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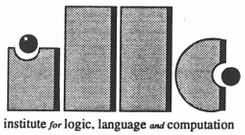

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Preface

The 1995 edition of the Amsterdam Colloquium was the tenth in a series which started in 1976. Originally an initiative of the Department of Philosophy, the Colloquium is now organized by the Institute for Logic, Language and Computation (ILLC), in which staff from the Department of Mathematics and Computer Science, the Department of Philosophy, and the Department of Computational Linguistics, all of the University of Amsterdam, cooperate.

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These proceedings contain 41 papers that were delivered at the Colloquium, of which 40 are accepted papers and one is an invited paper, by Anna Szabolcsi. The papers appear as the authors prepared them. The copyright resides with the individual authors.

Paul Dekker
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Amsterdam, February 1996

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Quantification over interdependent variables

Natasha Alechina [1]

University of Amsterdam

1 Introduction

In standard first order logic, individual variables take values in domains of points which do not have any structure except the one expressible in the language. Each variable can take any value; the assignments are completely independent. We are going to give up this assumption and look what happens to quantifiers in the new setting.

In many-sorted logic the assumption *is* already weakened: each sort has its own domain, therefore it is no longer true that any variable can take any value. In other words, while in the standard case $\exists x\varphi$ is true in a model M under an assignment s iff there is an element d in the domain of M such that $M, s^x_d \models \varphi$, for many sorted logic we should add that d should be in the domain associated with the sort of x. Within each sort the situation is as before. Moreover, it is easy to translate many-sorted logic back into ordinary predicate logic extended with unary predicates for each sort. There is almost no difference between the two, except for the fact that the translation mentioned above leads to increase in complexity.

A more radical variation would be the following (cf. (Németi 1992)): assume that not all assignments to individual variables are allowed. This has a clear intuitive motivation which can be illustrated by the following example from programming languages (cf. (Wirth 1976)). Consider a variable *date* which is an array of variables *day*, *month*, *year*. The values of *date* are triples of numbers, where the first number is the value of *day*, the second number is the value of *month*, and the third number is the value of *year*. The variable *day* may take values in the set $1, \ldots, 31$; the values of *month* come from $1, \ldots, 12$ and year can be any whole number. Each of these three variables can be updated (or, in our setting, quantified over) separately. It is clear however that if the value of *month* is 2, then the value of *day* cannot be 30; depending on the year, *day* may or may not take the value 29. Observe that this is not reducible to the case of many-sorted logic. Some assignments to *day, month* and *year* are not allowed while they respect the domains of the variables. In the truth definition for $\exists$ one might require that s^x_d is an allowed assignment. Here the *names* of variables become important; there can be a witness for $\exists x P(x)$ but no witness for $\exists y P(y)$ in a model.

Again this can be translated into first order logic by adding a special predicate for 'allowed assignment', in the example above, a ternary predicate *Date* would do. But in this case, the difference in complexity is even more striking. It was proved by Németi that the logic corresponding to the set of models equipped with arbitrarily restricted sets of assignments is decidable!

Yet another variation on the standard way of looking at quantifiers and variables is evaluating quantifiers 'locally' (cf. (Blackburn and Seligman 1995)). Consider the truth definition of a modal operator:

$$M \models \Diamond\varphi[w] \Leftrightarrow \exists v(wRv \wedge M, v \models \varphi)$$

[1] I would like to thank Hajnal Andréka, Johan van Benthem, Patrick Blackburn, Jaap van der Does, Dick de Jongh, Michiel van Lambalgen and Istvan Németi for their useful comments and suggestions when this work was in progress.

A modal operator can be seen as a quantifier binding a variable; the name of this variable is not important, but the range of a bound variable (the set of values it may take) depends on the point where the formula is evaluated.

In (van Lambalgen 1991) some generalized quantifiers were given an analogous interpretation, as quantifiers over objects dependent on the parameters of the formula: $Q^d x\varphi(x, y_1, \ldots, y_n)$ is interpreted as $\exists x(R(x, y_1, \ldots, y_n) \wedge \varphi(x, y_1, \ldots, y_n))$; the meaning of a quantifier depends on an assignment to the free variables and is therefore local.

In this paper we try to develop a uniform approach to the phenomena mentioned above.

2 Basic logic

2.1 Language and models

Here we investigate in general the idea that the range of a variable can be restricted by the values of some other 'relevant' variables. We introduce a logic corresponding to the following truth definition: $\exists x\varphi(x, y_1, \ldots, y_n)$ is true in a model M under an assignment of values d_i to y_i, if there is an object d, such that $\varphi(d, d_1, \ldots, d_n)$ holds in M and d is a possible value for x given the assignment of values to y_i. For the time being we assume that only the free variables of the formula $\exists x\varphi$ are relevant for determining the range of x.

Consider a language $\mathcal{L}(\Diamond)$, which contains countably many individual variables $v_0, \ldots, v_n, \ldots$, predicate symbols, $\neg$, $\wedge$, $=$ and a quantifier $\Diamond$. A well formed formula of $\mathcal{L}(\Diamond)$ is defined as follows:

1. if $x_1, \ldots, x_n$ are individual variables and P an n-place predicate symbol, then $P(x_1, \ldots, x_n)$ is a w.f.f.; if x, y are individual variables, then $x = y$ is a w.f.f.;
2. if φ_1 and φ_2 are w.f.f.'s, then $\neg\varphi_1$ and $\varphi_1 \wedge \varphi_2$ are w.f.f.'s;
3. if φ is a w.f.f. and x an individual variable, then $\Diamond_x\varphi$ is a w.f.f.;
4. nothing else is a w.f.f.

We define $\Box_x\varphi$ to denote $\neg\Diamond_x\neg\varphi$; $\vee, \rightarrow, \equiv$ are defined as usual.

Definition 1 A *structured dependence model* is a structure of the form $M = \langle D, R, V\rangle$ where D is a domain, V an interpretation function (a function assigning n-ary predicate symbols subsets of D^n), and R is a relation between a pair $\langle$*variable*, *object*$\rangle$ and a finite set of such pairs. Intuitively, R tells which value a variable can take given the values which other variables have taken. We could have demanded from the start that if $\langle x, d\rangle R\{\langle y_i, d_i\rangle : 1 \leq i \leq n\}$, then $x, y_1, \ldots, y_n$ are all different variables; cf. Lemma 6.

The relation $M, s \models \varphi$ ('φ is *true* in M under assignment s') is defined as follows:

- $M, s \models P_i^n(x_{j1} \ldots x_{jn}) \Leftrightarrow \langle s(x_{j1}) \ldots s(x_{jn})\rangle \in V(P_i^n)$;
- $M, s \models x = y \Leftrightarrow s(x) = s(y)$;
- $M, s \models \neg\varphi \Leftrightarrow M, s \not\models \varphi$;
- $M, s \models \varphi \wedge \psi \Leftrightarrow M, s \models \varphi$ and $M, s \models \psi$;
- $M, s \models \Diamond_x\psi(x, y_1, \ldots, y_n) \Leftrightarrow$ there exists $s' =_x s$ such that

$$\langle x, s'(x)\rangle \; R \; \{\langle y_1, s'(y_1)\rangle, \ldots, \langle y_n, s'(y_n)\rangle\}$$

and $M, s' \models \psi(x, \bar{y})$, where $\bar{y}$ are precisely the free variables of $\Diamond_x\psi$. (This will be denoted by $FV(\Diamond_x\psi) = \bar{y}$.)

We say that $M \models \varphi$ iff $M, s \models \varphi$ for all variable assignments s. □

If Γ and Δ are sets of formulas, we say that $\Gamma \models \Delta$ if for every model M and assignment s, if $M, s \models \Gamma$, then $M, s \models \Delta$. The logic corresponding to this notion of consequence is called L_{str}.

Observe that the truth definition for van Lambalgen's generalized quantifiers can be obtained from the truth definition above by erasing the first element in each pair (or dismissing some information about the variables):

$$s'(x) \; R \; \{s'(y_1), \ldots, s'(y_n)\}$$

instead of

$$\langle x, s'(x)\rangle \; R \; \{\langle y_1, s'(y_1)\rangle, \ldots, \langle y_n, s'(y_n)\rangle\}$$

We will see later that this move strengthens the logic by making the axiom $\Diamond_x\varphi \rightarrow \Diamond_y\varphi[x/y]$ valid (given that y is free for x in φ).

Another special case of structured dependence models are *partial assignments models* defined in section 3.

To finish this section, we prove that structured dependence models share a property which is taken for granted in classical first order logic but, as we shall see later, does not hold for all modal quantifiers.

Lemma 2 *(Locality) For every structured dependence model M, pair of assignments s and z and formula φ, if $s\lceil FV(\varphi) = z\lceil FV(\varphi)$, then*

$$M, s \models \varphi \Leftrightarrow M, z \models \varphi.$$

Proof. Completely analogous to the proof for the classical first order logic, the proof goes by induction on the complexity of φ. For atomic formulas the proposition obviously holds, and the steps for $\neg$ and $\wedge$ are also trivial.

Let $\varphi = \Diamond_x\psi(x, \bar{y})$, and the proposition hold for $\psi(x, \bar{y})$. Assume that $M, s \models \Diamond_x\psi(x, \bar{y})$. Then there is an assignment $s' =_x s$ such that the pair $\langle x, s'(x)\rangle$ is in the relation R to the set of pairs $\{\langle y_i, s'(y_i)\rangle : y_i \in FV(\Diamond_x\psi)\}$ and $M, s' \models \psi$. Consider an assignment $z' =_x z$ with $z'(x) = s'(x)$. Obviously, for every variable u in $FV(\psi)$, $z'(u) = s'(u)$; therefore by the inductive hypothesis $M, z' \models \psi$. Also, since $\langle x, s'(x)\rangle = \langle x, z'(x)\rangle$ and $\langle y_i, s'(y_i)\rangle = \langle y_i, z'(y_i)\rangle$ for every $y_i \in FV(\Diamond_x\psi)$, $\langle x, z'(x)\rangle$ is in the relation R to $\{\langle y_i, z'(y_i)\rangle : y_i \in FV(\Diamond_x\psi)\}$, which gives $M, z \models \Diamond_x\psi(x, \bar{y})$. □

2.2 Hilbert-style axiomatization

To give a Hilbert-style axiomatization for the logic of structured dependence models, we need to extend the language by countably many individual constant symbols. The idea is to repeat a standard Henkin argument; however, this extension is less trivial than in case of the first-order logic. The difficulty comes from the fact that the variables have much more 'individuality' than in first order logic. As a result, the constants should also acquire some.

We add to $\mathcal{L}(\Diamond)$ a set of individual constants d_j^i, countably many for each i. We call individual variables and constants *terms*. Intuitively, d_j^i 'behaves approximately as the variable v_i'. The precise meaning of this will become clear below. Formally, we introduce a function $\eta : TERMS \longrightarrow VARIABLES$, such that $\eta(v_i) = v_i$ and $\eta(d_j^i) = v_i$. We call a term t a *n-term*, if $\eta(t) = v_n$.

A well formed formula is defined as usual. A formula without free variables is called a *sentence*.

The interpretation of constants is the same as in first order logic. If $M = \langle D, R, V\rangle$, and d^i_j is a constant, then $V(d^i_j) \in D$.

The meaning of a term t in a model M given an assignment s, $[t]_{M,s}$ is as expected: if $t = v_i$, $[t]_{M,s} = s(v_i)$, and if $t = d^i_j$, $[t]_{M,s} = V(d^i_j)$. The following clauses of the truth definition are extended to the case of formulas containing constants:

- $M, s \models P^n_i(t_1 \dots t_n) \Leftrightarrow \langle [t_1]_{M,s} \dots [t_n]_{M,s}\rangle \in V(P^n_i)$;
- $M, s \models \Diamond_x \psi(x, t_1, \dots, t_n) \Leftrightarrow$ there exists $s' =_x s$ such that

$$\langle x, s'(x)\rangle \; R \; \{\langle \eta(t_1), [t_1]_{M,s'}\rangle, \dots, \langle \eta(t_n), [t_n]_{M,s'}\rangle\}$$

  and $M, s' \models \psi(x, \bar{t})$.

It is in the last clause that the idea that constants are instances of a particular variable becomes clear. For example,

$$M, s \models \Diamond_{v_3} P(d^1_1, v_3) \Leftrightarrow$$

$$\Leftrightarrow \exists d \in D(\langle v_3, d\rangle R\{\langle v_1, V(d^1_1)\rangle\} \wedge M, s^3_d \models P(d^1_1, v_3))$$

Note that $\Diamond_{v_3} P(d^4_1, v_3)$ can be false in the same model, even if $V(d^1_1) = V(d^4_1)$. Analogously, $\Diamond_{v_3} P(v_1, v_3)$ can be true and $\Diamond_{v_3} P(v_4, v_3)$ false even if $v_1 = v_4$. This corresponds to the failure of the following axiom (even in the language without constants):

$$t_1 = t_2 \wedge \varphi(t_1) \to \varphi(t_2),$$

given that t_2 is free for t_1 is φ. Instead, two weaker versions of this axiom are valid: the version where φ is an atomic formula and the version where t_1 and t_2 are terms of the same type.

Definition 3 *L_{str} is the following axiom system:*

A0 *propositional logic;*

A1 $\Box_x(\varphi \to \psi) \to (\Box_x\varphi \to \Box_x\psi)$, *given that* $\Box_x\varphi$ *and* $\Box_x\psi$ *contain the same parameters (free variables and constants);*

A2 $\varphi \to \Box_x\varphi$, *given that x is not free in φ;*

=1 $t = t$;

=2 $t_1 = t_2 \to t_2 = t_1$;

=3 $t_1 = t_2 \wedge t_2 = t_3 \to t_1 = t_3$;

=4 $t_1 = t_2 \wedge \varphi(t_1) \to \varphi(t_2)$, *given that t_2 is free for t_1 in $\varphi(t_1)$ and either* (=4a) *φ is atomic, or* (=4b) $\eta(t_1) = \eta(t_2)$;

R1 $\varphi,\ \varphi \to \psi / \psi$;

R2 $\varphi(t^i)/\Box_{v_i}\varphi[t^i/v_i]$, *where t^i is an i-term, i.e. v_i or d^i_j.*

The definition of a derivation in L_{str} is standard. We will write $\Gamma \vdash_{L_{str}} \varphi$ for 'φ is derivable from Γ in L_{str}'. In the sequel, the subscript is omitted when it is clear which system weare talking about.

In L_{str} the deduction theorem holds with restrictions (analogous to the restrictions in classical first order logic; cf. for example (Mendelson 1979)).

The following formulas are derivable in L_{str}:

A1b $\Box_x(\varphi \to \psi) \to (\varphi \to \Box_x\psi)$, given that $\Box_x\varphi$ and $\Box_x\psi$ contain the same parameters and x is not free in φ. This formula follows immediately from A1 and A2.

T $\Box_x(\Box_x\varphi \to \varphi)$.

Observe that $\Box_x\varphi \to \varphi$ is not valid in L_{str} since it is possible that φ is false on an assignment s, but true on every other assignment which can be obtained from s by changing the value of x to some d, so that $\langle x, d\rangle$ is in the relation R to the assignment of values to the rest of free variables of φ; this just means that $\langle x, s(x)\rangle$ does not satisfy this condition. Prefixing $\Box_x\varphi \to \varphi$ by $\Box_x$ means that we switch to the assignments s with $\langle x, s(x)\rangle$ accessible from the free variables of $\Box_x\varphi \to \varphi$; on such assignments, if $\Box_x\varphi$ is valid, so is φ.

Proof.

1. $\varphi(x) \to (\Box_x\varphi(x) \to \varphi(x))$ — A0
2. $\Box_x(\varphi(x) \to (\Box_x\varphi(x) \to \varphi(x)))$ — 1, R2
3. $\Box_x\varphi(x) \to \Box_x(\Box_x\varphi(x) \to \varphi(x)))$ — 2, A1, R1
4. $\neg\Box_x\varphi(x) \to (\Box_x\varphi \to \varphi(x))$ — A0
5. $\Box_x(\neg\Box_x\varphi(x) \to (\Box_x\varphi \to \varphi(x)))$ — 4, R2
6. $\neg\Box_x\varphi(x) \to \Box_x(\Box_x\varphi(x) \to \varphi(x))$ — 5, A1b, R1
7. $\Box_x(\Box_x\varphi(x) \to \varphi(x))$ — 3, 6, A0, R1 □

Observe that if ψ_1 and ψ_2 have the same free variables (with the possible exception of x), then from $\psi_1 \to (\psi_2 \to (\psi_1 \wedge \psi_2))$ by two applications of A1 we can derive $\Box_x\psi_1 \wedge \Box_x\psi_2 \to \Box_x(\psi_1 \wedge \psi_2)$. This implies that if all formulas χ_i have the same free variables, then by T the following formula (used in the proof below) is derivable:

$$\Box_x \bigwedge_i (\Box_x\chi_i \to \chi_i).$$

Now we prove completeness of L_{str} for the original language, not containing constants. We prove that for every set of formulas Γ and formula φ, if Γ and φ do not contain constants,

$$\Gamma \vdash \varphi \Leftrightarrow \Gamma \models \varphi.$$

Theorem 4 *L_{str} is sound and complete with respect to the class of structured dependence models: $\Gamma \vdash \varphi \Leftrightarrow \Gamma \models \varphi$.*

Proof. The proof of soundness is easy. As usual, we show that axioms are valid and inference rules preserve validity.

It is obvious that A1 is valid. Observe that A2 is valid due to the fact that structured dependence models satisfy locality. We have already given an example showing that unrestricted =4 is not valid. Now we prove that =4a and =4b are valid. Let φ be atomic. Observe that $M, s \models t_1 = t_2 \Leftrightarrow [t_1]_{M,s} = [t_2]_{M,s}$. $M, s \models P(\dots t_1 \dots)$ iff $\langle \dots [t_1]_{M,s} \dots\rangle \in V(P)$ iff $\langle \dots [t_2]_{M,s} \dots\rangle \in V(P) \Leftrightarrow M, s \models P(\dots t_2 \dots)$. This shows that =4a is valid. To prove that =4b is valid, assume that $\eta(t_1) = \eta(t_2)$, and reason by induction on complexity of φ. We have already proved the atomic case. The cases for $\neg$ and $\wedge$ are trivial. Finally,

$$M, s \models \Diamond_x\varphi(\dots t_1 \dots) \Leftrightarrow \exists d(\langle x, d\rangle \; R \; \{\dots \langle \eta(t_1), [t_1]_{M,s}\rangle \dots\} \wedge M, s^x_d \models \varphi);$$

since t_2 is free for t_1 and $\eta(t_1) = \eta(t_2)$, this is equivalent to

$$\exists d(\langle x, d\rangle \; R \; \{\ldots\langle\eta(t_2), [t_2]_{M,s}\rangle\ldots\} \wedge M, s^x_d \models \varphi(t_1/t_2)) \Leftrightarrow M, s \models \Diamond_x\varphi(\ldots t_2 \ldots).$$

To show that R2 preserves validity, assume that $\Box_{v_i}\varphi(v_i)$ is not valid, that is, there exist a model M and an assignment s such that $M, s \not\models \Box_{v_i}\varphi(v_i)$. Then there is a sequence s', which differs from s at most in i, such that $M, s' \not\models \varphi$. If t^i is v_i, we are done: $\varphi(v_i)$ is not valid. If t^i is some d^i, interpret d^i as $s(v_i)$ (observe that d^i is not free in $\varphi(v_i)$!). An argument analogous to the proof of validity of $= 4b$ above shows that $\neg\varphi(d^i)$ is satisfiable, thus $\varphi(d^i)$ is not valid.

For completeness, it is enough to show that every set Δ not containing constants and consistent with L_{str} has a model.

Assume that we have such a set Δ. We are going to construct a canonical model for Δ. The construction is analogous to the proof of completeness theorem for the minimal logic of dependence models by van Benthem given in (van Benthem and Alechina 1993).

We extend the language by adding countably many new constants

$$u^0_0, u^0_1, u^0_2, \ldots, u^0_n, \ldots$$

$$\ldots$$

$$u^m_0, u^m_1, u^m_2, \ldots, u^m_n, \ldots$$

$$\ldots$$

As usual, the domain of the model for Δ will consist of the equivalence classes of constants. Replace each free occurrence of each variable v_i in Δ by u^i_0. The resulting set of sentences is called Δ'. If we construct a model for Δ', then this model and the assignment $s_0 : v_i \mapsto [u^i_0]$ will satisfy Δ.

Make a list of sentences in the new language. Let us call this list Φ.

Let $\Sigma_0 = \Delta$, and $\Sigma_n = \Sigma_{n-1}$ if the nth formula in Φ is not consistent with Σ_{n-1}. If φ_n is consistent with Σ_{n-1}, we have two possibilities. If φ_n is not of the form $\Diamond_{v_i}\psi$, then $\Sigma_n = \Sigma_{n-1} \cup \{\varphi_n\}$.

If φ_n is of the form $\Diamond_{v_i}\psi$, we add a witness for $\Diamond_{v_i}$ as follows. We take the first new constant u^i_j which does not occur in Σ_{n-1} and add to Σ_{n-1} $\psi[v_i/u^i_j]$ and the set $\{\Box_{v_i}\chi \to \chi(v_i/u^i_j) : \Box_{v_i}\chi$ contains precisely the same constants as $\Diamond_{v_i}\psi\}$.

Let $\Sigma = \bigcup_n \Sigma_n$. Assume that Σ is inconsistent. Then a contradiction is derivable at some finite step n. Let Σ_{n-1} be consistent, and assume that Σ_n is not. Then for some $\Diamond_{v_i}\psi$,

$$\Sigma_{n-1} \cup \{\bigwedge_\chi(\Box_{v_i}\chi \to \chi(u^i_j)) \wedge \Diamond_{v_i}\psi \wedge \psi(u^i_j)\}$$

is inconsistent. Again, a contradiction is derivable from some finite conjunction of formulas in Σ_{n-1} and a finite conjunction of $\Box_{v_i}\chi_k \to \chi_k$ for $1 \le k \le m$.

By R2 we obtain

$$\Sigma_{n-1} \vdash \Box_{v_i} \bigwedge_{k\le m} (\Box_{v_i}\chi_k \to \chi_k(v_i)) \to \Box_{v_i}(\Diamond_{v_i}\psi(v_i) \to \neg\psi(v_i))$$

Now we apply A1 (all χ_k have the same parameters) and T to derive

$$\Box_{v_i} \bigwedge_{k\le m} (\Box_{v_i}\chi_k \to \chi_k).$$

Together with the statement above, this gives

$$\Sigma_{n-1} \vdash \Box_{v_i}(\Diamond_{v_i}\psi(v_i) \to \neg\psi(v_i)),$$

which by A1b implies

$$\Sigma_{n-1} \vdash \Diamond_{v_i}\psi(v_i) \to \Box_{v_i}\neg\psi(v_i),$$

that is,

$$\Sigma_{n-1} \vdash \Diamond_{v_i}\psi(v_i) \to \neg\Diamond_{v_i}\psi(v_i),$$

i.e. contrary to our assumption Σ_{n-1} is inconsistent (note contain u^i_j free, therefore the deduction theorem is applicable).

Now we have established that Σ is a maximal consistent set, i.e. it is consistent, it contains, for every formula $\varphi \in \Phi$, either φ or $\neg\varphi$, it is closed deductively, and it contains $\varphi_1 \wedge \varphi_2$ iff it contains both φ_1 and φ_2.

We define a model M_Σ for Δ using Σ as follows. Let the domain be the set of equivalence classes of constants, i.e. elements of the form $[u^i_j] = \{u : u = u^i_j \in \Sigma\}$.

For the interpretation function, we put

$$\langle[d_1], \ldots, [d_n]\rangle \in V(P) \text{ iff } P(d_1, \ldots, d_n) \in \Sigma.$$

This is well defined by the axiom =4a. The interpretation of constants is quite natural: $V(u) = [u]$.

The definition of R is slightly less straightforward:

$$\langle v_{i_1}, [d_1]\rangle R\{\langle v_{i_2}, [d_2]\rangle, \ldots, \langle v_{i_n}, [d_n]\rangle\}$$

holds in M_Σ iff for some $e_1, \ldots, e_n$, such that $e_j \in [d_j]$ and$\eta(e_j) = i_j$ and for all formulas χ, such that $\Box_{v_{i_1}}\chi$ contains precisely the constants $e_2, \ldots, e_n$,

$$\forall\chi(\Box_{v_{i_1}}\chi(v_{i_1}, e_2, \ldots, e_n) \to \chi(e_1, e_2, \ldots, e_n) \in \Sigma).$$

Observe that this is equivalent to

$$\forall\chi(\chi(e_1, e_2, \ldots, e_n) \to \Diamond_{v_{i_1}}\chi(v_{i_1}, e_2, \ldots, e_n) \in \Sigma)$$

Also, observe that by =4b, if

$$\forall\chi(\Box_{v_{i_1}}\chi(v_{i_1}, e_2, \ldots, e_n) \to \chi(e_1, e_2, \ldots, e_n) \in \Sigma)$$

holds for *some* $e_1, \ldots, e_n$ with $e_j \in [d_j]$ and $\eta(e_j) = i_j$, then it holds for *all* such constants.

Lemma 5 *(Truth Lemma) For every sentence φ, such that φ contains not more than oneterm of each type in the scope of each quantifier, $M_\Sigma \models \varphi \Leftrightarrow \varphi \in \Sigma$.*

Proof. The proof of the lemma goes by induction. The inductive hypothesis is somewhat stronger than the statement of the lemma; it says

$$\varphi(d_1, \ldots, d_n) \in \Sigma \Leftrightarrow M_\Sigma, s \models \varphi(v_{i_1}, \ldots, v_{i_m}, d_{m+1}, \ldots, d_n),$$

$0 \le m \le n$, where $s(v_{i_j}) = [d_j]$ and $\eta(d_j) = i_j$ for $j \le m$, for every formula φ which satisfies the condition of the lemma.

Without this condition, the assignment s above is not well defined. Namely, if φ contains two different constants d and e of type i, which are not in the same equivalence class, s would have to assign v_i two different objects $[d]$ and $[e]$.

since t_2 is free for t_1 and $\eta(t_1) = \eta(t_2)$, this is equivalent to

$$\exists d(\langle x, d\rangle \; R \; \{\ldots\langle\eta(t_2), [t_2]_{M,s}\rangle\ldots\} \wedge M, s^x_d \models \varphi(t_1/t_2)) \Leftrightarrow M, s \models \Diamond_x\varphi(\ldots t_2 \ldots).$$

To show that R2 preserves validity, assume that $\Box_{v_i}\varphi(v_i)$ is not valid, that is, there exist a model M and an assignment s such that $M, s \not\models \Box_{v_i}\varphi(v_i)$. Then there is a sequence s', which differs from s at most in i, such that $M, s' \not\models \varphi$. If t^i is v_i, we are done: $\varphi(v_i)$ is not valid. If t^i is some d^i, interpret d^i as $s(v_i)$ (observe that d^i is not free in $\varphi(v_i)$!). An argument analogous to the proof of validity of $= 4b$ above shows that $\neg\varphi(d^i)$ is satisfiable, thus $\varphi(d^i)$ is not valid.

For completeness, it is enough to show that every set Δ not containing constants and consistent with L_{str} has a model.

Assume that we have such a set Δ. We are going to construct a canonical model for Δ. The construction is analogous to the proof of completeness theorem for the minimal logic of dependence models by van Benthem given in (van Benthem and Alechina 1993).

We extend the language by adding countably many new constants

$$u^0_0, u^0_1, u^0_2, \ldots, u^0_n, \ldots$$
$$\ldots$$
$$u^m_0, u^m_1, u^m_2, \ldots, u^m_n, \ldots$$
$$\ldots$$

As usual, the domain of the model for Δ will consist of the equivalence classes of constants. Replace each free occurrence of each variable v_i in Δ by u^i_0. The resulting set of sentences is called Δ'. If we construct a model for Δ', then this model and the assignment $s_0 : v_i \mapsto [u^i_0]$ will satisfy Δ.

Make a list of sentences in the new language. Let us call this list Φ.

Let $\Sigma_0 = \Delta$, and $\Sigma_n = \Sigma_{n-1}$ if the nth formula in Φ is not consistent with Σ_{n-1}. If φ_n is consistent with Σ_{n-1}, we have two possibilities. If φ_n is not of the form $\Diamond_{v_i}\psi$, then $\Sigma_n = \Sigma_{n-1} \cup \{\varphi_n\}$.

If φ_n is of the form $\Diamond_{v_i}\psi$, we add a witness for $\Diamond_{v_i}$ as follows. We take the first new constant u^i_j which does not occur in Σ_{n-1} and add to Σ_{n-1} $\psi[v_i/u^i_j]$ and the set $\{\Box_{v_i}\chi \to \chi(v_i/u^i_j) : \Box_{v_i}\chi$ contains precisely the same constants as $\Diamond_{v_i}\psi\}$.

Let $\Sigma = \bigcup_n \Sigma_n$. Assume that Σ is inconsistent. Then a contradiction is derivable at some finite step n. Let Σ_{n-1} be consistent, and assume that Σ_n is not. Then for some $\Diamond_{v_i}\psi$,

$$\Sigma_{n-1} \cup \{\bigwedge_\chi(\Box_{v_i}\chi \to \chi(u^i_j)) \wedge \Diamond_{v_i}\psi \wedge \psi(u^i_j)\}$$

is inconsistent. Again, a contradiction is derivable from some finite conjunction of formulas in Σ_{n-1} and a finite conjunction of $\Box_{v_i}\chi_k \to \chi_k$ for $1 \le k \le m$.

By R2 we obtain

$$\Sigma_{n-1} \vdash \Box_{v_i} \bigwedge_{k\le m} (\Box_{v_i}\chi_k \to \chi_k(v_i)) \to \Box_{v_i}(\Diamond_{v_i}\psi(v_i) \to \neg\psi(v_i))$$

Now we apply A1 (all χ_k have the same parameters) and T to derive

$$\Box_{v_i} \bigwedge_{k\le m} (\Box_{v_i}\chi_k \to \chi_k).$$

Together with the statement above, this gives

$$\Sigma_{n-1} \vdash \Box_{v_i}(\Diamond_{v_i}\psi(v_i) \to \neg\psi(v_i)),$$

which by A1b implies

$$\Sigma_{n-1} \vdash \Diamond_{v_i}\psi(v_i) \to \Box_{v_i}\neg\psi(v_i),$$

that is,

$$\Sigma_{n-1} \vdash \Diamond_{v_i}\psi(v_i) \to \neg\Diamond_{v_i}\psi(v_i),$$

i.e. contrary to our assumption Σ_{n-1} is inconsistent (note that Σ_{n-1} does not contain u^i_j free, therefore the deduction theorem is applicable).

Now we have established that Σ is a maximal consistent set, i.e. it is consistent, it contains, for every formula $\varphi \in \Phi$, either φ or $\neg\varphi$, it is closed deductively, and it contains $\varphi_1 \wedge \varphi_2$ iff it contains both φ_1 and φ_2.

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$$\langle v_{i_1}, [d_1]\rangle R\{\langle v_{i_2}, [d_2]\rangle, \ldots, \langle v_{i_n}, [d_n]\rangle\}$$

holds in M_Σ iff for some $e_1, \ldots, e_n$, such that $e_j \in [d_j]$ and$\eta(e_j) = i_j$ and for all formulas χ, such that $\Box_{v_{i_1}}\chi$ contains precisely the constants $e_2, \ldots, e_n$,

$$\forall\chi(\Box_{v_{i_1}}\chi(v_{i_1}, e_2, \ldots, e_n) \to \chi(e_1, e_2, \ldots, e_n) \in \Sigma).$$

Observe that this is equivalent to

$$\forall\chi(\chi(e_1, e_2, \ldots, e_n) \to \Diamond_{v_{i_1}}\chi(v_{i_1}, e_2, \ldots, e_n) \in \Sigma)$$

Also, observe that by =4b, if

$$\forall\chi(\Box_{v_{i_1}}\chi(v_{i_1}, e_2, \ldots, e_n) \to \chi(e_1, e_2, \ldots, e_n) \in \Sigma)$$

holds for *some* $e_1, \ldots, e_n$ with $e_j \in [d_j]$ and $\eta(e_j) = i_j$, then it holds for *all* such constants.

Lemma 5 *(Truth Lemma) For every sentence φ, such that φ contains not more than oneterm of each type in the scope of each quantifier, $M_\Sigma \models \varphi \Leftrightarrow \varphi \in \Sigma$.*

Proof. The proof of the lemma goes by induction. The inductive hypothesis is somewhat stronger than the statement of the lemma; it says

$$\varphi(d_1, \ldots, d_n) \in \Sigma \Leftrightarrow M_\Sigma, s \models \varphi(v_{i_1}, \ldots, v_{i_m}, d_{m+1}, \ldots, d_n),$$

$0 \le m \le n$, where $s(v_{i_j}) = [d_j]$ and $\eta(d_j) = i_j$ for $j \le m$, for every formula φ which satisfies the condition of the lemma.

Without this condition, the assignment s above is not well defined. Namely, if φ contains two different constants d and e of type i, which are not in the same equivalence class, s would have to assign v_i two different objects $[d]$ and $[e]$.

The clause for atomic formulas follows from the definition of V_Σ. The clauses for negation and conjunction are trivial. Assume that φ is of the form $\Diamond_{v_{i_1}}\psi(v_{i_1}, d_2, \ldots, d_n)$ with $\eta(d_j) = i_j$.

Let $\Diamond_{v_{i_1}}\psi(v_{i_1}, d_2, \ldots, d_n) \in \Sigma$. By the construction of Σ, there is a witness for $\Diamond_{v_{i_1}}$, namely a constant d_1 with $\eta(d_1) = i_1$, such that $\psi(d_1, \ldots, d_n) \in \Sigma$ and

$$\forall\chi(\Box_{v_{i_1}}\chi(v_{i_1}, d_2, \ldots, d_n) \to \chi(d_1, d_2, \ldots, d_n) \in \Sigma).$$

By the definition of R, the latter means

$$\langle v_{i_1}, [d_1]\rangle R\{\langle v_{i_2}, [d_2]\rangle, \ldots, \langle v_{i_n}, [d_n]\rangle\}.$$

By the inductive hypothesis, $M_\Sigma, s \models \psi(v_{i_1}, \ldots, v_{i_m}, d_{m+1}, \ldots, d_n)$, where $s(v_{i_j}) = [d_j]$. Thus $M_\Sigma, s' \models \Diamond_{v_{i_1}}\psi(v_{i_1}, \ldots, v_{i_m}, d_{m+1}, \ldots, d_n)$, where s' is any assignment which differs from s in the i_1st coordinate.

Let $M_\Sigma, s \models \Diamond_{v_{i_1}}\psi(v_{i_1}, \ldots, v_{i_m}, d_{m+1}, \ldots, d_n)$, where $s(v_{i_j}) = [d_j]$ for $1 < j \leq m$, and $\eta(d_j) = i_j$ for $m < j \leq n$. Then for some $[d_1]$

$$\langle v_{i_1}, [d_1]\rangle R\{\langle v_{i_2}, [d_2]\rangle, \ldots, \langle v_{i_n}, [d_n]\rangle\}$$

and $M_\Sigma, s^{i_1}_{[d_1]} \models \psi(v_{i_1}, \ldots, v_{i_m}, d_{m+1}, \ldots, d_n)$. By the definition of R, there are $e_1, \ldots, e_n$, with $e_j \in [d_j]$ and $\eta(e_j) = i_j$, $1 \leq j \leq n$, such that

$$\forall\chi(\chi(e_1, e_2, \ldots, e_n) \to \Diamond_{v_{i_1}}\chi(v_{i_1}, e_2, \ldots, e_n) \in \Sigma).$$

By the inductive hypothesis, $\psi(e_1, \ldots, e_n) \in \Sigma$. Therefore $\Diamond_{v_{i_1}}\psi(v_{i_1}, e_2, \ldots, e_n) \in \Sigma$. By =4b, the same holds for any $e'_2, \ldots, e'_n$ in the eqivalence classes $[d_2], \ldots, [d_n]$ if $\eta(e_j) = i_j$.

This ends the proof of the truth lemma. □

Observe that the set Δ' consists of sentences satisfying the condition of the lemma; so, the lemma gives $M_\Sigma \models \Delta'$.

Coming back to the restriction on the set of formulas for which the truth lemma holds, note that since the extended language contains formulas of the form $\varphi(u^1_0, u^1_1)$, it may happen that the following set of pairs is in R:

$$\langle v_1, u^1_2\rangle R\{\langle v_1, u^1_0\rangle, \langle v_1, u^1_1\rangle\}.$$

This looks quite counterintuitive given that in accordance with the intended meaning of R this means: 'if the variable v_1 takes the value u^1_0 *and* the value u^1_1, u^1_2 is a possible value for v_1'. Later, when we interpret

$$\langle x_1, d_1\rangle R\{\langle x_2, d_2\rangle, \ldots, \langle x_n, d_n\rangle\}$$

as '$\{\langle x_i, d_i\rangle : 1 \leq i \leq n\}$ is a good *assignment*', i.e. a function, this becomes completely unacceptable. So the last step in the construction of the canonical model is to show that we may leave only such decent, or functional sets of pairs in the relation R and throw away all non-functional sets of pairs in R, where some variable occurs with different objects.

Lemma 6 *Let φ be a formula which contains at most one term of each type in the scope of each quantifier. Then for every structured dependence model M and assignment s, $M, s \models \varphi \Leftrightarrow M', s \models \varphi$, where M' is obtained from M by leaving only functional sets of pairs in the relation R.*

Proof. The proof goes by induction on the complexity of φ. The only non-trivial step is the quantifier clause. Let φ be $\Diamond_{v_i}\psi(v_i, t_1, \ldots, t_n)$. By assumption, $\eta(t_j) \neq v_i$ for $1 \leq j \leq n$, and $\eta(t_j) \neq \eta(t_k)$ for $1 \leq j \neq k \leq n$. But then there is an element d satisfying

$$\langle v_i, d\rangle\ R\ \{\langle \eta(t_1), [t_1]_{M,s}\rangle, \ldots \langle \eta(t_n), [t_n]_{M,s}\rangle\}$$

if and only if there is a corresponding element in M', since the set of pairs above is functional. Therefore

$$M, s \models \Diamond_{v_i}\psi(v_i, t_1, \ldots, t_n) \Leftrightarrow M', s \models \Diamond_{v_i}\psi(v_i, t_1, \ldots, t_n).$$

This finishes the proof of the lemma and of the Theorem 4. □

3 Obtaining stronger logics

3.1 In the direction of generalized quantifiers

Every generalized quantifier satisfies the axiom of alphabetic variants: $\Box_x\varphi \to \Box_y\varphi(x/y)$, with y free for x in φ. It turns out that this axiom is the only one which we need to add to L_{str} to obtain the minimal logic of *dependence models* defined in (van Benthem and Alechina 1996):

Definition 7 A *dependence model* is a structure of the form $M = \langle D, R, V\rangle$ where (D, V) is as before, and R is a relation between elements and finite sets of elements of D, called the dependence relation.

The relation $M, s \models \varphi$ is defined as usual, except for $M, s \models \Diamond_x\psi(x, y_1, \ldots, y_n) \Leftrightarrow$ there exists $s' =_x s$ such that $R(s'(x), \{s'(y_1), \ldots, s'(y_n)\})$ and $M, s' \models \psi(x, \bar{y})$, where $\bar{y}$ are precisely the free variables of $\exists x\psi$. □

Proposition 8 *$L_{str} + \Box_x\varphi \to \Box_y\varphi(x/y)$ is complete with respect to the class of dependence models.*

Proof. Follows from the completeness theorem for the minimal logic of dependence models, given in (van Benthem and Alechina 1996), (Alechina 1995). □

Observe that the *names* of the variables do not matter in dependence models, where R is a relation between an object and a finite set of objects. Not surpisingly, the axiom of renaming bound variables corresponds to a sort of permutation equivalence for variables:

Consequence 9 *$L_{str} + \Box_x\varphi \to \Box_y\varphi(x/y)$ is complete with respect to the class of structured dependence models satisfying the following property: for any permutation of variables in $\mathcal{L}$ τ,*

$$\langle v_{i_1}, d_{i_1}\rangle R\{\langle v_{i_2}, d_{i_2}\rangle, \ldots, \langle v_{i_n}, d_{i_n}\rangle\} \equiv$$

$$\equiv \langle \tau(v_{i_1}), d_{i_1}\rangle R\{\langle \tau(v_{i_2}), d_{i_2}\rangle, \ldots, \langle \tau(v_{i_n}), d_{i_n}\rangle\} \equiv R(d_{i_1}, \{d_{i_2}, \ldots, d_{i_n}\}).$$

Other examples of strengthening the logic to obtain 'real' generalized quantifiers are given in (Alechina and van Lambalgen 1995a).

The dependence relation in dependence models has a sort of built-in antisymmetry; in a derivation, relations between parameters can be represented as a directed acyclic graph (cf. (Alechina and van Lambalgen 1995b)). The situation is very different for assignments models.

3.2 Circular dependencies, or assignments

In this section we take a somewhat different approach to modal quantifiers, based on the intuition that the truth of a quantified formula depends on availability of a certain assignment (rather than a certain object). First, we consider logics for partial assignments models.

Definition 10 A *partial assignment* is a finite set of pairs $\langle x, d\rangle$, where x is a variable and d an element of the domain. □

There are two possibilities for defining partial assignments models. Andréka and Németi proposed (for assignments models) to distinguish models where only part of assignments are present (Crs) and models where all assignments are present, but only part of them is available for the quantifier (Crs^+). The first kind of models satisfies $\varphi \to \Diamond_x \varphi$, and the second kind does not. The latter +-variant is closer to the intuitions underlying the logic of structured dependence models; we shall see that one additional condition on R yields the logic corresponding to the +-variant of partial assignments models. To obtain the variant without +, we'll have to change the truth definition instead of just adding one more condition on R. Most of the proofs below go by way of first proving a certain property for the + variant and then extending the result to the proper (partial) assignments models. This is the reason why we consider both usual and + variants.

Definition 11 A *partial assignments model* is a structure of the form $M = \langle D, W, V\rangle$, where $\langle D, V\rangle$ is a first order model and W is an arbitrary non-empty set of partial assignments. The relation $M, s \models \varphi$, where $s : FV(\varphi) \longrightarrow D$ is a partial assignment, is defined as follows:

- The clause for atoms and negation is the same as for structured dependence models (with assignments restricted to the free variables of a formula);
- $M, s\lceil FV(\psi_1 \wedge \psi_2) \models \psi_1 \wedge \psi_2 \Leftrightarrow M, s\lceil FV(\psi_1) \models \psi_1 \ \wedge \ M, s\lceil FV(\psi_2) \models \psi_2$;
- $M, s\lceil(y_1, \ldots, y_n) \models \Diamond_x \psi(x, y_1, \ldots, y_n) \Leftrightarrow$
  $\Leftrightarrow \exists d \in D(s' = \{\langle x, d\rangle, \langle y_1, s(y_1)\rangle, \ldots, \langle y_n, s(y_n)\rangle\} \in W \ \wedge \ M, s' \models \psi(x, \bar{y}))$

We say that a formula φ is true in a partial assignments model M ($M \models \varphi$) if for all assignments s of the right length *in* W $M, s \models \varphi$. The set of formulas valid in this class of models is called $L_{p.a.}$. For a more familiar notion of validity, namely $M \models \varphi$ if *all* assignments of the right length satisfy φ, we use the notation $L^+_{p.a.}$, following the notation proposed by Andréka and Németi for Crs^+_n. □

This truth definition demands something stronger than that the pair $\langle x, d\rangle$ is accessible from the rest. The fact that

$$\{\langle x, d\rangle, \langle y_1, s(y_1)\rangle, \ldots, \langle y_n, s(y_n)\rangle\}$$

is an admissible partial assignment implies that also $\langle y_1, s(y_1)\rangle$ is accessible from $\{\langle x, d\rangle, \langle y_2, s(y_2)\rangle, \ldots, \langle y_n, s(y_n)\rangle\}$, and so on. The relation '$\langle x, d\rangle$ and the set of pairs D' form a good assignment' is not directed; in a sense, every pair in $\{\langle x, d\rangle\} \cup D'$ depends on the rest.

Lemma 12 *The class of structured dependence models satisfying the* circularity property*: if*

$$\langle x_{i_1}, d_{i_1}\rangle R\{\langle x_{i_2}, d_{i_2}\rangle, \ldots, \langle x_{i_n}, d_{i_n}\rangle\}$$

holds, then for every pair $\langle x_{i_j}, d_{i_j}\rangle$,

$$\langle x_{i_j}, d_{i_j}\rangle R\{\langle x_{i_1}, d_{i_1}\rangle, \ldots, \langle x_{i_n}, d_{i_n}\rangle\}\backslash\{\langle x_{i_j}, d_{i_j}\rangle\},$$

defines the same logic as $L^+_{p.a.}$.

Proof. Let φ be any formula, and assume there are a structured dependence model M and an ordinary assignment s, such that $M, s \models \varphi$. Define $M_{p.a.}$ to have the same domain and interpretation function as M and put

$$\{\langle x_1, d_1\rangle, \ldots, \langle x_n, d_n\rangle\} \in W \ \Leftrightarrow \ \langle x_1, d_1\rangle \ R \ \{\langle x_2, d_2\rangle, \ldots, \langle x_n, d_n\rangle\}$$

This is well defined given the circularity property. An easy induction shows that

$$M, s \models \varphi \Leftrightarrow M_{p.a.}, s\lceil FV(\varphi) \models \varphi.$$

Conversely, let M be a partial assignments model, s a partial assignment and $M, s \models \varphi$. We define M_{str} to have the same domain and interpretation, as M, and

$$\langle x, d\rangle \ R \ z \Leftrightarrow \{\langle x, d\rangle\} \cup z \in W,$$

where z is a partial assignment. Let s' agree with s on the free variables of φ, and assign some fixed element of D to the rest of the variables. Again, it is easy to show by induction that $M_{str}, s' \models \varphi$. □

Proposition 13 *L_{str} + $\Box_{v_i}(\varphi \to \Diamond_{v_j}\varphi)$, where $v_i, v_j \in FV(\varphi)$ (A3) is complete with respect to the class of structured dependence models satisfying circularity:*

$$\langle v_i, d_i\rangle R\{\ldots\langle v_j, d_j\rangle\ldots\} \Longrightarrow \langle v_j, d_j\rangle R\{\ldots\langle v_i, d_i\rangle\ldots\}$$

Proof. To show that the axiom is valid in the class of models satisfying circularity, consider an arbitrary model M from this class, an assignment s and a formula φ with $FV(\varphi) = v_i, v_j, \bar{v}$. Note that if $v_i = v_j$, the axiom is valid already in L_{str} (it is equivalent to T). Consider a sequence $s' =_i s$ with $\langle v_i, s'(v_i)\rangle \ R \ s\lceil\bar{v} \cup \{\langle v_j, s(v_j)\rangle\}$, such that $M, s' \models \varphi$. Then by circularity $\langle v_j, s(v_j)\rangle \ R \ s\lceil\bar{v} \cup \{\langle v_i, s'(v_i)\rangle\}$, that is, $M, s' \models \Diamond_{v_j}\varphi$. But then $M, s \models \Box_{v_i}(\varphi \to \Diamond_{v_j}\varphi)$.

To prove completeness, we use the fact that any set of sentences consistent with L_{str} + A3 has a canonical model M_Σ as constructed above. Assume that in M_Σ

$$\langle v_i, d_i\rangle R\{\langle v_1, d_1\rangle, \ldots, \langle v_n, d_n\rangle\}\backslash\{\langle v_i, d_i\rangle\}$$

holds, that is, for every formula χ with constants $d_1, \ldots, d_n$, $\chi \to \Diamond_{v_i}\chi \in \Sigma$ (or, equivalently, $\Box_{v_i}\chi \to \chi \in \Sigma$). It suffices to show that

$$\langle v_j, d_j\rangle R\{\langle v_1, d_1\rangle, \ldots, \langle v_n, d_n\rangle\}\backslash\{\langle v_j, d_j\rangle\},$$

that is, that for every formula χ with constants $d_1, \ldots, d_n$, $\chi \to \Diamond_{v_j}\chi \in \Sigma$. Take an arbitrary formula χ with constants $d_1, \ldots, d_n$. Consider the formula $\chi \to \Diamond_{v_j}\chi$. This formula (let us call it θ) contains constants $d_1, \ldots, d_n$. By assumption, $\Box_{v_i}\theta \to \theta \in \Sigma$. But $\Box_{v_i}\theta$ is an axiom, so $\theta \in \Sigma$. □

Due to Lemma 12, this proves the completeness of L_{str} + A3 wrt $L^+_{p.a.}$.

Proposition 14 *Adding to L_{str} the axiom $\varphi \to \Diamond_{v_j}\varphi$ (A4) yields a logic complete wrt $L_{p.a.}$.*

Proof. Note that A4 implies A3, so we can use the previous proof to show that there is a partial assignments model M_Σ and an assignment s_0 satisfying a formula φ consistent with A0 – A4, R1, R2. But we do not know yet whether this assignment is in W.

If A4 is true in the canonical model M_Σ, then every assignment

$$\{\langle v_{i_1}, d_{i_1}\rangle, \ldots, \langle v_{i_n}, d_{i_n}\rangle\}$$

is a good assignment given that all d_{i_j} are of type i_j. Note that the assignment s_0 satisfying the original formula φ (which contains only the variables and no constants) is of such type, therefore s_0 is in W. This gives us $M_\Sigma, s_0 \in W \models \Delta$. □

4 Crs_n-models

Now we move to classes of models where the notion of assignment stands really central; assignments have a fixed length and do not shrink and extend depending on a formula.

To define assignments models, in particular models with assignments of length α, we need first to define formally a language with at most α variables.

Definition 15 $\mathcal{L}_\alpha(\Diamond)$ is a first order language containing α many variables v_i, $0 \leq i < \alpha$, $=$, $\neg, \wedge$ and $\Diamond$ and countably many predicate symbols. Well formed formulas (w.f.f.'s) are defined in a standard way. □

In the meta-language we will use $x_1, \ldots, x_n, \ldots$ to denote arbitrary variables of $\mathcal{L}_\alpha(\Diamond)$.

Definition 16 A *Crs_α-model* for $\mathcal{L}_\alpha(\Diamond)$ is a structure M of the following form: $M = \langle D, W, V\rangle$, where D is a non-empty domain, $W \subseteq D^\alpha$ is non-empty, and V is an interpretation function. Since there are α many variables, an *assignment* of elements of D to the variables is any element of D^α. The relation 'a formula φ is true in M under assignment s $(M, s \models \varphi)$' is defined *only for assignments in W*. The clauses for atomic formulas, $\neg$ and $\wedge$ are as before, and

$$M, s \models \Diamond_x \psi \Leftrightarrow \exists s' =_x s (s' \in W \wedge M, s' \models \psi).$$ □

We write $M \models \varphi$ (φ true in M) for $\forall s \in W (M, s \models \varphi)$.

Definition 17 A *Crs^+_α-model* for $\mathcal{L}_\alpha(\Diamond)$ is a structure M of the following form: $M = \langle D, W, V\rangle$, where D is a non-empty domain, $W \subseteq D^\alpha$ is non-empty, and V is an interpretation function. The relation 'a formula φ is true in M under assignment s $(M, s \models \varphi)$' is defined inductively as above.

Now $M \models \varphi$ (φ true in M) stands for $\forall s \in D^\alpha (M, s \models \varphi)$. □

A formula is $Crs_\alpha(Crs^+_\alpha)$-valid, if it is true in all models. A formula is $Crs_\alpha(Crs^+_\alpha)$-satisfiable, if its negation is not $Crs_\alpha(Crs^+_\alpha)$-valid.

It is clear that for $\alpha = \omega$ assignments models cannot be reduced to a special case of structured dependence models, since R must become an infinite relation. For $\alpha < \omega$, there is a resemblance between partial assignments models and assignments models. But there is an important property of structured dependence models which holds also for dependence models and partial assignments models (being special cases of structured dependence models), but not for assignments models, namely locality.

To get a feeling why locality fails in assignments models, consider the following example for $\mathcal{L}_2(\Diamond)$. Let M be a Crs_2 (Crs^+_2)-model with domain $D = \{0, 1\}$, $V(P) = \{\langle 1\rangle\}$ and W consisting of the following assignments:

	v_0	v_1
$s =$	0	0
$s' =$	1	0
$s'' =$	0	1

Then $M, s \models \Diamond_{v_1} P(v_1)$ and $M, s' \not\models \Diamond_{v_1} P(v_1)$, although these assignments agree on the free variables of $\Diamond_{v_1} P(v_1)$.

The failure of locality is the reason why the axiom $\varphi \to \Box_x \varphi$, given that x is not free in φ, is not valid in assignments models. Namely, consider the model above; $M, s \models \Diamond_{v_1} P(v_1)$ but $M, s \not\models \Box_{v_0} \Diamond_{v_1} P(v_1)$.

Since we cannot give an axiomatization of assignments models based on L_{str}, we define an *embedding* of Crs^+_n-logic into $L^+_{p.a.}$ and of Crs_n-logic into $L_{p.a.}$. Both

embeddings use the following simple idea. Consider the formulas of the n variable language $\mathcal{L}_n$. The difference in the truth conditions for these formulas in assignments models and in partial assignments models is that in the former a quantifier takes into account the assignment of values to all n variables, whether they occur free under the quantifier or not, whereas in the latter only the free variables are important. For the formulas which contain all n variables free under a quantifier, the truth conditions are the same. We take a formula φ of $\mathcal{L}_n$ and add dummy variables under every quantifier in φ. The result is called φ^n. We show that φ^n is equivalent to φ in assignments models, and that φ^n is satisfiable in assignments models if, and only if, it is satisfiable in partial assignments models. This gives the desired embedding.

Observe that formulas which have all n variables under each quantifier satisfy locality in Crs_n:

Lemma 18 *Let φ be a formula of $\mathcal{L}_n(\Diamond)$ which contains all n variables in the scope of each quantifier. Then for every Crs (Crs^+)-model M and a pair of assignments s, z which agree on the free variables of φ, $M, s \models \varphi \Leftrightarrow M, z \models \varphi$.*

Proof. The proof goes by induction on the complexity of φ; for atomic formulas the claim is obviously true, the clauses for $\neg$ and $\wedge$ use the induction hypothesis in a standard way. Assume $M, s \models \Diamond_{v_i} \psi$. Then there is an assignment $s' =_i s$ in W such that $M, s' \models \psi$. Since all variables except v_i are free in $\Diamond_{v_i} \psi$, and z agrees with s on the free variables of $\Diamond_{v_i} \psi$, $z =_i s'$, which gives $M, z \models \Diamond_{v_i} \psi$. □

This property will be also used in the proof of embedding results below.

4.0.1 Embedding of Crs^+_n into $L^+_{p.a.}$

Define a translation function ${}^n : \mathcal{L}_n(\Diamond) \longrightarrow \mathcal{L}_n(\Diamond)$ ($\mathcal{L}_n(\Diamond)$ is the language using only the variables $v_0, \ldots, v_{n-1}$) as follows: ^{n} commutes with atomic formulas and propositional connectives, and

$$(\Diamond_{v_i} \psi)^n = \Diamond_{v_i} (\top(v_0, \ldots, v_{n-1}) \wedge \psi).$$

Proposition 19 $Crs^+_n \models \varphi \Leftrightarrow L^+_{p.a.} \models \varphi^n$.

Proof. We will prove $Crs^+_n \not\models \varphi \Leftrightarrow L^+_{p.a.} \not\models \varphi$. Assume that $Crs^+_n \not\models \varphi$. Then there is a Crs^+_n model for $\neg\varphi$. Since in Crs^+_n $\neg\varphi$ and $(\neg\varphi)^n$ are equivalent, the same model satisfies $(\neg\varphi)^n$. To prove the proposition, it is enough to show that for every $\theta \in \mathcal{L}_n(\Diamond)$, θ^n is Crs^+_n-satisfiable iff θ^n is $L^+_{p.a.}$-satisfiable.

Assume that θ^n is Crs^+_n-satisfiable, i.e. there is a Crs^+_n-model M and an assignment (of length n) s_0, such that $M, s_0 \models \theta^n$. Define a partial assignments model $M_{p.a.+}$ as follows: $V_{p.a.+} = V$, $D_{p.a.+} = D$, and $W_{p.a.+} = W$.

We prove by induction that for every subformula ψ of θ^n

$$M, s \models \psi \Leftrightarrow M_{p.a.+}, s\lceil FV(\psi) \models \psi.$$

The clauses for atoms and $\neg$ are obvious. The clause for $\wedge$ is also easy:

$$M, s \models \psi_1 \wedge \psi_2 \Leftrightarrow M, s \models \psi_1 \ \wedge \ M, s \models \psi_2 \Leftrightarrow$$

$$\Leftrightarrow M_{p.a.+}, s\lceil FV(\psi_1) \models \psi_1 \ \wedge \ M_{p.a.+}, s\lceil FV(\psi_2) \models \psi_2 \Leftrightarrow$$

$$\Leftrightarrow M_{p.a.+}, s\lceil FV(\psi_1 \wedge \psi_2) \models \psi_1 \wedge \psi_2$$

Finally, $M, s \models \Diamond_{v_i} \psi \Leftrightarrow \exists s' =_i s (s' \in W \ \wedge \ M, s' \models \psi)$; since ψ is a subformula of θ in the scope of a quantifier, $FV(\psi) = \{v_0, \ldots, v_{n-1}\}$, and $s'\lceil FV(\psi) = s'$. Therefore the condition above is equivalent to

$$\exists d (s' = s\lceil FV(\Diamond_{v_i} \psi) \cup \{\langle v_i, d\rangle\} \in W_{p.a.+} \ \wedge M_{p.a.+}, s' \models \psi),$$

i.e. $M_{p.a.+}, s\lceil FV(\Diamond_{v_i}\psi) \models \Diamond_{v_i}\psi$.

This shows $M_{p.a.+}, s_0\lceil FV(\theta^n) \models \theta^n$, i.e. if θ^n is Crs_n^+-satisfiable, then it has a $L_{p.a.}^+$-model.

To prove the opposite direction, suppose that θ^n is satisfiable in a partial assignments model M and an assignment s_0. We construct a Crs_n^+-model M' and an assignment $a(s_0)$ with $M', a(s_0) \models \theta^n$. Let M' have the same domain and interpretation function as M, and $W' = W \cap \{s : dom(s) = \{v_0, \ldots, v_{n-1}\}\}$. Of course, all assignments in M' are of length n.

Let a be an arbitrary function which gives a partial assignment s from M an assignment $a(s)$ in M', so that $a(s)\lceil dom(s) = s$.

Now we prove $M, s \models \psi \Leftrightarrow M', a(s) \models \psi$. The clause for atoms and negation is trivial. For the conjunction we have:

$$M, s \models \psi_1 \wedge \psi_2 \Leftrightarrow M, s\lceil FV(\psi_1) \models \psi_1 \ \wedge \ M, s\lceil FV(\psi_2) \models \psi_2 \Leftrightarrow$$

$$\Leftrightarrow M', a(s\lceil FV(\psi_1)) \models \psi_1 \ \wedge \ M', a(s\lceil FV(\psi_2)) \models \psi_2$$

Since ψ_1 and ψ_2 are subformulas of θ^n, and $a(s)\lceil FV(\psi_i) = a(s\lceil FV(\psi_i))\lceil FV(\psi_i)$, by lemma 18 this is equivalent to

$$M', a(s) \models \psi_1 \ \wedge \ M', a(s) \models \psi_2 \Leftrightarrow M', a(s) \models \psi_1 \wedge \psi_2.$$

The clause for the quantifier is easy. Since the assignments used in the quantifier clause are of length n, we use the fact that such an assignment $s' \in W'$ iff $s' \in W$:

$$M, s \models \Diamond_{v_i}\psi \Leftrightarrow \exists d(s' = s \cup \{\langle v_i, d\rangle\} \in W \ \wedge \ M, s' \models \psi) \Leftrightarrow$$

$$\Leftrightarrow \exists s' =_i a(s) \in W'(M', s' \models \psi) \Leftrightarrow M', a(s) \models \Diamond_{v_i}\psi.$$

This gives $M', a(s_0) \models \theta^n$, that is, if θ^n has a $L_{p.a.}^+$-model, then θ^n has a Crs_n^+-model. □

4.0.2 Embedding of Crs_n into $L_{p.a.}$

Given the result above, it is easy to show that the same embedding works for Crs_n and $L_{p.a.}$.

Proposition 20 $Crs_n \models \varphi \Leftrightarrow L_{p.a.} \models \varphi$.

Proof. The beginning of the argument is the same as above; the problem is reduced to proving that θ^n is Crs_n-satisfiable iff θ^n is $L_{p.a.}$-satisfiable.

Given the definition of a partial assignments model, it is clear that $L_{p.a}$- and $L_{p.a.+}$-models are the same, and only the notions of satisfiability are different. (We cannot reformulate the definition of partial assignments models so that the satisfaction relation would be defined only for the sequences in W; cf. the clause for conjunction.) Therefore, if φ is satisfiable in an $L_{p.a.}^+$-model and the satisfying sequence is in W, then φ is $L_{p.a.}$-satisfiable.

The corresponding fact for the assignments models is also true, but needs a bit more elaborating. Let M be a Crs_n-model, and M^+ be the corresponding Crs_n^+-model (with the same D, W and V). Then for $s \in W$

$$M, s \models \varphi \Leftrightarrow M^+, s \models \varphi.$$

The proof is an obvious induction.

Consider a formula θ^n from the previous embedding proof. Without loss of generality we may assume that θ^n has all n variables free (in all logics, θ^n is satisfiable iff $\theta^n \wedge \top(v_0, \ldots, v_{n-1})$ is satisfiable).

Assume that θ^n is $L_{p.a.}$ satisfiable, i.e. there is an $L_{p.a.}$ model M and an assignment (of length n) $s \in W$ such that $M, s \models \theta^n$. From the analysis above, M is an $L_{p.a.}^+$ model as well, and the embedding proof above gives a Crs_n^+ model M' and assignment $a(s) = s$ which satisfy θ^n. Since s is in W and s is of length n, we have $M', s \in W' \models \theta^n$. But then there is a Crs_n-model satisfying θ^n.

Assume that there is a Crs_n-model M satisfying θ^n. Then there is a Crs_n^+-model M^+ such that $M^+, s \in W \models \theta^n$. By the embedding argument, there is an $L_{p.a.}^+$-model $M_{p.a.+}$ with $W_{p.a.+} = W$, such that $M_{p.a.+}, s\lceil FV(\theta^n) \models \theta^n$. But $s\lceil FV(\theta^n) = s$ and $s \in W$. Therefore, θ^n is $L_{p.a.}$-satisfiable. □

From the following fact (which is well-known, cf. for example (Németi 1992), but which we restate here for the sake of completeness) follows that also Crs_ω is embeddable in $L_{p.a.}$:

Proposition 21 *For every w.f.f. with at most n first variables* φ, $\models_{Crs_n} \varphi$ *iff* $\models_{Crs_\omega} \varphi$.

Proof. We prove that $\neg\varphi$ is Crs_n-satisfiable iff $\neg\varphi$ is Crs_ω-satisfiable.

The direction from left to right is easy: one can always transform a Crs_n model M, satisfying $\neg\varphi$, into Crs_ω model, by adding the same infinite 'tail' to all sequences in M.

For the converse, let M be a Crs_ω model, and $M, s \models \neg\varphi$. Just 'cutting the tails' of the sequences in M would not work, since it may make some sequence s' for which $s' =_x s$ does not hold in M, accessible from s in the new model. Let, for example, $n = 3$ (which means that φ contains only the variables v_0, v_1 and v_2). Assume that s' differs from s in the 0th and 5th coordinates. Then $s' =_0 s$ does not hold. But if we consider only the first 3 coordinates of s and s', then $s' =_0 s$.

To make a Crs_n model from M, which satisfies φ, we proced as follows. Let M^- be a submodel of M obtained by leaving out all sequences which differ from s in some coordinate m, $m \geq n$. An easy induction shows that

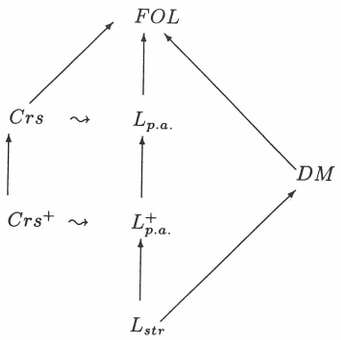
$$M, s \models \varphi \Leftrightarrow M^-, s \models \varphi.$$

(The inductive hypothesis being: for every s' which differs from s in at most n first coordinates, and every formula ψ containing at most n first variables, $M, s' \models \psi \Leftrightarrow M^-, s' \models \psi$).

M^- can be easily transformed into a Crs_n model by 'cutting the tails' of sequences in M^-, making them all of length n. □

5 The weakest logic above Crs and dependence models

So far, we have considered the following logics (with inclusions indicated by arrows, DM standing for the minimal logic of dependence models):



There are lots of non-trivial logics between DM and FOL (cf. Chapter 4). But it is interesting that there is no logic which is stronger than both Crs and DM and weaker than first order logic. In this section we are going to show that $Crs{+}DM = FOL$. This suggests that the intuitions underlying assignments models and dependence models are in a sense orthogonal.

Although we do not give an axiomatization of Crs_n-models, it is clear that at least the following axioms are valid there:

A1' $\Box_x(\varphi \to \psi) \to (\Box_x\varphi \to \Box_x\psi)$ (unrestricted monotonicity);

A4 $\varphi \to \Diamond_x\varphi$.

The following axioms are valid in the class of dependence models:

A0 propositional logic;

A1 $\Box_x(\varphi \to \psi) \to (\Box_x\varphi \to \Box_x\psi)$, given that $\Box_x\varphi$ and $\Box_x\psi$ contain the same parameters (free variables and constants);

A2 $\varphi \to \Box_x\varphi$, given that x is not free in φ;

A5 $\Box_x\varphi \to \Box_y\varphi(x/y)$, given that y is free for x in $\varphi(x)$;

R1 $\varphi,\ \varphi \to \psi/\psi$;

R2 $\varphi(x)/\Box_x\varphi(x)$.

Putting these axioms together and replacing $\Box$ by $\forall$ and $\Diamond$ by $\exists$ we obtain the following system:

Ax0 propositional logic;

Ax1 $\forall x(\varphi \to \psi) \to (\varphi \to \forall x\psi)$, given that x is not free in φ (from unrestricted monotonicity A1' and A2);

Ax2 $\varphi(x) \to \exists y\varphi(x/y)$, with the usual restriction (from A4 and A5);

MP $\varphi,\ \varphi \to \psi/\psi$;

Gen $\varphi(x)/\forall x\varphi(x)$ (from R1).

which axiomatizes first-order logic (cf., for example, (Mendelson 1979)). This proves the following proposition:

Proposition 22 *The weakest logic which derives all formulas valid in both dependence models and Crs_n-models, is classical first order logic.*

Both the logic of dependence models and Crs_n-models are much weaker than classical first order logic. In particular, they are decidable even in the presence of equality in the language (as we shall see below, this follows from the results of Németi (1992), Andréka and Németi (1994)).

6 Restricted fragments of predicate logic

A lot of insight in modal logic comes from considering it as a fragment of first order logic under the *standard translation* (cf. van Benthem (1983)). The same holds for the logics of modal quantifiers.

All logics introduced above can be embedded into fragments of first-order logic with restricted, or bounded, quantification (where every quantifier is restricted by an atomic formula).

In (Andréka, van Benthem and Németi 1995) the following fragment (called Fragment 2) is defined: a fragment of first-order language with equality where each subformula in the scope of $\exists$ is of the form $\exists\bar{x}(R(\bar{x},\bar{y}) \wedge \varphi)$, where R is any relational symbol (not necessarily fixed) and the free variables of φ are among $\bar{x},\bar{y}$. This fragment is shown to be decidable in (Andréka and Németi 1994). In this section, we show that the logics of partial assignments models and the logics of assignments models are embeddable into Fragment 2.

For Crs_n and Crs_n^+ this is a well-known fact (cf., for example, (Németi 1992)). The following translation tr from $\mathcal{L}_n(\Diamond)$ into the language of first-order logic enriched with a predicate R is used to define the embedding: tr commutes with atomic formulas and propositional symbols, and

$$tr(\Diamond_{v_i}\varphi) = \exists v_i(R(v_0,\ldots,v_{n-1}) \wedge tr(\varphi)).$$

Put for Crs_n^+ $\varphi^+ = tr(\varphi)$ and for Crs_n $\varphi^* = R(v_0,\ldots,v_{n-1}) \wedge tr(\varphi)$. Intuitively, $R(d_0,\ldots,d_{n-1})$ means that $\{\langle v_0,d_0\rangle,\ldots,\langle v_{n-1},d_{n-1}\rangle\}$ is an assignment in W. One can easily check that φ is Crs_n (Crs_n^+)-satisfiable iff φ^* (φ^+) is satisfiable in first-order logic.

