation. These sets may be vague and/or underspecified;
- (ii) Semantically atomic sets are derived via the  $COUNT_k$  operation and denote sets of entities which are atomic in a specified context  $k$ . These sets are grammatically countable.
- (iii) Naturally atomic sets are sets of inherently individuable entities, which may generate denotations for mass nouns as well as for plural count nouns. *Child* and *furniture* are naturally atomic count and mass predicates respectively, while *fence* and *mud* are examples of count and mass predicates which are not naturally atomic. As, we have seen, the distinction between semantic and natural atomicity is linguistically

relevant, since naturally atomic mass predicates allow quantity judgments based on number rather than volume (Barner and Snedeker[4]). They host distributive predicates such as *big* (Rothstein[5]), and in some languages they may provide antecedents for reciprocals (Rothstein[5], Pires de Oliveira and Rothstein[7]). However, natural atomicity is neither a necessary nor sufficient condition for count semantics, which makes use of semantically atomic predicates, thus making possible the counting of atomic elements whose atomicity is context dependent.

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## References

1. Chierchia, G.: Plurality of Mass Nouns and the Notion of Semantic Parameter. In: Rothstein, S. (ed.) *Events and grammar*, Kluwer, Dordrecht (1998)
2. Chierchia, G.: Mass Nouns, Vagueness and Semantic Variation. *Synthese* (in press)
3. Gillon, B.: Toward a Common semantics for English Count and Mass Nouns. *Linguistics and Philosophy* 15, 597–640 (1992)
4. Barner, D., Snedeker, J.: Quantity Judgments and Individuation: Evidence that Mass Nouns Count. *Cognition* 97, 41–66 (2005)
5. Rothstein, S.: Counting and the Mass Count Distinction. to appear in *Journal of Semantics*
6. Schwarzschild, R.: Stubborn Distributivity, Multiparticipant Nouns and The Count/Mass Distinction. In: *Proceedings of NELS 39* (in press)
7. Pires de Oliveira, R., Rothstein, S.: Bare Singular is Really Mass in Brazilian Portuguese. Ms Universidade de Federal de Santa Catarina and Bar-Ilan University (2009)
8. Krifka, M.: Thematic Relations as Links Between Nominal Reference and Temporal Constitution. In: Sag, I., Szabolsci, A. (eds.) *Lexical matters*, CSLI, Stanford (1992)
9. Mittwoch, A.: Aspects of English Aspect: On The Interaction of Perfect, Progressive and Durational Phrases. *Linguistics and Philosophy* 11, 203–254 (1988)
10. Rothstein, S.: *Structuring Events*. Blackwell, Oxford (2004)
11. Link, G.: The Logical Analysis of Plurals and Mass Terms: A Lattice-Theoretical Approach. In: Bäuerle, R., Schwarze, C., von Stechow, A. (eds.) *Meaning, Use and Interpretation*, pp. 303–323. de Gruyter, Berlin (1983); Reprinted in Portner, P., Partee, B. (eds.): *Formal Semantics: The Essential Readings*. Blackwell, Oxford (2002)
12. Landman, F.: Groups II. *Linguistics and Philosophy* 12, 723–745 (1989)
13. Krifka, M.: Nominal Reference, Temporal Constitution and Quantification in Event Semantics. In: Bartsch, R., van Bentham, J., van Emde Boas, P. (eds.) *Semantics and Contextual Expressions*, pp. 75–155. Foris, Dordrecht (1989)
14. Krifka, M.: Common Nouns: A Contrastive Analysis of English and Chinese. In: Carlson, G., Pelletier, F.J. (eds.) *The Generic Book*. Chicago University Press, Chicago (1995)
15. Rothstein, S.: Individuating and Measure Readings of Classifier Constructions: Evidence From Modern Hebrew. In: *Brill Annual of Afroasiatic Languages and Linguistics (BAALL)*, vol. 1, pp. 106–145 (2009)

# Donkey Anaphora in Sign Language I: E-Type vs. Dynamic Accounts<sup>\*</sup>

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**Abstract.** There are two main approaches to the problem of donkey anaphora (e.g. *If John owns a donkey, he beats it*). Proponents of dynamic approaches take the pronoun to be a logical variable, but they revise the semantics of quantification so as to allow an indefinite to bind a variable that is not within its scope. *Older dynamic approaches* took this measure to apply solely to indefinites; *recent dynamic approaches* have extended it to all quantifiers. By contrast, proponents of *E-type analyses* take the pronoun to go proxy for a definite description (with *it* = *the donkey*, or *the donkey that John owns*); in order to satisfy its uniqueness presupposition, they combine this approach with an analysis of *if*-clauses as quantifiers over situations. While competing accounts make very different claims about the coindexing relations that should be found in the syntax, these relations are not morphologically realized in spoken languages. But they *are* arguably realized in sign languages, namely through pointing. We argue that data from French and American Sign Language favor *recent dynamic approaches*. First, in those cases in which E-type analyses and dynamic analyses make different predictions about the formal connection between a pronoun and its antecedent, dynamic analyses are at an advantage. Second, it appears that the same formal mechanism is used irrespective of the indefinite or non-indefinite nature of the antecedent, which argues for recent dynamic approaches over older ones.

**Keywords:** anaphora, E-type anaphora, dynamic semantics, sign language.

## 1 The Debate

### 1.1 The Problem

We attempt to bring new light on the debate on donkey anaphora by investigating data from two sign languages, French Sign Language (LSF) and American Sign Language (ASL). Our enterprise is motivated by the following considerations:

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- (i) Competing approaches to donkey anaphora make different predictions about the patterns of coindexing that are found in different examples.
- (ii) In sign languages, coindexing is arguably realized overtly, by way of pointing (Sandler and Lillo-Martin 2006).
- (iii) Therefore sign languages could bring new light to this debate, which has remained open despite quite a bit of work in formal semantics.

The problem is illustrated in (1) and (2).

- (1) **Indefinites**
  - a. John owns a donkey. He beats it.
  - b. If John owns a donkey, he beats it.
- (2) **Non-Indefinites**
  - a. John owns fewer than 5 donkeys. He beats them
  - b. If John owns fewer than 5 donkeys, he beats them.

In each case, the pronoun is semantically dependent on the quantifier; but it is not c-commanded by it. This poses a problem if the following two standard assumptions are adopted:

- (i) Pronouns are logical variables.
- (ii) The semantics of quantifiers gives rise to a standard notion of scope, namely c-command.

Dynamic approaches preserve (i) but revise (ii) (e.g. Kamp 1981, Heim 1982, Groenendijk and Stokhof 1991). E-type approaches preserve a version of (ii) but revise (i), taking pronouns to go proxy for definite descriptions (e.g. *the donkey*, or *the donkey that John owns*; e.g. Evans 1980, Heim 1990, Ludlow 1994, Elbourne 2005). As we will see below, however, when all the necessary refinements of the E-type approach are taken into account, the two theories diverge considerably less than this cursory characterization suggests – so much so that under certain conditions the E-type approach can converge with the dynamic approach (Dekker 2004). Thus we cannot reasonably hope to distinguish empirically all conceivable E-type approaches from all dynamic analyses; but we will draw a distinction between some of those.

## 1.2 E-Type Approaches

E-type approaches have the following general ingredients:

- (3) **Pronouns as descriptions**  
Pronouns are treated as being (syntactically and/or semantically) definite descriptions in disguise. Depending on the approach, (1)b is analyzed as in a. or b. below, where *it* has the semantics of the definite description operator.  
a. If John owns a donkey, he beats it ~~donkey he has~~  
b. If John owns a donkey, he beats it ~~donkey~~ (Elbourne 2005)
- (4) **Quantification over situations / events**  
In order for the uniqueness presupposition of the definite description to be satisfied, *if*-clauses (and more generally all operators) must be taken to quantify over very fine-grained situations or events.

(5) **Formal Link**

In order to account for the classic contrast between a. and b. below, E-type theories must establish a 'formal link' between the pronoun and its antecedent.

a. Every man who has a wife is sitting next to her.

b. ?\* Every married man is sitting next to her (Heim 1990)

Elbourne 2005 takes the formal link to result, quite simply, from a syntactic ellipsis of the NP (e.g. *her = the ~~wife~~*).

Importantly, E-types accounts treat in a uniform fashion the case of indefinite and non-indefinite antecedents, as is illustrated in (6).

(6) a. If John owns a donkey, he beats it ~~donkey~~.

b. If John owns fewer than 5 donkeys, he beats them ~~donkeys~~.

**1.3 Dynamic Approaches**

Dynamic approaches share the following properties:

(7) **Pronouns as variables**

Pronouns are treated as logical variables, and can be coindexed with non-c-commanding indefinites, as illustrated in a. and b.

a. John owns [a donkey]<sub>i</sub>. He beats it<sub>i</sub>.

b. If John owns [a donkey]<sub>i</sub>, he beats it<sub>i</sub>.

(8) **Revision of quantification**

Quantification is revised so as to make it possible for a variable to depend on a non-c-commanding quantifier. This can be done in purely semantic terms, by way of quantification over assignment function (Heim 1983, Groenendijk and Stokhof 1991); or through syntactic stipulations such as those illustrated in a. and b., where  $\exists$  and  $\forall$  are unselective quantifiers (cf. Heim 1983, Kamp 1981; in these implementations, indefinites are taken to introduce variables).

a.  $\exists$  [John owns [a donkey]<sub>i</sub>. He beats it<sub>i</sub>.]

b.  $\forall$  [John owns [a donkey]<sub>i</sub>[he beats it<sub>i</sub>]

(9) **Formal Link: coindexing**

Coindexing provides a formal link between a pronoun and its antecedent, and it has a direct semantic reflex.

Dynamic approaches differ in their treatment of donkey pronouns that depend on quantifiers that are not indefinites. To see why there is an issue in the first place, consider the truth conditions that a simple-minded extension of (8)a would predict:

(10) **Problem**

a. John owns [at least 2 donkeys]. He beats them.

Bad prediction:  $\exists X$  [John owns  $X$  &  $\geq 2$  donkeys( $X$ ) & John beats  $X$ ]

b. John owns [fewer than 5 donkeys]. He beats them.

Bad prediction:  $\exists X$  [John owns  $X$  &  $< 5$  donkeys( $X$ ) & John beats  $X$ ]

It is immediate that the truth conditions captured by (10) are inadequate.

-Intuitively, (10)a entails that John beats *all the donkeys that he has*. But this entailment is not captured by the proposed truth conditions: the fact that the pronoun refers to the *maximal* group of donkeys that John owns is left unaccounted for.

-The same problem arises in (10)b: the inference that John beats all the donkeys that he has is not captured. But in addition, the proposed truth conditions do not even entail that John owns fewer than five donkeys (all they entail is that one can find a group of fewer than five donkeys that John owns – which is far too weak).

There are two broad solutions to the problem. One is a mixed approach (Kamp and van Eijck 1993): for indefinite antecedents, the standard dynamic line is adopted; for other antecedents, a version of the E-type approach is posited, one in which the quantifier has its ‘usual’ meaning but the pronoun goes proxy for a definite description – which directly accounts for the maximality condition observed in (10)a.

(11) **Mixed Solution** (Kamp & van Eijck 1993)

- a. Indefinites are treated in the dynamic way.
- b. Other quantifiers are treated with some version of the E-type account.

The alternative is a pure dynamic approach, one in which *all* quantifiers (not just indefinites) introduce discourse referents and can bind variables that they do not c-command. In order to address the problems seen in (10), quantifiers such as *at least two* and *fewer than five* are taken to introduce discourse referents *together with explicit maximality conditions*. As is illustrated in (10), this measure makes it possible to derive the correct truth conditions within a pure dynamic system.

(12) **Pure Solution** (e.g. van den Berg 1996, Nouwen 2003, Brasoveanu 2006)

- a. John owns [at least 2 donkeys]. He beats them.  
 $\exists X$  [John owns  $X$  &  $X = [\text{Max } Y: \text{donkey}(Y) \text{ \& John owns } Y] \text{ \& } \geq 2$   
donkeys( $X$ ) & John beats  $X$ ]
- b. John owns [fewer than 5 donkeys]. He beats them.  
 $\exists X$  [John owns  $X$  &  $X = [\text{Max } Y: \text{donkey}(Y) \text{ \& John owns } Y] \text{ \& } < 5$   
donkeys( $X$ ) & John beats  $X$ ]

## 1.4 The Complexity of the Debate

The debate between E-type and dynamic approaches is more subtle than it looks at first sight. In a nutshell, some recalcitrant examples have forced the E-type approach to adopt a mechanism of quantification over extremely fine-grained situations, which looks quite a bit like quantification over assignment functions (Dekker 2004). Consider the sentences in (13):

- (13) a. A bishop met a bishop. He blessed him.  
b. If a bishop meets a bishop, he blesses him.

The potential difficulty is immediate for Elbourne’s (2005) theory: if *he* and *him* are both construed as *the bishop*, it is not clear how their uniqueness presuppositions can be satisfied. But versions of the E-type analysis that resort to a longer descriptive content are no better off: resolving *he* as *the bishop that met a bishop* and *him* as *the bishop that a bishop met* won’t help a bit. The difficulty is that, to put it roughly, the two bishops of the antecedent clause play entirely symmetric roles.

How can the symmetry be broken? There are in fact two difficulties, which we illustrate on the example of (13)b (which is discussed in detail in Elbourne 2005).

(i) First, situations must be made fine-grained enough that the ‘symmetry’ between the bishops mentioned in the antecedent can in principle be broken. An old insight, called ‘Chierchia’s Conjecture’ in Dekker 2004, is that in the end situation-theoretic analyses might have to make situations as fine-grained as assignment functions. Dekker 2004 shows that with quite a few assumptions – which he takes to go against the spirit of the framework – situations are indeed isomorphic to assignment functions. One of these assumptions is that a situation in which *bishop B meets bishop B’* is different from a situation in which *situation B’ meets bishop B*. Such an assumption is accepted by Elbourne 2005; it is an important ingredient of his solution.

(ii) Second, *even* if situations are made extremely fine-grained, the situation-theoretic analysis must endow the pronouns in the consequent clause with enough descriptive content to pick out different individuals. Suppose for instance that we took *if*-clauses to quantify over situations that are just tuples of individuals – thus accepting Dekker’s isomorphism between situations and tuples of individuals. We would *still* have to explain how the pronouns *he* and *him* manage to pick out different individuals in the same situation (i.e. in the same tuple). One way to do so would be to stipulate that they come with some equivalent of indices, so that for instance  $he_1$  evaluated with respect to a situation  $s$  with  $s = \langle B, B' \rangle$  denotes  $B$ , while in the same situation  $he_2$  denotes  $B'$ . But it is immediate that such a radical step would make the situation-theoretic analysis even closer to its dynamic competitors.

These formal points are worth keeping in mind when one seeks to assess the donkey anaphora debate on empirical grounds. If indeed the two approaches can in principle converge, it might be hard to decide the debate in favor of one analysis and against all versions of the other. Rather, we can only hope to show something weaker, namely that a given theory is incorrect *or* must borrow essential formal tools from its competitor. This is what we will conclude about the E-type approach. To reach this result, we elicited examples from LSF with one main informant (Informant A), who became deaf around the age of 6, and two additional informants (Informants B and C) who were born deaf and are thus genuine native signers. We also elicited examples from ASL with a native signer (Informant 1, born deaf to deaf parents).

## 2 Predictions for Sign Language

### 2.1 Pronouns in Sign Language

In the sign languages that have been described, the relation between a pronoun and its antecedent is usually realized through the intermediary of *loci*, which are positions in signing space that are associated with nominal elements (Sandler and Lillo-Martin 2006). A pronoun that depends on a proper name will thus point towards (or ‘index’) the locus in which the proper name was signed. Since there appears to be an arbitrary number of possible loci, it was suggested that the latter are the morphological realization of indices (Sandler and Lillo-Martin 2006). This makes it particularly interesting to use sign language to investigate a theoretical debate that revolves around the nature of coindexing relations.

Of course, it could be that the anaphoric system of sign language is entirely different from that found in spoken languages. If so, we would be getting from sign language morphological evidence on a *different* system from the one that had originally prompted the donkey anaphora debate. But despite the difference in modality, there are some striking similarities between sign language pronouns and their spoken language counterparts.

In the following, sign language sentences are glossed in capital letters; subscripts correspond to the establishment of locations ('loci') in signing space; pronouns, glossed as IX (for 'index') as well as other expressions can then point back towards these locations. In such cases, the location is suffixed to the pronoun, so that IX-a is a pronoun that points words location a, while IX-b is a pronoun that points towards location b; the number 1 corresponds to the position of the signer (hence 1st person).

(i) First, in simple cases, the same ambiguity between strict and bound variable readings is found in both modalities, as is illustrated in (14) and (15).

(14) **LSF**

a. FANTASTIC. PIERRE LIKE WIFE POSS-a. IX-b JEAN TOO.

(Informant A 369; cf. Informant C, 193)

'It's fantastic. Pierre loves his wife, and Jean does too [= like Pierre's wife].'

b. COMPLICATED. PIERRE LIKE WIFE POSS-a. IX-b JEAN IX-b TOO.

(Informant A, 374; cf. Informant C, 201)

'Things are complicated. Pierre loves his wife, and Jean does too [= love Jean's wife]'

(15) **ASL**

IX-1 POSS-1 MOTHER LIKE. IX-a TOO. (Inf 1 108)

I like my mother. He does too [= like my / his mother]

(ii) Second, sign language pronouns appear to be constrained by at least some of the syntactic constraints on binding studied in syntax. For instance, versions of the following constraints have been described for ASL (Sandler and Lillo-Martin 2006): Condition A; Condition B; Strong Crossover<sup>1</sup>.

Still, it would be an overstatement to claim that *all* uses of indexing are pronominal. First, in some cases indexing serves to *establish* a locus rather than to *refer back* to one. Second, it has been argued in Bahan et al. 1995 that some uses of indexing in ASL correspond to a definite determiner. We are neutral on the latter matter, but we will seek to establish that for purposes of anaphora resolution indexing *also* plays the role of formal indices in dynamic semantics.

## 2.2 The Importance of Bishop Sentences

The simplest donkey sentences may seem to provide initial evidence in favor of dynamic accounts because pronouns appear to index antecedents that do not c-command them.

<sup>1</sup> In ongoing work, Gaurav Mathur and I have extended these results (originally due to Lillo-Martin) to Weak Crossover effects in ASL.

(16) **LSF**

a. <sub>a</sub>STUDENT <sub>b</sub>PRIEST BOTH-a,b DISCUSSED. IX-b KNOW BIBLE IX-a NOT-KNOW

'I talked to a student and a priest. The priest knew the Bible but the student didn't know it'. (Informant E; 2, 62)

b. EACH-TIME <sub>a</sub>LINGUIST <sub>b</sub>PSYCHOLOGIST ALL-THREE- b,a,1 TOGETHER WORK, IX-a HAPPY BUT IX-b HAPPY NOT.

'Whenever I work with a linguist and a psychologist, the linguist is happy but the psychologist is not happy.' (Informant E; 2, 63)

While these examples can be taken to display coindexing without c-command, they are by no means decisive. The E-type approach could account for them as follows:

(i) In sign language, a pronoun indexes the Noun Phrase that provides its descriptive content. In the implementation of Elbourne 2005, we may simply posit that a pronoun points towards the Noun Phrase which provides its antecedent under NP ellipsis. Since we already know from spoken languages that some formal link must be provided between the pronoun and its antecedent, it comes as no particular surprise that the same phenomenon can be observed in sign language.

(ii) In all cases such as (16), pronouns index exactly the syntactic element that they should – in particular under Elbourne's approach. Therefore (some) E-type approaches make exactly the same predictions as standard dynamic approaches.

When it comes to bishop sentences such as (13), however, things are different: some E-type accounts make different predictions from dynamic accounts. One conceivable E-type account (which corresponds to my understanding of Elbourne 2005) posits that *extra-linguistic material* is used to enrich the descriptive content of the pronouns to allow them to pick out different bishops. Following Elbourne 2005, we can introduce some additional material **D** and **N** to refer to the 'distinguished' and 'non-distinguished' bishop in a situation (one would need to say more about the *semantics* of **D** and **N**; but we assume, following Elbourne, that situations are fine-grained enough that the two bishops can indeed play asymmetric roles).

(17) If a bishop meets a bishop, he **D** ~~bishop~~ blesses him **N** ~~bishop~~.

The formal link between a pronoun and its antecedent is provided in this analysis by syntactic ellipsis. But in the case of (17) the very same results are obtained no matter which antecedent is used, since all that is elided is the noun *bishop*. For this reason, both pronouns could in principle take the same NP as their antecedent under ellipsis. There certainly are other cases in which two elided NPs can have the same antecedent, as is shown in (18); so this possibility should be open in (17) as well.

(18) If two bishops meet, one ~~bishop~~ blesses the other ~~bishop~~.

Thus we end up with the following prediction:

(19) a. E-type theories in which the denotations of the pronouns in (17) are distinguished by extra-linguistic material allow both pronouns to have the same antecedent under ellipsis. Thus if pointing in sign language realizes ellipsis resolution (Elbourne 2005), both pronouns should be allowed to index the same antecedent (while still denoting different individuals).

- b. For dynamic analyses, by contrast, coindexing is semantically interpreted, and thus the two pronouns cannot index the same antecedent given the intended truth conditions.

Let us now turn to the facts of LSF and ASL (see Sinha 2009 for relevant work on anaphora in Indian Sign Language within a dynamic framework).

### 3 Bishop Sentences in ASL and LSF

#### 3.1 Standard Cases

The patterns of indexing found in standard bishop sentences in ASL and LSF are in agreement with the predictions of dynamic analyses, and contradict the version of the E-type analysis discussed above:

(20) **ASL**

WHEN <sub>a</sub>ONE a-MEET-b <sub>b</sub>ONE

a. IX-a TELL IX-b HAPPY a-MEET-b (Inf 1, 2, 285; 111)

b. IX-b TELL IX-a HAPPY a-MEET-b (Inf 1, 2, 285; 111)

c. # Any patterns in which both pronominals index the same position.

‘When someone meets someone, he tells him that he is happy to meet him’

(21) **LSF**

a. PRIEST <sub>a</sub>IX <sub>b</sub>IX ONE PRIEST a-MEET-b. <sub>b</sub>IX BLESS-a.

‘A priest met a priest. He blessed him.’ (Informant **B**; 323)

b. WHEN ONE PRIEST <sub>a</sub>CL MEETS OTHER PRIEST <sub>b</sub>CL, a-GIVE-b book

‘When a priest meets another priest, he gives him a book.’ (Informant **A**; 28)

These patterns extend to cases in which several semantically parallel propositions are conjoined in the antecedent of a conditional:

(22) **ASL**

a. IF <sub>a</sub>FRENCH MAN HERE OTHER <sub>b</sub>FRENCH MAN HERE IX-a GREET IX-b (Informant 1, 2, 114)

‘If a Frenchman were here and another Frenchman were here, he would greet him’

b. IF <sub>a</sub>FRENCH MAN HERE OTHER <sub>b</sub>FRENCH MAN HERE OTHER <sub>c</sub>FRENCH MAN HERE IX-a GREET THE-TWO-b, c (Informant 1, 2, 115)

‘If a Frenchman were here and another Frenchman were here and yet another Frenchman were here, the first would greet the second and the third’.

(23) **LSF**

<sub>a</sub>PRIEST DISCUSS. ALSO OTHER <sub>b</sub>PRIEST DISCUSS. BOOK BIBLE IX-a a-GIVE-b

‘I talked to a priest. I also talked to another priest. The former gave a Bible to the latter.’ (Informant **E**; 2, 69)

The latter observation matters because it has sometimes been suggested within event semantics that the thematic roles corresponding to the subject vs. object of *meet* are crucial to break the symmetry between the indefinite antecedents in examples such as (20). It does not seem that this strategy can extend to cases of propositional

conjunction in (22)-(23), where the antecedents bear exactly the same thematic role – but can still be distinguished by pointing.

### 3.2 Intransitive Cases

Elbourne 2005 argues that in some cases a ‘symmetry problem’ does in fact arise in bishop sentences:

- (24) a. If a bishop meets a bishop, he greets him.  
b. #If a bishop and a bishop meet, he greets him.

Elbourne argues that the contrast in (24) is predicted by his E-type analysis, but not by its dynamic competitors. Without taking a stance on the analysis of the English data, we note that such examples appear to be unproblematic in ASL – as is predicted by dynamic analyses if pointing is the morphological realization of coindexing. Furthermore, all indexing patterns predicted by dynamic analyses are in fact realized:

- (25) **ASL**  
WHEN <sub>a</sub>ONE AND <sub>b</sub>ONE a-MEET-b  
a. IX-a TELL IX-b HAPPY a-MEET-b (Inf 1, 2, 306)  
b. IX-b TELL IX-a HAPPY a-MEET-b (Inf 1, 2, 306)  
‘When someone meet someone, he tells him that he is happy to meet him’
- (26) **ASL**  
WHEN <sub>a</sub>ONE AND <sub>b</sub>ONE AND <sub>c</sub>ONE MEET  
a. IX-a TELL THE-TWO-b, c HAPPY MEET  
b. IX-b TELL THE-TWO-a, c HAPPY MEET  
c. IX-c TELL THE-TWO-a, b HAPPY MEET (Inf 1, 2, 307)  
‘When someone meets someone, he tells him that he is happy to meet him’

## 4 Anaphora to Negative Quantifiers

Having determined that our sign language data favor dynamic approaches over (some) E-type accounts, it remains to see whether we can distinguish between the two main dynamic accounts. Let us remind ourselves of their main properties:

- (27) a. Mixed Solution (Kamp and van Eijck 1993)  
-Donkey pronouns with indefinite antecedents are treated as variables which are dynamically bound.  
-Donkey pronouns with non-indefinite antecedents are treated as E-type pronouns.  
b. Pure Solution (e.g. van den Berg 1996, Nouwen 2003, Brasoveanu 2006)  
-All donkey pronouns are treated as variables which are dynamically bound.  
-Non-indefinite quantifiers introduce not just discourse referents, but also maximality conditions.

Thus the prediction of pure dynamic accounts is that the formal link between a donkey and its antecedent should be the same when the latter is indefinite as when it is non-indefinite. Mixed dynamic accounts make no such prediction. We will now show that the same formal link is used whether the antecedent is indefinite or not. This does not strictly refute the mixed account – it could be that both types of anaphoric links are realized in the same way; but it makes this account less plausible.

The striking fact, then, is that in all the following examples the very same mechanism (establishment of a locus for the antecedent, pointing towards that locus for the pronoun) is used for non-indefinite antecedents as for indefinite antecedents.

(28) **LSF**

- a. LESS FIVE <sub>a</sub>STUDENT COME PARTY. IX-a-plural STAY.  
'Less than five students came to the party. They stayed.' (Informant A; 37)
- b. PIERRE FOUR LESS <sub>b</sub>STUDENTS. IX-b HATE IX-a.  
'Pierre has less than 4 students. They hate him.' (Informant B; 328)

(29) **LSF**

- a. IF LESS FIVE <sub>a</sub>STUDENT COME PARTY, IX-a-plural BE-BORED  
'If less than five students come to the party, they will be bored.' (Informant C; 210)
- b. IF FOUR <sub>a</sub>CL-plural LESS COME CLASS DANCE, IX-a-plural HAPPY NOT  
'If less than four people come to the dance lesson, they won't be happy.'  
(Informant A; 233)
- c. LESSON DANCE IF <sub>a</sub>PEOPLE FEW IX-a HAPPY NOT  
'If few people show up at the dance lesson, they won't be happy' (Informant E; 2, 73c)

(The same generalizations hold in ASL.)

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All in all, then, LSF and ASL data provide evidence in favor of dynamic accounts over (some) E-type accounts; and within dynamic accounts they favor pure accounts over mixed ones (without strictly refuting the latter).

## References

- Bahan, B., Kegl, J., MacLaughlin, D., Neidle, C.: Convergent Evidence for the Structure of Determiner Phrases in American Sign Language. In: Gabriele, L., Hardison, D., Westmoreland, R. (eds.) *FLSM VI, Proceedings of the Sixth Annual Meeting of the Formal Linguistics Society of Mid-America*, vol. 2, pp. 1–12. Indiana University Linguistics Club Publications, Bloomington (1995)
- Brasoveanu, A.: Structured Nominal and Modal Reference. PhD thesis, Rutgers, The State University of New Jersey (2006)
- Dekker, P.: Cases, adverbs, situations and events. In: Kamp, H., Partee, B. (eds.) *Context-dependence in the Analysis of Linguistic Meaning*. Elsevier, Amsterdam (2004)
- Elbourne, P.: *Situations and individuals*. MIT Press, Cambridge (2005)
- Evans, G.: Pronouns. *Linguistic Inquiry* 11(2), 337–362 (1980)
- Groenendijk, J., Stokhof, M.: Dynamic predicate logic. *Linguistics and Philosophy* 14(1), 39–100 (1991)
- Heim, I.: The semantics of definite and indefinite noun phrases. PhD Dissertation, University of Massachusetts, Amherst (1982)
- Heim, I.: E-type pronouns and donkey anaphora. *Linguistics and Philosophy* 13, 137–177 (1990)
- Kamp, H.: A theory of truth and semantic representation. In: Groenendijk, J.A.G., Janssen, T.M.V., Stokhof, M.J.B. (eds.) *Formal Methods in the Study of Language*. Mathematical Centre, Amsterdam (1981)

- Kamp, H., Reyle, U.: *From Discourse to Logic*. D. Reidel, Dordrecht (1993)
- Ludlow, P.: Conditionals, events, and unbound pronouns. *Lingua e Stile* 29, 165–183 (1994)
- Nouwen, R.: Plural pronominal anaphora in context. *Netherlands Graduate School of Linguistics Dissertations*, vol. 84. LOT, Utrecht (2003)
- Sandler, W., Lillo-Martin, D.: *Sign Language and Linguistic Universals*. Cambridge University Press, Cambridge
- Sinha, S.: *A Grammar of Indian Sign Language*. Ph.D thesis, Jawaharlal Nehru University, New Delhi, India (2009) (submitted)
- van den Berg, M.: *Some aspects of the internal structure of discourse: the dynamics of nominal anaphora*. PhD thesis, ILLC, Universiteit van Amsterdam (1996)

# Modality and Speech Acts: Troubled by German *Ruhig*

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**Abstract.** This paper aims to explain the distribution and effect of the German modal particle *ruhig*, which is argued to be licensed only in utterances that induce a particular change in the contextual status of a possible future course of events.

## 1 Introduction

### 1.1 Modal Particles in German

German makes abundant use of modal particles, especially in spoken discourse. While rarely obligatory, they can render utterances more natural. Most modal particles are tied to particular clause or speech act types. Therefore, it is often argued that they modify or specify the speech act (to be) executed (e.g. Zeevat [2003], Karagjosova [2004]). Characteristically, their semantic or pragmatic contribution is hard to pin down. In this paper I focus on the German modal particle *ruhig*, which has gained less attention than e.g. *ja*, *doch* or *wohl* (cf. Zimmermann [t.a.]). *ruhig* is particularly interesting in that its distributional restrictions raise intricate questions about the relation between modality and speech acts, as well as some core-distinctions in the realm of modal verbs (i.e., universal vs. existential modal force; performative vs. descriptive; strong vs. weak necessity).

### 1.2 The Friendly Particle

When *ruhig* is added to a sentence, mostly a flavor of reassurance is obtained, roughly ‘no worries’. Typically, the resulting sentences are used as permissions or recommendations. Examples are given in (1a) and (1b):

- (1) a. *Du kannst/solltest **ruhig** weiterschlafen.*  
you can/should     RUHIG sleep.on  
‘You can/should just go back to sleep, no worries.’     declarative

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- c. #Du schläfst **ruhig** weiter.  
 you sleep RUHIG on unmodalized declarative

Grosz concludes that this is an instance of modal concord. Two elements that match in modal force can sometimes give rise to a reading on which apparently only one of them is interpreted. The sentence in (3) contains a modal verb and a modal adverb that both express possibility. It can be interpreted either surface-compositionally with one modal operator in the scope of the other (**doubly modal reading** in (3a)), or as if there was just one modal element present (**modal concord reading** in (3b)).

- (3) You **may possibly** be familiar with my story.
- a. 'It is possible that you are allowed to be familiar with my story.'  
doubly modal reading
- b. 'It is possible that you are familiar with my story.'  
modal concord reading, standard view

No matter if the cancellation happens in syntax or in semantics, *ruhig* is interpreted as a test on whether it co-occurs (locally) with a modal verb of possibility (3.4). I think that this restriction to co-occurrence with a possibility modal is too strong and that, hence, *ruhig* should not be analysed in terms of modal concord. But before going into that, I would like to point out two issues that have to be added independently to the modal concord approach.

## 2.2 Addenda for Modal Concord

First, *ruhig* can never co-occur with epistemic possibility modals. 5

- (4) #Dieser Student könnte **ruhig** Peter sein.  
 this student could RUHIG Peter be  
 intended: ‘This student could be Peter, no worries.’

<sup>3</sup> Hedde Zeijlstra (p.c.) points out that the notion of such an ‘obligatory concorder’ is at odds with the original definition of modal concord on which both elements can also have modal meaning independently.

<sup>4</sup> Grosz (2009b) proposes an alternative analysis of modal concord on which the two elements jointly express a high degree of possibility/necessity.

<sup>5</sup> An anonymous reviewer points out apparent counter-examples like:

- (i) *Das kann ruhig so sein, aber ich glaube trotzdem, dass...*  
 that can RUHIG so be, but I believe still that...  
 'That may well be the case, yet, I believe that...'

I am not convinced that such concessions involve epistemic modality. Rather, they seem to involve some teleological notion of what assumptions are compatible with the argumentation the speaker wants to pursue. In particular, it seems to say that that the prejacent of the modal is compatible with the speaker's line of argumentation. In contrast to standard cases of epistemic modality (cf. Veltman (1996), such *ruhig*-sentences are possible even after the prejacent has come to be accepted. Further research is needed for a better understanding of such examples.

It seems that *ruhig* can only occur with modals that express possibility with respect to preferences or goals (deontic or teleological modality, cf. Kratzer [1981]).

Second, *ruhig* does not occur in interrogatives, even if they contain a possibility modal of the right flavor:

- (5) #*Kannst du ruhig weiterschlafen?*  
can.2SGIND you RUHIG sleep.on

The modal concord analysis can easily be combined with principles that take care of these two observations. Yet, we will see that it faces independent problems. The alternative analysis in section 3 offers a straightforward explanation for the incompatibility of *ruhig* with epistemic modality, as well as for its absence from interrogatives.

### 2.3 Problems with Modal Concord

**A Question of Modal Force.** Grosz ([2009a], [2009b]) claims that *ruhig* can never appear with necessity modals. Yet, *ruhig* often occurs with the modal verb *soll* (roughly ‘shall’/‘should’) as well as in imperatives. Both are not standardly assumed to constitute or contain possibility modals. *soll* is usually considered a necessity modal (Kratzer [1981]), but this is not undisputed: Ehrich ([2001]) argues that *soll* is truly ambiguous and is, on its weak reading, interpreted as a possibility modal.<sup>6</sup> For the presence of a modal operator in imperatives, Grosz draws on Schwager ([2006]), who argues that imperatives contain an operator that is interpreted like a necessity modal, so *Go home!*  $\approx$  *You should go home!*. This forces her to give a pragmatic account of **permission-imperatives**, i.e. imperatives that seem to constitute less an effort to get the addressee to do something, but rather open up the possibility for her to do so.

- (6) *Take a cookie (if you like).*  
a.  $\approx$  ‘You can take a cookie (if you like).’  
b.  $\not\approx$  ‘You should take a cookie (if you like).’

Schwager ([2006]) argues that this effect can and should be dealt with in pragmatics.<sup>7</sup> In contrast, Grosz argues that the modal operator in imperatives is semantically ambiguous between possibility and necessity. Moreover, for *sollen*, he argues that it passes Horn’s ([1972]) **tolerance test**. Conjunctions of possibility modals with contradictory prejacentes (cf. (7a)) express consistent modal states of affairs, while conjunctions of necessity modals with contradictory prejacentes (cf. (7b)) express contradictory states of affairs.

<sup>6</sup> Önnersfors ([1997]) argues that, in particular, all verb-first declaratives involving *sollen* as the main verb require an interpretation of *sollen* as possibility.

<sup>7</sup> Actually her story is more complex, as the necessity operator present in the imperative clause consists in exhaustified possibility. Nevertheless, this is relevant only in cases where exhaustification is blocked by elements like *zum Beispiel* ‘for example’. In the absence of such elements, imperatives contain a (complex) necessity operator and are thus not expected to license *ruhig* on the modal concord approach.

- (7) a. You may A and you may  $\neg A$ .  
 b. #You have to A and you have to  $\neg A$ .

According to Grosz ([2009a]), *soll* can behave like a possibility modal. This is contradicted by the data in Ehrich ([2001]) as well as the judgments of all ten native speakers I consulted.<sup>8</sup> Grosz does not test imperatives, but they fail likewise, cf. (8a). This cannot be blamed on the performativity of the imperative: explicit performatives that constitute permissions pass the test as expressions of possibility, cf. (8b). (For more natural utterances, with  $B \subseteq \neg A$ .)

- (8) a. #*Come in by the front door and come in by the back door (it's up to you, really).*  
 b. *I hereby allow you to come in by the front door, and I hereby allow you to come in by the back door.*

The German equivalents of (8a) and (8b) behave analogously.

**Absence of Modal Operator.** On closer examination, *ruhig* can even occur in sentences that do not seem to contain any modal operator at all. First, consider the free relative in (9): there is no modal verb and both finite verbs are marked as present indicative.

- (9) *Wer also eines der Hefte haben will,*  
 who therefore one of.the booklets have.INF wants.PRESIND,  
*schreibt ruhig schon mal eine Email.*  
 writes.PRESIND RUHIG already PRT an email  
 'Who wants to have one of the booklets should simply write an email.'

Second, *ruhig* occurs in subsentential constituents that are inserted in parenthesis and spell out how the details of a particular plan (presented in the present indicative) could be filled out.

- (10) *Du gehst einfach zu O2 (ruhig schon 2 Wochen bevor*  
 you go.PRESIND simply to O2 (RUHIG already 2 weeks before  
*dein Vertrag abläuft), schilderst denen das, dann*  
 your contract expires), explain.PRESIND to.them that, then  
*geht das mit der Rufnummernmitnahme.*  
 works.PRESIND that with the take-number-with-you  
 'You simply go to [the mobile phone company] O2 (RUHIG already 2  
 weeks before your contract expires), you explain it to them, and they'll  
 let you keep your number.'
- google

<sup>8</sup> Ehrich assumes that *sollen* is ambiguous between expressing necessity and possibility. It is not clear to me how she intends to account for (i) (her (45a)), which she judges as unacceptable. The conflict is not addressed in her paper.

- (i) \**Du sollst den Rasen mähen und du sollst den Rasen nicht mähen.*  
 you shall the lawn mow and you shall the lawn not mow

Third, *ruhig* occurs in unmodalized declaratives that serve to plan joint action:

- (11) *Weißt du was? Du gehst jetzt ruhig schon mal in*  
 know you what? you go.PRESIND now RUHIG already Q-PART in  
*den Speisewagen, und ich komm nach, sobald der*  
 the dining-car and I come.PRESIND VPART as-soon-as the  
*Schaffner die Tickets kontrolliert hat.*  
 conductor the tickets controlled has  
 ‘You know what? (We do the following:) you go to the dining car and I  
 will follow you as soon as the conductor has checked the tickets.’

**Intermediate Conclusions.** The distributional restrictions of *ruhig* cannot be explained in terms of modal concord with a possibility modal. *ruhig* occurs both with necessity modals and in the absence of modal operators as long as the sentences in question are used to guide future action.<sup>9</sup> This suggests that we should draw on the speech act theoretic side of the restriction. The idea is to endow *ruhig* with suitable restrictions on the speech act types/updates that can be performed with an utterance that contains the particle. The form restriction is then derived indirectly: sentences that cannot contain *ruhig* (e.g. *muss*-modalized ones) are sentences that cannot perform an update of the required type. To provide a formal account along these lines we have to bring together the semantics assigned to modalized declaratives with global notions relevant for speech acts (action alternatives, criteria for decision, ...). This is not without challenge as - in particular: non-epistemic - modal verbs are standardly interpreted pointwise.

### 3 Modeling Utterance Contexts for *ruhig*

#### 3.1 Conditions on *ruhig*

I introduce a simplified notion of an **utterance context with a decision problem** as a quadruple  $C = \langle s_c, a_c, CS_c, A_c^x \rangle$  where  $s_c$  is the speaker,  $a_c$  is the addressee,  $CS_c$  Stalnaker’s ([1978]) context set, i.e. the set of worlds compatible with mutual joint belief of  $s_c$  and  $a_c$ .  $A_c^x$  is a set of possible future courses of events (here, a set of propositions) that constitute a salient decision problem for some agent(s)  $x$ . Moreover,  $CS_c$  determines a set of criteria  $K_c$  that are known to constitute  $x$ ’s criteria for deciding among  $A_c^x$ . Given a precontext  $C$ , update with an utterance  $\phi$  (written  $C + \phi$ ) results in a postcontext  $C' = \langle s_{c'}, a_{c'}, CS_{c'}, A_{c'}^x \rangle$ , where  $s_{c'} = s_c$ ,  $a_{c'} = a_c$ ,  $A_{c'}^x = A_c^x$ , and  $CS_{c'} = CS_c \cap \llbracket \phi \rrbracket^c$  iff  $\llbracket \phi \rrbracket^c$  is defined.<sup>10</sup> Solving the decision problem means to establish a single  $\alpha \in A_c^x$  as optimal.

<sup>9</sup> If Kaufmann’s ([2005]) modal analysis of the English simple present were extended to German, ([9]), ([11]) may not be unmodalized. But they would still contain a necessity operator and should thus not license the presence of *ruhig*.

<sup>10</sup> This is a simplification:  $A^x$  could change as well; also, the context set should consist of world-assignment pairs to capture standard dynamic effects. My representation is inspired by Davis ([L.a.]), who proposes a similar analysis for Japanese *yo*. The similarity was pointed out to me by an anonymous reviewer for AC 2009.

An action  $\alpha$  is optimal in  $C$  iff it is optimal at all worlds in  $CS_c$ . Optimality at a given world is spelt out in terms of Kratzer's ([1981]) framework of graded modality that relies on two parameters,

- a **modal base**, e.g.  $f : W \rightarrow Pow(Pow(W))$  that assigns to  $w$  the set of propositions describing the relevant circumstances, and
- an **ordering source**, e.g.  $g : W \rightarrow Pow(Pow(W))$  that assigns to  $w$  the set of propositions constituting the relevant preferences.