For the partial assignments models, the idea is completely the same, but we have to introduce countably many relational symbols. The reason is that we must be able to distinguish assignments of the same length but to different variables (coordinates). So we introduce symbols like $R_{i_1,\ldots,i_n}$, $i_1 \le \ldots \le i_n$, with $R_{i_1,\ldots,i_n}(d_1,\ldots,d_n)$ meaning that d_j is assigned to the variable v_{i_j}. The translation is, as one would expect, the same as tr except for the quantifiers:

$$tr'(\Diamond_{v_{i_j}}\varphi(v_{i_1},\ldots,v_{i_n})) = \exists v_{i_j}(R_{i_1,\ldots,i_n}(v_{i_1},\ldots,v_{i_n}) \wedge tr'(\varphi))$$

For example, $\Diamond_{v_0}\Diamond_{v_2}P(v_0,v_2)$ is translated a

$$\exists v_0(R_0(v_0) \wedge \exists v_2(R_{0,2}(v_0,v_2) \wedge P(v_0,v_2)))$$

which is equivalent in first order logic to

$$\exists x\exists y(R_0(x) \wedge R_{0,2}(x,y) \wedge P(x,y));$$

$\Diamond_{v_3}\Diamond_{v_1}P(v_3,v_1)$ is translated (up to renaming of the variables) to

$$\exists x\exists y(R_3(x) \wedge R_{1,3}(y,x) \wedge P(x,y)),$$

i.e. to a different formula.

As before, we define $\varphi^+ = tr'(\varphi)$ and $\varphi^* = R_{i_1,\ldots,i_n}(v_{i_1},\ldots,v_{i_n}) \wedge tr'(\varphi))$, where $v_{i_1},\ldots,v_{i_n}$ are the free variables of φ. The claim is that φ is $L_{p.a.}(L_{p.a.}{+})$-satisfiable iff $\varphi^*(\varphi^+)$ is first-order satisfiable. From this follows

Proposition 23 *Both logics of partial assignments models are decidable.*

Proof. Follows from the embedding above and the result of Andréka and Németi (1994). □

The same idea applies to L_{str}. Since the difference between L_{str} and $L^{+}_{p.a.}$ is in the circularity property, we must show which variable was quantified. We add countably many predicates $R^{i_j}_{i_1,\ldots,i_n}$, where the superscript stands for the quantified variable. The translation becomes

$$tr''(\Diamond_{v_{i_j}}\varphi(v_{i_1},\ldots,v_{i_n})) = \exists v_{i_j}(R^{i_j}_{i_1,\ldots,i_n}(v_{i_1},\ldots,v_{i_n}) \wedge tr''(\varphi))$$

Again, $\varphi^{+} = tr''(\varphi)$ and φ is satisfiable in the structured dependence models iff φ^{+} is first order satisfiable. We prove that the translation is satisfiability-preserving only for this case; the proofs for partial assignments models are analogous.

Claim 24 *For any formula φ of $\mathcal{L}(\Diamond)$, φ is satisfiable in a structured dependence model iff $tr''(\varphi)$ is first-order satisfiable.*

Proof. Assume that there is a structured dependence model $M = \langle D, R, V\rangle$ and assignment s which satisfy φ. Let $M' = \langle D, V'\rangle$, where V' agrees with V on all predicates from $\mathcal{L}(\Diamond)$, and

$$\langle d_1,\ldots,d_n\rangle \in V'(R^{i_j}_{i_1,\ldots,i_n}) \Leftrightarrow \langle v_{i_j}, d_j\rangle\ R\ \{\langle v_{i_1}, d_1\rangle,\ldots,\langle v_{i_n}, d_n\rangle\} \setminus \{\langle v_{i_j}, d_j\rangle\}.$$

For example,

$$R^{4}_{2,4,5}(d_1,d_2,d_3) \Leftrightarrow \langle v_4, d_2\rangle\ R\ \{\langle v_2, d_1\rangle, \langle v_5, d_3\rangle\}.$$

It is easy to prove that for every formula ψ and assignment z,

$$M, z \models \psi \leftrightarrow M', z \models tr''(\psi).$$

The proof goes by induction on the complexity of ψ. The only non-trivial case is of ψ being of the form $\Diamond_{v_{i_j}}\theta(v_{i_1},\ldots,v_{i_n})$. $M, z \models \Diamond_{v_{i_j}}\theta(v_{i_1},\ldots,v_{i_n})$ means that

$$\exists d(\langle v_{i_j}, d\rangle\ R\ \{\langle v_{i_1}, z(v_{i_1})\rangle,\ldots,\langle v_{i_n}, z(v_{i_n})\rangle\} \setminus \{\langle v_{i_j}, z(v_{i_j})\rangle\} \wedge M, z^{i_j}_{d} \models \theta) \Leftrightarrow$$

$$\Leftrightarrow \exists d(R^{i_j}_{i_1,\ldots,i_n}(z(v_{i_1}),\ldots,d,\ldots,z(v_{i_n})) \wedge M', z^{i_j}_{d} \models tr''(\theta)) \Leftrightarrow$$

$$\Leftrightarrow M', z \models \exists v_{i_j}(R^{i_j}_{i_1,\ldots,i_n}(v_{i_1},\ldots,v_{i_n}) \wedge tr''(\theta)).$$

This proves the direction from left to right.

To prove the opposite direction, assume that $tr''(\varphi)$ is first-order satisfiable. We construct a model for φ by keeping the domain and interpretation function the same as in the model for $tr''(\varphi)$ and defining R as follows:

$$\langle v_{i_j}, d_j\rangle\ R\ \{\langle v_{i_1}, d_1\rangle \ldots \langle v_{i_n}, d_n\rangle\} \setminus \{\langle v_{i_j}, d_j\rangle\} \Leftrightarrow \langle d_1,\ldots,d_j,\ldots,d_n\rangle \in V(R^{i_j}_{i_1,\ldots,i_n}).$$

The rest of the argument is the same as above. □

Proposition 25 *The logic of structured dependence models is decidable.*

Proof. Follows from the embedding above and the result of Andréka and Németi (1994). □

The standard translation for dependence models is given in (van Lambalgen 1991) and (van Benthem and Alechina 1996). It also falls within Fragment 2 and is decidable.

7 Conclusion

The logics considered above can be seen as special cases of the same principal intuition: that the range of a variable bound by a quantifier can be dependent on the values of other variables. Different logics emerge when different conditions on the dependence relation are imposed. All such logics can be translated into first order logic. But it definitely makes sense to study them on their own or as *special fragments* of first order logic, because they may have properties which full first order logic does not posess. We have seen this on the example of decidability.

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Mathematical Treatments of Discourse Contexts*

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1. Introduction

This paper is concerned with some general questions about discourse interpretation: What are the key theoretical structures on which discourse interpretation should depend? How should a discourse context be thought of--as a structured representation, or model-theoretically? How should we model the updating of discourse contexts with new information?

An answer to the first question above, What are the key theoretical structures on which discourse interpretation should depend?, motivates the theory of discourse interpretation. If we take our cue from sentential semantics, the meaning of the discourse's parts should determine the meaning of the whole; some sort principle of compositionality of meaning must hold at the level of discourse interpretation. So a theory of discourse interpretation must develop from an account of discourse structure.

A theory discourse structure and, hence, discourse interpretation must integrate both semantic and pragmatic information. Recent work on discourse structure in AI, philosophy and linguistics has shown that discourse structure depends on numerous information sources--compositional semantic principles, lexical semantics, pragmatic principles, and information about the speaker's and interpreter's mental states. So a theory of discourse interpretation must in fact also be a theory of semantics and pragmatics and their interaction--a theory of the pragmatics-semantics interface.

This leads to several other questions: How should we model the interaction of these multiple knowledge sources needed to construct discourse structure, or the interaction between defeasible pragmatic principles and nondefeasible semantic principles?

I will review and lay out systematically one theory, Segmented Discourse Representation Theory or SDRT, which is one attempt at a theory of discourse interpretation and of the pragmatic-semantic interface.[1] Like DRT, SDRT exploits representations to model discourse contexts and to determine the semantic effects of discourse structure. After giving some applications of SDRT to problems of discourse interpretation, I develop a novel and alternative model-theoretic approach to discourse contexts for a theory of pragmatics and semantics, and then establish an equivalence with SDRT. The model-theoretic approach gives a more general description of the theory by abstracting away from irrelevant aspects of the representations. It also indicates the minimal amount of information about our representational structures needed to analyze the

* I would like to thank Tim Fernando and Alex Lascarides for extensive and helpful comments on previous drafts of this paper.

[1] This theory has already been introduced at some length in Asher(1993); I will give here a simpler and improved account.

phenomena SDRT has been concerned with. There is a lot of such information needed, and SDRT represents a minimal commitment to representational structures.

2. A Theory of Discourse Interpretation--SDRT

Previous work has established that a theory of discourse interpretation must give an account of discourse structure. But how do we characterize the structure of a discourse and how do we build it? SDRT furnishes some answers.

2.1 Introduction to SDRT

SDRT integrates and extends dynamic semantics as found in Kamp's Discourse Representation Theory (DRT). The output of a discourse is not a single DRS, as in DRT, but rather a complex propositional structure in which DRSs are related by discourse relations to form *segmented discourse representation structures* or SDRSs. SDRSs in turn may be related to other SDRSs by means of discourse relations to form a hierarchical structure. Now since SDRSs are abstract objects, the same SDRS could occur in more than one structural position. So I shall *label* SDRSs with positions with a set of labels $\{\pi_1, \pi_2, ...\}$. There is a surjective function from labels to SDRSs. I will assume that labels with distinct indices are distinct, though their "values" need not be; henceforth I shall write the value of π_n as α_n (if a variable) or k_n (if it is a constant and denotes a particular SDRS). Formally, an SDRS is a pair of sets, a set U of *constituents*, of labelled DRSs or SDRSs, and a set of *conditions* in which discourse relations apply to labels in U.

DEFINITION (SDRS and SDRS Conditions): If $\{\pi_1:\alpha_1,..., \pi_n:\alpha_n\} = U$ is a set of labelled SDRSs or DRSs, Con is a finite set of SDRS conditions all of whose arguments are among $\pi_1, ..., \pi_n$ and such that every π_i occurs in at least one element of Con, then <U, Con> is an *SDRS*.
If $\pi:\alpha$ and $\pi':\alpha'$ are labelled SDRSs and R is a Discourse Relation symbol, then $\underline{R(\pi, \pi')}$ is an *SDRS condition*.

SDRSs form a graph, in which the nodes or vertices are labelled SDRSs, while the edges are discourse relations of different types. $R(\pi_1, \pi_2)$ without underlines signifies that the condition in fact holds.[2]

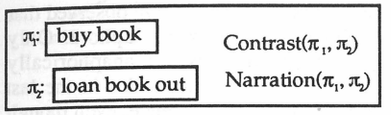
The SDRS definition is motivated by three concerns: first, an SDRS should furnish a structural constraint on the anaphoric potential of the context in a similar manner to the constraint of accessibility in DRT (for detailed arguments why see Asher 1993); second, an SDRS should represent the meaningful units or segments of a discourse, a goal which SDRT shares with many discourse theories; third, an SDRS should exhibit the semantic effects of discourse relations on the SDRSs they connect such as: temporal anaphora, focus/background articulation, and lexical disambiguation. SDRSs are designed to accomplish these aims, and they have several advantages over other proposals.

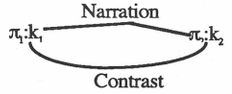
Unlike the structurally simpler discourse trees of, e.g., Polanyi and Scha or the structures of Mann and Thompson, two or more edges may relate vertices in an SDRS. Sometimes an appeal to at least two discourse relations is essential in accounting for the observed semantic effects. For example in treating the lexical ambiguity of *ausleihen* in the following pair of German discourses (1.a) and (1.b), we can account for the lexical disambiguation of *ausleihen* by appealing to the semantic effects of the discourse relations, if we assume that both the discourse relations Narration and Contrast relate the two constituents derived from the two

[2]For the semantics of various discourse relations see Asher (1993), chapter 7.

from the two clauses in the responses to (1.a-b). Narration relates the events in the clauses temporally, while Contrast says that the two clauses have incompatible implicatures. But the analysis of the disambiguation does not work if we are forced to choose only Narration or only Contrast to relate the constituents, and having a single relation that carries the semantic effects of both Narration *and* Contrast is not independently motivated (Knott, 1996). The SDRS for (1.b) is depicted below to the left, and the *constituent graph* at the right.

(1) Hast du das Buch gekauft, oder? (Did you buy the book?)
a. Nein, ich habe es nicht gekauft, aber ich habe es ausgeliehen
(No, I didn't buy it, but I borrowed it)
b. Ya, ich habe es gekauft aber ich habe es ausgeliehen.
(Yeah, I bought it, but I loaned it out)





I will return to this example once I have said more about discourse relations.

A second aspect of the complexity of SDRSs is that an SDRS--i.e. a graph--may make up a vertex of another graph. This is a direct consequence of the recursive definition of SDRSs. The justification for this additional complexity is the intuition that in discourses like (2) below, the sentences following the first introductory and topic sentence all contribute to the overall discourse structure *one* elaboration, one complex SDRS containing several SDRS constituents, *rather than separate* elaborations of the topic. The recursive definition of SDRSs allows us to distinguish between these two cases.

Like DRSs in DRT, SDRSs are constructed incrementally: Given a discourse consisting of the sentences $s_1, ..., s_n$ one first constructs an SDRS k_1 for s_1 and then uses k_1 as a context for the interpretation of s_2. One then updates k_1 with the interpretation of s_2, and then uses the resulting SDRS as a context to interpret s_3. The construction of a SDRS is more involved than DRS-construction. First, the SDRS construction procedure must specify a semantic representation for the sentence s_i. Second, the context SDRS for s_i in general offers a number of different attachment sites to which the representation of s_i could be attached; the SDRS construction procedure must say which sites these are and select one. Third, the construction must specify one or more discourse relations that relate the semantic representation of the new sentence to components of the context SRDS that are available at the chosen attachment site. Except when the processed sentence is the initial sentence of the discourse, its representation must be related to the context by at least one discourse relation. This is a *relevance* requirement for cohesive discourse. Finally, one must determine the the truth conditional effects of the attachment of the new information to an available attachment site by means of some particular discourse relation. The truth conditional content of a discourse comes in part from compositional and lexical semantics and in part from the effects of the discourse relations. Below, I will give examples with pronominal and temporal anaphora.[3]

[3]For other applications of SDRT, see Asher (1995b) for an analysis of focus/background, Asher et al. (1995) concerning spatio-temporal trajectories, and concerning lexical disambiguation --Asher & Lascarides (1995), Asher & Kamp (ms).

In SDRT, lexical and compositional, dynamic semantics (DRT) determine the semantic representation for each clause and, here, for each sentence.[4] To calculate the update of an SDRS with new information, SDRT uses three types of axioms: the axioms of attachment, axioms for inferring discourse relations, and axioms for the semantic effects of the discourse relations. Let us look at these more closely now.

2.2 SDRT Axioms of Attachment

The axioms for attachment constrain possible attachment sites. SDRT distinguishes a particular set of *attachment points* to which new information, forming a new constituent in the SDRS, may be attached by means of some discourse relation. Attachment sites are the labels of SDRSs. Only certain labels within an SDRS are attachment points. The motivation for this restriction comes from data on pronominal anaphora. Polanyi (1988) and others have observed that discourse structure establishes a constraint on pronominal anaphora. Only discourse entities introduced within certain constituents are anaphorically available; for instance in (2) (from Asher 1993), the pronoun *this* in the last sentence cannot refer to the Shuttle's launch in the second sentence, even though in terms of content this makes a certain amount of sense. The only antecedents for *this* are the entire Shuttle voyage or the launch of the TDRS satellite.

(2) After thirty eight months, America is back in space. The shuttle Discovery roared off the pad from Cape Kennedy at 10:38 this morning. The craft and crew performed flawlessly. Later in the day the TDRS shuttle communications satellite was successfully deployed. This has given a much needed boost to NASA morale.

To capture this observation, SDRT postulates that anaphoric relations must occur between SDRSs linked by some discourse relation *via* their labels. For this to be possible, the label of the SDRS that contains or is the antecedent must be a possible attachment site. Whether a discourse referent or SDRS α may be anaphorically linked to another discourse referent $\mathbf{x}$ introduced in an SDRS γ depends upon whether α is or occurs in a SDRS β that *via* the labels bears a discourse relation to γ. A slight complication to this occurs when the antecedent is a topic--in this case the anaphor can pick up the content not only of the topic but what it is a topic of. The meat of this constraint comes from the fact that possible attachment sites and thus discourse relation relata are restricted. They must either label: (i) the *current constituent* (the one added by processing the last sentence or clause) , (ii) a topic of the current constituent, or (iii) an SDRS that the current constituent is background to. I will assume that a function *current*, determined by SDRS update, takes an SDRS and returns the label of its last entered SDRS. $\pi_1 \Downarrow \pi_2$ means that π_1 labels a topic for the SDRS labelled by π_2. For labels π_1 and π_2, define: $\pi_1 < \pi_2$ in an SDRS δ, to be the smallest relation such that $\alpha_2\ \pi_1$: $\alpha_1 \in U_{\alpha_2}$ or $\exists\pi_j(\pi < \pi_2$ in δ & π_1: $\alpha_1 \in U_{\alpha_j})$. Since every SDRS is finite, $<$ is well-founded and every label has a unique $<$ predecessor. Let pred(π, τ) denote the predecessor of π in τ, and let $\alpha_{pred(\pi, \tau)}$ be the SDRS so labelled.

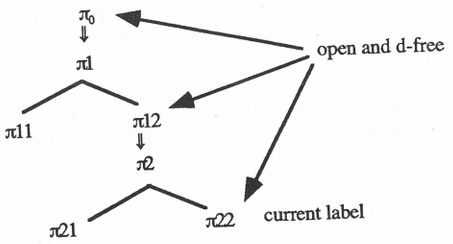
DEFINITION : *d-subordinate* = the transitive closure of $< \cup \Downarrow \cup$ Background.

[4]When s_i is complex (in that it has, for instance, one or more subordinate clauses), this representation could itself take the form of a complex SDRS, consisting of several DRS-like components; or, alternatively, the interpretation process could select one part of s_i, typically its main clause, attach its representation to the context SDRS and then proceed with the other parts. However, such complex sentences present a complication that is irrelevant for the examples in this paper, and so we ignore them here.

DEFINITION : A label π is *open* in β iff d-subordinate(current(β), π) or π = current(β).

DEFINITION (**D-Freedom**): A label π is *d-free* in β iff current(β) = π or $\neg\ \exists\pi'(\ \pi' \Downarrow \pi \in Con(\alpha_{pred(\pi, \tau)}))$.

An available attachment site in an SDRS is an *open and D-free* label; the diagram below depicts some (it's a constituent graph without the SDRSs); a line represents that the the labelled SDRS below belongs to the universe of the SDRS whose label is above.



Let's see how all this works by analyzing example (2). The first sentence yields a labelled DRS k_1; k_1 acts as a context within which to interpret the next sentence whose semantic representtionat is also a DRS k_2. k_1 is trivially open and D-free and so π_1 is an acceptable attachment site for k_2. Assuming a computation of the appropriate discourse relation to hold between k_1 and k_2, we have the SDRS below in which k_1 and k_2 are related by the discourse relation of Elaboration; and k_1 is a topic of k_2.

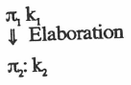
SDRS for first two sentences

π_1: k_1 π_2: k_2

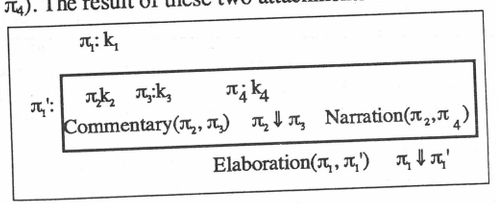
Elaboration(π_1, π_2)

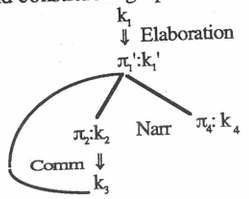
$\pi_1 \Downarrow \pi_2$

Constituent Graph



k_3 represents for the third sentence; it is related by Commentary to k_2, though π_1 and π_2 are both open and D-free and so available as attachment sites. The SDRS below shows the attachment of π_3 to π_2. The Commentary relation gives k_2 a primary discourse status, which we capture in SDRT by making k_2 a topic of k_3 (see the treatment of Background below). So after attaching k_3, the possible attachment sites are now π_2, π_3 or π_1. k_4 continues the story of America's being back in space. It also moves the narrative along from k_2; the adverbial *later that day* makes this clear. This narrative link is represented in SDRT by Narration(π_2, π_4). The result of these two attachments is this SDRS and constituent graph.





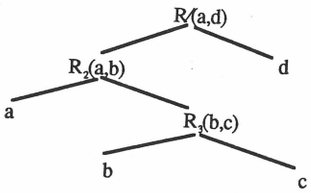
We can finish the example by attaching k_5 to the site π_1 of the topicby means of the discourse relation Result. This yields one possible intepretation of *this* by permitting the identification of the discourse referent z introduced by *this* with the entire episode--i.e. $z = k_1 \cup k_1'$. The only other available attachment point in the SDRS above is k_4. So SDRT predicts that k_4, pehaps e_4, and $k_1 \cup k_1'$ are the only possible antecedents of *this*.

A special feature of SDRT already implicit in our treatment of example (2) is that different discourse relations affect the structure of an SDRS differently. This helps account for the segmentation of discourse into complex structured units. But the differential treatment of discourse relations also has other effects--viz. on truth conditions. The anaphoric possibilities for pronouns are different, depending on the types of discourse relations used to relate various constituents. Consider the following discourses.

(3) A woman carrying a large envelope loomed out of the snow and approached the bar. John had been waiting there all morning. He had ordered expresso after expresso and read through all the newspapers on the rack. Now he went out into the cold to meet her.

(4) A woman with an enormous suitcase asked John to carry her bag to the taxi stand. Then a family with several pets and petulant children wanted his luggage cart. John was in a hurry to make his appointment.
a) But he was too polite to tell *her to lug her own suitcase.
b) But he was too polite to tell the lady to lug her own suitcase or the family to find their own luggage cart.

A theory of discourse structure that doesn't distinguish between the structural effects of different discourse relations, like a simple tree structure (cf. Gardent 1994), might give the following picture of (3) or (4.a) (R_i is a discourse relation).



Discourses (3) and (4.a) are quite different, however. For one thing, the anaphoric potential of the contexts created by the first four sentences in these two examples is different, yet their tree structures are isomorphic, if we don't distinguish between types of discourse relations in the structure and how these relations affect the structure. So if the structure dictates anaphoric availability, the theory that assigns (3) and (4) isomorphic structures is incorrect. We need to distinguish different sorts of relations in the discourse structure, and these relations must make a difference to the update of the discourse representation. We'll return to these examples in section 2.4. The point of (4.b) is that definite descriptions have a distint anaphoric potential from pronouns; a definite description can get an anaphoric interpretation in cases where a pronoun cannot.

2.3 SDRT Axioms for Calclulating Discourse Relations and their Effects

The previous section gave constraints on what were possible attachment points for new information in a given SDRS. These must be open and D-free labels. I will symbolize as $<\tau, \pi_1, \pi_2>$ the particular attachment of a labelled

SDRS $\pi_2:\alpha_2$ to an open and D-free label π_1 in an SDRS τ representing the discourse context prior to the integration of the new information in α_2. to establish a discourse relation between π_1 and π_2 and that relation's truth conditional and discourse structural effects, SDRT uses an axiomatic theory known as DICE (DIscourse in Commonsense Entailment).[5] We then define an SDRS update function that incorporates these effects into the SDRS. There will be an SDRS update function for each relation, and since more than one discourse relation may be inferred, the sum total effects of attaching $\pi_2:\alpha_2$ to π_1 in τ is the result of applying all the appropriate update functions.

A theory of discourse structure, so far as we understand at present, exploits and brings together many, separate sources of information. More specificially, it is the axioms relevant to the computation of a discourse relation that exploit multiple information sources--compositional and lexical semantics, the presence of certain "key words" that have been taken in the AI literature to signal the presence of a particular discourse relation, and even intonational stress. There is also an important interaction between these information sources and discourse structure. As we will see for instance, discourse structure sometimes determines temporal structure but in many cases too, temporal structure constrains discourse structure by licensing certain inferences by default or by blocking them.

The question of how these information sources are to be integrated within a formal, logical framework is a central question for a theory of the pragmatics/semantics interface. It is reasonable to suppose that these sources of information, each more or less autonomous, will from time to time support conflicting conclusions about interpretation. Thus the framework must be one in which conflicts of different sorts can be detected and resolved without the whole system going inconsistent. Further, the inference to a particular discourse relation is underdetermined by any given information source in general. But the other information sources may or may not resolve the question of which discourse relations definitively hold between two constituents. So we need to use these axioms in conjunction with a logic that allows us in the absence of information to the contrary to infer certain discourse relations by default.

To model defeasible implicatures and conflicting information sources in a principled manner, DICE uses a nonmonotonic logic in which generalizations with exceptions are represented by statements with a weak conditional operator $>$. $A > B$ means that "if A, then normally B." The effect of using a nonmonotonic logic is to generate in most cases only the preferred structure. An alternative would be to generate all the possible discourse structures for a given context and to impose a preferential ordering on them. But I'll only talk about the first here.

Temporal Relations: Narration

Narration, like other discourse relations in SDRT, is a "theoretical term." Its import for the content of a discourse is given by the axioms of the theory that spell out its truth conditional effects. The effect of Narration on the content of a discourse is summed up by the following axiom, where e_α is the main eventuality introduced in α (by the main verb of the clause that gives rise to α), o is temporal overlap, post(e_α) is the post-state of the eventuality e_α, pre(e_β) is the pre-state of e_β[6].

(A1) $\text{Narration}(\pi_1, \pi_2) \rightarrow \text{post}(e_{\alpha_1}) o \text{pre}(\alpha_2))$

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[5] DICE reasons about SDRSs and their properties and is part of SDRS construction, but it is not part of the discourse contexts as represented by SDRSs.

[6] When we take account of adverbials, (A1) gets more complicated. See Asher et. al. (1995).

This axiom reflects the intuition that Narrations must have cohesion; the events strung together by the Narration must fit consistently and without obvious gaps. The axiom has a number of consequences: by assuming that post(e) > e and pre(e) < e (where '<' is a temporal strict precedence relation such that e < e' iff e occurs prior to e'), we can infer from Narration(π_1, π_2) that $e_{\alpha_1} < e_{\alpha_2}$.

An additional effect of Narration is that elements connected together by Narration must have a common topic that is neither loically valid nor contradictory (i.e. contingent) once we update the SDRS--a policy stated in (A2). Update($\alpha_{pred(pred(\pi_1, \tau))}$, Narration($\pi_1$, π_2)), the update of the SDRS containing the SDRS in which π_1:α_1 is a constituent with the new Narration, is defined shortly. The intuition that a Narration must have a unifying theme motivates this axiom.

(A2) $<\tau, \pi_1, \pi_2>$ & Narration(π_1, π_2) → ($\exists\alpha_3$)(Contingent(α_3) & $\pi_3 \Downarrow \pi_1$, & $\pi_3 \Downarrow \pi_2$ & π_3: $\alpha_3 \in$ U(Update($\alpha_{pred(pred(\pi_1, \tau))}$, Narration($\pi_1$, π_2)) & $\pi_3 \Downarrow \pi_1$, $\pi_3 \Downarrow \pi_2 \in$ Con(Up-date($\alpha_{pred(\pi_1, \tau))}$).

Let us now look at how to infer Narration. Many of the laws for inferring discourse relations are defeasible generalizations. The basic SDRT axiom for discourse relations, e.g. of Lascarides and Asher (1993), is that without additional information we will infer that Narration holds between two constituents expressed by sentences in the past tense that are to be related together. Work on narrative structure by Canapael (1989) and Sandstrom (1995) indicates, however, that the original SDRT approach to Narration is perhaps not correct. Additional information seems needed to infer a narrative sequence in a text. For instance, in American English, Narration and its temporal structure may not be inferred in certain contexts, although our simple axiom would predict it.[7]

(5.a) John studied. Mary studied too.
(5.b) Mary studied, but John went drinking.
(5.c) Alexis was a good girl today She did her homework. She practiced piano, and she helped cook dinner.

Why does Narration fail in these examples? Given the logical framework of a nonmonotonic logic, there are two explanations: (i) something blocks or defeats the inference to Narration; (ii) the simple axiom of earlier work in SDRT is incomplete and the right default for Narration requires some extra condition in the discourse context that is missing in examples (5). It is hard to see what blocks Narration in these examples, so option (ii) seems more plausible. The requisite information for inferring Narration in some cases, exploiting some of Sandstrom's ideas, is that the first mentioned event e_{α_1} "occasions" the second mentioned event e_{α_2}--i.e., be part of a plan or a natural type of sequence of events in which events of the sort described by α_1 lead to events ot the type described by β. This seems intuitively plausible and supported by examinations of real texts. But obviously this can only be a default; interpreters don't have a plan or enough world knowledge to know about occasionings in every instance of Narration. Further, how do we express "occasioning" formally? The effects of Narration axiom in fact gives us the key. If one can deduce nonmonotonically from the world knowledge and the discourse context that the prestate of an event of ψing overlaps temporally the post-state of an event of φ-ing, then one can

[7]One might think (as I did earlier) that perhaps relations like Parallel and Contrast--as in (5.a-b)--preclude Narration. But I have no explanation for this and it seems in any case implausible given an example like (1) where we have both Narration and Contrast. At any rate this does not explain (5.c), which is acceptable in American English.

conclude Occasion(φ, ψ, τ). This inference is symbolized in the definition below by $\boxed{\sim}_{KB\&\tau}$ ($\varphi\&\psi$, post(e_φ) *o* pre(e_ψ)):

Definition: Occasion(φ, ψ, τ) ↔ (($\boxed{\sim}_{KB\&\tau}$ ($\varphi\&\psi$, post(e_φ) *o* pre(e_ψ))

Another key element to note about the examples in (5), however, is that other discourse relations besides Narration apply. In cases where we don't have clues that the author is trying to make some other sort of rhetorical structure, we want to assume that he is just telling us the order in which events happened--this "abductive" characteristic of the earlier approach seems desirable. To take account of these points, I propose two axioms for inferring Narration:

(A3a) [$<\tau, \pi_1, \pi_2>$ & Occasion(α_1, α_2, τ)] > Narration(π_1, π_2).
(A3.b) [$<\tau, \pi_1, \pi_2>$ & ¬$\boxed{\sim}$(($<\tau, \pi_1, \pi_2>$ & info(α_1, α_2)), (R(π_1, π_2) & R ≠ Narration))] > Narration(π_1, π_2)

(A3.b) says that if one cannot deduce nonmonotonically from presumed common background knowledge, the fact that π_2 is connected to π_1 in τ and facts about α and β that some other relation than Narration relates π_1 and π_2 then one may conclude Narration defeasibly (as long as (A2)'s constraints on topic are met).[8]

These are not the only axioms for inferring Narration. The use of certain adverbials like *later that day* as in example (2), discourse particles like *then*, or the *now* with the past tense as in example (3') below force us to conclude Narration as a logical consequence, not as a default consequence.

Applying these ideas to the examples in (5), the idea is that in (5.a) and in a context containing no prior knowledge about Mary and John, we cannot infer the Occasion relation. Further, since there are no clue words in (5.a)and since we can attach the second constituent by means of the Parallel relation, we cannot infer Narration by any of the axioms mentioned above. In (5.b) again we cannot infer Narration, but we do get a coherent SDRS because we infer Contrast.

We have seen the semantic effects of attachment with Narrationand how to compute Narration. Let's now investigate SDRS update for Narration. The update of an SDRS τ is defined by the resetting of the function Current needed to compute availability, and the changes that the new attachment makes in the SDRS labelled by the predecessor of the attachment point, if the update is local. There are several cases to consider because of the requirements that Narrations have a topic. If there is a topic, then Narration update is local; we substitute the update of pred(π, τ), which is defined below, for pred(π, τ).

(A4) ($\exists\pi_3$: $\alpha_3 \in$ U($\alpha_{pred(pred(\pi_1, \tau))}$)($\pi_3 \Downarrow pred(\pi_1, \tau) \in$ Con($\alpha_{pred(pred(\pi_1, \tau))}$) →
(i) Update(τ, Narration(π_1, π_2)) = τ[Update($\alpha_{pred(\pi_1, \tau)}$, Narration($\pi_1$, π_2))/$\alpha_{pred(\pi_1, \tau)}$]; & Current(Update($\tau$, Narration($\pi_1$, π_2))) = π_2
(ii) Update($\alpha_{pred(\pi_1, \tau)}$, Narration($\pi_1$, π_2)) = $<$(U($\alpha_{pred(\pi_1, \tau)}$) ∪ {$\pi_2$:$\alpha_2$}), Con($\alpha_{pred(\pi_1, \tau)}$) ∪ {Narration($\pi_1$, π_2)}>

This update rule was in fact used in constructing the SDRS for (2) viz. the attachment of constituent derived fourth sentence to the site π_2. It says ro add Narration(π_1, π_2) and π_2:α_2 to $\alpha_{pred(\pi_1, \tau)}$.

If there is no topic for α_1 and α_2 in the extant SDRS, then we must do something to ensure that (A2) is satisfied. There are two cases to consider. Here is case 1. Suppose α α has no topic. Then we are allowed to construct one,

[8]For more discussion see Asher and Lascarides (1995).

provided the constraints on topic are met. The following axiom specifies Update($\alpha_{pred(\pi_1, \tau)}$, Narration($\pi_1, \pi_2$)). Below Topic($\alpha$) denotes the topic of the constituents of α formed by computing the lowest common supertype of the conditions given by the main verbs in those constituents and summing their arguments (see Asher 1993 for details, and Prubst, Scha and van den Berg 1993 and Grover et al. 1995 for similar approaches):

(A5) $\neg\exists\alpha_2\ \pi_2 \Downarrow pred(\pi_1, \tau), \in (Con(\alpha_{pred(pred(\pi_1, \tau)} \cup Con(\alpha_{(pred(\pi_1, \tau))} \rightarrow$
(i) Update(τ, Narration(π_1, π_2)) = τ[Update($\alpha_{pred(\pi_1, \tau)}$, Narration($\pi_1, \pi_2$))/$\alpha_{pred(\pi_1, \tau)}$]; & Current(Update($\tau$, Narration($\pi_1, \pi_2$))) = π_2
(ii) Update($\alpha_{pred(\pi_1, \tau))}$, Narration($\pi_1, \pi_2$)) = <(U($\alpha_{pred(\pi_1, \tau)}$)[$\alpha_4/\alpha_1$] $\cup$ {π_5:α_5}), Con($\alpha_{pred(\pi_1, \tau)}$) $\cup$ {$\pi_1 \Downarrow \pi_5$, Elaboration($\pi_1 \Downarrow \pi_5$)}>, where α_5 = <{π_4:α_1, π_2:α_2}, {Narrationπ_1, π_2)}> and α_4 = Topic(α_5)

In the second case, we have Narration(π_1, π_2) and α_1 has a topic in $\alpha_{pred(\pi_1, \tau)}$ but this topic is not a topic for α_2. To do the update, we must know whether the topic was constructed by a previous update or not. If yes, we may revise $\alpha_{pred(pred(\pi_1, \tau))}$ so that (A2) is satisfied. The axioms for revision in this case is given in (A6). If the topic was not constructed but explicitly given, then we cannot attach with Narration to π_1, on pain of contradicting (A2). We must find another available attachment site. To illustrate, compare the following discourses.

(6.a) Alexis got out her music(1). She worked on her Mozart sonata(2). Then she went to get something to eat(3).
(6.b) Guy ate a wonderful meal (1). He started with foie gras on toast (2) Then he had a wonderful salmon, followed by cheese with port (3). Then he went dancing(4).

In (6.a) we infer Narration(π_1, π_2) and Narration(π_2, π_3), by using (A3a) and by exploiting the particle *then*. By (A5), we infer a constructed topic for k_1 and k_2, which we can then modify to take account of the new information in k_3. In (6.b), the topic of k_2 and k_3 is given by the first sentence. When we try to attach π_4: k_4 with Narration to π_3, however, the topic does not extend and so we cannot attach with Narration to π_3. We must instead attach to the topic label π_1.

(A6) $\pi_2 \Downarrow pred(\pi_1, \tau) \in Con(\alpha_{pred(pred(\pi_1, \tau))})$ & Constructed(α_3) & $\neg\pi_3 \Downarrow \pi_2) \rightarrow$
(i) Update(τ, Narration(π_1, π_2)) = τ[Update($\alpha_{pred(pred(\pi_1, \tau))}$, Narration($\pi_1, \pi_2$))/$\alpha_{pred(pred(\pi_1, \tau))}$]; & Current(Update($\tau$, Narration($\pi_1, \pi_2$))) = π_2
(ii) Update($\alpha_{pred(pred(\pi_1, \tau))}$, Narration($\pi_1, \pi_2$)) = $\alpha_{pred(pred(\pi_1, \tau))}$[$\alpha_4/\alpha_3$][$\alpha_{pred(\pi_1, \tau)}/\alpha_5$], where α_4 = Topic(α_5) &α_5 = $\alpha_{pred(\pi_1, \tau)}$ $\cup$ <{π_2:α_2}, {Narration(π_1, π_2)}>

The effects of Narration, codified in (A1)-(A6) capture something of the Gricean maxim, be orderly. They imply that by default the textual order of event reflects their real temporal order. But there are discourse contexts in which additional information about the structure and content of the related constituents may overturn this Gricean maxim. For example, if we change tenses in a discourse we may not be able to infer Narration even if Occasion follows from the event types involved. Only Contrast the Narration in (7.a) with (7.b), in which we cannot infer Narration due to a conflict between the temporal information conveyed by the sequence of tenses and the temporal implications of Narration.[9]

[9]The Narration in (7.a) might be inferred in either of two ways--either by (A3.a) if one supposes an occaasioning relation to be inferrable between waving and smiling, or as a "last resort" by (A3.b) since no other axiom concerning discourse relations is satisfied.

(7.a) John waved to Mary across the hall. She smiled at him.
(7b) John waved to Mary across the hall. She had smiled at him.

To say why Narration fails in (7.b), we must say a more about the interpretation of tenses in SDRT. SDRT uses a simplified DRT treatment of tense. The English simple past simply introduces an event in the past. The English pluperfect is "anaphoric" in that it anaphorically links the eventuality argument of the verb to some other temporal entity than those introduced by the tense form itself.[10] The contribution of the pluperfect in linear DRS notation is:

PP(φ) $\rightarrow$ [[s, e], [s = cs(e), φ(e), init(s) < ?, init(s) < now]]

In the DRS above s is the consequent state of e, and ? must be replaced with either an anaphorically available temporal or event discourse referent introduced outside the tense component in which ? is introduced.

In (7.b), the only available temporal discourse referent for the ? in the condition init(s_{k_2}) < ? introduced by the PP tense in α_2 is e_{k_1}. So we have init(s_{k_2})< e_{k_1}. Since s_{α_2} = cs(e'), where e' is the event of Mary's smiling, e'$\leq$ init(s_{k_2}) and so e' < e_k, which contradicts the temporal consequences of Narration. The tense sequence information here constrains the sort of discourse relation that can relate k_1 and k_2. This example an dthe others There is instead a complex interaction between compositional semantics, lexical semantics and discourse structure that determines temporal structure.

Examples like (8) were also taken to exemplify an overriding of Narration by other information.[11] But on the new treatment here (8) is not a true instance of the reversal of Narration but one where Narration simply does not apply.

(8) John stumbled. Mary pushed him.

There is no occasioning suggested between stumbling and pushing but rather as postulated in earlier work the speaker's invitation to see a causal connection between the two events, symbolized by means of a "triggering rule"; this rule allows us to infer D-permissible-Cause(k_2, k_1, τ) and with the approrpriate axioms, Explanation(π_1, π_2) and its temporal consequence--namely, that no temporal part of the explained effect precedes its explanatory cause.[12]

Narration can sometimes block Explanation. The account of Lascarides and Asher (1993) had trouble with the following, acceptable discourse, in which the inference to Explanation is blocked in favor of an inference to Narration even though there are no explicit clues for Narration:

[10] Arie Molendijk, I believe, came up with the idea that the French imparfait is anaphoric. Like the English pluperfect, the French plus que parfait and passé antérieur are also anaphoric in a way analogous to that made plain for the translation of the PP tense in English.
[11] See Lascarides and Asher (1993).
[12] Triggering rules exploit types in the lexicon. For instance, the D-permissible Cause axiom, formalized in Asher (1995a), concerns the juxtaposition of lexically defined event types and exploiting a particular pragmatic constraint about what forms speakers use that license certain discourse connections-- in this case causal and explanatory connections. They are once again laws of pragmatics, but they exploit various information sources beyond what might be traditionally considered pragmatics. Here is the rule for D-permissible cause
[<τ, α, β> & cause-change$_\pi$(ψ) & change$_\pi$(φ) & $\Box(\alpha_1 \rightarrow \varphi(e\alpha_1))$ & $\Box(\beta \rightarrow \psi(e\alpha_2))$] $\rightarrow$ D-Permissible-Cause(α_2, α_1, τ).
The rule may also be used to infer D-permissible-Cause(α_1, α_2, τ) when the information about φ and ψ above is reversed. Here is the relevant Explanation axiom:
<τ, π_1, π_2> & D-Permissible-Cause(α_2, α_1, τ) & ASPECT(α_1, α_2) > Explanation(π_1, π_2)

(9) John stumbled. Mary pushed him. Unable to stop his momentum, he rolled off the edge of the cliff to his death.

The third sentence offers us more information than present in (8) so that we can infer Occasion between the first two constituents. This example suggests a general pattern represented in (A7), which says that occasionings and Narrations win out over Explanations when conditions relevant to both are present. In the nonmonotonic logic used by SDRT, the axiom with the logically most specific antecedent (i.e. the one with the antecedent that entails the antecedents of the other applicable axioms) will win out in cases of conflict. Whenever the antecedents of the applicable axioms are such that none entails the others, then we "Nixon diamond" and get no conclusion concerning what discourse relation is applicable. This motivates the form of (A7). Concerning revision of already established SDRSs, I suppose that SDRS construction involves revision of previously established discourse relations, whenever new information satisfies the antecedents of more specific axioms that lead to conclusions in conflict with those previously established relations.

(A7) $[<\tau, \pi_1, \pi_2>$ & D-permissible-Cause$(\alpha_2, \alpha_1, \tau)$ & Occasion$(\alpha_1, \alpha_2, \tau)] >$ Narration(π_1, π_2).

Aspectually Sensitive Relations: Background

So far we have seen the two way interaction between temporal information and discourse structure in the calculation of discourse relations and their semantic effects. Aspectual information about the type of eventuality described in the constituents related may also affect reasoning about discourse relations . Background, for instance, exploits aspectual information; as shown in (A8), Background will be inferred if one of the constituents so related has a state as a main eventuality while the other has an event. (A8) is a default like (A3.a-b) But in comparing (10) to (7.a), we see that aspectual information wins out over the information used for Narration when both are present--this is represented by (A9). The temporal effects of Background are given in (A10); they are incompatible with those of Narration; so (A9) and (A3.a) conflict. Since (A9) has a more specific antecedent, DICE predicts Background for (10).

(10) John waved. Mary was smiling at him.

(A8): $(<\tau, \pi_1, \pi_2>$ & State(e_{α_2}) & $\neg$State$(e_{\alpha_1}))$ > Background(α_1, α_2)
(A9): $(<\tau,,\pi_1,\pi_2>$ & Occasion(α_1,α_2) & State(e_{α_2}) & $\neg$State$(e_{\alpha_1}))$ > Background(α_1, α_2)
(A10) Background$(\alpha_1, \alpha_2) \rightarrow e_{\alpha_1}$ O$_T$ e_{α_2}, where O$_T$ is temporal overlap.

The inference to Background may be overturned, however, when certain causal connections are inferred via the triggering rules. Such is the case in (11) from Hinrichs (1986). Assume that the French imparfait as in Kamp and Rohrer (1983) always introduces a state into a constituent. Further suppose that the lexicon is organized such that we can infer that shutting off a light is a cause of a change in the luminosity of a region and that *il faisait nuit noire* has an inchative interpreation as describing a change of luminosity. Then in (11) we will infer via the appropriate triggering rules D-permissible-cause(k_1, k_2, τ) (see footnote 12).

(11) Pierre éteigna la lumière (k_1) . Il faisait nuit noire, car les volets étaient fermés (k_2).

The information in (11) allows us to use the axiom for Result (A14), which is insensitive to the aspect of its constituents (this is represented by ASPECT).

(A11) $[<\tau,\pi_1,\pi_2>$ & Aspect(α_1,α_2) & D-PermissibleCause$(\alpha_1,\alpha_2)] >$ Result(α_1, α_2)
(A12) Result$(\alpha_1, \alpha_2) \rightarrow e_{\alpha_1} < e_{\alpha_2}$.

But from the compositional semantics and the aspectual calculation for the sentences, we have: Event(e_{k_1})--i.e. $\neg$State(e_{k_1})--and State(e_{k_2}). So the antecedents of the Background and Result axioms all are satisfied in this discourse context. Now these are all incompatible relations--by (A12) and (A10). Because (A11) has the most specific antecedent, DICE forces us in this case to infer Result rather than Background, and so we infer $e_{k_1} < e_{k_2}$ in (11), as intuitions dictate.

Like Narration, Background also has structural effects on the discourse context, creating a *Focus Background Pair* (FBP) with a topic with particular update rules. (A13) states a local update rule, similar to (A4).

(A13) Background(π_1, π_2) & $\neg\exists\pi_3{:}\alpha_3$ FBP$(\pi_3, \text{pred}(\pi,\tau)) \rightarrow$
(i) Update$(\tau, \underline{\text{Background}(\pi_1, \pi_2)}) = \tau[\text{Update}(\alpha_{\text{pred}(\pi_1, \tau)}, \underline{\text{Background}(\pi_1, \pi_2)})/ \alpha_{\text{pred}(\pi_1, \tau)}]$ & Current(Update$(\tau, \underline{\text{Background}(\pi_1, \pi_2)})) = \pi_2$
(ii) Update$(\alpha_{\text{pred}(\pi_1, \tau)}, \underline{\text{Background}(\pi_1, \pi_2)}) = \alpha_{\text{pred}(\pi_1, \tau)}[\alpha_1 \underline{\cup} \alpha_2/\alpha_1] \underline{\cup} <\{\pi_3{:}\alpha_3\}, \{\text{FBP}(\pi_1, \pi_3)\}>$, where $\alpha_3 = <\{\pi_4{:}\alpha_1, \pi_2{:}\alpha_2\}, \{\underline{\text{Background}(\pi_4, \pi_2)}\}>$
(A14) Background(π_1, π_2) & FBP$(\pi_3, \text{pred}(\pi_1,\tau)) \rightarrow$
(i) Update$(\tau, \underline{\text{Background}(\pi_1, \pi_2)}) = \tau[\text{Update}(\alpha_{\text{pred}(\text{pred}(\pi_1, \tau))}, \underline{\text{Background}(\pi_1, \pi_2)})/ \alpha_{\text{pred}(\text{pred}(\pi_1, \tau))}]$, & Current(Update$(\tau, \underline{\text{Background}(\pi_1, \pi_2)})) = \pi_2$
(ii) Update$(\alpha_{\text{pred}(\text{pred}(\pi_1, \tau))}, \underline{\text{Background}(\pi_1, \pi_2)}) = \alpha_{\text{pred}(\text{pred}(\pi_1, \tau))}[\alpha_1 \cup \alpha_2/\alpha_3][\alpha_4/\alpha_{\text{pred}(\pi_1, \tau)}]$, where $\alpha_4 = \alpha_{\text{pred}(\pi_1, \tau)} \underline{\cup} <\{\pi_2{:}\alpha_2\}, \{\underline{\text{Background}(\pi_1, \pi_2)}\}>$

(A14) is a non-local updating rule that governs what happens when one already has a FBP and a background relation attached to π_1; it says the topic of the FBP must always contain the foreground and the most recent background contstituent. (A13) and (A14) account for a peculiar observation about the availability of discourse referents introduced in Backgrounding constituents. If we consider one variant of (3) below, we find that attempting to refer anaphorically to a discourse entity introduced in the Background is difficult if more backgrounding material intervenes; but (3) indicates elements in the foreground remain available. Another variant (3") shows that elements in the last Background constituent(k2) are also available, even if the constituent (k3) in which the anaphor occurs is attached to the topic by Narration, thus "moving the action along."

(3') A woman carrying a large envelope loomed out of the snow and approached the bar. John had been waiting there all morning with an old male friend. John had ordered expresso after expresso and had read through all the newspapers on the rack. ?He had talked to him to pass the time. But now she was here.
(3") Mary entered the bar (k1). There was a man reading a newspaper over in the corner (k2). She went over to him. (k3)

I have examined axioms for inferring Narration and Background, their temporal effects, and their effects on discourse structure, as well as axioms

concerning discourse relations like Explanation and Result.[13] If this approach is at all on the right track, they require a relatively complex logic underlying the construction of discourse structure. Various features--tense, aspect, etc.--must be factored in and preferred interpretations result from complex interactions. Though I have no proof, it is dubious that simple mechanisms for discourse structure construction that do not make use of Boolean information like simple unification or tree adjunction will be able to do justice to the phenomena.

Structural Relations: Contrast

Some discourse relations, like Parallel, Contrast and Generalization, are inferred from the structure of the discourse and its constituents. Discourse theories that have exploited unification-like mechanisms have concentrated on Parallel where two bits of discourse structure are taken to be similar in relevant ways (Grover et al. Prubst, Scha and van den Berg 1993). What SDRT contributes to this work is a method whereby not only similarity but contrast and other types of relations can be computed to hold of discourse constituents. Though space does not permit a lengthy development, these relations exploit the logical structure of the constituents themselves as well as their contents. These relantions are all defined by means of an *embedding trees* $t(\alpha)$ for a constituent α.

Definition: $t(\alpha) = \langle X, Y\rangle$, where: $X = \{y: y \in_{tc} U_\alpha \vee y$ is a subDRS of α or of some $v \in_{tc} U_\alpha\}$ and $Y = (\{\in\restriction X\} \cup \{\Downarrow\restriction X\} \cup \{\text{subDRS}\restriction X\})$

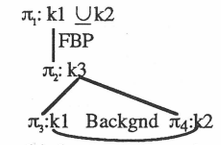
The approach of Asher (1993) yields scalar definitions for structural relations like Contrast. When a scalar relation like Contrast is inferred typically through the presence of a discourse particle or a grammatical construction like VP ellipsis, we disambiguate and coerce the interpretation so as to maximize the Contrast. The constraint about Contrast suffices to disambiguate (1.a); the *borrow* sense of *ausleihen* supports a stronger Contrast than the other possible interpretation. In (1.b), however, this is not the case; both interpretations of *ausleihen* support equally strong Contrasts. What disambiguates (1.b) is that we infer Narration in one interpretation (the *lend* interpretation) using (A3.a). But we cannot do so in the *borrow* interpretation, because that the pre-state of borrowing the book(in which the lender doesn't own the book) is incompatible with the "post-state" of buying the book in which one owns the book. What makes us opt for the *loan* interpretation in (1.b) is a principle that says that we disambiguate discourse content so as to maximize the number of discourse relations in an SDRS. Discourse coherence also has a scalar quality.[14]

2.4 Applications: Anaphoric Potential of Discourse Contexts

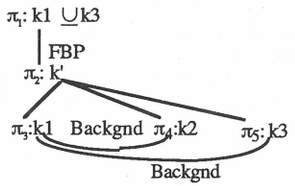
With this investigation of SDRT's discourse relations and SDRS construction, we can now go back to account for the difference in the anaphoric potential of examples (3) and (4). Let's consider (3) first. The first sentence yields an SDRS with just constituent$_\pi$ which is D-free and open. We attach the label π_2 of the DRS k2 derived from the second sentence to this site with Background using axiom (A9.a) since k2 has a state as a main eventuality and k1 has an event by the semantics of tense. The update axiom (A13) now forces us to construct a focus background pair, and the following SDRS:

[13] Though we have not discussed the update rules for these relations, the rule in each case is the same--they merely incoporate the new information into the SDRS in a way similar to that given by the consequent of the first Narration update rule (A4).

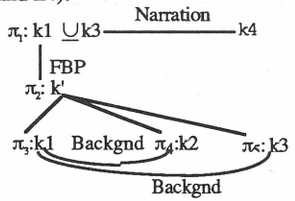
[14] For more details, see Asher & Kamp (ms.).



The DRS k3 derived from the third sentence continues the Background relation to k1. This is possible, because k1 as the foreground is still a topic in effect of the lower constituent. By (A19) we get a more specific sort of revision.

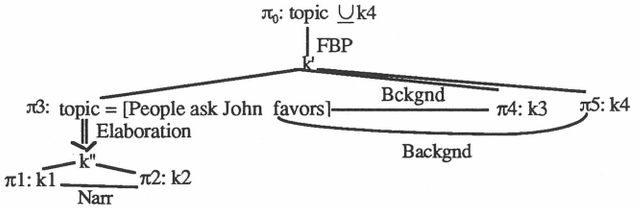


The constituent k4 forms a narration with its attachment point, because of the discourse particle *now*, which is often a cue that the author is resuming a Narration after some Background or Commentary. Crucially, it isn't background and so it cannot be attached underneath the FBP relation. The only constituent available for attachment that is D-free and Open is the topic of the FBP, in which according to our rules the discourse for a *a woman* occurs. The anaphoric link is predicted to be fine because the referent for *a woman* occurs in the topic to the Foreground Background pair. Here is the finished structure (I omit the topic constituent for k1 ⌣ k3 and k4):



In (3´), we proceed as in (3) until the fourth sentence, which yields a constituent that must by (A9a) be attached with Background to π_1, along with k2 and k3. But now the friend introduced in k2 is not available in the topic, which has been updated to k1 ⌣ k4 according to (A14), or in k3. So the anaphoric link in the fourth sentence is predicted to sound funny. In (3''), however, after update we have $k_1 \cup k_2$ in topic position, and we can attach k_3 to the label of this constituent using Narration. So the man introduced in k2 is available.

In example (4), different discourse relations figure in the construction of the SDRS, shown below. k1 and k2 continue a narrative underneath a constructed topic, which contains a discourse referent representing the group of people consisting of the woman and the family because of topic formation rules, but crucially not an individual discourse referent for the woman. When we attempt to attach either at the topic or at k2, SDRT predicts the woman to be inaccessible.



Another interesting discourse context with a peculiar anaphoric potential is the so called list structure discussed by Polanyi and Scha (1985). SDRT predicts that these examples have a distinctive temporal anaphoric potential. (5.c) above is an example. Another set of axioms and triggering rules lead us to conclude that the first constituent is elaborated upon by the three following constituents (each one derived from a clause). But we cannot relate k3 to k2 and k4 to k3 by Narration, because we cannot infer Occasion, nor are the right sort of discourse particles present to necessitate Narration. SDRT predicts that Alexis's doing her homework, her practicing piano and her helping cook dinner are all part of her being good today and so occurred today, but not that they occured in any particular temporal orders. This is to be distinguished from the discourse structure for (4) in which the Elaboration has a narrative structure.

The SDRT updating mechanisms thus account for the different anaphoric potential of discourses in terms of distinct discourse structures that result from the different discourse relations that bind together the constituents.

3. The Semanticized Version of the Pragmatics/Semantics Interface

The preceding section has developed a representational theory of pragmatics semantics, SDRT. SDRT in contrast to pure dynamic semantic theories like DRT defines a new semantic-pragmatic conception of context and a new notion of the context change potential (CCP) of new information that includes pragmatic and semantic information. Let us abbreviate the SDRT-CCP of a DRS φ as:

Definition: $\alpha \neg\exists(\varphi)\beta$ iff **K'** is an SDRS-update of **K** with the appropriate revision of φ.

Because of the possiblity of multiple attachment points in **K**, $\neg\exists(\varphi)$ is not a function. But we can construct a functional SDRT-CCP of φ, $\underline{\neg\exists}(\varphi)$, using $\neg\exists(\varphi)$ to define a CCP between sets of SDRSs.

I now turn to a semanticized version of this notion of CCP and an accompanying notion of context. Given certain assumptions about the tasks of SDRSs, SDRSs are equivalent to their semanticized counterparts.

To see how this might be done, let us look at the correspondence between representational and model-theoretic versions of dynamic semantics. Two types of discourse contexts used in dynamic semantics are the so-called "model theoretic" contexts of Groenendijk and Stokhof (1987) and the representational contexts of DRT. A model-theoretic context is a set of world sequence pairs (WSPs) and the meaning or context change potential (CCP) of a an unambiguous sentence or here DRS φ is a function $P(\varphi)$:Pow(WSP) → Pow(WSP) that is defined distributively or pointwise over the set, exploiting φ's effect on an input WSP (w, g) to produce a certain output WSP (w', g'), where w = w' and $g \subseteq g'$. Each w has a domain of entities, Dom(w). A representational context is a recursively structured object (a DRS); the CCP R(φ) of a DRS φ is the partial function: $[R(\varphi)](\psi) = \psi \underline{\cup} \varphi$, if Consistent(φ, ψ) ($\underline{\cup}$ is DRS-merge).

Fernando (1995) shows that for the simple core fragment of DRT, for all *accessible* information states (those that we can produce from the empty

information state by some finite number of formula updates), the representational level and the model theoretic level yield bisimulation-equivalent notions of context and CCP.[15] Let $\Phi_\perp$ be the set of absurd formulas, and let W be a sufficiently large collection of worlds (to represent all logical possibilities). Define Acc as the smallest set of states such that, where $[P(\varphi)](\sigma)$ represents the application of the function $P(\varphi)$ to σ:

$\{<w, 0>: w \in W\} = \sigma_0 \in Acc$ and $\sigma \in Acc \Rightarrow [P(\varphi)](\sigma) \in Acc$

So we can define within Acc for any consistent φ, $\sigma_\varphi = [P(\varphi)](\sigma_0)$
Suppose that $P([R\varphi](\psi)) = P(\psi) \circ P(\varphi)$ and define $\varphi \underline{\leftrightarrow} \psi$ iff $\forall\varphi' \in \Phi$ ($\varphi R(\varphi') \in \Phi_\perp$ iff $\psi R(\varphi') \in \Phi_\perp$).

Theorem (Fernando 1995): Suppose in addition to the assumptions above that every formula has a characteristic formula in first order logic.
Then: $\varphi \underline{\leftrightarrow} \psi$ iff $\sigma_\varphi = \sigma_\psi$; and further $\underline{\leftrightarrow}$ and $\Phi_\perp$ are r.e.

The two notions of CCP in dynamic semantics are equivalent. Far from "eliminating representationalism," the inclusion in WSPs of functions from discourse referents to objects makes them essentially representational; this information is not about the world but about how our information about the world is structured. But using WSPs is useful because they indicate the minimal amount of representational information needed to treat phenomena for which DRT was designed. I apply this strategy now to pragmatic-semantic contexts.

3.1. Model Theoretic Contexts for the Pragmatics/Semantics Interface

In SDRT, each SDRS has a model theoretic content--a set of WSPs or state. In fact an SDRS is a notational variant for a formula of logic, where discourse referents become variables. In SDRT, the variables are typed; there are variables and terms for SDRSs (which can be anaphoric antecedents as in (2) above), variables labels, eventuality variables and individual variables. In SDRT models, we have the corresponding types--thus states, or rather their representatives, are elements the domains of our models. There are a number of ways to do this familiar from the literature on the semantic paradoxes. We could just make SDRSs the internal correlates to WSPs (see e.g. Asher 1993). But we can also exploit the following observation for our limited purposes here. Consider a language L with first order formulas built up out of atomic formulas of the form $R(\pi, \pi', \pi_1, ...\pi_n)$ and π:φ, where φ is a DRS (or again a first order formula). The intended interpretation of $R(\pi, \pi', \pi_1, ...\pi_n)$ is that $R(\pi, \pi')$ is a condition in an SDRS whose label is π_1 and that π_1 occurs in an SDRS π_2 and so on *and* that the condition $R(\pi, \pi')$ is satisfied in that context. This requires that in the first order language the discourse relation symbols have variable polyadicity. We would also have to say more about what it is for $R(\pi, \pi')$ to hold (see Asher 1993 for more details). The interpretation for π: α is that $f(\pi)$ is the set of worlds in which α is true. In light of this equivalence (which will not work for a DRT treatment of the attitudes), we can identify SDRSs with sets of worlds expressible by some formula in the first order language just sketched.

To define the update of discourse contexts model-theoretically, we need a set states, a set of available attachment points, and information concerning the structure of the formulas whose interpretation constitute these states--for temporal anaphora, we need to have access to the main eventualities of WSP's;

[15] A similar representation theorem for a slightly different conception of model theoretic context is proved in Asher (1993).

and for phenomena exploiting Parallel and Contrast, we need the embedding trees of constituents. A model-theoretic characterization of discourse context then will consist of a set of labelled states with a well-founded, pre-ordering < on labels. Associatedwith each label are two functions correlating it with an embedding tree and a main eventuality, which depends on how the state associated with the label was expressed by the formula φ in the discourse.

Definition: A discourse context is a tuple (X, <, Current, e, f) where: X ⊂ (Labels × Pow(WSP)); < is a well-founded pre-ordering on LABELS s.th. $\pi_i < \pi_j$ iff ∀<w, f> ∈ s_i ∃x ∈ Dom(f) s.th. f(x) = {w: ∃h <w,h> ∈ s_j}. Current is a label in X; and for π_i:s_i ∈ X, ∀<w, f> ∈ s_i ∃u ∈ Dom(f) e(π_i) = u and u is the main eventuality in φ_i, while f(π_i) = the embedding tree of φ_i.[16]

The < minimal element in LABELs corresponds to the outermost SDRS. Using <, we isolate as before pred(π_i, τ) and the associated state $s_{pred(\pi_i, \tau)}$. If s_i has a topic s_j, then either $s_{pred(s_i, \tau)} \models \pi_j \Downarrow \pi_i$. or else pred(s, τ) = s_j and $s_{pred(pred(s_i, \tau), \tau)} \models s_{pred(s, \tau)} \Downarrow s_i$. By defining the function Current for model-theoretic contexts, we can define the set of available attachment sites as in section 2.2.

3.2 A Model Theoretic Analysis of the Pragmatics-Semantics CCP:
Given a context and a set of attachment sites, we now define the context change potential of new information within such a context. We need first to compute the discourse relation and the semantic effects of the new attachment. We may use the DICE axioms to compute discourse relations by replacing the α's in those axioms with variables ranging over states or perhaps their internal correlates (the names first order formulas that express SDRSs).

A difficulty with the pragmatics-semantics notion of CCP is that it is not always local. As we have seen, updates have effects beyond simply the attachment position and the contents of the predecessor of the attachment site. Let us suppose now that an attachment point π_i in X given a discourse structure τ = (X, <, Current, e, f) has been chosen. The discourse referents for the topic position must be accessible to discourse referents in a subordinate state. This must be reflected in the semantics. So we must constrain the subordinate states to respect assignments made in topics. Let

Top(π_i, τ) = s_j, where ∃s' >τ s_i s. th. s' ⊨ $\pi_j \Downarrow \pi_j$
= {<w, ∅>: w ∈ W} otherwise.

We now define the model-theoretic content of new information φ, $\|\varphi\|_{\pi_j, \tau}$, relative to an attachment point π_j in a context τ, using the semantic update function P(φ) (section 3.1). This content must respect the assignments in states that are discourse related to the new constituent, and so I require that P(φ) operate on a set of WSPs that encode information not only from the constituent whose label is the attachment site but also from the topic.

$\|\varphi\|_{(\pi_i, \tau)}$ = P(φ) [<w, g>: ∃h∃k∃w' (g = h ∪ k & <w, h> ∈ $\|\alpha_i\|_{\pi', \tau}$ & <w', k> ∈ $\|Top(\pi_i, \tau)\|_{\pi', \tau}$)}

In those cases where φ is the first sentence of a discourse, we attach to the null discourse context τ_0 = <{π_0: σ_0}, ∅, π_0, ∅, ∅> (σ_0 is defined in 3.1). In this case, $\|\varphi\|_{(\pi_0 \tau_0)}$ = [P(φ)]σ_0 = σ_φ.

[16] Note that all the assignments in a state have the same domain.

To revise the model-theoretic context after adding the new information by means of some discourse link, the Update axioms of section 2.3 may be used. Here is the translation of (A4) into the model-theoretic setting. Define for a set of variables, f $\supseteq_Y$ g iff dom(g) - dom(f) ∪ Y. (o is state composition here.)

(A4') $s_{pred(\pi_1, \tau)} \models (\pi_3 \Downarrow pred(\pi_1, \tau)$ & π_3: $\wedge\alpha_3$) →
(i) Update(X_τ, Narration(π_1, π_2)) = X_τ[Update($s_{pred(\pi_1, \tau)}$, Narration(π_1, π_2))/$s_{pred(\pi_1, \tau)}$];
(ii) Update($s_{pred(\pi_1, \tau)}$, Narration(π_1, π_2)) = {<w, g>: ∃f∃x(<w, f> ∈ ($s_{pred(\pi_1, \tau)}$ o s_2)& g $\supseteq_{\{x\}}$ f & g(x) = {w: ∃h <w, h> ∈ s_2} & <w,g> ⊨ (Narration(π_1, π_2) & π_2: x)}

In updating we encode the new state as the value of a new variable. The other update axioms for the other discourse relations are analogous and allow us to define the update of model-theoretic counterparts to SDRT contexts.

I now define the CCP of a formula φ with label π_j, given an attachment point π_i and a DICE computation of R(π_i, π_j) in a context τ = <X, <, Current, e, f>.

$\Rightarrow_{\varphi, \pi_i}$ (τ) = (X', Current, <', f', e'), where
(i) X' = Update(X_τ, R(π_i, π_j))
(ii) <' = the well-founded pre-ordering on labels in X' subject to the constraints on < in the definition of model-theoretic contexts.
(iii) Current := π_j
(iv) e' = e ∪ {<π_j, e>} where e is the main eventuality of φ;
(v) f' = f ∪ {<π_j, t(φ)>}, where t(φ) is the embedding tree of φ.

Using $\Rightarrow_{\varphi, \pi_i}$, and generalizing over π, $\underline{\Rightarrow}_\varphi$ is a function over sets of discourse contexts. The set of accessible pragmatic-semantic model-theoretic contexts **S** contains the empty discourse context τ_0 and is closed under $\underline{\Rightarrow}_\varphi$.

This representation of CCP is simpler than that of SDRT, because we have abstracted away from the details of the representational language which don't matter for the update. A second disparity with SDRT update is that the updates are strictly monotonic. The assignments to earlier values are always carried along and so we predict that any propositional antecedent or earlier discourse referent will be available--contrary to fact. One of the functions of discourse structure as in the examples (3) and (4) is to make us "forget" about certain antecedents when they are no longer in an appropriate part of the discourse structure. Now they can, as (4) shows, be usually accessed by anaphoric, definite descriptions; so we need the information encoded in the update for definites. To make the model-theoretic CCP equivalent to that of SDRT, I require that the *translation of any pronoun* in a sentence into a formula φ *only use a variable that is in Dom(f)* for some <w, f> ∈ s, where s:π and π is either the attachment site or some state < π.

$\underline{\exists}(\varphi')$ is a partial function from SDRSs into SDRSs. It is partial because DICE is not guaranteed to yield a discourse relation R holding between π_φ and the chosen attachment site. Let us partialize it further so that there is a set of formulas Φ ⊥ characterized as absurd (i.e. those that are not satisfiable in at any world in any model) and that $\underline{\exists}(\varphi')$ returns a value only if the result is not in Φ ⊥.

Theorem: (i) Let φ and ψ be a finite sequence of formulas (φ_1, ...φ_n), and let φ' be a formula. (ii) Let σ_φ = {s: (...($\tau_0 \underline{\Rightarrow} \varphi_1$) $\underline{\Rightarrow} \varphi_2 \underline{\Rightarrow}$ ... $\underline{\Rightarrow} \varphi_n$)} ∈ **S** and define
(iii) $\varphi \underline{\Leftrightarrow} \psi$ iff ∀φ' ∈ Φ ($\varphi \underline{\exists}(\varphi') \in \Phi \perp$ iff $\psi \underline{\exists}(\varphi') \in \Phi \perp$). (iv) suppose DICE yields the same discourse relations for $\underline{\exists}$ as it does for $\underline{\Rightarrow}$. Then:
$\varphi \underline{\Leftrightarrow} \psi$ iff $\sigma_\varphi = \sigma_\psi$.