$g$  induces the preorder  $\leq_{g(w)} \subseteq W \times W$  ('at least as good') in [12a]. Under the assumption that  $g$  is always finite, this allows us to define the set of optimal worlds w.r.t.  $w$ ,  $f$ , and  $g$  as in [12b]:

- (12) a. For all worlds  $w_i, w_j$ :  $w_i \leq_{g(w)} w_j$  iff  
 $\{p \in g(w) \mid p(w_j)\} \subseteq \{p \in g(w) \mid p(w_i)\}$   
 b. The optimal worlds in  $w$  w.r.t.  $f$  and  $g$  are  $O(f, g, w) :=$   
 $\{w_1 \in \bigcap f(w) \mid \neg \exists w_2 \in \bigcap f(w) [w_2 \leq_{g(w)} w_1 \ \& \ \neg w_1 \leq_{g(w)} w_2]\}$

As usual, *can* and *must* express compatibility and entailment w.r.t.  $O(f, g, w)$ . Pointwise optimality is now used to define optimality in a context  $C$ . For this, the relevant ordering source is fixed as  $g_c = \lambda w. \lambda p. p \in K_c$  (i.e., each world is mapped to  $K_c$ , the criteria for decision in  $C$ ). Moreover, in deciding one has to take into account all possibilities the world could be like, therefore, as a modal base, we use  $f_c = ' \lambda w. \text{the relevant circumstances in } w '$ , where  $\bigcup_{w \in CS_c} \bigcap f_c(w) = CS_c$ .

- (13) a. An action  $\alpha \in A_c^x$  is **optimal** in context  $C$  iff  
 $(\forall w \in CS_c)[O(f_c, g_c, w) \subseteq \alpha]$ .  
 b. An action  $\alpha \in A_c^x$  is **as-good-as-it-gets** in context  $C$  iff  
 $(\forall w \in CS_c)$   
 $[(\exists w_i \in \bigcap (f_c(w) \cup \alpha))[(\forall w_j \in \bigcap f_c(w))[w_j \leq_{g_c(w)} w_i \rightarrow \alpha(w_j)]]]$ .

That is, in a context  $C$ ,  $\alpha$  is optimal if it is a human necessity (cf. Kratzer [1981]) at each world of  $CS_c$ , and it is as-good-as-it-gets, if at all worlds in  $CS_c$ ,  $\alpha$  follows from the modal base plus one maximally consistent combination of ordering source propositions. Any optimal action is also as-good-as-it-gets.

I assume that *ruhig* occurs in contexts  $C$  where  $K_c$  does not suffice for  $x$  to resolve the issue of what course of events to choose from  $A_c^x$ . This lack of an optimal candidate can be due to a lack of knowledge about the facts, or to conflicting preferences in  $K_c$ . The requirements of *ruhig* are spelt out by making it a partial identity function on propositions: *ruhig* returns its prejacent  $\alpha_{st}$  iff they occur in a sentence  $\phi$  whose LF is a sequence  $[\psi_1[ruhig[\alpha]]\psi_2]$  (with  $\psi_1$  and  $\psi_2$  possibly empty) and  $C + '\psi_1\alpha\psi_2' = C'$  s.t. <sup>11</sup>

<sup>11</sup> In my implementation, Davis' ([1.1]) condition on Japanese *yo* (his (23a)) reads as follows (only that Davis allows for  $A_c^x$  and  $A_{c'}^x$  to differ):

$(\exists \alpha \in A_c^x)[(\forall w_i, w_j \in CS_{c'})[(\alpha(w_i) \ \& \ w_j <_{g'_c(w_j)} w_i) \rightarrow \alpha(w_j)]]]$ .

If this is correct for *yo* (cf. McCready [2006] for an analysis that suggests little similarity to *ruhig*), *ruhig* and *yo* differ at least as follows: (a) *yo* does not require  $\alpha$  to be mentioned in the sentence; (b) *ruhig* requires that  $\alpha$  become globally optimal.

- (14) (i)  $\alpha \in A_c^x$  and  $\alpha \in A_{c'}^x$ , (ii)  $\alpha$  is locally optimal in  $C$ , and (iii)  $\alpha$  is optimal in  $C'$ <sup>12</sup>

We can now see why *ruhig* occurs in permissions: permitting  $\alpha$  requires that the addressee is (taken to be) interested in realising  $\alpha$ , but also in not violating the rules (Searle 1969). Therefore, as long as either  $\alpha$  is prohibited, or it is unknown if  $\alpha$  is prohibited,  $\alpha$  cannot be optimal. Uttering (15) (under a deontic reading of the modal) rules out worlds at which following the rules and having a cookie is incompatible. This is why *ruhig* is acceptable.

- (15) *Du kannst dir von mir aus ruhig ein Keks nehmen.*  
 you can yourself by me of RUHIG a cookie take  
 ‘You can take a cookie, no worries.’

In the absence of *ruhig*, the possibility modal could be understood teleologically, and would then express a trivial truth: evaluated at a  $w'$  s.t. having a cookie is permitted, the best worlds are worlds where the addressee has a cookie and follows the rules. Evaluated at a  $w''$  s.t. having a cookie is prohibited, the best worlds are partitioned into ones where she has a cookie (but violates the rules) and ones where she follows the rules (but doesn’t have a cookie). Hence, at both  $w'$  and  $w''$ , the optimal worlds have a non-empty intersection with having a cookie. The update does not eliminate any worlds ( $CS_c = CS_{c'}$ ), taking a cookie is not globally optimal in  $C'$  and *ruhig* is not licensed. It is not absolutely clear what governs the interpretation of contextual parameters of modal verbs, but the pressure for a consistent interpretation is clearly a decisive factor. Consequently, even if *ruhig* does not tinker with semantic meaning, it can help to trigger a particular ordering source (here, speaker deontic rather than hearer teleological). Updates that constitute recommendations, plans and suggestions each work slightly differently, but can meet the conditions in (14) and can thus license *ruhig*. For reasons of space I cannot discuss further examples.<sup>13</sup>

### 3.2 The *must*-Problem

Besides sentences containing *kann* ‘can’, also those containing *soll* ‘shall’ or imperatives can give rise to the required update effect, in particular when occurring in recommendations. Consider a scenario described by Grosz: a cable car is about to depart. A passenger wants to use the restroom, but is unsure if he can make it back in time. The conductor issues (16):

<sup>12</sup> In contrast to the speakers I consulted, an anonymous reviewer accepts clauses of the form ‘You may RUHIG  $\alpha$  and you may RUHIG also  $\neg\alpha$ ’. If these cases are not to be resolved pragmatically (as corrections), there might be a variety of German where clause (iii) is replaced by a notion that lies between optimal and as-good-as-it-gets in strength.

<sup>13</sup> Note that the unacceptability of *ruhig* in interrogatives, even if used as indirect speech acts, indicates that the update-conditions imposed by *ruhig* pertain to the minimal (‘automatic’) update and ignore additional effects as mediated by pragmatic considerations. I am indebted to Eric McCready (p.c.) for pointing this out.

- (16) *Gehen Sie ruhig noch auf die Toilette!*  
 go.IMP you.POLITE RUHIG still to the toilet  
 ‘Just go to the restroom (if you like).’

Apparently,  $CS_c$  contains worlds  $w'$  where the preference ‘go to the toilet’ is compatible with the goal ‘reach the cable car’, and worlds  $w''$  where this is not so. By saying that all the optimal worlds according to the circumstances and preferences verify that the addressee goes to the toilet, the speaker rules out worlds at which the two events are incompatible. Clearly, in the post-, but not in the precontext going to the toilet is optimal. Given that this *ruhig*-permissible update was achieved by a necessity modal, why is it that it could not be achieved by a sentence containing *must*? In principle, two lines of reasoning suggest themselves, both of which have to do with the question what (kind of) ordering sources are involved. First, von Stechow & Iatridou (2008) point out that **weak necessity modals** like *ought* and *should* (historical subjunctives) differ from **strong necessity modals** like *must* in that they involve two ordering sources of different status. Loosely speaking, one of them is not actually binding, but only counterfactually. This idea merits closer investigation. Second, Ninan (2005) considers *must* inherently **performative**<sup>14</sup> because it is infelicitous with follow-ups that indicate that the particular necessity will not be respected:

- (17) *Sam {#must, has to} go to confession; but he won't go.*

This contrast could also be accounted for if we require that *must* comes with an ordering source  $g$  that is considered ‘binding’ ( $CS_c$  entails that all  $g$ -optimal events will be realised). In contrast, *ruhig* seems to require an ordering source for which it is not given that it is being followed (e.g. the speaker’s rules in the cookie case, the hearer’s preferences in the cable car case). A satisfactory implementation of this, as well as an answer to why imperatives and *sollen* ‘should’ are nearly but not fully interchangeable, have to await further insight into the nature of modal bases and, in particular, of ordering sources.

## 4 Conclusions

I have argued that the German modal particle *ruhig* imposes both formal and functional restrictions on its contexts of occurrence. On the one hand, it has to combine with the description of a possible future course of events  $\alpha$  that belongs to a set of contextually given alternatives. On the other hand, it has to occur in a sentence that gives rise to an update that renders  $\alpha$  a globally optimal choice. Some of the problems discussed show that we need a better understanding of the parameters involved in the standard Kratzer semantics for modality.

<sup>14</sup> Roughly, inducing a change in a modal state of affairs rather than describing it.

## References

- Davis, C., et al.: Decisions, dynamics, and the Japanese particle *yo*. *Journal of Semantics*
- Ehrich, V.: Was *nicht müssen* und *nicht können* (nicht) bedeuten können: Zum Skopus der Negation bei den Modalverben des Deutschen, pp. 148–176 (2001)
- Grosz, P.: German particles, modality, and the semantics of imperatives. In: Lima, S., et al. (eds.) *Proceedings of NELS 39. GLSA*, Amherst (2009a)
- Grosz, P.: Grading modality: A new approach to modal concord and its relatives. Talk at SuB 14, University of Vienna, September 28–30 (2009b)
- Horn, L.: On the semantic properties of logical operators in English. Ph. D. thesis, UCLA (1972)
- Karajosova, E.: The Meaning and Force of German Modal Particles. Saarbrücken Dissertations in Computational Linguistics and Language Technology (2004)
- Kaufmann, S.: Conditional predictions. *Linguistics and Philosophy* 28, 181–231 (2005)
- Kratzer, A.: The notional category of modality. In: Eikmeyer, H., Rieser, H. (eds.) *Words, Worlds, and Contexts*. de Gruyter, Berlin (1981)
- McCready, E.: Japanese *yo*: Its semantics and pragmatics. *Sprache und Datenverarbeitung* 30, 25–34 (2006)
- Ninan, D.: Two puzzles about deontic necessity. In: Gajewski, J., et al. (eds.) *New Work on Modality*, pp. 149–178. MITWPL, Cambridge (2005)
- Önnerfors, O.: Verberst Deklarativsätze. *Grammatik und Pragmatik*. Almqvist & Wiksell, Stockholm (1997)
- Sadock, J.M., Zwicky, A.M.: Speech act distinctions in syntax. In: Shopen, T. (ed.) *Language Typology and Syntactic Description*, vol. I, pp. 155–196. CUP, Cambridge (1985)
- Schwager, M.: Interpreting Imperatives. Ph.D.thesis, University of Frankfurt (2006)
- Searle, J.: *Speech Acts*. CUP, Cambridge (1969)
- Stalnaker, R.: Assertion. In: Cole, P. (ed.) *Syntax and Semantics* 9, pp. 315–332. Academic Press, New York (1978)
- Veltman, F.: Defaults in update semantics. *Journal of Philosophical Logic* 25, 221–261 (1996)
- von Fintel, K., Iatridou, S.: How to say *ought* in foreign. In: Guéron, J., Lecarme, J. (eds.) *Time and Modality*, vol. 75, pp. 115–141. Springer, Heidelberg (2008)
- Zeevat, H.: The syntax semantics interface of speech act markers. In: Kruijff-Korbayová, I., et al. (eds.) *Proceedings Diabrock 2003* (2003)
- Zimmermann, M., et al.: Discourse particles. In: Portner, P., Maienborn, C., von Heusinger, K. (eds.) *Handbook of Semantics*. Mouton de Gruyter, Berlin

# German *Noch So*: Scalar Degree Operator and Negative Polarity Item\*

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**Abstract.** This paper describes a puzzle introduced by German *noch so*, a degree operator and Negative Polarity Item. *Noch so* sentences allow for paraphrases containing a scalar particle (like *even*), suggesting that its polarity sensitivity can receive an analysis along the lines of Lahiri ([4]). While such an analysis is indeed straightforward to state, it fails to capture the perceived meaning contribution of *noch so* in good cases. Conversely, an account generalizing from the perceived meaning contribution of *noch so* does not seem to derive its polarity sensitivity.

## 1 Introduction

An approach pioneered by [1] relates polarity sensitivity to semantic triviality. [2], [3], and [4] let unlicensed negative polarity items (NPIs) yield contradictory presuppositions or implicatures. In these proposals, an NPI requires the asserted proposition to be stronger, or less likely, than certain alternative propositions. This requirement is designed to be satisfiable in licensing contexts only.

The present paper describes a problem for applying such a contradiction based analysis to the German expression *noch so*, a polarity sensitive degree operator. It is first shown that under standard assumptions about gradable predicates and degree operators, a contradiction based account is straightforward to state. It merely requires a minor innovation concerning the nature of alternative propositions: where in previous relevant work propositions are generated by alternative predicates or domain restrictions, *noch so* invokes propositions generated by alternative degrees.

However, while such a contradiction based account helps derive polarity sensitivity of *noch so*, it is shown not to capture the contribution to meaning it makes in cases where it *is* licensed. An alternative analysis is presented, designed to capture this contribution. This analysis, however, does not without further assumptions derive polarity sensitivity. Ways of reconciling the requirements imposed by the distribution of *noch so* and its perceived contribution to meaning are discussed, although with no conclusive outcome.

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## 2 Some Properties of *Noch So*

*Noch so* is a degree operator. It shares a signature distributional restriction with familiar comparative, equative or superlative morphemes: as (1) and (2) illustrate, *noch so* can combine with gradable predicates such as *lang* ‘long’, but not with non-gradable predicates such as *zweiköpfig* ‘two-headed’.

- (1) Peter hatte vor keiner [ **noch so** lang.en ] Schlange Angst.  
*Peter had of no NOCH SO long.infl snake fear*  
 ‘Peter wasn’t even afraid of any LONG snake.’
- (2) # Peter hatte vor keiner [ **noch so** zweiköpfig.en ] Schlange Angst.  
*Peter had of no NOCH SO two-headed.infl snake fear*

*Noch so* is also “scalar”. As the translation in (1) indicates, *noch so* sentences allow for paraphrases with the scalar particle *even*.<sup>1</sup> Finally, *noch so* is a NPI. In (1), *noch so* appears in the restrictor of the downward entailing determiner *kein* ‘no’; (3), where upward entailing *einige* ‘some’ replaces *kein*, is bad.

- (3) \* Peter hatte vor einigen [ **noch so** lang.en ] Schlangen Angst.  
*Peter had of some NOCH SO long.infl snakes fear*

Also, (4) and (5) show that *noch so* can appear in the scope of downward entailing *keiner* ‘no one’, but not in the scope of upward entailing *jeder* ‘everyone’.

- (4) Keiner hat ein [ **noch so** lukrativ.es ] Angebot angenommen.  
*no one has a NOCH SO lucrative.infl offer accepted*  
 ‘No one even accepted a LUCRATIVE offer.’
- (5) \* Jeder hat ein [ **noch so** lukrativ.es ] Angebot angenommen.  
*everyone has a NOCH SO lucrative.infl offer accepted*

Building most directly on [4], the following explores how, under standard assumptions about gradable predicates and degree phrases, the polarity sensitivity of *noch so* can be derived from its scalarity.

## 3 A Contradiction Based Account

The scalarity of certain polarity items is exploited in several existing accounts of polarity sensitivity (e.g. [4, 5, 6, 7]). Most relevant here is Lahiri’s work [4], where the scalarity of certain NPIs in Hindi is argued to yield contradictions in upward entailing contexts. This section formulates a Lahirian analysis of *noch so*.

<sup>1</sup> However, *even* not being a degree operator, the translations given are mere approximations. Note that the English translation given in (1) remains acceptable if non-gradable *TWO-HEADED* replaces gradable *LONG*.

### 3.1 Lahirian Analysis of Scalar Indefinites

Lahiri is concerned with NPI indefinites in Hindi, but English *even ONE* makes much the same point, participating in contrasts like the one between (6) and (7).

- (6) \* Even ONE student called.  
 (7) Not even ONE student called.

Following [10], Lahiri assumes that a scalar particle triggers the presupposition that the sentence minus the scalar particle expresses a proposition which is less likely than all relevant alternative propositions. In the case at hand, alternatives are obtained by replacing the focused numeral *one* with alternative numerals *two*, *three*, etc. Under an *at least* semantics for numerals, (6) is then predicted to presuppose that the proposition that one or more students called is less likely than the proposition that two or more students called, etc. But this presupposition is contradictory: the asserted proposition is entailed by each of its alternatives, and so it has to be at least as likely as any of them. So the unacceptability of (6) can be credited to a necessarily false presupposition.

The contradiction is correctly predicted to dissipate in (7), where negation reverses the direction of entailment. The asserted proposition now entails each of its alternatives, making it possible for it to be less likely than each of them, hence rendering the presupposition satisfiable.

### 3.2 Gradable Predicates, Degree Phrases, and Monotonicity

In a standard view (e.g. [8,9]), gradable adjectives relate individuals to degrees under an *at least* semantics. (8), for example, lets the denotation of *lang* ‘long’ map a degree  $d$  and individual  $x$  to the proposition that  $x$ ’s length is at least  $d$ .

- (8)  $||lang|| = \lambda d.\lambda x.\lambda w. x\text{'s length in } w \geq d$

Given such lexical meanings, degree phrases like *noch so*, can be considered generalized quantifiers over degrees, which can move covertly from the adjective’s degree argument position to combine with a derived degree property (cf. [9]). (9) and (10) are then conceivable L(ogical) F(orms) for (3) and (1).

- (9) *noch-so*  $\lambda d$ [Peter hatte vor einigen [ $d$  langen] Schlangen Angst]  
 (10) *noch-so*  $\lambda d$ [Peter hatte vor keiner [ $d$  langen] Schlange Angst]

The *at least* semantics in (8) ensures that the degree property denoted by the lambda abstract in (9) is downward monotone in the sense of (11) (cf. [8]). That is, propositions that it maps higher degrees to entail propositions that it maps lower degrees to. If Peter is afraid of some snakes of length 60cm or more, then he is thereby also afraid of some snakes of length 50cm or more.

- (11) *P is downward monotone*  $:\Leftrightarrow \forall d_1, d_2 [d_1 \leq d_2 \rightarrow P(d_2) \subseteq P(d_1)]$   
 (12) *P is upward monotone*  $:\Leftrightarrow \forall d_1, d_2 [d_1 \leq d_2 \rightarrow P(d_1) \subseteq P(d_2)]$

In contrast, the degree property denoted by the lambda abstract in (10) is upward monotone in the sense of (12). If Peter is not afraid of any snake of length 50cm or more, then he is also not afraid of any snake of length 60cm or more.

### 3.3 *Noch So* and Contradictory Presuppositions

Assuming an *at least* semantics for gradable adjectives, the scalarity of *noch so* invites a straightforward Lahirian analysis of its polarity sensitivity. The entry in (13) lets *noch so* impose a condition on the common ground comparing propositions obtained by applying its degree property argument to different degrees.

$$(13) \text{ noch-so } \phi \text{ presupposition: } \forall d_1, d_2 [d_1 < d_2 \rightarrow |\phi|(d_1) \ll_c |\phi|(d_2)]$$

Here  $p \ll_c q$  conveys that  $p$  is less likely than  $q$  relative to the common ground  $c$ . So (13) requires that the degree property argument of *noch so* maps higher degrees to contextually likelier propositions than lower ones.

This condition has the intended effect for (3). Recall that the degree property expressed by the lambda abstract in (9), is downward monotone: propositions for higher degrees entail propositions for lower degrees. Given this, the presupposition that (13) assigns to (9) is necessarily false, again because a proposition is bound to be at least as likely, in any context, as any proposition entailing it. The unacceptability of (3) can be credited to this contradiction. In contrast, no offending presupposition is predicted for (1). Recall that in the LF (10), the lambda abstract denotes an upward monotone degree property, with propositions for lower degrees entailing propositions for higher degrees. As a consequence, the presupposition triggered according to (13) is consistent and (1) is correctly expected to be acceptable. The Lahirian analysis, then, offers a straightforward account of the polarity sensitivity of *noch so*.