The proof uses the same conversion from trace equivalence to bisimulation appealed to in Fernando's theorem and found in Park (1981), and it exploits fact that we can encode an SDRS by means of a characteristic first order formula.

The complexity of the characteristic formulas encoding SDRSs is perhaps r.e., but its complexity depends upon the exact nature of satisfaction conditions for discourse relations, which still require further investigation. Nevertheless, the model-theoretic notion of context can model SDRT's analysis of VP ellipsis, simple propositional anaphora and temporal anaphora, as well as lots of cases of disambiguation. We have shown that SDRSs are equivalent to the model-theoretic notions of discourse context and CCP. Thus, SDRT does not make any representational commitments over and above those given in the model-theoretic version. Of course these are still considerable.

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Modal Correspondence for Models

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Abstract

This paper considers the correspondence theory from modal logic and obtains correspondence results for models as opposed to frames. The key ideas are to consider infinitary modal logic, to phrase correspondence results in terms of substitution instances of a given modal formula, and to identify bisimilar model-world pairs.

1 Introduction

Let χ be a formula of standard modal logic. A *first-order correspondent for* χ is a first-order sentence c_χ for which the following *frame correspondence* holds:

($\mathbf{F}[\chi, c_\chi]$) For all frames F, the following are equivalent:

1. $F \models \chi$
2. $F \models c_\chi$

We hasten to point out that the symbol $\models$ is being overloaded here. In (1), we mean that for all valuations V based on the frame F and all worlds $w \in F$, $(F, V, w) \models \chi$ in the sense of modal logic. In (2), we mean that F satisfies c_χ in the sense of first-order logic. We'll continue this overloading throughout the paper, counting on context to make clear which use we have in mind.

The main examples of pairs (χ, c_χ) such that ($\mathrm{F}[\chi, c_\chi]$) holds come from the Sahlqvist Theorem and its generalizations and refinements. As it happens, we do not need any of the details of this work, so we only refer the interested reader to the original paper (Sahlqvist 1975) and to two other works (van Benthem 1984, 1985) where it is discussed.

One example of this correspondence, the one we'll use as an illustration later, is the following:

$$\begin{array}{lcl} \rho & = & \Box((p \wedge \Box p) \rightarrow q) \ \vee \ \Box(\Box q \rightarrow p) \\ c_\rho & = & (\forall x)(\forall y)(\forall z)(R(x,y) \wedge R(x,z) \rightarrow (R(y,z) \vee R(z,y) \vee z = y)) \end{array}$$

A frame satisfying c_ρ, is said to be *locally connected*. A proof that the equivalence ($\mathrm{F}[\rho, c_\rho]$) holds can be found in (van Benthem 1985), page 47.

The point of this paper is to discuss analogs of the frame correspondence results for *models*. One's first guess at such a result might be the following:

($\mathrm{M}[\chi, c_\chi]$) Assume ($\mathrm{F}[\chi, c_\chi]$). For all models M, the following are equivalent:

1. $M \models \chi$
2. $M \models c_\chi$

However, $(\mathrm{M}[\chi, c_\chi])$ is false for almost all χ. The problem is that while (2) implies (1), there are typically too many models for (1) to imply (2). For a concrete example, we return to local connectedness. Consider a model M with three points, u, v, and w. The relation R is defined by $R(u,v)$ and $R(u,w)$ (and only these). This frame is clearly not locally connected. But we can turn it into a model of the modal sentence ρ. Just let V be defined by $V_p = V_q = \emptyset$. It is not hard to check that M does satisfy the modal formula ρ at each point. So the correspondence results for frames do not go through if one simply substitutes "models" for "frames."

There are several possible diagnoses for this failure. One might be that in the case of the model correspondence results, one must alter either χ or c_χ. We do not believe that this would work. In fact, if we take the example of c_χ from above, then there are no formulas χ' giving an equivalence as in $(\mathrm{M}[\chi, c_\chi])$; we present related ideas in Section 7.

A second intuition behind the failure of $(\mathrm{M}[\chi, c_\chi])$ has to do with the meaning and treatment of the atomic propositions p and q as they are used in the frame results. If $F \models \varphi$, then not only will F satisfy φ at all worlds and under all valuations, but F will also satisfy *every substitution instance of* φ. We exploit this kind of satisfaction in this paper. Indeed, we feel that part of the failure of $(\mathrm{M}[\chi, c_\chi])$ is due to the fact that (1) is interpreted as usual in modal logic while it seems equally reasonable to require the satisfaction of all substitution instances of φ.

Finally, it is well-known that the satisfaction of modal formulas is preserved under bisimulations: if $M \models \chi$ and M is bisimilar to M' then $M' \models \chi$. However, first-order satisfaction lacks this important property. Thus to get a correspondence between (1) and (2) we should not insist that $M \models c_\chi$ but only that some model M' bisimilar to M satisfies c_χ.

To summarize our strategy for getting model correspondence results: on one hand we consider substitution instances of the modal formulas, and on the other we allow the first-order correspondent to hold not on the original model but on some model bisimilar to it. This leads us to two results, one which applies to all models and one which applies only to image finite models.

Theorem 1.1 *If a modal formula χ has a first-order frame correspondent c_χ, then for all* models *M, the following are equivalent:*

1. *M satisfies all substitution instances of χ in infinitary modal logic.*
2. $\mathrm{Fr}(\mathrm{StrEx}(M)) \models c_\chi$.

In part (2), $\mathrm{Fr}(\mathrm{StrEx}(M))$ refers to the frame underlying the strongly extensional quotient of M. (All terms are defined in Section 2.) In (1), we make use of infinitary modal logic. Our second theorem, by contrast, applied to the standard modal logic (which we call the *finitary part* of the full infinitary logic):

Theorem 1.2 *If a finitary modal formula χ has a first-order frame correspondent c_χ, then for all* image finite models *M, the following are equivalent:*

1. *M satisfies all substitution instances of χ in standard modal logic.*
2. $\mathrm{Fr}(\mathrm{StrEx}(M)) \models c_\chi$.

Our main purpose in this paper is to present these two and some related results. Our inspiration for these results grew out of looking at modal logic in the context of non-wellfounded sets, as presented in Barwise-Moss (1996). We have presented them here in more familiar terms.

2 Background

We collect here a few points of background having to do with frames and models, bisimulation, the strongly extensional quotient, and substitution.

2.1 Infinitary Modal Logic, Frames, and Models

Fix a set AtProp of *atomic propositions*. (It will be important that AtProp is a set and not a proper class, but you should ignore this for now.) We use letters like p and q to range over atomic propositions. The infinitary modal language $\mathcal{L}$ is the smallest collection such that

1. $\mathsf{AtProp} \subseteq \mathcal{L}$
2. The constant T (for *true*) belongs to $\mathcal{L}$.
3. If $\varphi \in \mathcal{L}$, then $\neg\varphi \in \mathcal{L}$.
4. If Φ is any subset of $\mathcal{L}$, then $\bigwedge\Phi \in \mathcal{L}$.

A *frame* is a pair $F = (W, R)$ consisting of a set W of *worlds* together with a binary relation R on W. R is called the *accessibility relation* of F. A *valuation* on F is a map $V : \mathsf{AtProp} \to \mathcal{P}(W)$, where $\mathcal{P}(W)$ is the power set of W.

A *model* is a pair $M = (F, V)$ consisting of a frame F and a valuation V on F. When we have to display everything, we write (W, R, V) rather than $((W, R), V)$. If M is a model, then $\mathrm{Fr}(M)$ is the *frame underlying* M.

A *model-world pair* is a pair (M, w) such that $w \in M$. (We occasionally use M for the name of its set of worlds, rather than W.) We define the satisfaction relation $(M, w) \models \varphi$ by recursion on $\varphi \in \mathcal{L}$ as follows:

$$
\begin{array}{ll}
(M,w) \models p & \text{if } w \in V(p) \\
(M,w) \models \mathsf{T} & \text{for all } (M,w) \\
(M,w) \models \neg\varphi & \text{if } (M,w) \not\models \varphi \\
(M,w) \models \Diamond\varphi & \text{if for some } v \in W \text{ such that } R(w,v),\ (M,v) \models \varphi \\
(M,w) \models \bigwedge\Phi & \text{if } (M,w) \models \varphi \text{ for all } \varphi \in \Phi
\end{array}
$$

The finitary boolean connective $\wedge$ is a special case of $\bigwedge$: we write $\varphi \wedge \psi$ for $\bigwedge\{\varphi, \psi\}$. We also introduce F, $\bigvee$, $\vee$, $\rightarrow$, and $\leftrightarrow$ as abbreviations as usual.

2.2 Bisimulation

Let $M = (W, R, V)$ and $M' = (W', R', V')$ be models. A *bisimulation between M and M'* is a relation $\equiv$ from W to W' (that is a subset of $W \times W'$) with the following properties:

- If $w \equiv w'$, then for all $p \in \mathsf{AtProp}$, $w \in V(p)$ iff $w' \in V'(p)$.
- If $w \equiv w'$ and $w\,R\,v$, then there is some v' such that $w'\,R'\,v'$ and $v \equiv v'$.
- If $w \equiv w'$ and $w'\,R'\,v'$, then there is some v such that $w\,R\,v$ and $v \equiv v'$.

Further, we write $(M,w) \equiv (M',w')$ if there is some bisimulation $\equiv_0$ between M and M' such that $w \equiv_0 w'$. It is not hard to check that $\equiv$ itself is a bisimulation, and indeed it is the largest bisimulation between M and M'. When $M = M'$, this relation $\equiv$ is an equivalence relation. (However, it is not true that every bisimulation is an equivalence relation.)

The last fact we need is that if $(M,w) \equiv (M',w')$, then for all modal formulas φ, finitary or infinitary, $(M,w) \models \varphi$ iff $(M',w') \models \varphi$. This fact is usually proved for finitary formulas by induction, but the same proof goes through for infinitary formulas; the limit steps of the induction are trivial.

One important fact is that the converse to this is false; it is possible to have pairs (M,w) and (M',w') which satisfy the same *finitary* formulas but which are not bisimilar. Nevertheless, there is a reasonable hypothesis on M and N which, together with the assumption that (M,w) and (M',w') satisfy the same finitary formulas, allows us to conclude that $(M,w) \equiv (M',w')$. This is that M and M' are *image finite*. This means that each world in these models has only finitely many successors in the accessibility relation R. Other names for this concept are *locally finite* and *finitely branching*.

It is also the case that if (M,w) and (M',w') satisfy all of the same *infinitary* formulas, then they are bisimilar. As it happens, there is a stronger result: there is a single infinitary formula $\theta(M,w)$ so that for all (M',w'), $(M',w') \models \theta(M,w)$ iff $(M,w) \equiv (M',w')$. This is not at all immediate. We discuss this result and the work behind it in Section 3 below.

2.3 The Strongly Extensional Quotient

A model M is *strongly extensional* if $(M,u) \equiv (M,v)$ implies $u = v$, for all worlds u and v of M. Every model is bisimilar to a strongly extensional model, and this is unique up to isomorphism. **Definition** For every model M, we

define a model $\mathrm{StrEx}(M)$, the *strongly extensional quotient* of M, and its underlying frame $\mathrm{Fr}(\mathrm{StrEx}(M))$, as follows. Their common set of worlds is the set of equivalence classes of M under the bisimulation relation. Writing $\bar{v}$ for the equivalence class of the world v, we define an accessibility relation $\bar{R}$ on these equivalence classes by:

$$\bar{R}(\bar{v},\bar{w}) \quad \text{iff} \quad R(v,x) \text{ for some } x \in M \text{ such that } (M,x) \equiv (M,w).$$

This is easily seen to be a well-defined relation on $\mathrm{StrEx}(M)$ and gives us our desired frame. This frame is made into a model by taking

$$V(p) \quad = \quad \{\bar{w} \mid w \in V_M(p)\}.$$

We sometimes denote the set of world also by $\mathrm{StrEx}(M)$ also.

Consider, for example, the three-point model M from Section 1. Notice that $(M,v) \equiv (M,w) \not\equiv (M,u)$, so $\mathrm{StrEx}(M)$ will be a two-point model with one edge and an empty valuation. In contrast to the frame of M, the frame of $\mathrm{StrEx}(M)$ *is* locally connected.

Proposition 2.1 *Let $M = (W,R,V)$ be a model, and let w and v belong to W.*

1. *$(M,w) \equiv \mathrm{StrEx}(M,\bar{w})$. Hence $\mathrm{StrEx}(M)$ and (M,w) satisfy the same modal sentences.*
2. *If $(M,w) \equiv \mathrm{StrEx}(M,\bar{v})$, then $\bar{w} = \bar{v}$.*
3. *$\mathrm{StrEx}(M)$ is strongly extensional.*

Remarks on hypersets. While not needed for the rest of this paper, we hint at the relationship between strongly extensional models and sets. Call a frame $F = (W,R)$ strongly extensional if every model on F is strongly extensional. It is easy to see that if W is any wellfounded, transitive set, then the frame $(W,\in^{-1})$ is strongly extensional. Thus, working in ZFC, every transitive set W gives rise to a strongly extensional frame. The theory ZFA of hypersets due to Azcel, and independently and somewhat earlier, to Honsel and Forti, implies that every strongly extensional frame is isomorphic to a frame of the form $(W,\in^{-1})$ for some transitive set W. In general, of course, such sets will not be wellfounded. For example, if $F = (W,R)$ has just one world w and wRw then the corresponding set, known as Ω, satisfies $\Omega = \{\Omega\}$ and is certainly not wellfounded. The work on modal logic reported here began in our book Barwise-Moss (1966), where we study this universe of hypersets.

2.4 Facts on Substitution

Definition A *substitution* is a map $s : \mathsf{AtProp} \to \mathsf{Prop}$. We write the value of s at p either by $s(p)$ or as s_p. We say that s is *finitary* if each s_p is a finitary sentence. Each s extends homomorphically to a map $[s] : \mathsf{Prop} \to \mathsf{Prop}$, and we write $\varphi[s]$ for the action of this map on the formula φ.

Our first goal in this direction is a Substitution Lemma.

Definition Let V be a valuation on F, and let s be a substitution. Define a valuation V^s on F by

$$V^s(p) \quad = \quad \{w \in W : (F,V,w) \models s(p)\}.$$

Lemma 2.2 (Substitution Lemma) *For all F, V, w and s, and all infinitary formulas φ,*

$$(F,V,w) \models \varphi[s] \quad \textit{iff} \quad (F,V^s,w) \models \varphi.$$

Proof By induction on φ. For atomic p, the result is immediate from the definition. The induction steps are trivial. ⊣

Definition We write $M \models_\infty \varphi$ if for all substitutions s, $M \models \varphi[s]$. Further, we write $M \models_\omega \varphi$ if for all finitary s, $M \models \varphi[s]$.

Proposition 2.3 *If $F \models \varphi$, then for all valuations V, $(F, V) \models_\infty \varphi$.*

Proof Fix V and a substitution s. Consider the valuation V^s. Our assumption tells us that $(F, V^s) \models \varphi$. By Lemma 2.2, $(F, V) \models \varphi[s]$. ⊣

The converse of Proposition 2.3 is false in general, but we'll see that it holds when φ is especially simple in Proposition 8.1.

3 Modal Characterization Theorem

In this section, we present a basic result for infinitary modal logic.

Definition Let (M, w) be a model-world pair. An infinitary modal formula θ *characterizes* (M, w) if for all model-world pairs (N, v),

$$(N, v) \models \theta \quad \text{iff} \quad (M, w) \equiv (N, v).$$

The main result of this section is that every (M, w) has a characterizing formula $\theta(M, w)$; of course, $\theta(M, w)$ is unique up to semantic equivalence.

Let (M, w) be a model-world pair. We define a formula $\theta^\alpha(M, w)$ by recursion on the ordinal number α as follows:

$$\begin{array}{lcl} \theta^0(M,w) & = & \bigwedge_{M,w\models p} p \ \wedge \ \bigwedge_{M,w\models\neg p} \neg p \\ \theta^{\alpha+1}(M,w) & = & \bigwedge_{wRv} \Diamond\theta^\alpha(M,v) \ \wedge \ \Box \bigvee_{wRv} \theta^\alpha(M,v) \\ \theta^\lambda(M,w) & = & \bigwedge_{\beta<\lambda} \theta^\beta(M,w) \end{array}$$

Proposition 3.1 *For all (M, w) and (N, v), and all α, either*
1. *$\theta^\alpha(M, w) = \theta^\alpha(N, v)$, or*
2. *$\models \theta^\alpha(M, w) \leftrightarrow \neg\theta^\alpha(N, v)$.*

We need to define a cardinal that measures the "productivity" of a world in a model. Given a model M and a world w of M, the *children* of w are the worlds $v \in M$ such that wRv. The set of *descendants* of w is the smallest set X containing w and such that if $v \in X$ then all of v's children are in X. Define $\kappa(M, w)$ to be the smallest infinite regular cardinal such that all descendants of $\bar{w}$ in $\mathrm{StrEx}(M)$ have $< \kappa$ children.

Theorem 3.2 *For all (M, w), $\theta^{\kappa(M,w)}$ characterizes (M, w) up to bisimulation. That is,*

$$(N, v) \models \theta^{\kappa(M,w)} \quad \text{iff} \quad (M, w) \equiv (N, v).$$

We are especially interested in the case when M is image finite. In this case, $\kappa(M, w) = \omega$ for all $w \in M$. So (M, w) is characterized by the conjunction of the sentences $\theta^n(M, w)$. If the set of atomic propositions is finite, each of

these is a finitary sentence. This tighter result holds not only when M is image finite, but also when $\mathrm{StrEx}(M)$ is image finite.

We shall not give the proofs of the results of this section here, since they are merely restatements of results from Chapter 11 of our book (Barwise and Moss 1996). That chapter has a number of results on modal characterization, including different proofs of some of the results of this paper. It also discusses the relation of set theory and modal logic at length.

In the remainder of this section, we discuss some corollaries of Theorem 3.2. One of the most basic is the following result:

Corollary 3.3 *The following are equivalent for model-world pairs (M, w) and (M', w'):*
1. *$(M, w) \equiv (M', w')$.*
2. *For all infinitary φ, $(M, w) \models \varphi$ iff $(M', w') \models \varphi$.*

Characterization by Finitary Formulas Although we shall not need the following result, we present it for completeness.

Proposition 3.4 *Assume that* AtProp *is finite. Then (M, w) is characterizable up to bisimulation by a finitary modal formula iff the accessibility relation in $\mathrm{StrEx}(M)$ is finite and wellfounded below $\bar{w}$.*

Proof First, we show by induction on the wellfounded part of a relation R that if R is finite and wellfounded below w, then (M, w) is characterizable by a finitary modal formula. If w has no successors, then (M, w) is characterized by

$$\bigwedge\{p \mid w \in V(p)\} \ \wedge \ \bigwedge\{\neg p \mid w \notin V(p)\} \ \wedge \ \neg\Diamond\mathsf{T}.$$

Since we are assuming that the language has only finitely many atomic propositions, this sentence is finitary. It clearly characterizes (M, w). Assuming that we have finitary sentences $\theta(M, v)$ characterizing the successors of w, we get $\theta(M, w)$ by

$$\theta(M,w) \quad = \quad \bigwedge_{wRv} \Diamond\theta(M,v) \ \wedge \ \Box \bigvee_{wRv} \theta(M,v).$$

So if (M, w) is any model-world pair and in $(\mathrm{StrEx}(M), \bar{w})$ is in the wellfounded part of $\bar{R}$, then (M, w) can be characterized by a finitary formula.

For the converse, suppose that $(\mathrm{StrEx}(M), \bar{w})$ is characterized by a modal formula ψ of depth n. It is easy to check that for every model-world pair (N, v) and each natural number n, there is a wellfounded strongly extensional model N' and world $v' \in N'$ so that (N, v) and (N', v') satisfy the same modal formulas of depth $\leq n$. Applying this to $(\mathrm{StrEx}(M), \bar{w})$, we get a wellfounded (N, v) which also satisfies ψ. But this means that $(\mathrm{StrEx}(M), \bar{w}) \equiv (N, v)$. It is not hard to show that if (N, v) is wellfounded and strongly extensional, and if $(N, v) \equiv (N', v')$, then (N, v) is isomorphic to (N', v'); this fact is proved by induction on the height of v in N. However, this is a contradiction, since the subframe generated by $\bar{w}$ is not wellfounded while the subframe generated by v is wellfounded. Therefore, $(\mathrm{StrEx}(M), \bar{w})$ is in the wellfounded part of $\bar{R}$. ⊣

Classes of Model-World Pairs Closed Under Bisimulation A result of van Benthem's states that a *first-order definable* class C is closed under bisimulation iff there is a single finitary modal formula characterizing it (see van Benthem 1985). Here is a version of this for infinitary logic.

Proposition 3.5 *Let K be any class of model-world pairs. Then the following are equivalent:*

1. *K is closed under bisimulation.*
2. *There is a class C of infinitary modal formulas such that for all (M,w), $(M,w) \in K$ iff $(M,w) \models C$.*

Proof (2) $\Rightarrow$ (1) is immediate from the fact that bisimilar pairs satisfy the same infinitary formulas. In the other direction, for each model N, let ψ_N be:

$$\bigvee_{v \in N} \theta(N,v) \quad \rightarrow \quad \bigvee_{\substack{v \in N \\ (N,v) \in K}} \theta(N,v).$$

We remark that in each ψ_N, the disjunction in the consequent really is a (set-sized) modal formula: the worlds of N form a set and v ranges over this set. It is easy to check that $(N,v) \models \psi_N$ iff $(N,v) \in K$. Let C be the class of all formulas ψ_N, as N runs through all models. We then check that $(N,v) \models C$ iff $(N,v) \in K$. $\dashv$

(We do not know a condition on K that would allow us to change (2) to require that C be a set.)

On the ordering of infinitary modal formulas The infinitary modal formulas, like the formulas in any semantically-presented logical system, have a natural order $\leq$ defined on them by

$$\varphi \leq \psi \quad \text{iff} \quad (M,w) \models \varphi \rightarrow \psi \text{ for all model-world pairs } (M,w).$$

This order is a preorder, and we usually identify equivalent formulas to get a partial order. The infinitary boolean operators imply that this partial order is a complete lattice. We write L for this lattice.

Proposition 3.6 *The maximal elements of $L - \{\mathsf{F}\}$ are exactly the characterizing formulas $\theta(M,w)$.*

Proof This is easy to check, since a formula φ is maximal in $L - \{\mathsf{F}\}$ iff φ is satisfied at exactly one model-world bisimulation class. This last condition means that φ is (equivalent) to a characterizer. $\dashv$

4 An Example: Connectedness

Recall the connectedness formula ρ and its first-order correspondent c_ρ from the introduction. In this section, we prove a correspondence result for these

formulas in detail. The results in this section are special cases of Theorem 1.1 and 1.2 and the fact that ρ and c_ρ correspond for frames. Our hope is that the concrete reasoning in this section will make the generalizations easier to follow in Sections 5 and 6 below.

Let M be a model that satisfies all infinitary instances of

$$\Box((p \wedge \Box p) \rightarrow q) \ \vee \ \Box(\Box q \rightarrow p)$$

at each of its worlds w. We want to show that the frame of StrEx(M) is locally connected, that is, satisfies

$$(\forall x)(\forall y)(\forall z)(R(x,y) \wedge R(x,z) \rightarrow (R(y,z) \vee R(z,y) \vee z = y)).$$

Fix a world $\bar{w}$ of StrEx(M), and fix u and v such that $\bar{R}(\bar{w},\bar{u})$ and $\bar{R}(\bar{w},\bar{v})$. We want to see that one of the following alternatives holds: $\bar{R}(\bar{u},\bar{v})$, $\bar{R}(\bar{v},\bar{u})$, or $\bar{u} = \bar{v}$. Suppose towards a contradiction that none of these hold. Let s be the following substitution:

$$\begin{array}{lcll} s_p & = & \theta(M,v) \ \vee \ \bigvee\{\theta(M,v') \mid R(v,v')\} & (1) \\ s_q & = & \bigvee\{\theta(M,u') \mid R(u,u')\} & \end{array}$$

For this s, we have two cases: either $(M,w) \models \Box((s_p \wedge \Box s_p) \rightarrow s_q)$ or $(M,w) \models \Box(\Box s_q \rightarrow s_p)$.

In the first case, $(M,v) \models (s_p \wedge \Box s_p) \rightarrow s_q)$. Since (M,v) does satisfy $s_p \wedge \Box s_p$, we see that $(M,v) \models s_q$. This means that for some u' such that $R(u,u')$, $(M,v) \equiv (M,u')$. Therefore, in StrEx(M), $\bar{R}(\bar{u},\bar{v})$. In the second case, $(M,u) \models s_q \rightarrow s_p$. But $(M,u) \models \Box s_q$, and so we have $(M,u) \models s_p$. This means that either $(M,u) \equiv (M,v)$, or for some successor v' of v, $(M,u) \equiv (M,v')$. These last conditions translate to: $\bar{u} = \bar{v}$ or $\bar{R}(\bar{v},\bar{u})$. Either way, we have verified that the first-order condition c_ρ holds in Fr(StrEx(M)).

Now we want to go the other way: suppose that Fr(StrEx(M)) is locally connected. We can now use the known frame correspondence result to see that at all valuations V, the model (Fr(StrEx(M)), V) satisfies ρ and at all worlds. In particular, so does the model StrEx(M). Since M and StrEx(M) are bisimilar, $M \models \rho$.

To continue this example, let us examine the result for image finite models M. In this case, we assume that M satisfies all finitary instances of ρ, and again we want to prove that M satisfies the first-order condition. At first glance, we seem to be out of luck because the substitution s in (1) is ineluctably infinitary. However, we can get around this problem.

Let the successors of v be $v_1, \ldots, v_j$, and let the successors of u be $u_1, \ldots, u_k$. For each n, let s^n be the following substitution:

$$\begin{array}{lcll} s^n_p & = & \theta^n(M,v) \vee \theta^n(M,v_1) \vee \cdots \vee \theta^n(M,v_j) & (2) \\ s^n_q & = & \theta^n(M,u_1) \vee \theta^n(M,u_2) \vee \cdots \vee \theta^n(M,u_k) & \end{array}$$

Since the set of atomic sentences is finite (there are just two) each s^n is a finitary substitution.

For each n, we have two cases:

1. $(M,w) \models \Box((s_p^n \wedge \Box s_p^n) \rightarrow s_q^n)$
2. $(M,w) \models \Box(\Box s_q^n \rightarrow s_p^n)$

Since there are infinitely many n and two choices possible for each, we see that one of the two alternatives must occur infinitely often. We'll give the argument in the second case, the first being similar (and in fact a bit easier).

So assume that for infinitely many n, $(M,w) \models \Box(\Box s_q^n \rightarrow s_p^n)$. Applying this to u, and using the fact that $(M,u) \models s_q^n$ for all n, we see that $(M,u) \models s_p^n$ for infinitely many n. ¿From this, we have that for each n either $(M,u) \models \theta^n(M,v)$ for infinitely many n, or there is a successor v_k of v such that $(M,u) \models \theta^n(M,v_k)$. If $(M,u) \models \theta^n(M,v)$ for infinitely many n, then $(M,u) \models \theta(M,v)$; this is because $\theta(M,v)$ is the conjunction of its finite approximations, and each approximation is stronger than its predecessors. So $(M,u) \equiv (M,v)$ in this case.

If for infinitely many n there is some v_k as above, then as v has only finitely many successors, there must be a fixed v_k so that for infinitely many n, $(M,u) \models \theta^n(M,v_k)$. And as we have seen, this implies that $(M,u) \equiv (M,v_k)$.

This illustrates the direction (1) $\Rightarrow$ (2) in Theorem 1.2. Going the other way, assume that $\mathrm{StrEx}(M)$ is locally connected. Then as we have seen, M satisfies all infinitary substitution instances of ρ. A fortiori, $M \models_\omega \rho$.

5 Proof of the First Correspondence Theorem

In this section, we prove Theorem 1.1. The proof is a consequence of two results having to do with the relation $\models_\infty$ (see Section 2.4) and the strongly extensional quotient. The first of these uses the characterization results of Section 3.

Lemma 5.1 *Suppose that $M \models_\infty \varphi$. Then $\mathrm{Fr}(\mathrm{StrEx}(M)) \models \varphi$ as well.*

Proof Let V be any valuation on $\mathrm{Fr}(\mathrm{StrEx}(M))$. Define the substitution s by

$$s(p) \quad = \quad \bigvee\{\theta(\mathrm{StrEx}(M),\bar{w}) \mid \bar{w} \in V(p)\}.$$

We need two claims at this point. First, for all v and w in M, if $v \equiv w$, then $(M,v) \models \psi[s]$ iff $(M,w) \models \psi[s]$. This is by induction on ψ. For the atomic case, the important subsidiary fact is that $(M,v) \models \theta(\mathrm{StrEx}(M),\bar{w})$ iff $v \equiv w$. The boolean steps are trivial, and the $\Diamond$ step comes from the definition of bisimulation.

The second thing we need is that

$$(M,w) \models \varphi[s] \quad \text{iff} \quad (\mathrm{Fr}(\mathrm{StrEx}(M)),V,\bar{w}) \models \varphi.$$

(This would establish the lemma, of course.) The proof here is by induction on φ. The atomic case is by the definition of s. The induction step for $\Diamond$ is worth looking at: if $(M,w) \models \Diamond\varphi[s]$, then there is some v so that $R(w,v)$ and $(M,v) \models \varphi[s]$. By induction hypothesis, $(\mathrm{Fr}(\mathrm{StrEx}(M)),V,\bar{v}) \models \varphi$. But in $\mathrm{StrEx}(M)$, $R(\bar{w},\bar{v})$. So $(\mathrm{Fr}(\mathrm{StrEx}(M)),V,\bar{w}) \models \Diamond\varphi$.

Going the other way, suppose that $(\mathrm{Fr}(\mathrm{StrEx}(M)),V,\bar{w}) \models \Diamond\varphi$. Let $\bar{v}$ be such that $R(\bar{w},\bar{v})$ and $(\mathrm{Fr}(\mathrm{StrEx}(M)),V,\bar{v}) \models \varphi$. By induction hypothesis, $(M,v) \models \varphi[s]$. Now v is bisimilar to some child v' of w, and by our first claim above, $(M,v') \models \varphi[s]$. Hence $(M,w) \models \Diamond\varphi[s]$. $\dashv$

Lemma 5.2 *If $\mathrm{Fr}(\mathrm{StrEx}(M)) \models \varphi$, then $M \models_\infty \varphi$.*

Proof Let s be any substitution; we show that $M \models \varphi[s]$. Let V be the valuation defined on $\mathrm{StrEx}(M)$ by

$$V(p) \quad = \quad \{\bar{w} \mid (M,w) \models s(p)\}.$$

We claim that for all φ, and all worlds w,

$$(M,w) \models \varphi[s] \quad \text{iff} \quad (\mathrm{Fr}(\mathrm{StrEx}(M)),V,\bar{w}) \models \varphi.$$

The proof is pretty much as in Lemma 5.1. For atomic propositions, the result is immediate. The only thing to check is that if $(\mathrm{Fr}(\mathrm{StrEx}(M)),V,\bar{w}) \models \Diamond\varphi$, then $(M,w) \models \Diamond\varphi[s]$. To see this, suppose that $\bar{v}$ is such that $R(\bar{w},\bar{v})$ and $(\mathrm{Fr}(\mathrm{StrEx}(M)),V,\bar{v}) \models \varphi$. By induction hypothesis, $(M,v) \models \varphi[s]$. Then in M, w has a child v' such that $v' \equiv v$. Since bisimilar worlds satisfy the same infinitary formulas, $(M,v') \models \varphi[s]$. Hence $(M,w) \models \Diamond\varphi[s]$. $\dashv$

Now we prove Theorem 1.1. Suppose that χ has c_χ as a frame correspondent. By Lemmas 5.1 and 5.2, $M \models_\infty \chi$ iff $\mathrm{Fr}(\mathrm{StrEx}(M)) \models \chi$. By the definition of a frame correspondent, $\mathrm{Fr}(\mathrm{StrEx}(M)) \models \chi$ iff $\mathrm{Fr}(\mathrm{StrEx}(M)) \models c_\chi$. This completes the proof.

6 The Image Finite Case

We now prove the second main result of the paper, Theorem 1.2. We need a variation on Lemma 5.1, since the proof there will not go through.

Lemma 6.1 *Assume that the set of atomic sentences is finite. Let M be a model such that $\mathrm{StrEx}(M)$ is image finite, V be a valuation on $\mathrm{StrEx}(M)$ with the property that $V(p)$ is finite for all p, and s be the substitution*

$$s(p) \quad = \quad \bigvee\{\theta(\mathrm{StrEx}(M),\bar{w}) \mid \bar{w} \in V(p)\}.$$

and for each natural number n, let s^n be the substitution

$$s^n(p) \quad = \quad \bigvee\{\theta^n(\mathrm{StrEx}(M),\bar{w}) \mid w \in V(p)\}.$$

Then for all finitary formulas φ and all worlds w, the following are equivalent:

1. *$(M,w) \models \varphi[s]$.*
2. *For all but finitely many n, $(M,w) \models \varphi[s^n]$.*
3. *For infinitely many n, $(M,w) \models \varphi[s^n]$.*

Proof By induction on φ. The atomic case is most interesting. Clearly if $(M,w) \models p[s]$ then for all n, $(M,w) \models p[s^n]$. If $(M,w) \models \neg p[s]$, then for each $\bar{v} \in V(p)$ there is some n so that for all $m > n$, $(M,w) \models \neg\theta^m(\mathrm{StrEx}(M),\bar{v})$. Since $V(p)$ is finite, we see that for almost all n, $(M,w) \models \neg p[s^n]$.

The steps of the induction for negation are easy; it is for them that we need both (2) and (3). For conjunction, note that since φ is finitary, we have a finite conjunction. The rest follows easily.

The induction step for the operator $\Diamond$ is similar to the atomic case. $\dashv$

We now prove Theorem 1.2, which we restate for the reader's convenience.

Theorem 6.2 *If a finitary modal formula χ has a first-order frame correspondent c_χ, then for all* image finite models M, *the following are equivalent:*

1. $M \models_\omega \chi$
2. $\mathrm{Fr}(\mathrm{StrEx}(M)) \models c_\chi$.

Proof As before, (2) $\Rightarrow$ (1) is by Lemma 5.2. In the other direction, assume that $M \models_\omega \chi$. We show that $\mathrm{Fr}(\mathrm{StrEx}(M)) \models \chi$ as well. Suppose towards a contradiction that this is false. Then there is a valuation V and a world w such that $(\mathrm{Fr}(\mathrm{StrEx}(M)), V, w) \models \neg\chi$ with the additional property that each $V(p)$ is a finite set. (This is a general fact concerning image finite frames: if a sentence of depth k can be invalidated at a world w by some V, then it can be invalidated using some V such that each $V(p)$ is included in the (finite) set of worlds whose distance from w is at most k.) Now we use Lemma 6.1, with s and s^n as defined above. By our assumption that $(M, w) \models_\omega \chi$ we see that $(M, w) \models \chi[s^n]$ for all n. So $(M, w) \models \chi[s]$ as well. But now by the second claim in Lemma 5.1, $(\mathrm{Fr}(\mathrm{StrEx}(M)), V, \bar{w}) \models \chi$. This is a contradiction. $\dashv$

7 Remarks on the Model Correspondence Theorems

In this section, we want to present a few results which indicate that the model correspondence theorems cannot be easily improved. (This is not to say that there are no other possible model correspondence results. It just means that in our basic results it does not seem to be possible to narrow the gap between the two halves.)

All of our examples in this section are presented for the correspondence concerning local connectedness. Similar examples hold for most other instances of frame correspondence. (One source of exceptions is the case of modal formulas in which all atomic propositions either have all positive or all negative occurrences. We deal with this situation in Section 8 below.)

Why image finite models are used in Theorem 1.2 First, we give an example to show that the restriction to image finite models in Theorem 1.2 is necessary. Recall the sentence ρ from Section 1. We want a model M which satisfies all finitary instances of ρ without being locally connected.

We take

$$W \quad = \quad \{a, b, c\} \cup \{b_n \mid n \in N\} \cup \{c_n \mid n \in N\}.$$

On this, we put

- $R(a, b)$, $R(a, c)$, $R(b, b)$.
- For all n, $R(b, b_n)$ and $R(c, c_n)$.
- For all $n < m$, $R(b_n, b_m)$ and $R(c_n, c_m)$.

We'll work with a language with countably many atomic propositions, say p_n $(n \in \omega)$. The valuation V is given as follows:

$$V(p_n) \quad = \quad \{b\} \cup \{b_m \mid m \geq n\} \cup \{c\} \cup \{c_m \mid m \geq n\}.$$

Note that b and c differ on the formula $\Diamond \bigwedge_n p_n$. It follows that (M, a) is not bisimilar to any locally connected model. We claim, however, that M satisfies all finitary instances of ρ at all points. The only point worth considering is a. It is straightforward to show that b and c agree on all finitary modal formulas. Let s be a finitary substitution. Since the submodels generated by b and c are locally connected, (M, b) and (M, c) satisfy $\rho[s]$. And since (M, b) and (M, c) satisfy all of the same formulas, they indeed satisfy the same disjunct of $\rho[s]$. Suppose that they satisfy the second disjunct $\Box(\Box s_q \rightarrow s_p)$. Since $R(b, b)$, $(M, b) \models \Box s_q \rightarrow s_p$. Therefore, (M, c) also satisfies this formula. It follows that $(M, a) \models \rho[s]$.

Are all substitution instances necessary? Second we have a remark on Theorem 1.1. We show that there is no *set* S of substitutions such that the following are equivalent:

1. For all $s \in S$, $M \models \rho[s]$.
2. $\mathrm{Fr}(\mathrm{StrEx}(M)) \models c_\rho$.

To see this, fix a set S of substitutions. Let κ be an infinite regular cardinal such that all boolean combinations of all formulas in each s_p are of size $< \kappa$. We consider a language where $\mathsf{AtProp} = \{p_\alpha \mid \alpha < \kappa\}$. For this, we build two models M_1 and M_2. M_1 is the obvious generalization of the model M from above in this section. M_2 is the same as M_1 except that we do not include the edge $R(b, b)$. M_2 is bisimilar to a locally connected model, since $(M_2, b) \equiv (M_2, c)$. Thus, for all p (in S or otherwise), and all $w \in M_2$, $(M_2, w) \models \rho[s]$.

It is reasonably easy to see that at corresponding points, M_1 and M_1 and M_2 agree on all formulas of modal depth $< \kappa$. Therefore, since M_2 has property (1) above, so does M_1. But M_1 is strongly extensional but not locally connected, so (2) fails. Thus (1) and (2) are not equivalent.

On finitary substitution instances and Theorem 1.2 Finally, we have a parallel remark on Theorem 1.2. As it is formulated, part (1) seems to require a check of infinitely many substitution instances. We show here that this is necessary. In other words, we show that there is no single finitary φ so that for all image finite models M the following are equivalent:

1. $M \models \varphi$
2. $\mathrm{Fr}(\mathrm{StrEx}(M)) \models c_\rho$

Suppose towards a contradiction that φ exists. Let n be its modal depth. Consider the models M_1 and M_2 from the discussion above, with $\kappa = \omega$. Let N_1 be the submodel determined by

$$\{a, b, b_1, \ldots, b_n, c, c_1, \ldots, c_n\}.$$

Then N_1 and N_2 are image finite. As before, N_1 is not bisimilar to a locally connected model while N_2 is. And N_1 and N_2 agree on φ. This is a contradiction.

8 A Full Correspondence Result

Recall from Section 1 that the putative correspondence result (M[χ, c_χ]) is false for the case of local connectedness. (M[χ, c_χ]) also fails for most other correspondence results. However, there is one class of cases where (M[χ, c_χ]) does hold.

Proposition 8.1 *Let φ be a formula in which every atomic proposition p has only positive occurrences or only negative occurrences. Let V be any valuation on a frame F. If $(F, V) \models_\infty \varphi$, then $F \models \varphi$.*

Proof Let s be the following substitution: If p occurs only positively in φ, then $s(p) = \mathsf{F}$. If p occurs only negatively in φ, then $s(p) = \mathsf{T}$. So $(F, V) \models \varphi[s]$. By Lemma 2.2, $(F, V^s) \models \varphi$. Note also that $V^s(p) = \emptyset$ if p occurs positively in φ, and $V^s(p) = F$ if p occurs negatively.

Let W be any valuation; we show that $(F, W) \models \varphi$. If p occurs positively in φ, $V^s(p) \subseteq W(p)$. And if p occurs negatively, then $W(p) \subseteq V^s(p)$. It follows easily by induction on φ that $(F, W) \models \varphi$. ⊣

Theorem 8.2 *Let φ be a formula in which every atomic proposition p has only positive occurrences or only negative occurrences. There is a first-order sentence c_φ such that for all models M the following are equivalent:*

1. *$M \models_\infty \varphi$*
2. *$M \models c_\varphi$ in first-order logic.*

Proof Let $ST(\varphi)$ be the standard translation of φ. Take $ST(\varphi)$ and make the following replacements: If p occurs positively, replace each $P(x)$ by F; if p occurs negatively, replace each $P(x)$ by T. Then delete the second-order quantifiers. This defines c_φ.

Let F be the frame underlying M. By Proposition 8.1, (1) is equivalent to (1′): $F \models \varphi$. The standard frame correspondence result is that (1′) is equivalent to (2′): $F \models c_\varphi$. (In the notation from Section 1, this frame correspondence result is the equivalence (F[χ, c_χ]) for this sentence.) But also it is a general fact that if a first-order sentence only has R (and no unary relations $P(x)$) then truth in a model is the same as truth in the underlying frame. So (2′) is equivalent to (2). ⊣

9 A characterization of infinitary model logic

If we have a sentence φ of infinitary modal logic, then the class

$$Mod(\varphi) \quad = \quad \{(M, w) \mid (M, w) \models \varphi\}$$

is closed under bisimulation. The converse is false, however. Consider, for example, the class $\mathcal{K}$ of all models with frames of the form $(\kappa, <^{-1})$, where κ is an infinite cardinal, construed as an initial ordinal with ordering $<$. Since each

of these frames is strongly extensional, this class is closed under bisimulation. However, it is not the class of models of any infinitary sentences, as a simple downward Lowenheim-Skolem argument demonstrates.

The aim of this section is to show that Theorem 3.2, in combination with well-known facts about the familiar sequence of approximations $\equiv_\alpha$ to bisimulation, yields a simple characterization of the classes definable in infinitary modal logic. (The result of this section was independently observed by Alexandru Baltag.)

We recall the relations $\equiv_\alpha$ defined by induction on α as follows:

$(M, w) \equiv_0 (N, v)$ iff for all $p \in \mathsf{AtProp}$, $w \in V(p)$ iff $w' \in V'(p)$.
$(M, w) \equiv_{\alpha+1} (N, v)$ iff $(M, w) \equiv_\alpha (N, v)$,
$\forall$ child w_0 of w $\exists$ child v_0 of $v (M, w_0) \equiv_\alpha (N, v_0)$, &
$\forall$ child v_0 of v $\exists$ child w_0 of $w (M, w_0) \equiv_\alpha (N, v_0)$.
$(M, w) \equiv_\lambda (N, v)$ iff for all $\alpha < \lambda$, $(M, w) \equiv_\alpha (N, v)$.

We summarize some well-known and easily established facts about these relations.

Proposition 9.1

1. *Each $\equiv_\alpha$ is an equivalence relation on model-world pairs.*
2. *The equivalence relations get increasingly fine as α increases.*
3. *$(M, w) \equiv (N, v)$ iff for all ordinals α, $(M, w) \equiv_\alpha (N, v)$.*
4. *$(M, w) \equiv_\alpha (N, v)$ iff for all modal formulas φ of modal depth at most α, $(M, w) \models \varphi$ iff $(N, v) \models \varphi$.*

Let $\mathcal{K}$ be a class of model-world pairs. We say that $\mathcal{K}$ is *bounded by* α if $\mathcal{K}$ is closed under the equivalence relation $\equiv_\alpha$; that is, if for all $(M, w) \equiv_\alpha (N, v)$, if $(M, w) \in \mathcal{K}$ then $(N, v) \in \mathcal{K}$. $\mathcal{K}$ is *bounded* if it is bounded by some ordinal.

Every class of model-world pairs definable by some infinitary modal sentence is bounded (indeed, it is bounded by the modal depth of any sentence φ defining the class). The point of this section is to observe that the converse is also true.

Theorem 9.2 *A class $\mathcal{K}$ of model-world pairs is definable by an infinitary modal sentence iff $\mathcal{K}$ is bounded by some ordinal.*

Proof Suppose $\mathcal{K}$ is bounded by α. It is easy to see that the class of all formulas of the form $\theta^\alpha(M, w)$ is a set. Hence, so is

$$\Theta = \{\theta^\alpha(M, w) \mid (M, w) \in \mathcal{K}\}.$$

Let $\varphi = \bigvee \Theta$. Clearly every $(M, w) \in \mathcal{K}$ is a model of φ. We claim that the converse is true. For suppose $(N, v) \models \varphi$. Then there is an $(M, w) \in \mathcal{K}$ such that $(N, v) \models \theta^\alpha(M, w)$. But then, as is easily checked by induction on α, $(N, v) \equiv_\alpha (M, w)$. Hence $(N, v) \in \mathcal{K}$ since $\mathcal{K}$ is bounded by α. ⊣

We can look at this as a characterization theorem for infinitary modal logic. Any extension of infinitary modal logic that had a sentence not equivalent

to one in infinitary modal logic would define an unbounded class. Hence, infinitary modal logic is the largest logic such that every definable class is bounded.

Note, by contrast, that it is easy to obtain proper extensions of infinitary modal logic that have the same notion of equivalence. Let $\mathcal{K}$ be any unbounded class of model-world pairs closed under bisimulation. (We gave an example of such a class above.) Add a new atomic proposition $In(\mathcal{K}$ and closed under the same syntactic operations. For the semantics, define $(M, w) \models In(\mathcal{K})$ iff $(M, w) \in \mathcal{K}$. Since $\mathcal{K}$ is closed under bisimulation, it is clear that this new logic, while able to define classes not definable in infinitary modal logic ($\mathcal{K}$ for example), has the property that (M, w) and (N, v) are equivalent in this logic iff $(M, w) \equiv (N, v)$.

10 Is that all there is?

We have shown in this paper that every frame correspondence result $(\mathrm{F}[\chi, c_\chi])$ gives rise to a model correspondence result. The natural question at this point is whether *every* model correspondence result arises in this way. That is, let $(\mathrm{M}'[\chi, c_\chi])$ be given as follows:

$(\mathrm{M}'[\chi, c_\chi])$ For all *models* M, the following are equivalent:

1. $M \models_\infty \chi$
2. $\mathrm{Fr}(\mathrm{StrEx}(M)) \models c_\chi$

A very natural question at this point would be: does $(\mathrm{M}'[\chi, c_\chi])$ imply $(\mathrm{F}[\chi, c_\chi])$? Results on this have been obtained by Giovanna d'Agostino, and by Johan van Benthem.

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Local Satisfaction Preferred

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1 Introduction

Previously I have followed e.g. Karttunen, Heim and Zeevat in assuming that presuppositions must always be satisfied in their local context: Geurts [Geu95] has caricatured this type of model as offering *Local Satisfaction Guaranteed*. In this paper I will consider how the guarantee might be relaxed somewhat, so that whilst there remains a *preference* for presuppositions to be satisfied, there is no guarantee. I will firstly introduce a cut-down cancellation theory of presupposition, where presuppositions place defeasible constraints not on the local context, but on the global context. One could say that this was a model of *Global Satisfaction Preferred*. I will then show how the cancellation model can be altered so that presuppositions place defeasible constraints not on the global context, but on their local context of evaluation. This would presumably be a *Local Satisfaction Preferred* model of presupposition (*Local Satisfaction Guaranteed or Your Money Back*, perhaps).

One way to think about the purpose of this paper is as a way of combining two traditions in presupposition theory, cancellationist and dynamic. This combination is very much in the spirit of Soames' proposals [So82], but improves on his account in two respects: (1) unlike Soames' account, or for that matter any of the standard cancellationist accounts such as [Gaz79a, vdS88, Me92], the system allows a treatment of quantified open presuppositions; (2) common-sense reasoning is allowed to influence the behaviour of the system, and can, for instance, strengthen the notoriously weak presuppositions found in Soames' account and in others in the dynamic tradition [Kar74, Hei83a].

Readers will probably not be surprised that my real purpose in writing this paper is not to achieve historical synthesis, but to extend and defend the dynamic account of presupposition I have advocated elsewhere. This account consists of a dynamic semantics which (as well as providing the meaning of a fragment of English including quantifiers, pronouns and epistemic modalities) defines what it is for a local context to satisfy a presupposition, and also an account of global accommodation enabling an initially unsatisfied presupposition to be added to the global context of interpretation.

Other dynamic models, Heim's CCP account and van der Sandt's DRT-based theory [vdS92], allow for yet more forms of accommodation. Apart from global accommodation, there is also a possibility of presuppositions being added in local contexts involved in the interpretation of a sentence. If sentence S has the form "If A then B", and B carries a presupposition P, then there are three possibilities. (1) *Global accommodation.* If P itself is accommodated, we derive the same effect as if S was replaced by "P and if A then B". (2)

Local Accommodation, i.e. accommodation in the local context of the trigger itself. Local accommodation of P would give the same effect as replacing S by "If A then P and B". (3) *Intermediate accommodation*, in my terminology this is accommodation at any site in between the global and local context along the standardly stated path of anaphoric accessibility. In the case of S there is only one intermediate site, in the antecedent of the conditional, so that intermediate accommodation has an effect much like replacing S with "If P and A then B".

Elsewhere [Bea95, Bea94a, Bea94b] I have provided extensive arguments against intermediate accommodation. Or, more precisely, I have argued that presuppositions do not themselves trigger intermediate accommodation, although in certain discourse contexts the same effects will be achieved through other processes, such as domain restriction of quantifiers [vF94] and accommodation through modal subordination [Rob87]. To give the flavour of the argument, without rehearsing it completely, observe that the intermediate accommodation reading of E1, below, would correspond to E2.

E1 If a woman buys a truck, she always sells her cadillac.

E2 If a woman who owns a cadillac buys a truck, she always sells her cadillac.

The point is that E1 seems, in the absence of any special context, rather strange. It seems to be taken for granted that women who buy a truck will have a cadillac to sell. Yet E2 is clearly felicitous. If E1 had an intermediate accommodation reading as in E2, there would be no explanation for the infelicity. I conclude that E1 has no such intermediate accommodation reading.[1]

If these arguments against intermediate accommodation are accepted, there remain two forms of accommodation, global and local. As I have said, the model I have proposed previously describes global accommodation. In this paper, I discuss one approach to local accommodation, an approach which draws on a parallel first observed by Heim [Hei83a] between local accommodation and cancellation.

Describing a system allowing some form of local accommodation is easy. One can simply allow presuppositional constructions to be ambiguous between a presupposing reading and a non-presupposing reading where what would otherwise be presupposed is instead asserted. But, that would also not take us very far. What is more problematic, both empirically and technically, is specifying under what circumstances local accommodation occurs. Empirically, I will make a simple assumption also made by other authors, such as van der Sandt and Heim, an assumption which is implicit in cancellation models, although expressed differently: local accommodation occurs when global accommodation is ruled out. I will only allow global accommodation to be ruled out if it would cause inconsistency, that is, if globally accommodating would leave the comprehending agent with contradictory information. It may well be that the determination of the appropriate accommodation strategy is more complex, but it is not the aim of this paper to study such empirical issues in detail. Rather, I will concentrate on the technical problem.

What is the nature of the technical difficulty associated with describing a combined model of local and global accommodation? A principle goal of presupposition theory is to account for

[1] I do accept that in a context where a set of cadillac owning women was already salient, E1 might become markedly more felicitous, and be understood as restricted to such women much as in E2. In this case the effects of intermediate accommodation would be derived through a separate process of domain restriction. This is not to say that presuppositions play no role in domain restriction, but that presuppositions do not in and of themselves license such restriction.

inferences that we make when understanding a sentence containing presupposition triggers. To me, this means that we must ultimately aim to formally describe an inference relation saying which sentences of English follow from which other sentences in which contexts of utterance. In Heim's terms, allowing alternative forms of accommodation could mean that a single sentence has multiple context change potentials (CCPs). This means that an inference relation will in effect involve some form of quantification over alternative possible CCPs. Furthermore, we have a working assumption as to which CCPs are preferred, i.e. that those involving global accommodation are *ceteris paribus* preferred to those involving local accommodation. The definition of the inference relation must utilise whatever information is available, and so must be defined in terms of these preferences. The technical problem, then, is to describe an inference relation which not only accounts for the dynamic effects of different CCPs, but also accounts for preferences between alternative CCPs. The goal of this paper is to present a declarative description of such inference relations.[2]

The effects of local accommodation will be obtained by allowing presuppositions also to be asserted in their local context (an assumption first found in Gazdar's model) and by then stating a preference for the presupposition also to be satisfied in the incoming context. If the presupposition is not satisfied in the incoming context, assertion of the presupposition will produce the effect of local accommodation. If the presupposition is already satisfied, the assertion of the presupposition will have no further effect on the information state.[3] In the next section, I will define a static system, so as to concentrate on the nature of preferences rather than on the interaction between preferences and dynamic semantics. in the next three sections I will show how dynamic effects of various sorts can be obtained, showing in outline how it is possible to add (1) sensitivity to local context (2) a dynamic model of global accommodation, and (3) a complete CCP account of meaning extending to first order.

2 Global Satisfaction Preferred

A system will now be defined in which presuppositions are interpreted as providing global preferences, which is essentially the idea of cancellationist accounts. The formalisation is over

[2] Note that both van der Sandt [vdS92] and Zeevat [Zee92] have provided dynamic models in which the preferences for one form of accommodation over another are described. But the descriptions are given procedurally. A procedural or algorithmic statement of the theory can have its advantages: it can be compact, and it can make for straightforward computer implementation. But such a statement can also make formal study of the effects of the theory difficult. It is typically awkward to check general properties of an inference relation which is given partly algorithmically. It is presumably for this reason that van der Sandt and Geurts [SG91] have investigated how van der Sandt's theory may be declaratively restated. A major difference between van der Sandt and Geurts' approach on the one hand, and the system proposed in the current paper on the other, is that van der Sandt and Geurts are committed to a particular form of representation (i.e. DRSs), whereas the current proposal provides a more abstract description of information states.

[3] There are cases where presuppositions apparently do not hold in their local context (see [Kay92]), although this only occurs with a limited range of triggers. I will not deal with such cases in the current paper. More generally, we might like to vary exactly what was asserted according to what propositions hold in the incoming context. For instance we might like the assertion associated with "the handlebar" to vary as follows: if there is already a salient handlebar, there should be no assertion; if there is no handlebar but there is a salient bicycle then a handlebar marker should be introduced and linked to the marker for bicycle appropriately; and if there is neither bike nor handlebar then perhaps a handlebar marker should just be added. I see no reason why CCPs should not be defined in this way, and combined with the model given in this paper such that various of these options are preferred over others. But such a general account of local accommodation goes beyond what I will actually formalise here.

the language of propositional logic augmented with a unary *presupposition operator*, ∂ (as introduced in my Amsterdam Colloquium paper of 4 years ago).

Models are pairs $\langle W, F\rangle$, where W is a set of worlds and F is an interpretation function mapping atomic formulae onto sets of worlds. It may be assumed that $[\![.]\!]$ is an essentially classical valuation, mapping formulae onto the set of worlds where they would be classically true, and giving formulae $\partial\phi$ the same valuation as ϕ. An information state will just be a set of worlds, the update of an information state σ with a formula ϕ, written $\sigma + \phi$, will just be given by $\sigma \cap [\![\phi]\!]$, and σ will be said to satisfy ϕ, written $\sigma \models \phi$, if σ is a fixed point of ϕ, i.e. $\sigma + \phi = \sigma$[4]. If μ is a pair of states $\langle \sigma, \tau\rangle$, I will write $\mu + \phi$ to mean $\langle \sigma + \phi, \tau + \phi\rangle$.

The preferences of a formula ϕ, written $\uparrow \phi$, will be defined in terms of an information ordering, which, as before, is a set of pairs of states (epistemic alternatives). Definition D1 below specifies the set of *good orderings* (GO) relative to some model. Firstly, the minimally constrained ordering $\odot$, which is the set of pairs of any two non-absurd states, is a good ordering (the set of non-absurd states defined over a set of worlds W being written W^*). Secondly, an ordering $\omega(\phi)$, which is one where there is a preference for states where ϕ holds, is a good ordering. For a pair $\langle \sigma, \tau\rangle$ to be in $\omega(\phi)$, it must be the case that if τ satisfies ϕ, then σ does too. Thirdly, the intersection of any two good orderings is also a good ordering. I will not explore the algebraic structure of the Good Orderings here.

Definition D1 **(Good Orderings)** *The set of good orderings is the smallest set GO such that (1) $\odot$ is in GO, (2) for any ϕ, $\omega(\phi)$ is in GO, and (3) for any O and O' in GO, $O \cap O'$ is in GO, where $\odot$ and $\omega(\phi)$ are defined by:*

$$\begin{aligned} W^* &= \wp(W)\backslash\emptyset \\ \odot &= W^* \times W^* \\ \omega(\phi) &= \{\langle \sigma, \tau\rangle \in \odot \mid \tau \models \phi \rightarrow \sigma \models \phi\} \end{aligned}$$

It is natural to write $\sigma \geq_O \tau$ for $\langle \sigma, \tau\rangle \in O$, and $\sigma > \tau$ to mean that $\sigma \geq_O \tau$ but $\tau \not\geq_O \sigma$. Since any formula which is satisfied by the minimally informative state W is a tautology, and is thus also satisfied by every other state, it follows that every non-absurd state is at least as preferable as W in any ordering $\omega(\phi)$. Indeed, this property holds for any good ordering O: $\forall \sigma \in W^*, \sigma \geq_O W$.

For each formula ϕ an associated set of preferences $\uparrow \phi$ will now be recursively stipulated. Here the only clause which actually introduces preferences is that for presuppositional formulae, a formula $\partial\phi$ setting up a preference for states where ϕ holds. Atomic formulae carry the trivial preference ordering $\odot$, which means that no state is strictly preferred to any other. The negation of a formula carries the same preferences as its argument, and the conjunction of two formulae carries a combination of the input preferences of the two conjuncts. Here two orderings are combined simply by taking their intersection.

[4]In this and the coming two sections I will not take advantage of the generality of this definition of satisfaction, and could equally well have defined $\sigma \models \phi$ as equivalent to $\sigma \subseteq [\![\phi]\!]$.

Definition D2 **(GSP Orderings)**

$$\begin{aligned} \uparrow \phi &= \odot \quad \textit{for atomic } \phi \\ \uparrow \neg\phi &= \uparrow \phi \\ \uparrow (\phi \wedge \psi) &= \uparrow \phi \cap \uparrow \psi \\ \uparrow (\phi \rightarrow \psi) &= \uparrow \neg(\phi \wedge \neg\psi) \quad [=\uparrow \phi \cap \uparrow \psi] \\ \uparrow (\phi \vee \psi) &= \uparrow \neg(\neg\phi \wedge \neg\psi) \quad [=\uparrow \phi \cap \uparrow \psi] \\ \uparrow \partial\phi &= \omega(\phi) \end{aligned}$$

When defining a notion of entailment it will be important to consider the case of formulae containing explicitly denied presuppositions. So the set of preferred inputs for a formula ϕ, written $\Uparrow\phi$ will be defined as the set of the preferred inputs out of those that are actually consistent with ϕ. I write $c(\phi)$ for the set of states which are consistent with ϕ, and $\sigma \not<_O \tau$ to mean that τ is not higher in the ordering O than σ:

Definition D3 **(Consistency, Non-Preference, and Preferred Inputs)**

$$\begin{aligned} c(\phi) &= \{\sigma \mid (\sigma + \phi) \neq \emptyset\} \\ \sigma \not<_O \tau \quad &\textit{iff} \quad \tau \geq_O \sigma \rightarrow \sigma \geq_O \tau \\ \Uparrow\phi &= \{\sigma \in c(\phi) \mid \forall \tau \in c(\phi)\ \sigma \not<_O \tau\} \end{aligned}$$

Now one formula ϕ can be defined to presuppositionally entail another ψ, written $\phi \rightsquigarrow \psi$, if and only if with respect to all of ϕ's preferred inputs ϕ contextually entails ψ:

Definition D4 **(Presuppositional Entailment)**

$$\phi \rightsquigarrow \psi \quad \textit{iff} \quad \forall \sigma \in \Uparrow\phi\ (\sigma + \phi) \models \psi$$

A logician peering at this definition for the first time might perhaps be worried. What strange new fangled relation is $\rightsquigarrow$, and why is it being described as a notion of logical consequence? I believe it could be termed a *conservative* notion of consequence in at least two senses.

Firstly, for the sublanguage without the ∂-operator, $\rightsquigarrow$ is equivalent to the classical relation of consequence. This is so because none of the operators apart from ∂ introduce non-trivial preferences if their arguments have only trivial preferences, so that the set of preferred inputs for a consistent non-presuppositional formula is just the set of all states. In that case $\phi \rightsquigarrow \psi$ holds if and only if the set of worlds in the extension of ϕ is a subset of those in the extension of ψ, which is just the condition for classical entailment. Inconsistent formulae have an empty set of preferred states, and will presuppositionally entail all other formulae, just as in classical propositional logic.

Secondly, under this consequence relation all classical entailment patterns should remain valid for the full language of PL+∂ with entailment defined by $\rightsquigarrow$. To see this, initially let us consider the conclusion. Observe that since the preference ordering associated with the conclusion is not involved in the definition of $\rightsquigarrow$, and since the valuation of $\partial\phi$ is just that for ϕ, we may safely erase all occurrences of ∂ on the right of an entailment. Now the premise. Suppose that some formula ϕ has a sub-formula ψ. Clearly replacing ψ by $\partial\psi$ will not affect the semantic interpretation of ϕ, and *a fortiori* will not affect the set of states with which

ϕ is consistent. Now suppose that ϕ is classical, i.e. does not contain any instances of the ∂-operator. Then, as shown above, all its classical consequences will also be presuppositional entailments. Now modifying any number of sub-formulae by enclosing them in (any number of) ∂-operators can only restrict the set of preferred inputs to ϕ. It is easily seen that the set of entailments of the modified premise must be a strict superset of the entailments of the original premise. So, $\rightsquigarrow$ is a strictly weaker notion of consequence than classical entailment, in that more formulae are presuppositionally entailed than classically entailed.

Next we consider how the presuppositions of a formula may be defined using the ideas underlying presuppositional entailment. The following definition says that one formula presupposes another if and only if all preferred states of the first satisfy the second:

Definition D5 **(GSP Presupposition)**

$$\phi \gg_{\text{GSP}} \psi \quad \textit{iff} \quad \forall \sigma \in \Uparrow\!\phi \;\; \sigma \models \psi$$

If ϕ is consistent (i.e. $c(\phi) \supset \emptyset$) then for any state σ in $\Uparrow\!\phi$, $\sigma + \phi$ must be nonempty. Further, since $+$ is defined in terms of intersection, and since the set of propositions which are satisfied in $\sigma \cap \tau$ is just the union of those which are satisfied in σ and those which are satisfied in τ, it follows that the earlier definition of presuppositional entailment could be rewritten: $\phi \rightsquigarrow \psi$ *iff* $\forall \sigma \in \Uparrow\!\phi$ $(\sigma \models \psi$ *or* $[\![\phi]\!] \models \psi$. Given that additionally $[\![\phi]\!] \models \psi$ holds if and only if $\phi \models \psi$, where $\models$ is the classical entailment notion (extension of consequent containing extension of premise), it follows that $\rightsquigarrow$ can here be expressed in terms of preferential presupposition and classical entailment: $\phi \rightsquigarrow \psi$ *iff* $\phi \gg_{\text{GSP}} \psi$ *or* $\phi \models \psi$[5]. The following relatively weak properties of the system show that it is natural to think of ∂-formulae as *potential presuppositions* in Gazdar's sense, for they emerge as presuppositions when nothing conflicts, and disappear otherwise:

1. The presupposition operator yields presuppositions even when embedded: if ϕ contains a subformula $\partial\psi$ and no other ∂-formulae, and the conjunction of ϕ and ψ is consistent, then $\phi \gg_{\text{GSP}} \psi$.
2. Presuppositions persist if there is no conflict: if $\phi \gg \psi$ and χ is a non-presuppositional formula such that the conjunction of ϕ, ψ and χ is consistent, then $\phi \wedge \chi \gg_{\text{GSP}} \psi$.
3. Presuppositions disappear when conflicting material is added: if $\phi \gg \psi$, and although the conjunction of ϕ and χ is consistent, that of ϕ, ψ and χ is inconsistent, then $\phi \wedge \chi \not\gg_{\text{GSP}} \psi$.

What this shows is that we have here the beginnings of a cancellationist account of presupposition. Of course, much would need to be shown before we could really be confident about this system, and much needs to be added (e.g. cancellation by implicatures) before the system would be very potent as a theory of presupposition. But the system still manifests at least the most basic property of a cancellationist theory. That is, presuppositions project if there is no conflict and are canceled otherwise. Given that the developments in the current article are only intended to be proofs of concept, this provides sufficient foundation to move on to the next stage, of showing how a preference for global satisfaction can be replaced by a preference for local satisfaction.

[5]This provides another demonstration that $\rightsquigarrow$ is conservative with respect to classical entailment in the senses discussed above.

Example

Suppose that E3, below, is represented as $\neg K \wedge \neg(\partial K \wedge B)$, where K is the proposition that there is a unique King of France, and B is the proposition that there is a unique King of France who is bald. Abbreviate this formula as ϕ.

E3 There is no King of France, so it's not the case that the King of France is bald.

The preferences associated with ϕ formula can be calculated as follows:

(1) $\uparrow K$

(2) $= \odot$

(3) $\uparrow(\neg K)$

(4) $= \uparrow K = \odot$

(5) $\uparrow B$

(6) $= \odot$

(7) $\uparrow(\partial K)$

(8) $= \omega(K)$

(9) $\uparrow(\partial K \wedge B)$

(10) $= \omega(K) \cap \odot = \omega(K)$

(11) $\uparrow(\neg(\partial K \wedge B))$

(12) $= \uparrow(\partial K \wedge B) = \omega(K)$

(13) $\uparrow(\neg K \wedge \neg(\partial K \wedge B))$

(14) $= \odot \cap \omega(K) = \omega(K)$

So, the sentence is associated with a presuppositional preference for contexts where K holds. However, the set of preferred inputs $\Uparrow\!(\phi)$ only involves those preferences which are consistent with assertion of the formula. Since ϕ classically entails $\neg K$, the set of contexts consistent with ϕ, $c(\phi)$, contains no contexts where K holds. In fact no context in $c(\phi)$ is higher in the $\uparrow \phi$ ordering than any other context in $c(\phi)$, so that $\Uparrow\!(\phi) = c(\phi)$. Since the set of preferred inputs for ϕ does not contain contexts where K holds, it follows that K the proposition that there is a French King, is not presupposed (or entailed).

3 Local Satisfaction Preferred

In the Global Satisfaction Preferred (GSP) system above, conjunction is symmetrical, its preferences not reflecting any ordering of the conjuncts. The only change now made to that system is an alteration of these preferences associated with conjunction[6]. In the following

[6]The preferences associated with implication and disjunction are taken to be still defined in terms of the equivalences with conjunction and negation used above. Thus disjunction becomes asymmetric once conjunction is asymmetric. But in this system it might make sense to define the preferences for disjunction symmetrically, as the intersection of the preferences of the disjuncts. For now problematic cases of cancellation of presuppositions occurring in a disjunct need not be handled by defining disjunction so as to enforce local satisfaction, but can instead be handled by the cancellation mechanism.

definition, whilst the presuppositions of the left conjunct become preferences of the whole conjunction, those of the right conjunct only impose a preference for satisfaction in a context which has been first augmented with the left conjunct:

Definition D6 (Local Satisfaction Preference for Conjunction)

$$\uparrow(\phi \wedge \psi) \;=\; \{\langle\sigma,\tau\rangle \in \uparrow \phi \mid \langle \sigma + [\![\phi]\!], \tau + [\![\phi]\!]\rangle \in \uparrow \psi\}$$

It is easily checked that this operation does not introduce any non-good orderings[7].

Let presupposition in the LSP system ($\gg_{\text{LSP}}$) be defined as for the GSP system excepting that the preferences associated with conjunction are altered. The system's properties can be described as follows, where $\gg_K$ denotes the relation of presupposition found Karttunen's [Kar74] system:[8]

1. When there is no inconsistency, Karttunen presuppositions are LSP presuppositions: if ϕ has just one presuppositional subformula, $\phi \gg_{\text{K}} \psi$, and the conjunction of ϕ and ψ is consistent, then $\phi \gg_{\text{LSP}} \psi$.