It will be useful to also consider a minor variant of this account. The entry in (13) follows (4) in the assumption that “scalar” implications compare the (contextual) likelihood of propositions. This view goes back to the analysis of *even* in (10). In an alternative approach, taken in (11), *even* compares propositions in terms of (contextual) semantic strength or informativity; also, (2) presents a general perspective on NPI licensing that is based on (4) but substitutes informativity for likelihood as the semantic relation operative in scalar implications. With that substitution, (13) becomes (14), where  $p \subset_c q$  conveys that  $p$  asymmetrically entails  $q$  relative to the common ground  $c$ , that is,  $p \cap c \subset q \cap c$ .

$$(14) \text{ noch-so } \phi \text{ presupposition: } \forall d_1, d_2 [d_1 < d_2 \rightarrow |\phi|(d_1) \subset_c |\phi|(d_2)]$$

Entry (14) also supports a contradiction based account of the polarity sensitivity of *noch so*, and even more straightforwardly so than (13). The condition is obviously contradictory in (9), where the degree property is downward monotone. Evidently, no proposition can be asymmetrically entailed, in any context,

by a proposition it entails. Again, the contradiction is avoided in (10), where the degree property is upward monotone.<sup>2</sup>

## 4 But the Lahirian Presupposition Is Not Right

While the conditions in (13) and (14) derive the polarity sensitivity of *noch so*, the question is whether they correctly characterize the meaning of acceptable *noch so* examples such as (1). Note that the degree property in (10) is not merely upward monotone, but *strictly* upward monotone in the sense of (15). That is, it maps lower degrees to (strictly) stronger propositions than higher degrees.<sup>3</sup>

$$(15) \quad P \text{ is strictly upward monotone} :\Leftrightarrow \forall d_1, d_2 [d_1 \leq d_2 \rightarrow P(d_1) \subset P(d_2)]$$

As a consequence, it does not take much for (10) to satisfy the condition in (14), which requires strict upward monotonicity under common ground assumptions. For the condition not to be met, the context would have to obliterate the asymmetric entailment between some or all relevant propositions, rendering them contextually equivalent. This would be the case, for example, if the common ground entailed that Peter either was afraid of all snakes (of all lengths) or was not afraid of any snakes (of any lengths). A felicitous use of (1) should merely require that no such assumption is established.

More formally, asymmetric entailment between propositions  $p$  and  $q$  is obliterated in common ground  $c$  if  $p \cap c = q \cap c$  despite  $p \subset q$ . The condition that no such obliteration obtains is the requirement that  $c$  contain a possible world where  $q$  is true while  $p$  is false, for all propositions  $p$  and  $q$  of the relevant form. It is worth noting that this condition is not a presupposition in the usual sense. If a sentence is said to presuppose a proposition  $p$ , this is understood as the requirement that the common ground entail  $p$  (e.g. [13]). In the case at hand, in contrast, the common ground is required *not* to have certain entailments.

The situation is much the same if the condition in (13) is assumed instead of (14), assuming, as seems plausible, that greater (contextual) semantic strength is a sufficient condition for lesser (contextual) likelihood.<sup>4</sup> Then (13) too is guaranteed to be satisfied in (10) unless the common ground contains assumptions rendering some or all of the relevant propositions contextually equivalent.<sup>5</sup>

<sup>2</sup> Apart from the issue discussed in the next section, a potential problem for this analysis (which it shares with the proposal in [4]) concerns possible licensing by expressions that are non-upward entailing without being downward entailing. The account predicts that a contradiction can be avoided if *noch so* scopes over a non-upward entailing expression, whether downward entailing or not. The data do not seem to bear out this predication for *noch so*. For example, *noch so* is never licensed by a quantifier like *genau drei Studenten* ‘exactly three students’.

<sup>3</sup> That degree properties like the ones in (9) and (10) are strictly monotone, even if the scale in question is dense, is proposed explicitly in [8].

<sup>4</sup> Whether or not semantic strength and likelihood relate as suggested, it is clear that (13) fails to derive the meaning contribution of *noch so* described below.

<sup>5</sup> Related discussion is found in [6] (p. 95ff), where proposals on *even* in [7] and [12] are evaluated.

To be sure, these predictions do not square well with the actual interpretation of (II). A felicitous utterance of the sentence does certainly not require that assumptions of any kind *not* be established in the conversation. Nor is it consistent with intuitions that the absence of established assumptions of any sort could be a sufficient condition for a felicitous use of the sentence.

Instead, an utterance of (II) is judged to introduce a “scalar” implication similar to the one associated with the (merely approximate) English translation given: it suggests that fear of shorter snakes is less likely, or perhaps entails, fear of longer snakes.

This type of scalar implication is even more salient in (I16), where *kein* ‘no’ is replaced by (equally downward entailing) *jeder* ‘every’. In this case, a scalar implication is perceived that is diametrically opposed to default assumptions on how fear of snakes might correlate with their length. Implausibly, (I16) suggests that fear of longer snakes is less likely, or entails, fear of shorter snakes.

- (16) ! Peter hatte vor jeder [ **noch so** lang.en ] Schlange Angst.  
*Peter had of every NOCH SO long.infl snake fear*  
 ‘Peter was even afraid of every LONG snake.’

It is worth noting that the contradiction based accounts given above are correct in one respect: scalar implications are indeed presuppositions. This, at least, is what their projection behavior suggests. For example, scalar implications are judged to survive embedding under the modal adjective *möglich* ‘possible’, and the plausibility contrasts described above persist under such embedding. The task, then, is to determine the proper content of this presupposition, generalizing from the examples presented in this section.

## 5 Presupposing Downward Monotonicity

The observations reported in the last section suggest that the correlation between the ordering of degrees and likelihood or informativity of propositions is roughly the reverse of what contradiction based accounts would posit. Settling on informativity as the relevant semantic notion, an entry like (I17) suggests itself.

- (17) noch-so  $\phi$  **presupposition:**  $\forall d_1, d_2 [d_1 < d_2 \rightarrow |\phi|(d_2) \subseteq_c |\phi|(d_1)]$

Here  $p \subseteq_c q$  conveys that  $p$  entails  $q$  relative to the common ground  $c$ , that is,  $p \cap c \subseteq q \cap c$ . Thus, (I17) requires that the degree property in question is downward monotone under common ground assumptions.

Assuming as before that *noch so* must outscope its licenser, the entry in (I17) makes sense of the observations reported in the last section. According to (I17), the LF of (II) in (I10) presupposes that Peter’s not being afraid of any longer snakes entails his not being afraid of any shorter snakes, which is plausible. The corresponding LF for (I16) in (I18) is assigned a less plausible presupposition, viz. that Peters being afraid of all longer snakes entails his being afraid of all shorter snakes.

- (18) noch-so  $\lambda d$ [Peter hatte vor jeder [d langen] Schlange Angst]

Similarly, (17) seems to apply correctly to example (4) above. The presupposition assigned to the LF in (19) is that no one accepting a more lucrative offer entails no one accepting a less lucrative offer. Again, this presupposition is plausible and fits with intuitions on what the sentence conveys.

- (19) noch-so  $\lambda d$ [keiner hat ein [d lukratives] Angebot angenommen]

Intuitions on meaning, then, support the condition in (17). Moreover, this condition turns out to derive a distributional constraint on *noch so* not mentioned so far. While correctly characterized as a NPI in so far as it must occur in a downward entailing context, *noch so* is subject to an additional distributional restriction that it does not share with more familiar NPIs. Consider (20).

- (20) \* Kein Gedicht ist [ **noch so** lang ].  
*no poem is NOCH SO long*

This case is structurally similar to the acceptable sentence in (4), the only relevant difference being that the adjective phrase hosting *noch so* is a modifier in (4) but the main predicate in (20). Why would this difference matter?

Under (17), a semantic explanation of the contrast becomes available. According to (17), the LF of (20) in (21) presupposes that, if there is a given length that no poem reaches, then no poem reaches a lesser length either.

- (21) noch-so  $\lambda d$ [ [kein Gedicht] ist [d lang] ]

This scalar presupposition is in obvious conflict with the assumptions that every poem has some length or other and that there are lengths that no poem (not even the longest) reaches. The former assumption can in fact be considered another presupposition of the sentence: degree predicates can be assumed to only be defined for individual arguments in the domain of the underlying measure function (e.g. (14)); hence the denotation of *lang* ‘long’ will only be defined for individuals that have a length, and this presupposition is expected to project universally in (21). So the unacceptability of (20) can be blamed on inconsistent presuppositions.

This type of inconsistency is expected to be avoided in the otherwise similar case in (4). After all, while every poem necessarily has a length, it is not necessary for every, or any, person to have accepted a (more or less lucrative) offer. Likewise, no inconsistency is expected to arise in (1), as it is not necessary for Peter to have fear of any snakes. (22) and (23) further illustrate the point.

- (22) \* Kein Student war [ **noch so** vorsichtig ].  
*no student was NOCH SO tentative*

- (23) Kein Student hat sich [ **noch so** vorsichtig ] beschwert.  
*no student has refl NOCH SO tentatively complained*  
 ‘No student complained even TENTATIVELY.’

The contrast is again expected. While (22) is expected to presuppose that every student was tentative (to some degree), (23) clearly does not presuppose that every student complained (with any degree of tentativeness).

## 6 Back to Polarity Sensitivity

The last section continued to assume, without discussion, that *noch so* is a NPI that must outscope its downward entailing licenser. The question is whether the current analysis provides a rationale for this requirement. Another open question concerns the truth conditional content of *noch so*. These two related issues are addressed below, although with no conclusive outcome.

In the account explored here, a *noch so* sentence presupposes that the degree property combining with *noch so* is downward monotone under common ground assumptions. But, unless *noch so* scopes over a non-upward entailing operator, the relevant degree property is guaranteed to be downward monotone, irrespectively of assumptions established in the common ground. So, unless *noch so* scopes over a non-upward entailing operator, the presupposition derived is tautologous.

It is tempting to relate this observation to the polarity sensitivity of *noch so*. However, it is not clear that a necessarily true presuppositions alone can be the source of unacceptability. After all, a sentence that carries no (non-tautologous) presupposition might still be informative by virtue of its asserted content. This raises to salience a question skirted so far: what *is* the asserted content of a *noch so* sentence? The following briefly explores two options, concluding that neither is able to derive the polarity sensitivity of *noch so*.

Sentence (1) is judged to convey that Peter was not afraid of snakes of any lengths. Similarly, (4) conveys that none of them accepted any offer of any degree of lucrativity. This could be captured straightforwardly by making *noch so* a universal quantifier, as in (24).

$$(24) \text{ noch-so } \phi \text{ \textbf{assertion:}} \quad \lambda w. \forall d[ |\phi|(d)(w) = \text{T} ]$$

One might hope that letting *noch so* quantify universally sheds light on its polarity sensitivity. In fact, (24) would render a sentence like (25) below contradictory, assuming that a poem's length must be finite. Since the scale of length has no upper bound, the segment of the scale characterized by the relevant degree property in (25) is guaranteed to be a proper subset of the scale of length, which contradicts the condition in (24).<sup>6</sup>

$$(25) \quad * \text{ Das Gedicht ist [ \textbf{noch so} \text{ lang} ].}$$

*the poem is NOCH SO long*

Much the same applies to the unacceptable case in (3). One might hope, then, that all cases of unlicensed *noch so* can be excluded on the grounds of being contradictory. Note that, in contrast to the contradiction based accounts discussed

<sup>6</sup> As a AC09 reviewer notes, (25) is acceptable in an irrelevant reading where *noch* 'still' and *so* 'so' are interpreted separately. The same is true for (26) below (and possibly other examples marked as ungrammatical.).

above, here the contradiction would be in the asserted, not the presupposed, content.

Unfortunately, such an explanation falls short of excluding all cases of unlicensed *noch so*. One case that (24) would not render contradictory is (26). Assuming that the scale of fullness contains a maximal degree (see e.g. [15]), (24) predicts (26) to merely convey that the glass was full. Similarly, the truth conditions that (24) assigns to (27) could be met if the relevant modal base fails to set an upper bound for the permitted length of the poem.

(26) \* Das Glas war [ **noch so** voll ].  
       *the glass was NOCH SO full*

(27) \* Das Gedicht darf [ **noch so** lang ] sein.  
       *the poem may NOCH SO long be*

The fact that (26) and (27) are nevertheless unacceptable suggests that the universal analysis is insufficient to derive the polarity sensitivity of *noch so*.

An alternative analysis makes *noch so* quantify over degree existentially as in (28), rather than universally.

(28) *noch-so*  $\phi$  **assertion:**  $\lambda w.\exists d[|\phi|(d)(w) = T]$

Note that (28) is still consistent with the intuition that acceptable *noch so* sentences effectively quantify over degrees universally. Consider again (1) and the LF in (10). There the degree property combining with *noch so* is necessarily upward monotone. Moreover, in order to satisfy the presupposition in (17), the degree property in (10) must also be downward monotone under common ground assumptions. According to (17), then, the degree property in (10) is implied to be either true of all degrees of length or false of all such degrees. So in conjunction with the scalar presupposition, the existential quantification in (28) acquires universal force.

Under the semantics in (28), a possible reason for the unacceptability of (25) is that it is tautologous at the level of both presupposition and assertion. As said, downward monotonicity of the relevant degree predicates guarantees satisfaction of the condition in (17). Under the assumption that every poem has some length, the existential condition is necessarily satisfied as well. Under (28), the examples in (26) and (27) arguably come out trivial as well. Perhaps, then, *noch so* must scope over a downward entailing licenser in order to avoid a trivially true interpretation.

Unfortunately, however, this rationale too does not extend to all examples of unlicensed *noch so*. Example (3) is a case in point. According to (28), the LF in (9) has the contingent entailment that there are some snakes (of some length) that Peter was afraid of. Similarly, (28) does not derive the contrast between (23) above and (29). Under (28), (29) should have the non-trivial entailment that some student complained.

- (29) \* Ein Student hat sich [ **noch so** vorsichtig ] beschwert.  
*some student has refl NOCH SO tentatively complained*

As present, then, it is unclear whether the meaning contribution of *noch so* detectable in acceptable examples can help one derive its distribution.

## 7 Conclusion

This paper has identified a dilemma for the analysis of *noch so*. A contradiction based account of its distribution along the lines of [4] fails to account for its meaning in good cases. Conversely, an account that derives the perceived meaning of *noch so* does not seem to derive its polarity sensitivity. Hopefully future work will resolve the dilemma.

## References

1. Kadmon, N., Landman, F.: Any. *Linguistics and Philosophy* 16, 353–442 (1990)
2. Chierchia, G.: Broaden Your Views. *Linguistic Inquiry* 37, 535–590 (2006)
3. Krifka, M.: The Semantics and pragmatics of polarity items. *Linguistic Analysis* 25, 1–49 (1995)
4. Lahiri, U.: Focus and Negative Polarity in Hindi. *Natural Language Semantics* 6, 57–123 (1998)
5. Abrusàn, M.: Even and Free Choice Any in Hungarian. In: Puig-Waldmüller, E. (ed.) *Proceedings of Sinn und Bedeutung 11*, Universitat Pompeu Fabra, Barcelona, pp. 1–12 (2007)
6. Guerzoni, E.: Why Even Ask? PhD dissertation, MIT (2003)
7. Heim, I.: A Note on Negative Polarity and Downward Entailingness. In: Jones, C., Sells, P. (eds.) *Proceedings of NELS 14*. GLSA, University of Massachusetts, Amherst, pp. 98–107 (1984)
8. Fox, D., Hackl, M.: The Universal Density of Measurement. *Linguistics and Philosophy* 29, 537–586 (2006)
9. Heim, I.: Degree Operators and Scope. In: *Proceedings of SALT X*, pp. 40–64. CLC Publications, Cornell University (2000)
10. Karttunen, L., Peters, S.: Conventional Implicature. *Syntax and Semantics* 11, 1–56 (1979)
11. Kay, P.: Even. *Linguistics and Philosophy* 13, 59–112 (1990)
12. Schwarz, B.: Notes on even. Stuttgart University (2000) (Unpublished manuscript)
13. Stalnaker, R.: Presuppositions. *Journal of Philosophical Logic* 2, 447–457 (1973)
14. Gajewski, J.: More on Quantifiers in Comparative Clauses. In: Friedman, T., Ito, S. (eds.) *Proceedings of Semantics and Linguistic Theory XVIII*, pp. 551–568 (2009)
15. Kennedy, C., McNally, L.: Scale Structure Degree Modification, and the Semantics of Gradable Predicates. *Language* 81, 345–381 (2005)

# Some New Observations on ‘Because (of)’

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**Abstract.** *Because (of)* is ambiguous between a ‘reason’ and a ‘plain cause’ interpretation. Presenting a semantic analysis within the framework of Discourse Representation Theory, I argue that *because (of)* always denotes a causal relation between causing facts and caused entities of various sorts and that its interpretational variance is dependent on the ontological nature of the caused entity. Finally, I point to a difference between sentential-complement *because* and nominal-complement *because of* with regard to their interaction with modals. Whereas both *because* and *because of* may outscope e.g. deontic necessity modals, only *because* may outscope epistemic modal operators.

## 1 Introduction: Plain Causes and Reasons

Causal *because (of)* adjuncts are ambiguous between a *reason* and a *plain cause* interpretation as exemplified by the sentences in (1)(2)<sup>1</sup>

(1) Reason:

- a. The dog was put down **because of** its aggressiveness.
- b. I picked out the painting **because** it matches my wall.

(2) Plain cause:

- a. Last winter, a homeless person died **because of** low temperatures.
- b. The stunt plane crashed **because** it ran out of petrol.

In (1a), the complement of *because of*, *its aggressiveness*, is interpreted as a prominent part of the reason or motive of some agent for putting down the dog. Similarly in (1b), the picking out of the painting is motivated by the fact that it matches the wall of the speaker. Assuming that *because (of)* introduces

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<sup>1</sup> In the below discussion of the general semantic properties of *because (of)*, I will mostly use examples involving nominal-complement *because of*. The overall analysis carries over to sentential-complement *because*, though. See Section 3 for a difference in interpretation between the two variants.

a causal relation, I will assume that what is caused in the case of the reason interpretation, is an attitudinal state.

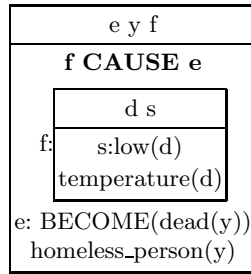
With regard to the plain cause interpretation in (2a), on the other hand, the complement of *because of*, *low temperatures*, is interpreted as the direct or indirect cause of the death of the homeless person. Similar remarks apply to (2b). The plain cause interpretation emerges when the caused event is not under the control of an agent. It should be noted that ‘plain’ does not refer to a certain complexity of the causal chain involved. It is intended to highlight the difference between this interpretation of *because (of)* and the reason interpretation, which, as will become clearer below, is also a cause of sorts.

Despite its frequent occurrence in the literature on causation in general, there exist surprisingly few formal analyses of sentential-complement *because* (cf. e.g. Hara 2008, Johnston 1994, Kratzer 1998), and – to my knowledge – no such analyses of nominal-complement *because of*. Thus, one of the primary goals of this paper is to contribute towards a better understanding of the semantics of *because (of)* and a precisification of some of the factors which determine its interpretation. Accordingly, the paper consists of two main parts: I first provide a discourse representation theoretic analysis of *because (of)* which is more elaborate than previous ones, showing what an account of the interpretational variation of *because (of)* must encompass. I then discuss some subtle differences between *because* and *because of* having to do with their interaction with deontic and epistemic modal operators.

The remainder of this paper is organized as follows: In Section 2, I present the semantics for *because (of)* including a discussion of the lexical organization of the interpretational variants. In Section 3, I discuss the differences between the sentential and nominal complement variants with regard to scopal interaction with modals. Section 4 concludes the paper.

## 2 A DRT Semantics for ‘Because of’

The semantic analysis of *because (of)* is framed within the framework of Discourse Representation Theory (DRT; for an overview, see Kamp & Reyle 1993, van Genabith et al. to appear; for a treatment of some aspects of event-based causality within DRT, see Kamp & Roßdeutscher 1994, Solstad 2007). Before presenting the semantic analysis, I would like to make some brief remarks on the relation between *because* and *because of*. Basically, I regard the two to make a very similar semantic contribution, representing them both by means of the same, underspecified semantics. The variation which may be observed (cf. Section 3), I contend, is due to the fact that *because* takes sentential, whereas *because of* takes nominal complements. I will however remain neutral with regard to the issue whether the preposition *because of* and the conjunction *because* may also be regarded to be one lexeme, only subject to morphosyntactic variation with respect to the realisation of their complement. When discussing the distributional



**Fig. 1.** DRS showing plain cause interpretation of *because (of)*

differences with regard to modal operators mentioned above (cf. Section 3), this issue will be of some importance.<sup>2</sup>

On my analysis, *because (of)* introduces a causal relation. I further claim that the interpretational variance which can be observed with regard to plain cause or reason interpretations is determined by the ontological nature of the arguments which enter into this causal relation. As we will see below, a reason interpretation can – not very surprisingly – only occur if intentionality is present. However, we will also see that this feature alone is not sufficient to predict the interpretation of *because (of)*.

Although approaching the semantics of *because (of)* from a different perspective than Kratzer (1998), who focuses on the interaction of (sentential-complement) *because* with indefinites, I share with Kratzer the idea that the ambiguity of *because (of)* can be dealt with by means of a common underspecified semantic representation. However, it is an important objective of this paper to enrich the insights offered by Kratzer (1998) and e.g. Johnston (1994), showing more explicitly how the different interpretations of *because (of)* emerge.

I assume that *because (of)* always involves a causal relation between a causing fact (as introduced by its syntactic complement) and some caused entity, which may be a state, an event, an attitudinal or a modal state (corresponding to the syntactic phrase to which the PP is adjoined).

I first turn to the plain cause reading in (2a), repeated below for convenience:

(2a) Last winter, a homeless person died **because of** low temperatures.

As modifiers of predicates which designate states or unintentionally performed events, such as *die*, *because (of)* phrases can trivially only be interpreted as plain causes. The semantics of (2a) is provided in the Discourse Representation Structure (DRS) in Figure 1 (ignoring tense and other aspects not relevant to my present purpose). The causal relation introduced by *because* is printed in boldface

<sup>2</sup> More than once, it has been suggested to me that *because* and *because of* differ with regard to the availability of phonological reduction, which could be taken to provide an argument in favour of treating the two variants differently both morphosyntactically and semantically. However, data from conversations and informal writing show that both variants are subject to such reduction. Thus, one may find both *cos* as a reduction of *because* on the one hand, and *cos of* as a reduction of *because of* on the other.

as the topmost condition of the DRS. The causing fact of this relation occurs below this condition. In the case of (2a), the fact *f* of the temperatures being low causes the event of dying, which is the only eventuality that can be modified by the *because of* phrase. At the bottom of the DRS, the simple event (not involving intentionality) which enters the causal relation as the caused entity, is represented. Admittedly, there is a lot to be said about the nature of the CAUSE predicate itself. For reasons of space, I cannot delve into that matter here and will leave the discussion of the simple case of the plain cause interpretation here.

Before discussing the reason interpretation, let me make some brief remarks on the nature of the causing entity in the causal relation introduced by *because (of)*. As stated above, this causing entity always needs to be of fact type. Informally, facts are taken to be true propositional entities involving existential quantification. Although this is certainly not wholly uncontroversial (cf. e.g. Fine 1982, Kratzer 2002), it is a useful approximation that helps explain a number of distributional facts with respect to the possible arguments of nominal-complement *because of*. Since only very few nouns can be claimed to have referential arguments of fact type (possible exceptions include *fact*, *circumstance* among others, cf. Asher 1993), we expect the occurrence of most nouns as complements of *because of* to be accompanied by a process of reinterpreting the referential arguments as a fact, cf. (3):

(3) People are telling us they are using the bus **because of** the gas prices.

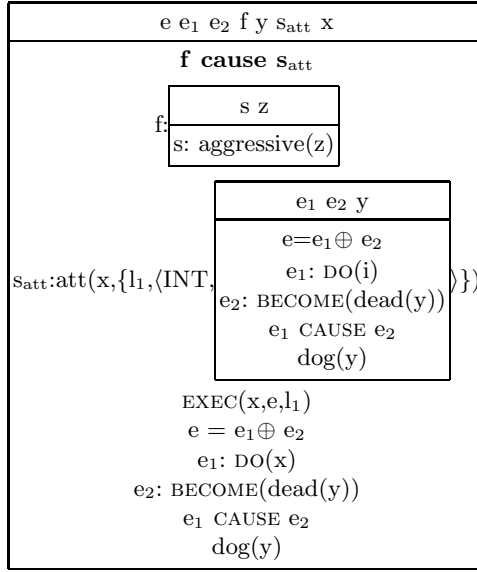
In (3), the *because of* phrase cannot be adequately interpreted as simply saying that the fact that gas is priced causes people to take the bus. Rather, it is a particular quality of that price, very likely that it is high (or far above its normal level), which is the cause of people taking the bus. Notice, however, that this interpretational specification can only occur by way of reinterpretation or similar mechanisms since it is not explicitly expressed. I take a sortal conflict, according to which the DP *the gas prices* does not meet the selectional restrictions of *because of* out of the lexicon, to be the trigger of reinterpretation (Egg 2005).

Turning now to the reason interpretation of *because (of)*, consider again the example in (1a), repeated below for convenience:

(1a) The dog was put down because of its aggressiveness.

In combination with predicates of intentional action such as *put down*, *because (of)* phrases are interpreted as reasons or motives (subject to certain, well-defined restrictions which will be discussed below). In this case, as can be seen from the DRS for (1a) in Figure 2, the discourse referent which enters the causal relation as the caused entity, is an attitudinal state *s<sub>att</sub>* of some agent *x* (the argument of DO), which consists of an intention INT to put down the dog (van Genabith et al. *to appear*)<sup>3</sup>. Importantly, this analysis amounts to viewing *reasons*

<sup>3</sup> The repetition of conditions which can be observed in Figure 2 is indeed intended. However, for reasons of space, it is a matter which I cannot discuss here. It may be noted that the EXEC predicate provides information that the intention of the attitude-holder is actually realized. See van Genabith et al. (*to appear*) for details.



**Fig. 2.** DRS for reason interpretation of *because (of)*

as caused attitudinal states.<sup>4</sup> What is more, assuming the causative relation to be transitive, the fact *f* may also indirectly be seen as a cause of death.

The ambiguity of *because (of)* between the plain cause and reason interpretations is not directly derivable from the representations in Figures 1 and 2. In both figures, the (underspecified) relation CAUSE occurs. Ultimately, the above analysis needs to be complemented by an appropriate theory of causality which makes clear how a causal relation may be subject to interpretational variation depending on the ontological nature of its argument. Short of being able to present such a theory, I will for the sake of simplicity assume that this theory allows us to state meaning postulates such as (4) specifying how the occurrence of certain arguments in the causal relation leads to a difference in realisation of the causal relation.<sup>5</sup> Thus, (4) should be taken to state that whenever an attitudinal state is caused by a fact – the latter ontological category not being subject to any variation in the case of *because (of)* – the fact is a reason of the holder of the attitudinal state for being in whatever state this is:

$$(4) \quad f \text{ CAUSE } s_{att} \implies f \text{ REASON } s_{att}$$

Having presented the basic characteristics of the plain cause and reason interpretations of *because (of)*, I want to point at some data which show that the

<sup>4</sup> This is certainly a view which will be too strong to cover the various uses of the notion of *reason* in the philosophical literature. Still, I believe that it offers a perspective which could be of interest beyond the discussion of the semantics of *because (of)*.

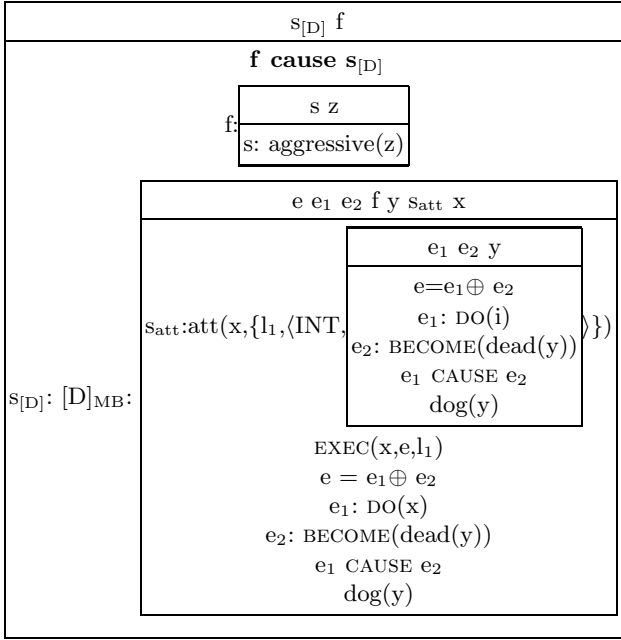
<sup>5</sup> In particular, due to the singular character of attitudinal states, one needs a more sophisticated theory of causation than the standard counterfactual approach of e.g. Lewis (1973).

interpretational variation is not only dependent on the presence or absence of an agent capable of intentional action. Interestingly, the presence of intentionality is not enough to predict a reason interpretation of the *because (of)* phrase. Thus, in contexts involving modals expressing deontic necessity (5a), possibility (5b) or ability (not exemplified here), a reason interpretation is not available for *because (of)*: the *because of* phrases in (5) can only be interpreted as plain causes:

- (5) a. The dog had to be put down **because of** its aggressiveness.  
 b. **Because of** the high crystallisation energies it is possible to measure the crystal growth in transdermal patches even at 25°C.

Sentence (5a) can only be interpreted as saying that the aggressiveness of the dog caused the necessity to put it down. The *because of* phrase cannot target the attitudinal state associated with *put down*. There is no reading available for (5a) according to which it is necessary for the agent to put the dog down for the reason of the dog being aggressive (as opposed to it being three-legged, for instance). Since (5a) is identical to (1a) apart from the presence of the deontic necessity modal *had to*, it seems reasonable to make the modal responsible for the unavailability of the reason interpretation. As indicated by the DRS for (5a) in Figure 3, this is accounted for by assuming that in this case, the causal relation persists between the fact *f* and the modal state  $s_{[D]}$  consisting of a deontic modal operator  $[D]_{MB}$  (MB is short for *modal base*) which takes scope over the DRS for (1a) in Figure 2. The modal blocks access to the attitudinal state in its scope. Consequently, the map in (4) is not applicable in the case of Figure 3. The absence of a reason interpretation is seen to have its rationale in the observation that if something is necessarily the case, reasoning or motives are of no importance. Put differently, if an obligation pertains, it does so regardless of someone’s attitudinal state.

So far, I have said very little about the syntax of *because (of)*. Unfortunately, I cannot go into details here, but it may be noted that the behaviour of *because (of)* in combination with deontic modals, not allowing a reason interpretation, seems to indicate that the assumptions in Johnston (1994) are not adequate for my purposes. Johnston assumes that *because* adjuncts (he does not discuss *because of*) have two possible adjunction sites, one at IP level and one at VP level. If this were the case, one should expect the excluded reason interpretation to be available e.g. in the case of (5a) after all, assuming that the VP level constitutes a position below any modal operators. Unless one wants to make use of any semantic filtering mechanism, I think it is fair to conclude that the semantic observations above suggest that it is more plausible that *because (of)* has a fixed adjunct position which is above not only VP, but also above any projections where (deontic) modals are introduced. The data which Johnston seeks to explain (having to do with the interaction between negation and *because* adjuncts) must thus be explained differently. In the next section, I will make some remarks on possible syntactic adjunction sites for *because* and *because of* adjuncts.



**Fig. 3.** DRS representation of ‘because of’ outscoping a deontic necessity modal

Summarizing the analysis so far, I claimed that *because (of)* denotes a causal relation between a causing fact and a caused entity of various ontological categories. If what is caused is a (modal) state or a simple event not involving intentionality, a ‘plain cause’ interpretation results, if what is caused is an intentionally performed event or otherwise involves an attitudinal state, a ‘reason’ interpretation results.

### 3 ‘because of’ vs. ‘because’ in Epistemic Modal Contexts

I claimed that the above observations are valid for both sentential-complement *because* and nominal-complement *because of*. However, there is one interesting aspect for which the two variants differ with respect to their possible interpretations. Consider the examples in (6), in which *must* should be interpreted epistemically:

- (6) a. Bill must have gone back home **because** the jacket is missing.  
b. Bill must have gone back home **because of** the missing jacket.

Whereas (6a) is ambiguous, (6b) is not. In (6a), the *because* phrase specifies either (i) Bill’s reason or motive for going back home, parallel to (1a) as analysed in the DRS in Figure 2, or (ii) the speaker’s reason (evidence) for inferring that Bill must have gone back home, i.e. the speaker sees that the jacket is missing and concludes from this that Bill must have gone back home. In (6b), however,

only the former interpretation (i) is available for the *because of* phrase. Thus, although one might expect that the semantics of *because of* should principally be identical to that of *because* if the complement of *because of* is reinterpreted as a proposition-like fact, the examples in (6) show that this cannot be the whole story.

Although the ambiguity of *because* in (6a) is well-known (cf. e.g. Sweetser 1990), the difference in interpretational possibilities displayed between (6a) and (6b) is not very well understood. In fact, the only discussion of the contrast in (6) that I am aware of, is found in Degand (2000), which I will return to below. Admittedly, I do not have a final answer to why this difference shows up, either, but in what follows, I want to point out how my analysis could be extended towards explaining the difference displayed in (6).

One possibility to deal with the data in (6) would be to assume that *scopal differences with regard to different modal operators* are involved in the varying behaviour in (6). Thus, one could assume that *because* and *because of* do display a difference in syntactic behaviour, despite their semantic parallels, where only *because* adjuncts may outscope epistemic modals. Thus, Degand (2000: p. 692) assumes that *because of* adjuncts appear at an “intra-clausal” level where no epistemic modal operators are available. On the other hand, *because* clauses are adjoined at an “inter-clausal” level, where epistemic operators may be embedded under the causal relation introduced by *because*. In line with this, it is also imaginable that what matters for the difference is the possibility of *modifying speech act operators*, as discussed by Scheffler (2005), thus making a parallel to Austin’s (1961) famous biscuit conditionals (cf. e.g. Siegel 2006 and Predelli 2009 for discussion). In this case, *because* would unite the split behaviour of German *denn* and *weil*, the latter of which does not allow the evidential reading in (6a). The matter is not quite clear to me though, since the evidential interpretation in (6a) seems intuitively different from the “movie causal” in (7):

- (7) What are you doing tonight, because there’s a good movie on. (Sweetser 1990: p. 77)

Additionally, Scheffler’s ‘conventional implicature’ approach demands that German *denn* does not denote a causal relation, which is hardly plausible for *because of*.

A last possibility to explain the difference in (6) that I would like to mention is exploiting the categorical difference between the complements of *because* and *because of*: On this view, only the (syntactically) clausal complements of *because* and no DPs such as the complements of *because of* may be interpreted evidentially as it would be needed for the reading which (6b) lacks. This could be connected to the process of reinterpretation which is involved in the case of nominal-complement *because of*. According to this view, the proposition-like reinterpretation of the complement of *because of* would only seemingly lead to an interpretation parallel to the truly propositional sentential complements of *because*. However, this solution cannot be any less murky than the assumptions concerning the nature of facts.

As it stands now, the difference in adjunction sites seems to me to be the most plausible and promising option for solving the problems posed by the data in (6).

## 4 Conclusion and Outlook

Summarizing, I argued that the factive causal relation introduced by *because (of)* phrases, **f cause e** or **f cause s** in the DRSs in Figures 1, 2 and 3, is assumed to be neutral with regard to its interpretation as a plain cause or reason. If what is caused is a non-intentional state or event, a plain cause interpretation results, whereas whenever an attitudinal state is caused, the *because (of)* complement is interpreted as a reason. In case what is caused is a deontic modal state, only plain cause interpretations are possible, regardless whether the modal embeds an attitudinal state or not. I also discussed some data involving epistemic modals for which *because* and *because of* differed: only *because* adjuncts seem able to outscope epistemic modal operators.

Future work needs to involve an attempt at clarifying the notion of facts and an explication of the theory of causation, at least to the extent that it is needed for the purpose of the present analysis. In addition, *because* should be compared with other causal expressions in English such as *since*, which only seems to have the reason interpretation. Finally, comparing *because (of)* to similar expressions in other languages would be helpful. Whereas the German preposition *wegen* ‘because of’ seems to behave completely parallel to nominal-complement *because of*, sentential-complement *because* unites the function of *denn* and *weil* in German.

## References

- Asher, N.: Reference to Abstract Objects in Discourse. Kluwer, Dordrecht (1993)
- Austin, J.L.: Ifs and cans. In: Austin, J.L. (ed.) *Philosophical Papers*, pp. 153–180. Oxford University Press, Oxford (1961)
- Degand, L.: Causal connectives or causal prepositions? Discursive constraints. *Journal of Pragmatics* 32, 687–707 (2000)
- Egg, M.: *Flexible Semantics for Reinterpretation Phenomena*. CSLI Publications, Stanford (2005)
- Fine, K.: First order modal theories III – Facts. *Synthese* 53, 43–122 (1982)
- Johnston, M.: *The Syntax and Semantics of Adverbial Adjuncts*. Ph.D. Thesis, University of California Santa Cruz (1994)
- Hara, Y.: Evidentiality of Discourse Items and Because-Clauses. *Journal of Semantics* 25, 229–268 (2008)
- Kamp, H., Reyle, U.: *From Discourse to Logic*. Kluwer, Dordrecht (1993)
- Kamp, H., Roßdeutscher, A.: Remarks on lexical structure and DRS construction. *Theoretical Linguistics* 20, 97–164 (1994)
- Kratzer, A.: Scope or pseudoscope? Are there wide-scope indefinites? In: Rothstein, S. (ed.) *Events and Grammar*, pp. 163–196. Kluwer, Dordrecht (1998)
- Kratzer, A.: Facts: Particulars or information units? *Linguistics and Philosophy* 25, 655–670 (2002)

- Lewis, D.: Causation. *Journal of Philosophy* 70, 556–567 (1973)
- Predelli, S.: Towards a semantics for biscuit conditionals. *Philosophical Studies* 142, 293–305 (2009)
- Scheffler, T.: Syntax and semantics of causal denn in German. In: Dekker, P., Franke, M. (eds.) *Proceedings of the Fifteenth Amsterdam Colloquium*, pp. 215–220. ILLC/Department of Philosophy, Amsterdam (2005)
- Siegel, M.E.A.: Biscuit conditionals: quantification over potential literal acts. *Linguistics and Philosophy* 29, 167–203 (2006)
- Solstad, T.: Lexical Pragmatics and Unification: The Semantics of German Causal 'durch' ('through'). *Research on Language and Computation* 5, 481–502 (2007)
- Sweetser, E.: *From Etymology to Pragmatics: Metaphorical and cultural aspects of semantic structure*. Cambridge University Press, Cambridge (1990)
- van Genabith, J., Kamp, H., Reyle, U.: *Discourse Representation Theory*. In: Gabbay, D. (ed.) *Handbook of Philosophical Logic*. Kluwer, Dordrecht (to appear)

# *Much* Support and *More*

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**Abstract.** This paper examines the semantics of *much* when it occurs as a dummy element, in so-called *much* support (*Fred is diligent; in fact he is too much so*) and *more* comparatives (*more intelligent*, where *more* = *much* + *-er*). It is shown that far from being anomalies, *much* support and *more* comparatives provide a clue to the correct analysis of *much* more generally: *much* is essentially contentless, serving only as a carrier of degree morphology. In short, *much* always acts as *much* support. These findings provide support for a theory of quantity adjectives (*many*, *few*, *much* and *little*) as predicates of scalar intervals, with the remainder of the content traditionally ascribed to them contributed instead by null syntactic elements and operations. The vacuous nature of *much* itself is also argued to account for its infelicity in unmodified form in many contexts (e.g. ??*We bought much rice*).

## 1 Introduction

### 1.1 The Problem

A curiosity about *much* is its ability to act as a dummy element. *Much* otherwise has uses as a quantifier meaning ‘a large quantity of’ (1), and as an adverbial element meaning ‘to a high degree’ (2):

- (1) a. Much alcohol was consumed last night  
b. Much office work is tedious  
c. We don’t have much rice
- (2) a. I much prefer wine to beer  
b. Isabelle doesn’t work much

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But consider cases such as (3), an example of what Corver (1997) refers to as *much* support. If *so* is a pronominal copy of the adjective *diligent* (or of some projection of the adjective), which is modified by the degree modifier *too*, *much* does not appear to make any semantic contribution at all.

- (3) John is diligent; in fact, he is too much so

A similar issue is posed by comparatives formed with *more*. In (4), *more intelligent* and *smarter* seem parallel in interpretation, involving the comparative forms of *intelligent* and *smart*, respectively. This would suggest that the comparative morpheme *-er* and *more* are semantically equivalent. But *more* in its quantificational uses has been analyzed as the comparative of *much* (and *many*) (5) (cf. Bresnan 1973). If this approach is extended to cases such as (4a) (i.e. *more intelligent* = *much* + *-er* + *intelligent*), we again have an extra *much* without apparent semantic content.

- (4) a. Sue is more intelligent than Fred  
b. Sue is smarter than Fred
- (5) I have more [much + *-er*] rice than I need

## 1.2 Previous Treatments

Within the literature, there are two prominent approaches to the facts outlined above. On the one hand, Bresnan (1973) posits an underlying *much* in adjectival projections generally, such that the adjective phrase in (6a) is underlyingly (6b). In pre-adjectival contexts, *much* is then deleted via a rule of *much*-deletion (7):

- (6) a. Mary is too intelligent  
b. [AP[QP too much] intelligent]
- (7)  $\text{much} \rightarrow \emptyset$  [AP... — A]

In this way, a parallel can be reestablished between cases such as (4a), where *much* is present (in its comparative form *much* + *-er* = *more*), and (4b), where *much* has been deleted via (7).

Corver (1997), on the other hand, distinguishes two *much*'s: the lexical contentful *much* of examples such as (2) and the 'dummy' *much* of *much* support (3). The former is an adjectival element that introduces its own degree argument; the latter is a dummy element that is inserted as a last resort to establish a local relationship between a degree operator (e.g. *too* in (3)) and the degree argument of the pro-form *so*. While Corver's analysis has been challenged (notably by Doetjes 1997, to be discussed further below), the notion of a separate dummy *much* has been adopted by later authors, including Kennedy & McNally (2005) and Rett (2006).

But both of these approaches – Bresnan's *much* deletion and Corver's posited ambiguity – add complexity to the grammar. I argue here that when *much* itself receives the correct analysis, neither is in fact necessary.

### 1.3 Main Claim

The central proposal developed in this paper is that there is nothing anomalous about the *much* of *much* support and *more* comparatives. *Much* in these contexts has precisely the same semantics as it does in the apparently contentful cases (11) and (12). Specifically, *much* in its lexical semantics is essentially contentless, serving only as a carrier of degree morphology, which can be inserted as needed for morphological or syntactic reasons, without affecting the compositional semantics.