2. Non-Karttunen presuppositions are not LSP presuppositions: if ϕ has just one presuppositional subformula, and $\phi \not\gg_{\text{K}} \psi$, then $\phi \not\gg_{\text{LSP}} \psi$

3. Inconsistency with presuppositions causes cancellation: if ϕ has just one presuppositional subformula, $\phi \gg_{\text{LSP}} \psi$, and although the conjunction of ϕ and χ is consistent, that of ϕ, ψ and χ is inconsistent, then $\phi \wedge \chi \not\gg_{\text{LSP}} \psi$.

These results, which I hope with further study could be strengthened, show that the LSP system is Karttunen-like when there is no inconsistency, and thus typically produces weaker presuppositions than the cancellationist GSP model. However, under threat of inconsistency, LSP presuppositions vanish into thin air, in the manner of a cancellationist rather than a dynamic semantic account of presupposition[9].

[7]Define the localisation of O to ϕ as $L(O,\phi) = \{\langle\sigma,\tau\rangle \in \odot \mid \langle\sigma+\phi,\tau+\phi\rangle \in O\}$. Now the above clause for conjunction may be written: $\uparrow(\phi \wedge \psi) = \uparrow \phi \cap L(\uparrow \psi, \phi)$

To determine whether this produces only good orderings, we need to check that if O is a good ordering then $L(O,\phi)$ is a good ordering. But if O is a good ordering then either $O = \odot$, or $O = \omega(\chi)$ for some χ, or O is an intersection of two other good orderings O_1 and O_2. The three cases break down as follows:

1. If $O = \odot$ then $L(O,\phi) = \odot$
2. If $O = \omega(\chi)$ then $L(O,\phi) = \omega(\phi \rightarrow \chi)$
3. If $O = O_1 \cap O_2$ then $L(O,\phi) = L(O_1,\phi) \cap L(O_2,\phi)$

It follows that the preferences associated with the LSP system are still good orderings.

[8]The presuppositional properties of Karttunen's 1974 system are also found in Karttunen and Peters' joint proposal [KP79], and in Heim's [Hei83a] if local and intermediate accommodation (details of which she does not specify in detail) are ignored. In [Bea92] a semantics for the formal language used here is proposed which manifests those properties, and $\gg_K$ can be taken as the presupposition relation in that system. Assuming that $\phi \gg K\pi$, this relation can be summed up extensionally as follows: (1) $\neg\phi \gg K\pi$; (2) $\phi \wedge \psi \gg K\pi$; (3) $\phi \rightarrow \psi \gg K\pi$; (4) $\psi \wedge \phi \gg K\psi \rightarrow \pi$;(5) $\psi \rightarrow \phi \gg K\psi \rightarrow \pi$. Also, unsurprisingly, we have that for all ϕ, $\partial\phi \gg_K \phi$.

[9]One noteworthy aspect of LSP concerns the fact that whilst the presuppositions of a formula may be canceled by explicitly denying them, denying the truth of the actual arguments of ∂-operators can still leave non-trivial conditional presuppositions. For instance, let α be $(\phi \rightarrow \neg(\partial(\psi) \wedge \chi)) \wedge \neg\psi$. The left-hand conjunct introduces a preference for states where $\phi \rightarrow \psi$ holds. But this preference is not canceled by the conjunction with $\neg\psi$. So the formula still ends up presuppositionally entailing more than it would classically entail. In particular α presuppositionally entails $\neg\phi$.

An example may clarify. Consider the discourse 'If Mary closed her eyes then she did not realise that Bill was smiling. He wasn't.', which I find rather odd. If we take the two sentences to be conjoined, and we understand the 'realise' clause as $(\partial(\text{bill-is-smiling}) \wedge \text{mary-realises-bill-is-smiling})$, then the discourse has the logical form of α, and LSP predicts that it entails that Mary did not close her eyes. Given the oddity of this discourse, I am not sure whether this result is problematic. But it does show that there may be cases where LSP predicts *stronger* presuppositions than GSP, for in this case LSP predicts a conditional presupposition, and GSP predicts no presupposition. I would note that, at least in this case, the inference could presumably be removed by stipulating, as is standard, that the conditional introduced an implicature that the truth of its antecedent was unresolved. Whether we could then produce a variant of LSP that did produce strictly weaker presuppositions than (some similar variant of) GSP, and whether this would be desirable, I cannot say.

[10]In the calculation of the preferences for ϕ, note the passage from line 12 to 13. This relies on an analogue of the deduction theorem allowing us to deduce from $\sigma + D \models N$ that $\sigma + D \models N$. To see that this is so, observe that $\sigma + D$ is that subset of worlds in σ which classically satisfy D. Then $\sigma + D \models N$ says that $\forall w \in \sigma$, if w classically satisfies D then w classically satisfies N. It follows that $\forall w \in \sigma$,, w classically satisfies $D \rightarrow N$, and thence the desired result.

Example

The following example contains the factive "the knowledge that" in the consequent of a conditional. Use D for "David wrote the article", N for "no decent logician was involved in writing the article", and T for "the belief that no decent logician was involved in writing the article will disturb the editors". Under the suspect but broadly attractive assumption that knowledge is belief plus presupposition of truth, we derive the translation $D \rightarrow (\partial N \wedge T)$: call this ϕ.

E4 If David wrote the article then the knowledge that no decent logician was involved in writing the article will disturb the editors.

The preferences associated with ϕ can be calculated as follows:

(1) $\uparrow(\partial N \wedge T)$
(2) $= \{\mu \in \uparrow \partial N \mid \mu + \partial N \in \uparrow T\}$
(3) $= \{\mu \in \omega(N) \mid \mu + \partial N \in \odot\}$
(4) $= \omega(N)$
(5) $\uparrow(D \rightarrow (\partial N \wedge T))$
(6) $= \uparrow(\neg(D \wedge \neg(\partial N \wedge T))$
(7) $= \uparrow(D \wedge (\partial N \wedge T))$
(8) $= \{\mu \in \uparrow D \mid \mu + D \in \uparrow(\partial N \wedge T)\}$
(9) $= \{\mu \in \odot \mid \mu + D \in \omega(N)\}$
(10) $= \{\langle\sigma,\tau\rangle \mid \langle\sigma + D, \tau + D\rangle \in \omega(N)\}$
(11) $= \{\langle\sigma,\tau\rangle \mid \langle\sigma + D, \tau + D\rangle \in \{\langle\sigma',\tau'\rangle \mid (\tau' \models N) \rightarrow (\sigma' \models N)\}\}$
(12) $= \{\langle\sigma,\tau\rangle \mid (\tau + D \models N) \rightarrow (\sigma + D \models N)\}$
(13) $= \{\langle\sigma,\tau\rangle \mid (\tau \models D \rightarrow N) \rightarrow (\sigma \models D \rightarrow N)\}$
(14) $= \omega(D \rightarrow N)$

So associated with the formula $D \rightarrow (\partial N \wedge T)$ is a preference for contexts where $D \rightarrow N$ holds.[10] This mirrors the behaviour of systems proposed e.g. by Karttunen [Kar74], Heim

[Hei83a] and myself [Bea95]. In all of these systems, a sentence "If A then B", where B has a presupposition P, itself carries a presupposition equivalent to "If A then P". It is easily verified that the preferred proposition $D \rightarrow N$ is consistent with the ϕ, and so becomes a full presupposition.

4 Adding Common Sense to LSP

In earlier work [Bea95, Bea:MS] I have proposed a model for global accommodation which is intended to take preferential knowledge about the world into account. Once again, information orderings are at the heart of the model.

The idea is that global accommodation is modeled not as a repair strategy on "faulty" information states (i.e. states that do not satisfy presuppositions), but instead as a monotonic *filtering* operation. A hearer cannot be sure of the assumptions that the speaker is making, and so maintains a number of alternative context sets, or *epistemic alternatives*, all of which might, as far as the hearer knows, correctly represent the speakers' assumptions. As discourse proceeds, each use of a presuppositional expression provides the hearer with more information about what the speaker was taking for granted, and so allows the hearer to filter out various epistemic alternatives. After each uttered sentence, some alternatives are filtered out, and the remainder are updated with the content of the utterance. This produces a new set of alternatives which is both compatible with what the speaker has presupposed, and with what the speaker has asserted.

To complicate the picture slightly, it is allowed that the hearers knowledge of the speaker is not of an absolute character, in the sense that of the various things that the speaker might have assumed, some alternatives are more plausible than others. To account for this, the hearer's knowledge of the speakers' assumptions is modeled using a preference ordering over epistemic alternatives. These orderings should have certain structural properties (e.g. transitivity, reflexivity), but in general will not be *good orderings* in the above technical sense. Intuitively, a pair $\langle \sigma, \tau \rangle$ will be in the ordering just in case σ is considered at least as plausible a model of the speaker's assumptions as τ. In terms of information orderings, update proceeds by filtering out those pairs for which at least one of the alternatives is incompatible with the speaker's presuppositions, and otherwise updating both members of each pair with the new information.

It must now be determined how such a plausibility ordering may be combined with the LSP system, which involves its own ordering determined not by common sense but by a preference for satisfaction of conventionally stipulated presuppositions. This combination will take the form of a definition of presuppositional entailment which is sensitive to an external plausibility ordering, so that $\phi \rightsquigarrow_O \psi$ if ψ is a presuppositional consequence of ϕ relative to the ordering O.

One obvious possibility would be to take the intersection of the two orderings, and use that in the new definition of consequence. But I do not think this would give the right results empirically. Suppose that states are preferred where Mary does not own a rhinoceros, and I say 'I didn't realise that Mary owned a rhinoceros.' Taking the intersection of an ordering encoding a preference for states where Mary owns a rhinoceros with one where there is a preference for states where Mary does not own a rhinoceros will have a canceling effect. The set of maximal states in the resulting ordering would contain both states where Mary had a rhinoceros and states where she did not, and so there would be no net preference

for states where Mary did own a rhinoceros. However, my strong impression is that even though it is *prima facie* unlikely that Mary owns a rhinoceros, if I say 'I didn't realise that Mary owned a rhinoceros' and do not explicitly mention any caveat, then I am committed to Mary's having a rhinoceros. Thus, when interpreting what someone has said, satisfaction of the presuppositional defaults could be said to have a higher priority than satisfaction of common sense preferences about how the world ought to be.

We can model this by firstly finding the maximal states associated with the presuppositions of the utterance, and then using the plausibility ordering to select amongst the resulting states. In the following, $max(O, S)$ defines the elements of the set of states S which are maximal in the ordering O, and $\phi \rightsquigarrow_O \psi$ means that ϕ presuppositionally entails ψ relative to the ordering O:

Definition D7 (Plausible Presuppositional Entailment)

$$\begin{array}{rcl} max(O,S) & = & \{\sigma \in S \mid \forall \tau \in S\ \sigma \not< \tau\} \\ \phi \rightsquigarrow_O \psi & \textit{iff} & \forall \sigma \in max(O, \Uparrow\phi)\ \ (\sigma + \phi) \models \psi \end{array}$$

This defines a notion of entailment which in principle can account for aspects of all the desiderata above. This notion allows for:

1. Filtering of presuppositions entailed in their local context.
2. Cancellation of presuppositions that are explicitly denied.
3. Strengthening of weak presuppositions such as those generated when a presupposition is located in the consequent of a conditional.

Example

Represent the following example as $\phi = L \rightarrow \partial S \wedge C$, where L is "I land at Heathrow", S is "I have a sister", and C is "I will be collected by some sister of mine":

E5 If I land at Heathrow, my sister will collect me.

By analogy with the example in the previous section, we know that $\Uparrow\phi$ contains all and only contexts where $L \rightarrow S$ holds, i.e. where it is established that if I land at Heathrow then I have a sister. Now consider two classes of context: class A contains contexts where it is established that if I land at Heathrow then I have a sister, but it is not established that I have a sister; class B contains contexts where it is established that I have a sister (and thus *a fortiori* that if I land at Heathrow then I have a sister. that is, $\forall \sigma \in A, \sigma \models L \rightarrow S$ *and* $\sigma \not\models S$, and $\forall \sigma \in B, \sigma \models S$. Our common sense knowledge of the world means that it is *a priori* implausible that a speaker would take for granted that my having a sister is in any way causally dependent on whether I land at Heathrow. Therefore, it could be argued that type A contexts will tend to be low down in the plausibility ordering. On the other hand, it is *a priori* relatively plausible that I do actually have a sister, so type B contexts could be relatively high in the ordering. Following this line of reasoning, let us make the following assumption about the plausibility ordering O:

For every class A contextS, there is a class B context which is higher in the O-ordering. That is $\forall\sigma$, if (i) $\sigma \models L \rightarrow S$ and (ii) $\sigma \not\models S$, then $\exists\tau, \tau \models S$ and $\tau >_O \sigma$.

Given this, given also that O does contain some A and B type contexts, and that all contexts in ϕ are either in class A or in class B, it follows that relative to the ordering O, ϕ presuppositionally entails S: $\phi \sim_O S$. That is to say, relative to such an ordering, the sentence "If I land at Heathrow, my sister will collect me." entails that I do have a sister.

5 Notes on a First Order Extension

In this section, two changes are proposed to LSP: the classical semantics used above is replaced by a dynamic semantics, and the whole system is extended to first order level. The dynamic semantics is inspired by Heim [Hei83a, Hei83b] and Dekker [Dek93]. Limitations of space unfortunately prevent me from providing adequate motivation and introduction to the semantics itself: the reader is referred to the discussions of related systems in the papers by Heim and Dekker, to Groenendijk and Stokhof's [GS91a], and especially to my [Bea94a].

Definition D8 **(Models for FO LSP)** *Models are triples $\langle W, D, \|.\| \rangle$ where W is a nonempty set of worlds, D is a nonempty domain of individuals (assumed constant across worlds), and $\|.\|$ is an interpretation function which relative to a world maps each n-ary predicate onto a set of n-tuples of individuals from D.*

Definition D9 **(Assignment Functions)** *I will use partial assignment functions f, g, mapping a subset of variables onto elements of D:*
(1) G denotes the set of all possible partial assignments for a given model;
(2) $f \geq g$ means that g agrees with f on valuations for all variables in the domain of g, but that f may have a larger domain;
(3) $f >_x g$ means f extends g by having the variable x in its domain.

Definition D10 **(Information States and Updates)** *Information states or contexts σ, τ are sets of world-assignment pairs.*
(1) $\sigma + \phi$ denotes the update of σ with ϕ;
(2) if σ contains no assignments valuing x, then σ_x is an information state differing from σ by having all the assignments extended to value x, defined by: $\sigma_x = \{\langle w, f\rangle \mid \exists g \langle w, g\rangle \in \sigma$ and $f \geq_x g\}$, and if σ contains assignments valuing x then σ_x is undefined;
(3) $\sigma * \phi$ is a restricted update of σ with ϕ in which no new variables are introduced, defined by $\sigma * \phi = \{\langle w, f\rangle \in \sigma \mid \exists g \geq f, \langle w, g\rangle \in \sigma + \phi\}$.

Definition D11 **(Satisfaction and Ordinary Dynamic Entailment)**
(1) Satisfaction of a formula relative to an information state is defined by:
$\sigma \models \phi$ iff $\sigma * \phi = \sigma$.
(2) A non-presuppositional form of dynamic entailment is defined by:
$\phi \models \psi$ iff for all σ containing assignments with uniform domain, $\sigma + \phi \models \psi$.

Definition D12 **(Heim/Dekker semantics)** *In all of the following, it is possible that one of the component terms of the right hand side is undefined (e.g. because σ_x is undefined in the clause for existentials): in this case the effect of update is left undefined. Clauses for universals, disjunctions and implications are taken to follow standard (dynamically motivated) equivalences.*

$$\begin{aligned}
\sigma + P(x_1, \ldots, x_n) &= \{\langle f, w\rangle \in \sigma \mid \langle w, f(x_1), \ldots, f(x_n)\rangle \in \|P\|_w\} \\
&\quad \textit{(if } x_1 \ldots, x_n \textit{ are in the domain of all assignments} \\
&\quad \textit{in } \sigma\textit{, otherwise undefined.)} \\
\sigma + \phi \wedge \psi &= (\sigma + \phi) + \psi \\
\sigma + \neg\phi &= \sigma \backslash (\sigma * \phi) \\
\sigma + \exists x \phi &= \sigma_x + \phi \\
\sigma + \partial\phi &= \sigma + \phi
\end{aligned}$$

In defining the preferences associated with formulae of this system, most of the definitions from the earlier sections of this paper can be carried over, modulo reinterpretation of some fundamental constants, and of the range of variables. In particular, states σ are now sets of world assignment pairs instead of worlds, and W^* is redefined as $\wp W \times G\backslash\emptyset$, and new interpretations follow immediately for $\odot$, the set of good orderings, $c(\phi)$ (in terms of the new notion of update), $\not<_O$, $\Uparrow$, $\rightsquigarrow$, $\gg_{folsp}$ (i.e. first order lsp presupposition). The preferences of formulae can be defined as in LSP, except over the new notion of information state, and with simple predications defined to carry the trivial preference relation $\odot$ and existentials defined as follows:

Definition D13 **(Preferences for Existentials (first version))**

$$\uparrow(\exists x \phi) = \{\langle \sigma, \tau\rangle \mid \langle \sigma_x, \tau_x\rangle \in \uparrow \phi\}$$

Example

Let us consider the treatment of $\exists x(f(x) \wedge \partial(h(x)) \wedge r(x))$, a crude translation of Heim's E6, with f for "fat man", h for "has bike" and r for "is riding his bicycle".

E6 A fat man is riding his bike.

The preferences can be calculated as follows[11]

(1) $\uparrow(\partial(h(x))$
(2) $= \{\langle \sigma, \tau\rangle \mid (\tau \models h(x)) \rightarrow (\sigma \models h(x))\}$
(3) $\uparrow(\partial(h(x)) \wedge r(x))$
(4) $= \uparrow(\partial(h(x))$
(5) $\uparrow(f(x) \wedge \partial(h(x)) \wedge r(x))$

[11]The derivation assumes that $\phi \rightarrow \psi$ is defined as $\neg(\phi \wedge \neg\psi)$ and that $\forall x\phi$ is defined as $\neg\exists x\neg\phi$, and also uses a couple of lemmas that require demonstration. Firstly it is taken to follow from $\sigma + \phi \models \psi$ that $\sigma \models \phi \rightarrow \psi$, and secondly it is taken to follow from $\sigma_x \models \phi$ that $\sigma \models \forall x\phi$.

(6) $= \{\langle\sigma,\tau\rangle \mid (\tau + f(x) \models h(x)) \rightarrow (\sigma + f(x) \models h(x))\}$

(7) $= \{\langle\sigma,\tau\rangle \mid (\tau \models f(x) \rightarrow h(x)) \rightarrow (\sigma \models f(x) \rightarrow h(x))\}$

(8) $\uparrow (\exists x f(x) \wedge \partial(h(x)) \wedge r(x))$

(9) $= \{\langle\sigma,\tau\rangle \mid \langle\sigma_x,\tau_x\rangle \in \uparrow (f(x) \wedge \partial(h(x)) \wedge r(x))$

(10) $= \{\langle\sigma,\tau\rangle \mid (\tau_x \models f(x) \rightarrow h(x)) \rightarrow (\sigma_x \models f(x) \rightarrow h(x))\}$

(11) $= \{\langle\sigma,\tau\rangle \mid (\tau \models \forall x f(x) \rightarrow h(x)) \rightarrow (\sigma \models \forall f(x) \rightarrow h(x))\}$

This shows that the system as defined manifests the projection of open presuppositions familiar from Heim's analysis: an existentially bound presupposition trigger can yield a universal presupposition. However, this universal is now only a default: if the input context guarantees that not all fat men have bikes, then the presupposition could be canceled/locally accommodated. Although the universal presupposition does not seem appropriate in this case, even as a default, there are a number of ways to alter the system such that an existentially bound trigger yielded an existential presupposition. In this regard, see the developments of related systems in [Bea94a, Bea95]: one can alter the preferences associated with the existential quantifier, or those associated with the ∂-operator so as to derive existential instead of universal presuppositions, but I will not attempt such modifications here.

To the extent that LSP is a cancellationist account at all, it is the first such account which provides any treatment at all of the interaction of presupposition and quantification, even if that treatment still requires further work, and thus LSP improves significantly on the cancellationst models of Gazdar [Gaz79a], van der Sandt [vdS88], Mercer [Me87, Me92] and Soames [So82].

6 Last Words

There is considerable space for further study and development. In general, one can see approximately what behaviour to expect of the systems proposed here because they combine several existing proposals. Yet it remains necessary to study the properties of the systems in detail. As far as further development is concerned, I can think of two directions of research which might be particularly beneficial. Firstly, one might consider making the notion of local accommodation more sophisticated, allowing the locally accommodated material to go beyond a simple assertion of the presupposition. This would be necessary if the system was to deal with cases where the insertion of bridging material associated with a definite description happens locally.[12] Secondly, it would be natural to introduce a treatment of implicature, the interaction between implicature and presupposition being one of the cornerstones of the cancellation models. Here it seems appropriate to think in terms of the reconstrual of Gricean principles adopted by van der Sandt [vdS92] in terms of informativity constraints.[13]

[12] C.f. footnote 3, above. We might want to analyse "If you ride a bike, hold the handlebar tightly." as involving the local insertion of bridging material linking the handlebar marker to the bicycle marker.

[13] In brief, the effects of van der Sandt's treatment could be obtained by making it a (possibly defeasible) requirement of update with any non-presuppositional formula that the local context neither satisfies the formula nor satisfies its negation. Thus instead of quantifying over contexts which are consistent with the formula, as in the LSP notion of presuppositional entailment, we would be able to quantify over contexts in which all sub-formulae played a non-trivial role in the update. Thus we would be able to avoid globally accommodating a presupposition which would render any subformula uninformative, and thus achieve the same result as Gazdar achieves using potential implicatures.

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Degree Questions, Maximal Informativeness, and Exhaustivity*

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1. Introduction

Our aim in this paper is to develop a satisfactory account of degree questions like (1).

(1) How many books did John read?

We focus on the fact that (1) requires an answer naming the maximal number of books John read. In Rullmann (1995) it is argued that maximality in degree questions is one and the same thing as exhaustivity in questions involving individuals. While we adopt this idea as our starting point, we discuss problems for his account with respect to examples like (2), which requires naming the minimal number of eggs that are sufficient.

(2) How many eggs are sufficient to bake this cake?

We solve this problem by generalizing Rullmann's (1995) approach. This then leads to a more general discussion of exhaustivity in questions, in which we argue for a flexible approach to exhaustivity. Our paper can be regarded as an extension of a Karttunen (1977) semantics for interrogatives and as a defense of this extended system against various criticisms of Karttunen based on exhaustivity and maximality.
The structure of the paper is as follows. In section 2 we present Rullmann's analysis, which is stated in terms of Karttunen's semantics for questions. Section 3 deals with empirical problems for this analysis. While those problems do not affect the central idea, they do affect the specific implementation in terms of a maximality operator. We will suggest that the various data should receive a uniform explanation in terms of maximal informativeness. To this effect we will adopt in section 4 a proposal by Heim (1994). She reanalyzes exhaustivity not as a property of the semantics of a question itself, but as a property of the notion of answerhood. We will show that (and why) her proposal carries over to degree questions. This leaves us with a satisfactory analysis of degree questions in a Karttunen framework. Section 5 is concerned with the relation between this analysis and Rullmann's (1995) proposal, as well as an analysis along the lines of Groenendijk & Stokhof (henceforth: G&S) (1982, 1984). We will show how in some sense our analysis can remodel their results. However, the differences between our proposal and G&S's semantics are not merely a matter of implementation. Section 6 is devoted to the differences between the two proposals and their potential empirical implications. This includes

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discussion of well-known phenomena (like different kinds of question embedding verbs and mention-some interpretations) as well as some new data (in particular a type of degree question that involves *at least* and *at most*). Section 7 discusses two potential ways of implementing our suggestions formally. We conclude that our proposal ought to be considered as an alternative, flexible approach to exhaustivity, and maximality.

2. Rullmann (1995)

2.1. Karttunen (1977)

Before discussing Rullmann's (1995) implementation of maximality, we will briefly sketch Karttunen's semantics for questions on which his account is based. For a full exposition the reader is referred to Karttunen (1977). In Karttunen's semantics the denotation of a question is a set of propositions, namely the set of all those propositions that are true answers to the question. If for instance Mary, Sue and Jane were at the party and no one else was, then the denotation of the question (3a) will be the set of propositions given in (3b):

(3) a. Who was at the party?
b. {Mary was at the party, Sue was at the party, Jane was at the party}
c. $\lambda p \exists x[person(x) \& p(w) \& p=\lambda w'[x$ was at the party in w'$]]$

More generally, (3a) denotes the set of propositions (3c), which we can think of informally as the set of propositions of the form 'x was at the party', where x is a person (though in reality of course propositions can't be said to have a 'form', because they are sets of possible worlds).
We will usually use the term "Karttunen denotation" to refer to the extension of a question (i.e. to an object of type <<s,t>,t>). When we are talking about Karttunen intensions (i.e. objects of type <s,<<s,t>,t>>), this should be clear from the context.

2.2. Degree questions and maximality

In chapter 3 of his dissertation, Rullmann (1995) notes that degree questions like (4a) and (4b) require an answer that is in some sense maximal:

(4) a. How many books did John read?
b. How high can John jump?
c. Jill knows how high John can jump.

Someone who utters (4a) wants to know the maximal number n such that John read n books. Similarly, (4b) asks for the maximal (degree of) height d such that John can jump d-high. The embedded case (4c) is parallel: Jill has to be aware of the maximal height John can jump. Note that if John read five books and not more than five books, then the only possible true answer to (4a) will be "five", even though the proposition that John read four books is literally speaking true in that situation. We will call this effect maximality.
Rullmann´s idea is that (4a) and (4b) really mean something like (5a,b):

(5) a. Which number n is such that n is the greatest number of books that John read?
b. Which degree d is the greatest degree such that John can jump d-high?

Quasi-formally, the interpretations of (4a) and (4b) can be represented as in (6a) and (6b), where *max* is an operator that picks out the maximal degree (or number) from a set of degrees and "?" is a question-operator whose semantics is spelled out below:

(6) a. ?n: n=max(λn'[John read n' books])
b. ?d: d=max(λd'[John can jump d'-high])

This basic idea can be implemented in a Karttunen-style semantics of questions, where the denotation of a question is the set of propositions that are true answers to the question:

(7) a. $\lambda p \exists n[p(w) \& p=\lambda w'[n=$ max(λn'[John read n' books in w'])]]
b. $\lambda p \exists d[p(w) \& p=\lambda w'[d=$ max(λd'[John can jump d'-high in w'])]]

(7a,b) are basically the Karttunen denotations of the paraphrases in (5a,b). Note that these formulas will always denote a singleton set (or the empty set), because there is at most one maximal degree or number. The single element is the maximum answer, so Rullmann´s theory accounts for the maximality effect.[1]
In the rest of the paper we will mostly give examples of degree questions involving numbers (*how many*) rather than degrees, for simplicity.

2.3. Individual questions and exhaustivity

Rullmann shows that his analysis can be extended to questions involving individuals rather then degrees or numbers, if we adopt a Link-style analysis in which the domain of discourse contains not only atomic individuals, but also their mereological sums, or groups. Maximality should then be interpreted with respect to the part-of relation on groups. A question like (8) can be analyzed as asking for the maximal group of individuals such that this group was at the party:

(8) a. Who was at the party (last night)?
b. Which x is such that x is the largest group that was at the party?
c. ?x: x=max(λx'[x' was at the party])
d. $\lambda p \exists x[p(w) \& person(x) \& p = \lambda w'[x =$ max(λx'[x' was at the party in w'])]]

[1] One immediate objection that might be raised against this analysis is that it builds maximality into the semantics of questions with no role to play for pragmatics, thereby separating this phenomenon from that of scalar implicatures in declarative sentences. We will leave this issue aside for the moment. The theory we eventually argue for is one which does leave more room for pragmatics, especially in the analysis of unembedded questions (see section 7.3.).

Following a suggestion by Jacobson (1995), Rullmann shows that maximality can be used to account for the property of strong exhaustivity argued for by G&S (1982, 1984). G&S distinguish two kinds of exhaustivity in questions, weak and strong exhaustivity. Weak exhaustivity is the property which licenses inferences of the following form:

(9) John knows who was at the party.
Mary was at the party.
∴ John knows that Mary was at the party.

Strong exhaustivity is the property of questions which makes it possible to draw inferences of the following type (in addition to ones like (9)):

(10) John knows who was at the party.
Mary was not at the party.
∴ John knows that Mary was not at the party.

G&S (1982) propose a semantics for questions which is an alternative to Karttunen's theory but which accounts for both weak and and strong exhaustivity (unlike Karttunen's analysis, which captures only weak exhaustivity).
Rullmann (1995) argues that by introducing maximality into the Karttunen semantics of questions we get a theory that like G&S's theory accounts for both weak and strong exhaustivity. First, as noted above, maximality guarantees that a question will always denote a singleton set of propositions. Because there is a one-to-one relation between singleton sets and their elements, we may therefore as well identify the denotation of a question with the proposition that is the unique member of this set (at least for the class of question-embedding verbs which can take that-clause complements and which G&S call extensional, such as *know*). This means that the denotation of (8a) will be the proposition in (11), rather than the singleton set containing it:

(11) $\iota p \exists x[p(w) \ \& \ person(x) \ \& \ p = \lambda w'[x = max(\lambda x'[x' \text{ was at the party in } w'])]]$

Now suppose that in the actual world w, Mary, Sue and Jane were at the party and no one else was. Then the proposition denoted by (11) will be:

(12) $\lambda w'[Mary+Sue+Jane = max(\lambda x'[x' \text{ was at the party in } w'])]$

This proposition contains all and only those worlds in which Mary, Sue and Jane were at the party and no one else was. Now if John stands in the know-relation to this proposition this will imply that (i) for every x such that x is a member of {Mary, Sue, Jane}, John knows that x was at the party, and that (ii) for every x such that x is not a member of {Mary, Sue, Jane}, John knows that x was not at the party. (Crucially, just like G&S we assume that knowing p entails knowing every proposition entailed by p.) In other words, maximality accounts for both weak and strong exhaustivity. Thus, by adding maximality to Karttunen's theory of questions, we end up with a theory that - though not formally equivalent to it - is able to account for the intuitions that motivate G&S's theory.

Note that strong exhaustivity plays a role in degree questions in just the same way that it does in individual questions. If John knows how many books Bill read, and in fact Bill read five books and not more than five, then by strong exhaustivity, John knows that Bill did not read n books, for any n>5. Rullmann's (1995) analysis of degree questions accounts for this implication in the same way that it does for individual questions.

In this paper, we build on and substantially modify Rullmann's proposal in order to deal with a class of degree questions in which minimality rather than maximality seems to be called for. We show that if we interpret maximality as maximal informativeness and apply it at the level of propositions rather than degrees or individuals, we can account for this class of examples, while at the same time preserving the intuition that maximality and exhaustivity are two sides of the same coin. In the second half of the paper, we then show that by adopting this perspective we get a theory which allows for a more flexible approach to weak and strong exhaustivity than the one defended by G&S. In this respect we largely follow Heim (1994). In support of this approach we will discuss data (in part derived from the existing literature) which show that degree and non-degree questions are not uniformly interpreted exhaustively. Finally, we suggest two ways in which non-exhaustive and exhaustive interpretations of questions may be related in a theory either lexically (by means of meaning postulates or lexical decomposition) or by type shifting.

3. Problems with the maximality operator

3.1. Degree questions requiring a minimal answer

Consider a question like (13):

(13) How many eggs are sufficient (to bake this cake)?

Intuitively, if you ask (13), you want to know the smallest number n, such that n eggs would be enough. The interpretation we would get for (13) in Rullmann´s (1995) analysis, however, is given in (14):

(14) ?n: $n=max(\lambda n'[n' \text{ eggs are sufficient to bake this cake}])$

This is not a satisfactory interpretation for (13), for two (related) reasons. Firstly, the maximum is likely to be undefined in this case: Suppose that in fact, three eggs are sufficient to bake the cake, but fewer than three eggs are not. Then four eggs are also sufficient, and so are five, six etc. So there is no largest number of eggs that would be sufficient. Let us ignore this problem for a moment, though. Maybe the set of numbers is contextually restricted in some way, so that a largest element is defined. Even then, we do not end up with the desired interpretation for the question, because this gives us the largest number of eggs sufficient, while we intuitively want the smallest such number, namely three.So if we formalize (13) in a way analogous to (4a) in section 2.2, a more appropriate solution would be (15):

(15) ?n: $n=min(\lambda n'[n' \text{ eggs are sufficient to bake this cake}])$

There are a few other predicates that behave in the same way as *be sufficient*:

(16) a. Mit wieviel Geld kann ein Professor auskommen?
With how much money can a professor make do
"On how much money can a professor live?"
b. Wie weit zu schwimmen ist ausreichend?
How far to swim is sufficient
"How far is it sufficient to swim?"
c. Wieviel Arsen kann einen Menschen umbringen?
How much arsenic can a man kill
"How much arsenic is enough to kill somebody?"
d. How big a difference (in light intensity)is perceivable?

In all these examples, an appropriate answer would name a minimum (the minimal amount of money on which a professor can live, the minimal distance it suffices to swim etc.), rather than a maximum. Why should that be the case?
The "minimum" interpretation crucially depends on what we will call the question predicate. We will somewhat informally use this term to refer to what is the argument of the max operator in formulas like (14). In the degree questions in section 2 that required maximal answers, we always had question predicates that allowed inferences from larger dergrees to smaller degrees. So for instance in (17) the question predicate (17b) allows inferences from a number n to numbers m smaller than n; i.e. if John has read five books, then he has also read four books, three books etc.

(17) a. How many books did John read?
b. λn'[John read n' books in w']

So in (17) the question predicate has the following property:

(18) A predicate P is downward scalar iff
For all n,m: P(n) & m≤n -> P(m)

In the minimality inducing examples (13) and (16), on the other hand, the question predicate had the reverse property:

(19) A predicate P is upward scalar iff
For all n,m: P(n) & n≤m -> P(m)

So for instance if three eggs are sufficient, then four eggs, five eggs etc. will also be sufficient.
The upward scalar predicates seem to be considerably rarer. We do not at present know why this should be so; it explains, however, why they were first overlooked.

We think that the difference between the maximality inducing examples and the minimality inducing ones boils down to informativity. In case the question predicate allows inferences from a large number to smaller ones, the most informative answer to the question will be to name the maximum, since this implies all other true answers. In the minimality case, it is most

informative to give the minimum answer because here the minimum implies all other true answers.
Therefore, we believe that it is misguided to give the maximum (or, for that matter, the minimum) any special status. We have come to the conclusion that we should have neither a maximum nor a minimum operator in the semantics of degree questions. Note that we do not get an ambiguity; what type of answer is required seems fixed for a given predicate. The fact that we choose the maximum answer in the case of downward scalar predicates should follow from general principles. The same principles should account for the fact that upward scalar predicates require a minimum answer.
So far, our remarks on informativity have been completely informal. Before we turn to a proper formalization of our idea, we would like to discuss another type of question predicate that behaves in yet another way with degree questions.

3.2. Degree questions with nonscalar predicates

Consider (20):

(20) a. With how many people can you play this game?
b. How many courses are you allowed to take per semester?
c. How high can a helicopter fly?

A complete answer to (20a) could be, for instance, between 4 and 6. This is, in effect, a complete list of all true answers to the question, or to put it differently, their conjunction. Similarly for the other examples.
The question predicates in (20) are predicates that do not allow inferences either from large degrees to smaller ones or the other way around. If it is permissible to take five courses per semester, for example, then nothing follows about the possibility of taking six courses or four courses. You might be required to take at least five courses. On the other hand, six might be too many. In other words, in cases like (20) we know that there might be a lower bound as well as an upper bound for the degrees that the predicate applies to. More complicated scenarios are conceivable, for example that you are allowed to take either 4 courses or else between 6 and 8. Or a game may be played with any even number of players.
Hence, the question predicates in (20a-c) are neither downward scalar nor upward scalar. We will refer to them as nonscalar predicates. Since in these cases naming one true answer does not allow any inferences, the only fully informative answer is the conjunction of all true answers. So this is a case where neither a maximum nor a minimum operator would get us anywhere. Resorting to informativeness, however, is still a natural thing to do.

4. Maximal informativeness of answers

We are now in a position to formalize our idea that informativeness is the crucial notion in describing the types of answers you get in degree questions. Our strategy will be to incorporate informativeness not into the semantics of the question, but into the definition of answerhood to a question. We will take as our starting point the ordinary Karttunen semantics for questions. On the basis of that we will define the concept of a maximally informative answer. As it turns out, the notion of maximally informative answer that we need for degree questions, has already been

formalized as a concept of answerhood in the Karttunen system by Heim (1994), who calls it answer1.

(21) the answer1 to a question Q in w answer1(Q)(w) = ∩(Q(w))

Later on, we will see a second concept of answerhood, answer2. To see how answer1 works, we will now consider an example for each of our three types of question predicate.
The easiest case are the nonscalar predicates. Intersection of all propositions in the Karttunen denotation of the question is just conjunction of all those propositions in which the question predicate is truthfully applied to its argument. So for instance in (22a), given the Karttunen denotation (22b)

(22) a. How many courses are you allowed to take?
b. λp∃n[p(w) & p=λw'[you are allowed in w' to take n courses]]

the intersection of the propositions in (22b) will be the conjunction of all the true propositions of the form "you are allowed to take n courses". So for instance if you are actually allowed to take either four or between six and eight courses, answer1(||(22a)||)(w) would be the following proposition:

(23) λw[you are allowed to take 4 courses in w and you are allowed to take 6 courses in w and you are allowed to take 7 courses in w and you are allowed to take 8 courses in w]

In the case of a downward scalar question like (24a) answer1(||(24a)||)(w) would be as in (24b):

(24) a. How many books did John read?
b. ∩(λp∃n[p(w) & p = λw'[John read n books in w']])
c. λw'[John read 5 books in w']

Now suppose that John actually read five books (and not more than five). The proposition that John read four books (which is also in the Karttunen denotation of (24a)) is actually a superset of the proposition that he read five books. Similarly for the other true propositions of the form "John read n books". So the intersection of all these true propositions is the same set as the proposition that John read five books, (24c). answer1(||(24a)||)(w) is thus identical to the maximum answer.
Finally, consider a minimum case like (25a). Answer1(||(25a)||)(w) would be constructed as in (25b):

(25) a. How many eggs are sufficient?
b. ∩(λp∃n[p(w) & p= λw'[n eggs are sufficient in w']])
c. λw'[3 eggs are sufficient in w']

Let us assume once more that three eggs are sufficient (and fewer than three eggs are not sufficient). The true propositions in the Karttunen denotation are of the form "n eggs are sufficient" for n ≥ 3. The proposition that three eggs are sufficient is a subset of all the propositions in that set. Therefore, the intersection of all the propositons in the set is identical to the

proposition that three eggs are sufficient, (25c). We thus end up with the minimum answer as answer1(||(25a)||)(w).
Note that answer1 can only be empty if the original Karttunen denotation was empty already. Otherwise, it will always contain at least the actual world.
Note also that it can only make a difference whether we take the maximal, minimal or intersective answer if there is more than one proposition in the original Karttunen denotation (i.e. if there is more than one true "simple" answer to the question). Frequently, there is only one proposition in that set, for instance in cases involving modal necessity. Cases with modal possibility are frequently cases in which the Karttunen denotation is not a singleton. That is why so many of our examples involve modals like *can* or *be allowed to*. What type of answer we get then depends on the inferential properties of the predicate.
So for the three types of degree questions that we have looked at, the notion of answer1 seems to give good results. We will now relate this notion to Rullmann's original proposal as well as to G&S's semantics for questions. In the course of doing that we will also get back to the issue of exhaustivity.

5. Strong exhaustivity and answer2

Compare our denotation of answer1 for example (26a), (26b), to Rullmann´s (1995) semantics in (26c) (assuming the same facts about the actual world as before):

(26) a. How many books did John read?
b. λw[John read five books in w]
c. λw[max(λn[John read n books in w])=5]

The two propositions are not identical. While (26c) contains the information that five is the maximal number of books John read, (26b) expresses just the proposition that John read five books. Rullmann´s semantics and answer1 also differ in (27):

(27) a. Who was at the party?
b. λw[Mary+Sue+Jane were at the party in w]
c. λw[max(λx[x was at the party in w])=Mary+Sue+Jane]

(27c) expresses the proposition that the maximal group that was at the party consists of Mary, Sue and Jane. (27b) just says that Mary, Sue and Jane were at the party, without any information as to whether there were other people there or not. In other words, (27b) gives the complete true answer, while (27c) gives the complete true answer plus the information that this is the complete true answer to the question.
Rullmann shows that (27c) in a sense captures the same information as the G&S denotation (28) of (27a)(unfortunately, we cannot introduce G&S´s theory here, for reasons of space. The reader is referred to G&S (1982)):

(28) λw'[λx[x was at the party in w']=λx[x was at the party in w]]

So essentially, while Rullmann´s proposal and G&S´s semantics incorporate strong exhaustivity, our notion of answer1 only yields weak exhaustivity.

Discussion of this issue in the last decade has made clear at least that we need to have strong exhaustivity at some points, for example in questions embedded under the verb *know*. Fortunately, Heim´s (1994) paper already contains a proposal of how to get strong exhaustivity from answer1, namely by means of her notion of answer2.

(29) answer2(Q)(w) = λw'[answer1(Q)(w') = answer1(Q)(w)]

Heim (1994) shows that answer2 will in general produce the same truth conditions as the G&S-semantics. However, in certain cases this equivalence breaks down. See Heim (1994) for discussion.
This means that we can obtain the information needed to capture strong exhaustivity from the original Karttunen denotation, by applying answer1 and answer2.
Answer2 in a sense remodels a G&S semantics. Moreover, we think that a G&S semantics gives a fairly good result for degree questions, although we don´t have the room to show this.[2] This raises the question of why we went through all this trouble of defining notions of answerhood on the basis of a Karttunen semantics, if we could have had a satisfactory result in a G&S semantics straightforwardly without such a fuss.[3] This question will be addressed in the next section.

6. Arguments for a flexible approach to exhaustivity

Summarizing our discussion so far, we have a theory of questions which makes available at least three distinct semantic objects that are associated with a question. Firstly, there is the Karttunen denotation, the set of all true propositions that count as (not necessarily exhaustive) answers to the question. Let's call this set Q(w) (Q being the Karttunen intension). Secondly, we have answer1(Q)(w), the proposition that is the intersection of all the members of Q(w). This constitutes the weakly exhaustive true answer to the question. Thirdly, we have answer2(Q)(w), which is the strongly exhaustive answer to the question and which is (almost) the same as the denotation that G&S assign to questions. An important question that arises then is whether we really need all three of these notions. Couldn't we just use the strongly exhaustive answer2, as G&S have argued so forcefully throughout their work? In this section we will address this question, arguing that having all three notions allows us to adopt a more flexible theory that takes into account cases in which insisting on strong exhaustivity gives rise to truth conditions that appear to be stronger than is supported by our intuitions. In taking this position we have been greatly inspired by Heim (1994) who also provides many of the arguments discussed in this section.
Our discussion is primarily intended to motivate a flexible approach to exhaustivity, rather than to argue against a specific alternative like G&S's. The points we are going to make are not necessarily problematic for G&S

[2]G&S don´t treat degree questions explicitly.

[3]In addition to strong exhaustivity, G&S use another argument against Karttunen's theory, namely the distinction between what they call the de re and the de dicto readings of the head noun of the wh-phrase. We do not deal with this important issue in this paper, because it is orthogonal to the problem of exhaustivity. We hope to work out a suggestion in future work.

when taken individually - G&S explicitly discuss and account for some of them, in particular the mention-some interpretations. We nonetheless think that the global picture that emerges supports a rich and flexible system which provides a range of interpretations for questions with various degrees of exhaustivity, because of its greater overall simplicity and elegance. In addition, at certain points we will present facts that we do judge problematic for G&S, or in fact any theory that treats strong exhaustivity as a property of the basic question denotation. We will mention that explicitly in each case.

6.1. Embedded questions without strong exhaustivity

As Heim points out, although it is possible to define answer2 in terms of answer1, and answer1 in terms of the Karttunen denotation, this is crucially a one-way street. When we have only answer2 (or in fact any strongly exhaustive question interpretation), it's not possible to get back answer1 or the Karttunen denotation. So in a certain sense, answer2 contains less information than answer1 and the Karttunen denotation. For instance, because a question and its negation impose the same partition on the set of possible worlds, (30a) and (30b) will have the same answer2, but their Karttunen denotation and answer1 will generally differ:[4]

(30) a. Who was at the party?
b. Who was not at the party?

One potential problem for strong exhaustivity will therefore be question-embedding verbs which discriminate between an embedded question and its negation. A case in point are emotive factives such as *surprise* (Berman 1991, Heim 1994). (31b) may very well be true although (31a) is false, for instance in a situation in which everyone who was at the party was expected to be there by the speaker, but some people who were also expected to be there did not show up:

(31) a. It surprised me who was at the party.
b. It surprised me who was not at the party.

This can be captured easily if we have the notion of answer1 at our disposal, but not if the only thing we have is answer2.
Another example illustrating the same problem are propositional attitude verbs which refer to ways of conveying information. The verbs we have in

[4]To see this, suppose that Mary, Sue and Jane were at the party and Bill, Graham and Marc were not, and that they are all the people in the context. Then answer1((30a))(w) will be (i), and answer1((30b))(w) will be (ii).
(i) λw[Mary+Sue+Jane were at the party in w]
(ii) λw[Bill+Graham+Marc were not at the party in w]
Answer2((30a),w) will be (iii), and answer2((30b),w) will be (iv):
(iii) λw[answer1((30a))(w) = λw'[Mary+Sue+Jane were at the party in w']]
(iv) λw[answer1((30b))(w) =λw'[Bill+Graham+Marc were not at the party in w']]
However, these two sets of possible worlds will be identical, since whenever the answer1 to (30a) will be the proposition that Mary, Sue and Jane were at the party, the answer1 to the negated question will be that the complement of Mary, Sue and Jane in the universe of discourse were not at the party.

mind include *tell, read, write down* and *list.* These verbs seem to have two distinct senses (although it's not at all clear whether we are dealing with an actual ambiguity here), a transparent and a non-transparent one (cf. Heim 1994). On the transparent sense of *read*, reading who was at the party implies reading who was not at the party. This is the sense that G&S seem to have in mind when they argue for strong exhaustivity. However, although we acknowledge the existence of the transparent sense, we believe that there also is a sense in which (32a) and (32b) are not equivalent - and in fact tend to think this is the sense in which this class of verbs is ordinarily understood:

(32) a. John read/wrote down who was at the party.
b. John read/wrote down who was not at the party.

We agree with Heim (1994) that there is an interpretation for these verbs (which is still fairly transparent, but not completely) in which for instance *tell* means something like 'cause one to know answer2 by uttering answer1'. The point is that in this sense the verbs would make use of answer1 as well as answer2.

A third case in which answer1 seems to play a role in embedded wh-constructions is based on the semantics of the noun *answer* (Heim (1994)). She notes that (33) may be true in a situation where John just happens to know a proposition which constitutes the weakly exhaustive answer to the embedded question, even if he is not aware that it is the weakly exhaustive answer.

(33) John knows the answer to the question who was at the party.

So suppose that Mary and Sue were the only party guests then (33) is true if John knows the propositions that Mary and Sue were at the party, even if he believes (wrongly) that others attended the party as well. The noun *answer* must therefore mean answer1. But because answer1 cannot be retrieved from answer2, this implies that the embedded question itself cannot be strongly exhaustive.

6.2. Mention-some readings

Another argument showing that sometimes questions are not even weakly exhaustive can be based on what G&S call the mention-some interpretation of questions (see especially G&S 1984, chapter 6). Some examples which favor the mention-some interpretation are the following:

(34) a. John knows where you can buy the New York Times.
b. Mary told me how to get to the train station.

(34a) for instance has a reading on which it is true even if John isn't able to provide a complete list of places where one can buy the NYT, but only one particular location, say, the newsstand at the train station. G&S account for the existence of the mention-some interpretation in terms of disjunctions of questions -- an analysis which we don't have the space to discuss here. What is relevant for our present purposes is that the mention- some interpretation can be straightforwardly captured if we can avail ourselves of the Karttunen interpretation of the embedded question. On the mention some

interpretation (34a) will be true iff John knows at least one of the propositions in the Karttunen denotation of the embedded question. Note that the mention-some interpretation represents a case in which not only strong, but also weak exhaustivity fails.
An important question is when the mention-some interpretation is actually available. For many speakers it only seems to be possible in principle if the question contains a modal element of some sort. We don´t know how to explain this apparent influence of modality. Other factors that obviously play a role in the availability of mention-some interpretations are of a pragmatic nature, including considerations of plausibility and world knowledge.

6.3. (Non-)exhaustivity markers

Yet another argument in favor of an approach which recognizes the existence of non-exhaustive readings derives from the use of various linguistic expressions to explicitly mark a question as being understood either exhaustively or non-exhaustively. One such marker is the expression *for example* in (35):

(35) Who for example was at the party last night?

By adding *for example* the speaker makes explicit that she will be satisfied with a non-exhaustive (in the weak sense) answer to the question. *For example* cannot easily occur in embedded questions. However, there are other non-exhaustivity markers in other languages that can. In Dutch we find *zoal* (see (36)), and in German its cognate *so* (see (37)).

(36) Jan wil weten wie er zoal (niet) op het feest waren.
Jan wants know who there zoal (not) at the party were
"John wants to know who for example were (not) at the party"

(37) Hans will wissen, wer so (?nicht) auf dem Fest war.
Hans wants to know who so (not) at the party was
"John wants to know who for example were (not) at the party"

What these sentences express is that Jan/Hans wants a few representative examples of people who were at the party. While this interpretation is a rather vague one, it is clearly non-exhaustive. Although we will not provide a formal interpretation for non-exhaustivity-markers, we believe that intuitively their existence shows that questions in natural language in principle have the option of being interpreted non-exhaustively. The question can be marked to enforce this interpretation.

In addition to non-exhaustivity markers we find expressions in natural language which can be used to indicate exhaustivity. German has the word *alles* which has exactly that function (Reis 1992, Beck to appear), and in Dutch we find the corresponding *allemaal*:

(38) Hij weet wie er allemaal op het feest waren.
He knows who there all at the party were
"He knows who all were at the party"

(39) Er weiss, wer alles auf dem Fest war.
He knows who all on the party was
"He knows who all were at the party"

To these examples from our own native languages we add (40) from a dialect of Irish English discussed by McCloskey (to appear):

(40) a. What all did you get for Christmas?
b. What did you get all for Christmas?

What these expressions do is force a (weakly) exhaustive interpretation of the question in which they are contained. They are incompatible with a mention-some interpretation:

(41) Hans weiss wo man alles/ueberall die NYT kaufen kann
Hans knows where one all/everywhere the NYT buy can
"Hans knows where all you can buy the NYT."

In contrast to (34a), (41) does not have the mention-some interpretation and can only be interpreted exhaustively. It should be pointed out though that *alles* and *allemaal* do not force strong exhaustivity, which explains why they are not incompatible with the class of verbs mentioned earlier like *surprise* which disprefer a strongly exhaustive interpretation:

(42) Es hat mich überrascht, wer alles auf dem Fest war.
it has me surprised who all at the party was
"It surprised me who all was at the party."

We suggest the following semantics for *allemaal/alles/all*:

(43) $alles(Q)(w) = \lambda p[p=\cap(Q(w))]$

Alles operates on a question denotation and gives us the weakly exhaustive interpretation, i.e. the set containing answer1. Since we propose to deal with mention-some interpretations via the elements in the Karttunen denotation, from (43) there will be no way back to a real mention-some interpretation. The only element in the set denoted by the question is already weakly exhaustive.
We believe that just like non-exhaustivity markers such as *for example* exhaustivity markers like German *alles* pose a challenge to a theory that uniformly gives every question an exhaustive interpretation. If the basic meaning of questions already were an exhaustive one, exhaustivity markers would be superfluous and the question with the exhaustivity marker should have exactly the same interpretation as the corresponding question without it. However, we feel that this is not the case: (41) differs in meaning from (34a) in that the former does not allow a nonexhaustive interpretation whereas the latter does. A rigid approach to exhaustivity will have no way to deal with this difference (for instance the G&S approach to the mention-some interpretation could not, in our judgment, predict that (41) does not have a mention-some interpretation, since *alles* could make no difference to the original question interpretation).

6.4. Degree questions with *at least/at most*

The next argument gets us back to the issue of degree questions. Consider the paradigm in (44):

(44) a. Wieviele Leute waren da?
How many people were there
"How many people were there?"
b. Wieviele Leute waren mindestens da?
How many people were at least there
"How many people were there at least?"
c. Wieviele Leute waren hoechstens da?
How many people were at most there
"How many people were there at most?"

The intuition is clear that (44a-c) mean something different. This holds also for the embedded case:

(45) a. Hans weiss, wieviele Leute da waren.
Hans knows how many people there were
"Hans knows how many people were there"
b. Hans weiss, wieviele Leute mindestens da waren.
Hans knows how many people at least there were
"Hans knows how many people were there at least"
c. Hans weiss, wieviele Leute hoechstens da waren.
Hans knows how many people at most there were
"Hans knows how many people were there at most"

(45b) and (45c) are actually a bit odd. We will come to a tentative explanation for that in a minute.
For (45a) to be true, Hans has to know the exact number of people who where there. For (45b) to be true, he has to know a reasonable lower bound of the number of people who were there, for (45c) a reasonable upper bound. So for example if in fact 86 people were there, and Hans knows that definitely no more than 90 people were there, one could truthfully utter (45c).
For the following formal discussion we will assume that *at least* and *at most* mean exactly what they normally do, namely (46)

(46) a. at least n (N) (P) <=> $card(\lambda x[N(x) \& P(x)]) \geq n$
b. at most n (N) (P) <=> $card(\lambda x[N(x) \& P(x)]) \leq n$

(47) a-c are the Karttunen denotations of (44) a-c:

(47) a. $\lambda p \exists n[p(w) \& p=\lambda w'[\exists x[people(x) \& card(x)=n \& were_there(x)(w')]]]$
b. $\lambda p \exists n[p(w) \& p=\lambda w'[card(\lambda x[people(x) \& were_there(x)(w')]) \geq n]]$
c. $\lambda p \exists n[p(w) \& p=\lambda w'[card(\lambda x[people(x) \& were_there(x)(w')]) \leq n]]$

Now suppose that actually 86 people were there. Then (47) a-c are the sets given in (48) a-c:

(48) a. { 86 people were there, 85 people were there, ...}
b. { at least 86 people were there, at least 85 people were there, at least 84 people were there, ...}
c. { at most 86 people were there, at most 87 people were there, at most 88 people were there, ...}

Applying answer1 to these sets will result in the following propositions:

(49) a. λw[86 people were there in w]
b. λw[at least 86 people were there in w]
c. λw[at most 86 people were there in w]

This would mean that for Hans to know how many people were there at most, he would have to know that at most the actual number of people were there. This is not the result we intuitively want: It is sufficient for Hans to know that definitely no more than a reasonable upper bound of the actual number of people were there. The same holds for (45b). The ordinary G&S interpretation runs into the same problem.[5]
What is going on here? We think that in (45b,c) the mention-some interpretation is the only one that makes sense. An exhaustive interpretation of any kind will always lead to unintuitive results in that the resulting interpretation predicts truth-conditions that are too strong. So technically (44)b,c and (45)b,c are just more instances of a mention-some interpretation. We have discussed them separately because (i) the data are quite interesting by themselves, and (ii), because they show that non-exhaustivity in the case of degree questions will be non-maximality and non-minimality, and that that is in fact possible in degree questions. Another example demonstrating this might be (50) in an appropriate context (e.g. an artist wanting to make a realistic life-size sculpture of a polar bear).

(50) How tall can a polar bear be?

The enforced mention-some interpretation might be what makes (45)b,c, odd: A predicate like *know* seems to favour exhaustive interpretations. So in order to interpret (45b,c), one might have to use a slightly disfavoured way of combining the question meaning with *know*.

7. Two approaches to flexible exhaustivity

We have reviewed a number of arguments, partly taken from the existing literature, which show that questions do not uniformly receive a (weakly or strongly) exhaustive interpretation. Jointly and separately, these arguments undercut an approach in which exhaustivity is built directly into the basic meaning of the question. However, we are also convinced by G&S's arguments that at least in some cases (strong) exhaustivity is called for, especially when we are dealing with an embedding verb like *know*. We therefore conclude, following Heim (1994), that a flexible approach to exhaustivity is called for, one in which the basic denotation of questions is a

[5]Note that here also, a maximality operator would give the wrong results: (45b) would come out as (45a), while (45c) is undefined.

non-exhaustive one, but where exhaustivity may arise as a result of several factors that are so to speak external to the question itself. We believe that the three formal notions that we have discussed in this paper (the Karttunen denotation, answer1, and answer2) may play a key role in articulating such an approach. The general approach we advocate immediately raises several important questions as to when and how exhaustivity of either variety comes into play. We do not have a definitive answer to these questions, but in the remainder of this paper we would like to explore two possible ways one may go about answering them. The two approaches have in common that they assign to the question the non-exhaustive Karttunen denotation as its basic interpretation. They differ however in the way in which weak and strong exhaustivity come into play.

7.1. Lexical semantics of question-embedding predicates

On the first approach, exhaustivity is built into the meaning of certain question-embedding predicates. So for instance, we can distinguish the exhaustive interpretation of the verb *know* from its mention-some interpretation as follows:

(51) a. $know_{exhaust}(Q)(x)(w)$ iff $know_{prop}(answer2(Q)(w))(x)(w)$
b. $know_{mentionsome}(Q)(x)(w)$ iff $\exists p[Q(w)(p) \ \& \ know_{prop}(p)(x)(w)]$

Here $know_{prop}$ is the denotation of the propositional attitude verb *know* that takes a *that*-complement.
$know_{exhaust}$ and $know_{mentionsome}$ are relations between a person and a question intension (and a possible world) which are defined in terms of $know_{prop}$. Whether we do this with a meaning postulate or by means of lexical decomposition is immaterial for our present purposes. A person x stands in the $know_{exhaust}$-relation to a question-intension Q in a world w iff x stands in the $know_{prop}$-relation to answer2(Q)(w) in w. x bears the $know_{mentionsome}$-relation to Q in w iff in w x stands in the $know_{prop}$-relation to at least one of the propositions in Q(w).
Similarly, we can account for the contrast between the "transparent" and the "non-transparent" sense of *write* as follows:

(52) a. $write_{transp}(Q)(x)(w)$ iff $write_{prop}(answer2(Q)(w))(x)(w)$
b. $write_{nontransp}(Q)(x)(w)$ iff $write_{prop}(answer1(Q)(w))(x)(w)$

7.2. Type shifts

The second approach treats the operations that turn the Karttunen denotation into either answer1, answer2, or a mention-some interpretation as type-shifting operations that turn a set of propositions (the Karttunen denotation) into a proposition. Type-shifting is triggered whenever there is a mismatch between the type of argument required by the embedding predicate and the basic type of the embedded question. Some predicates like *wonder* (G&S's intensional verbs) inherently take a complement of the type of a question-intension, <s,<p,t>> (where p is the type of a proposition, <s,t>). For such verbs, no type-shifting is necessary. Other verbs - which are extensional in G&S's sense - take propositional complements, of type p. If their complement is an embedded question, it is necessary to apply a type shift. We can now view answer1 and answer2 as type-shifting operations which

lower an object of type <s,<p,t>> to one of type p. To get the mention-some interpretation, a somewhat more complex operation is needed. Intuitively, what we want to do is to existentially quantify over the propositions that are in the extension of the embedded question. Technically one way of implementing this is by turning the question-intension into a generalized quantifier over propositions, of type <<s,<p,t>>,t>. Let's call this operation answer3:

(53) answer3(Q)(w) = λP[∃p[P(w)(p) & Q(w)(p)]]

This generalized quantifier can now be combined with the question-embedding verb by the standard techniques that are used to combine a quantified object-NP with an extensional verb (QR, quantifying-in, storage, or type shifts).

(54) know (answer3(Q)(w))(John)(w)
iff λP∃p[P(w)(p) & Q(w)(p)] (λw'λq[know(q)(John)(w')])
iff ∃p[know(p)(John)(w) & Q(w)(p)]

In principle, each of the three typeshifting operations is always available, but ideally various external factors can be identified to explain why in fact we find only certain specific readings in many examples. At this point, we have no concrete proposals to make as to what these factors might be, however.
The relative pros and cons of the two approaches we have sketched will be clear. In the first approach it is possible to specify exactly for each (extensional) question-embedding predicate what sort of interpretation it gets. But since in a sense this is done by brute force, this approach gives up the hope of achieving a really explanatory account of when we get which reading. The second approach aims to provide just that, but it would be fair to say that at this point this is not much more than a promissory note.
It is possible that the truth is somewhere in the middle, i.e. that there is a lexical as well as a grammatical possibility to type shift. Certain shifts seem pretty much lexicalized (e.g. *know* plus answer2), others seem to apply in a more flexible way. (55) might be a case in point, since *believe* does not normally take an interrogative argument.

(55) You won´t believe who I met last night.

7.3. Unembedded questions

We have not considered so far the case of unembedded questions and their relation to answers. We will not formally relate them in this paper. We believe rather uncontroversially that the relation is in essence pragmatic. So when a speaker S asks a question Q, a hearer under most circumstances (though not all) infers that S wants to know a satisfactory answer to Q. What is satisfactory for S depends on the specific context. It may be answer1, answer2, or just an example answer, an element of Q(w). The hearer will provide what information s/he can in accordance with Gricean maxims, so in particular the answer will be true and as informative as necessary, but no more than that. If the context suggests that S will be satisfied by an example answer, a hearer will not bore S with a complete list. On the other hand, it seems a natural strategy to provide a maximum of

information, that is, answer1. This in turn carries the implicature that the hearer really was as informative as possible, i.e. given an answer A it is often inferred that A is the complete answer. S thus concludes answer2 from answer1. If this inference is not desired, the answer provided must be marked as partial (by adding something like *for example, among others,...*).
What we have sketched above should extend to degree questions in particular: If for instance John in fact read five books, no well-informed person would answer the question "how many books did John read?" with "John read four books", since the answer is, while true, not the most informative one. Giving the most informative answer involves no extra trouble, so it should be very highly preferred. Since the answer actually given carries the implicature that it is the most informative answer, such an answer would even be very misleading.
In the case of unembedded questions, there are various other formal relations possible between question and answer. See for example G&S for discussion. What is important for our purposes is that we believe that the Karttunen denotation will work ok for unembedded questions as well as embedded questions. That is, we believe that the various relations of pragmatically "good" answer to Q can be defined given the information that the Karttunen denotation Q provides. Although we do not formally show this, we feel justified in that assumption since from the Karttunen denotation the G&S denotation (more or less) can be derived, and G&S have demonstrated in detail the usefulness of that in defining question-answer relations.

8. Summary and conclusion

We have suggested a Karttunen semantics for degree questions which captures the maximality effect without using a maximality operator in the question semantics. In our semantics, the maximal answer has no special status in the question denotation at all. This gives us a greater flexibility, which, we claim, is needed. This is demonstrated by degree questions requiring a minimum answer, or ones requiring an explicit list of all true members of the Karttunen question denotation. What type of answer we get is determined by semantic properties of the question predicate: Upward scalar predicates determine minimum answers, downward scalar predicates determine maximum answers, and non-scalar predicates require complete lists. We always get the most informative answer. We therefore link questions to their most informative answers by a notion of answerhood defined in Heim (1994), thus accounting for maximality without assuming a maximality operator, plus minimality, plus list answers. In the context of individual questions, our strategy to this point can account for weak exhaustivity, but not strong exhaustivity. This can be remedied by defining a second notion of answerhood (cf. Heim).
We are thus left with a fairly rich system of semantic notions concerning interrogatives: the Karttunen denotation, answer1 and answer2. We believe that there are good arguments that this rich system is in fact useful: There seem to be predicates using answer1 instead of or in addition to answer2, and we think that there are ones which use the Karttunen denotation either directly or indirectly.
By way of presenting arguments for our richer system, we have considered a number of data indicating flexibility in the way an interrogative sentence gets interpreted. The intended interpretation can be marked syntactically in certain ways (*alles, for example, zoal*), but it need not be. It is clear that

semantic theories of interrogatives need to capture this flexibility. It is less clear what is the best way to go about that task.
Our overall strategy is to have a rather weak question denotation, from which other information is recovered by applying certain semantic operations to it. Starting from a Karttunen denotation, it is possible to strengthen the information contained in it in various ways. This gives one a range of interpretational possibilities, all of which, we argue, play a role in natural language. We hope that external factors can be identified that determine which interpretation is actually chosen. G&S pursue the opposite strategy: They start with a question denotation that contains the maximum of information one can get (i.e. what we end up with in the "optimal" case). There are thus cases in which they have to get rid of surplus information, so to speak.
We believe that our strategy is more suited to treat natural language interrogatives. It allows for a greater flexibility in what we think is a more natural way. However, we acknowledge that this is in a sense a plausibility argument, and that technically it might be feasible to do things the other way around.
In addition to that, we have argued that semantic notions might play a role that are not, as far as we can see, recoverable in a G&S system. If our arguments are considered valid, we have a real advantage over a G&S system (or in fact any system that treats strong exhaustivity as a property of the basic semantics of interrogative sentences). In this respect, our proposal can be seen as a defense of a Karttunen semantics for interrogatives.

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Discourse Grammar and Dynamic Logic[1]

Martin H. van den Berg

1 introduction

Nowadays little will doubt that the large scale structure of natural language is as recursive as the sentence-syntactic structure that is the subject of more traditional linguistics. These large scale structures, which is the subject of our investigations, include stories, texts, conversations, doctors consults, etc. Following common practice, we will denoted any such structure with the general term **discourse**.

In this paper, I will make a first stab at combining the discourse parsing mechanisms developed in (Prüst 1992, Prüst et.al. 1994) and applied in (Polanyi and Scha 1983, Polanyi 1988) with the dynamic logic for plurals presented at the Ninth Amsterdam Colloquium (van den Berg 1994) and in my dissertation (van den Berg 1996a).

1.1 Anaphora

In the formal treatment of anaphoric linking, two separate stages have to be distinguished. First, the antecedent of the anaphor (pronoun or definite) has to be identified (*resolution*). Second, the way the antecedent gives a value to the anaphor has to be formalized (*use*). Existing theories of discourse semantics provide us with a very powerful formal apparatus to formalize the latter, but the former is not touched upon at all by Dynamic Predicate Logic and its descendants (DPL), and only marginally by Discourse Representation Theory (DRT), which concentrates on cases where the value picked up is identical to an existing antecedent or to one that is easilly calculated from existing antecedents.

Let me start with explaining what the formalism for the *use* of anaphors is. First, let me characterize what an anaphor is. The following quote from the "competition" gives a definition:

> *"The term anaphoric is used here in a liberal way. It is not narrowly restricted to cases where an expressions can be linked via coindexing with a preceding phrase. It applies to all cases where an expression is used in such a way that its interpretation depends on one or more foregoing phrases."*
> [fn. 1. Groenendijk et.al. 1995]

The last of this I agree with. In fact, this should be read very loosely. Even when only part of a meaning depends on one or more foregoing phrases, that part is said to be anaphoric. This includes not only pronouns and definites, but also contexts sets of arbitrary quantifiers, the way events relate to each other in narratives and embedding an probably also VPs. Although the latter can

1. This paper forms a pair with (van den Berg and Polanyi 1996).

often just as well be described in terms of copying of sufficiently abstracted out material.

The way I implement this is by a dynamic logic that is capable of dealing with plurals in a generalized quantifier setting, and can deal with the fact the elements of plural objects often have relations with elements of other plural objects; relations that can be carried by the anaphoric links. This logic, called FDPL, is defined in (van den Berg 1996, *and an earlier version in 1994*). This is essentially a coindexing approach, hence my disagreement with the first part of the quote. The following examples illustrate the kind of nominal cases this logic can deal with correctly:

(1) $A^1_{\cdot}$ *man walked in the park.* He^2_1 *whistled.*
(2) $Some^1_{\cdot}$ *people entered the bar.* $They^2_1$ *ordered a beer.*
(3) $Some^1_{\cdot}$ *people entered the bar.* The^2_1 *men sat down at the bar and* the^3_1 *women went to the juke-box.*
(4) $John^1_{\cdot}$ *went to Paris.* $Mary^2_{\cdot}$ *went to Paris too.* $They^3_{1+2}$ *had a great time.*
(5) *If* $a^1_{\cdot}$ *student marries* $an^2_{\cdot}other_{2\neq1}$ *student,* the^3_{1+2} *man usually finishes earlier, but* the^4_{1+2} *woman has higher grades.*
(6) $Every^1_{\cdot}$ *car that entered the village was dirty. Especially* the^2_1 *wheels were full of dirt. If* it^3_1 *had a backdoor,* it^4_3 *too was a mess.*

This serves illustrates the notation: superscripts represent newly introduced indices (referents), subscripts represent old (bound) indices. When an expression is co-indexed with more than antecedent, a + sign is put between them; the^4_{1+2} *woman* (5) means that the woman is selected from the set constructed from referents 1 and 2, and the unique woman in this set is introduced as referent 4. Note the use of "$\neq$" $an^2_{\cdot}other_{2\neq1}$ to express that the introduced referent 2 is not equal to 1. A dot (·) is used to denote an underspecified position, this will correspond to a unification variable in the semantics.

1.2 Prüst et.al. 1995

In (Prüst et.al. 1995), from now on denoted by DGVP, it is observed, that the translation of language into logic results in type logical expressions that have a syntactic form that encode the information structure of the translated expression, without the idiosyncrasies the actual language syntax.

Because that paper deals with VP-anaphora, a modified copying approach is all that is needed to explain anaphoric linking. For example, in

(7) *John went to Paris. Bill did too.*

merely copying *went to Paris* into *did* gives the right meaning:

(8) *John went to Paris. Bill went to Paris.*

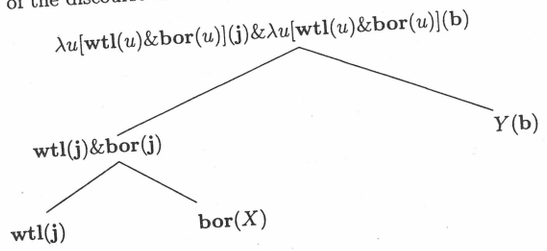
The DGVP formalism is versatile enough to encode a simple form of abstraction:

(9) *John went to the library. He borrowed a book on computer science. Bill did too.*

The unification mechanism is defined in a way that the first two sentences are taken together as *John went to the library and borrowed a book on computer science* and then the relevant part is copied to *did*:

(10) *John went to the library. He borrowed a book on computer science. Bill went to the library and borrowed a book on computer science too.*

In terms of the discourse tree:



This is best illustrated by a simplified[2] example:

(11) *John gave Mary a book.*
(12) $\exists x(x = \mathbf{j}, \exists y(y = \mathbf{m}, \exists z(\mathbf{b}(z), \mathbf{g}(x, y, z))))$
(13) *A book was give to Mary by John.*
(14) $\exists z(\mathbf{b}(z), \exists y(y = \mathbf{m}, \exists x(x = \mathbf{j}, \mathbf{g}(x, y, z))))$

Which shows that formalism is capable of distinguishing between active and passive sentences.

1.3 The logical language

The logic I will use in this paper is a simplified version of FDPL, dynamic binding is encoded in a language that has a simple predicate logical syntax with generalized quantifier symbols. For every variety (distributive, pseudo-distributive,

2. One important simplification is the use of first order quantifier logic rather than type logic. Although expressions translate into type-logic, most examples can just as well be given in quantifier logic (predicate logic with quantifier symbols). This has the advantage of being more compact and more easily readable. The order of application of the quantifiers is supposed to mirror the order of application of lambda's in the actual translation. For example, the first case given here might have as the actual translation (depending on the actual translation formalism chosen):

$\lambda P_1.\exists x[x = \mathbf{j} \wedge P_1(x)](\lambda S\lambda T\mathbf{g}'(x)(S)(T)(\lambda P_2.\exists y[y = \mathbf{m} \wedge P_2(y)])(\lambda P_3.\exists z[(b)(z) \wedge P_3(z)]))$

$\mathbf{g}' := \lambda\mathbf{P}\lambda\mathbf{Q}[\lambda x\mathbf{Q}(\lambda y\mathbf{P}(\lambda z\mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{z})))]$

The complexity of which illustrates why it makes sense to cheat a bit and use simplified formulas.

collective and singular) there exists, where needed, a quantifier with that variety. The only difference is the way context sets are identified with earlier introduced dynamic variables. Example (1) is translated as:

(15) $\exists^s x_1 \subseteq \cdot[\mathbf{m}(x_1), \mathbf{w}(x_1)] \wedge \forall x_2 \subseteq x_1[x_2 = x_2, \mathbf{wh}(x_2)]$

Because in this paper, we are really dealing with the syntactic form of these formulas, I will only give a sketch of a simplified version of FDPL.

Definition (Semantics of FDPL) *Dynamic formulas are interpreted as 3-valued relations between states, states are assignments (G, H, K) of dynamic variables to plural objects (sets). The definitions have a form in which we define when a formula is defined and when it is true, rather than in terms of false and true.*

The interpretation is relative to a model $\langle \wp(E), \mathcal{I} \rangle$.

(16) $[\![Px_1 \ldots x_n]\!]^d := \{< G, H > \mid G = H\}$

$[\![Px_1 \ldots x_n]\!]^+ := \{< G, H > \mid G = H \& < G(x_1), \ldots, G(x_n) > \in \mathcal{I}(P)\}$

(17) $[\![x \subseteq y]\!]^d := \{< G, H > \mid G = H\}$

$[\![x \subseteq y]\!]^+ := \{< G, H > \mid G = H \& G(x) \subseteq G(y)\}$

(18) $[\![\varepsilon_x]\!]^d := \{< G, H > \mid G \approx_x H \& G(x) = \emptyset\}$

$[\![\varepsilon_x]\!]^+ := [\![\varepsilon_x]\!]^d$

(19) $[\![+\phi]\!]^d := [\![\phi]\!]^+$

$[\![+\phi]\!]^+ := [\![\phi]\!]^+$

(20) $[\![\phi \wedge \psi]\!]^d := [\![\phi]\!]^d \circ [\![\psi]\!]^d$

$[\![\phi \wedge \psi]\!]^+ := [\![\phi]\!]^+ \circ [\![\psi]\!]^+$

(21) $[\![\neg\phi]\!]^d := \{< G, H > \mid G[\![\phi]\!]^+ H \text{ or } G[\![\neg\phi]\!]^+ H\}$

$[\![\neg\phi]\!]^+ := \{< G, H > \mid G[\![\phi]\!]^d H \ \& \ \neg\exists K \ G[\![\phi]\!]^+ K\}$

Quantifiers are defined by essentially writing out the traditional definition using the dynamic possibilities and the following maximization operator (again: simplified).

(22) $G[\![\mathsf{M}_{x\subseteq y}(\phi)]\!]^d H := G[\![\mathsf{M}_{x\subseteq y}(\phi)]\!]^+ H$ or $(G[\![\phi]\!]^d H$ and not $G[\![\phi]\!]^+ H)$

$G[\![\mathsf{M}_{x\subseteq y}(\phi)]\!]^+ H := G[\![\phi]\!]^+ H$ and for no K, L

$K \approx_x G, L \approx_x H (K(x) \subseteq G(y) \& K[\![\phi]\!]^+ L)$

This defines a formula $G[\![\mathsf{M}_{x\subseteq y}(\phi)]\!]^d H$ which is true for an input G, giving an output H, iff $G[\![\phi]\!]^d H$ holds between them and for no larger set (within the set $G(y)$) ϕ is also true. Then quantifiers are defined by the following:

(23) $Qx \subseteq y(\phi, \psi) := \varepsilon_{x'\subseteq y} \wedge \varepsilon_{x\subseteq x'} \wedge \mathsf{M}_{x'\subseteq y}(\phi[x'/x]) \wedge \mathsf{M}_{x\subseteq x'}(\psi) \wedge Q(x', x)$

x' is the dynamically introduced referent denoting the CN anaphor, x is the subset of these that denotes the normal anaphor. Non of these need be unique, though the CN usually is. I refer to (van den Berg 1996a) for details.

1.4 Simple Examples

As a simple example, take

(24) A^1_1 *man walked in the park. He*$^2_1$ *whistled.*

The local translations of these sentences are $\exists x \subseteq \cdot(\mathbf{m}(x), \mathbf{witp}(x))$ for the first, and $\exists y \subseteq \cdot(y = y, \mathbf{wh}(y))$ for the second sentence. The discourse rules are then applied, and the second translation is default unified with the first resulting in $\exists y \subseteq x(y = y, \mathbf{wh}(y))$. So how can we set up the rules to achieve this? Note one thing that will become a problem very soon. In DGVP, we essentially used term-unification, mapping identical positions in the formulas on identical positions. But here the referent x when introduced is in a different position (to the left in $x \subseteq z$) than when it is used (to the right in $y \subseteq x$). So we have to remember to built in something that takes care of this.

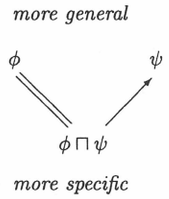
2 Unification Structures

To use FDPL in a unification grammar, we have to add unification variables X. We will add variables for all objects of the language X^e for unification variables that take the position of dynamic variables, $X^{p,n}$ to take the position of predicates of arity n, Q^q to take the position of quantifiers, and X^t to take the position of formulas. I will always omit the superscripts of unification variables, because their use will make their type clear.

2.1 Unification

2.1.1 Unification Ordering

Suppose we have an algebra of expressions $\mathcal{A}$ and a relation of unification $\sqcap$ on the terms of $\mathcal{A}$. Then we also have ordering on the terms: $\phi \preceq \psi$ iff $\phi = \phi \sqcap \psi$. in words: ϕ is at least as specific as ψ, if when we unify ϕ with ψ, we do not get anything new:



This can also be put around, if we have a pre-ordering, than the unification of ϕ and ψ is the most general object χ that is is at least as specific as both ψ and ϕ, if it exists. The generalization of ϕ and ψ is the most specific object χ that is at least as general as both ψ and ϕ. Neither of these need be unique. If the ordering was derived from a notion of unification, the unification one gets by applying this definition is the original one. This means that any notion of unification can be defined by giving the corresponding ordering. Because defining an ordering involves two expressions, whereas defining unification directly involves three, the ordering is easier to define.

2.1.2 Ordering of Syntacto-Semantic Structures

The ordering of terms of dynamic type logic, written as $\phi \preceq \psi$ is defined indirectly. First I define an ordering $\phi \preceq_\xi \psi$, which means that ϕ is at least as specific than ψ given some substitution (by ξ, called the unifier) of values for unification variables and some renaming of variables (also done by ξ). We then say that $\phi \preceq \psi$ iff $\phi \preceq_\xi \psi$ for some unifier ξ that only renames bound variables (but may substitute anything).

Definition (Unifiers) *A unifier $\xi \in$UNIF is a function that maps unification variables of a type α on meaningful expressions of the same type.*

Unifiers are used in the definition of the ordering. To simplify the below definitions, we do not assume transitivity of the ordering these definitions define. This means that the following is slightly more complex than you might expect:

Definition (Ordering Rules) *The ordering on expressions of dynamic type logic, relative to a unifier, is defined indirectly. First an ordering $\preceq_\xi$ is defined, which orders expressions relative to a unifier ξ, then a global ordering $\preceq$ is defined by*

(25) $\phi \preceq \psi$ *iff there is a unifier ξ and expressions $\phi_1 \ldots \phi_n$ such that $\phi \preceq_\xi \psi_1 \preceq_\xi \cdots \preceq_\xi \psi_n \preceq_\xi \psi$*

When $\phi \preceq \psi$, the expression ϕ is said to be at least as specific as ψ.

There is one last ingredient to the logic. It is assumed that there is some basic ordering $\leq$ on the predicates, called the Aristotelian ordering. For example

woman$\leq$**person**$\leq$**animate**$\leq \ldots$. This can be used to define a generalization between **walk(john)** and **run(mary)** that results in **move(person)**.

The definition is in two parts. First, basic rules are given to impose a term-unification-like ordering on the terms. Then extra rules are given to impose semantically contentful relations.

Predicates are ordered using the pre-given Aristotelian ordering: for P, P' predicates:

(26) $P \preceq_\xi P'$ *iff* $P \leq P'$.

Variables are ordered by identity, but an extra condition serves to make substitution by unification variables simulate real abstraction (cf. (2.1.2) below): it makes that if a dynamic variable is "replaced" by a unification variable, it has to be replaced throughout the formula that we are comparing. For x, x' dynamic variables:

(27) $x \preceq_\xi x'$ *iff* $x = x'$ *and for no unification variable* X, $\xi X = x$.

Quantifiers should be the same:

(28) $Q \preceq_\xi Q'$ *iff* $Q = Q'$.

Unification variables do what they are supposed to do. For ϕ any expression, X a unification variable of the same type:

(29) $\phi \preceq_\xi X$ *iff* $\xi X = \phi$.

Finally, the recursive rules finish the term-unification part. For P, P' n-ary predicates, $t_1, \ldots, t_n, t'_1, \ldots, t'_n$ terms (dynamic variables or unification variables that take their position):

(30) $P(t_1, \ldots, t_n) \preceq_\xi P'(t'_1, \ldots, t'_n)$ *iff* $P \preceq_\xi P'$ *and* $t_1 \preceq_\xi t'_1$ *and* $\ldots$ *and* $t_n \preceq_\xi t'_n$.

For ϕ, ψ, ϕ', ψ' formulas, R, R' logical connectives:

(31) $\phi R \psi \preceq_\xi \phi' R' \psi'$ *iff* $\phi \preceq_\xi \phi'$ *and* $\psi \preceq_\xi \psi'$ *and* $R = R'$.

For s, t, s', t' terms:

(32) $(t \subseteq s) \preceq_\xi (t' \subseteq s')$ *iff* $t \preceq_\xi t'$ *and* $s \preceq_\xi s'$

Finally, for quantifiers:

(33) $Qt \subseteq s(\phi, \psi) \preceq_\xi Q't' \subseteq s'(\phi', \psi')$ *iff* $Q \preceq_\xi Q'$ *and* $(t \subseteq s) \preceq_\xi (t' \subseteq s')$ *and* $\phi \preceq_\xi \phi'$ *and* $\psi \preceq_\xi \psi'$.

In the case of VP-anaphora term unification-like results from the above would already enable us to give some simple examples, but as already remarked above, for NPs we have the problem of the different positions for new and old referents. The following rule, which I will call the **new/old** rule, deals with this:

(34) $(x \subseteq t) \preceq_\xi (X \subseteq x)$ *provided* $\xi X = x$.

Note how this rule interacts with the rule for ordering variables. We are now bound to a unifier that assigns x to X, so we will never have $x \preceq_\xi x$. Hence if $\phi \preceq_\xi \psi$ is achieved by applying this rule, ψ cannot contain x (of course, not all x need be replaced by X).

This completes the discussion of the basic ordering rules, resulting in basic term-unification rules (plus the rule for coordinating "old" an "'new" positions). Here are some simple example that can be seen immediately.

(35) $\exists^s x_1 \subseteq X'[\mathbf{m}(x_1), \mathbf{w}(x_1)] \preceq \exists^s X \subseteq x_1[\mathbf{m}(X), \mathbf{w}(X)]$

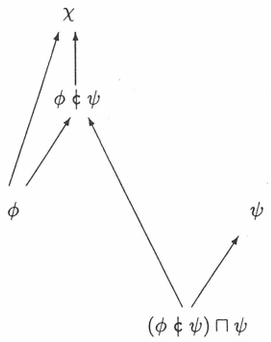
(36) $\exists^s X \subseteq x_1[\mathbf{m}(X), \mathbf{w}(X)] \preceq \exists^s X \subseteq x_1[Y, Z]$

(37) $\forall x_2 \subseteq x_1[x_2 = x_2, \mathbf{wh}(x_2)] \preceq \forall x_2 \subseteq Y'[x_2 = x_2, \mathbf{wh}(x_2)]$

The next step is to define the default unification mechanisms that are used to add information to an underspecified meaning.

2.2 Common Denominator and Default Unification

Although the above pre-order is given to be able to define unification and generalization, other relations also suggest themselves. The following diagram defines a notion called **common denominator**:



Informally: the common denominator of ϕ relative to ψ is that part of ϕ that unifies with ψ. How this works for term-unification is easy to see. Compare corresponding positions in the formulas, when two disagree, replace them by a unification variables, otherwise, take the value in ϕ. In general, the definition is as follows:

Definition (Common Denominator) *The common denominator of ϕ relative to ψ, written as $\phi \natural \psi$, is the most specific expression, more general than ϕ (or equivalent), that unifies with ψ.*

The common denominator need not be unique. A notion of default unification can be defined using this.

Definition (Default Unification) *The default unification of ϕ with ψ, written as $\phi \gg \psi$, is the unification of ψ with $\phi \natural \psi$.*

As generalization and unification, default unification need not be unique[3]. This now gives us enough to deal with a large number of simple examples. Note that example (1), already discussed as an example to inspire us, works directly. Suppose we have the meaning of the first sentence:

(38) *A man*[1] *walked in the park.*
$\exists^s x_1 \subseteq Z[\mathbf{m}(x_1), \mathbf{w}(x_1)]$

And we interpret the second sentence, to begin with out of context:

(40) *He whistled.*
$\forall x_2 \subseteq Y[x_2 = x_2, \mathbf{wh}(x_2)]$

We now default unify this with the earlier sentence. As it turns out, because we added the extra rule making this not exactly term-unification, the common denominator is not unique in an interesting, non-trivial way. If this were only term-unification, the only common denominator would be

(42) $QY \subseteq Z[Z', Z'']$

leaving not much content, unifying this with the second sentence and conjoining the result with the first gives:

(43) $\exists^s x_1 \subseteq Z[\mathbf{m}(x_1), \mathbf{w}(x_1)] \wedge \forall x_2 \subseteq Z[x_2 = x_2, \mathbf{wh}(x_2)]$

Although, as we will see in the discussion of example (3), this is not a reading we can dismiss in general, a more reasonable resolution follows from applying the new/old rule: the common denominator when this is applied becomes

(44) $QY \subseteq x_1[Z', Z'']$

Now unifying this with the second sentence and conjoining the result with the first makes:

(45) $\exists^s x_1 \subseteq Z[\mathbf{m}(x_1), \mathbf{w}(x_1)] \wedge \forall x_2 \subseteq x_1[x_2 = x_2, \mathbf{wh}(x_2)]$

which is the more common reading: the second sentence introduces x_2 as the largest subset of x_1 that is identical to itself (i.e. $x_2 = x_1$) and claims that the elements of this set (= the man) whistles.

3. The complexity of both operations is of the same order as the unification operation.

3 Discourse Grammar

The above is anecdotal at best. The next thing to do is to give actual rules for constructing discourse, and apply these to the examples. Only when that works can it be said that we have an actual discourse grammar. to do this, we first have to decide what the actual objects of the grammar are.

3.1 Discourse Units and Discourse Operators

In sentence syntax, the basic components are words of many different categories and constructs made of these. The components of discourse consist of control material, called *discourse operators* (DOs), and the actual content: propositional material called *discourse constituent units* (DCUs).

In the Linguistics Discourse Model (LDM), originally proposed by Polanyi, and worked out by her and a number of other authors, the interpretation of discourse is a recursive structural and semantic representation of a source text in terms of the DCU and DO components, built by incrementally parsing the text.

Discourse Operators are typically expressed as lexical items or idiomatic expressions. They include vocatives, affirmations/disaffirmations and many types of particles, exclamations, connectives, etc. Discourse Operators express the attitude of the speaker towards what is being said, meta-information about the state of the discourse, specification of the relationships among DCU's, etc. To achieve all this, DO's form a mixed bunch. There are discourse operators that take one or more DCU's and form a new, construct, DCU (these are called DCU-modifiers), DO's that change a DCU in a DO and stand alone DO's like *anyway* and idiomatic expressions.

An elementary DCU is a (minimal) utterance encoding a single event or state of affairs indexed for physical and socially constructed situation of utterance (real or modeled), modality, specificity, point of view, etc. Typically, an elementary DCU takes the form of a simple sentence or clause but that need not be the case. Semantically, elementary DCU's are simple quantifications ("predications').

Discourse Constituent Units are defined recursively over elementary DCU's, by applying discourse operators to one or more DCUs. Important special cases are embedding a DCU under an operator that changes one or more of the indices and composition of two DCUs, forming *coordination*, *subordinations* and *binary attachments*. I refer to (Polanyi and van den Berg 1996) for further details.

3.2 digression: DCU representations are not DRS's

In the linguistic discourse model, DCUs are typically represented by boxes. Take the example

(46) *John entered the bar. He sat down at a table.*

This corresponds to the (simplified) DCU:

Interaction: Speaker, Addressee *Speech Event:* Conversation *Genre Unit:* Story *modality:* Realis *event:* t_0
John entered the bar (at t_1 in t_0) *John sat down at a table* (at $t_2 > t_1$ in t_0)

This contains all kind of information: that it is a story, who tells the story and to whom, whether this is about the real world and when, etc. The most prominent part, and a part that gets special attention, is the actual meaning, here represented by the sentences themselves. This looks a bit like a DRS in DRT, but the resemblance is only accidental. This can be seen if it is made more formal as follows (assuming some suitable dynamic logic, like the one defined in the next section):

Interaction: Speaker = **s**, Addressee = **a** *Speech Event:* Conversation *Genre Unit:* Story *modality:* Realis
$\exists x(\mathbf{john}(x), \exists t[t \subseteq t_0, \mathbf{enter}(t, x))]$ $\exists y \subseteq x(x = y, \mathrm{NEXTt}[\mathrm{t} \subseteq \mathrm{t_0}, \mathbf{sitdown}(\mathrm{t}, y)])$

Here we see the similarity between this notation and standard interpretations of logical formulas (I suppressed the temporal/event variables for the moment):

(47) $[\![\exists x(\mathbf{john}(x), \mathbf{enter}(x)) \wedge \exists y \subseteq x(x = y, \mathbf{sitdown}(y))]\!]^{\mathbf{s},\mathbf{a},\mathbf{w_0},\ldots}$

In DRT the boxes are part of the language that makes the dynamics of the logic tick. Here, the boxes are simply a convenient way of writing down a meaning together with its context. This makes easier to think of the components of the box as features in a grammar, which is what I will do below. I will, however, refrain from using the box notation. In (Polanyi and van den Berg 1996), only the end result of a parse is written down, both as the resulting parse tree and the interpreting box. Here, the focus lies on the construction of that tree, and boxes on every node would have to be considered. I will therefore use an abbreviated notation, omitting all unchanged (or orthogonally given) features, in particular the index material in the top half. This is yet another simplification.

3.3 Semantic Features

The first question that may be raised is: what semantic values are stored as features of the DCU and how are they stored? For the purposes of this paper, two features are important. First of all, there is the semantics itself, which is

stored in the Sem attribute. Next to this, there is a feature that encodes what the DCU *is about*, called the Schema attribute. Essentially, it is the common denominator of all still relevant sub-DCUs[4]. for example, for

(48) *John comes from San Francisco. He lives downtown.*

The schema is (the logical equivalent of) *John LOCATED*, where *LOCATED* is some appropriate, more general predication. In the case of subordination, the subordinated clause does not contribute to the schema, as in the case of interruption:

(49) *John comes from San Francisco. You met him yesterday. He can tell you were you can get the best dim sum.*

The schema of the first two sentences is the first sentence itself, the second sentence does not make a contribution.

The schema of a basic DCU is identical to the semantics, how the schema is calculated for a complex DCU has to be defined in the discourse rules.

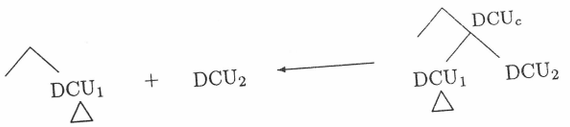
Now we know what the features are, we can look at how the actually look. As already discussed before, they will have the form of **syntacto-semantic** structure, logical expressions stored as syntactic objects and with operations on them as syntactic objects (as the unification order defined above illustrates).

The grammar formalism defined in DGVP and repeated in a simpler form below, will work on such structures independently of the choice of logical language provided (1) the syntax of the language is rich enough to encode different information structures as different expressions and (2) the translation is consistent and assigns like structures to like expressions. Any recursively defined translation scheme will satisfy the latter, and the former is easily satisfied as long as the construction of the expression can be read back from the logical formula that results from this construction, as is the case for classic Montague Grammar and formalisms that succeeded it. The predicate logic language used here is sufficient to deal with the simple examples discussed here, but when more complex phenomena, like topic/focus, active/passive and sub-sentential relations are to be treated, we will need type-theory. I will leave that to a later place.

3.4 Right-attachment

When a new DCU is added to the discourse structure being constructed, it can only be added to a DCU that is still being constructed. This means that the DCU has to be attached to the right edge of the discourse tree. There is essentially only one way of attaching a DCU:

4. In (Polanyi and van den Berg 1996), the "meanings" written in the tree are examples of the Schema not the Sem atribute, as the following two examples will also illustrate: In a subordination, the subordinated material disappears. This is exactly what the Schema is supposed to encode.



Within this scheme, a number of rules are defined, corresponding to different ways of constructing DCU_c out of the two component DCU's.

3.5 List-rules

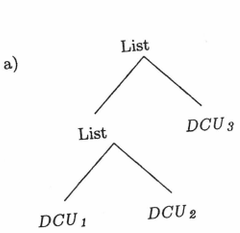
List-rule

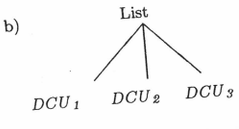
(50) $$\begin{bmatrix} \text{Dcu} & = & \mathbf{R}\text{–List} \\ \text{Sem}_f & = & \text{Sem}_1[\![\mathbf{R}]\!]\text{Schema}_1 \gg \text{Sem}_2 \\ \text{Schema}_f & = & \text{Schema}_1 \gg \text{Sem}_2 \end{bmatrix} \Rightarrow \begin{bmatrix} DCU_1 \\ \text{Sem}_1 \\ \text{Schema}_1 \end{bmatrix} \mathbf{R} \begin{bmatrix} DCU_2 \\ \text{Sem}_2 \\ \text{Schema}_2 \end{bmatrix}$$

Application of this rule is called **list construction** when:
- $\text{Schema}_1 \not\diamond \text{Sem}_2$ is non-trivial (does not only contain variables) and
- $\mathbf{R} \in \{and, or, \ldots\}$ a coordination, $[\![\mathbf{R}]\!]$ its meaning.

Application of this rule is called **list extension** when:
- $\text{Schema}_1 \not\diamond \text{Sem}_2 = \text{Schema}_1$,
- $DCU_1 = \text{List}$, and
- $\mathbf{R} \in \{and, or, \ldots\}$ a coordination, $[\![\mathbf{R}]\!]$ its meaning.

In the case of extension, the discourse tree, which has the form (a) below, is often written as (b), because the two nodes have equivalent features (except, obviously, the semantics):





And similar for n-ary lists.

We already saw an application of the list rule, although it wasn't yet clear that it was, in the discussion of example (1).

Dcu	=	List
Sem	=	$\exists^s x_1 \subseteq Z[\mathbf{m}(x_1), \mathbf{w}(x_1)] \wedge \forall x_2 \subseteq x_1[x_2 = x_2, \mathbf{wh}(x_2)]$
Schema	=	$QY \subseteq x_1[Z', Z'']$

Dcu	=	X
Sem	=	$\exists^s x_1 \subseteq Z[\mathbf{m}(x_1), \mathbf{w}(x_1)]$
Schema	=	$\exists^s x_1 \subseteq Z[\mathbf{m}(x_1), \mathbf{w}(x_1)]$

Dcu	=	Y
Sem	=	$\forall x_2 \subseteq Z[x_2 = x_2, \mathbf{wh}(x_2)]$
Schema	=	$\forall x_2 \subseteq Z[x_2 = x_2, \mathbf{wh}(x_2)]$

Example (2) has exactly the same structure.

Example (3) brings in something new, and illustrates list-extension. What is new is that it shows that the dividing line between sentence grammar and discourse grammar may not be where you expect it to be. This example consists of three DCUs:

(51) (a) *Some*$^1_1$ *people entered the bar.* (b) *The*$^2_1$ *men sat down at the bar and* (c) *the*$^3_1$ *women went to the juke-box.*

this is parsed in two steps. First DCU_{a} and DCU_{b} are combined,

Dcu	=	List
Sem	=	$\exists x_1 \subseteq Z[\mathbf{p}(x_1), \mathbf{e}(x_1)] \wedge \forall x_2 \subseteq x_1[\mathbf{m}(x_2), \mathbf{sd}(x_2)]$
Schema	=	$QY \subseteq x_1[\mathbf{p}(Y), Z']$

Dcu	=	X
Sem	=	$\exists x_1 \subseteq Z[\mathbf{p}(x_1), \mathbf{e}(x_1)]$
Schema	=	$\exists x_1 \subseteq Z[\mathbf{p}(x_1), \mathbf{e}(x_1)]$

Dcu	=	Y
Sem	=	$\forall x_2 \subseteq Z[\mathbf{m}(x_2), \mathbf{sd}(x_2)]$
Schema	=	$\forall x_2 \subseteq Z[\mathbf{m}(x_2), \mathbf{sd}(x_2)]$

Where I assumed that **m** (men) constitutes a subsort of **p** (people): $\mathbf{m} \leq \mathbf{p}$. Now this DCU is combined with the third DCU:

Dcu	=	Y'
Sem	=	$\forall x_3 \subseteq Z'[\mathbf{w}(x_3), \mathbf{wjb}(x_3)]$
Schema	=	$\forall x_3 \subseteq Z'[\mathbf{w}(x_3), \mathbf{wjb}(x_3)]$

Resulting in the top DCU:

Dcu	=	List
Sem	=	$\exists x_1 \subseteq Z[\mathbf{p}(x_1), \mathbf{e}(x_1)] \wedge \forall x_2 \subseteq x_1[\mathbf{m}(x_2), \mathbf{sd}(x_2)] \wedge \forall x_3 \subseteq x_1[\mathbf{w}(x_3), \mathbf{wjb}(x_3)]$
Schema	=	$QY \subseteq x_1[\mathbf{p}(Y), Z']$

Note that this is indeed a list extension; the schema does not change:

$$(QY \subseteq x_1[\mathbf{p}(Y), Z']) \notin (\forall x_3 \subseteq Z'[\mathbf{w}(x_3), \mathbf{sd}(x_3)]) = (QY \subseteq x_1[\mathbf{p}(Y), Z']).$$

Also note that the Sem-formula expresses the right meaning: There is a set x_1 of people that entered, the maximal subset (x_2) of men of x_1 sat down at the bar and the maximal subset (x_2) of men of x_1 went to the juke-box.

4 More Complicated Linkage

Examples (4–6) constitute co-indexing that cannot be resolved by the above system, for different reasons.

4.1 Sum-linkage

Let's look at (4).

(4) *John*$^1_{\cdot}$ *went to Paris. Mary*$^2_{\cdot}$ *went to Paris too. They*$^3_{1+2}$ *had a great time.*

The first two sentences constitute an example of a common denominator that has to be calculated not using the new/old rule, but plain term-unification. The construct-DCU resulting from the first two sentences will then become:

Dcu	=	List
Sem	=	$\exists^s x_1 \subseteq Z[\mathbf{j}(x_1), \mathbf{toP}(x_1)] \wedge \forall x_2 \subseteq Z'[\mathbf{j}(x_2), \mathbf{toP}(x_2)]$
Schema	=	$QY \subseteq Z''[\mathbf{p}(Y), \mathbf{toP}(Y)]$

But this DCU does not contain enough information to resolve the link needed for the third sentence. Default unification of the schema of this DCU with the semantics of the third DCU does not add the required anaphoric link.

What is needed here is another refinement of the ordering, resulting in a refinement of unification. Look at the new/old rule again:

$$(x \subseteq t) \preceq_\xi (X \subseteq x) \text{ provided } \xi X = x.$$

The informal content of this rule is as follows:

Suppose a quantification holds for x in t, then if we "rename" x by X throughout the formula, then we can choose X from x and the quantification still holds.

This cannot be easily maintained formally because (1) the "bound" variable is a unification variable and (2) this a dynamic logic, so after stating a quantification, the formulas in its scope may have introduced variables that will conflict with a repetition of the same quantification with only X renamed. all other variables would have to be renamed too.

This informal analysis inspires us to try another ordering rule. This is called the summed-extension rule[5].

$$(X \subseteq x) \preceq_\xi (X \subseteq x + y)$$

Note that this rule has no direct constraint. However, the X has to come from somewhere, and this means an implicit constraint (from a previous rule) of $\xi X = x$.

If we are allowed to apply this rule, one possible common denominator of the meanings of *John*$^1_{\cdot}$ *went to Paris* and *Mary*$^2_{\cdot}$ *went to Paris too* is:

(53) $\exists X \subseteq x_1 + x_2[P(X), \mathbf{toP}(X)]$

5. The expression $x \subseteq y + z$ is true in an assignment G when $G(x)$ is a subset of $G(y) \cup G(z)$.

And the default unification of this with the semantics of the third sentence *They*$^3_{1+2}$ *had a great time.* gives the required resolution:

(54) $\exists x_3 \subseteq x_1 + x_2[x_3 = x_3, \mathbf{hagt}(X)]$

4.2 Virtual Discourse Referents

Example (5) constitutes an intriguing variation on the previous analysis:

(5) *If a*^{1} *student marries an*^{2} *other*$_{2\neq1}$ *student, the*$^3_{1+2}$ *man usually finishes earlier, but the*$^4_{1+2}$ *woman has higher grades.*

Whereas in the case of (4), the referent $x_1 + x_2$ is created and named x_3, here the $x_1 + x_2$ referent is never really introduced, it only exists during its use, it is a virtual referent. Despite this strange fact, this does correspond to what is the naive analysis of this sentence. When asked, the usual paraphrase of this sentence that people give is something like

(56) *If a student marries another student, the male one of them usually finishes earlier, but the female one of them has higher grades.*

where the discourse referent is real (*them*).

Another problem that is made obvious by this example is that the border between sentence and discourse grammar is even vaguer than already suggested. Here two referents from one sentence have to be combined. It is not clear how to that, without adding more of sentence grammar to our discourse grammar. This goes far beyond the scope of this paper. Note that when we were able to derive $x_1 + x_2$ from the first sentence, the analysis is essentially a combination of the analyses of (3) and (4).

4.3 Indirect Linkage

Example (6) illustrates that linkage need not always be mediated by the subset relation.

(6) *Every*^{1} *car that entered the village was dirty. Especially the*$^2_1$ *wheels were full of dirt. If it*$^3_1$ *had a backdoor, it*$^4_3$ *too was a mess.*

However, except for the obvious observation that this means that the $\subseteq$-relation that until now was hardwired into the local meaning may itself have to be calculated relative to the context, this example is fairly similar to the previous ones. Instead, I will discuss the following example, that Asher discussed during the conference[6]. He observed that the sentences

(58) (a) *A woman, who had just flown in from New York, went into the bar.*
(b) *A family joined her.*

6. It only have my memory of this at the moment, so I am reinventing the example, hopefully without missing some essential points.

Can be followed by a number of different kinds of sentences. First we can do a simple sum-reference

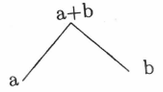
(59) *They started talking.*

which can be treated identically to the sum-referents discussed earlier. However, in

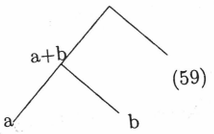
(60) *They were going to Majorca on a holiday.*

The plural pronoun(*they*) seems only to refer to the family. This of course guided by world knowledge. Someone just flown in from New York will not likely be leaving immediately on a holiday (although it is not impossible). But that just tells us something about the different weights the probability of different resolutions will get from world-knowledge, it does not add anything to the discussion of what readings are possible grammatically.

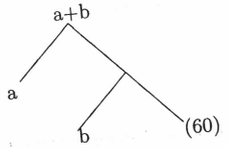
Both above analysis are straightforwardly analyzed in terms of the discourse grammar rules discussed so far. Assuming the first two sentences form a discourse tree as follows:



Then the (59) sentence attaches at the (a+b) node,



whereas the (60) sentence attaches to the (b) node.



However, there are also sentence that seem to defy our analysis:

(61) *The woman was carrying a black purse.*
(62) *The women started talking.*

The problems with these are related, but the solutions may appear at first to conflict in an interesting way.

In (61) something strange seems to be happening. We pick up the woman, but that referent is not introduced in a right edge. So what is going on here? The answer is, that this is just another example of sum-reference. The construct DCU, consisting of the first two sentences introduces two referents One (x_1) refers to the woman, the other (x_2) refers to the family *as a plural object*. This means that we can select the woman as the maximal subset x_3 of $x_1 + x_2$ that satisfies **w**. The sum only contains two objects, only one of which is a woman:

(63) $\forall x \subseteq X(\mathbf{w}(x), \mathbf{cbp}(x))$

But The women intended in (62) seem to be the one woman mentioned explicitly with the female members of the family. But this last group is not explicitly present in the context. There is only a discourse referent to the woman, and a discourse referent to the plural-entity which is the family. To be able to unwrap this entity, we have to have the member-of relation in our language. We will write this as $\in^{\cdot}$. Using this I will write

(64) $G(\downarrow x) = G(x) \cup \{d \mid \exists e \in G(x).d \in^{\cdot} e\}$

The **flattening** of the set referred to by x[7]. Defining a flattened subset relation is now easy

(65) $x \sqsubset y$ *is true when* $G(\downarrow x) \subseteq G(\downarrow y)$.

And when this is an allowed restriction to quantifiers, the translation of (62) is easy

(66) $\forall x \sqsubset X(\mathbf{w}(x), \mathbf{st}(x))$

and x can then be resolved to the sum of the woman and the family.

This example, and the original example (6) suggests that for anaphoric linkage the subset-relation is only a default-relation, which can be overridden when an explicit other relation is given (as in a phrase like *the wheels of the car*) or there is a prominent implicit relation while the default would result in an ill-defined link.

5 Conclusions

The above only scratches the surface of discourse resolution. I do think that what it shows is that fairly simple means, not deviating too much from term-unification, can already achieve a number of different cases. Also, the way of

7. To be a real flattening, this should be defined as a limit, because theoretically, plural entities may themselves contain plural entities, but for the moment, I'll assume, as simplification, that they don't.

defining unification in terms of an ordering that we can have independent intuitions about gives a controlled way of introducing new properties. Because of the theory neutral way default unification and the discourse rules formulated in terms of it are defined, changing logical representation will not easily break the rules.

5.1 Simplifications

There were a number of simplifications and things that were brushed under the table. Non of these are fatal, most of them would contribute a long discussion for which there is no room.

5.1.1 Type-logic

I restricted myself to a predicate logical language, rather than the type-logic that a sentence grammar would actually produce. This was done for several reasons. First of all, the type-logic would not contribute any extra insights for the cases discussed here. Predicate logic can already express enough structure to distinguish different positions/roles in the translation. Type logic *is* needed to explain why quantifiers are constructed the way they are, but that is well-known territory. Another reason not to mess with type logic is the extra complications that dynamic logic poses. Although in no way really problematic (cf. Muskens 1990) it would constitute a long and really different discussion.

5.1.2 Other Rules

in this paper, I only discussed the construction and extension rules of DGVP. The other rules function similarly, and can also be applied directly to noun-phrase cases[8]. We expect no surprises in this direction.

5.2 Further Research

Real work has to be done in a number of directions. First and foremost, the artificial distinction between sentence and discourse grammar makes that phenomena that play on the borderline of these cannot be easily understood. There is some hope that this will change in the near future, because it has become clearer and clearer, that as Bod remarks in the last chapter of his 1995 dissertation, structural disambiguation in sentence grammar needs clues from the context to work. A more union of sentence and discourse seems to be something that both sides might profit from.

Another extension is in the direction of other anaphora than nominal anaphora. The similarity between events and noun-phrase for example should lead to a similar treatment in discourse grammar.

8. All this is not surprising. When we discussed the forms the rules should take, we discussed as many NP-examples as VP-examples to argue for the particular forms they take.

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Discourse Structure and Discourse Interpretation[1]

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1 Introduction

In this paper, we will address the well known problem of the apparent non-linear flow of temporal information in narrative. We will show that many seeming exceptions to the *Strong Narrative Hypothesis* (SNH) (Polanyi, 1987) which states that the order of event clauses in a text is isomorphic with the order of events in the model of the text can be accommodated within the SNH when the hierarchical structure of the narrative text itself is understood. In our discussion in the present paper we will use the formal machinery of the *Linguistic Discourse Model* (LDM), a recursive discourse grammar developed by Polanyi and Scha in a series of papers (Polanyi and Scha 1984; Polanyi 1987, 1988, 1996; Prüst, Scha and van den Berg 1994; Scha and Polanyi 1988) to suggest a proper treatment of some classes of putative counterexamples to the SNH. In doing so, our aim is to demonstrate the utility of the LDM framework in helping to identify and resolve complex issues in the structure and interpretation of natural discourse.

2 Interpretation of temporal relations in discourse

A common view on narrative texts is to regard them essentially as linear sequences of event clauses which push the "position" on the time-line further and further. These **event clauses** encode non-habitual, specific states of affairs which are to be interpreted as instantaneous. are distinguished in the world's languages from non-event clauses morphosyntactically, using tense marking, particles, etc. (Labov and Waletzsky 1967; Labov 1972; Longacre, 1976; Kamp and Rohrer 1983; Hinrichs 1986; Dowty 1986). This view of narrative restated in Polanyi (1987) as **The Strong Narrative Hypothesis** (SNH) holds that:

The order of event clauses in a narrative is isomorphic with the order of events in the semantic model of the text.

The temporal interpretation of a simple *flat narrative* text such as (1):

(1) (a) *John came to the door.*
(b) *He left the groceries in the kitchen.*
(c) *I put them away.*
(d) *Then he left.*

1. This paper forms a pair with (van den berg 1996b).

can be schematically represented as in (2) in which the text of (1) expresses some states of affairs in some world W and interpreting T is the incremental process of constructing a model C from the information in the text.

(2)

(in W)	*T1*	*T2*	*T3*	*T4*
	COME	*LEAVE*	*PUT AWAY*	*EXIT*
(in text)	*a*	*b*	*c*	*d*
(in C)	*come*	*leave*	*put away*	*exit*
	t1	*t2*	*t3*	*t4*

The semantic representation of the narrative itself is represented informally in a simple box notation in which the top part of the box shows the social and textual indexes at which interpretation occurs and the bottom part of the box shows the propositions interpreted at these indexes in the order they would be assigned in a recursive data structure resulting from a procedure of formal discourse interpretation[2].

(3)

interaction–Speaker/Addressee *speech event–Conversation* *genre unit–Story* *modality–realis* *polarity–positive* *point of view-p*
John came to the door (at t_1) John left the groceries (at t_2) I put the groceries away (at t_3) John left (at t_4)

While the simple narrative in (1a–d), a simple listing of events in time, presents no problems to the SNH, other structures commonly found in narratives such as *flashbacks, causal elaborations, commentaries, governed main clauses, reported speech, repairs and entropic actions* present more difficulty. In the remainder of this section, we will give an example of each of these phenomena in turn and discuss the problems each presents for the initial formulation of the SNH. In the next section, we will present an overview of the machinery of the *Linguistic Discourse Model* (LDM) which we will use to analyze each of these classes of apparent counterexample in order to determine how the SNH as given above must be amended to provide a more adequate generalization of the flow of narrative time.

2. Cf. (van den berg 1996b), section 3 for some more details.

2.1 Putative counterexamples to the SNH

Probably the best known apparent counterexample to the SNH is the *flashback*. As exemplified in the following example, there are cases when the forward progression of time is interrupted and an event clause or sequence of event clauses intended to be interpreted as having taken place previous to the last mainline timepoint is interpolated between event clauses intended to be interpreted at immediately adjoining timepoints:

(4) *John came home and left the groceries.* ***He'd stopped for gas on the way home. And then visited his grandmother.*** *I put them away and then he left.*

Using the box notation, the interpretation of this narrative containing a flashbacked interlude is represented as shown below:

(5)

Interaction: Speaker, Addressee *Speech Event: Conversation* *Genre Unit: Story*	
Episode 1 (at t_0) *modality: realis* *point of view: 1*	
John came to the door (at t_1) John left the groceries (at $t_2 > t_1$)	*Flashback Episode* (at $t'_0 < t_2$) *modality: realis* *point of view: 1*
I put the groceries away (at $t_3 > t_2$) John left (at $t_4 > t_3$)	John stopped for gas (at t'_1) John visited his grandmother (at $t'_2 > t'_1$)

In examples such as the following discussed in Moens and Steedman (1988) and Lascarides and Asher (1991) among other places, a second event may provide an explanation of how a first event came about:

(6) *John fell. Bill pushed him.*

In the cases of *causal subordination* as we term examples of this sort, the second event clause receives an interpretation in which it preceded the first in time. When we return to discuss this example later on in the paper, the interpretation indexes which may appear merely decorative and straightforward in the two preceding examples will take on a critical role.[3]

In the case of *commentary* as shown here, an entire explanatory dialogue may be interpolated in the middle of a linear temporal sequence:

(7) *John came home and left the groceries.* ***Oh. Why did he do that? Because he'd stopped for gas on the way home. And then visited his grandmother.*** *Anyway, I put them away and then he left.*

3. This is the case for all of the subsequent classes of counterexample many of which function as explanations as well and so we shall omit using our boxes until we take up the examples again in some more detail.

The interpolated dialogue itself may contain event clauses, as this example does, which appear to provide counter examples to the SNH since an event mentioned in as interpolated dialogue need not follow in time the last event before the interrupting discussion took place, The temporal order of the text which we intuitively assign to this discourse does not seem to obey the constraints of the SNH.

Similar problems may arise in the interpretation of text including *reported speech* such as the following example:

(8) *John came home and left the groceries. I asked why he was late. He said,* ***I stopped for gas on the way home. And then visited my grandmother.*** *So I said "OK". I put them away and then he left.*

In this case, as was the case for our *commentary* example, the segment of *reported speech* may itself contain event clauses. However,just as in the *commentary*, we do not interpret **John's stopping for gas.** in this example as having taken place after **John's leaving the groceries.**

Cases of the *Governed main clause* (GMC) such as in example (9) are elaborations much as the *flashback* or *causal elaboration* cases we have already discussed. However, in GMC constructions, one event is expanded upon in a series of subsequent event clauses which are interpreted as reporting the same event at a different level of granularity so that several sub-events which had been earlier packaged together into one event are each presented alone. (See 4.2 below):

(9) *John came home and left the groceries in the pantry. Then he went to the kitchen. He got the ice cream out of the freezer.* ***He opened the door found the unopened container way in the back and grabbed it. Then he jerked it out with one powerful motion.*** *He got himself a bowl from the cupboard, served out a big helping and went to the living room to watch TV. I put away the groceries. Then he left.*

There remain two other classes of counterexamples to discuss: *repairs* and the cases of *entropic action*. *Repairs* involve the replacement of one piece of information in the semantic representation of a text by another piece of information which is encoded subsequent to the information to be replaced. In such *repairing* segments include event clauses such as example (10), once again the SNH appears to make an incorrect prediction.

(10) *John came home and left the groceries. Then he wrapped the present for his grandmother.* ***No. He called Mary. And then he wrapped the present for his grandmother.*** *Anyway, I put them away and then he left.*

However, analyzing of all of the phenomena presented so far under the LDM will allow us to salvage the SNH with relatively minor adjustments. Cases of *entropic action* such as the example below present more of a challenge for our approach.

(11) *But as soon as the Mariner, who was a man of infinite-resources-and-sagacity, found himself truly inside the whale's dark inside cupboards, he stumped and he jumped and he thumped and he bumped, and he pranced and he danced, and he banged and he clanged, and he hit and he bit, and he leaped and he creeped, and he prowled and he howled, and he hopped and he dropped, and he cried and he sighed, and he crawled and he bawled, and he stepped and he lept,, and he danced hornpipes where he shouldn't and the whale felt most unhappy indeed.*

[Rudyard Kipling. *How the whale got his throat*, p. 11]

In this case, we do not assign a unique and linearized order to the Mariner's many actions after finding *himself truly inside the whale's inside cupboards.* We assume some instances of **stumping, jumping, prowling, sighing etc. on the part of the Mariner**, but we do not assume that the text is instructing us that **a prowling** following **a jumping** and preceded **a sighing**, each of which occurred at a discrete instant in the world modeled by the text. On the contrary, we assume that a somewhat chaotic scene is being reported in which, over a period of time, the Mariner might have engaged in one or another of these activities in no set order. The *Linguistic Discourse Model* to which we now turn will provide us with many notions for accounting for all of these challenges to the SNH but to give us clues about the interpretation of *entropic action* we shall have to reach somewhat beyond formal mechanisms.

3 Overview of the Linguistic Discourse Model

The *Linguistic Discourse Model* (LDM) is designed to provide the tools needed to understand discourse syntactic and semantic processes. The model consists of a context free grammar implementing incremental discourse parsing and a language to describe the structured semantic component that results from the parsing process. Under the LDM discourse interpretation is treated as the incremental construction of a recursive data structure representing the interpretation of the "discourse".

The LDM is a theory of discourse interpretation modeled as a parser which incrementally builds a structural and semantic representation of a source text. Under the LDM, the surface structure of discourse is composed of **discourse constituent units** (DCUs) and **discourse operators** (DOs). DCUs carry propositional information (they are "of type t"), while DOs carry non-propositional information. Both DCUs and DOs are defined in a joined recursion starting with basic expressions.

An **elementary Discourse Constituent Unit** is a (minimal) utterance encoding a single event or state of affairs indexed for physical and socially constructed situation of utterance (real or modeled), modality, polarity, specificity, point of view, etc. An elementary DCU may have the form of a simple sentence

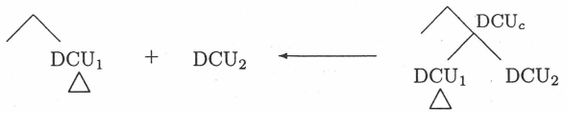
or clause but that need not be the case. They translate as *basic quantificational expressions.*

Elementary Discourse Operators express non-propositional information, specifying relationships among DCUs, the attitude of the speaker towards what is being said, meta-information about the state of the discourse, etc. These operators include *logical operators*, *vocatives*, *affirmations/disaffirmations* and many types of *particles*, *exclamations* and *connectives*. They also include *modifiers*, which make a complex DO out of a (usually elementary) DCU.

Complex DCUs are composed from elementary DCUs by:

- Coordination (which creates *lists* of many types including topic chains and narratives.
- Subordination (of "interrupting" elements whether semantically related or not).
- Binary attachment (Questions, asymmetric logical operations,...).
- Embedding under an appropriate (DCU-modifying) DO (modals, temporal modifiers,...).

In accordance with rules of discourse grammar, incoming DCUs are attached to an appropriate existing or newly created node, labelled **Coordination, Subordination** or **Binary**, on the right edge of a discourse parse tree. Only DCUs on the right edge of the parse tree are available for further processing, because these are the only once that can still be said to be "under construction":



3.1 Sentence Grammar and Discourse Grammar

The above characterization of the most basic units of discourse as consituting "elementary predications" poses a problem for the interaction of discourse grammar with sentence grammars. Traditionally, the process of parsing a discourse is split into two parts. First the discourse units —in such case usually considered to be sentences— are parsed in some standard syntactic theory of sentence grammar, and then these units are combined together following the rules of discourse grammar.

Recently, this two step approach has also come under attack from the sentence grammar side. It has been observed that disambiguation, not only of the meaning, but also of the structure of a sentence, is guided by recent

utterances (cf. Lang 1977 for observations concerning meaning, and Bod 1995 §8.2, for suggestions about structure).

From the side of discourse grammar the following examples illustrate the problem. All three contain two elementary DCUs:

(12) *John went inside. He ordered a beer.*
(13) *John went inside and he ordered a beer.*
(14) *John went inside because he needed a beer.*

In the last two case, standard sentence syntax will analyze this as one unit: there is an overlap between the realms of sentence and discourse grammar. The only solution to both the problem of context sensitivity of sentence grammar and the problem of partitioning the discourse in natural units seems to be to define one uniform grammar for both sentences and discourse. Achieving this is still far away, currently it seems best to rely on existing sentence grammars.

3.2 Discourse semantics

So far, we have not said anything specific about discourse semantics and how this interacts with the discourse construction rules. and it is not our intention here to provide a treatment of this complex subject. However, it is adressed in (van den berg 1996b), where the resolution of nominal anaphora is discussed. The approach in that paper can be adopted without much adjustment to cover the temporal cases we deal with in this paper.

4 Discourse structure and temporal interpretation

In this section, we will present analyses of the classes of apparent counterexamples to the SNH presented earlier. We will show how the LDM framework sheds light on the regulation of interpretation of time in narrative discourse as well as how an orderly interpretations of temporal development emerges from seemingly complex surface orderings of event clauses.

4.1 Discourse subordination and the flow of Narrative Time

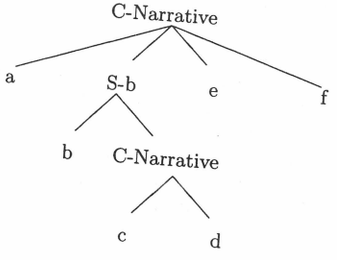
Under an LDM, many of the classes of apparent counterexamples to the SNH presented above are **subordinated** to the narrative timeline which they interrupt; the interrupted mainline is represented as a **Narrative list** at the **Coordination** node dominating the subordinated constituents while the mainline narrative remains accessible for resumption. Once closed off by attachment of a DCU to a higher node to its right, however, the subordinated "interrupting" constituent is no longer available for continuation – syntactically or semantically. In this way, the forward motion of narrative time which the SNH suggests results from a sequence of event clauses is seen to be an effect of event clauses interpreted relative to the same domain structurally (i.e. as daughters of the same dominating narrative **Coordination**) and, as we shall see, semantically

as well as only DCUs interpreted relative to the same set of indexes participate in the same narrative coordination and thus can be constituent actions on the same narrative mainline. (The classes of counter example which are not treated as discourse subordinations, **repair**, a **Binary** operation, and **entropic action** are discussed in sub-sections (4.4) and (4.5) below.

The Treatment of flashback in the LDM is straightforward. For an example like:

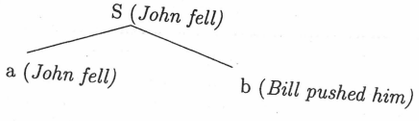
(15) (a) *John came home* (b) *and left the groceries.* (c) ***He'd stopped for gas on the way home.*** (d) ***And then visited his grandmother.*** (e) *I put them away* (f) *and then he left.*

the rules of discourse syntax assign a structure like this:



As is shown here, a flashbacked segment containing a narrative, forms its own independent narrative line. When the embedded narrative is completed, a comitant shift in interpretative indexes as signalled, in spoken language, by a shift in intonation[4] forces an abandonment of the embedded constituent and a return at the next event clause to develop the narrative mainline.

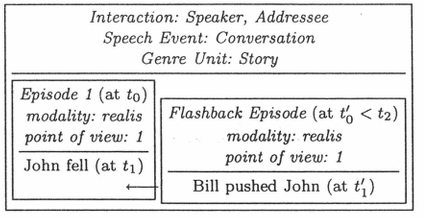
The treatment of causal elaboration in the LDM like that of the *flashback* is straightforward and hardly surprising. Causal elaboration is simply an elaborating constituent and is embedded under the appropriate **S** node in the parse tree. For the example discussed above, (a) *John fell* (b) *Bill pushed him.* the parse tree



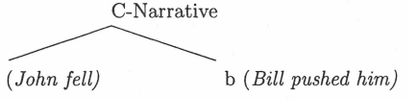
4. Often accompanied by a shift in body positioning as well.

is interpreted by the semantic component to result in the following representation:

(16)

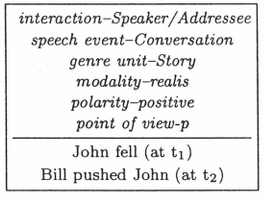


This differs from the narrative case of the same sequences of clauses in which the parse tree can be drawn as follows:



In the narrative case, the instructions to semantics result in the construction of the following representation which differs significantly from the representation for its *causal elaboration* counterpart:

(17)



In our view, there is no ambiguity between these two counterparts which consist of identical syntactic arrangements of identical morphemes. Despite the similarities in the case of these written English examples we maintain these two strings are not identical and that one need not bring in world knowledge or specialized reasoning to assign the correct readings to the strings. Our arguments are based on evidence from Dutch and prosodic evidence from English.

When we consider the translation of these counterparts into Dutch, we notice that they receive very different translations.

1. The Causal Elaboration Case

(a) *John fell.* **Bill pushed him.**
(a′) *Jan viel. Bill had 'm geduwd.*

2. The Narrative Case
(b) *John fell. Bill pushed him.*
(b′) *Jan viel. Bill duwde 'm.*

It is not possible to translate the *narrative* case with *Jan viel. Bill had 'm geduwd.* nor is it possible to translate the *causal elaboration* with {*Jan viel. Bill duwde 'm.* The inability to translate the phenomenon into Dutch[5] suggests that what we are seeing here is not some very deep universal semantic phenomenon but possibly merely a superficial phenonemon of English. Even this status, however, is rather more than the "ambiguity" deserves since when one considers spoken English rather than written English the apparent identity in form between the two cases disappears in this language as well. Prosodically, the *narrative case John fell. Bill pushed him* is realized using a listing intonation while, when these strings are communicating a case of *causal elaboration, John fell. Bill pushed him.* are prosodically realized so that the elaborating clauses are encoded with markedly lower pitch and volume.

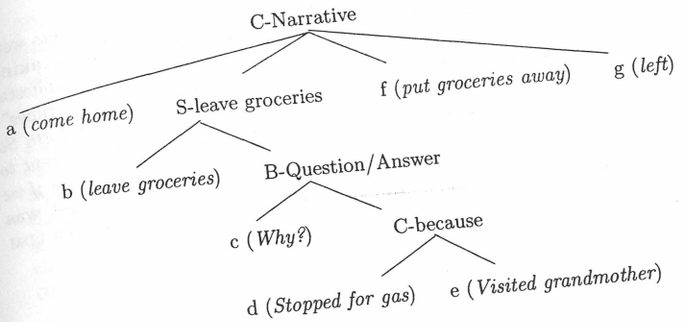
There is thus really no more "ambiguity" in these forms than in the past and present tense forms of the verb *read* in *I read* (/rid/) *a lot of books everyday* versus *I read* (/rɛd/) *a lot of books in the past.* While in written English they may be spelled identically, they are not realized phonologically in the same way. Perhaps we need not, therefore, develop complex theories of disambiguation to account for differences in assigning meanings to these two forms.

A commentary which is interpolated in the middle of a linear temporal sequence may be topically unrelated to the ongoing narrative or, as in the case of the example discussed earlier, may contain material relevant to the narrative:

(18) (a) *John came home* (b) *and left the groceries.* (c) ***Why did he do that?*** (d) ***Because he'd stopped for gas on the way home.*** (e) ***And then visited his grandmother.*** *Anyway,* (f) *I put them away* (g) *and then he left.*

In this case, the interpolated commentary will be subordinated to the interrupted narrative which will remain structurally accessible for completion. The parse tree the LDM assigns to this example is shown below:

5. Or any number of other languages.



In the cases of elaboration covered so far, the elaborated information added substantially new information to the discourse, often providing motivation for actions on the main narrative line. In the case of the *governed main clause*, the information added merely expands upon an event already known by presenting it at a greater degree of magnification so that what occurred in the narrative world can be observed in more detail.

4.2 Governed main clause

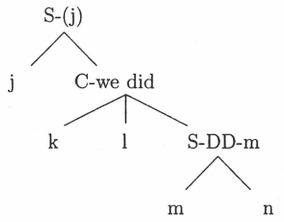
Governed main clause (GMC) constructions consist of well formed event clauses which are related to the events of the main narrative but do not advance its time line. Like *flashbacks* and *causal elaborations*, GMCs are elaborations. In narrative GMC constructions, the granularity of description of some events differs in the governing and governed structures: the governed DCUs provide a more complex, finer grained breakdown of an event reported in the structurally superior DCU. The common factor in flashbacks, explanatory/evidential DCUs, and the GMC is the shift in indexes. The SNH holds only for event clauses interpreted at the same indexes. Consider the following story adapted from Linde, 1980:

Renting the apartment

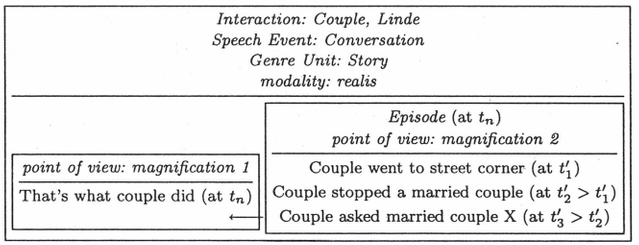
(19) (a) *We SETTLED DOWN for a cup of coffee in a drugstore.* (b) *The druggist STROLLED over to us.* (c) *and we TOLD him our story.* (d) *and ASKED him.* (e) *what to do.* (f) *He TOLD us* (g) *"STOP someone on the street,* (h) *ASK him* (i) *if the apartment is a good price."* (j) ***So that's what we did*** (k) *We went to the street corner* (l) *We stopped a married couple* (m) *We asked them* (n) *if we should take the apartment* (o) *They said* (p) *"yes"*

(20) (q) *So we WENT back* (r) *and RENTED it.*

Having read the story, let us consider some basic facts. How many times were **people stopped on the street and asked about the wisdom of taking the apartment**? Yes. Just once. But how many times are various manifestations of the **stoppings and askings** invoked? The answer is "Three times". Once in the reported speech of the druggist, once in the PRO-verbal form *So that's what we did* and once more in the elaboration on (j) —(k) *We went to the street corner* (l) *We stopped a married couple* (m) *We asked them* (n) *if we should take the apartment.* This extended narrative elaboration on **what was done** is an example of a *narrative* GMC. Since it is an elaboration, the LDM treats it as a subordinated constituent as shown below:



The corresponding semantic representation of this GMC is shown below:



Narrative structure and NP-anaphora Our analysis of the GMC supports Partee's (1984) observation of the similarity between entities and events as can be seen from the following examples:

(21) a *There is some food the freezer.*
There is some spinach. There are some noodles. And there is some delicious ice cream.

b *John took some food from the freezer. He took some spinach. Then he took some noodles. Finally he helped himself to some delicious ice cream.*

(21a) has a simple structure: first, the food is mentioned as a whole. Then an elaboration of its content is given enumerating types of food. Therefore in (21′):

21′ *There is some food the freezer. There is some spinach. There are some noodles and there is a large pink elephant.*

The pink elephant must be a type of food, which is unlikely. Accordingly, the discourse is judged marked. Similarly, in the narrative case, the event-clause *John took some food from the freezer* introduces an event, which is further specified in the subsequent narrative. Just as the list of things in (21′) cannot escape being a food because each item on the list is necessarily part of the entity the list elaborates upon, the embedded narrative can never push the time point beyond the interpretation time of the dominant event.

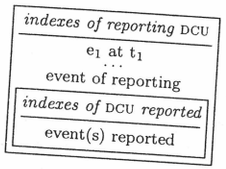
With this analysis of the GMC the narrative mainline is seen to emerge from the surface complexities of the text once the complex hierarchical structure of the discourse is revealed. The SNH continues to obtain in cases of discourse subordination it is clear to which (sub)text the constraint must apply. The case of *reported speech* may also be treated as a discourse subordination, as we shall see, with the DCU corresponding to the reporting clause dominating a structurally subordinated segment of reported speech. However, a more elegant and complete solution which treats the reporting clause as a DO is arguably preferable to the simpler DCU/DCU embedding structure.

4.3 Treatment of Reported Speech in the LDM

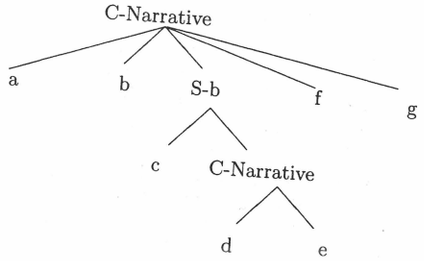
Under the simpler treatment, syntactically, the reported DCUs are embedded as the right daughters of a Subordination node of the type **Reported Speech**, semantically, the entire reported Interaction in which the reported DCUs were uttered is embedded relative to the Interaction in which the reporting event took place as is shown here[6]:

6. This is a case which corresponds to *modal subordination* as discussed by Roberts 1987 etc. The modal subordination case is to be distinguished from the case of simple structural subordination in which the modal indexes of the embedded constituent differ from that of the matrix but do not correspond to a context within its modal scope. A third case of discourse subordination under the LDM is the case of true interruption. In this case, no semantic relationship obtains between the syntactically dominant constituent and the embedded interrupting constituent. These semantic differences are reflected in the architectures of the boxes assigned to each type of subordination. In the case of modal subordination such as *reported speech*, the box corresponding to the embedded constituent is inside the box of the dominant constituent. In the case of *elaboration*, the boxes of matrix and subordinate are placed side by side with an arrow indicating the predication for which the embedded constituent provides more detail. In the interrupting case, the boxes corresponding to syntactically dominant and syntactically embedded segments are side by side. There is no arrow between them. In all cases, relationships among the indexes at the top of each box makes clear the semantic relationships among the predications.

(22)



Therefore,

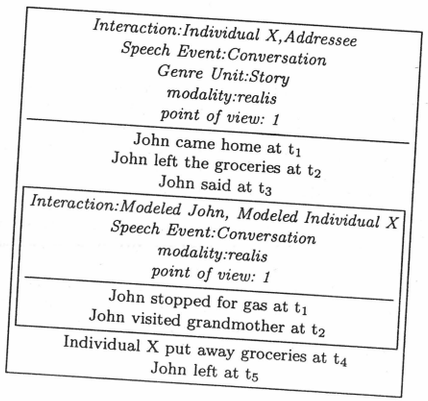


is the parse tree corresponding to example (23), *Reported Speech*:

(23) (a) *At 7 o'clock John came home* (b) *and left the groceries.* (c) *He said,* (d) ***I stopped for gas on the way home.*** (e) ***And then visited my grandmother.*** (f) *I put them away* (g) *and then he left.*

while the semantics can be represented as shown:

(24)



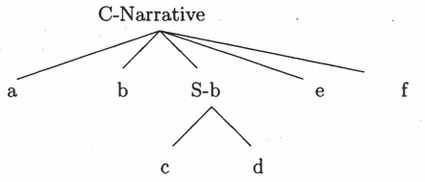
Although this treatment is adequate for the examples of *reported speech* we have been working with here, it is not adequate for the general case of reported speech in which often there is no explicit reporting clause but who is speaking is indicated tone of voice in spoken narration and change of speaker is signalled only through a simple alternation of quoted blocks on a printed page. A more unified treatment of reported speech would treat what appear to be a clausal structures (the *He said. She said*) as DOs which serve to mark a shift in indexes of interpretation at a given moment in an unfolding discourse rather than as DCUs which represent events on the event line. Developing this line of analysis further is beyond the scope of the present paper, however.

4.4 Treatment of Repair in the LDM

Unlike the other apparent counterexamples to the SNH which were uniformly treated from a discourse syntactic point of view as subordinations, **repair** is handled as a **Binary** relation holding between a faulty constituent and the constituent which repairs it. Operating on the semantic representation, *repair* replaces the information in need of repair with new information. Consider example (25):

(25) (a) *John came home and* (b) *left the groceries.* (c) *Then he wrapped his present for his grandmother.* (d) ***No. He called Mary.*** (e) ***And then he wrapped the present for his grandmother.*** (f) *Anyway, I put them away* (g) *and then he left.*

The LDM assigns the following structural description to this discourse:



Notice that in the discourse parse tree all information is preserved while, unlike the subordinated case, the rightmost constituent in a *repair* dominates as indicated by the extension of the **R** node which corresponds to its right daughter rather than to the left daughter as in the case of an **S** node. In the semantic representation corresponding to the *repaired* discourse not all information is present. The instruction which discourse attachment at an **R** node sends to semantics is to replace information already in the representation with the newer, more correct, information. This property of discourse leads us to claim that

while discourse syntax is monotonic, discourse semantics is non-monotonic; information is lost from the semantic representation in the case of REPAIR as is shown in the example below.

interaction–Speaker/Addressee *speech event–Conversation* *genre unit–Story* *modality–realis* *polarity–positive* *point of view-p*
John came to the door (at t_1)
John left the groceries (at t_2)
John called Mary (at t_3)
John wrapped present (at t_4)
I put the groceries away (at t_5)
John left (at t_6)

4.5 Treatment of Entropic Action in the LDM

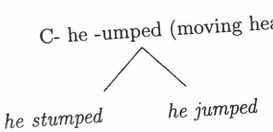
The question of what we have termed here *entropic action* —sequences of verbs which fulfill all morphological and syntactic criteria for encoding instantaneous non-habitual and non-iterative sequential action and which nonetheless describe an unordered and iterative non-punctual swirl of activity — present a problem for the SNH which can not be dealt with neatly by demonstrating how an embedded sequence of events does not interfere with the orderly unfolding of narration along a sequentially organized timeline. Something rather different is going on here and it is worth exploring some of the issues a bit, even if very briefly, informally and somewhat inconclusively. Consider once again this small paragraph from Rudyard Kipling's *How the whale got his throat*:

But as soon as the Mariner, who was a man of infinite-resources-and-sagacity, found himself truly inside the whale's dark inside cupboards, he stumped and he jumped and he thumped and he bumped, and he pranced and he danced, and he banged and he clanged, and he hit and he bit, and he leaped and he creeped, and he prowled and he howled, and he hopped and he dropped, and he cried and he sighed, and he crawled and he bawled, and he stepped and he lept,, and he danced hornpipes where he shouldn't and the whale felt most unhappy indeed.