To be clear, I do not intend to claim that a sentence including *much* has the same meaning as the equivalent sentence without *much*. That is obviously not the case: (8a), for example, means something different than (8b), and (9a) allows a different answer than (9b).

- (8) a. We didn't buy much rice
- b. We didn't buy rice
- (9) a. Did you buy much rice?
- b. Did you buy rice?

Rather, the claim is that to the extent that there is a difference in meaning in the pairs above, none of the extra meaning present in the (a) examples can be attributed to the content of *much* itself. *Much* instead serves merely to support or signal the presence of other contentful elements, which may be phonologically null. Put differently, *much* in essence is always *much* support.

## 2 Proposal

### 2.1 The Decomposition of *much*

The broader context for the present analysis is a theory according to which the adjectives of quantity (Q-adjectives) *many*, *few*, *much* and *little* are taken to denote predicates of scalar intervals, an approach that builds on Schwarzschild (2006) and Heim (2006). This is independently motivated by the need to account for their differential uses, as in (10) and (11), where they occur as modifiers in comparatives.

- (10) a. We have much more than 10 kg of rice
- b. We have little more than 10 kg of rice
- c. We have much less than 10 kg of rice
- (11) a. John is much shorter than Fred
- b. John is much younger than Fred

Q-adjectives are most commonly analyzed as quantifying determiners (Barwise & Cooper 1981), as in (12a). Alternately, building on analyses of cardinal numerals as cardinality predicates (e.g. Landman 2004), one might analyze them as predicates over groups or portions of matter (12b) (along these lines, see Pardee 1989 for a predicative treatment of *many* and *few*).

- (12) a.  $\llbracket \text{much}_{\text{quant}} \rrbracket = \lambda P \lambda Q. \exists x [P(x) \wedge Q(x) \wedge \mu_{\text{DIM}}(x) > d_{\text{Std}}]$   
 b.  $\llbracket \text{much}_{\text{pred}} \rrbracket = \lambda x. \mu_{\text{DIM}}(x) > d_{\text{Std}}$   
 where  $\mu_{\text{DIM}}(x)$  is a measure function that associates a portion of matter with a degree on some dimension  $\text{DIM}$  (e.g. weight, volume, etc.) and  $d_{\text{Std}}$  is a context-dependent standard of comparison

However, neither of the entries in (12) can be applied to examples such as (10) and (11). Here *much* and *little* first of all cannot be analyzed as quantifying determiners (per (12a)), in that there are not two predicates that could serve as arguments. But they also cannot be analyzed as predicates of portions of matter (per (12b)). In (10a), we might be tempted to say that *much* is predicated of that portion of the rice we have in excess of the first 10 kg; but in (10c), there is no equivalent portion of rice of which *much* could be predicated. This same issue applies even more clearly in the case of (11), where there is no stuff of any sort that could provide an argument for the Q-adjective.

Instead, from an intuitive perspective, *much* and *little* in these cases describe the gap between two values or degrees on a scale (cf. Klein 1982). That is, (10c) specifies that the gap between the amount of rice we have and 10 kg is large; (11b) specifies that the gap between John's age and Fred's is large. This can be formalized as follows: First, the gap between two scalar values is represented as a scalar interval, that is, a convex set of degrees (13). *Much* and *little* are then taken to denote predicates of scalar intervals. As a first approximation, this may be represented as in (14), where *much* is true of an interval if its length exceeds some context-dependent standard, while *little* is true of an interval if its length falls short of some (possibly different) standard.

- (13) A set of degrees  $I_{\langle dt \rangle}$  is an interval iff  
 $\forall d, d', d'' \text{ such that } d < d' < d'', (d \in I \wedge d'' \in I) \rightarrow d' \in I$
- (14) a.  $\llbracket \text{much} \rrbracket = \lambda I_{\langle dt \rangle}. \text{length}(I) > d_{\text{Std}}$   
 b.  $\llbracket \text{little} \rrbracket = \lambda I_{\langle dt \rangle}. \text{length}(I) < d_{\text{Std}}$   
 where  $\text{length}(I) = \max(I) - \min(I)$

With this in place, differential examples such as (10) and (11) now receive a straightforward analysis. For example:

- (15)  $\llbracket (10c) \rrbracket = 1$  iff  $\llbracket \text{much} \rrbracket(\{d : \text{amount of rice we have} \leq d \leq 10 \text{ kg}\})$   
 iff  $\text{length}(\{d : \text{amount of rice we have} \leq d \leq 10 \text{ kg}\}) > d_{\text{Std}}$

Thus to accommodate the differential uses of *much* and other Q-adjectives, some of the semantic content typically ascribed to these terms (as in (12)) must be stripped away. Specifically, *much* and *little* as defined in (14) do not introduce quantification over individuals; and second, *much* and *little* do not in their lexical semantics include a measure function, that is, a function that associates portions of matter with degrees (cf. (12)). This would seemingly leave the entries in (14) unable to handle the use of Q-adjectives as quantifiers, as in (1). But these too can be accommodated with the interval-based semantics given above,

by attributing the missing semantic content to other elements. To this end, I first propose that quantificational force arises via existential closure. Second, I follow Schwarzschild (2006) (and less directly Kayne 2005) in proposing that the measure function role is played by a functional head *Meas* (for ‘measure’), in whose specifier position the quantifier phrase headed by quantificational *much* or *little* occurs. *Meas* has the semantics in (16):

$$(16) \quad \llbracket \text{Meas} \rrbracket = \lambda x \lambda d. \mu_{DIM}(x) = d$$

To work out a relevant example, (11a) has the surface structure in (17a). But *much* cannot be interpreted in situ due to a type mismatch, so raises at LF (17b), leaving a trace of type *d* in its base position. The semantic derivation proceeds as in (18).

- (17) a. SS:  $\llbracket_{[DP [QP \text{ much}] \text{ Meas alcohol}] \text{ was consumed}} \rrbracket$   
 b. LF:  $\llbracket_{[QP \text{ much}]_1 \llbracket_{[DP t_1 \text{ Meas alcohol}] \text{ was consumed}} \rrbracket} \rrbracket$

$$(18) \quad \begin{aligned} & \llbracket \text{much}_1 \rrbracket (\llbracket t_1 \text{ Meas alcohol was consumed} \rrbracket) \\ &= \llbracket \text{much}_1 \rrbracket (\lambda d_1. \exists x [\text{alcohol}(x) \wedge \text{consumed}(x) \wedge \mu_{DIM}(x) = d_1]) \\ &= \text{length}(\{d : \exists x [\text{alcohol}(x) \wedge \text{consumed}(x) \wedge \mu_{DIM}(x) = d]\}) > d_{Std} \end{aligned}$$

The end result in (18) specifies that length of the interval from 0 to the degree corresponding to amount of alcohol consumed exceeds  $d_{Std}$  (or in simpler terms, that the amount of alcohol consumed exceeds  $d_{Std}$ ).

Note also that while the examples discussed here involve *much* and *little*, the same approach can be extended to their count counterparts *many* and *few*, by taking the degrees in question to be degrees of cardinality.

However, the analysis outlined here is not, in the present form, quite adequate. *Much* and *little* are gradable expressions, able to combine with degree modifiers (*too much*, *so much*, *as much as*, etc.). In their modified forms, they do not have the ‘greater than standard’ interpretation that characterizes the positive (i.e. unmodified) form (for example, ‘I have as much rice as Fred’ does not entail ‘I have much rice’). This is not captured by the entries in (14), in which the standard of comparison  $d_{Std}$  is part of the lexical semantics of *much* and *little*.

Within the literature on gradable adjectives (e.g. Cresswell 1977, Heim 2000), which is extended to Q-adjectives in particular by Hackl (2000), the usual approach to this issue is to remove the standard of comparison from the semantics of the positive form itself. Instead, the gradable expression is given a degree argument as its first argument, which may be saturated or bound by a degree modifier (e.g. by *too* or *as*). In the case of the bare positive form, where there is no overt degree morphology, a phonologically null degree operator POS (for ‘positive’) plays this role.

Marrying this approach with the interval-based semantics for Q-adjectives developed above, we can give *much* and *little* the revised entries in (19), with (20) (taken from von Stechow 2006) representing a possible semantics for the null positive morpheme POS:

- (19) a.  $\llbracket \text{much} \rrbracket = \lambda d \lambda I_{\langle dt \rangle} . d \in I$   
 b.  $\llbracket \text{little} \rrbracket = \lambda d \lambda I_{\langle dt \rangle} . \neg d \in I$
- (20)  $\llbracket \text{POS} \rrbracket = \lambda I_{\langle dt \rangle} . N_S \subseteq I$

Here POS introduces as a standard of comparison the range  $N_S$  consisting of values that would be considered neither large nor small with respect to the context.<sup>1</sup>

According to the revised definitions in (19), *little* is interpreted as degree negation (a conclusion argued for on independent grounds by Heim 2006), associating an interval with the degrees not contained within it. But *much* simply associates an interval with the degrees within it. The result is that it functions essentially as an identity function on intervals. This becomes evident through an example. With the semantics for *much* and POS introduced above, (11a) has the revised LF in (21), where both *much* and POS have raised from their DP-internal surface positions for type-driven reasons. The semantic derivation proceeds as in (22):

- (21) LF:  $[\text{DegP POS}]_2 [[\text{QP t}_2 \text{much}]_1 [[\text{DPT}_1 \text{Meas alcohol}] \text{ was consumed}]]$
- (22)  $\llbracket \text{POS}_2 \rrbracket (\llbracket \text{t}_2 \text{much}_1 \rrbracket (\llbracket \text{t}_1 \text{Meas alcohol was consumed} \rrbracket))$   
 $= \llbracket \text{POS}_2 \rrbracket (\llbracket \text{t}_2 \text{much}_1 \rrbracket (\lambda d_1 . \exists x [\text{alc}(x) \wedge \text{consumed}(x) \wedge \mu_{DIM}(x) = d_1]))$   
 $= \llbracket \text{POS}_2 \rrbracket (\lambda d_2 . \exists x [\text{alcohol}(x) \wedge \text{consumed}(x) \wedge \mu_{DIM}(x) = d_2])$   
 $= N_S \subseteq \{d : \exists x [\text{alcohol}(x) \wedge \text{consumed}(x) \wedge \mu_{DIM}(x) = d]\}$   
 ‘The amount of alcohol consumed exceeds  $N_S$ ’

Here, *much* takes as argument the set of degrees (interval) formed by lambda abstraction over the trace of type  $d$  in its base position. Subsequently, lambda abstraction over the trace of POS again produces a set of degrees. But as can be verified above, the second set of degrees is identical to the first. Under this analysis, *much* is essentially semantically inert, simply mapping a set of degrees (interval) to itself. It makes no other contribution to the semantics of the sentence.

Thus in analyzing *much* as a gradable expression, yet more of its content must be stripped away, and transferred instead to POS, leaving *much* itself as a pure identity element. Put differently, *much* has no content of its own, but serves only as a carrier of degree morphology (in the case above, of POS).

There is an obvious question that follows: if *much* is semantically contentless, why is it required in at all? For instance, in an example such as (21), why can the DegP headed by POS not combine directly with Meas, eliminating the QP layer (and thus *much*) entirely. Here I follow Doetjes (1997), who considers a similar set of facts from a more syntactic perspective, and take the reasons to be selectional

<sup>1</sup> Nothing in the analysis that follows hinges on the specific definition of POS in (20); what is crucial is that *much* have the semantics in (19a). Note also that with the definition of POS in (20), differential cases such as (10) and (11) must be handled slightly differently than above. This can be accomplished by defining the comparative morpheme *-er* in such a way to produce an interval of the same length as the original gap, but lower bounded by 0. This is worked out in detail in Solt (2009).

in nature. Specifically, degree modifiers (*-er*, *too*, POS, etc.) are restricted to combining with gradable terms (gradable adjectives and Q-adjectives); to occur in the extended noun phrase they must first compose with *much*, creating a QP that has more flexible selectional properties.

## 2.2 Applied to *much* Support and *more* Comparatives

Having concluded that *much* is transparent to semantic composition, we should not be surprised that it is able to function as a dummy element. An analysis of *much* support and *more* comparatives now follows quite simply.

I begin with *much* support. With regards to the syntactic structure, I assume that in the case of a degree modifier plus gradable adjective (e.g. *too diligent*), the degree modifier constitutes a Degree Phrase (DegP) located in the specifier position of the adjective phrase AP (Heim 2000). I take *so* to be a pro-form standing in for the AP, such that SpecAP is not available as a position for a degree modifier. To remedy this, *much* is inserted, in the form of a QP headed by *much* in the specifier position of a higher functional projection of the adjective:

- (23) a. He is [AP [DegP *too*] diligent]  
 b. He is [FP [QP [DegP *too*] *much*] F<sup>0</sup> [AP *so* ~~diligent~~]]

However, due to the transparent nature of *much*, the resulting interpretation is semantically equivalent to what would obtain if *much* were not present:

- (24)  $\llbracket \text{too}_2 \rrbracket (\llbracket \text{t}_2 \text{much}_1 \rrbracket (\lambda d_1. \text{he is } d_1 \text{ diligent}))$   
 $= \llbracket \text{too}_2 \rrbracket (\lambda d_2. \text{he is } d_2 \text{ diligent})$

*More* comparatives can be treated similarly. Adjectives that form *more* comparatives (e.g. *intelligent*) cannot compose directly with the comparative morpheme *-er* (presumably for morphological reasons). I propose that while *-er* comparatives feature a DegP *-er* in SpecAP, *more* comparatives feature a QP *more* (i.e., [QP [DegP *-er*] *much*]), again located in the specifier position of a higher functional projection FP.

- (25) a. Sue is [AP [DegP *-er*] smart]  
 b. Sue is [FP [QP [DegP *-er*] *much*] F<sup>0</sup> [AP *intelligent*]]

But as in the case with *much* support, the interpretation is parallel to that which would obtain without *much*:

- (26)  $\llbracket \text{-er}_2 \rrbracket (\llbracket \text{t}_2 \text{much}_1 \rrbracket (\lambda d_1. \text{Sue is } d_1 \text{ intelligent}))$   
 $= \llbracket \text{-er}_2 \rrbracket (\lambda d_2. \text{Sue is } d_2 \text{ intelligent})$

Thus in both cases, *much* can be inserted to host degree morphology (e.g. *too* or *-er*), without affecting the compositional semantics (cf. Doetjes 1997 for a related conclusion).

## 2.3 Summary

In summary, *much* support and *more* comparatives are not anomalies at all. We do not require a separate semantics for *much* in these cases, nor is it necessary to posit an underlying *much* in constructions where it is not overtly realized. Instead, the same semantic analysis that is required for apparently contentful uses of *much* – one which renders it essentially vacuous – allows *much* to also function as a dummy element.

## 3 The Infelicity of Bare *much*

It has often been noted that bare *much* is only marginally acceptable in many contexts (see Zwicky 2006a, 2006b for discussion). The (carefully chosen) original example (11a) is itself somewhat awkward; and other examples of unmodified quantifier *much* are typically quite bad (27a). By contrast, in the same contexts, *much* in combination with an overt degree morpheme (*-er*, *too*, *that*, etc.) is perfectly felicitous (27b-e).

- (27) a. ??I bought much rice  
       b. I bought more rice than I needed  
       c. I bought as much rice as I could  
       d. I bought too much / so much / that much rice  
       e. How much rice did you buy?

The present analysis suggests an account for this. *Much* is semantically vacuous; its primary role is as a carrier of degree morphology. As such, it is infelicitous in the absence of an overt degree morpheme. In combination with the null morpheme POS, whose interpretation is entirely context dependent, *much* does not have enough content to stand on its own. This is particularly the case because *much* does not even specify a dimension of measurement (e.g. *too much rice* could be an excessive amount in terms of weight, volume, etc.).

The picture is, however, somewhat more complicated, in that bare *much* is not always infelicitous. Specifically, *much* in its unmodified form is awkward (if not outright ungrammatical) in quantificational use (27a), (28a,b), as a post-verbal modifier (28c) and in *much* support (28d). But by contrast, it is quite acceptable in the differential use (29a,b), in partitives (29c) and headless nominals (29d), as a pre-verbal adverb with a small group of verbs (29e), and as a modifier of deverbal adjectives (29f) and two ordinary adjectives, *alike* and *different* (29g).

- (28) a. ??Much wine is left  
       b. ??Sue lost much money in the stock market crash  
       c. ??John slept much  
           (cf. John slept too much; John slept as much as he wanted)  
       d. ??I'm tired; in fact, much so  
           (cf. ...in fact, too much so to go to the party)

- (29) a. We have much more/much less than 10 kg of rice  
 b. John is much taller/much shorter than Fred  
 c. Sue lost much of her money in the stock market crash  
 d. Much of what has been written about the topic is wrong  
 e. I much prefer wine to beer  
 f. a much improved effort; a much loved teacher  
 g. Mice and moles are much alike/much different

Is this pattern evidence that there are in fact two *much*'s, contra the unified account developed in this paper? Corver (1997) uses data such as these as support for his proposed distinction between dummy and lexical *much*: the former must occur with degree morphology (in that its role is to establish a local relationship between a degree operator and the degree argument of another element), while the latter, being itself contentful, may occur bare. However, a closer look at the data in (28) and (29) shows that the distribution of bare *much* does not in any obvious way line up with a contentful versus dummy divide. For example, as noted above, bare *much* is typically awkward in when it occurs as a quantifier (28a,b); yet it seems implausible to align the quantifier *much* to the dummy category. The contrast between *much* as a pre-verbal versus post-verbal adverbial ((29e,f) vs. (28c)) would also be puzzling on this analysis. Note also that in the contexts where unmodified *much* is infelicitous in positive sentences (29), it is perfectly acceptable in the equivalent negative sentences (e.g. *I didn't buy much rice*; *John didn't sleep much*). This suggests that *much* in contexts such as (28) has the status of an NPI (cf. Zwicky 2006b), and that the infelicity of the positive examples cannot be attributed simply to the absence of a degree operator.

If the contrast exemplified above does not reflect the existence of two distinct *much*'s, how can it be accounted for? While I do not have a conclusive explanation for these facts, two (not necessarily incompatible) possibilities suggest themselves. First, consider a headless nominal such as (29d). Here *much* is required to give the noun phrase phonetic content. This points to the possibility that in other of the acceptable cases (e.g. the partitive and differential), bare *much* is allowed by virtue of making a layer of syntactic structure overt. Second, a comparison of the quantificational (28c) to the partitive (29c) suggests that bare *much* is more felicitous when its interpretation is more constrained. What counts as *much money* is entirely context dependent. But partitives are interpreted proportionally; *much of her money* means a large proportion of her money, an interpretation which is much less free than that available in the quantificational case. Differential uses of *much* are also constrained in interpretation. While only the context determines what counts as *much rice* (27a), in the case of *much more than 10 kg of rice* (29a), the difference must be significant in comparison to 10 kg (i.e. the extra amount required to count as *much more than 10 kg* is different than that needed to count as *much more than a ton*). I leave the relative role of these two factors, and their potential to explain other of the contrasts between (28) and (29), as a topic for future research.

## 4 Conclusions

In this paper, I have argued that there is nothing particularly anomalous about *much* support and *more* comparatives. Rather, these constructions provide a clue to the true nature of *much*: a semantically contentless carrier of degree morphology, which may be inserted as needed as a dummy element, and which is (in many contexts) infelicitous in the absence of an overt degree morpheme. This analysis thus provides further support towards a theory of Q-adjectives as predicates of scalar intervals, with much of the content typically ascribed to them instead contributed by phonologically null elements and semantic operations.

## References

- Barwise, J., Cooper, R.: Generalized quantifiers and natural languages. *Linguistics and Philosophy* 4, 159–219 (1981)
- Bresnan, J.: Syntax of the comparative clause construction in English. *Linguistic Inquiry* 4, 275–343 (1973)
- Corver, N.: *Much* support as a last resort. *Linguistic Inquiry* 28, 119–164 (1997)
- Cresswell, M.J.: The semantics of degree. In: Partee, B. (ed.) *Montague grammar*, pp. 261–292. Academic Press, New York (1977)
- Doetjes, J.S.: Quantifiers and selection: On the distribution of quantifying expressions in French, Dutch and English. Ph.D. Dissertation, University of Leiden (1997)
- Hackl, M.: Comparative quantifiers. Ph.D. Dissertation, Massachusetts Institute of Technology (2000)
- Heim, I.: Degree operators and scope. In: *Proceedings of Semantics and Linguistic Theory (SALT) X*. CLC Publications, Ithaca (2000)
- Heim, I.: *Little*. In: *Proceedings of Semantics and Linguistic Theory (SALT) XVI*. CLC Publications, Ithaca (2006)
- Kayne, R.S.: On the syntax of quantity in English. In: Kayne, R.S. (ed.) *Movement and silence*. Oxford University Press, New York (2005)
- Kennedy, C., McNally, L.: The syntax and semantics of multiple degree modification in English. In: *Proceedings of the HPSG 2005 Conference*. CSLI Publications, Stanford (2005)
- Klein, E.: The interpretation of adjectival comparatives. *Journal of Linguistics* 18, 113–136 (1982)
- Landman, F.: *Indefinites and the type of sets*. Blackwell, Oxford (2004)
- Partee, B.H.: Many quantifiers. In: Powers, J., de Jong, K. (eds.) *Proceedings of the 5th Eastern States Conference on Linguistics*, pp. 383–402. Ohio State University, Columbus (1989)
- Rett, J.: How *many* maximizes in the Balkan Sprachbund. In: *Proceedings of Semantics and Linguistic Theory (SALT) XVI*. CLC Publications, Ithaca (2006)
- Schwarzschild, R.: The role of dimensions in the syntax of noun phrases. *Syntax* 9, 67–110 (2006)
- Solt, S.: The semantics of adjectives of quantity. Ph.D. Dissertation, City University of New York (2009)
- von Stechow, A.: Times as degrees: früh(er) ‘early(er)’, spät(er) ‘late(r)’, and phrase adverbs. Tübingen (2006) (Unpublished manuscript)
- Zwicky, A.: Much ado about a lot. *Language Log*, April 5 (2006a)
- Zwicky, A.: Big much squib. *Language Log*, May 17 (2006b)

# Quantifiers and Working Memory

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**Abstract.** The paper presents a study examining the role of working memory in quantifier verification. We created situations similar to the span task to compare numerical quantifiers of low and high rank, parity quantifiers and proportional quantifiers. The results enrich and support the data obtained previously in [123] and predictions drawn from a computational model [45].