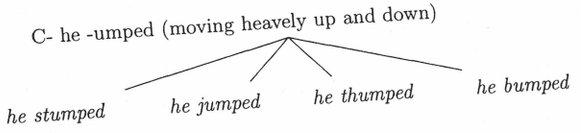
One thing is certain, all of this complex action is happening more or less at once and more or less in an unordered fashion and it is only because language forces linearization are these actions presented in any given order at all. One impression one may have is of an observer trying to characterize just what sort of fit *the Mariner is throwing*, trying to get it right, the observer tries first one term and then another and then another over a period of time noticing still more dimensions to the man's behavior. Even though there are a multiplicity of verbs used, the impression one gets from reading this passage, especially in reading this

passage aloud, is that there is not a multiplicity of actions being predicated but only one complicated rhythmic noisy physical engagement between *the Mariner and the whale's interior*. Where could this impression come from? Is it inference and world knowledge we would want to call upon particularly to aid us in understanding what is going on in this passage more than in understanding language in general? Or are there other sources of information which aid in building this impression?

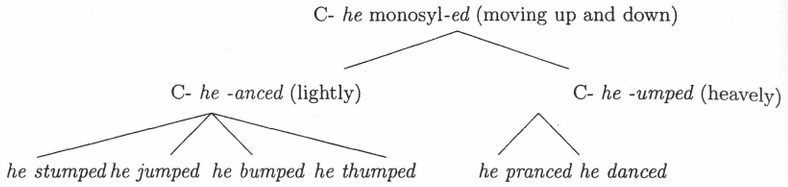
To answer this question, let us consider the language of the piece beginning with *he stumped and he jumped and he thumped and he bumped* and continuing to *he pranced and he danced, and he banged and he clanged*. Take the very first clausal pair *he stumped and he jumped*, they rhyme, of course, and their rhythm is enhanced by the syntactically unnecessary repetition of the pronoun *he*. Under the LDM, this couplet is coordinated under a **C** node which captures what these two DCUs hold in common. In this case most of their syntactic structure, most of their phonological structure and a good deal of their lexical meaning:



When the next clause is encountered *he thumped* and then the next *he bumped* these are easily accommodated under the same C- node:



By now the rhythm emerging from the repeated structure is very strong, strong enough to carry us across the breakdown of the *he -umped* paradigm and into the *he -anced* paradigm which still shares a great deal of structure with its sister – as both the *he X-ed* structure continues and the nature of the *X-ing* is basically the same as is the rhyming and the rhythm carried by the monosyllabicity of the couplet members. At this point, the parse tree looks like this:



While *thumped, bumped, pranced,* and *danced* primarily describe movement, they also suggest sounds. This connection of sound with movement is picked up in the next couplet *he banged and he clanged* but the balance between the two is reversed: in *bang and clang* there is rhythmic movement but predominately rhythmic loud sound. A higher level coordination is possible at this point and increasingly higher level **Coordinations** will be possible on and on until *and he danced hornpipes where he shouldn't* which breaks the pattern syntactically by introducing an object, *hornpipes*, and a long embedded clause, *where he shouldn't*, and breaks the pattern phonologically by introducing multisyllabicity (again with *hornpipes and the long embedded clause* and then semantically with *shouldn't*, which, by changing the polarity, signals the end of the entire interlude and moves the discourse to where the ***whale*** can participate as an experiencing subject and react to all of the *Mariner's continuous rhythmic noisy behavior*. Interestingly, this impression of ceaseless repetitive sound and motion is captured by the LDM as it recursively cranks through coordinating equivalent structures until all of those actions are captured by one **C**-DCU which is interpreted not at a series of moments in time, but at one moment. Thus, while the LDM demonstrates how in discourse a moment of time may be expanded with a GMC so that a number of actions can be seen to be constituitive of a higher level more general action, so, too, can it show how an apparent string of separate events are constituitive of one single higher level predicate which might obtain at one instant or over an interval of time depending on the precise nature of the actions described.

5 Conclusion: Revised SNH

In conclusion, we now consider once again the SNH which we stated earlier held that:

The order of event clauses in a narrative is isomorphic with the order of events in the semantic model of the text.

Considering our analysis here, we conclude that the SNH is essentially correct provided that

(1) Proper components for the linear sequence are chosen, and

(2) Constituents of the linear sequence are allowed to be complex.

In our view, an understanding of the intricate workings of naturally occurring complex discourse is within reach. An adequate approach to the complexities of discourse must accommodate three essential insights:

- Discourse syntax and semantic structure is not flat.
- A discourse is interpreted (parsed) incrementally. The result of the interpretation procedure is a recursive data structure.
- Discourse syntax is interpreted monotonically. Corresponding semantics update may be non-monotonic (repair).

These are the three premises upon which the LDM is being developed.

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Predicate Logic Unplugged

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1 Introduction

In this paper we describe the syntax and semantics of a description language for underspecified semantic representations. This concept is discussed in general and in particular applied to Predicate Logic and Discourse Representation Theory.

The reason for exploring *underspecified representations* as suitable semantic representations for natural language expressions emerges directly from practical natural language processing applications. The so-called *Combinatorial Explosion Puzzle*, a well known problem in this area, can succesfully be tackled by using underspecified representations. The source of this problem, scopal ambiguities in natural language expressions, is discussed in section 2.

The core of the paper presents *Hole Semantics*. This is a general proposal for a framework, in principle suitable for any logic, where underspecified representations play a central role. There is a clear separation between the object language (the logical language one is interested in) and the meta language (the language that describes and interprets underspecified structures). It has been noted by various authors that the meaning of an underspecified semantic representation cannot be expressed in terms of a disjunction of denotations, but rather as a *set* of denotations (cf. Poesio 1994). We support this view, and use it as underlying principle for the definition of the semantic interpretation function of underspecified structures. Section 3 is an informal introduction to Hole Semantics, and in section 4 things are formally defined. In section 5 we apply Hole Semantics to Predicate Logic, resulting in an "unplugged" version of (static and dynamic) Predicate Logic. In section 6 we show that this idea easily carries over to Discourse Representation Structures.

A lot of attention has been paid to "underspecified semantics" recently. Strongly related to the work presented here are Quasi Logical Forms (Alshawi 1992, Alsahwi and Crouch 1992), Underspecified Discourse Representation Structures (UDRSs) (Reyle 1993), Minimal Recursion Semantics (Egg and Lebeth 1995), and further (Poesio 1994, Muskens 1995, Pinkal 1995). The work presented here provides a straightforward syntax and semantics for a *general* kind of scopally underspecified representations.

2 Natural Language Ambiguities

In every day life, people communicate with each other by uttering true statements, or to put this more generally: they say things that make sense. In a situation where a speaker utters an utterance p, the hearer tries to interpret p in such a way that p denotes truth (rather than falsity). This probably strongly affects the way ambiguous utterances are processed by human beings. Imagine a situation where someone utters (1):

(1) Do not sleep and pay attention, please!

Utterance (1) is in isolation ambiguous. There is a reading where the negation outscopes "sleep and pay attention" and a reading where negation only has scope over "sleep". Normally, it is context, intonation, or world knowledge that enables a person to select the appropriate reading. Disambiguation is not the topic of this paper. What we are interested in is what introduces these ambiguities, how

we represent ambiguities in a logical representation, and how we interpret these representations.

Ambiguities in natural language are caused by different sources, such as predicative ambiguities or structural syntactic ambiguities, but in this paper we will restrict ourselves to semantic scope ambiguities. Among these we find all natural language expressions that, when translated into some logical form, introduce boolean operators, quantifiers, modals, questions, and many more. We will refer to these as operators. When at least two operators appear related to each other in a natural language expression, there is a chance that the expression is ambiguous. In (1) it is the scope of negation ("not") and conjunction ("and") that cause the ambiguity.

In the following examples the ambiguity is caused by the scope of implication and conjunction (2), and the scope of the intensional verb and disjunction (3). The absence of prosodic information in these examples make them ambiguous.

(2) If a man walks then he whistles and a woman is happy

(3) Do you want tea or coffee?

Standard examples in the literature on quantifier scope ambiguities and underspecification are (4) and (5). These kinds of examples are traditionally used to provide evidence that human beings do not disambiguate while processing natural language input. While (4) is said to have thousands of readings, it seems very unlikely that humans generate and test every one of them.

(4) A politician can fool most voters on most issues most of the time, but no politician can fool all voters on every single issue all of the time

(5) Everybody is not here

In the previously mentioned references to underspecified semantics, most authors seem to agree on an approach where an underspecified representation plays a central role. Scope ambiguities are not resolved but are put together in a very compact representation. Of interest is a kind of representation that describes the (complete and sound) logical translations of ambiguous expressions. In this paper we define a semantic representation that is able to express underspecification for any kind of object language. First we sketch the basic idea of underspecified representations, then we move on to precisely defining its ingredients and properties.

3 Underspecified Representations

Semantic representations of natural language expressions are traditionally constructed on the basis of their syntactic analysis. Since expressions can be semantically ambiguous, this is a one-to-many mapping. The idea of underspecified representations is to make this mapping functional, i.e., a one-to-one mapping from syntactic to semantic structure. The interpretation of an underspecified semantic representation is (hence) the set of interpretations that are expressed in it.

The way we define underspecified representations is as follows. Take an object language (the logic in which you are interested), and define the syntax of its basic formulas. We label these formulas for an obvious reason: it will be very easy to talk about them on a meta level. Labels are used as constants. Then we introduce variables over labels, which we will refer to as *holes*, as arguments of scope bearing operators. The last step is to add a set of constraints on the labels and holes, that tell

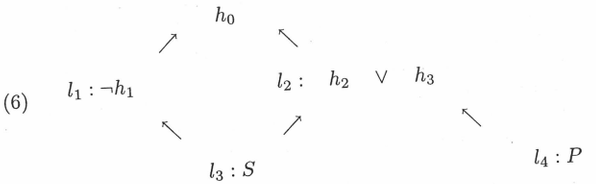
how the different pieces of structures fit together, in such a way that all readings are covered. So what we end up with is a set of labeled formulas, a set of meta variables (holes), and a set of constraints. This is our underspecified representation (UR).

Constraints state relations between the different formulas in UR with respect to scope. For example, it is possible to say that a formula (with label l) is in the scope of an operator (with hole h) by $l \leq h$. This constraint forces l to be directly or indirectly in the scope of h of the relevant operator (e.g., l is in the scope of an operator with label l', and l' is in the scope of h).

So, metaphorically speaking, *holes* underspecify scope in an UR. In order to give URs a non-ambiguous interpretation, the holes should be plugged with the (labeled) formulas of UR in such a way that all the constraints of UR are satisfied. We illustrate this idea with a simple example, where we take Propositional Logic as object language. We use the following notational conventions: holes are represented by h_i, i an index. We label a formula ϕ as l_i: ϕ, where l is a label with index i.

Consider again (1). Assume that there is some syntactic analysis for it on which we build our UR. Translate the negation ("do not") as "$l_1 : \neg h_1$", and the disjunction ("or") as "$l_2 : h_2 \vee h_3$". Take "$l_3 : S$" as translation for "sleep", and "$l_4 : P$" for "pay attention". As variable for "widest scope" we take "h_0". Finally, we set our constraints in the following way: "sleep" should be in the scope of negation ($l_3 \leq h_1$) and in the scope of the left disjunct ($l_3 \leq h_2$), "pay attention" in the scope of the right disjunct ($l_4 \leq h_3$), and the operators l_1 and l_2 can both take wide scope ($l_1 \leq h_0$ and $l_2 \leq h_0$). Then a graphical representation of the UR for (1) is (the constraints are graphically realized by arrows):

(6)



Now we will pay attention to the interpretation of (6), by taking into consideration the possible mappings from holes to labels (pluggings). In other words: a plugging is a bijective assignment function, with the set of holes as scope and the set of labels as range. In this example, there are exactly two possible pluggings, P_1 and P_2:

(7) $P_1 : \{h_0 = l_1, h_1 = l_2, h_2 = l_3, h_3 = l_4\}$

(8) $P_2 : \{h_0 = l_2, h_1 = l_3, h_2 = l_1, h_3 = l_4\}$

The reader may check that these are indeed the only admissible pluggings: for hole h_0 label l_3 does not come into question since it will never be in the scope of h_1 or h_2 and hence not all constraints would be satisfied; for hole h_3, the only suitable candidate is l_4. The two pluggings (7) and (8) correspond to the object language formulas in (9) and (10) respectively. The interpretation of (6) is a set, containing the interpretation of (9) and that of (10).

(9) $(\neg(sleep \vee pay_attention))$

(10) $((\neg sleep) \vee pay_attention)$

We summarize and discuss this section shortly. An UR consists of a set of

labeled formulas, a set of meta variables that represent scope (holes), and a set of constraints on these. The idea of labeling is taken directly from Reyle's Underspecified DRSs (Reyle 1993). A notable difference is that Reyle uses labels for DRSs, whereas we use them for smaller logical units, since this gives us an advantage with respect to the descriptive power of URs. In this section we sketched by way of an example what URs are. The next section formalizes these ideas.

4 Hole Semantics

The underspecified representations proposed in the previous section are now subject to more formal specifications. We define the syntax and semantics of an UR, and also the notions *proper* UR, *consistent* UR, and *possible pluggings*. Let us start with the syntax.

Definition 1: **SYNTAX UR**

Let H a set of variables over formulas, L the set of labeled X-formulas, and C the set of constraints on $H \cup L$. Then an UR $U = \langle H, L, C \rangle$

In the rest of the paper, we will use U for an underspecified representation and H_U, L_U, and C_U to refer to the holes, labeled formulas and constraints of U respectively. The syntax of expressions in L_U obviously depends on the object language, therefore we do not pay any attention to it just now, but postpone it to the next section, where we take predicate logic as our target language. We use P (sometimes indexed) for pluggings, which are total assignments from holes to labels.

Let us first make a brief excursion to lattice theory, from which we borrow some principles. We can view a $U = \langle H, L, C \rangle$ as a join semi-lattice $\langle H_U \cup L_U, \leq \rangle$. $\leq$ is reflexive, transitive and antisymmetric and therefore a partial order. For any $k_i, k_j \in H_U \cup L_U$, there is a k such that k is the least upper bound of k_i and k_j. End of excursion. Now we define subordination for labels or holes in U as SUB(k,k'), meaning "k is subordinated to k'", or k' subordinates k.

Definition 2: **SUBORDINATION (SUB)**

Let l be a label, h a hole, k a hole or a label of U. Then:

1. $\text{SUB}_U(k, k)$;
2. $\text{SUB}_U(k, k')$ if there is a $k \leq k' \in C_U$
3. $\text{SUB}_U(h, l)$ if there is a ϕ such that $l : \phi \in L_U$ and h is an argument of $l : \phi$ and it is not the case that $\text{SUB}_U(l, h)$;
4. $\text{SUB}_U(k, k')$ if there is a k'' such that $\text{SUB}_U(k, k'')$ and $\text{SUB}_U(k'', k')$;
5. SUB is only defined on the basis of 1-4.

The second clause is the explicit way of defining subordination: if there is a constraint $\leq$ present in U. The first clause represents reflexivity, the third clause defines subordination on labeled formulas that have holes as arguments. The fourth clause expresses transitivity. With SUB we can define a *proper* UR.

Definition 3: **PROPER UR**

An UR U is *proper* iff for all $k, k' \in H_U \cup L_U$ it is the case that there is a k'' such that $\text{SUB}_U(k,k'')$ and $\text{SUB}_U(k',k'')$.

A proper UR is one which describes a join semi-lattice. Yet we are able to define what, with respect to a plugging, a *consistent* UR is, using the following notational convention: for any $k \in H_U \cup L_U$, we define $I_P(k) = P(k)$ iff $k \in H_U$,

and $I_P(k) = k$ iff $k \in L_U$. A consistent UR is an UR which is proper, taking pluggings into account.

Definition 4: **CONSISTENT UR**

$\text{CONS}_{U,P}$ iff for all k, k' such that SUB(k,k'), it is the case that either $I(k) = I(k')$, or $I(k) \neq I(k')$ and SUB(k',k) is not supported.[1]

We have not yet defined what possible pluggings are. Pluggings are, as we have discussed in the previous section, bijective functions from holes to labels. A plugging for an UR U is *possible*, if the UR, with respect to this plugging, is consistent. In other words, when the underspecified representation, taking the plugging into account, has the properties of a join semi-lattice. Since we have already defined what a consistent UR is, defining possible pluggings is an easy job.

Definition 5: **POSSIBLE PLUGGING (PP)**

$\text{PP}_U = \{P \mid \text{CONS}_{P,U}\}$

A plugging is possible, if U is consistent with respect to this plugging. We will illustrate this with two examples. First example: suppose that $U = < \{h_0\}, \{l_1 : \phi\}, \{l_1 \leq h_0\}$ for some formula ϕ. Hence, SUB(l_1,l_1), SUB(h_1,h_1), and SUB(l_1,h_1) are valid. Then a possible plugging P for U is one such that $\text{P}(h_0){=}l_1$, since $\text{CONS}_{U,P}$ holds.

Second example: consider the following constraints of an UR: $\{h_1 \leq l_1, h_2 \leq l_2, l_2 \leq h_0, l_1 \leq h_0, l_3 \leq h_1, l_3 \leq h_2\}$, then a plugging P where $P(h_0){=}l_3$, $P(h_1){=}l_2$, and $P(h_2){=}l_1$ is not possible. The UR to which these constraints belong is not consistent, since, for example, SUB(l_3,h_2) and SUB(h_2,h_0) are valid and with P lead to "SUB(l_3,l_1)" and "SUB(l_1,l_3)", violating antisymmetry.

So far, so good. We have defined the syntax, properness, and consistency of an UR. For the semantic interpretation of an UR we need to be able to address the label or hole that subordinates all others. We call this TOP, and define it as follows.

Definition 6: **TOP**

$\text{TOP}_{P,U} = I(k)$ iff $k \in H_U \cup L_U$ and there is no k' such that $k' \in H_U \cup L_U$, $k \neq k'$, and CONS-$\text{SUB}_{P,C_U}(k, k')$.

The semantic interpretation of an UR is that of its TOP. As interpretation function for URs, with respect to a model $\mathbf{M}$ we will use $[\![\cdot]\!]_{\mathbf{M}}$, as to avoid confusion with the interpretation function of the object language, for which we will adopt the traditional $[\![\cdot]\!]_{\mathbf{M},P}$.

Definition 7: **SEMANTICS UR ($[\![\cdot]\!]$)**

$[\![U]\!]_{\mathbf{M}} = \{\, [\![\text{TOP}_{P,U}]\!]_{\mathbf{M},P} \mid P \in \text{PP}_U \,\}$

This definition states that the interpretation of an underspecified representation U^X, is the set of object language denotations, as many as there are possible pluggings for U^X. For some reasons it might be an advantage to redefine this function. For example, when the object denotations are truth values, the interpretation of an UR for this particular object language has three different values: $\{0\}$, $\{1\}$, and $\{0, 1\}$. This approach is too weak to capture the fact that an UR might have more than one interpretation with the same denotation. This situation can be avoided

1. Read this giving disjunction scope over conjunction!

if we relate the object denotation to a plugging, as we do in our revised semantic interpretation function.

Definition 8: **SEMANTICS UR (revised) ($[\![.]\!]^*$)**

$$[\![U]\!]^*_{\mathbf{M}} = \{ < P, [\![\mathrm{TOP}_{P,U}]\!]_{\mathbf{M},P} > \mid P \in \mathrm{PP}_U \}$$

Here $[\![.]\!]^*$ is defined as a function from URs to a set of pairs of pluggings and object language denotations. And this ends the general specification of Hole Semantics. In the next section we will apply Hole Semantics to Predicate Logic.

5 Predicate Logic "Unplugged"

In this section we take Predicate Logic as object language, resulting in Predicate Logic Unplugged (PLU). Given the framework of Hole Semantics described in the previous section, we only need to define the syntax of PLU formulas and their model interpretation. Taking as convention that terms (written as $t_1, ..., t_n$) are either object language variables or constants, PLU formulas are defined as follows:

Definition 9: **Syntax PLU formulas**

1. If h_i, h_j are holes, then $h_i \rightarrow h_j$, $h_i \vee h_j$, $h_i \wedge h_j$ are PLU formulas;
2. If h is a hole, then $\neg h$ is a PLU formula;
3. If x is an object language variable, h a hole, then $\forall x\ h$ and $\exists x\ h$ are PLU formulas;
4. If R is a predicate symbol for an n-place predicate, and $t_1, ..., t_n$ are terms, then $Rt_1, ..., t_n$ is a PLU formula;
5. Nothing else is a PLU formula.

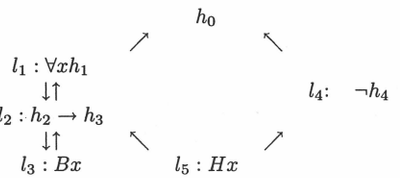
The syntax of PLU formulas is in principle the same as that of ordinary Predicate Logic, with the exception that holes in places, where normally PL-formulas are found, are introduced. We will illustrate Predicate Logic Unplugged with an example. Consider (5), repeated here for convenience as (11), and its translation (12) in U^{PL}.

(11) Everybody is not here

$$(12) \quad < \left\{ \begin{array}{l} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \end{array} \right\}, \left\{ \begin{array}{l} l_1 : \forall x\ h_1 \\ l_2 : h_2 \rightarrow h_3 \\ l_3 : Bx \\ l_4 : \neg h_4 \\ l_5 : Hx \end{array} \right\}, \left\{ \begin{array}{l} l_1 \leq h_0 \\ l_2 \leq h_1 \\ h_1 \leq h_2 \\ l_3 \leq h_2 \\ h_2 \leq l_3 \\ l_4 \leq h_0 \\ l_5 \leq h_3 \\ l_5 \leq h_4 \end{array} \right\} >$$

Note that $I(h_0)$ is TOP, and the label which is plugged into this h_0 will receive widest scope. The other holes in (12) are introduced by the scope bearing operators (universal quantifier, negation, and implication). Further, notice that we constrain l_3 to be directly in the scope of h_2 via the constraints $l_3 \leq h_2$ and $h_2 \leq l_3$, and this is also the case for l_2 and h_1. These extra constraints exclude unwanted readings. In a graphical representation, the UR looks like:

(13)



There are two pluggings (the interested reader may verify this). Plugging (14) interprets (12) as giving the universal quantifier wide scope, outscoping negation. The corresponding formula in predicate logic is (15), which is true in a model where all persons (in the relevant domain, of course) do not have the property being at the speaker's location.[2]

(14) $P_1 : \{h_0 = l_1, h_1 = l_2, h_2 = l_3, h_3 = l_4, h_4 = l_5\}$

(15) $\forall x(Bx \rightarrow \neg Hx)$

Plugging (16) interprets (12) as negation outscoping the universal quantifier. In a model where there is some person that is not at the speaker's location this interpretation denotes truth. A corresponding formula in predicate logic is (17).

(16) $P_2 : \{h_0 = l_4, h_1 = l_2, h_2 = l_3, h_3 = l_5, h_4 = l_1\}$

(17) $\neg\forall x(Bx \rightarrow Hx)$

The model interpretation of PLU can be sketched as follows. Call $[\![.]\!]^{PLU}_{M,P}$ the interpretation function for PLU formulas, and $\mathbf{M} = < D, F >$ a model. D is the domain (a nonempty set) and F an interpretation function ($F(d) \in D$ if d is a constant, and $F(R) \subset D^n$ for an n-place predicate symbol R). As usual, we use g and g' for total assignment functions. For a term t, $[\![t]\!]_{M,g}$ is $g(t)$ if t is a variable, and $F(t)$ is t is a constant.

Definition 10: **Interpretation Function for PLU ($[\![.]\!]^{PLU}$)**

1. $[\![h_i \rightarrow h_j]\!]^{PLU}_{\mathbf{M},P,g} = 1$
   iff $[\![h_i]\!]^{PLU}_{\mathbf{M},P,g} = 0$ or $[\![h_j]\!]^{PLU}_{\mathbf{M},P,g} = 1$
2. $[\![h_i \vee h_j]\!]^{PLU}_{\mathbf{M},P,g} = 1$
   iff $[\![h_i]\!]^{PLU}_{\mathbf{M},P,g} = 1$ or $[\![h_j]\!]^{PLU}_{\mathbf{M},P,g} = 1$
3. $[\![h_i \wedge h_j]\!]^{PLU}_{\mathbf{M},P,g} = 1$
   iff $[\![h_i]\!]^{PLU}_{\mathbf{M},P,g} = [\![h_j]\!]^{PLU}_{\mathbf{M},P,g} = 1$
4. $[\![\neg h_i]\!]^{PLU}_{\mathbf{M},P,g} = 1$
   iff $[\![h_i]\!]^{PLU}_{\mathbf{M},P,g} = 0$
5. $[\![\forall x\ h_i]\!]^{PLU}_{\mathbf{M},P,g} = 1$
   iff for all $d \in D$ it is the case that $[\![h_i]\!]^{PLU}_{\mathbf{M},P,g[d/x]} = 1$
6. $[\![\exists x\ h_i]\!]^{PLU}_{\mathbf{M},P,g} = 1$
   iff for at least some $d \in D$ it is the case that $[\![h_i]\!]^{PLU}_{\mathbf{M},P,g[d/x]} = 1$

2. We translate the deictic property "being here" simply as H, for convenience.

7. $[\![Rt_1,...,t_n]\!]^{PLU}_{\mathbf{M},P,g} = 1$
iff $< [\![t_1]\!]_{M,g},...,[\![t_n]\!]_{M,g} > \in F(R)$

Using definition 11, we are able to define an interpretation of underspecified representations of PLU itself, since this definition does not depend on the object language. In fact, using the syntax of PLU, it is fairly easy to define a *dynamic* underspecified Predicate Logic.

Suppose that $[\![.]\!]^{DPLU}_{\mathbf{M},P}$ is the interpretation function that interprets labeled PLU-formulas as in Dynamic Predicate Logic (DPL) (Groenendijk and Stokhof 1991). An assignment g is a function from variables to elements of D, $g'[x]g$ means that g' is an x-variant of g.

Definition 11: **Interpretation Function for DPLU** ($[\![.]\!]^{DPLU}$)

1. $[\![h_i \rightarrow h_j]\!]^{DPLU}_{\mathbf{M},P} =$
$\{\langle g,g\rangle \mid \forall g'(\langle g,g'\rangle \in [\![h_i]\!]^{DPLU}_{\mathbf{M},P} \rightarrow \exists g''(\langle g',g''\rangle \in [\![h_j]\!]^{DPLU}_{\mathbf{M},P}))\}$
2. $[\![h_\vee h_j]\!]^{DPLU}_{\mathbf{M},P} =$
$\{\langle g,g\rangle \mid \exists g'(\langle g,g'\rangle \in [\![h_i]\!]^{DPLU}_{\mathbf{M},P} \vee \langle g,g'\rangle \in [\![h_j]\!]^{DPLU}_{\mathbf{M},P})\}$
3. $[\![h_i \wedge h_j]\!]^{DPLU}_{\mathbf{M},P} =$
$\{\langle g,g'\rangle \mid \exists g''(\langle g,g''\rangle \in [\![h_i]\!]^{DPLU}_{\mathbf{M},P} \& \langle g'',g'\rangle \in [\![h_j]\!]^{DPLU}_{\mathbf{M},P})\}$
4. $[\![\neg h_i]\!]^{DPLU}_{\mathbf{M},P} =$
$\{\langle g,g\rangle \mid \neg\exists g'(\langle g,g'\rangle \in [\![h_i]\!]^{DPLU}_{\mathbf{M},P})\}$
5. $[\![\forall x\ h_i]\!]^{DPLU}_{\mathbf{M},P} =$
$\{\langle g,g\rangle \mid \forall g'(g'[x]g \rightarrow \exists g''(\langle g',g''\rangle \in [\![h_i]\!]^{DPLU}_{\mathbf{M},P}))\}$
6. $[\![\exists x\ h_i]\!]^{DPLU}_{\mathbf{M},P} =$
$\{\langle g,g'\rangle \mid \exists g''(g''[x]g \& \langle g'',g'\rangle \in [\![h_i]\!]^{DPLU}_{\mathbf{M},P})\}$
7. $[\![Rt_1,...,t_n]\!]^{DPLU}_{\mathbf{M},P} =$
$\{\langle g,g\rangle \mid < [\![t_1]\!]_{M,g},...,[\![t_n]\!]_{M,g} > \in F(R)\}$

6 Underspecified Discourse Representation Structures

As stressed before, Hole Semantics is in principle independent of the object language. Besides Predicate Logic, we could also take Discourse Representation Structures (DRSs, as proposed in Discourse Representation Theory (DRT) Kamp and Reyle 1993) as object language, resulting in DRT Unplugged (DRTU). We first define DRTU formulas:

Definition 12: **Syntax DRTU formulas**

1. If h_i, h_j are holes, $k_1,...,k_n$ holes or labels, then $[\ |h_i \rightarrow h_j]$, $\otimes\{k_1,...,k_n\}$, $[\ |\neg h_i]$, $[\ |h_i \vee h_j]\}$ are DRTU formulas;
2. If x is a discourse marker, P a symbol for an n-place predicate, then $[x|\ ]$ and $[\ |P(x_1,...,x_n)]$ are DRTU formulas.
3. Nothing else is a DRTU formula.

Here a DRS is represented as $[D|C]$, D the set of discourse markers, C the set of conditions. The merger ($\otimes$) makes one DRS out of several by taking the union

of the domains and the conditions respectively of its argument, a set of DRSs. The definition of $[\![.]\!]^{DRTU}$ can for example be realised along the lines presented in Kohlhase et al. 1995 or Muskens 1993. We will not present it here, but instead give an example. Consider again (2), repeated here as (18). The UR translation is shown in (19).

(18) If a man walks then he whistles and a woman is happy

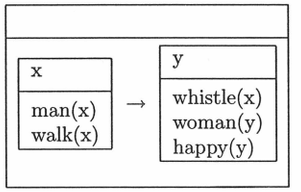
$$(19)\quad < \left\{ \begin{array}{l} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \end{array} \right\}, \left\{ \begin{array}{l} l_1 : [\ |\ h_1 \rightarrow h_2\ ] \\ l_2 : \otimes\{h_3, h_4\} \\ l_3 : \otimes\{l_4, l_5, l_6\} \\ l_4 : [\ x\ |\ ] \\ l_5 : [\ |\ man(x)\ ] \\ l_6 : [\ |\ walk(x)\ ] \\ l_7 : [\ |\ whistle(x)\ ] \\ l_8 : \otimes\{l_9, l_{10}, l_{11}\} \\ l_9 : [\ y\ |\ ] \\ l_{10} : [\ |\ woman(y)\ ] \\ l_{11} : [\ |\ happy(y)\ ] \end{array} \right\}, \left\{ \begin{array}{l} l_3 \leq h_1 \\ l_7 \leq h_2 \\ l_7 \leq h_3 \\ l_8 \leq h_4 \\ l_1 \leq h_0 \\ l_2 \leq h_0 \end{array} \right\} >$$

There are two possible pluggings for (19), and therefore two readings for (2) available. The first reading (paraphrased as a linear DRS in 21, and for convenience in the more familiar boxed notation 25) triggered by plugging (20) corresponds to the "wide scope disjunction" reading.

(20) $P_1 : \{h_0 = l_1, h_1 = l_3, h_2 = l_2, h_3 = l_7, h_4 = l_8\}$

(21) $[\ |\ [\ x\ |\ man(x)\ walk(x)\ ] \rightarrow [\ y\ |\ whistle(x)\ woman(y)\ happy(y)\ ]\ ]$

(22)

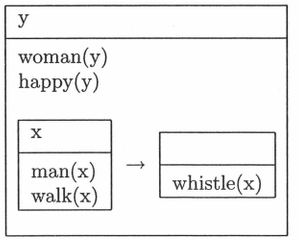


The other possible plugging (23) results in a reading where conjunction outscopes disjunction. The DRS for this reading is shown in (24) and (25).

(23) $P_1 : \{h_0 = l_2, h_1 = l_3, h_2 = l_7, h_3 = l_1, h_4 = l_8\}$

(24) $[\ y\ |\ woman(y)\ happy(y)\ [\ x\ |\ man(x)\ walk(x)\ ] \rightarrow [\ |\ whistle(x)\ ]\ ]$

(25)



7 Conclusion

We proposed a framework for underspecified semantics representations in general, called *Hole Semantics*, and claimed that, due to a clear seperation of object and meta level, it is independent of the object languages. Underspecified Representations in Hole Semantics correspond to a partial descriptions of the semantics, using meta variables (holes) and subordination constraints. We have shown that Hole Semantics can be applied both to Predicate Logic (static and dynamic) and Discourse Representation Theory, with respect to semantically ambiguous scope.

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The Role of Context in the Interpretation of Generic Sentences[1]

Lawrence Cavedon[2] and Sheila Glasbey[3]

Abstract

We argue that *context* plays an important role in the interpretation of generic sentences. The examples we use to demonstrate this have also been used to argue that normative approaches (upon which logics of normality are based) to generics have some fundamental problems related to scope. We use Barwise and Seligman's *channel theory*—a model of information-flow based on ideas from situation theory—as the basis for an analysis of generics that provides a context for each generic sentence and thereby avoids the problems raised. The channel-theoretic analysis provides the basis for a framework for reasoning both *with* generics (about the default properties of individuals) and *about* them (to infer new generics). Modelling the context of a generic also allows us to model the sort of misunderstandings that may occur in a dialogue, whereby a listener misinterprets the scope of an uttered generic sentence.

1 Generics and Context

It is generally agreed that generic sentences are used to convey information about regularities of one kind or another, and that the generalizations thus made may have exceptions which do not invalidate the truth of such statements. Consider, for example, the generic sentence:

(1) Swans are white.

This appears to convey some kind of general relationship between the property of being a swan and the property of being white, such that we might expect any swan we encountered to be white, while being prepared to accept that this particular swan might be exceptional in some sense with respect to the property of whiteness—for example it might still have the dark feathers of a very young swan. The existence of such exceptions has made it difficult to give an adequate semantics for generic sentences. The problem has been extensively discussed in the literature and a number of attempts made to overcome it. It has become clear that we cannot capture the semantics of generics adequately by postulating a generic quantifier which corresponds to something like 'most' or 'usually'. There are well-known examples such as:

(2) Turtles live long lives.

which speakers accept as true even when they know that most turtles are killed by predators in their youth.[4] A comprehensive survey of the problem of exceptions and the various solutions proposed in the literature is given in ([Pelletier and Carlson, 1995], Introduction).

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[4] [Morreau, 1992] argues that the generic "Potatoes contain vitamin C" would be true even if *all* potatoes were boiled to the extent that none of them actually did contain vitamin C.

A less frequently-discussed feature of generics is the following. Suppose our speaker has visited Australia, and knows that the swans indigenous to that country are black. She may well reply to (1) by saying:

(3) That's not true of Australian swans. Swans are black in Australia.

And a native Australian a couple of centuries ago might have truthfully asserted the equivalent in her own language of:

(4) Swans are black.

A closely-related pair of examples is the following:

(5) Peacocks have brightly coloured tail feathers.

(6) Peacocks lay eggs.

It appears that (5) and (6) are both accepted as true, even by someone who knows that only the males have bright tail feathers and only the females lay eggs.[5]

These examples reveal the importance of **context** to the interpretation of generic sentences. The truth of a generic seems to depend on context in the sense of "what the speaker is talking about". If it is clear that she is talking about Australia, then we will probably accept (4) as true. It is interesting, however, that (5) and (6) seem acceptable when given in reply to a question like "Tell me what you know about peacocks"—i.e., we need not already be "focusing" on male peacocks in order for us to accept (5) as true.

(5) and (6) are problematic for standard "normative" accounts of generics (e.g. [Morreau, 1992]) which claim that generics are about "normal" individuals. The problem is that there are no peacocks that qualify as normal here—i.e., no peacocks that both have bright tail feathers and lay eggs. Intuitively, (5) refers to "just the males" and (6) to "just the females".

We need an account that can capture precisely this dependency on context. Also, we need to capture the kind of **reasoning** that is/isn't licensed on the basis of the content of (5) and (6). Notice, for example, that, while accepting the truth of (5), we cannot reason correctly that if A is a peacock, A will (almost certainly) have brightly coloured tail feathers. This, of course, is because A may be a female peacock.

The challenge is to provide an account of generics which allows for exceptions and the kind of context-dependency discussed above, and which also models the reasoning which is or is not licensed on the basis of a true generic. In this paper we propose such an account, based on Barwise and Seligman's **Channel Theory** ([Barwise and Seligman, 1994], [Seligman and Barwise, 1993]).

2 Channel Theory

Channel theory is a mathematical framework which provides an account of **regularities** (i.e., law-like dependencies) and is an attempt to provide a comprehensive

[5] Occasionally someone disputes (6) and points out that in fact it is the *peahens* which lay eggs. However most speakers seem happy to use 'peacock' to refer to both the male and female of the species.

model of **information flow.** Building on the situation-theoretic model of **constraints,** Barwise and Seligman take regularities to reflect the structure of the world (as discriminated by some agent) without attempting to reduce these informational entities to some "more primitive" relationship (e.g. quantification over "normal" individuals). As such, the channel-theoretic analysis of generics below does not provide any sort of reductionist account of **why** true generics are so, but it does provide a **structural** account which is used to explain some problematic issues in generics.

Barwise and Seligman are particularly concerned with modelling the paradoxical issue that regularities seem to be both *reliable* (i.e., underly inference) yet *fallible* (i.e., may admit exceptions). Such properties are, of course, also required of the semantic objects used to model generic sentences. Barwise and Seligman resolve the problem by ensuring that relationships between tokens (or individuals) play a central role, as well as the relationships at the type level. As we see below, this gives rise to the concept of a **channel**— a classification of token-level connections by type-level relationships. In our model of generics, a generic sentence effectively describes a channel—i.e. a generic can only be true or false relative to some channel—and this channel thereby provides a context against which the generic is evaluated. This model is discussed further in the next section.

At the heart of channel theory is a relativistic theory of information, based on the concept of a **classification.** A classification is a structure that carves up (part of) the world into **tokens** and assigns various **types** to these tokens.

Definition *A* classification *A is a structure $\langle tok(A), typ(A), :_A \rangle$ consisting of a set of* tokens *$tok(A)$, a set of* types *$typ(A)$ and a relation $:_A$ on $tok(A) \times typ(A)$. For $t \in tok(A)$ and $\phi \in typ(A)$, we say t is* classified by *ϕ in A if $t :_A \phi$ holds.*[6]

The tokens of a classification may be objects, individuals, situations, or even other classifications, while the types are properties appropriate for classifying (along some dimension) these tokens. A classification can be seen as a representation of information of a particular sort. This representation is highly relativistic—intuitively, one could view a classification as being relativised to a specific agent, epistemic state, point of view, or perspective on the world.

The second main concept is that of a **channel.** A channel $\mathcal{C}$ is an informational link between two classifications A and B, licensing the flow of information of a particular sort. Formally, a channel is itself a classification—the tokens of $\mathcal{C}$ are **connections,** each linking tokens of A and B.[7]

Definition *Let A and B be classifications. A channel $\mathcal{C} : A \Rightarrow B$ linking A and B is a classification $\langle tok(\mathcal{C}), typ(\mathcal{C}), : \rangle$. The tokens of $\mathcal{C}$ are* connections, *denoted $s \mapsto s'$, with $s \in tok(A)$ and $s' \in tok(B)$. The types of $\mathcal{C}$ are* constraints, *denoted $\phi \rightarrow \psi$, with $\phi \in typ(A)$ and $\psi \in typ(B)$.*

If $(t \mapsto t' : \phi \rightarrow \psi)$ holds in $\mathcal{C}$, this signifies that t being of type ϕ carries the information that t' is of type ψ. The type-token distinction allows for characterisation of different sorts of exceptions to the general rule. If a connection $t \mapsto t'$ fails to be classified by a constraint $\phi \rightarrow \psi$ and $t : \phi$, then $t \mapsto t'$ is said to be an **exception** to that constraint. Exceptions may lead to erroneous inference (i.e., a conclusion that $(t' : \psi)$ holds may be reached whereas the channel does not support this).

[6] In general, there are two classification relations, one for positive and one for negative classification. However, we do not make use of negative classification in this abstract, so it is omitted for simplicity. We drop the subscript from the relation when this causes no confusion.

[7] We slightly abuse Seligman and Barwise's definition here—connections and constraints should not be identified with the tokens/types they connect. However, this abuse simplifies matters somewhat.

Alternatively, a token t may be exceptional with respect to a constraint simply because the constraint is not applicable to it—this corresponds to the case whereby no connection in the pertinent channel emanates from t.

It should be noted that channel theory does not require any relationship between the connections of a given channel and the assertibility of its constraints—as far as the theory is concerned, every connection may well constitute an exception, without invalidating the constraints within that channel. Of course, if this was found to be too flexible, then the framework could accommodate an imposed requirement that exceptions were minimised. However, this flexibility seems to reflect the sort of behaviour highlighted by the examples of the previous section.

3 Modelling Generics in Channel Theory

In this section, we describe a simple channel-theoretic model of generic sentences. This simple model will suffice for illustrating our approach to the particular issues relating to context that we discussed earlier. Later, we embellish the model by providing a more complex (situation-theoretic) semantic framework for representing types.

A simple channel-theoretic model of generics is obtained by using the type-level of a classification to model properties and the token-level to model individuals. A type-level constraint is used to model a generic relationship, while a connection between a token t and itself[8] is included in a channel if t is appropriate to the generic relationship represented by any constraints in that channel.

For example, "Swans are white" describes a channel $\mathcal{C}$ containing connections of the form $s \mapsto s$ and a constraint $swan \rightarrow white$, where s is taken from some collection of swans, *swan* is a type that holds of swan-objects and *white* is a type that holds of white-objects. Under this model, we would expect that Australian (black) swans are not involved in tokens contained in $\mathcal{C}$, since this generic is not applicable to them. Other swan-tokens (e.g. dirty swans or young swans) may correspond to exceptions in $\mathcal{C}$. The distinction between exceptions and inapplicable tokens is a subtle one, but the the distinction turns out to be a useful one and we discuss it further below.

In the channel-theoretic model of generics, a generic statement has both a **demonstrative content**, which is a channel, and a **descriptive content**, which is the regularity (i.e., constraint) supported by that channel. This is very similar to J. L. Austin's notion, incorporated into situation semantics, that certain kinds of statements are "about" situations. In situation semantics, the situation is the demonstrative content, and an infon supported by that situation is the descriptive content of the statement—thus a situation is effectively classified as being of the type of the appropriate infon. See ([Cooper, 1995]) for a recent discussion of these ideas.

There is thus a clear parallel between our proposed semantics for generic statements and the situation semantics analysis of non-generic statements. Basically, the content of a generic is the assertion that a given channel $\mathcal{C}$ supports a certain constraint $\phi \rightarrow \psi$—i.e., the assertion that $\mathcal{C}$ is of type $\phi \rightarrow \psi$.[9] The collection of tokens associated with a channel effectively captures the "scope" over which the associated generic is asserted to hold. Sentences (5) and (6) involve different channels—the first channel

[8] In general, of course, a connection may involve two different tokens, but for simplicity here we assume that we are only dealing with "reflexive" connections.

[9] By allowing classifications whose tokens are channels and types are constraints we also allow a uniform treatment of nested generics.

supports a regularity involving male peacock-tokens while the second supports a regularity involving female peacock-tokens.

The channel-theoretic view of the semantic content of generics inherits several features from the channel-theoretic notion of regularities. In particular, channel-types (i.e., constraints) are primitive entities and there is no direct relationship between the fact that a channel supports a particular regularity, and the properties required of the associated individuals. Some/many/all[10] of the tokens satisfying the antecedent type of the constraint may be exceptions without undermining the channel's support of that constraint. This property seems supported by several of the examples well-known in the literature and described earlier.

The channel-theoretic view ignores the issue of precisely what it is that makes a particular generic true, other than that the generic describes the way (a part of) the world happens to be structured. However, this is not as unsatisfactory as it may seem: the channel-theoretic view can be seen as consistent with any explanation regarding which generics are or are not true, and the formal framework provides important properties of its own.[11] The challenge for accounts of generics that do not directly relate the truth of the generic to the properties of the individuals is in providing a means for reasoning defeasibly about individuals: e.g. given "Birds fly" and a bird t, we should be able to reach the default conclusion that t can fly. Defeasible reasoning using the channel-theoretic model is outlined in some detail below.

Another question that needs answering concerns *which* channel it is that a given generic describes. The short answer is that it is the one which the speaker has in mind when she asserts the generic sentence. This does not allow us to specify the "appropriate" channel objectively—i.e., for a given generic sentence such as "Birds fly", we cannot determine in isolation exactly which channel is the demonstrative content. In fact, there is often more than one channel that can be deemed "appropriate", containing different numbers of exceptions (e.g. the channel ranging over all birds; the channel ranging over individuals of the species of birds that fly, ruling out penguins and emus; the channel ranging over individual birds that fly, ruling out penguins and sick birds).[12] What is important from the point of view of our formal model is that the supporting channel—and the associated collection of "relevant" tokens—plays an important role in the interpretation of generics. This is further demonstrated below, when we discuss how difference between speaker and hearer as to what is the appropriate supporting channel can lead to misunderstanding in dialogues involving generic sentences.

4 Channels and Context

In this section, we further discuss the role played by a channel in providing a scope to a regularity. As explained above, the channel-theoretic model of generics views a generic sentence as describing a channel. The described channel effectively provides

[10] For example, the channel supporting Morreau's generic involving boiled potatoes would contain only exceptions.

[11] The normative view of generics (e.g. [Morreau, 1992]) itself does not provide a particularly conclusive account as to which generics are true and which are false. Under the normative view, a true generic is one which holds universally over all "normal" individuals; however, it is never clear as to what exactly constitutes a "normal" individual other than the fact that it has the properties claimed of it by the generic.

[12] The example channels described would contain decreasing numbers of exceptions, since sick birds would be exceptions in the second channel, and penguins and emus would be exceptions in the first channel.

a "scope of applicability" for the generic relationship. A channel has a collection of tokens associated with it, and it is only those tokens which are "relevant" to the generic relationship.

This is illustrated by considering example (1): "Swans are white". By uttering this sentence, the speaker is asserting that the relationship *swan→white* is supported by some channel $\mathcal{C}_1$. We expect $\mathcal{C}_1$—the channel which the speaker is describing—is concerned with European swans, which are, by and large, white in colour. This is reflected by taking the set of tokens associated with $\mathcal{C}_1$ to be European-swan-tokens—i.e. for all $s \mapsto s \in tok(\mathcal{C}_1)$, s is a European swan token.

The problem of conflicting generics, each of which seems applicable, is also addressed by the channel-theoretic model. For example, the sentences (5) and (6) involving peacocks can simply be seen as describing different channels: (5) describes a channel that contains only male-peacock tokens, while (6) describes a channel containing only female-peacock tokens. Another pair of examples which illustrates the problem is the following:

(7) Dutch make good sailors.
Dutch make good farmers.

Even if we assume that farmers make poor sailors and sailors make poor farmers, we can make sense of the above sentences by ensuring that the first sentence describes a channel which is restricted to sailor-tokens and the second sentence a channel which is restricted to farmer-tokens. This corresponds with our intuitions that these sentences can be paraphrased as 'Dutch sailors are good sailors' and 'Dutch farmers are good farmers' respectively.

As with the pair of peacock sentences, normative accounts of generics are stretched by such pairs of generics being simultaneously acceptable, since this may rule out the possibility of there being any "normal" individuals at all.

The channel-theoretic analysis is also flexible enough to allow "arbitrary" exceptions to generic relationships. For example, not all European swans are white; hence, there will be some European swan-tokens which are exceptions in the channel $\mathcal{C}_1$. For example, if s is such a token then $s \mapsto s$: *swan→white* does not hold in $\mathcal{C}_1$; however, this in no way invalidates the fact that $\mathcal{C}_1$ supports *swan→white*.

In our model, the distinction between exceptions and "irrelevant" tokens (i.e., those outside the scope of applicability of a generic) parallels an agent's expectations regarding default reasoning. An appropriate channel supporting "Peacocks lay eggs" would not contain any male-peacock tokens; in the channel-theoretic model of defeasible reasoning described below, no default conclusion regarding a male peacock's egg-laying abilities would be drawn. On the other hand, female-peacock tokens are relevant to the channel supporting this generic; hence, in the absence of other information, a female peacock token would be defeasibly assumed to be an egg-layer. Of course—as discussed above—the "appropriate" channel for a given use of this generic sentence may not be the one that applies to all and only female-peacock tokens. For example, it may exclude sick-peacock, old-peacock and young-peacock tokens. What is seen as the exact scope of the generic, defined by the channel being described, may differ between different agents—for example between two participants in a dialogue. This is the source of potential miscommunication described later. However, the channel that the agent describes with her utterance determines in part those tokens about which she is prepared to draw default conclusions, using the generic relationship.[13]

[13]The importance of "scoping" when performing default reasoning has been discussed in the AI literature—see ([Etherington et al., 1991]).

5 Reasoning with Generics

An important property of generics is that they are used for default reasoning. This can involve both the inference of new generics and the inference of properties of the associated individuals. For example, given the following:

"Birds fly"
"Tillermids are birds"[14]

we would probably conclude by default that "Tillermids fly". Furthermore, if we were told Tweety was a bird, then we would conclude by default that Tweety can fly. However, if we were additionally told that Tillermids were a kind of penguin, then we would probably withdraw our earlier conclusion. Similarly, if we knew that Tweety was a penguin, we would not conclude that he can fly.

5.1 Inferring new generics from old

Barwise and Seligman have defined a number of operations on channels which form the basis of a channel-theoretic calculus of information flow. For example, given channels $\mathcal{C}_1$ and $\mathcal{C}_2$ which respectively contain the constraints $\phi\rightarrow\psi$ and $\psi\rightarrow\tau$, the **serial composition** $(\mathcal{C}_1;\mathcal{C}_2)$ of these channels contains the constraint $\phi\rightarrow\tau$. [Cavedon, 1995] showed that these operations need to be modified in order to display the behaviour required of systems of default reasoning (e.g. lack of Transitivity and Monotonicity). In particular, Cavedon showed that by modifying the operations so that they take account of the **background assumptions** of a channel, a powerful logic of conditionals is obtained, leading to a logic of generics with certain desirable properties.[15]

A channel implicitly reflects a constraint's background assumptions via the context it provides. For example, the channel supporting the generic "Birds fly" will reflect the implicit assumption that this generic excludes penguins by virtue of the fact that penguin tokens are either excluded or are exceptions in the channel. However, we define a method that makes these assumptions more explicit, allowing the channel operations to be modified so as to account for them. Most importantly, the assumptions behind a regularity supported by a channel $\mathcal{C}$ are made explicit via $\mathcal{C}$'s relationship to other channels, and *not* within $\mathcal{C}$ itself. This is captured via the **subchannel** relationship, defined stepwise as follows. We assume each classification A has a (primitive) type-entailment ordering $\leq_A$ associated with its types—thus $\phi \leq_A \psi$ if ψ entails ϕ.[16]

Definition *Classification A is a* subclassification *of classification B, written $A \sqsubseteq B$, iff $tok(A) \subseteq tok(B)$, $typ(A) \subseteq typ(B)$, $\leq_B$ agrees with $\leq_A$ when restricted to $typ(A)$, and for each $a \in tok(A)$ and $\phi \in typ(A)$, $a : \phi$ holds in A iff $a : \phi$ holds in B.*

Definition *Let $\phi\rightarrow\psi, \phi'\rightarrow\psi'$ be constraints contained in channels $\mathcal{C} : A \Rightarrow B$ and $\mathcal{C}' : A' \Rightarrow B'$, respectively. We write $\phi\rightarrow\psi \leq_{(A',B')} \phi'\rightarrow\psi'$,* wrt $\mathcal{C}$ and $\mathcal{C}'$,[17] *if:*

[14]Tillermids are an imaginary kind of bird.

[15]Taking account of background assumptions in this way has some similarities to the situation-theoretic approach to a logic of conditionals described in ([Barwise, 1985]). However, as discussed below, the channel-theoretic approach does not require the provision of a type which captures all background assumptions of a given constraint.

[16]We don't impose any particular properties on $\leq_A$, but there are some obvious minimum requirements.

[17]We drop the reference to the specific channels when this causes no confusion.

1. $A \sqsubseteq A', B \sqsubseteq B'$,
2. $\phi \leq_{A'} \phi'$, *and*
3. $\psi' \leq_{B'} \psi$.

Definition *Let $\mathcal{C} : A \Rightarrow B$ and $\mathcal{C}' : A' \Rightarrow B'$ be channels, and f a function from $typ(\mathcal{C})$ to $typ(\mathcal{C}')$. We say $\mathcal{C}$ is a $\leq_{(A',B')}$ –*subchannel *of $\mathcal{C}'$* wrt *f, written $\mathcal{C} \sqsubseteq_{(f,A',B')} \mathcal{C}'$, if*

1. $A \sqsubseteq A', B \sqsubseteq B'$;
2. $tok(\mathcal{C}) \subseteq tok(\mathcal{C}')$;
3. *for all $c \in tok(\mathcal{C})$ and $\gamma \in typ(\mathcal{C})$, if $c : \gamma$ in $\mathcal{C}$ then $c : f(\gamma)$ in $\mathcal{C}'$;*
4. *for all $\gamma \in typ(\mathcal{C})$, $\gamma \leq_{(A',B')} f(\gamma)$ wrt $\mathcal{C}$ and $\mathcal{C}'$.*

We write $\mathcal{C} \sqsubseteq_{(A',B')} \mathcal{C}'$ if there exists an f such that $\mathcal{C} \sqsubseteq_{(f,A',B')} \mathcal{C}'$.

The subchannel ordering reflects a notion of "increasing discrimination", in that $\mathcal{C} \sqsubseteq \mathcal{C}'$ means that $\mathcal{C}'$ makes explicit some of the background assumptions of $\mathcal{C}$. For example, $\mathcal{C}$ may be described by "Birds fly" (i.e. $(bird \rightarrow flies) \in typ(\mathcal{C})$) whereas $\mathcal{C}'$ is described by "Birds which aren't penguins fly" (i.e. $(bird \wedge \neg penguin \rightarrow flies) \in typ(\mathcal{C}')$). The requirement that $tok(\mathcal{C}) \subseteq tok(\mathcal{C}')$ ensures that we are dealing with the same (or increasing) context as we move up the ordering. Note, however, that $\mathcal{C}'$ is not to be seen as a somehow "truer" version of $\mathcal{C}$—both channels are equally valid entities.

The subchannel ordering allows the background assumptions of a channel $\mathcal{C}$ to be explicated *without* representing those assumptions in $\mathcal{C}$ itself. Further, $\mathcal{C}$'s assumptions need not be fully represented in any one channel (i.e. they may be distributed across different super-channels). The channel operations of Barwise and Seligman are then evaluated against a background hierarchy of channels ordered by subchannel. For example, the serial composition channel $(\mathcal{C}_1; \mathcal{C}_2)$ defined earlier will now only support $\phi \rightarrow \tau$ if the background assumptions of $\mathcal{C}_1$ and $\mathcal{C}_2$ do not conflict. This is determined by checking whether there are any superchannels $\mathcal{C}'_1, \mathcal{C}'_2$ such that:

1. $\mathcal{C}_1 \sqsubseteq_{f_1} \mathcal{C}'_1$, $\mathcal{C}_2 \sqsubseteq_{f_2} \mathcal{C}'_2$; and
2. $ante(f_1(\phi \rightarrow \psi))$ type-conflicts with $ante(f_2(\psi \rightarrow \tau))$, where $ante(\phi' \rightarrow \psi')$ denotes ϕ'.[18]

By modifying the operations in this way, a powerful conditional logic is obtained. For reasoning with generics, [Cavedon, 1995] defines an algorithm for constructing a hierarchy from a starting set of channels. In particular, if the consequents of two constraints type-conflict, then super-channels are constructed which encode the negation of the antecedents as implicit assumptions. For example, if $\mathcal{C}_1$ supports $bird \rightarrow flies$ and $\mathcal{C}_2$ supports $penguin \rightarrow \neg flies$, then a channel $\mathcal{C}'_1$ supporting $bird \wedge \neg penguin \rightarrow flies$ is inserted as a superchannel of $\mathcal{C}_1$—i.e. $\neg penguin$ is recognised as an implicit background assumption to $bird \rightarrow flies$.[19] Cavedon shows that using the modified operators leads to a powerful logic of generics, satisfying the usual requirements of non-monotonicity, the Penguin principle and graded normality. See ([Cavedon, 1995]) for further properties and details.

[18] In the full model, background assumptions need not be represented in the antecedent of a super-channel's constraints.

[19] The definition avoids inserting $\neg bird$ as an implicit assumption to $penguin \rightarrow \neg flies$ because of the presence of the constraint $penguin \rightarrow bird$. See ([Cavedon, 1995]) for the full definition.

5.2 Reasoning defeasibly about individuals

In the channel-theoretic model of generics, generics are "simply true"—i.e., whether or not a generic is supported by a channel is not at all determined by the properties of individuals associated with that channel. In order to be able to reason defeasibly about individuals, we therefore need to impose a concept of "maximal normality" on channels. That is, when an agent reasons about individuals using a constraint supported by a channel, she assumes that as many individuals as possible associated with that channel are *normal*. Earlier, we stated that, given a channel $\mathcal{C}$ supporting a generic G, the individuals that an agent was willing to draw (default) inference using G on were those that participated in connections in $\mathcal{C}$. Hence, the maximal normality condition ensures that channels contain as many connections as possible—only the individuals which an agent considers "abnormal" with respect to a generic have no associated connection in the pertinent channel.

Maximal normality is defined against a collection of channels ordered by subchannel and some given collection of information. This information is a set of propositions of the form $(s : \phi)$, denoting the assertion that s is of type ϕ. To ensure that normality is **maximal**, the only time an exception is permitted is when one of the following conditions holds:

1. the given information conflicts with the information that would otherwise be inferred;
2. inheritance from separate premises would conflict.

The second case is taken care of by referring to the channel hierarchy since conflicting consequences lead to encoded background assumptions, as described above. This leads to the following definitions.

Definition *Let $\mathcal{C} : A \Rightarrow A$ be a channel (which is part of a channel hierarchy) supporting a constraint $\phi \rightarrow \psi$, Ψ a collection of propositions (formed from the tokens and types of A), and $t \in tok(A)$ a token such that $(t : \phi) \in \Psi$. The token t is said to be* inappropriate in $\mathcal{C}$ wrt Ψ *if either:*

1. *$(t : \psi') \in \Psi$ and ψ type-conflicts with ψ', or*
2. *there exists a channel $\mathcal{C}'$ such that $\mathcal{C} \sqsubseteq_f \mathcal{C}'$, where $\phi \rightarrow \psi \in typ(\mathcal{C})$ and $f(\phi \rightarrow \psi) = \phi' \rightarrow \psi'$ and $(t : \tau) \in \Psi$, where τ type-conflicts with ϕ'.*

To illustrate the second case, if Ψ contains $(tweety : penguin)$ then $tweety$ is inappropriate in the channel $\mathcal{C}_1$ (in the last paragraph of the previous subsection) supporting $bird \rightarrow flies$ since the information in Ψ conflicts with the antecedent of the constraint in $\mathcal{C}'_1$. On the other hand, $tweety$ is "normal" in $\mathcal{C}_2$. Maximal normality is defined by requiring that for any channel $\mathcal{C} : A \Rightarrow A$, every token in A which is not inappropriate in $\mathcal{C}$ participates in a connection in $\mathcal{C}$.

Inference via channels proceeds as follows. If $t \mapsto t \in tok(\mathcal{C})$, $\phi \rightarrow \psi \in typ(\mathcal{C})$ and $t : \phi$, then we infer $t : \psi$.[20] Hence, in our example, we *cannot* infer $tweety : flies$ in $\mathcal{C}_1$ because $tweety$ is inappropriate in $\mathcal{C}_1$ and therefore there is no connection $tweety \mapsto tweety$ in $\mathcal{C}_1$. On the other hand, $tweety$ is not inappropriate in $\mathcal{C}_2$, meaning we can infer $tweety : \neg flies$.

We have only sketched the channel-theoretic model for default reasoning here. See ([Cavedon, 1995]) for further details and a discussion of the properties of the model.

[20] Note that the connection may not be **classified** by the constraint—i.e. it may be that $(t \mapsto t : \phi \rightarrow \psi)$ does not hold in $\mathcal{C}$. In this case, $t \mapsto t$ is said to be an **exceptional connection** and leads to the inference of incorrect information. Information which an agent concludes by default may in fact turn out to be incorrect. However, models of default reasoning are more concerned with determining what an agent can *reasonably* conclude, given only partial information.

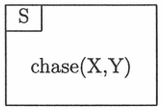
6 More Complex Types

We will now examine how we can incorporate more complex types into our constraints. We may use **situation-theoretic types** for this purpose, such as can be obtained from the grammatical processing of sentences by a situation-theoretic grammar of the kind developed by Cooper and described in the FraCaS deliverable D8 ([FraCaS, 1994], ch.3). Using such situation-theoretic types in our constraints will facilitate the development of a situation-theoretic grammar for the processing of generic sentences, and we intend to pursue work on this in the future.

Furthermore, the use of situation-theoretic types in our constraints seems particularly appropriate given that channel theory developed in part from earlier situation-theoretic notions of information flow and constraints, described, for example, in ([Barwise and Perry, 1983]).

A further advantage of using situation-theoretic types is that they allow us to combine our analysis of generic sentences with a situation-theoretic analysis of bare plurals, which enables us to account for the availability of generic v. existential readings of bare plurals (see [Glasbey, 1995] for a proposal along these lines). We do not, however, attempt to do this here.

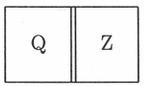
We will use the Extended Kamp Notation (EKN) of ([Barwise and Cooper, 1993]), which represents situation-theoretic objects using a graphical notation similar to DRT ([Kamp and Reyle, 1993]). A box notation is used for situation-theoretic objects such as infons, situations and propositions. Propositions in EKN include objects of the form:



{P}

which is the proposition P that a situation S supports the infon $\boxed{\text{chase(X,Y)}}$ (the latter being written in standard linear notation as $\ll$ chase, X, Y; 1 $\gg$). X and Y are parameters[21] which fill the argument roles of the relation **chase**. A proposition like this one, that a situation supports an infon,[22] is called an **Austinian** proposition.

Situation-theoretic objects may have **restrictions** imposed on them, as shown below, where the object Q is restricted by a proposition or conjunction of propositions Z.

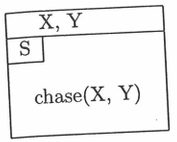


Q denotes an object only if Z is true.

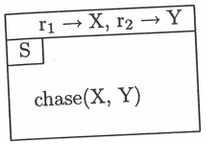
A situation-theoretic **type** is formed by simultaneously abstracting over one or more of the parameters of a proposition, using the technique of **simultaneous abstraction** developed in ([Aczel and Lunnon, 1991], [Lunnon, 1991]). For example, we may abstract over X and Y in the proposition P above to give:

[21] We denote parameters by capital letters.

[22] The situation is also said to be of the type represented by the infon, if the proposition is true.



which is the binary type of an X, Y pair such that situation S supports the infon $\boxed{\text{chase(X, Y)}}$. Following [Barwise and Cooper, 1993], we can **index** the **roles** corresponding to abstracted-over parameters with any kind of situation-theoretic object. For example, we can index the roles corresponding to X and Y above with r_1, r_2 respectively:

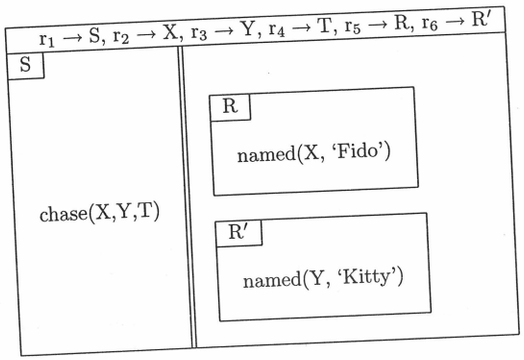


In any assignment of parameters to the roles of an abstract, all roles with the same role index must be assigned to the same parameter.

Grammatical processing of a sentence such as:

(8) Fido chased Kitty.

using a situation-theoretic grammar would give the type:[23]



We assume here that the relation **chase** has a time argument, filled here by the parameter T. Such a time argument need not necessarily be filled in the syntax. See ([Glasbey, 1994], ch.3) for discussion of the time arguments of relations.

R and R′ are **resource situations** supporting information about the naming of individuals (see, for example, [Cooper, 1993] for discussion of resource situations).

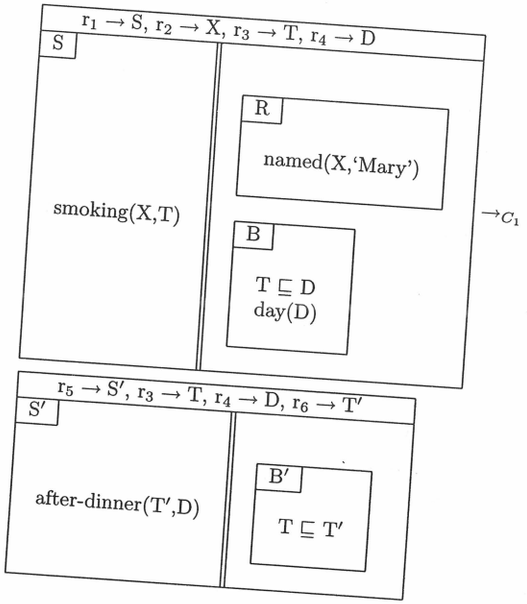
It is straightforward to use such types as the antecedent and consequent of a channel-theoretic constraint. Consider, for example, the generic sentence:

[23] We ignore tense information here, as well as certain other kinds of information that are not central to our analysis.

(9) Mary smokes after dinner.

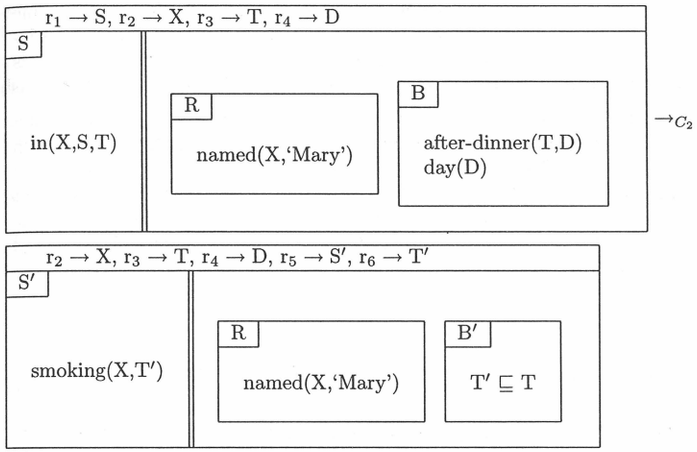
(9) has (at least) two readings—one which can be paraphrased 'If Mary is smoking, then it's likely to be after dinner' and another which can be paraphrased 'If it's after dinner, then Mary is likely to be smoking (at some time during that period)'. The example is taken from ([Krifka, 1995]). We will not give a detailed discussion of this example here—see ([Cavedon and Glasbey, ress]) for a justification of this analysis. We show in that paper that the two readings can be captured by the following constraints:

First reading:



The above tells us that there is a constraint in a channel C_1 between two types. The antecedent type is a quaternary type of a situation S, an individual X named 'Mary', a time T and a day D, where S supports the fact that Mary is smoking at T. The succedent type is a quaternary type of a situation S′, the same time T, the same day D and a time T′ that temporally includes T, such that S′ supports the fact that T′ is the (whole) after-dinner period of day D.

Second reading:



The above tells us that there is a constraint in a channel C_2 between two types. The antecedent is a 4-ary type of a situation S, an individual X named 'Mary', a time T which is the after-dinner period of D, and a day D, where S supports the information that X is in that situation for the duration of time T. The succedent is a quaternary type of the individual named 'Mary', the same time T, a time T′ temporally included in T, and a situation S′.

Notice that in each of these constraints there are certain parameters which occur in the succedent but not in the antecedent type. In the constraint corresponding to the first reading, these are S′ and T′. We paraphrased the content of the succedent by referring to 'a time T′' and 'a situation S′'. Clearly we want to think of these parameters as being existentially quantified over.

What we need is to ensure that parameters appearing in the succedent but not in the antecedent of a constraint get an existential interpretation (similar to what happens to variables in DRT).

This is not difficult to achieve, using the apparatus of channel theory and situation theory. Suppose we have a channel that supports the constraint $\phi \rightarrow \psi$, and contains connections of the form $p \mapsto q$. Now suppose we have the information that $[r_1 \rightarrow X] : \phi$. From this, we can infer ("by default") that there must be an assignment $[r_2 \rightarrow Y]$ such that $[r_2 \rightarrow Y] : \psi$. That is:

$$[r_1 \rightarrow X] : \phi \quad \rightarrow_d \quad \exists Y \text{ s.t. } ([r_1 \rightarrow X] \mapsto [r_2 \rightarrow Y]) \;\&\; [r_2 \rightarrow Y] : \phi$$

where '$\rightarrow_d$' is some kind of default inference.

In general, if we have a constraint $\phi \rightarrow \psi$, and f_1 is an assignment to the roles of ϕ, we need to be able to infer the existence of an appropriate extension f_2 to f_1, such that f_2 is an assignment to the roles of ψ. Thus we need to quantify over assignments, in such a way that, if we have a channel supporting the constraint $\phi \rightarrow \psi$, then:

$$\forall f \; (f : \phi \quad \rightarrow_d \quad \exists f' \text{ s.t. } (f \sqsubseteq f') \;\&\; (f' : \psi) \;\&\; (f \mapsto f'))$$

which gives us the required existential quantification over roles in f' that are not in f. (For further discussion, see [Cavedon, 1995].)

7 Generics in Dialogue

Finally, we turn briefly to the use of generic sentences in dialogue. We discuss some of the ways in which misunderstandings may occur between participants in a dialogue involving generic sentences—particularly where the "scope" of the generic is concerned. The work outlined here is described in more detail in ([Cavedon and Glasbey, 1996]).

We discussed earlier how we take the content of a generic statement to consist of a **demonstrative content** (which is a channel) and a **descriptive content** (which is the regularity or constraint supported by that channel). The content of a generic statement is the assertion that a given channel $\mathcal{C}$ supports a particular constraint $\phi \rightarrow \psi$. The collection of tokens associated with a channel effectively captures the "scope" over which the generic is asserted to hold. Sentences (5) and (6), repeated here as (10) and (11):

(10) Peacocks have brightly coloured tail feathers.

(11) Peacocks lay eggs.

involve different channels—the first supports a regularity involving male peacock tokens and the second supports a regularity involving female peacock tokens. We then have to consider the issue of which channel is the "correct' (or most appropriate) one for a given use of a generic sentence. As discussed earlier, we take the appropriate channel to be the one which the speaker has in mind when she makes the generic statement.

Now consider a dialogue situation. It may happen that the speaker does not supply enough information for the hearer to fully identify the channel that the speaker is talking about. The hearer may remain uncertain, may ask for further information, or may take the speaker to be describing some channel other than the one the speaker had in mind. This channel may contain more or fewer connections than the one the speaker intended. For example, the hearer may misunderstand an utterance of (10), where the speaker was referring to the channel containing only male peacock tokens, and assume that she was referring to the channel containing *all* peacock tokens, male and female.

It is also possible that the hearer may misunderstand the descriptive content of the utterance. Consider, for example, an utterance of (9), repeated here as (12), which we saw above has at least two readings.

(12) Mary smokes after dinner.

It may happen that the speaker utters (12) with the intention of conveying the first reading, whereas the hearer instead obtains the second reading.[24]

Here, we are solely concerned with modelling the first kind of misunderstanding—where the hearer is mistaken about the demonstrative content (the channel) but not about the descriptive content (the constraint).

We will not present a detailed model of dialogue in this paper. Instead, we will point out some of the features that are required of a model for a dialogue involving generic statements, and offer a sketch of such a model. We begin with a short

[24]Perhaps this is unlikely in a dialogue between native speakers, where intonational clues are usually sufficient to distinguish between such alternative readings. The misunderstanding is probably much more likely to occur in written text, in cases where sufficient context is not provided to identify the intended reading.

dialogue between two conversational participants, T (a teacher) and S (a student). T has given S an assignment to find out as much as he can about a species of bird unfamiliar to them both—the Tillermid.

T(1) : Tell me what you've found out so far about Tillermids.

S(1) : Well—Tillermids are black, with scarlet wing-tips. They lay pale blue eggs and they catch fieldmice to feed to their young.

T(2) : Tell me, are they all black with scarlet wing-tips?

S(2) : Well, no. Actually just the males. The females are a sort of dingy brown all over.

The dialogue begins with a request by T for S to give her some information about Tillermids. Notice that, as far as we can tell, T appears to be seeking information about "Tillermids in general"—she makes no reference to any particular subset of the species such as "the females". S(1) replies with three generic clauses, each giving a property of Tillermids. There is no indication of whether S intends the demonstrative content of the clause in each case to be the channel involving all Tillermid tokens, or one involving a subset of these.

T asks for this clarification in the case of S's first generic statement—she asks whether Tillermids are all "black with scarlet wing-tips". In our model of generics, we take this to be T asking which channel is the demonstrative content of S's first generic statement.

S(2) replies, telling T that the generic applies to "just the males"—i.e., the channel that he has in mind for this generic is the one involving only male Tillermid tokens. He adds the further statement that 'The females are a sort of dingy brown all over', which clearly refers to the channel involving only female Tillermids.

We thus have what appears to be a rather common kind of conversational interchange, where one speaker offers some information and the hearer accepts this while requesting further information and/or clarification, which the first speaker then supplies. We would not want to say that S "misled" T by saying 'Tillermids are black with scarlet wing-tips'—but, rather, that the information he gave was insufficient for T's purposes.

We clearly need to model the fact that what occurs initially here is a "partial transfer" of information from S to T. The part of the information that is successfully tranferred is the descriptive content of the generic—the constraint $TILL \rightarrow BSWT$ where $TILL$ is the property of being a Tillermid and $BSWT$ is the property of being black with scarlet wing-tips. The information that is not conveyed is the demonstrative content—the channel which supports this constraint.

A range of alternative responses by T might have occurred here. T could, for example, have assumed that S was talking about the channel involving *all* Tillermids. The misunderstanding might have come to light only later—say, when T, seeing a brown bird nearby, says 'Well, that's obviously not a Tillermid' and S says 'It could be a female'. Another possibility is that T might have used her world knowledge about birds (the fact that the male of the species is often more brightly-coloured than the female) to infer that S was probably talking about male Tillermids only.

Remember, however, that T is a teacher, and teachers can be over-pernickety about the use of language. Suppose that T replies as follows:

T(3) : Hmm. Isn't it rather misleading, then, to say that Tillermids are black with scarlet wing-tips? You really must be careful to specify whether you are talking about the whole species, or only part of it.

S(3) : Sorry.

Now suppose T continues addressing a suitably-chastised S:

T(4) : OK, let's go on. What was the second thing? You say Tillermids lay pale blue eggs?

S(4) : Well, about half of them do.

T(5) : That's very odd. What colour eggs do the other half lay?

S(5) : They don't lay any eggs at all, because they're males. Only the females lay eggs.

T(6) : Do you think I don't know that? Don't be impertinent!

S(6) : Sorry.

Now we have a misunderstanding. S, taking T's reprimand to heart (or, perhaps more likely, seeing a chance to get back at his teacher) spells out that "about half of them" lay pale blue eggs. He is, of course, technically correct—only female Tillermids lay eggs so he is correct to say that "about half of them", meaning "about half of the species", do so. T's misunderstanding arises because she has already used her world knowledge (that only females lay eggs) to infer that S's statement 'They lay pale blue eggs' refers to the channel involving female Tillermids only. Thus when S(4) spells out that 'About half of them [lay pale blue eggs]', T takes this to mean about half of the *female* Tillermids. Puzzled, she asks what colour eggs the other half (of the females) lays. S(5) explains, and T takes his reply to be a piece of impertinence.

Now that the meanings of the first two of S's generic statements have been successfully negotiated (S and T can be said to have the same channel-constraint pairs "in mind"), T moves on to the third, and asks:

T(7) : What was the other thing?

S(7) : They catch fieldmice to feed to their young.

T(8) : Ah yes. Now, is that all of them, or just the males, or just the females?

S(8) : I don't know.

Interestingly, it turns out that S did not have a "fully-specified" channel in mind here as the demonstrative content of the generic 'They catch fieldmice to feed to their young'. It seems quite reasonable that this should be the case—that we may sometimes make generic statements without knowing exactly which channel is the appropriate one. A formal model of the dialogue will therefore need to be able to capture this underspecification on the part of the speaker, as well as other factors discussed above like the interaction of the hearer's prior knowledge with the new information received.

We thus require a model which can capture what we might call "partially successful communication"—which includes cases where some but not all of the relevant

information in the speaker's mind is transferred to the hearer. A suitable model for this purpose appears to be the channel-theoretic model of dialogue developed by [Healey and Vogel, 1994]. Healey and Vogel (H&V) present a semantic framework for modelling the information flow between participants in a dialogue which takes account of the idea that it is unlikely that any two speakers will understand "exactly the same thing" by an utterance, even in cases where there is no perceived ambiguity or misunderstanding. H&V consider that one important source of this variation is the fact that it is unlikely that any two individuals will have exactly the same "ontology"—they will almost certainly "carve up the world in different ways". They point out that this raises problems for traditional semantic theories which assume an interpretation function mapping the meanings of words onto parts of the world. Our dialogue, while not particularly highlighting matters of ontology, would nevertheless be difficult to model in a traditional framework given that, as we have seen, the transfer of information is not an all-or-nothing matter. We have seen how it is possible for only part of the speaker's intended meaning to "reach" (or be understood by) the hearer.

We will not give details of the H&V model here, but refer the interested reader to ([Healey and Vogel, 1994]), and to ([Cavedon and Glasbey, 1996]) where we apply this model to the above dialogue. The essence of the H&V model is a "communication channel" which exists between two participants A and B in a conversation.[25] The communication channel is composed of two channels—a "production channel" which supports constraints between idea types (in A's head) and types of utterances (as made by A), and a "reception channel" which supports constraints between characteristics of utterances (as made by A) and idea types in B's head. The model allows communication to take place even when the idea that B ends up with is not (exactly) the idea that A set out to convey. Communication can thus be partially successful. We can model, for example, the fact that a generic statement by S may succeed in conveying the descriptive content (the constraint) but not the demonstrative content (the channel) to T. We can therefore also model the requests for clarification and the negotiation which result eventually in all S's pertinent information being conveyed to T. We could also model cases where the descriptive content is "misunderstood"—for example where the wrong reading for an ambiguous generic sentence like (12) is conveyed.

8 Conclusion

We have shown that generic statements make reference to a "context", and that this context plays an important role in their interpretation. The context defines the intended "scope" of the generic—which can be thought of as the collection of individuals which the generic statement is "about".

We described a channel-theoretic model of generics which endows constraints with a context—this context being the **channel** supporting the constraint. This enables us to model the information content of a generic statement in a way which captures in a precise way the observed reference to context. We can thus give an adequate semantics for a number of generic sentences that have posed problems for previous accounts. In fact, we believe that these problematic examples provide empirical support for channel theory's particular structural account of regularities.

We showed, too, how the channel-theoretic analysis provides the basis of a framework for reasoning both *with* generics (i.e., about the default properties of individ-

[25] Healey and Vogel actually consider multi-agent communication but for simplicity we restrict ourselves to two participants.

uals) and *about* them (to infer new generics). This gives a powerful logic of generics supporting the usual requirements of non-monotonicity, the Penguin principle and graded normality.

By linking the channel-theoretic framework to a situation-theoretic analysis of sentences, we can obtain a rich ontology of semantic objects which allows powerful formal models of a wide range of generic phenomena.

Finally, we discussed some of the kinds of misunderstandings that can occur in dialogues involving generic statements. We outlined how our account of generics may be combined with a formal model of dialogue (such as that of Healey and Vogel) in a way which allows us to model misunderstandings about the demonstrative content (i.e., the channel) of a generic statement. Further work is needed to develop and extend such models to include other kinds of misunderstanding and an account of belief revision on the part of the dialogue participants.

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Generics and Frequency Adverbs as Probability Judgments*

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Abstract
This paper argues that several seemingly unrelated properties of generics and frequency adverbs have a single underlying cause. Generics and frequency adverbs are unbounded, they cannot be applied to temporary generalizations, and they require a regular distribution of events along the time axis. A generic may be false even if the vast majority of individuals in its domain satisfy the predicated property, whereas a freqeuncy statement using e.g. *usually* would be true. Truth judgments of generics vary considerably across speakers, whereas truth judgments of frequency statements are much more uniform. These phenomena can all be accounted for if we assume that generics and frequency adverbs express probability judgments, and if, in turn, we interpret probability judgments to be statements of relative frequency in the limit.

1 Five Puzzles

Generics and frequency statements occur frequently in natural language. Much of our knowledge about the world is expressed using such sentences—one needs only to glance at an encyclopedia to encounter myriads of examples. Yet it is far from clear what such sentences mean, i.e. what a given sentence entails, what it presupposes, and what it is that makes it true or false.

With a view towards a solution to this general conundrum, we will consider five more specific puzzles in this paper. We will argue that these puzzles can be solved if we assume that generics and frequency statements express probability judgments, and if these probability judgments, in turn, are interpreted as relative frequencies in the limit.

1.1 Unboundedness

Declerk (1986) has noted that generics are *unbounded*, and the same seems to hold of frequency staements as well. According to Declerk, "a sentence refers to a bounded number of events... if that sentence somehow involves reference to something that acts as a boundary for the number of events" (p. 172). Thus, (1.a) is acceptable, because it is about an unbounded set of events, whereas (1.b) is ruled out, because it sets an explicit limit on the size of its domain, namely 365:

(1) a. John always/Ø walks to school.
b. *On 365 occasions John always/Ø walks to school.

The "something" which serves as a boundary may be an explicit limit, as in (1.b), but need not be; sentences which refer to specific times or places, or which hold of relatively short durations, are bounded too. For example, (2.a) (Henriette de Swart, personal communication) is fine, whereas (2.b) is not:[1]

(2) a. This year, John always/Ø walks to school.

*. I would like to thank Bob Carpenter, Greg Carlson, Clark Glymour, Chris Manning, Carolyn Penstein-Rosé, Henriette de Swart and the audience at the *Amsterdam Colloquium* for helpful comments and suggestions.

1. Cf. Comrie's (1976) claim that a habitual reading results if a sentence refers to a situation holding through an extended period of time.

b. *This morning, John always/Ø walks to school.

Generics and frequency statements which quantify over individuals seem to be unbounded in the same way. Thus, for example, (3.a) (from Carlson 1977) easily receives a generic interpretation, (3.b) is not generic, and (3.c) is not even grammatical:

(3) a. Texans are often/Ø tall.

b. Three Texans are tall.

c. *Three Texans are often tall.

While both (3.b) and (3.c) entail that the size of the domain of the quantifier is 3, sentence (3.a) expresses no such limit, and is thus unbounded.

It should be emphasized that being unbounded is not the same as being finite. The size of the domain of the quantifier in (3.a) is, of course, finite; there is a finite number of Texans, and, in all likelihood, there will be a finite number of Texans throughout eternity, assuming that the earth will not exist forever; however, no limit on the number of Texans is expressed by (3.a).

Being bounded, then, is a property of the whole sentence, not just the noun phrase; it is dependent on a variety of factors in addition to the noun phrase, among them tense, aspect, the use of modals and adverbial phrases (see Declerk 1986). A numerically quantified noun phrase, for example, may sometimes be interpreted generically, as the following examples (from Krifka *et al.* 1995) indicate:

(4) a. Two canaries can be kept in the same cage if it is large enough.

b. Two magnets either attract or repel each other.

c. Two's company; three's a crowd.

It should be noted that the sentences in (4) are not about the bounded domains containing, respectively, two canaries, two magnets or two people; rather, these sentences are about the *unbounded* domains containing unspecified numbers of pairs of canaries, magnets, and people.

Unlike the domain of a determiner, the domain of a generic or frequency adverb cannot be restricted by the context to a bounded set (Croft 1986). For example, when immediately following (5.a), sentence (5.b) would be only about the lions in the circus, and may very well be true; sentence (5.c), on the other hand, would be false, since it expresses a statement about lions in general:

(5) a. There are three lions in the circus.

b. Every lion is tame.

c. Lions are usually/Ø tame.

Context can, in fact, restrict the domain of a generic to an *unbounded* set. For example, when (6) is uttered in the context of a discussion of North America, it would be true, but when uttered in the context of a discussion of the Mediterranean—false:

(6) Trees display a variety of vivid colors in the fall.

Declerk and Croft discuss the phenomena described here, but they do not provide an explanation; why is it that the domains of generics and frequency adverbs are unbounded in this way? In this paper we will propose an answer.

1.2 Temporary Generalizations

Generics and frequency statements do not hold of generalizations which are perceived to be temporary. Suppose it so happened that all Supreme Court judges had a prime Social Security number; this would not suffice for (7) to be true:

(7) Supreme Court judges always/usually/Ø have a prime Social Security number.

It seems that the truth of (7) requires that Supreme Court judges have a prime Social Security number not just at present, but that this property can reasonably be expected to hold in the future with some regularity, say because a law were enacted which posed restrictions on the Social Security number of judges.

This property of generics has been observed by many researchers. It is usually claimed that the problem with sentences such as (7) is not that they express a temporary generalization, but rather an accidental one. Generics, it is proposed, have a modal character; they only express properties which are, in some sense, "necessary," "normative," "essential," or "inherent."[2]

What precisely is meant by these terms has yet to be made precise; but even if such a formulation were possible, it seems that modal approaches miss the mark. The following sentences are true, although they do not seem to express necessary (essential, inherent, etc.) properties:

(8) a. A cheetah outruns any other animal.

b. Spices are affordable.

c. Gold cubes are smaller than 10 cubic meters.

d. Dogs annoy Sam.

Perhaps running fast is an inherent property of cheetahs, but certainly not the property of running faster than any other animal, since some other animal might have been faster. Affordability is not a necessary property of spices—in fact, throughout much of history, spices were extremely expensive; yet (8.b) is true nonetheless. Similarly, we would be hard-pressed to claim that gold cubes are inherently smaller than 10 cubic meters, or that annoying Sam is an essential property of dogs.

Even if a modal approach were appropriate for generics, this would still leave frequency statements unaccounted for. Advocates of the modal view of generics either ignore the latter, or treat them as lacking a modal character. But this leaves unexplained the unacceptability of frequency adverbs in sentences such as (7) above.

1.3 Regular Distribution in Time

Stump (1981) has noticed that a frequency statement implies a regular distribution of events in time. Thus, for example, for (9.a) to be true, it is not sufficient that there exist *some* events of John's jogging in the park:

(9) a. John sometimes jogs in the park.

b. John jogs in the park.

Rather, these events should be distributed in time with some repeated regularity, say once a week, a month or a year. The same seems to hold for generics,

2. For modal approaches see, among others, Lawler (1973); Dahl (1975); Burton-Roberts (1977); Heim (1982); Schubert and Pelletier (1989).

as (9.b) has the same reglarity implication as that of (9.a).

This phenomenon has been addressed by de Swart (1991), who proposes to capture it by using a dynamic evaluation procedure. She does not, however, provide an explanation for *why* frequency adverbs and generics behave in this way.

1.4 Majority Is Not Enough

Often, a property may hold of the vast majority of individuals in the domain of a generic, and yet the sentence would not be true:

(10) a. Israelis usually/Ø live on the coastal plane.
b. People in southeast Asia usually/Ø speak Chinese.
c. People usually/Ø have black hair.
d. People are usually/Ø over three years old.
e. Crocodiles usually/Ø die before they attain an age of two weeks.
f. Primary school teachers are usually/Ø female.
g. Bees are usually/Ø sexually sterile.
h. Books are usually/Ø paperbacks.
i. Prime numbers are usually/Ø odd.[3]

The sentences in (10) are true when they contain the frequency adverb *usually* (or *generally, mostly,* etc.), but false[4] as generics (i.e. with the phonologically null generic quantifier instead). The majority of Israelis live on the coastal plain, yet (10.a), as a generic, is not true; the majority of people in southeast Asia speak Chinese, yet (10.b), as a generic, is not true; and so on.

This phenomenon has been noticed by many researchers, and has been used to argue that a quantificational account of generics is impossible (Carlson 1977). Among those who believe, as we do, that generics *are* quantificational, a widely held view is that generics express universal quantification, but only over *normal* individuals (Delgrande 1987; Morreau 1992; Krifka 1995 among others). This view deserves to be discussed in some detail.

While the specifics of the various proposals differ, the basic idea is as follows: a relation of *normality* over possible worlds is assumed; this relation induces a partial order on worlds. Then a generic sentence like (11) is claimed to be true iff in all worlds which are "most normal," all birds fly.[5]

(11) Birds fly.

The falsity of each of the generics in (10) is then explained on the grounds that not all normal individuals satisfy the predicated property.

This approach, however, suffers from some serious drawbacks. In order for such a theory to make testable empirical claims, an independent account of normality is required. In other words, we need to go beyond stating that (11) is true because all normal birds fly, and that a normal bird is a bird which flies... Some definition of normality which is neither circular nor stipulative is called for, but none seems to be forthcoming. It may be claimed that normality

3. Examples (10.e), (10.g), (10.h) and (10.i) are from Carlson (1977); examples (10.c) and (10.d) are due to Chris Manning and Henk Zeevat, respectively.
4. Or, perhaps, lacking a truth value; judgments on such cases vary.
5. Or, more precisely, for every world w, there is a more normal world w', such that in all worlds more normal than w', all birds fly.

exists as some sort of abstract, Platonic classification of entities into normal and abnormal ones, but if that is the case, how do people obtain knowledge of this classification when they make their truth judgments?

In fact, the prospects for finding a definition of normality which would fit all cases seem rather dim. Consider the following sentences (from Krifka *et al.* 1995):

(12) a. Two and two equals four.
b. A lion has a mane.
c. Six apples cost one dollar.
d. A turtle is long-lived.
e. A pheasant lays speckled eggs.

Krifka *et al.* point out that with regard to (12.a), the most normal worlds are those where the laws of mathematics hold (arguably all worlds); with regard to (12.b)—those where stereotypical properties hold; and with regard to (12.c), only one world is considered normal—the real world. For some cases it is hard to come up with any reasonable definition of normality. For example, it might be suggested that for (12.d) and (12.e), the most normal worlds are those where the laws of biology hold; however, a world where all turtles reach old age or where all pheasants lay eggs (hence no males, and no one to fertilize the eggs) would be a highly abnormal one, biologically speaking. Krifka *et al.* suggest that the definition of normality in every given case is accommodated by the hearer, but no account of such accommodation is proposed.

Whether or not the normality view can account for generics, it cannot be applied to the case of frequency adverbs. The latter do not quantify over normal individuals, otherwise (13) would be false, since, by hypothesis, all normal birds do fly:

(13) Birds are sometimes incapable of flying.

A completely different theory, then, would have to be developed in order to explain frequency adverbs, yet a unifying account of the two phenomena would be highly desirable, given the many similarities between them, some of which are discussed in this paper.

1.5 Uncertain and Conflicting Truth Judgments

Generic sentences such as *Birds fly* are often presented as being unproblematically true. It is rarely noted that truth judgments of such sentences are often uncertain, and vary considerably among individuals. When, in an informal study, informants were asked to judge the truth of (14.a), some agreed that it was true, some were uncertain, and said things like, "Well, it's sort of true, but then there is the penguin," and others claimed it was outright false. In contrast, there was almost unanimous agreement when a judging a frequency statement; virtually all informants agreed that (14.b) was true.

(14) a. Birds fly
b. Birds usually fly.

The fifth puzzle, then, is why truth judgments of generics and frequency statements differ in this manner.

2 Probability

2.1 Probability-Based Truth Conditions

This paper argues the thesis that all five puzzles above can be accounted for if we assume that generics and frequency statements express probability judgments. Before presenting the arguments for this view, let us be more precise about the relation between truth conditions and probability judgments. We will assume that frequency adverbs and the generic operator act like generalized determiners, in that they denote binary relations between properties.[6] The logical form of (15.a), then, would be (15.b):

(15) a. Birds always fly.

b. **always**(**bird**, **fly**)

It seems plausible that for (15.a) to be true, the probability that an arbitrary bird flies needs to be 1. Similarly, *never* would require this probability to be 0, *sometimes* would require it to be non-zero, and *often* would require it to be fairly large. We will follow most researcher s in interpreting *usually* as being the counterpart of *most*, and require the probability to be greater than 0.5. For now, we will treat the phonologically null generic operator, **gen**, as synonymous with *usually*; both require the probability to be greater than 0.5 (this simplifying assumption will be revised below).

We can now propose the following truth conditions for generics and frequency statements (where $P(A|B)$ is the conditional probability of A given B):

Definition 1 (Truth conditions, first attempt) *A sentence $Q(\psi, \phi)$ is true iff*

$$\begin{cases} P(\phi|\psi) = 1 & \text{if } Q = \textbf{always} \\ P(\phi|\psi) = 0 & \text{if } Q = \textbf{never} \\ P(\phi|\psi) > 0 & \text{if } Q = \textbf{sometimes} \\ P(\phi|\psi) \gg 0 & \text{if } Q = \textbf{often} \\ P(\phi|\psi) \ll 1 & \text{if } Q = \textbf{seldom} \\ P(\phi|\psi) > 0.5 & \text{if } Q = \textbf{usually} \\ \quad \ldots & \\ P(\phi|\psi) > 0.5 & \text{if } Q = \textbf{gen} \end{cases}$$

2.2 Interpretations of Probability

As it stands, definition 1 does not really give us truth conditions. For it to do so, we need to specify the meaning of the term $P(A|B)$. What does it mean to say, for example, that the probability that a fair coin comes up "heads" is 0.5?

The meaning of probability has been the topic of much debate since at least the time of Laplace (see L. J. Cohen 1989 for an overview). It is not our goal here to provide a general solution to this philosophical problem; our aim is a more modest one. We will attempt to provide an account of the probability judgments expressed by generics and frequency statements. To this end, we will examine three theories, two which have already been applied to this problem, and one which, perhaps, ought to be.

6. This is, in fact, a simplification. We will return to this issue in section 4.1.

Logical Relation. The idea underlying logical relation theories is that $P(A|B)$ is the ratio of possible worlds in which both A and B hold to those where B holds. The probability that a coin comes up "heads," then, is the ratio of worlds in which it comes up "heads" to worlds in which it is tossed. Of course, in order to make sense of the idea of ratios of possible worlds, one needs to define some measure function on worlds, and a variety of such functions have been proposed in the literature. Schubert and Pelletier (1989) apply this framework to the problem of generics, and they assume (but do not specify) a measure function which favors worlds close to the real world in terms of the "inherent" or "essential" nature of things.

Ratio of Sets. A different approach is proposed by Åquist *et al.* (1980) in their account of frequency adverbs. They suggest that if A and B are sets of individuals,[7] the probability $P(A|B)$ is the ratio of the cardinality of $A \cap B$ to the cardinality of B. Hence the probability of a fair coin to come up "heads" would be 0.5 just in case in half of all observed tosses, the coin came up "heads."

Relative Frequency. A somewhat similar view is that probability judgments do not express simple ratios of finite sets, but statements of relative frequency in the limit, i.e. the mathematical limit of the ratio as the number of Bs approaches infinity.

The underlying idea is simple. If we want to know how likely smokers are to get lung cancer, we count the number of cancer patients among smokers, and divide by the total number of smokers in our sample. We do this for large samples, over long periods of time. The larger the sample and the longer the duration of the study, the closer the ratio will get to the desired probability. In the limit, as time goes by, and the sample size approaches infinity, the ratio will *be* the probability. Of course, we cannot actually examine an infinite number of smokers; but we can extrapolate from a finite sequence of examinations to what the limit might be. The longer the actual sequence is, the more confidence we should have in the correctness of the extrapolation.

2.3 Characterizations of Probability Judgments

The question is, of course, which, if any, of the interpretations above should we choose? L. J. Cohen 1989 argues that different interpretations of probability are appropriate for different types of probability judgments. He characterizes probability judgments using four parameters, and points out that different interpretations are appropriate for different settings of these parameters. We will follow his proposal and examine the behavior of generics and frequency statements with respect to the four parameters.

2.3.1 Necessity vs. Contingency

The first parameter indicates whether a probability judgment expresses a necessary or contingent statement. In general, generics and frequency statements are true or false contingently; they may be true in the real world, yet false in other worlds, and vice versa. It just happens to be the case that, in the real world,

7. Or tuples of individuals.

Birds usually/Ø fly is true, but it might have been otherwise. Logical relation theories, according to which probability judgments are necessarily true or false, are not, as they stand, applicable here. Schubert and Pelletier (1989) try to solve this problem by assuming a measure function which takes into account "necessary" or "essential" properties of the real world. However, since they do not define this function, and candidly admit that they "leave open what the possible worlds similar to w [the real world] with respect to the inherent nature of things are" (p. 260), it is not clear to what extent they are successful.

Ratio and relative frequency theories, on the other hand, are not problematic in this regard, since there is no necessity for a ratio or a frequency to have a certain value.

2.3.2 Propositions vs. Properties

L. J. Cohen's second parameter indicates whether the terms A and B stand for propositions or properties. Generics and frequency statements relate properties;[8] we relate the property of flying to the property of being a bird, rather than relating the proposition that Tweety flies to the proposition that Mary said so.

Logical relation theories will not, on the face of it, be appropriate here, because only propositions can be said to hold in a given world, not properties. Schubert and Pelletier (1989) solve this problem by considering pairs of worlds and individuals, rather than simply worlds.

Ratio and frequency theories are, once again, unproblematic; they require A and B to designate properties, since these theories are concerned with the ratio or the frequency with which individuals which satisfy B also satisfy A.

2.3.3 Substitutivity

The third, and, perhaps, crucial parameter involves the conditions under which other terms can be substituted for A or B without changing the probability $P(A|B)$. Suppose we have two extensionally equivalent properties, ψ_1 and ψ_2. That is to say, the set of individuals which satisfy ψ_1, at this moment in time and in the real world, is identical to the set of individuals which satisfy ψ_2. If generics and frequency adverbs behave extensionally, we would expect $Q(\psi_1, \phi)$ and $Q(\psi_2, \phi)$ to have the same truth conditions for every adverb Q and property ϕ.

It seems that this does not hold in general. Consider (16):

(16) A computer usually/Ø computes the daily weather forecast.

Carlson (1989) observes that

> "the daily weather forecast" requires an *intensional* interpretation, where its meaning cannot be taken as rigidly referring to the present weather forecast, e.g. the one appearing in today's copy of the *Times* predicting light rain and highs in the upper thirties (p. 179, emphasis added).

The intensionality exhibited here, it is important to note, is with respect to the time index, but not with respect to possible worlds.[9] Suppose that the weather report is John's favorite newspaper column. Then (17) would have the same truth conditions as (16), although there are any number of worlds where John never so much as glances at the daily weather forecast:

(17) A computer usually/Ø computes John's favorite newspaper column.

We should make it clear what we are not saying here. We are definitely not claiming that the property of being the daily weather report, or of being John's favorite newspaper column, has the same extension in all possible worlds; clearly, in different worlds, there may be different weather conditions, and John may have different preferences. What we *are* claiming is that the truth condition of a generic sentence do not depend on the extensions of the properties it relates in any other world but the real one, though the truth conditions do depend on the extensions of the properties at different times.

To give another example, suppose that John is fears all bats, but no other animal. The set of bats is equivalent to the set of animals John fears, though the intensions of the respective terms differ; there are any number of possible worlds where John feels nothing but love towards bats. However, it seems that we can substitute the term *animals which John fears* for *bats* without changing truth conditions:

(18) a. Bats usually/Ø fly.
b. Animals which John fears usually/Ø fly.

Similarly, there is no logical necessity for the whale to be the largest animal on earth, or for the quetzal to be Guatemala's national bird; yet (19.a) and (19.b) have the same truth conditions as (20.a) and (20.b), respectively:

(19) a. The whale suckles its young.
b. The quetzal has a magnificent, golden-green tail.

(20) a. The largest animal on earth suckles its young.
b. Guatemala's national bird has a magnificent, golden-green tail.

Generics and frequency statements, then, are parametric on time, but not on possible worlds; if two properties have the same extension throughout time, they can be freely exchanged in a generic sentence *salva veritate*. An account of probability is needed, then, where different descriptions of the same timeless property can be substituted for each other without changing the probability.

Ratio theories are clearly unsuitable, since sets are purely extensional entities. Logical relation theories are not appropriate either, since they would predict that generics and frequency statements are parametric on possible worlds—one term can be substituted for another only if the two are logically equivalent.

Relative frequency theories, on the other hand, satisfy the requirement exactly. The relevant frequency of As among Bs is not the one that happens to hold at present, but at the "end of time," i.e. in the limit of an infinite number of observations. On the other hand, possible worlds play no role in the relative frequency interpretation of probability; what matters is the frequency of As among Bs in the real world, regardless of what may happen in other worlds.

8. Or rather, as we will claim in section 4.1 below, open formulas.

9. For the distinction between the two types of intensionality, depending on the index involved, cf. Landman (1989).

2.3.4 Extensibility

The fourth parameter indicates whether or not a probability judgment would remain the same if the number of Bs were greater than it actually is.[10] For example, the judgment that an asbestos worker has a 0.8 probability of getting cancer is extensible, since it would be expected to hold if there were more asbestos workers than there actually are; otherwise, it could not be used as an argument for not working in an asbestos factory. On the other hand, the judgment that a person attending the Tenth Amsterdam Colloquium has a 0.6 probability of staying at the Hotel Excelsior is not extensible; perhaps, if there had been additional participants, they would have had to stay elsewhere, since the Hotel Excelsior would have been full. Generics and frequency statements are extensible; *Birds usually/Ø fly*, it seems, would be true even if there were more birds than there actually are.

Logical relation theories would pass this test, since they take probability judgments to express necessary statements, which hold regardless of the number of instances. Ratio theories, in contrast, would predict that generics and frequency statements are not extensible, since changing the number of instances might very well change the ratio. Relative frequency theories, once again, are appropriate here, since they are concerned with the relative frequency as the number of Bs approaches infinity, rather than with the number of actual Bs.

3 Generics and Frequency Statements as Relative Frequencies

3.1 Problems with the Relative Frequency Interpretation

We conclude from the above that the interpretation of probability which is most appropriate for the study of generics and frequency statements is that of relative frequencies in the limit. This interpretation, however, is not widely held by philosophers of science, because it suffers from at least two severe problems. Both problems have to do with the size of the reference class, i.e. the class of Bs in the probability judgment of $P(A|B)$. The difficulties lie on opposite sides of the spectrum, so to speak: one is the problem of finite reference classes, the other is that of infinite ones.

When the reference class is finite, limits are meaningless—the size of the reference class cannot increase without bounds. This problem is particularly striking when the size of the class is zero—in this case, the relative frequency is undefined. Suppose we have a fair coin which has never been tossed, and is about to be melted down, so it will never be tossed in the future either. We would still like to say that the probability that this coin, if tossed, would land "heads" is 0.5, even though the relative frequency is undefined. Even when the reference class is nonempty but finite, the frequency theory faces difficulties. The relative frequency can never be equal to 0.5 if the number of coin tosses is odd, and even if the number if even, it might very well not be precisely 0.5; yet the probability would still be 0.5. Intuitively, the problem is that if the reference class is finite, we cannot make further observations to "correct" the relative frequency and move it as close to the limit (i.e. the probability) as we wish.

The second major difficulty with relative frequency theories is encountered in the case of infinite reference classes. Imagine, for example, an infinite sequence of coin tosses. The probability that a coin lands "heads" is claimed to be the limit of the ratio of "heads" outcomes to the total number of coin tosses, as the number of tosses approaches infinity. However, not all sequences are guaranteed to have limits; it is quite possible for a given sequence not to converge at all.

An even more serious objection is that there may be more than one limit, depending on the order of elements in the sequence. Suppose, for example, that we construct an infinite sequence of coin tosses by choosing the first nine "tails" outcomes, then the first "heads" outcome, then the following nine "tails" outcomes, etc. Then the relative frequency of "heads" outcomes will be 0.1, rather than 0.5; yet the probability that a fair coin comes up "heads" is surely 0.5, regardless of the way in which the coin tosses are sampled.

Taken from the perspective of a general theory of probability, these are, indeed, weighty problems. Every problem, however, is also an opportunity; and from the point of view of a linguistic theory of generics and frequency statements, these difficulties are actually advantages. If we assume that the probability judgments expressed by these constructions are relative frequencies in the limit, we would predict that generics and frequency statements should be interpreted in such a way that the problems discussed above do not occur—they will simply be legislated out. In other words, we could use the shortcomings of the relative frequency view to predict constraints on the class of acceptable generics and frequency statements.

3.2 The Unboundedness Constraint

Since finite reference classes pose a problem for the relative frequency view, let us simply get rid of them, and treat the reference class as if it were infinite. That is to say, we will regard the reference class as if it were possible to examine more and more of its members at will, without ever "running out" of individuals. The reference class need not actually *be* infinite; but it must be consistent to treat as if it were. This assumption would be consistent unless the sentence entails some limit on the size of the reference class. In other words, the reference class can be treated as infinite only if the sentence is unbounded, otherwise the sentence would be unacceptable. This is, of course, precisely the phenomenon described in section 1.1 above, i.e. that generics and frequency statements are unbounded.

The unboundedness constraint, then, suggests a solution to our first puzzle. It also accounts for the second puzzle, namely the one concerning temporary generalizations. As we have noted in section 1.2 above, even if all current Supreme Court judges happened to have a prime Social Security number, (21) would not be true:

(21) Supreme Court judges always/usually/Ø have a prime Social Security number.

Because of the unboundedness constraint, (21) cannot be interpreted to be about the bounded domain of current Supreme Court judges, but rather about

10. L. J. Cohen refers to this property as *counterfactualizability*, but we believe this term is somewhat confusing, as well as being rather cumbersome.

the unbounded domain of such judges over an extended period of time. If the number of judges is unbounded, i.e. if one can, so to speak, sample more and more Supreme Court judges and examine their Social Security numbers, the temporary discrepancy will disappear after a while, and judges with prime Social Security numbers will be in the minority. Since probabilities are relative frequencies "in the long run," rather than at present, sentence (21) is not true.

3.3 The Homogeneity Constraint

The second problem with the view of probabilities as relative frequencies is, as mentioned above, the fact that an infinite sequence might have more than one limit, depending on the manner in which this sequence is constructed. Most attempts to combat this problem involve postulating some constraints on the possible sequences constructed out of a given reference class, such that the sequences be *random* or *homogeneous*. This means that the relative frequencies over all "suitable" subsequences would converge to the same limit.

The idea of homogeneity can apply to the reference class itself, rather than to a sequence based on it. The following definition is based on Salmon (1977):

Definition 2 (Homogeneity) *A reference class ψ is homogeneous with respect to a property ϕ, iff there is no set of properties Ω such that:*

1. *Ω induces a partition on ψ, i.e. $\forall x : (\psi(x) \rightarrow \exists! \omega \in \Omega : \omega(x))$.*
2. *The partition induced by Ω is* relevant, *in the sense that*
$$\exists \omega \in \Omega : P(\phi|\psi \wedge \omega) \neq P(\phi|\psi).$$

Thus, a reference class ψ is homogeneous iff it cannot be partitioned in such a way that the probability of ϕ given one of the subsets of the reference class induced by the partition is different from the probability of ϕ given ψ as a whole.

We would like to require that the domain of generics and frequency adverbs be homogeneous. The requirements of definition 2, however, are far too strong. Salmon notes that there are two trivial cases of homogeneous reference classes: when all ψs are ϕs and when no ψs are ϕs. In the former case, the probability of ϕ given any subset of the reference class would be equal to 1, and in the latter case to 0. He observes that, as the definition stands, it will only be satisfied in the trivial case, as a relevant partition can always be found in any non-trivial case. Suppose that at least one ψ is a ϕ, and at least one ψ is not a ϕ; now let $\Omega = \{\phi, \psi \wedge \neg\phi\}$. Ω is clearly a partition of ψ; moreover, it is a relevant partition, since $P(\phi|\psi \wedge \phi) = 1$ and $P(\phi|\psi \wedge (\psi \wedge \neg\phi)) = 0$. Clearly, then, the class of possible partitions needs to be constrained in some way, so that only "suitable" ones be considered, if the concept of homogeneity is to play any useful role at all. A considerable body of work has been devoted to formalizing such constraints,[11] but the level of success achieved is debatable.

We would like to suggest that, at least insofar as generics and frequency statements are concerned, the suitability of a given partition is a pragmatic matter. A partition is suitable to the extent that it is considered *salient*, given the context and the language user's model of the world. In other words, it

11. See Salmon (1977) for an overview.

cannot be any arbitrary partition; it needs to "make sense," given the way we view the world. The domain of a generic or a frequency adverb, then, needs to be homogeneous with respect to the predicated property and a set of salient partitions.

Salmon's definition requires that for every $\omega \in \Omega$, $P(\phi|\psi \wedge \omega)$ be the same, but this requirement seems unnecessarily strong. We would require only that for every $\omega \in \Omega$, $P(\phi|\psi \wedge \omega)$ satisfy the condition as determined by the frequency adverb (e.g. equal to 1 for *always*, greater than 0.5 for generics, etc.), but not necessarily that the actual value of the probability be the same for every $\omega \in \Omega$. We can now revise definition 1 as follows:

Definition 3 (Truth conditions, revised version) *A sentence $Q(\psi, \phi)$ is true iff for every Ω, a* salient *partition of ψ, and for every $\omega \in \Omega$,*

$$\begin{cases} P(\phi|\psi \wedge \omega) = 1 & \text{if } Q = \textbf{always} \\ P(\phi|\psi \wedge \omega) = 0 & \text{if } Q = \textbf{never} \\ P(\phi|\psi \wedge \omega) > 0 & \text{if } Q = \textbf{sometimes} \\ P(\phi|\psi \wedge \omega) \gg 0 & \text{if } Q = \textbf{often} \\ P(\phi|\psi \wedge \omega) \ll 1 & \text{if } Q = \textbf{seldom} \\ P(\phi|\psi \wedge \omega) > 0.5 & \text{if } Q = \textbf{usually} \\ \quad\cdots & \\ P(\phi|\psi \wedge \omega) > 0.5 & \text{if } Q = \textbf{gen} \end{cases}$$

3.4 Some Salient Partitions

It is usually fairly clear whether a partition is salient or not. For example, Jorge Luis Borges's famous taxonomy,[12] in which animals are divided into such groups as *those that belong to the Emperor* and *stray dogs*, is clearly not salient; in fact, its sole purpose is to demonstrate a classification which humans would find extremely unnatural.

Nevertheless, judgments on whether a partition is considered salient in a given context may vary considerably across cultures, languages and individuals. A speaker of Dyirbal, for example, may consider a partition which groups together women, dogs, crickets, water and fire to be quite natural (Dixon 1982); a speaker of English may find such a partition very bizarre.

We will not attempt here to present an exhaustive list of salient partitions, nor will we attempt to identify the conditions under under which a particular partition is salient. Plausibly, these conditions have to do with the nature of the properties related by the predicate, as well as the linguistic and extralinguistic context. Some illustrative examples, however, are in order.

3.4.1 Time

A partition which seems salient for almost all speakers in many cases is that of *time*, i.e. a partition of the domain into temporal intervals. For example, (9.a), repeated here as (22), expresses quantification over events:

(22) John sometimes/Ø jogs in the park.

If (22) is required to be homogeneous, given the *time* partition, it would mean that the probability of John's jogging in the park is greater than 0 (or greater

12. From "The Analytical Language of John Wilkins," in *Other Inquisitions* (University of Texas Press, 1964). Of course, Borges's taxonomy is not really a partition, since some categories overlap.

than 0.5 for the generic) in each one of a series of temporal intervals. The size of the intervals would vary from sentence to sentence, depending on what is perceived to be a reasonable sample size. Thus, given (22), the interval size might be on the order of weeks; given (23.a), each interval would be several years long; and given (23.b), the interval size would be on the order of several minutes:

(23) a. Karen sometimes/Ø spends her holidays in Switzerland.
b. This broadcast is often/Ø interrupted by commercials.

It is the homogeneity requirement, then, which accounts for the fact, discussed in section 1.3 above, that frequency adverbs and generics require their domain to be regularly distributed in time. Sentence (22), then, implies that John's jogging in the park occurs with some regularity, because its truth requires that in each one of the intervals into which the set of events is partitioned, John jogs in the park.

3.4.2 Space

Another partition which is often perceived to be salient is that of *space*, i.e. a partition of the individuals in the domain according to the regions in which they are located. This would explain why (10.a), repeated here as (24), is not true, even though the majority of Israelis do live on the coastal plane:

(24) Israelis live on the coastal plane.

If the domain of Israelis is partitioned according to the geographical regions in which they live, there will obviously be subsets of this domain whose members do not live on the coastal plane, e.g. those who live in Jerusalem. Hence, the domain is not homogeneous with respect to the property of living on the coastal plane, and (24) is not true.[13]

Partitioning individuals according to their locations, or, perhaps, their ethnic groups, would also account for (10.b) and (10.c), repeated here as (25.a) and (25.b), respectively:

(25) a. People in southeast Asia speak Chinese.
b. People have black hair.

Although the majority of people in southeast Asia do speak Chinese, there are regions (and peoples) where Chinese speakers are in the minority. As a consequence, the set of southeast Asians is not homogeneous with respect to the property of speaking Chinese, and (25.a) is not true. Similarly, while the majority of people in the world have black hair, this does not hold in all regions or of all ethnic groups. Most Scandinavians, for example, do not have black hair. If such a partition is taken into account, the domain of people fails to be homogeneous with respect to the property of having black hair.

3.4.3 Age

Sentences (10.d) and (10.e), repeated here as (26.a) and (26.b), respectively, can be accounted for if *age* is considered a salient partition:

(26) a. People are over three years old.

13. According to definition 3, the sentence is predicted to be false. Some would judge sentences which violate the homogeneity constraint, such as (24), to be lacking a truth value. To accommodate such judgments, all we need to do is change definition 3 so that the homogeneity requirement is a presupposition rather than an entailment of a sentence.

b. Crocodiles die before they attain an age of two weeks.

The majority of people are clearly over three years old, and the majority of crocodiles do, indeed, perish in their infancy; yet (26.a) and (26.b) are not true. If we partition people according to their ages, there will be subsets of people such that the probability for their members to be over three year old is zero. Hence the domain of the generic quantifier is not homogeneous, and sentence (26.a) is not true. Similarly, the domain of crocodiles is not homogeneous with respect to the property of dying before the age of two weeks. If we partition crocodiles according to their ages, there will be subsets of crocodiles all of whose members are older than two weeks; hence the homogeneity constraint is violated and (26.b) is not true.

3.4.4 Gender

Gender is often a salient partition too. Thus, although the majority of primary school teachers are female, (10.f), repeated here as (27), is not true:

(27) Primary school teachers are female.

The reason seems to be that, if we partition the set of teachers according to their genders, there will obviously be a set, the probability of whose members to be female is zero—the set of male teachers. Therefore, the set of teachers is not homogeneous with respect to the property of being female.

The same kind of partition would account for (10.g), repeated here as (28):

(28) Bees are sexually sterile.

Although the vast majority of bees are, indeed, sterile, there is a subset of bees which is not, if bees are partitioned according to their genders—the drones. Since their probability to be sexually sterile is rather low, the homogeneity constraint is not satisfied.

3.4.5 Subject Matter

The majority of books are probably printed in paperback rather than in hardcover, yet (10.h), repeated here as (29), is false:

(29) Books are paperbacks.

We suggest that in this case, the relevant salient partition involves dividing books according to their subject matter. Detective stories, for example, are much more likely to be published as paperbacks, whereas reference books are more likely to be printed in hardcover. The domain of books, then, is not homogeneous with respect to the property of being a paperback; any browser in a bookstore will see shelves where the majority of books are paperbacks, and shelves where the majority are hardcover. Consequently, the homogeneity constraint is not satisfied, and (29) is not true.

3.4.6 Abstract Domains

Sentence (10.i), repeated here as (30), is an interesting example:

(30) Prime numbers are odd.

Although the vast majority of prime numbers, indeed, infinitely many of them except for 2, are odd, (30) is false. This seems to be the case in general for generics involving mathematical statements—they do not allow any exceptions. The explanation for this fact, we claim, is that in mathematical domains, *every*

partition is salient. Mathematics is an abstract realm, where we do not have solid intuitions about what is or is not "natural"; every way to divide up the domain seems to make just as much sense as any other way. Since some partitions of the set of primes (e.g. the partition into even and odd numbers) include the singleton set {2}, the probability of whose sole member to be odd is, of course, zero, the homogeneity constraint is violated.

Note that sentences similar to (30), but which are divorced from the realm of mathematics, may be judged true. Suppose that, in order to reduce the risk of forgery, it were decided that the identifying number on $1000 bills be prime. Given such a scenario, (31) would be true:

(31) $1000 bills have an odd identifying number.

The domain of $1000 bills is not abstract, and some partitions (e.g. a partition based on the date on which a bill was printed) would be more salient than others. A partition which would isolate the one specific $1000 bill with the identifying number 2 would not normally be considered salient; hence the domain of $1000 bills is homogeneous, and sentence (31)—true.

3.4.7 Frequency Adverbs vs. Generics

We have noted in section 1.4 above that sentences (24)–(30) would be true if the frequency adverb *usually* were used instead of the phonologically null generic quantifier. We propose that the reason for this difference between generics and frequency adverbs stems from different saliency criteria for partitions. For frequency adverbs, only the *time* partition is considered salient; only temporal partitions are relevant for satisfying the homogeneity constraint. Thus, while frequency statements, just like generics, require regular distribution of events in time, they do not have similar requirements with respect to other partitions. This, perhaps, is the source of the intuition that the temporal sense of frequency adverb is somehow primary, and the atemporal sense is somehow secondary or derived. We agree with Lewis (1975) that frequency adverbs may bind both temporal[14] and atemporal variables; but they only require homogeneity with respect to the *time* partition, regardless of what type of entities they quantify over.

3.4.8 Biological Family

Judgments of the saliency of partitions might vary among individuals. For a zoologist, very fine distinctions among birds might be salient; not so for the layperson. Indeed, a zoologist might judge *Birds fly* to be false for just this reason. For such a speaker, there would be a salient partition of the set of birds, such that some of its subsets would have a majority of non-flying birds. For example, if we partition the domain of birds into biological families, the penguin family (Spheniscidae) would be just such a subset. For other speakers, a partition into families may not be salient, and they would judge *Birds fly* to be true. Variations in saliency judgments among people explains the phenomenon discussed in section 1.5 above, namely that truth judgments of generics vary considerably among individuals. Indeed, different people may have different intuitions about some of the other partitions mentioned in this section.

14. Or quasi-temporal, e.g. events, situations, occasions, and the like.

Recall that frequency adverbs only require their domains to be homogeneous with respect to the *time* partition. Since, given this partition, the domain of birds is, indeed, homogeneous with respect to the property of flying (the majority of birds at any given time, since they first evolved and, as far as can be determined, until they become extinct, do fly), *Birds usually fly* is unproblematically true. Since judgments of the truth of frequency statements do not depend on the saliency of any partition other than *time*, these truth judgments are more certain, and enjoy wider agreement among individuals, than truth judgments of generics.

4 Extensions to the Theory

There are several ways in which the theory presented here can be extended. Limitations of space permit us to do little more than point out the problems and sketch the solutions; the reader is referred to A. Cohen (forthcoming) for the details.

4.1 Probabilities over Arbitrary Open Formulas

In the discussion so far, we have presented the logical form of generics and frequency statements as $Q(\psi, \phi)$, where ψ and ϕ are properties. This representation, however, would face difficulties in accounting for sentences such as (32), which seems to quantify over more than one variable:

(32) Women live longer than men.

Sentence (32) is not simply about women or about men, but rather about pairs of men and women. For this and other reasons, a more adequate representation seems to be that of a tri-partite structure, following Heim (1982). This raises a problem, however: Heim views quantifiers as relating open formulas, rather than properties—we need, therefore, a way to define probabilities over arbitrary open formulas. While the traditional probability calculus does not let us do this, there are formalisms (Halpern 1990; Bacchus 1990) which make it possible, and which, in fact, can be incorporated quite naturally into the theory presented here.

4.2 Domain Restriction

A generic or a frequency adverb does not always quantify over *all* the individuals in its domain:

(33) Mammals usually/Ø bear live young.

Sentence (33) does not mean that any arbitrary mammal is likely to bear live young; only fertile, adult females are likely to do so. In other words, only mammals which procreate in some fashion are relevant for the truth or falsity of (33). The theory presented here should be integrated with an account, such as the one proposed in A. Cohen (1994), of how to obtain (34.b), rather than (34.a), as the truth conditions of (33):

(34) a. $P(\textbf{live-young}|\textbf{mammal}) > 0.5$
b. $P(\textbf{live-young}|\textbf{mammal} \wedge \textbf{procreate}) > 0.5$

4.3 Distinction Readings

The following well known sentences provide a problem for the account of generics presented here (and other theories of generics as well):

(35) a. The Frenchman eats horsemeat.
b. Dutchmen are good sailors.

Sentences (35.a) and (35.b) might be true even if an arbitrary Frenchman is unlikely to eat horsemeat, or if an arbitrary Dutchman is unlikely to be a good sailor. What the sentences in (35) seem to say is that a Frenchman is more likely to eat horsemeat than an arbitrary person is, and that a Dutch sailor is more likely to be a good one than an arbitrary sailor is. Note that (35.a) might be true even if a horsemeat eater is more likely to be, say, Belgian, and (35.b) might be true even if a good sailor is more likely to be, say, British.

We refer to such readings of generics as *distinction* readings. The analysis of generics as probability judgments provides a natural way to represent the respective truth conditions of (35.a) and (35.b):

(36) a. $P(\textbf{eat-horsemeat}|\textbf{French}) > P(\textbf{eat-horsemeat}|\textbf{person})$
b. $P(\textbf{good-sailor}|\textbf{Dutch} \wedge \textbf{sailor}) > P(\textbf{good-sailor}|\textbf{sailor})$

It turns out that the account of domain restriction can also be applied to the problem of distinction readings, thus providing a unified theory of the two readings of generics.

4.4 Quantificational Predicates

The term *quantificational predicate* is due to Krifka *et al.* (1995), who use it to designate a class of predicates such as *be common*, *be rare*, and *be widespread*:

> Consider the sentence *A rhino (with blue eyes) is common.* It does not express a property of a specific rhino, nor does it express a generalization over individual rhinos. Instead its meaning is 'The *chance* of encountering a (blue-eyed) rhino is high', that is, it is a statement about the distribution of rhinos (p. 96, emphasis added).

A quantificational predicate, then, is a statement about the chance of observing individuals satisfying a certain property; this seems to call for an analysis in terms of probability. We propose to represent the meaning of (37.a) as (37.b):

(37) a. Rhinos are common in Africa.
b. $P(\textbf{rhino}|\textbf{animalin-Africa}) \gg 0$

Krifka *et al.* go on to note that

> in *be common*, as well was [sic] in the similar *be widespread*, there is a meaning component implying that one can come across the entities in question at many places all over the universe; a sentence such as *Rhinos are common in Africa* seems to be false in case there are many rhinos in Africa, but all of them gathered at a single place, say, in the Ngorongoro crater (p. 97).

This requirement would follow naturally from the homogeneity constraint; the reference class needs to be homogeneous with respect to the property of being a rhino, given the *space* partition. Therefore, for (37.a) to be true, rhinos need to be distributed more or less evenly across Africa.

5 Summary

Our goal in this paper has been to account for a variety of properties of generics and frequency adverbs without appealing to mysterious devices such as essential

properties or normality relations. Instead, we have argued for a unified generics and frequency statements as expressions of relative frequency in the limit, and proposed that this provides explanations for five puzzling phenomena:

1. The domain of generics and frequency adverbs is unbounded because for limits to be meaningful, it must be consistent to regard the reference class as infinite.
2. Generics and frequency adverbs do not hold of temporary generalizations, because the reference class may not be bounded in time either.
3. Generics and frequency adverbs require regular distribution of events along the time axis because *time* is a salient partition, and the domain of these quantifiers needs to be homogeneous with respect to it.
4. Generic sentences may fail to be true even if the majority of instances support the predicated property, because for a generic sentence to be true, it is also necessary that the domain of the quantifier be homogeneous with respect to all salient partitions.
5. People vary considerably in their judgments of the truth or falsity of generics, because judgments of saliency of partitions vary across individuals.

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Quantifiers, Contexts, and Anaphora

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Abstract

In this article we study the interpretation of nominal anaphora by means of generalized quantifier theory. One way of viewing first-order DRT *and* DPL *is that they interpret singular anaphora outside the scope of their antecedent by extending the antecedent's scope. Moving to the general case, we therefore want to know how general these scope principles are. We shall argue that they can be sustained for all singular anaphora, but that they may fail for plural anaphoric noun phrases. Some recent dynamic semantics of quantifiers are reviewed in this light. Our own proposal stands in the E-type tradition. It treats the relevant anaphora as generalized quantifiers which are contextually restricted by material inherited from their antecedent. Indeed, all E-type anaphora are interpreted by one and the same mechanism. To make this precise, we present a labelled, many dimensional categorial system, which generates the meaning of a wide class of discourses in a compositional way.*

1 Introduction

DRT's logic for singular anaphora (Heim 1982, Kamp 1981) can be seen as exploiting the equivalence of (1a) and (1b).

(1) a. A man walks in the park. He whistles.
b. A man walks in the park and whistles.

Due to this equivalence, (1a) can be transformed into (2) without loss of meaning.

(2) $\exists x[\mathrm{M}x \wedge \mathrm{W}x \wedge \mathrm{Wh}x]$

As is well-known, (2) is obtained as follows. The antecedent sentence of (1a) induces the DRS (3a), where x is a discourse referent restricted by the conditions in the body.

(3) a. $x[\mathrm{M}x \wedge \mathrm{W}x]$
b. $x[\mathrm{M}x \wedge \mathrm{W}x \wedge \mathrm{Wh}x]$

Next the anaphor sentence '$\mathrm{Wh}x$' is merged with this DRS to give (3b). Finally, (3b) is interpreted as (2) by means of the discourse operator $\exists$. This strategy complies with Geach's view that pronouns are bound variables. Since the meaning of the antecedent sentence on its own is $\exists x[\mathrm{M}x \wedge \mathrm{W}x]$, the process can also be viewed as extending the scope of $\exists x$ to include the anaphor sentence. That this is so, is seen even more clearly in DPL's compositional reconstruction of DRT (Groenendijk and Stokhof 1991). A crucial feature of DPL is that the formalization (4a) of (1a) is equivalent to (4b), which has its standard meaning.

(4) a. $\exists x(\mathrm{M}x \wedge \mathrm{W}x) \wedge \mathrm{Wh}x$
b. $\exists x(\mathrm{M}x \wedge \mathrm{W}x \wedge \mathrm{Wh}x)$

A related principle of scope extension corresponds to the synonomy of (5a) and (5b).

(5) a. If a man walks in the park, he whistles.
b. Every man who walks in the park, whistles

In DPL, and *mut. mut.* in DRT, these sentences can be obtained from each other since (6a) and (6b) are equivalent.

(6) a. $\exists x(\mathrm{M}x \wedge \mathrm{W}x) \rightarrow \mathrm{Wh}x$
b. $\forall x(\mathrm{M}x \wedge \mathrm{W}x \rightarrow \mathrm{Wh}x)$

However, the generality of scope extension as principle of anaphor resolution was already challenged by Evans (1977, 1980). In his polemics with Geach he held that not all pronouns with an antecedent are bound variables. His main observation was that pairs like (7a) and (7b) are unequivalent.

(7) a. Quite a few men drove a Jaguar. They whistled.
b. Quite a few men drove a Jaguar and whistled.

The difference between (7a) and (7b) cannot be explained if 'they' in (7a) is bound by 'quite a few' in the standard sense. For then both sentences would mean that about half of the men who drove a Jaguar whistled. Instead, (7a) implies (7b) but not conversely. In particular, (7b) does not imply the anaphor sentence of (7a), which means that each man who drove a Jaguar whistled.

Observations such as these make us face a number of questions. How general are the principles of anaphor resolution embodied in the first-order versions of DRT and DPL? Are there extensions of these logics which treat quantifiers so that the scope principles remain fully general? Or should we perhaps distinguish in a principled way between the noun phrases which do and those which do not allow their scope to be extended?

The results in section 2 imply that scope extension can be sustained for all singular but not for all plural anaphora. This is shown in detail in section 3 and 4, respectively. It gives a logical reason of why it is non-trivial to incorporate dynamic generalized quantifiers within DRT and DPL. Section 4 also reviews some of the proposals made in this direction.

Since we prefer to treat anaphora outside the scope of their antecedent in a uniform way, we leave the idea of scope extension all together. Instead, we treat the relevant anaphora as generalized quantifiers which are contextually restricted by material inherited from their antecedent. This approach developed out of Evans 1977, 1980, Neale 1990, Ludlow and Neale 1991 using the context sets of Westerståhl 1984. Cf. also Van Deemter 1991 and Van den Berg 1991, 1996, among others. To make this precise, section 5 presents a labelled, many dimensional categorial system which generates the meaning of a wide class of discourses in a compositional way.

2 Anaphora and scope extension

The proper setting for studying scope extension is generalized quantifier theory, provided we first lay down how anaphoric links influence meaning. A convenient way is to view anaphoric noun phrases as contextually restricted quantifiers. Then the anaphoric element is a determiner $\mathbf{A}$, left unspecified for the moment, while the anaphoric link between $\mathbf{D}^i$ and $\mathbf{A}_i$ in (8a) means (8b).[1]

(8) a. $\mathbf{D}^i AB \ \ldots \ \mathbf{A}_i C$
b. $\mathbf{D}AB \ \ldots \ \mathbf{A}A \cap BC$

This treatment seems far removed from the dynamic semantics of DRT and DPL, but we shall indicate how the theories fit within this framework.

In view of (4) and (6), we want to know for which $\mathbf{D}$ and $\mathbf{A}$ (9a–9b) and (10a–10b) are equivalent:

[1] In this article I often speak of determiners, but *mut. mut.* the same remarks can be made for non-complex noun phrases.

(9) a. $\mathbf{D}^i AB \ \& \ \mathbf{A}_i C$
b. $\mathbf{D}AB \cap C$

(10) a. $\mathbf{D}^i AB \Rightarrow \widetilde{\mathbf{A}}_i C$
b. $\widetilde{\mathbf{D}}A \cap BC$

$\widetilde{\mathbf{D}'}$ is the dual of $\mathbf{D}'$. The reason for using $\widetilde{\mathbf{A}}$ in (10a) is that the unary quantifiers in DRT and DPL leave some room in formalizing the principles of scope extension. Recall that DPL defines $P \Rightarrow Q$ as $\neg(P \& \neg Q)$. This means that the equivalence of (10a–10b) amounts to the same thing as the equivalence of (11a–11b).

(11) a. $\mathbf{D}^i AB \ \& \ \mathbf{A}_i C$
b. $\mathbf{D}A \cap BC$

Accordingly, we should study two related but quite different ways of scope extension: Roberts' telescoping in (9) vs. what might be called 'absorption' in (11).

The question concerning (9a–9b) and (10a–10b) is one of inverse logic: for which $\mathbf{D}$ are the equivalences $\langle\mathbf{A}\rangle$ or $[\mathbf{A}]$ valid?[2]

$\langle\mathbf{A}\rangle$ $\quad \mathbf{D}^i AB \ \& \ \mathbf{A}_i C \Leftrightarrow \mathbf{D}AB \cap C$

$[\mathbf{A}]$ $\quad (\mathbf{D}^i AB \Rightarrow \widetilde{\mathbf{A}}_i C) \Leftrightarrow \widetilde{\mathbf{D}}A \cap BC$

$\mathbf{D}$ in $\langle\mathbf{A}\rangle$ and $[\mathbf{A}]$ is called the antecedent determiner and $\mathbf{A}$ the anaphoric determiner. The full paper studies $\langle\mathbf{A}\rangle$ and $[\mathbf{A}]$ separately. For now we just observe that they collapse in case of intersective $\mathbf{D}$.

Definition 1 A determiner $\mathbf{D}$ is *intersective* iff for all A, B, A', B':[3]

$$A \cap B = A' \cap B' \Rightarrow (\mathbf{D}AB \Leftrightarrow \mathbf{D}A'B')$$

Notice that for an intersective $\mathbf{D}$: $\mathbf{D}AB \cap C \Leftrightarrow \mathbf{D}A \cap BC$, whence the collapse. Section 2.2 has a formal treatment of $\langle\mathbf{A}\rangle$, but the examples in section 2.1 will first give us a feel for what we might expect.

2.1 Some examples

The most familiar pattern of Evans' inference uses a pronoun for the anaphoric element; cf. the equivalence in (1).

(1) All men walked in the park. They whistled.
$\Leftrightarrow$ All men walked in the park and whistled.

Here, 'they' is interpreted as a universal quantifier restricted to the context set 'men walking in the park'. So, (1) is an instance of $\langle\mathbf{all}\rangle$. For conservative $\mathbf{D}$ one direction of $\langle\mathbf{all}\rangle$ is always valid.

(2) $\mathbf{D}AB \quad A \cap B \subseteq C$
$\Rightarrow \ \mathbf{D}AB \cap C$

But as we shall see, in almost all other cases the converse is false. For instance, (3) is invalid.

(3) Some men walked in the park and whistled.
$\not\Rightarrow$ Some men walked in the park. They whistled.

The first sentence of the conclusion is implied, which is the case iff $\mathbf{D}$ in $\langle\mathbf{A}\rangle$ is MON↑. But clearly the premisse does not imply the second sentence which states that all men walking in the park whistle.

[2] Cf. Van Benthem (1984) who asked similar questions for syllogistic patterns of reasoning.
[3] Alternative definitions are: $\mathbf{D}$ is intersective *iff* $\mathbf{D}$ is both conservative and co-conservative *iff* $\mathbf{D}$ is conservative and symmetric. ($\mathbf{D}$ is *co-conservative* iff for all A, B: $\mathbf{D}AB \Leftrightarrow \mathbf{D}A \cap BB$. And $\mathbf{D}$ is *symmetric* iff for all A, B: $\mathbf{D}AB \Leftrightarrow \mathbf{D}BA$.) Keenan (1987) identifies intersective determiners with the indefinites.

Once anaphora are viewed as contextually restricted quantifiers, there are quite a few variants of (1) without pronouns. Cf. Van Deemter 1991. Take (4), for example.[4]

(4) Five men bought whiskey. Five got drunk.
⇔ Five men bought whiskey and got drunk.

As we already noted, the antecedent determiner in $\langle \mathbf{A} \rangle$ must be MON↑. And indeed, (4) is valid if 'five' is read as the monotone increasing 'at least five', but invalid if read as the non-monotonic 'exactly five'. The use of the monotone decreasing 'at most five' would result in invalidity as well. Moreover, (5) is invalid on each option for the numeral.

(5) Five men bought whiskey. Some got drunk.
$\not\Leftrightarrow$ Five men bought whiskey and got drunk.

The above examples suggest that as soon as the antecedent $\mathbf{D}$ and the anaphoric $\mathbf{A}$ are one and the same MON↑ determiner a valid equivalence results. That this is not true is shown by (6).

(6) Most men went to the party. Most whistled.
$\not\Leftrightarrow$ Most men went to the party and whistled.

Although the top discourse follows from the sentence at bottom, the converse direction fails (consider four men of which three walk and two whistle). Therefore, we need to look for special MON↑ determiners, and we shall give the relevant properties shortly.

2.2 The validity of $\langle \mathbf{A} \rangle$

We are going to characterize the logical determiners which validate $\langle \mathbf{A} \rangle$ for a certain choice of $\mathbf{A}$.[5] Relative to $\mathbf{A}$, these are the determiners for which it is all one whether they are linked anaphorically to $\mathbf{A}$ (the lefthand side of $\langle \mathbf{A} \rangle$), or whether they have their scope extended to include the descriptive material of the anaphor sentence (the righthand side of $\langle \mathbf{A} \rangle$).