**Keywords:** working memory; generalized quantifiers; computational semantics; span test.

## 1 Introduction

The role of working memory in language comprehension has been extensively studied (see e.g. [6]). The theory of the specific aspects of memory has been developed by Baddeley and colleagues [78]. They proposed to extend the concept of a short-term memory, suggesting that it could be divided into three separable components. It has been assumed that working memory consists not only from temporary storage units (phonological or visual) but also from a controlling system (central executive). Working together, these components form a unified memory system that is responsible for the performance in complex tasks.

Danemamn and Carpenter (1980) developed span test to asses the working memory construct proposed in [7]. In the task subjects read series of sentences and are asked to remember the final word of each sentence. Data suggest that the result of the span test (the number of correctly memorized words) is a good predictor of language comprehension and other language-processing tasks [9,10,11,12]. The main idea of the span test is that solving it requires engagement of both processing and storage functions. In an experimental study a trade-off between them is usually observed. There are two possible explanations of this phenomenon. One is a computational theory according to which storage and processing use the same cognitive resource and compete for a limited capacity [13,11]. The second is ‘multiple resource’ theory, where the working memory is viewed as a group of cognitive subsystems each having a specialized function

[8,14]. According to that account performance in a particular task relies on one or more subsystems acting together.

The paper presents a study examining the role of working memory in quantifier verification. We created situations similar to the span task [15]. The aim of our research is to verify the contribution of working memory for a few specific natural language quantifiers.

1.1 Quantifier Verification Model

In [1] the pattern of neuroanatomical recruitment while subjects were judging the truth-value of statements containing natural language quantifiers have been examined using neuroimaging methods. The authors were considering two standard types of quantifiers: first-order (e.g., ‘all’, ‘some’, ‘at least 3’), and higher-order quantifiers (e.g., ‘more than half’, ‘an even number of’). They presented the data showing that all quantifiers recruit the right inferior parietal cortex, which is associated with numerosity, but only higher-order quantifiers recruit the prefrontal cortex, which is associated with executive resources, like working memory.

The distinction between first-order and higher-order quantifiers does not coincide with the computational resources, like working memory, required to compute the meaning of quantifiers. Cognitive difficulty of quantifier processing might be better assessed on the basis of complexity of the minimal corresponding automata [4,5]. Taking this perspective, in [3] an analogical reaction time experiment carefully differentiating between the following classes of quantifiers has been conducted (see Table 1). The study has shown that the increase in reaction time is determined by the minimal automata corresponding to the quantifier. Among others, the results indicate that the numerical and parity quantifiers are processed faster than the proportional quantifiers. This is consistent with computational analysis as only proportional quantifiers demand a recognition mechanism with unbounded internal memory, like a stack in push-down automata (see [3]). Therefore, there is not only a quantitative but also qualitative difference between memory resources which are necessary to compute these two types of quantifiers. This conclusion also follows from the differences in the brain recruitments observed in [1].

Table 1. Quantifiers and complexity of minimal automata

Quantifiers	Examples	Minimal automata
logical	‘all cars’,	acyclic 2-state FA
numerical	‘at least $k$ ’	acyclic FA with number of states depending on $k$
parity	‘an even number of balls’	2-state FA with loops
proportional	‘most lawyers’	PDA

## 1.2 The Present Study

The data obtained so far support the assumption that the difficulty of mental processing of quantifiers depends on the complexity of the corresponding minimal automata. This complexity can be explained by a difference in needed memory resources, e.g., different number of states in the case of various numerical quantifiers. The present paper extends previous results by studying the engagement of working memory during quantifier verification tasks.

We examined three groups of quantifiers: proportional, parity and numerical (high and low rank). We predicted that when subjects are asked to maintain arbitrary information in short-term memory then similar differences between quantifiers should be revealed as those described in [3] as well as in [16]. In particular, the difficulty (indicated by reaction time and accuracy) should decrease as follows: proportional quantifiers, numerical quantifiers of high rank, parity quantifiers, numerical quantifiers of low rank. Additionally, processing of the proportional quantifiers should influence the storage functions. The effect should be stronger in more demanding situation, for instance when the number of elements to be stored in the memory is increasing.

## 2 Method

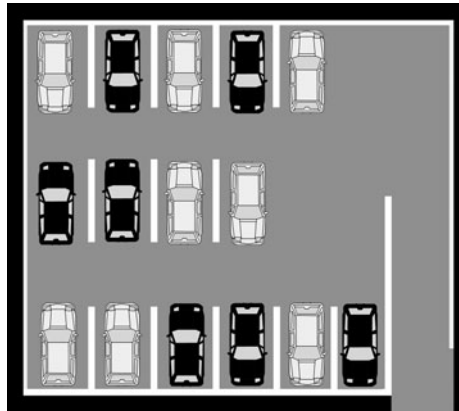
### 2.1 Participants

Sixty native Polish-speaking adults took part in the study. They were volunteers from Warsaw University of Finance and Management undergraduate population. Of these, 18 were male and 42 were female. The mean age was 24 years ( $SD = 4.75$ ) with a range of 21-40 years. Each subject was tested individually in exchange for partial fulfillment of course credits.

### 2.2 Materials and Procedure

The general aim of this study was to assess how subjects are judging the truth-value of statements containing natural language quantifiers with an additional memory load. The experiment was a combined task and consisted of two elements. It required participants to verify sentences and to memorize a sequence of single digits for the later recall.

**Sentence Verification Task.** The task consisted of sixty-four grammatically simple propositions in Polish containing a quantifier that probed a color feature of a car on a display, e.g., ‘Więcej niż połowa samochodów jest czerwona’ (More than half of the cars are red) or ‘Parzysta liczba samochodów jest niebieska’ (An even number of cars are blue). The same number of color pictures presenting a car park with 15 cars were constructed to accompany the propositions. The colors used for the cars were red, blue, green, yellow, purple and black. Each picture contained objects in two colors (see Figure 1).



**Fig. 1.** An example of stimulus used in the first study

Eight different quantifiers were used in the study. They were divided into four groups:

- (1) parity (divisibility) quantifiers (odd, even), DQ;
- (2) proportional quantifiers (less than half, more than half), PQ;
- (3) numerical quantifiers of relatively low rank (less than 5, more than 4), NQ4/5;
- (4) numerical quantifiers of relatively high rank (less than 8, more than 7), NQ7/8.

Each quantifier was presented in 8 trials. Hence, there were in total 64 tasks in the study.

Half of each type of items was true and half false. Propositions were accompanied with a quantity of target items near the criterion for validating or falsifying the proposition. Therefore, these tasks required a precise judgment (e.g. seven targets in 'less than half'). Debriefing following the experiment revealed that none of the participants had been aware that each picture consisted of exactly fifteen objects.

Each quantifier problem involved one 15.5 s event. In the event the proposition and a stimulus array containing 15 randomly distributed cars were presented for 15000 ms followed by a blank screen for 500 ms. Subjects were asked to decide if the proposition accurately described the presented picture. They responded by pressing the button with letter 'p' if true; the button with letter 'f' was pressed if false. The letters refer to first letters of Polish words for 'true' and 'false'.

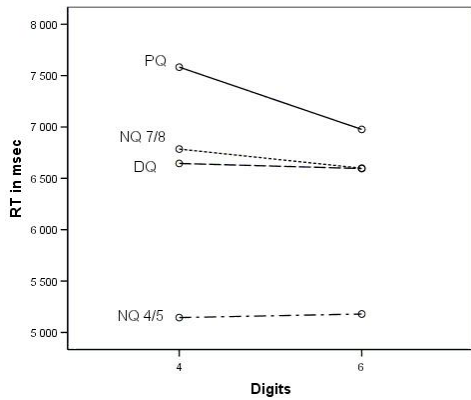
**Memory Task.** At the beginning of each trial the subjects were presented a sequence of digits consisting of four or six elements from the range between 0 and 9. After completing the sentence verification task they were asked to recall the string. Each quantifier type was accompanied by the same number of four and six digits.

### 3 Results

#### 3.1 Sentence Verification Task

ANOVA with type of quantifier (4 levels) and number of digits to memorize (2 levels) as two within-subject factors was used to examine differences in means in reaction time and accuracy of sentence verification task. Greenhouse-Geiser adjustment was applied where needed.

The analysis of reaction time indicated that two main effects – of quantifier type ( $F(2.282, 134.62) = 41.405$ ;  $p < 0.001$ ;  $\eta^2=0.412$ ) and of number of digits ( $F(1, 59) = 4.714$ ;  $p < 0.05$ ;  $\eta^2=0.075$ ) as well as quantifier  $\times$  digits interaction ( $F(2.544, 150.096) = 2.931$ ;  $p < 0.05$ ;  $\eta^2=0.05$ ) – were significant (see Figure 2).



**Fig. 2.** Mean RT in 4- and 6-digit memory load conditions

For simple effects we analyzed differences between quantifiers separately for two memory conditions. We found that mean reaction time was determined by quantifier type while subjects were maintaining 4 digits in memory. Pairwise comparisons among means revealed that PQ were solved longer than other types of quantifiers while NQ 4/5 were processed shorter than the rest of quantifiers; finally, there was no difference between DQ and NQ 7/8. In 6-digit condition we also found a significant effect – NQ 4/5 had shorter average RT than other quantifiers. One-way ANOVA revealed that only PQ differed between memory load conditions (see Table 2).

**Table 2.** Mean RT in milliseconds for each quantifier type

Quantifier	M (4-digit)	M (6-digit)
PQ	7582	6976
DQ	6644	6595
NQ 7/8	6784	6598
NQ 4/5	5144	5179

The main effects of quantifier type ( $F(2.574, 151.867) = 22.238$ ;  $p < 0.001$ ;  $\eta^2=0.275$ ) and of digits ( $F(1, 59) = 4.953$ ;  $p < 0.05$ ;  $\eta^2=0.078$ ) were found in accuracy. All four types of quantifiers differed significantly from one another besides DQ and NQ 7/8 (see Table 3 for mean score). In 4-digit condition all quantifiers were performed worse ( $M = 6.22$ ) than in 6-digit condition ( $M = 6.43$ ).

**Table 3.** Mean (M) of the accuracy for each type of quantifier

Quantifier M	
PQ	5.57
DQ	6.36
NQ 7/8	6.45
NQ 4/5	6.93

Summing up, we observed that in 4-digit memory load condition proportional quantifiers were solved longer and poorer than other types of quantifiers. On the other hand, numerical quantifiers with low rank were performed shorter and better than others. There was no difference between parity quantifiers and numerical quantifiers of high rank.

In 6-digit condition we observed lower average reaction time of numerical quantifiers of low rank in comparison with proportional, parity and numerical quantifiers of high rank, which had equal means. Analysis of accuracy showed the following increase of difficulty: numerical quantifiers of low rank, then parity quantifiers and numerical quantifiers of high rank (the same level), and finally proportional quantifiers.

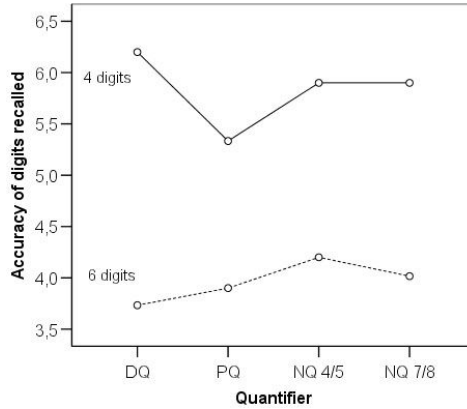
Finally, the accuracy on all types of quantifiers was better in 6-digit condition. However, as we will see in the next section there was a significant drop in recalling task.

### 3.2 Memory Task

ANOVA with two within-subject factors was used to examine how strings of digits (2 levels: four and six elements) were recalled with respect to quantifier type (4 levels) they were accompanied by. Greenhouse-Geiser adjustment was applied where needed.

The analysis indicated main effect of digits ( $F(1, 59) = 90.646$ ;  $p < 0.001$ ;  $\eta^2=0.606$ ) and digits  $\times$  quantifier interaction ( $F(3, 177) = 4.015$ ;  $p < 0.05$ ;  $\eta^2=0.065$ ) (see Figure 3).

To examine the interaction effect we compared recall accuracy for 4 and 6 digits. Significant differences between two situations for each level of second variable were obtained. Performance on digit recall with respect to quantifier type was also analyzed separately for 4- and 6-digit strings. In the former condition digits accompanying PQ were memorized worse in comparison with other determiners, while in the latter condition we did not observe any differences (see Table 4).



**Fig. 3.** Accuracy of 4- and 6-digit recall with respect to quantifier type

**Table 4.** Means of recalling accuracy with respect to quantifier type

Number of digits	M (PQ)	M (DQ)	M (NQ 7/8)	M (NQ 4/5)
4	5.33	6.20	5.90	5.90
6	3.90	3.73	4.01	4.20

4 Discussion

Our study assessed quantifier verification task with additional memory load conditions. Obtained data revealed that in the 4-digit load condition the most difficult were proportional quantifiers (the longest RT and the poorest accuracy). Subjects performed better on numerical quantifiers with low ranks than on the other determiners, and finally there were no differences between parity quantifiers and numerical quantifiers of high rank. The results support our predictions and are consistent with the previous findings in [3] and [17].

We expected similar effects in 6-digit memory load condition. This hypothesis was only confirmed with respect to sentence verification accuracy. The score increased in all types of quantifiers but differences between them remained at the same level as in 4-digit condition. Moreover, we observed that numerical quantifiers of low rank had the lowest average reaction time. Proportional, parity and numerical quantifiers of high rank had equal means.

The discrepancy between performances under two memory load conditions needs explanation. We believe that the analysis of digits retrieval sheds some light on the obtained data. The real differences between quantifiers occurred only in 4-digit condition. Holding six elements in memory was probably too difficult in face of processing secondary task. The decrease of accuracy in digits recall with simultaneous increase in performance on quantifier verification task could be described as a trade-off between processing and storage (see [15,14]).

Another interesting observation concerns proportional quantifiers. In 4-digit condition the strings of numbers accompanying this class of quantifiers were recalled worst. However, in the case of 6-digit memory load there were no differences among quantifier types. It is worth to put those results together with the data on the reaction time for proportional quantifier verification. The mean RT decreased because subjects focused only on the sentence verification task ignoring the recalling task. This may be interpreted as supporting the hypothesis, following from the computational model, that working memory engagement in the case of proportional quantifier processing is qualitatively different than in the processing of quantifiers corresponding to finite-automata.

An interesting result is tied up with numerical quantifiers and the number of states in the corresponding minimal automata. In [5] it has been hypothesized that the number of states is a good predictor of cognitive load. Indeed, our current results show the difference between numerical quantifiers of low and high ranks. This fact strongly supports that claim.

Finally, let us briefly discuss a problematic case. The relation between parity and numerical quantifiers of high rank is somewhat unclear. In our previous study [3] we observed a significant difference in reaction time between those two types of quantifiers. However, the size effect of the difference was smaller than in other pairwise comparisons among quantifiers. Can the computational model account for the discrepancy? It draws an analogy between states and stack, on the one hand, and working memory resources, on the other hand. The difference between parity and numerical quantifiers can not be explained in that way. Minimal automata corresponding to parity quantifiers have two states while in the case of numerical quantifiers one needs in principle more. However, the critical factor might be that numerical quantifiers unlike parity quantifiers correspond to automata without loops (see Table 1). Clearly, 2-state automata with loops are more complex than 2-state acyclic machines (corresponding to Aristotelian quantifiers) and indeed our previous research has shown a difference between the two quantifier groups [3]. However, drawing only from the computational model it is by no means obvious which factor adds more to cognitive difficulty: additional states or loops<sup>1</sup>. This constitutes one of the most interesting problems for our approach (see [2] for a more detailed discussion). A future research focusing on neurocognitive modeling of quantifier comprehension could help in clarifying the interrelations among computational aspects and their cognitive correlates. The aim would be to pin down the specific cognitive mechanisms responsible for quantifier comprehension, taking into account factors like the role of central executive, attentional costs, storage functions as well as aspects of representing and approximating quantities, like distant effect (see e.g. [18]). After all, quantifiers

<sup>1</sup> Notice, that necessity of using loops suggests more complex verification strategies, e.g., people can try to pair object or just count all of them and then divide by 2. There might be no obvious minimal startegy corresponding to the one coded by the automata. The hypothesis would be that if train in using the minimal startegy after a while subjects will improve on parity quantifiers, even performing better than on numerical quantifiers.

might be viewed as a way of embedding number system [19] in natural language. The perspective needs to be carefully investigated in the future.

## References

1. McMillan, C., Clark, R., Moore, P., Devita, C., Grossman, M.: Neural basis for generalized quantifiers comprehension. *Neuropsychologia* 43, 1729–1737 (2005)
2. Troiani, V., Peelle, J., Clark, R., Grossman, M.: Is it logical to count on quantifiers? Dissociable neural networks underlying numerical and logical quantifiers. *Neuropsychologia* 47, 104–111 (2009)
3. Szymanik, J., Zajenkowski, M.: Comprehension of simple quantifiers. Empirical evaluation of a computational model. *Cognitive Science* 34, 521–532 (2010)
4. van Benthem, J.: *Essays in logical semantics*. Reidel, Dordrecht (1986)
5. Szymanik, J.: A comment on a neuroimaging study of natural language quantifier comprehension. *Neuropsychologia* 45, 2158–2160 (2007)
6. Baddeley, A.: Working memory and language: an overview. *Journal of Communication Disorders* 36, 189–208 (2003)
7. Baddeley, A., Hitch, G.: Working memory. In: Bower, G. (ed.) *The psychology of learning and motivation*, pp. 47–90. Academic Press, New York (1974)
8. Baddeley, A.: *Working memory*. Oxford University Press, Oxford (1986)
9. Conway, A., Engle, R.: Individual differences in working memory capacity - more evidence for a general capacity theory. *Memory* 6, 122–125 (1996)
10. Daneman, M., Green, I.: Individual differences in comprehending and producing words in context. *Journal of Memory and Language* 25, 1–18 (1986)
11. Just, M., Carpenter, P.: A capacity theory of comprehension: Individual differences in working memory. *Psychological Review* 99, 122–149 (1992)
12. King, J., Just, M.: Individual differences in syntactic processing: The role of working memory. *Journal of Memory and Language* 30, 580–602 (1991)
13. Daneman, M., Merikle, P.: Working memory and language comprehension: A meta-analysis. *Psychonomic Bulletin and Review* 3, 422–433 (1996)
14. Duff, S., Logie, R.: Processing and storage in working memory span. *The Quarterly Journal of Experimental Psychology* 54, 31–48 (2001)
15. Daneman, M., Carpenter, P.: Individual differences in working memory and reading. *Journal of Verbal Learning and Verbal Behavior* 19, 450–466 (1980)
16. Troiani, V., Peelle, J., Clark, R., Grossman, M.: Magnitude and parity as complementary attributes of quantifier statements. *Neuropsychologia* 47, 2684–2685 (2009)
17. Szymanik, J., Zajenkowski, M.: Improving methodology of quantifier comprehension experiments. *Neuropsychologia* 47, 2682–2683 (2009)
18. Pietroski, P., Lidz, J., Hunter, T., Halberda, J.: The meaning of ‘most’: semantics, numerosity, and psychology. *Mind and Language* 24(5), 554–585 (2009)
19. Dehaene, S.: *The Number Sense: How the Mind Creates Mathematics*. Oxford University Press, USA (1999)

# Pluractionality and the Unity of the Event

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**Abstract.** This paper exposes shortcomings of an analysis to single-event plural verbs (Cusic’s event-internal pluractionals) based on temporal discontinuity. It shows how to ground discontinuity on the participant used to measure out the event, by forcing breaches in the property of Mapping-to-SubObject that its theta role should have. This provides an explanation for why phases cannot be described by the same predicate applying to the whole event, which in turn exposes differences with respect to semelfactive verbs and the minimal units they are made of.

## 1 Introduction

The Italian sentence in (1a) describes the eating of one apple by Luisa that differs from the event described by (1b) because it takes place on and off. Culmination is an implicature in both sentences.

- (1) a. Luisa ha mangiucchiato la mela                    ‘L. ate the apple’  
      b. Luisa ha mangiato la mela                      ‘L. ate the apple’

*Mangiucchiare* is an event-internal pluractional verb [8][1][14], and the multiplicity of subevents it describes, aka ‘phases’ [1], does not impact on the singularity of the event or of its participants. On the contrary, event-external pluractional verbs denote pluralities of units higher up in Cusic’s hierarchy, namely ‘events’ or ‘occasions’. Lasersohn’s [7] definition in (2) concerns the latter. Sources of multiplicity are subsequent times, distinct places or participants, which can be seen as the key in a form of distribution where the event predicate would be the share. Such a key is provided in (2) by a non-overlap condition and a function  $f$  that is a temporal or spatio-temporal trace, or a thematic role. Lasersohn adds the constraints  $P=V$  unless  $X$  is a plurality of phases, and  $n \geq 2$  on cardinality, that relies on pragmatic contextual information for fixing the value of  $n$ .

- (2)  $V\text{-}PA(X) \Leftrightarrow \forall e \ e' \in X [P(e) \& \neg f(e) \circ f(e')] \& \text{card}(X) \geq n$   
PA=pluractional affix,  $f$  is a temporal/spatio-temporal trace function or a thematic role assigned by  $V$

Cases like (1a) are not covered by (2) precisely because multiplicity has an event-internal source, cf. [15] for criticisms. Recently, two proposals have been put forth for this and similar cases. One seeks to identify phases via their temporal trace [11], the other via their local participants [12; 13; 15]. This paper discusses shortcomings in grounding phases on time and shows how to ground them on the participant that is used to measure out the event. This provides an explanation

for why phases cannot be described by the same predicate applying to the event and how they differ from minimal units of an event.

## 2 Event-Internal Plurality

Tatevosov has argued that the morpheme *-kala-* in Chuvash (Altaic, Turkic) introduces a form of verb plurality arising from discontinuity of subevents in a single event.<sup>1</sup> This morpheme is analysed as a degree modifier that lowers below standard a contextually determined gradable property of an event predicate.

- (3)  $||\text{-kala-}|| = \lambda P \lambda e \exists d [F_c(P)(e) = d \wedge d < \text{STANDARD}(F_c)(C)]$   
 $F_c$  = variable over degree functions specifying the degree of a gradable property of an event,  $P$  = type of event,  $C$  = comparison class

Continuity is the property discussed, and  $F_c$  in (3) is assigned the function  $F_{\text{CONTINUITY}}$  as a value. The standard of comparison for the scale of continuity is the maximal degree, because this is an upper closed scale. A function  $\tau^C$ , called COVERING TIME, is defined as a temporal trace function that can return the duration of an event even if it is discontinuous, because it returns the total minimal interval of duration by identifying the initial and final moments of the event. Any subinterval of the covering time of a maximally continuous event is the temporal trace of a subpart of such an event. The covering time of an event with a degree of continuity less than maximal has at least a gap in it, i.e. a subinterval which is the temporal trace of no subevent of  $e$ , cf. (4).

- (4)  $F_{\text{CONTINUITY}}(P)(e) < 1 \rightarrow \forall e [P(e) \rightarrow \exists t [t < \tau^C(e) \wedge \neg \exists e' [e' < e \wedge t = \tau(e')]]]$

Crucially, the definition of continuity relies on the possibility for  $\tau^C(e)$  of identifying the initial and final moments of  $e$ . This is quite a standard assumption if one has a continuous event to start with, cf. [5], and gaps are subsequently added to its trace, whereas the reverse order of action is problematic, i.e. identifying the event by taking away gaps in an interval, as discussed below. Assuming a VP modifier status for *-kala-*, instead of V modifier, gets around the problem of carving out a subevent  $e'$  in an event  $e$  with gaps as in (4). But no evidence nor explicit motivation for the VP level of modification are provided. The data show that *-kala-* combines with the verb stem before perfective affixes and verb inflection, which is common for aspectual affixes.

<sup>1</sup> Tatevosov rejects the label ‘pluractionality’ for the case of plurality he describes. However, the objection he raises concerning the singularity of the participants in the event, applies to event-external pluractionality, and on the contrary, supports a characterisation as event-internal plurality. Another objection he raises is that *-kala-* does not produce the full spectrum of readings typically associated with pluractionality. Note that having a partial spectrum is rather the standard case for a (pluractional) affix, and that the readings produced by *-kala-* mentioned in the paper are typical forms of diminutive event-internal plurality. It is not clear how well his approach generalises to cases where more than one dimension is affected at the same time, quite a common situation in event-internal plurality.

Another way of characterising single events with an internal form of multiplicity has been explored by Tovenà, in single and joint work, w.r.t. a class of Italian and French verbs derived by suffixation, e.g. It. *mordicchiare* (nibble), *tagliuzzare* (cut into small pieces). Plurality, she argues, arises from distributing over the cells of a cover applied to a participant. The levels of the event  $e$  and of the plurality of phases  $e'$  are specified separately in (5), the plurality is given an explicit status at event level through groupification, and the equation  $e=\uparrow e'$  links levels in the representation. From this, we can work out that the suffix *-icchi* contributes a constraint like in (6) on the internal structure of the event.

- (5)  $\|mordicchiare\| = \lambda x \lambda y \lambda e [(mordicchiare(e) \& Ag(e, y) \& Pat(e, x)) \Leftrightarrow \exists e' (*morderePhase(e') \& e=\uparrow e' \& *Ag(e', y) \& {}^M Pat(e', x))]$
- (6) a.  $\|mordere\|(bite) = \lambda x \lambda y \lambda e [mordere(e) \& Ag(e, y) \& Pat(e, x)]$   
 b.  $\|-icchi\| = \lambda O[\lambda x \lambda y \lambda e [(O(x)(y)(e)) \Leftrightarrow \exists e' (*VPhase(e') \& e=\uparrow e' \& *Ag(e', y) \& {}^M Pat(e', x))]]]$

Two operations are exploited to bring about the distribution effect within one event: i) at least one participant is fragmented via a mass role  ${}^M R$  [6], and ii) this role is related to a predicate of phases that denotes in a plural domain, but it is the event predicate that is the basic one and that assigns roles to the participants.<sup>2</sup> Tovenà shows that the verb does not make phases accessible at discourse referent level so they cannot be counted, the cardinality of the plurality cannot be compared even when left unspecified, and there are thematic restrictions that do not apply to pluralities of other levels. The proposal in (5) generalizes by considering property scales measuring an abstract dimension instead of physical entities, e.g. the volume of a physical entity, as well as paths and scales associated to the event because of implicit arguments. In the canonical case, the unfolding of the event is measured by adjacent isomorphic transitions of the theme along a scale related to the event by Krifka's [5] Movement Relation. Whereas, in event-internal pluractional verbs, the correlation between a dynamic predicate and a form of gradability is disrupted. Fragmenting means to cancel the homomorphism between the mereological structures of (one or more) scales and the event.

Both proposals are about single events that exhibit a form of plurality and describe a situation as non canonical. They share the view that the source of multiplicity is to be sought somewhere in the unfolding of the event, and differ in the conception of plurality invoked.

### 3 Grounding Phases

Grounding phases on time raises several problems. First and foremost, total absence of discontinuity—i.e. strict continuity—is generally not enforced in

<sup>2</sup> The parts of the participant work as the distributive key, whose plurality becomes visible only within the event, via the mass role  ${}^M R$ . This role uses a cover necessarily weaker than the cover that has the atom as its unique cell.

canonical events described by unmodified predicates and cannot be taken as a discriminating criterion. Let's call *disc-V* the class of verbs describing discontinuous events. The type of subevents that distinguish events of the *disc-V* type from canonical ones are no trivial gaps where the applicability of the predicate to the interval is suspended, but constitutive parts of the event type under definition. They are not a zooming-in effect. Let's call them lulls. Since they are a required component, the trace function applied to the event must return an interval that contains them too, otherwise the event is not of the *disc-V* type. This is the interval Tatevosov called covering time and that corresponds to the minimal convex interval that has the temporal trace of the V type event as subinterval.

Once we acknowledge that lulls are definitional for *disc-V* type of events, strictly speaking, there is no temporal discontinuity anymore, hence no source of multiplicity. In this respect, the characterisation of *-kala-* in Chuvash as VP modifier is instrumental, because it allows one to have what looks like a single discontinuous event that instantiates *e* in (4) but is not directly defined as an event in itself. Yet,  $\tau^C$  is assumed to correctly identify its initial and final moments, and it is the original property of event that is used to vouch for the composition of all the relevant discontinuous subevents into one event. But why invoking a property that needed to be modified precisely because it was not able to adequately characterise the event under examination? The general question is whether and how can we properly define/identify subevents other than by using temporal intervals, which by themselves do not define event properties.

Furthermore, what said for the event applies to subevents. Either, subevents are of the *disc-V* type and thus contain lulls, which means that *disc-V* events cannot be made of just two *disc-V* subevents, because these subevents themselves must contain lulls.<sup>3</sup> We have a sorites paradox of the heap type here. Or, being two continuous subevents as Tatevosov assumes, they cannot be of *disc-V* type. They actually are of the V type. But this is a way to stipulate that all V-type subevents cannot cluster somewhere within the temporal trace of a *disc-V* event. Moreover, lulls cannot last too long lest the connectedness of the event is jeopardised. Something should be said about what prevents discontinuity from disrupting the event in (11a) into a collection of distinct events of eating (parts of) the same apple.

Let us look at (11a) with the insight that taking out lulls from the temporal trace of a *disc-V* event gives us the trace of a V-type event in its canonical realisation, and explore the alternative possibility of grounding discontinuity on a measure of increment. If events of the *disc-V* type must have discontinuous subevents, i.e. phases, this means that there must be some subevent(s) in which no part of the apple is in a  $\theta$  relation with the event and the event's temporal progression is not suspended. As said above, events described by event-internal pluractional verbs are perceived as non-canonical instantiations of an event type. Instead of using the problematic notion of continuity, the notion of non-canonical event can be defined as the case of an event exhibiting localised

<sup>3</sup> Recall Lasersohn's cardinality constraint  $n \geq 2$  endorsed by Tatevosov.

losses of mapping-to-subobjects (MSO [5]) for a  $\theta$  role whose corresponding  $\theta'$ , which would be assigned by V to the same entity if the event were realised (or described as realised) in its canonical form, does have MSO. This localised effect is characterised by saying that a *disc*-V event must contain subevents that are V events—in all of which  $\theta$  satisfies MSO—and subevents that are not, and therefore are lulls. Generalising, i) the  $\theta$  assigned by the pluractional-V verb to the object is the closure of  $\theta'$  under sum formation of the object, where  $\theta'$  is the role assigned by simplex V to the object, and ii)  $\theta'$  has MSO and MO (mapping-to-object), while  $\theta$  has not. The mapping must be extended to include measures used in the definition of event incrementality.

Discontinuity comes out from a modification of a property of a  $\theta$  role instead of using a temporal definition. The multiplicity of phases perceived in the event is anchored in the parts of an entity/value relevant to the conceptualisation of the event and not in its temporal interval. The non-canonical event is characterised as a predictable modification of the semantic characterisation of the canonical form. This modified semantic characterisation matches the understanding that we are dealing with a class of verbs that have morphologically derived forms. The approach defended in this paper provides explicit motivation for the disequation  $P \neq V$  for event-internal pluralities that Lasersohn had to stipulate. As we have seen, the nature of the event, i.e. its being non-canonical in a way that allows the expression of a local form of plurality, does not lend itself to a recursive definition, and phases cannot be described by the predicate that describes the event. This impossibility may be seen to lurk behind Tatevosov's requirement that subevents be continuous. Next, the issue of the duration of lulls, but not of their presence, boils down to the general, albeit non-trivial, issue of defining continuity for an event.

The complexity of the issue of where the pluractional marker does its modification highlights the fact that, as we just said, the event property used for describing the pluractional event cannot be the same as that used to describe phases, but also that the modification of the verb has consequences at various levels. On the one hand, properties of the theta roles assigned by the verb naturally belong to the content of the verb, and altering them is tantamount to modifying the ingredients of the aspectual characterisation of the verb, the so-called lexical or inner aspect. On the other hand, aspectual consequences spread in the whole VP up to sentential level. A change in the nature of the event description is the most salient output of the use of a pluractional form. We can add two language specific pieces of data from Italian and French to this discussion against an analysis of event-internal pluractional markers as VP modifiers. These languages use evaluative suffixes to form pluractional verbs of the event-internal type.<sup>4</sup> First, evaluative suffixes can form denominal pluractional verbs, beside deverbal ones, see (7). Hence the impact of the suffix must be assessed also below VP level.

<sup>4</sup> This word-formation option is available in other Romance languages, although with varying degrees of productivity. Evaluative morphology typically belongs to the nominal domain, but has a variety of uses [4].

- (7) a. Italian: *sorseggiare* (sip) ← *sorso*<sub>N</sub> (sip)  
 b. French: *pointiller* (dot) ← *point*<sub>N</sub> (dot)

Second, the suffix can affect the conjugation of the verb in deverbal cases, as discussed in [14]. Simplex verb forms may belong to any class of conjugation, whereas the derived pluractional forms all belong to the first class, e.g. 3rd conjugation *tossire* (cough) > 1st conjugation *tossicchiare*. Evaluative suffixes bring about the same effect of normalisation on nouns. All modified nouns belong to the broadest inflection class, independently of the class of the base, e.g. *poeta* (poet) vs. *poetino*.

## 4 On Phases and Minimal Units

In a plurality, phases are parts described all by the same predicate and endowed with some form of atomicity, that makes it possible for us to appreciate their multiplicity but does not warrant their identification. Thus, they cannot be counted, e.g. a claim of victory like *Luisa ha mordicchiato più di Daniele* (Luisa nibbled more than Daniele) cannot be foiled by replying *No, perché lui è più veloce* (no, because he is faster) with the intention of saying that he gave more little bites in the same time span. It is far from clear that a minimal cardinality can be defined unambiguously for these verbs, pace Tatevosov and Lasersohn. Again, the situation hints at a sorites paradox, where the beginning of a sorites series does not coincide with the beginning of a series. There may be a cut-off separating *nibbling* from *biting* when going in this direction—as one may concede that an event of two little bites is still a *nibbling*, yet an event with one is certainly not. There isn't necessarily a cut-off when going the other way. The issue is more complex than just having two series with non-coinciding beginnings. It involves the status of the units. A plurality of phases described by an event predicate P is homogeneous because phases are not identifiable, but P is not properly cumulative for reasons we have seen. First, not just any set of phases is an instance of a P event, in particular the singleton set is not. Second, given what we said on subevents in section 3, only subevents understood as bigger than single phases can be properly called subevents of the same event type P and be added in a series. Third, lulls are subintervals on which we do not want to define subevents. In sum, it appears that cumulativity, and divisivity for that matter, are properties of events and are standardly tested by working with subevents that can qualify as events, not with phases.

Event-internal pluractionals are reminiscent of semelfactives, e.g. *jump*, *cough*, in their iterated/activity reading, but should be kept separate. Pluractional verbs may be derived from semelfactive bases, but they no longer have a semel use, cf It. *tossire-tossicchiare* (cough). In order to expose the differences, let's first recall some claims about semelfactives. Semelfactives are dynamic, as they can occur in the imperative form, and atelic instantaneous, as they combine with punctual adverbs, according to Smith [10], and their peculiarity is the absence of change. In the light of the discussion in section 2 this means that the realisation of the event does not modify the preconditions, and iteration without gaps is possible

as a consequence. As for the activity reading specifically, Rothstein [9] points out that semelfactives are event predicates that denote extended events, as they combine with duration adverbs, take progressive form and are said to induce the imperfective paradox. As for the single event reading, Rothstein, who takes semel verbs to be telic interval predicates related to homonymous activity predicates, is keen to recall that Smith herself acknowledges that semelfactive are events conceptualised as instantaneous though they take time to reach a completion in reality. Thus, semelfactives denote events that have internal structure—as opposed to achievements, which are analysed as near-instantaneous changes from  $\neg\phi$  to  $\phi$ . For Rothstein, the occurrence in the progressive and the imperfective paradox is linguistic evidence for such a structure. Taking scores, we can observe that the combination of being instantaneous and allowing seamless sequences makes the activity reading of semelfactives look like a plurality of phases. The possibility of spatio-temporally locating events in the semel reading or units of events in their activity reading, makes these events and units different from phases and clearly sets apart semelfactives from pluractional verbs.

Events denoted in semel uses may be complex, but so do phases. The series of movements that must occur as part of an event denoted by a semelfactive, in Rothstein terms, are constitutive of its internal structure. It is a single instantiation of this structure that Rothstein takes as paralleling the minimal parts of an activity in Dowty's [2] terms, i.e. the smallest events in P that count as events of P. The difference with ordinary activities, I suggest, is in the non-arbitrary way of dividing minimal events/units that is peculiar to semelfactives. We can see activities as characterised by cycles of parts, but in general their cycles do not have specific first/prominent elements, whereas in semelfactives they do. Rothstein assumes that activities denote in a domain where minimal events are not in an atomic set but in a singular set, whereas, in the semelfactive use of a predicate, a natural atomic function picks out the atomic set. The difference is that a singular set contains minimal singular but overlapping entities, and this, in Rothstein's words, means that two minimal events of walking may overlap but two minimal events of jumping will not. The point is delicate and we do not need to take a stand on it. For our concerns, it suffices to assume that the minimal event in a singular set may have structure inside, but that no part in it qualifies as the beginning or the apex of the cycle, contrary to the minimal event in an atomic set. The recursion of the distinguished part signals the completion of a cycle, what precedes or follows it qualifies as part of the cycle. I won't try to define what counts as a full cycle. What matters is that the prominent part may act as the identifier for the whole event, as usual with achievements, and be relevant to count units/cycles that are viewed as events. Since the whole cycle is short, one-cycle events may be conceptualised as instantaneous.

In short, first, the identifying capacity of a minimal unit is an artefact in an activity and cannot be used to count events. It is the series that we count, leaving aside the important but independent issue of its continuity. Second, the combination of having well defined minimal units, which are perceived and characterised as such by a language, and having output conditions that meet input

conditions, which amounts to what has been dubbed as absence of change in the literature, are the ingredients for the double behaviour of semelfactives. Minimal units are events. Semelfactive predicates denote single events in the *semel* use, and sums of events in the *activity* use. Minimal events can be added one at the time and yield an incremental process. Third, by contrast, event-internal pluractionals share the second ingredient at phase level, but crucially differ with respect to the first. Phases may still be viewed as cycles containing a prominent part that makes it possible to see a multitude, but are not individually mapped onto events in the hierarchy of Cusic, hence cannot be counted. They do not partake in the same event type as the grouped plurality they form.

## 5 Conclusion

Phases of a discontinuous internally plural event appear as a result of altering a  $\theta$  relation of a canonical event description in a way that reduces cohesion. More specifically, at least one theta role lacks a property that is found in the description taken as canonical. The loss of MSO has to do with multiplication, hence it is coupled by a loss of MO. The same pattern of modification can be used to create new verbs from nominal bases.

Phases are viewed as the reflect of the application of the predicate to the parts of the participant demoted to a sum that works as the distributive key. This gives them a form of atomicity sufficient for event-internal plurality. But the modification makes the recursive application of the event property impossible, and phases do not qualify as minimal units of the event they are part of. Therefore, semelfactives and event-internal pluractional verbs are two different types of predicates.

## References

- [1] Cusic, D.: Verbal plurality and aspect. PhD thesis, University of Stanford (1981)
- [2] Dowty, D.R.: Word meaning and Montague grammar. Reidel, Dordrecht (1979)
- [3] Dressler, W.: Studien zur verbalen Pluralität. Österreichische Akademie der Wissenschaften, Phil-Hist (1968)
- [4] Jurafsky, D.: Universal tendencies in the semantics of the diminutive. *Language* 72, 533–578 (1996)
- [5] Krifka, M.: The origins of telicity. In: Rothstein, S. (ed.) *Events and Grammar*, pp. 197–235. Kluwer, Dordrecht (1998)
- [6] Landman, F.: *Events and plurality*. Kluwer Academic Press, Dordrecht (2000)
- [7] Lasnik, P.: *Plurality, conjunction and events*. Kluwer Academic Press, Dordrecht (1995)
- [8] Newman, P.: The classification of Chadic within Afroasiatic. *Universitaire Pers.*, Leiden (1980)
- [9] Rothstein, S.: *Structuring events*. Blackwell Publishing, Oxford (2004)
- [10] Smith, C.: The parameter of aspect. *Studies in Linguistics and Philosophy*. Kluwer Academic Publishers, Dordrecht (1991)
- [11] Tatevosov, S.: Pluractionality vs. discontinuity. In: *Proceedings of the Sixteenth Amsterdam Colloquium*, pp. 217–222 (2007)

- [12] Tovenà, L.M.: A class of pluractional verbs in Italian and French. handout. Semantics beyond set theory, paris, October 25 (2007)
- [13] Tovenà, L.M.: Pluractional verbs that grammaticise number through the part-of relation. In: Proceedings of Going Romance (2008) (to appear)
- [14] Tovenà, L.M.: Issues in the formation of verbs by evaluative suffixation. In: Proceedings of XLIII Congresso della Società di Linguistica Italiana (to appear)
- [15] Tovenà, L.M., Kihm, A.: Nibbling is not many bitings in French and Italian: A morphosemantic analysis of internal plurality. In: Proceedings of the 34th Annual Meeting of the Berkeley Linguistics Society (2008)

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