$\langle \mathbf{A} \rangle$ $\mathbf{D}AB$ & $\mathbf{A}A \cap BC$, iff $\mathbf{D}AB \cap C$.

As it happens, in this characterization the choice of $\mathbf{A}$ cannot be fully arbitrary, but it is sufficient to require that $\mathbf{A}$ is restrictive. This notion corresponds to one of the three inferences which make up $\langle \mathbf{A} \rangle$ for the special case where $\mathbf{A}$ is $\mathbf{D}$.

(7) (a) $\dfrac{\mathbf{D}AB \cap C}{\mathbf{D}AB}$ (b) $\dfrac{\mathbf{D}AB \cap C}{\mathbf{D}A \cap BC}$ (c) $\dfrac{\mathbf{D}AB\ \mathbf{D}A \cap BC}{\mathbf{D}AB \cap C}$

We leave the import of (7c) for the full paper. Property (7a) is familiar: $\mathbf{D}$ has it iff $\mathbf{D}$ is MON↑. The most relevant property for the characterization we are after is (7b). A determiner $\mathbf{D}$ has (7b) iff it is *restrictive*.[6]

(8) $\dfrac{\mathbf{D}AB\ B \subseteq C}{\mathbf{D}A \cap CB}$

For instance, from right to left we have: $\mathbf{D}AB \cap C$; (res) $\mathbf{D}A \cap BB \cap C$; (cons) $\mathbf{D}A \cap BC$. Notice that restrictiveness is weaker than intersectivity and ↓MON. Prime cases such as the anaphoric use of 'some' and 'all' will therefore be covered by our result. An example of a restrictive quantifier which is neither intersective nor ↓MON is 'most'; witness the validity of (9):

(9) $\dfrac{\text{Most logic books are yellow and expensive}}{\text{Most yellow logic books are expensive}}$

We now prove theorem 4, which states that logical $\mathbf{A}$ have $\langle \mathbf{A} \rangle$ iff the only determiners which validate $\langle \mathbf{A} \rangle$ are those obtained from $\mathbf{A}$ by restricting the cardinality of its first argument.

Lemma 1 *Let* $\mathbf{A}$ *be restrictive. For each* $\mathbf{D}$ *with* $\langle \mathbf{A} \rangle$ *we have for all* A *such that* $\mathbf{D}A \neq \emptyset$: $\mathbf{A}A = \mathbf{D}A$.

PROOF. Let $\mathbf{D}$ have $\langle \mathbf{A} \rangle$. Then $\mathbf{D} \subseteq \mathbf{A}$, as is shown by the following implications: $\mathbf{D}AB$; (cons) $\mathbf{D}AA \cap B$; $\langle \mathbf{A} \rangle$ $\mathbf{A}A \cap AB$; $\mathbf{A}AB$. Conversely, let A be such that $\mathbf{D}A \neq \emptyset$, and assume $\mathbf{A}AB$. Choose an X with $\mathbf{D}AX$. Since $\mathbf{D}$ has $\langle \mathbf{A} \rangle$ it is MON↑, whence $\mathbf{D}AX \cup B$. Since $\mathbf{A}$ is restrictive: $\mathbf{A}A \cap (X \cup B)B$. Now $\langle \mathbf{A} \rangle$ implies: $\mathbf{D}A(X \cup B) \cap B$; i.e., $\mathbf{D}AB$. □

The proof of this lemma already gives some information on the kind of $\mathbf{D}$ which validate $\langle \mathbf{A} \rangle$.

Corollary 2 *If* $\mathbf{D}$ *has* $\langle \mathbf{A} \rangle$ *then* $\mathbf{D}$ *is* MON↑ *and* $\mathbf{D} \subseteq \mathbf{A}$ □

For logical determiners the restriction in lemma 1 to $\mathbf{D}A \neq \emptyset$ can be removed by means of the following concept.

Definition 2 For a class of cardinals Θ the determiner $\mathbf{D}^{\Theta}$ is defined by:[7]

$$\mathbf{D}^{\Theta}AB \Leftrightarrow |A| \in \Theta \ \&\ \mathbf{D}AB$$

In case $\Theta = \{\kappa\}$ we write $\mathbf{D}^{\kappa}$ for $\mathbf{D}^{\Theta}$.

Lemma 3 *Let* $\mathbf{A}$ *be logical and restrictive. If a logical determiner* $\mathbf{D}$ *has* $\langle \mathbf{A} \rangle$, *then* $\mathbf{D} = \mathbf{A}^{\Theta}$ *for some class of cardinals* Θ.

PROOF. Let $\mathbf{D}$ have $\langle \mathbf{A} \rangle$, and define:

$$\Theta := \{\kappa : \exists A(\mathbf{D}A \neq \emptyset \ \&\ |A| = \kappa)\}$$

To show $\mathbf{D} \subseteq \mathbf{A}^{\Theta}$, assume $\mathbf{D}AB$. Then, $|A| \in \Theta$, and by corollary 2: $\mathbf{A}^{\Theta}AB$. Conversely, to show $\mathbf{A}^{\Theta} \subseteq \mathbf{D}$ assume $\mathbf{A}^{\Theta}AB$. Then $|A| \in \Theta$ and $\mathbf{A}AB$. By definition of Θ, there is an A' with $|A'| = |A|$ and $\mathbf{D}A' \neq \emptyset$. Lemma 1 gives:

(#) For all B': $\mathbf{D}A'B' \Leftrightarrow \mathbf{A}A'B'$.

Since $|A| = |A'|$, there is a bijection $g : A \longrightarrow A'$. Clearly:

$$|A \cap \overline{A \cap B}| = |A' \cap \overline{g(A \cap B)}| \text{ and } |A \cap B| = |A' \cap g(A \cap B)|$$

We have the following equivalences: $\mathbf{A}AB$; (logicality) $\mathbf{A}A'g(A \cap B)$; (#) $\mathbf{D}A'g(A \cap B)$; (logicality) $\mathbf{D}AB$. □

Since (almost?) all important instances of $\mathbf{A}$ are logical and restrictive, lemma 3 makes plain that for most antecedent determiners in $\langle \mathbf{A} \rangle$ scope extension does not preserve meaning. Lemma 3 also enables a characterization of the logical $\mathbf{A}$ which have $\langle \mathbf{A} \rangle$.

Theorem 4 $\mathbf{A}$ *is logical and* $\langle \mathbf{A} \rangle$, *iff for all logical* $\mathbf{D}$: $\mathbf{D}$ *has* $\langle \mathbf{A} \rangle$ *iff* $\mathbf{D} = \mathbf{A}^{\Theta}$ *for some class of cardinals* Θ.

PROOF. First assume that $\mathbf{A}$ is logical and $\langle \mathbf{A} \rangle$. We have already observed that the $\mathbf{A}$ with $\langle \mathbf{A} \rangle$ are restrictive, and it is clear that $\mathbf{A}^{\Theta}$ is logical if $\mathbf{A}$ is. So with a

[4] In the discussion at AcTen after my talk, Barbara Partee said to disagree with the walk-in-the park variant of (4) because it requires the second *five* to introduce a new quintet. According to Hans Kamp this variant would be acceptable after some reflection. Herman Hendriks suggested the present version which raised no linguistic objections.

[5] Recall that a determiner is *logical* iff it is conservative (CONS), and invariant under suitable changes and permutations of the domain (EXT and ISOM). Cf. Westerståhl 1989, or Van der Does & Van Eijck 1996 for an introduction.

[6] Van Benthem (1984) proposed this principle as part of *confirmation* in his study of conditional relations. Cf. also Lapierre 1996.

[7] Those who feel uncomfortable with cardinals and classes may assume the domains E to be finite, and may read 'natural number' for 'cardinal' and 'set' for 'class'.

view to lemma 3 the only thing which remains to prove the equivalence for logical **D** is that the $\mathbf{A}^{\Theta}$ have $\langle\mathbf{A}\rangle$. But this property of **A** is preserved as well:

$$\begin{aligned} & \mathbf{A}^{\Theta}AB \;\&\; \mathbf{A}A \cap BC \\ \Leftrightarrow\; & |A| \in \Theta \;\&\; \mathbf{A}AB \;\&\; \mathbf{A}A \cap BC \\ \Leftrightarrow\; & |A| \in \Theta \;\&\; \mathbf{A}AB \cap C \\ \Leftrightarrow\; & \mathbf{A}^{\Theta}AB \cap C \end{aligned}$$

Conversely, if the logical determiners with $\langle\mathbf{A}\rangle$ are precisely the $\mathbf{A}^{\Theta}$ for some Θ, then **A** must have $\langle\mathbf{A}\rangle$ too: $\mathbf{A} = \mathbf{A}^{\{\kappa:\kappa=\kappa\}}$ □

The next sections apply theorem 4 to show its impact for current discourse logics. In section 3 it is used to study the semantics of singular pronouns, and in section 4 to study plural definites and numerals.

3 Singular anaphora and their antecedents

Singular anaphora are perhaps the most puzzling of all, since their meaning may change with the place of occurrence. Nevertheless, we shall argue that they can be treated as contextualized quantifiers in a principled way. Besides, this section gives (i) a full description of the determiners which satisfy the principle of scope extension for singular pronouns (section 3.1), and (ii) a universal to single out the determiners which may be antecedent to a singular pronoun (section 3.2). Section 3.3 is a digression on the interpretation of singular pronouns in terms of choice fuctions. The picture which emerges is that it does not make much difference for the semantics of singular anaphora whether it is given by means of scope extension or in terms of contextual restriction. Mainly plural anaphora distinguish the two.

3.1 Singular anaphora

Let us first make a few observations to show that if we interpret singular pronouns as contextually restricted existential quantifiers they behave much as in DRT and DPL. To this end recall that the singular version of 'some' has a numberless and a singular interpretation. They are given in (10), where $\mathbf{some}^s$ is equivalent to a Russellian singular description $\mathbf{the}^s$.

(10) $\mathbf{some}AB \;\Leftrightarrow\; A \cap B \neq \emptyset$
$\mathbf{some}^sAB \;\Leftrightarrow\; A \cap B \neq \emptyset \;\&\; |A| = 1$

Similarly, on a Russellian view the semantics of 'a' is either **some** or $\mathbf{some}^s$. Since **some** has $\langle\mathbf{some}\rangle$ we know from theorem 4 that $\mathbf{some}^s$ has $\langle\mathbf{some}\rangle$ as well. Moreover, $\mathbf{some}^s$ has $\langle\mathbf{some}^s\rangle$. Thus we see that interpreting singular pronouns as existentials gives what is required.

(11) A man walks in the park. He whistles.
A man walks in the park and whistles

To be precise, the equivalence (11) holds iff 'a' and 'he' are both interpreted as **some**; or both as $\mathbf{some}^s$; or 'a' is interpreted as the description $\mathbf{some}^s$ and 'he' as the existential **some**. It is invalid if 'a' is an existential and 'the' a singular definite description.

Since **some** is intersective the remarks in section 2 make plain that it has [**some**], too. So, donkey-like sentences can be handled along these lines as well:.[8]

(12) If a man walks in the park, he whistles.
Every man who walks in the park, whistles.

[8]It would be interesting to compare this view on singular descriptions with that in the philosophical literature. Neale (1990, chapter 5 & 6) shows that donkeys, sage plants, bathrooms and MIGs, among other anaphoric sentences, can be handled by interpreting singular E-type pronouns as numberless descriptions; i.e., univeral quantifiers with a non-empty first argument. This works well in all the cases were a universal reading is required, but it fails for such texts as in 11.

It is worth to notice that the difference in quantificational force of the singulars—existential in (11), universal in (12)—is closely related to the polarity of the antecedent and the anaphor. By default we have that an occurrence of a singular pronoun is existential (*weak*) if its polarity and that of its antecedent 'a' are the same (both positive or both negative); it is universal (*strong*) otherwise. This generalizes Kanazawa's obervation concerning monotonic determiners to arbitrary constructions, and can be formalized by means of a monotonicity calculus. Cf. Van Benthem 1986, Sanchez 1991, Kanazawa 1994. However, in case of other antecedents than 'a' it seems that singular pronouns favour a weak reading.

DRT and DPL The existential semantics for singular pronouns appears far removed from compositional versions of DRT, but it is not. A denotation for singular pronouns which is essentially the same as that in DPL can be phrased in terms of choice functions. Before we give this denotation, let us quickly recall the treatment of pronouns in DPL.

In DPL the pronoun 'he' in (1a) is a variable which is captured dynamically by the preceding existential quantifier.

(1a) A man walks in the park. He whistles.

The basic idea is that the existential associated with the indefinite 'a' introduces an object d stored under x to give assignment a^d_x.

(1a') $a \xrightarrow{\exists x} a^d_x \xrightarrow{Mx} a^d_x \xrightarrow{Wx} a^d_x \xrightarrow{Whx} a^d_x$

Next, the atomic sentences within the scope of the quantifier test one after the other whether d is a man and a walker. If so, d is picked up by the pronoun 'he'. When d is a whistler as well, (1a) is true.

A slightly different but logically equivalent approach would hold that 'he' chooses an object from the set of walking men. For the case at hand, this can be made precise as follows.[9]

Definition 3 A function $g : \wp(A) \longrightarrow A$ is a *choice function* for A iff for all non-empty $X \subseteq A$: $g(X) \in X$. In terms of a choice function f the determiner $\langle f\rangle$ is defined by:

$$\langle f\rangle_E AB \Leftrightarrow A \neq \emptyset \wedge f_E(A) \in B$$

for all $A, B \subseteq E$. The dual $[f]$ of $\langle f\rangle$ is defined by: $[f]_E AB \Leftrightarrow \neg\langle f\rangle_E A\overline{B}$.

Using $\langle f\rangle$ the anaphoric link in (12a) can be interpreted as (12b).

(12) a. $\mathbf{D}^iAB \;\&\; \mathbf{A}_iC$
b. $\exists f : \mathbf{D}AB \;\&\; \langle f\rangle A \cap BC$

We should therefore ask for the **D** which satisfy (**cf**).

(**cf**) $\exists f : \mathbf{D}AB \;\&\; \langle f\rangle A \cap BC$, iff $\mathbf{D}AB \cap C$.

But since $\exists f : \langle f\rangle AB$ iff $A \cap B \neq \emptyset$, for all A, B, it is the same to ask for the **D** with $\langle\mathbf{some}\rangle$. We therefore know from theorem 4:

Corollary 5 *The logical determiners* **D** *which have* (**cf**) *are precisely those with* $\mathbf{D} = \mathbf{some}^{\Theta}$ *for some class of cardinals* Θ. □

The result shows that as soon as pronouns are treated by means of choice functions DRT's and DPL's mechanism of extending the scope of **some** only carries over to the indicated subparts of this quantifier. To know whether or not this is too restrictive,

[9]We gloss over the fact that the choice function should be dependent on the variable. Cf. Van der Does 1994. Another important issue is that the restricting domains may be dependent on each other. Cf. Van den Berg 1991, 1996, and section 5, among other places. The idea to use choice functions to interpret singular pronouns is found at many different places: Urs Egli, Meyer-Viol 1992, Chierchia 1992, Gawron, Nerbonne, Peters 1991show, Von Heusinger & Peregrin, Van Eijck 1993. Also: Muskens 1989.

we have to determine the class of possible antecedents of singular pronomina. This is done in section 3.2. Here it is shown that the only other possible antecedents are the analogous subparts of *one*. Hence, corollary 5 does not prohibit intuitively acceptable linkings, for such antecedents can be defined in terms of **some** in the familiar way. Consider for instance (13).

(13) a. Just one man drove a Jaguar. He whistled.
b. Just one man drove a Jaguar and whistled.

As for (7), the difference between (13a) and (13b) cannot be explained if 'he' in (13a) is bound by 'just one' in the standard sense. However, this observation does not suffice to show that a bound variable analysis of (13a) is impossible. One could read the antecedent sentence as stating: $\exists x.$'x is the unique man driving a Jaguar'. This first-order definition of 'just one' splices the quantifier into an existential and a uniquely satisfied predicate. And scope extension would still hold for the first. Cf. Groenendijk, Stokhof, Veltman 1995. However, in case of quantifiers like 'quite a few' and 'most' this resort to first-order definitions is impossible. Cf. Barwise and Cooper 1981.

3.2 Antecedents for singular anaphora

Corollary 5 states that the interpretation of singular pronouns by means of scope extension should be limited to certain subparts of 'some'. So, adopting scope extension as the main principle of pronoun resolution one would like to know whether this gives us all the cases we want. To this end we have to determine which noun phrases may figure as antecedent to singular anaphora (hence: s-antecedent). We shall give an answer to this question by proposing constraints on their denotations. They imply that the only logical determiners which may be antecedent to a singular anaphor are those of form:'some^s', 'a', 'just one', 'the', 'one of the', ' one...out of −'. These can all be interpreted as $\mathbf{some}^{\Theta}$ or $\mathbf{one}^{\Theta}$.

A strong intuition concerning singular antecedents is that their denotation introduces a possible referent for the anaphor. The singular dynamicity constraint SDYN corresponds to this intuition. Given CONS, which is assumed throughout, the following two formulations are equivalent:

SDYN1 $\mathbf{D}_E AB \Rightarrow \exists b \in B : \mathbf{D}_E A\{b\}$
SDYN2 $\mathbf{D}_E AB \Rightarrow \exists b \in A \cap B : \mathbf{D}_E A\{b\}$

We propose the following 'universal'.

Constraint 1 *All s-antecedents satisfy* SDYN.

In fact, the constraint can be taken locally relative to given E, A, B or globally relative to all E, A, B (global variant). This makes a difference for determiners such as 'every' and 'at most one' which for particular E, A, B may respectively be equivalent to the SDYN 'the^s' and 'just one', although they are clearly not SDYN on all E, A, B. In such situations the local variant would allow:

(14) Every/at most one man cried. He was unhappy.

But (14) is ruled out on the global variant. It has as an immediate consequence that no non-empty SDYN determiner is MON↓. For if $\mathbf{D}$ is globally SDYN then it is *positive* in the sense that: $\mathbf{D}_E AB$ always implies $A \neq \emptyset \neq B$. And of course each non-empty MON↓ $\mathbf{D}$ has $\mathbf{D}A\emptyset$ for some A. This corresponds to our judgement that there can be no link between the antecedents and 'he' in (15).

(15) No/at most one man walked his dog. *He is lazy.

From now on we work with the global version of SDYN.

What about logical s-antecedents with a different monotonicity behaviour? Proposition 6 answers the MON↑ case. For simplicity we now assume FIN: the

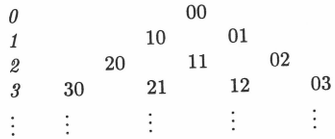
domains considered are finite. Let X be a set of natural numbers, and define as before: $\mathbf{D}^X AB \Leftrightarrow_{\text{def}} |A| \in X \ \&\ \mathbf{D}AB$.

Proposition 6 *The only* MON↑ *s-antecedents are those of form* $\mathbf{some}^X$ *with* X *a set of natural numbers.*

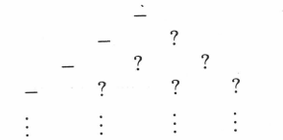
PROOF. It is clear that $\mathbf{some}^X$ SDYN, and MON↑. So, take a logical $\mathbf{D}$ with these properties. Since $\mathbf{D}$ is logical, we may view it as a relation $\mathbf{d}$ between numbers according to the equivalence:

$$\mathbf{D}AB \Leftrightarrow \mathbf{d}|A - B||A \cap B|$$

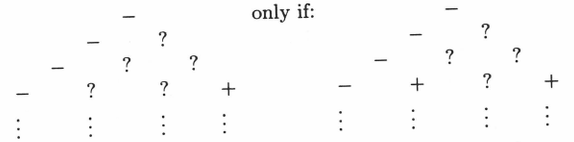
Notice that $\mathbf{D}$ is a relation among sets and $\mathbf{d}$ the corresponding relation among numbers. Restricting our attention to finite domains, the properties of $\mathbf{D}$ become regularities in the number tree:



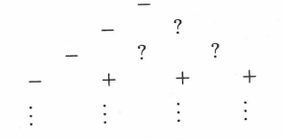
SDYN corresponds to: $\mathbf{d}nm \Rightarrow m \geq 1 \ \&\ \mathbf{d}(n+m-1)1$. E.g., if $\mathbf{d}03$ then also $\mathbf{d}21$. And according to MON↑ all positions to the right of a + for $\mathbf{d}$ are + as well: $\mathbf{d}nm \Rightarrow \forall k(m \leq k \leq n+m \Rightarrow \mathbf{d}n+m-k,k)$. Similarly for MON↓ and +'s to the left. Using these regularities we see first that not: $\mathbf{d}m0$ for all m, due to SDYN.



Second, for each line $m+1$ at which $\mathbf{d}$ has a '+' SDYN gives $\mathbf{d}m1$. For example:



In combination with MON↑ this implies for that m and all $n \geq 1$: $\mathbf{d}mn$.



Now, let $X = \{n : \mathbf{d} \text{ has a '+' at row } n\}$. From the previous remarks it is clear that the resulting pattern is that of $\mathbf{some}^X$. □

In principle, this simple result already gives many possible antecedents for singular antecedents. But presumably not all of them are realized in natural language: X should not be too complex to describe. E.g., $\mathbf{some}^{\omega}$, ω the set of natural numbers, is the meaning of *some*, *a*, *at least one* (which are equivalent according to quantifier theory). Also, (*at least*) *one of the* n denotes: $\mathbf{some}^n$, which for $n = 1$ gives the meaning of *the*^{sg}. And so forth, and so further...

Proposition 6 is in perfect harmony with corollary 5, which said that scope extension for singular pronouns is just possible for the $\mathbf{some}^X$. So, if there are

any problematic SDYN antecedents they should be non-monotonic. The behaviour of such determiners can be quite chaotic. But as the previous proof has already shown, they will always subsume 'exactly one^X' for some set X of natural numbers.

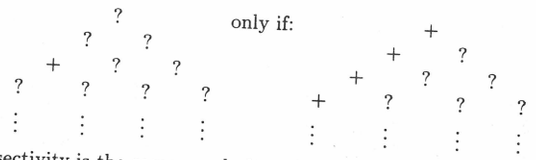
Corollary 7 *For each logical* SDYN $\mathbf{D}$*:* **exactly one**$^X \subseteq \mathbf{D}$*, X some set of numbers.*

PROOF. Just observe that $\mathbf{D}$ is $\mathbf{D}^{\{n\,:\,\mathbf{d} \text{ has a '+' at row } n\}}$, and reason as in the previous proof. □

Moreover, it is often claimed that not just any determiner is suitable as an antecedent but that only the indefinites are. Keenan 1987 proposes to identify indefinites with the intersective determiners. But as soon as we turn an intersective $\mathbf{D}$ into one of form $\mathbf{D}^X$ this has to be replaced by *weak* intersectivity (relative to X):[10]

$$A \cap B = A' \cap B' \;\&\; (|A| \in X \Leftrightarrow |A'| \in X) \Rightarrow (\mathbf{D}AB \Leftrightarrow \mathbf{D}A'B')$$

Intersectivity corresponds to the following regularity in the tree of numbers: $\mathbf{d}nm \Rightarrow \forall k : \mathbf{d}km$. For instance:



Weak intersectivity is the same regularity relativized to X:

$$\mathbf{d}nm \Rightarrow \forall k(k + m \in X \Rightarrow \mathbf{d}km)$$

Let P be a set of natural numbers, and define $\langle P\rangle AB \Leftrightarrow_{\text{def}} |A \cap B| \in P$. We have the following characterization.

Proposition 8 *The only non-empty, logical, non-monotonic, weakly intersective,* SDYN $\mathbf{D}$ *are those of form $\langle\{1\} \cup P\rangle^X$ with $X \neq \emptyset, P$ sets of numbers such that (i) $0 \notin P$, (ii) $\exists k > 1 : k \notin P$.*

PROOF. Let $\mathbf{D}$ be of form $\langle\{1\} \cup P\rangle^X$. That $\mathbf{D}$ is SDYN is immediate from the fact that it contains the relevant parts of the 1-diagonal (i.e., $\mathbf{d}m1$ for all $m+1 \in X$). It is also clear that $\mathbf{D}$ is weakly intersective. The non-monotonicity of $\mathbf{D}$ now follows from non-emptiness, (i) and (ii). For by (i) $\mathbf{D}$ has no points $m0$ so that it is not MON↓. And by (ii) each row $k+l$ at which $\mathbf{D}$ has a '+' lacks at least one point $\mathbf{d}kl$ with $l > 1$, which blocks MON↑.

Conversely, let $\mathbf{D}$ have the properties named. Define $P := \{k : \exists l.\mathbf{d}lk\}$, and $X := \{n : \mathbf{d} \text{ has a '+' at row } n\}$. It is clear that $\mathbf{D} \subseteq \langle P\rangle^X$, so assume $\langle P\rangle^X AB$. Then $|A\cap B| \in P$ and $|A| \in X$. So $\exists l.\mathbf{d}l|A\cap B|$ and $|A-B|+|A\cap B| \in X$. By weak intersectivity: $\mathbf{d}|A - B||A \cap B|$, and hence $\mathbf{D}AB$. It remains to show that $1 \in P$, and that P has (i) and (ii). Since $\mathbf{D}$ is non-empty, $1 \in P$ follows from SDYN, as does (i). Finally, $\mathbf{D}$ cannot contain all m-diagonals with $m \geq 1$, for then it would be the monotonic **some**^{X}. So (ii) is true as well. □

There are of course many determiners $\langle\{1\} \cup P\rangle^X$ which do not figure as s-antecedents; the converse of constraint 1 is false. For instance, *one or five* has this form, and (16) is marked.

(16) One or five boys sold records. *He was rich.

[10] This has ramifications for Keenan's proposal. The complex indefinite *two of the twelve*—as in: 'there are two of the twelve men in the garden'—has meaning: $\mathbf{2}^{12}$, but is not intersective. So in this case we seem to need weak intersectivity. However, proportional determiners like 'ten percent of the records' are also weakly intersective but disallowed in existential sentences. Thus, neither intersectivity nor weak intersectivity properly explains the distribution of indefinite noun phrases. Note, by the way, that both forms of intersectivity imply conservativity.

In fact it seems that the cases were $P = \emptyset$ are the only non-monotonic s-antecedents, besides the MON↑ **some**^{X} that comes from $P = \{n : n \geq 1\}$. In short:

Constraint 2 *For each X, only the strongest and the weakest $\langle\{1\} \cup P\rangle^X$ with $0 \notin P$ figure as s-antecedents.*

Here, strength is measured by $\subseteq$: $\langle\{1\}\rangle^X \subseteq \langle\{1\} \cup P\rangle^X \subseteq$ **some**^{X}. It is attractive that the constraint couples logical strength to simplicity of linguistic expression. Still, it is descriptive rather than explanatory.

In section 4 we consider distributive plural anaphora and their antecedents. But first we digress a bit further on the determiners obtained from choice functions.[11]

3.3 Digression: determiners from choice functions

Fix a domain E and let CF_E be the set of choice functions f_E for E with $f(\emptyset) \in E$ arbitrary. In lifting f of type *et.e* to a determiner $\langle f\rangle$ of type *et.ett* the following requirements are reasonable:

1. $\langle f\rangle$ should be stronger than **some**;
2. $\langle f\rangle$ should be as logical as possible.

The lifting given in definition 3 behaves well in these respects:

$$\langle f\rangle_E AB \Leftrightarrow f_E(A) \in B \;\&\; A \neq \emptyset$$

We need $A \neq \emptyset$, for otherwise $\langle f\rangle$ is neither part of **some**, nor CONS. In particular, without this requirement we have $\langle f\rangle\emptyset\{f(\emptyset)\}$, but not **some**$\emptyset\{f(\emptyset)\}$ and not $\langle f\rangle\emptyset\emptyset$. On the other hand, $\langle f\rangle$ does have these properties. In fact, it is not difficult to see that:

1. **some**$_E = \bigcup_{f\in CF_E}\langle f\rangle$;
2. For an arbitrary f of type *et.e*: $f \in \text{CF}_E$ iff $\langle f\rangle$ is CONS!

As to the latter, if $f \notin \text{CF}_E$ there exist a non-empty X with $f(X) \notin X$. So we can use the previous observation with X instead of $\emptyset$ to see that $\langle f\rangle$ is not CONS.

To establish logicality of $\langle f\rangle$, this determiner should be a functor which is invariant under growth of suitable domains. Such functors could be derived from choice *functors*, which assign a choice function f_E to each domain E. These functors should have EXT in the following sense: $f_E(X) = f_{E'}(X)$, for $\emptyset \neq X \subseteq E, E'$. Then, for a choice functor f:

f has EXT iff $\langle f\rangle$ has EXT in the corresponding sense.

For let $\langle f\rangle$ have EXT and $\emptyset \neq X \subseteq E, E'$. Then $\langle f\rangle_E X\{f_E(X)\}$ and by EXT for $\langle f\rangle$: $\langle f\rangle_{E'} X\{f_E(X)\}$. So: $f_{E'}(X) \in \{f_E(X)\}$, and $f_{E'}(X) = f_E(X)$.

This is as near as we can get to logicality, since in general $\langle f\rangle$ will not be closed under permutations. E.g., let $f\{1,2\} = 1$ and $f\{2,3\} = 2$, then the bijection π which maps 1 onto 3 gives: $\langle f\rangle\{1,2\}\{1\}$ and not $\langle f\rangle\pi\{1,2\}\pi\{1\}$. Indeed, $[f]$ and $\langle f\rangle$ can only be logical on singleton domains (for a permutation π with $\pi(a) = b$: $a = f(\{a\}) = f(\pi\{a\}) = b$).

Along similar lines we see that the denotations for indefinites that arise from choice functions do not comply with Keenan's thesis that the indefinites in type *et.ett* are exactly the intersective determiners. For an f with $f\{1,2\} = 1$ and $f\{2,3\} = 3$ has: $\langle f\rangle\{1,2\}\{2,3\}$ and not $\langle f\rangle\{3,2\}\{2,1\}$. However, in the course of the present discussion we have already seen other assumptions that counter this claim. For instance, the quantificational E-type approach to singular pronouns in section 3.1 holds that the definite singular pronouns denote the intersective **some**, against the thesis of Keenan (and that of Barwise and Cooper 1981 concerning definites).

[11] This section is inspired by discussions with Yoad Winter on his work concerning indefinites and choice functions. Cf. his contribution to the proceedings.

It would be nice to have a characterization of the determiners $\langle f \rangle$ in terms of properties of type *et.ett* elements. As it happens, a natural cluster of properties delimits the set of determiners of this form. Call $\mathbf{D}$:

1. *positive* iff $\mathbf{D}XY$ implies $X \neq \emptyset \neq Y$
2. *almost reflexive* iff $\mathbf{D}XX$, for $X \neq \emptyset$
3. *continuous* iff $\mathbf{D}X\bigcup_I Y_i \Leftrightarrow \exists i \in I : \mathbf{D}XY_i$
4. *functional* iff: $\mathbf{D}X\{a\}$ and $\mathbf{D}X\{b\}$ imply $a = b$

Proposition 9 *A determiner* $\mathbf{D}$ *in type et.ett is positive, almost reflexive, continuous and functional iff there is a choice function* f *such that* $\mathbf{D} = \langle f \rangle$.

PROOF. It is clear that $\langle f \rangle$ is positive, almost reflexive, continuous and functional. Conversely, note that $\mathbf{D}$ is continuous and functional only if for $Y \neq \emptyset$:

(*) $\quad \mathbf{D}XY \Leftrightarrow \exists! a \in Y : \mathbf{D}X\{a\}$

('$\exists!$' means: exactly one.) Hence an almost reflexive $\mathbf{D}$ allows us to define the choice function f by: $f(X)$ is the unique $a \in X$ with $\mathbf{D}X\{a\}$ for $X \neq \emptyset$, and arbitrary elsewhere. Now we have the following equivalences:

- $\langle f \rangle XY$
- $f(X) \in Y$ and $X \neq \emptyset$
- $\mathbf{D}X\{f(X)\}$, $f(X) \in Y$, and $X \neq \emptyset$
- $\mathbf{D}XY$

From top to bottom, the second step uses $X \neq \emptyset$, almost reflexivity, (*) and the definition of f, while the third step uses continuity (hence: MON↑). From bottom to top the first step uses positivity, (*), and the definition of f. □

Since the proof does not depend on the domain E we also have the analogous result for functors f and $\mathbf{D}$.

With a view to the purported scopelessness of the indefinite 'a' as interpreted by $\langle f \rangle$, the lifting proposed has a familiar hitch. The noun phrase $\langle f \rangle A$ is scopeless with respect to conjunctions and disjunctions in its second argument. But it is scopeless with respect to complementation and quantificational verb phrases only if A is *presupposed* rather than stated to be non-empty. (In case of the quantifiers one could also require that they be positive.) For the same reason generalization to **some**A by means of wide scope existential quantification over f may sometimes fail. However, changing the lifting to:

$$\langle f \rangle_E AB \Leftrightarrow f_E(A) \in B$$

provided that $A \neq \emptyset$ together with the use of a partial logic gives *mut. mut.* the results above, and does make $\langle f \rangle$ scopeless. Cf. Fodor and Sag 1982.

4 Plural anaphora and some of their antecedents

4.1 Definite anaphora

As is well-known, definite noun phrases such as 'they', 'the man', 'those four records' are often interpreted by means of an anaphoric link. There is an ongoing debate as to their precise semantical status. In DRT a definite $\mathbf{D}A$ is treated as an open expression. And in quantification theory it is interpreted as a principal filter of form $\{Y : A \cap X \subseteq Y\}$ (cf. Barwise and Cooper 1981. The context set X is sometimes disregarded in what follows.) This means that each definite $\mathbf{D}$ is ↓MON. Therefore, each such $\mathbf{D}$ has $\langle \mathbf{D} \rangle$, since this holds for each ↓MON determiner. Let us have a brief look at some examples.

There are numberless, singular, and plural forms of the quantifier **all**:

(13) $\quad \mathbf{all}AB \Leftrightarrow A \subseteq B$
$\quad\quad \mathbf{all}^s AB \Leftrightarrow A \subseteq B \;\&\; |A| = 1$
$\quad\quad \mathbf{all}^p AB \Leftrightarrow A \subseteq B \;\&\; |A| > 1$

The semantics of 'the' is $\mathbf{all}^s$ (singular) or $\mathbf{all}^p$, (plural), and that of 'all' and 'they' is **all** (numberless) or $\mathbf{all}^p$ (plural). Each of these versions satisfy $\langle \mathbf{all} \rangle$. In fact we have for all Θ: $\mathbf{all}^\Theta$ has $(\mathbf{all}^\Theta)$, even though these determiners need not be ↓MON. This accords with our intuitions. For instance, (14) is valid if 'the' and 'they' are both numberless, both plural, or 'the' plural and 'they' numberless:

(14) The books are boring. They are dusty.
The books are boring and dusty.

In case 'the' is numberless and 'they' plural (14) is invalid. For conservative antecedent $\mathbf{D}$ the direction from top to bottom is always valid, but for (14) the converse fails (the bottom sentence need not imply that at least two boring books are dusty). Theorem 4 implies the following characterization for the numberless case.

Proposition 10 *Call a determiner* $\mathbf{D}$ *maximal iff for all* E: $\mathbf{D}_E \subseteq \mathbf{all}_E$. *The only determiners with* $\langle \mathbf{all} \rangle$ *are the maximal* MON↑ *ones.*

Notice that the maximal MON↑ determiners are precisely those of form $\mathbf{all}^\Theta$. Natural language examples of maximal MON↑ determiners would be: *the four*, *the five or more*, ... And (15) is indeed valid.

(15) The four statutes were expensive. They sold quickly.
The four statutes were expensive and sold quickly.

This would also be true on the plural form of 'they', for another consequence of theorem 4 is:

Proposition 11 *The only logical* $\mathbf{D}$ *with* $(\mathbf{all}^\Theta)$ *are those with* $\mathbf{D} = \mathbf{all}^{\Theta'}$ *for some* $\Theta' \subseteq \Theta$.

Next we consider numerals, as main examples of quantifiers in discourse logic.

4.2 Numerals

An important category for discourse logics are the indefinite or weak noun phrases which often figure as antecedents for plural anaphora. Numerals are prime examples of indefinites. As in the case of definites there is an ongoing debate concerning their semantic status. In DRT they are modeled as open expressions. They are essentially different from quantificational noun phrases. Discourse operators bind them depending on the place at which they occur. In quantifier theory indefinite noun phrases are mostly identified with intersectives.

Call a determiner INT↑ iff it is intersective and MON↑. It is not hard to see that each INT↑ $\mathbf{A}$ has $\langle \mathbf{A} \rangle$. Well-known examples of INT↑ determiners are 'a', 'some', 'at least n', 'infinitely many', besides conjunctions and disjunctions of such determiners. For example, (16) is valid, iff 'five' means: at least five.

(16) Five women went to a bar. Five had a beer.
Five women went to a bar and had a beer.

Theorem 4 shows that the only other antecedent determiners with this property are the parts of 'five' with the size of their first argument restricted. In English some of these determiners are realized in partitive constructions like 'five of the ten' $(\mathbf{5}^{10})$ or 'five of the ten or more' $(\mathbf{5}^{\{\kappa : 10 \leq \kappa\}})$, as in (17).

(17) Five of the ten or more women went to a bar. Five had a beer.
Five of the ten or more women went to a bar and had a beer.

Scope extension for plural anaphora is best studied by prototypical discourses such as (18).

(18) Four women went to a bar. They had a beer.
Four women went to a bar and had a beer.

Is (18) valid? The answer varies with our view on numerals, which could be adjectival, referential, or quantificational. Cf. Van der Does 1992, 1994 for the issues at stake.

It is clear that (18) is invalid if 'four' is the quantifier **four**: it is not of form $\mathbf{all}^{\Theta}$. By contrast, (18) would hold on a referential treatment. For then the semantics of 'four books appeared' is:

(19) $\mathbf{all}^4 B \cap C A \Leftrightarrow B \cap C \subseteq A \;\&\; |B \cap C| = 4$

with C some context set. This referential use of numerals is close to their adjectival use, where they also put a cardinal restriction on the noun; e.g., (17a) is true iff (17b) is.

(17) a. Four books appeared.
b. $\exists X[|X| = 4 \wedge X \subseteq B \wedge X \subseteq A]$

Yet, there is a crucial difference: (17b) would make the equivalence in (18) invalid. This is best seen from a slightly more general stance. Logically speaking any determiner **D** could be treated in an adjectival way (whether or not this is desirable empirically). The recipe in (18) is basic to the above proposal Cf. also Verkuyl 1981, Van Eijck 1985, Kamp and Reyle 1993.

(18) $\mathrm{adj}_1(\mathbf{D})AB \Leftrightarrow \exists X \subseteq A[\mathbf{D}AX \wedge X \subseteq B]$

Van Benthem (1986) observes that $\mathrm{adj}_1(\mathbf{D})$ is always MON↑. Indeed: $\mathrm{adj}_1(\mathbf{D}) = \mathbf{D}$ iff **D** is conservative and MON↑. Now, $\mathrm{adj}_1(\mathbf{just\ n}) = \mathrm{adj}_1(\mathbf{at\ least\ n}) = \mathbf{at\ least\ n}$, so we are back to the quantificational use which invalidates (18).

The semantics of numerals in second-order DRT can best be seen as a combination of the referential and the quantificational use. In DRT (17b) results from interpreting the DRS (20) by means of the discourse operator $\exists$.

(20) $X[|X| = 4 \wedge X \subseteq B \wedge X \subseteq A]$

One could think of X as a discourse referent (cf. the contextually restricted noun in (19)). It lives during certain stages in the construction of a DRS, but becomes a bound variable as soon as a discourse operator is applied. In case of the discourse in (18) the DRS (20) is transformed into (21).

(21) $X[|X| = 4 \wedge X \subseteq D \wedge X \subseteq B \wedge X \subseteq S]$

Therefore, (18) should be valid, since (19a) is of course equivalent to (20).

(19) a. $\exists X[|X| = 4 \wedge X \subseteq D \wedge X \subseteq B \wedge X \subseteq S]$
b. $\exists X[|X| = 4 \wedge X \subseteq D \wedge X \subseteq B \cap S]$

A problem for this approach is that does not explain why (22) is invalid.

(22) Just four books appeared and sold like hot cakes.
Just four books appeared. They sold like hot cakes

In current versions of DRT 'just four' is either a quantifier or it is more like an adjective. In the first case it does not allow anaphoric links outside its scope. Such a link is perfectly in order in the conclusion of (22), so the use should be adjectival. But then we come accross the identity named earlier: $\mathrm{adj}_1(\mathbf{just\ n}) = \mathrm{adj}_1(\mathbf{at\ least\ n}) = \mathbf{at\ least\ n}$. The anaphoric link is in place but at the expense of altering the meaning of the premisse to: at least four books appeared and sold like hot cakes.

A proposal which circumvents these problems is in (20). It is a second-order analogue of the solution given for the dynamics of 'just one' in (13).

(20) $\mathrm{adj}_2(\mathbf{D})AB \Leftrightarrow \exists X[\mathbf{D}AX \wedge X = A \cap B]$

This gives $\mathrm{adj}_2(\mathbf{D}) = \mathbf{D}$ iff **D** is conservative, and hence includes almost all natural language determiners. Following the same route as before we now arrive at the interpreted DRS in (23a).

(23) a $\exists X[|X| = 4 \wedge X = B \cap A \wedge X \subseteq S]$
b $|A \cap B| = 4 \wedge A \cap B \subseteq S$

Notice that (23a) is equivalent to the E-type approach in (23b), which invalidates (22). It clearly shows that it is not the quantifier itself whose scope is extended but rather that of the logically superflous second-order discourse operator. Indeed, a DRT-like formalization of these anaphora is possible, and can be summarized in the slogan: plural anaphora are bound to be E-type. The following section has a categorial system for E-type anaphora.

5 E-type anaphora and Categorial Semantics

The sentences with pronominal noun phrases which were problematic for Montagovian grammars before the eighties mostly have the property that any 'standard' formalization leaves the anaphora and their antecedent noun phrase independent of each other. This counters the intuition that the interpretation of such E-type anaphora should somehow be affected by the meaning of its antecedent. As we have seen, all E-type anaphora can be treated as contextually restricted generalized quantifiers. Once more, (24) can be rendered as (25) with interpretation (26).

(24) Just^i two men walk in the park. They$_i$ whistle.

(25) $\mathbf{two}^i MW \wedge \mathbf{all}_i Wh$

(26) $\mathbf{two}MW \wedge \mathbf{all}M \cap W\,Wh$

Notice that the E-type approach is independent of noun phrase meaning. Basically the same mechanism can be used for all noun phrases which are antecedent to anaphora outside their scope. To make this precise, we shall present a labelled, many dimensional categorial system which generates the meaning of a wide class of discourses in a compositional way.

5.1 The proof system

Since deductions use a lot of space this short article concentrates on the general form of the lexical items and the rules. We conclude with some worked examples.

5.1.1 Lexical items

The basic items in our natural deduction system are vectors of form:

$$n : \tau : \alpha : i : \beta : m$$

Cf. Oehrle 1988, 1995. Here, τ is a lambda-term of type α, i is an index taken from an infinite set I, n and m denote functions from indices to lambda-terms with n figuring as input and m as output, and β is the type of input term $n(i)$. In general lexical specifications are of form:

(27) $n : \tau : \alpha : \mathrm{TYPE}(n(i)) : n[i := \tau]$

where $n[i := \tau]$ is n with i set to τ. For the fragment considered here, the only exceptions to this scheme are the lexical entries for determiners (type $((et)((et)t))$, or $et.ett$). For simplicity, let us assume that all determiners are sensitive to context and contribute to it. Such determiners come with an extra argument for a context set: if **D** is of type $et.ett$, then its context dependent form in type $et(et.ett)$ is given by the lift: $\lambda C \lambda X \lambda Y.\mathbf{D}C \cap XY$. Cf. Westerståhl 1984.[12] Further, their output

[12] Since not all quantifiers in type $((et)t)$ can be relativized in this way, context sensitive quantifiers are restricted to those of form $\mathbf{D}(\top)$ with **D** a determiner and $\top$ the domain of discourse.

should be a partially specified context set, so that $n[i := \lambda C\lambda X\lambda Y.C \cap X \cap Y]$. Notice that this term asks for the same number and the same kind of arguments as the determiner itself. But whereas a determiner combined with its arguments yields a truth-value, the output term yields a set.

5.1.2 General rules

The main difference between the present many-dimensional deduction system and the familiar one for simple types, is that some rule schemes concern the interaction between the elements of a vector. For instance, a determiner should be able to 'pop' a context set from the input dimension. Also, the context set contributed by a determiner may be specified by a complex term, and we should be able to 'push' such terms into the output dimension. The following three rules take care of this:

$$\frac{n : \tau : \alpha : i : (\alpha\beta) : m}{n : n(i)(\tau) : \beta : i : (\alpha\beta) : m[i := n(i)m(i)]}$$

$$\frac{n : \tau : (\alpha\beta) : i : \alpha : m}{n : \tau(n(i)) : \beta : i : \alpha : m[i := m(i)n(i)]}$$

$$\frac{n : \tau : \alpha : i : \text{TYPE}(n(i)) : m}{n : \tau : \alpha : j : \text{TYPE}(n(j)) : m[j := \tau]}$$

Application requires that the output of one premiss is equal to the input of the other (cf. conjunction as composition in DPL, Groenendijk and Stokhof 1991):

$$\frac{n : \tau : (\alpha\beta) : i : \text{TYPE}(n(i)) : m \quad m : \sigma : \alpha : i : \text{TYPE}(m(i)) : k}{n : \tau(\sigma) : \beta : i : \text{TYPE}(n(i)) : k[i := m(i)k(i)]}$$

The output of the lexical items ensure that the output applications in the above rules are possible. The rule of abstraction is the most subtle of all, since we should also abstract over some of the terms in the output:

$$\frac{\begin{array}{c}[- : x : \alpha : i : - : -] \\ \mathcal{D} \\ n : \tau : \beta : i : \text{TYPE}(n(i)) : m\end{array}}{n : \lambda x.\tau : (\alpha\beta) : i : \text{TYPE}(n(i)) : \lambda(m)}$$

The use of '$-$' indicates that these items in the vector may differ per assumption occurrence. An index i *depends* on variable x if i occurs in a vector along a path in derivation $\mathcal{D}$ which has x as an open assumption. In terms of this notion, $\lambda(m)$ is defined by: $m(i) := \lambda x.m(i)$ if i depends on x in $\mathcal{D}$, $m(i)$ is unchanged otherwise.

5.2 An example

Given the above proof system, fairly standard deductions generate the meaning of sentences with E-type anaphora. For example, the phrase *Just one*$^{\mathbf{i}}$ *man who walks two*$^{\mathbf{j}}$ *dogs* has as a type *ett* meaning $\mathbf{one}(\mathbf{who}(\mathbf{two}(D)(Wx)))(M)$, where **who** denotes the type *et*.(et.et) term $\lambda Y\lambda X\lambda x[Xx \wedge Yx]$. Figure 1 has a deduction which leaves the types of the terms implicit (the changes to in- and output in this deduction are given below).

Since each subexpression comes with an input/output pair, these items tend to proliferate. But in each case the functions from indices to term only alter at the indicated index. We now show that the phrase in Figure 1 has output $15(i) \equiv 11(i) \equiv \lambda Y(\mathbf{who}(\mathbf{two}(D)(Wx)))(M) \cap Y)$, and $15(j) \equiv \lambda x.D \cap Wx$ be giving the relevant deductions for the output states in Figure 2 and 3. (We are looking for a more perspicious and economical representation, but hope that the notation is sufficiently clear.)

Figure 4 shows that using the input rules $15(j)$ can be applied to give *calls them*$_j$ the type *et* meaning: $\lambda x.\mathbf{all}(D \cap Wx)(Cx)$. So, *Just one*$^{\mathbf{i}}$ *man who walks two*$^{\mathbf{j}}$ *dogs calls them*$_j$ means $\mathbf{one}(\lambda x.Mx \wedge \mathbf{two}(D)(Wx))(\lambda x.\mathbf{all}(D \cap Wx)(Cx))$, and

FIGURE 1 derivation of: *Just one*$^{\mathbf{j}}$ *man who walks two*$^{\mathbf{i}}$ *dogs*

$$\dfrac{\dfrac{0 : \mathbf{one} : j : 1 \quad \dfrac{1 : M : j : 2 \quad \dfrac{2 : \mathbf{who} : j : 3 \quad \dfrac{\dfrac{\dfrac{\dfrac{3 : \mathbf{two} : i : 4 \quad 4 : D : i : 5}{3 : \mathbf{two}(D) : i : 6} \quad \dfrac{6 : W : i : 7 \quad 7 : x : i : 8^{\mathbf{i}}}{6 : Wx : i : 9}}{3 : \mathbf{two}(D)(Wx) : i : 10}}{3 : \lambda x.\mathbf{two}(D)(Wx) : i : 11}\, i}{3 : \lambda x.\mathbf{two}(D)(Wx) : j : 12}}{2 : \mathbf{who}(\lambda x.\mathbf{two}(D)(Wx)) : j : 13}}{1 : \mathbf{who}(\lambda x.\mathbf{two}(D)(Wx))(M) : j : 14}}{0 : \mathbf{one}(\mathbf{who}(\lambda x.\mathbf{two}(D)(Wx))(M)) : j : 15}$$

FIGURE 2 output for: *walks two*$^{\mathbf{i}}$ *dogs*

$$\dfrac{\dfrac{\dfrac{4(i) := \lambda XY.X \cap Y \quad 5(i) := D}{6(i) := \lambda Y.D \cap Y} \quad \dfrac{7(i) := W \quad 8(i) := x^{\mathbf{n}}}{9(i) := Wx}}{10(i) := D \cap Wx}}{11(i) := \lambda x.D \cap Wx}\, n$$

FIGURE 3 output for: *just one*$^{\mathbf{j}}$ *man who walks two dogs*

$$\dfrac{1(j) := \lambda Y\lambda X.Y \cap X \quad \dfrac{2(j) := M \quad \dfrac{3(j) := \mathbf{who} \quad 12(j) := \lambda x.\mathbf{two}(D)(Wx)}{13(j) := \mathbf{who}(\lambda x.D \cap Wx)}}{14(j) := \mathbf{who}(\lambda x.D \cap Wx)(M)}}{15(j) := \lambda X.\mathbf{who}(\lambda x.\mathbf{two}(D)(Wx))(M) \cap X}$$

FIGURE 4 derivation of: *calls them*$_{\mathbf{i}}$

$$\dfrac{\dfrac{\dfrac{\dfrac{15 : x : i : 16^{\mathbf{m}}}{15 : 15(i)(x) : i : 17} \quad 17 : \mathbf{pro} : i : 18}{15 : \mathbf{pro}(15(i)(x)) : i : 19} \quad \dfrac{19 : C : i : 20 \quad 20 : x : i : 21^{m}}{19 : Cx : i : 22}}{15 : \mathbf{pro}(15(i)(x))(Cx) : i : 23}}{15 : \lambda x.\mathbf{pro}(15(i)(x))(Cx) : i : 24}\, m$$
$$15 : \lambda x.\mathbf{pro}(15(i)(x))(Cx) : j : 25$$

has output $25(i) \equiv \lambda x.Mx \wedge \mathbf{two}(D)(Wx) \wedge \mathbf{all}(D \cap Wx)(Cx)$, and $25(j) \equiv \lambda x.D \cap Wx \cap Cx$. Similarly for other choices of determiner. Notice that output $25(j)$ is of type *eet*, which captures a dependency of antecedent *two*$^{\mathbf{j}}$ on *just one*$^{\mathbf{i}}$. Accordingly, we are correctly forced to interpret the continuation *They*$_j$ *bark* by means of the polyadic iteration $\mathbf{pro}_i\mathbf{pro}'_jB$, meaning: $\mathbf{some}(25(i))(\lambda x.\mathbf{all}(25(j)(x))(B))$.

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Aspect and quantification: an iterative approach*

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Abstract

This paper presents an approach to the aspectual influence of DP arguments in order to unify two diverging traditions of aspectual semantics, which seem to contradict each other in the aspectual classification of verbs like *to eat*: [Krifka 1992] or [Verkuyl 1995] assign no aspectual class to these verbs in isolation. Instead, the aspect of whole verbal projections of these verbs is calculated. The influence of DP arguments (semantically, generalized quantifiers) is considered in this calculation. But linguists like [Vendler 1967] and [Dowty 1979] classify verbs like *to eat* without taking into account the influence of different kinds of DP arguments. These two traditions are reconciled in terms of the proposed approach. It is based on an elaborate notion of iteration that involves pragmatic reasoning in a well-defined way. The coverage of aspectual theories can be extended in this approach to expressions whose classification was problematic till now, e.g., expressions like *to see twenty zebras* or verbal projections with the quantifier *all the*.

1 Introduction

Verbal projections that contain generalized quantifiers (and larger constituents that contain such verbal projections) are still a rather troublesome issue for aspectual semantics. Intuitions on the aspectual classification of such constituents differ. Consequently, the aspectual theories that model these intuitions classify verbs like *to eat* in seemingly conflicting ways: If one regards the verb in isolation or only, like in (1), in combination with singular object DPs, as in the aspectual classifications of [Vendler 1967] and [Dowty 1979], *to eat* involves a change of state, viz., the disappearance of food caused by a consumer:

(1) to eat a tasty hamburger

As changes of state belong to the extension of bounded verbal expressions[1] only, *to eat* emerges as bounded in these classifications.

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1. No bounded expression applies to proper parts of elements of its extension. Formally, boundedness can be defined as the second-order property **BD**:

But if one considers different kinds of object DPs like [Krifka 1992] and [Verkuyl 1995], the aspect of VPs whose verb is *to eat* depends on its object DP: The (un-)boundedness of the semantic representation of this DP carries over to the representation of the whole VP. Hence, such verbs should themselves have no fixed aspect. At this point, intuitions – and the theories based upon them – clash. It is the aim of the present paper to reconcile these aspectual theories and their insights.[2]

In the following sections, I will present an approach that combines the two trends in aspectual semantics. First, the notion of iteration will be introduced. In the analysis of the meaning of iterative expressions, detailed reasoning is needed to distinguish semantics proper from the effects of interacting pragmatic phenomena. Then I will explain the influence of DP arguments on the aspect of their verbal projection in terms of an underlying iteration. As will be shown in section 5, the approach can be generalized to account for hitherto neglected problematic cases for aspectual classification. Two such cases will be investigated in detail: Expressions like *Max see twenty zebras* are compatible both with time frame adverbials (e.g., *in five hours*) and durative adverbials (as *for five hours*). Following standard assumptions of aspectual semantics, the former signals boundedness and the latter, unboundedness. This seeming contradiction can be explained in my approach. The second case involves DPs with the quantifier *all the* in aspectually relevant argument positions. The fact that they make their predicate bounded follows directly from the proposed approach and an appropriate semantics of *all the*.

2 Technicalities

Before turning to the subject of iteration, I will conclude the introduction by defining some terminology and presenting a number of basic assumptions, which I will presuppose in this paper, but cannot go into here in detail.

First of all, I will use the term 'predicate' to refer to verbs, their projections, and to larger constituents that contain verbal projections, on the condition that these constituents do not yet include sentence mood and tense. Maximal constituents that fulfill this condition will

(i) $\forall P\ (\mathbf{BD}(P) \leftrightarrow \forall x\ \forall y\ (P(x) \wedge y \sqsubset x \rightarrow \neg P(y)))$

2. Note, however, that both these theories classify verbs without aspectually relevant arguments themselves (like the stative *to see*). These verbs determine the aspect of their verbal projections in each case. Thus, both *to see a plane* and *to see planes* are unbounded.

be called 'sentence radicals'; in the following they will be characterized in the examples by a verb without inflection (e.g., *Max sing-*).

The semantics of predicates will be described in the tradition of [Davidson 1967]. That is, the argument frame of each verb contains an additional argument that represents the state of affairs, situation or eventuality (I will use these expressions interchangeably in this paper) involved in the denotation of the verb. Sentence radicals, then, turn out to be sets of eventualities semantically.

At this point invariably and inevitably one question arises: What is the ontological status of eventualities? Discussing this problem would far exceed the range of this paper (but see [Bäuerle 1987], [Galton 1984], [Herweg 1990], [Krifka 1992], and others for discussion). As the approach proposed in this paper does not depend on this ontological choice, I have opted for a very simple approach and have represented eventualities as times. This is not to be taken as a claim about the actual ontological status of eventualities. It is possible to adapt the proposed approach to ontologically richer theories on eventualities. Formally, times are distinguished from other individuals by means of a sorted logic, in which the variables t, t', t'', ... are restricted to times.

3 The semantics of iteration

The proposed approach accounts for the aspectual influence of generalized quantifiers in terms of iteration. In this section, I will spell out the theory of iteration on which the treatment of aspectual influence of DPs is built.

3.1 The analysis

As there are no pure directly visible iterative operators in English, I will approach the subject of iteration by discussing frequency adverbials like *three times*. As these adverbials are semantically more complex but still very close to the iteration operator, it is possible to investigate their semantic properties and then generalize the findings in this domain to derive the semantics of the iteration operator.

The decomposition I advocate for *times*, which forms the basis of frequency adverbials like *n times* (where $n \in \mathbb{N}$) is surprisingly simple:

(2) $\lambda n\ \lambda P\ \lambda t\ \exists t'\ (t = \bigcup t' \wedge card(t') = n \wedge \forall t''\ (t'' \in t' \rightarrow P(t'')))$

Expressions of the type *n times* map predicates on times P onto sets of times t such that t is the underlap of a set of times t' with the cardinality n, each of whose members belongs to the extension of P.

Some ancillary definitions are necessary here. Underlap of a set of times t' is defined as the smallest convex time t that comprises every member of t':

(3) $\forall t\, \forall t'\, (t = \bigcup t' \leftrightarrow$
$\forall t''\, (t'' \in t' \rightarrow t'' \sqsubseteq t \wedge \forall t'''\, ((t'' \in t' \rightarrow t'' \sqsubseteq t''') \rightarrow t \sqsubseteq t')))$

Convexity is defined in (4). In between two parts of a convex time, there can only be other parts of the same time:

(4) $\forall t\, (\mathbf{CONV}(t) \leftrightarrow$
$\forall t'\, \forall t''\, \forall t'''\, (t' \sqsubseteq t \wedge t'' \sqsubseteq t \wedge t' < t''' < t'' \rightarrow t''' \sqsubseteq t))$

3.2 Interplay of semantics and pragmatics in the field of iteration

While this decomposition seems to give the right results for bounded predicates modified by frequency adverbs (as in (5)), it looks as if it is too weak a representation for cases like (6):

(5) Nigel go- to the pub three times

(6) Amélie be- in Oxford three times

According to (2), (6) is assigned the following decomposition:

(7) $\lambda t\, \exists t'\, (t = \bigcup t' \wedge \mathit{card}(t') = 3 \wedge \forall t''\, (t'' \in t' \rightarrow \mathbf{in'(a, o)}(t')))$

The decomposition (7) denotes the set of all times t that are the underlap of a set of times t'. This second set has three members, each of which belongs to the extension of *Amélie be- in Oxford*.

The problem with (7) is that, due to the cumulativity and the divisivity[3] of unbounded predicates like *Amélie be- in Oxford*, the extension of (7) also contains continuous, uninterrupted stays of Amélie in Oxford: Nothing in the semantics of (7) precludes the three involved subintervals to be adjacent or even overlapping. In these cases, the cumulativity of (7) entails that the three subintervals boil down to a single one for which this predicate holds, too. To state the problem the other way round: Because of the divisivity of *Amélie be- in Oxford*, any single uninterrupted stay of Amélie in Oxford may be split up arbitrarily in three adjacent chunks that likewise fall under the predicate (7). However, there is a clear intuition that (6) can only refer to (underlaps of) triples of stays of Amélie in Oxford whose members are interrupted by periods of her absence.

3. These properties can be represented as closedness with respect to a suitable part-of relation (i) and a suitable sum operation (ii), respectively:
(i) $\forall P\, (\mathbf{DIV}(P) \leftrightarrow \forall t\, \forall t'\, (P(t) \wedge t' \sqsubseteq t \rightarrow P(t')))$
(ii) $\forall P\, (\mathbf{CUM}(P) \leftrightarrow \forall t\, \forall t'\, (P(t) \wedge P(t') \rightarrow P(t \sqcup t')))$

I will rule out this unwanted 'continous' interpretation of unbounded predicates modified by frequency adverbials in terms of Gricean maxims rather than treating this phenomenon semantically by including an additional operator in the semantics of iterative adverbs.[4]

The conversation maxim that is at the core of the explanation has been derived in [Krifka 1992] from maxims out of [Grice 1975]. I have dubbed it the 'maxim of informativeness':

(8) Out of two true, equally informative expressions, choose the less complex one

(8) entails that the usability of the semantic denotation of an expression may be narrowed down pragmatically: If elements of this extension could also be expressed by means of a less complex expression, it is infelicitous to refer to these elements by the more complex expression, although the elements still belong to the denotation of the more complex expression. Hence, the use of a more complex expression (assuming cooperativity on the part of the speaker) entails that it is more informative than less complex expressions.

Applied to the problematic case (6), the maxim of informativeness entails that predicates that include a frequency adverb are used only if they bear more information than the same predicate without the frequency adverb. In other words, it would be infelicituous to use a predicate that includes a frequency adverb in order to denote an eventuality that could also be referred to by the same predicate without the frequency adverb. Nevertheless, the more complex and the simpler predicate may have overlapping denotations.

This reasoning explains the strong intuition that (6) cannot be used to refer to underlaps of triples of adjacent or overlapping times. Although the semantics of (6) comprises underlaps of such triples, the maxim of informativeness rules out the 'continuous' interpretation of (6), that is, the use of this predicate for underlaps of these triples: Such underlaps could be referred to by a simpler expression, viz., *Amélie be in Oxford*. Hence, it is possible to keep the representation (2) for frequency adverbs in spite of examples like (6) and let pragmatics do the work of excluding the 'continuous' interpretation of sentences like

4. Note that this step was not motivated by the lack of a suitable operator. A variant of the 'S-phase' of [Herweg 1990, 103] could be employed here:
(i) $\forall P\, \forall t\, (S - \mathit{phase}(t) \leftrightarrow P(t) \wedge \forall t'\, (t \sqsubset t' \rightarrow \neg P(t')))$
In prose: The phase operator maps predicates on times P on the predicate of being maximal with respect to P. Hence, if t is a phase of P, no t' t is a proper part of belongs to the extension of P, too. This operator would at a first glance be useful for cases (5) and (6), as it bounds unbounded predicates by maximizing them while it is an identity mapping for bounded predicates (bounded predicates are their own phase-predicates).

(6).

Other cases that strengthen the case for this pragmatic reasoning involve unbounded predicates like (9) and (10), although these predicates may – unlike (6) – be interpreted 'continuously' if they are modified by frequency adverbs. However, enforcing a 'discontinuous' reading need not be the only way of complying to the maxim of informativeness: A 'continuous' interpretation should be possible for predicates with frequency adverbials if there are other ways in which this interpretation is more informative than the predicate without the frequency adverb. 'Discontinuous' interpretations as for (6) are just one way of complying to the maxim of informativeness. Hence, if there is such a difference in informativeness, the pragmatic account given for (6) holds good for these examples, too.

(9) Max remain- seated three times

(10) A professor be- in the SCR three times

Applying (9) to underlaps of triples of times whose members are adjacent is felicitous. This is so because *to remain seated* carries a presupposition apart from its basic meaning that some sitting eventuality is not interrupted (this would be rendered in terms of an unbounded predicate): Very roughly, if someone remains seated something must have happened that should have caused him to get up but failed to do so.

Hence, although the three uninterrupted seating eventualities involved in the semantics of (9) might add up to one if they are adjacent, the three pairs of presupposed unsuccessful attempts at making Max get up and his uninterrupted sitting cannot add up to one single pair. Thus, the involved presupposition prevents a 'continuous' interpretation of predicate (9) from falling prey to the maxim of informativeness. (9) is more informative than the corresponding predicate without the frequency adverbial: The former involves three pairs of presupposed eventualities that should have caused Max to get up (but haven't) and seating eventualities, the latter, only one pair.

Similar reasoning is involved in an explanation of example (10). This predicate denotes complex eventualies (underlaps of triples of eventualities) that involve up to three professors. If only one professor is involved, the members of the triples involved in the denotation of (10) may neither overlap nor be adjacent, according to a reasoning similar to the one employed in the case of (6). However, there are other eventualities in the denotation of (10), the reference to whom by (10) is not barred by the maxim of informativeness, namely, eventualities that involve three different professors. If (10) is interpreted as referring to such an eventuality, it carries information not present in the less com-

plex *A professor be in the SCR*; hence, in this case, (10) does not fall prey to the maxim of informativeness no matter whether the involved three subeventualities are disjointed, adjacent or even overlap.

These two examples show that the barring of 'continuous' readings is just one strategy of complying to the maxim of informativeness. Hence, it would not just be superfluous to include a maximalization operator into the semantics of frequency adverbials, introducing such an operator would give the wrong results in a number of cases: This inclusion would predict wrongly that the n eventualities involved in predicates that include the frequency adverbial *n times* must be neither adjacent nor overlapping, which has been shown to be too strong a restriction.

3.3 Iteration and convexity

This section finishes with a few words on (non-)convexity of iterative eventualities. I will argue that even iterative expressions like (11) involve convex rather than discontinuous eventualities because of the way they interact with durative adverbials like *for two hours*.

(11) Amélie go- to the letter-box five times

The argument hinges on the observation that durative adverbials do not count gaps within situations. Consider e.g. a situation in which Max was in the office from eight to twelve and from two to five. It is possible to refer to this situation by (12) but not by (13):

(12) Max be- in the office for seven hours

(13) Max be- in the office for nine hours

Hence, durative adverbials may serve as a kind of litmus test for the convexity of situations. If one applies a durative adverbial to an iterative predicate, the denoted iterative situation is convex if the durative adverbial measures the whole span from the beginning of the first subeventuality involved in the interative eventuality to the last of these subeventualities. Should the adverbial measure only the sum of the durations of these subeventualities, iterative predicates stand for non-convex eventualities.

Imagine a situation which lasts three hours and in which Amélie goes to the letter-box every five minutes. Each going lasts one minute. Then, the sum of the durations of the goings is 36 minutes. The fact that (14) but not (15) is adequate as a description of this situation, although durative adverbials would not count gaps within a situation, shows that iterative predicates denote convex eventualities.

(14) Amélie go- to the letter-box (repeatedly) for three hours

(15) Amélie go- to the letter-box (repeatedly) for 36 minutes

This concludes the discussion of frequency adverbials like *three times*. Now that their semantics has been established, the step to the semantics of the iteration operator is very simple: All one has to do is to drop the cardinality restriction on the underlapped set. The iterative operator is formalized as '$ITER_S$' and defined in (16):

(16) $\forall P\, \forall t\, (ITER_S(P,t) \leftrightarrow \exists t'\, (t = \bigcup t' \wedge \forall t''\, (t'' \in t' \rightarrow P(t''))))$

According to (16), iteration is a relation between a predicate on times P and a time t if and only if t is the underlap of a set of times t' each of whose members belongs to the extension of P. Such iterative predicates are unbounded, they are cumulative and divisive.[5]

In this section, I have presented an account of iteration that is based on observations about frequency adverbs like *three times*. The semantic representations of both these adverbs and iteration share a basic structure of an underlapped set of eventualities that belong to the extension of the same predicate (the predicate modified by such adverbs or the iteration operator, respectively). I have shown that there is considerable interplay with pragmatics that allows a very simple decomposition of frequency adverbials and of the general iteration operator.

4 Generalized quantifiers as triggers for iterative readings

4.1 Informal description

Now that we have dealt with iteration, we are in a position to tackle the influence of generalized quantifiers on the aspectual classification of their predicate. This influence will be represented in terms of an operator $ITER_N$ that is part of the decomposition for verbs that allow this aspectual influence. The following subsection is devoted to an informal presentation of the operator $ITER_N$.

The proposed approach for the description of the influence of generalized quantifiers on the aspectual class of their predicate hinges on two basic assumptions:

5. Iterative predicates are divisive only to a certain lower bound. In an aspectual classification that distinguishes between stative predicates like *to love* or *to be in Oxford* and process predicates like *to walk* and *to tremble*, this would entail that iterative predicates belong to the group of process predicates, which share this property.

- there is a partition of the main individual I_1[6] that is involved in the semantics of the aspectually relevant DP
- argument positions in verbs that allow this aspectual influence are determined in the lexicon

The proposed approach will be introduced on the example (17).

(17) Max eat- five apples

For the purpose of exposition, I will start the introduction with verbal decompositions in the style of [Dowty 1979] and derive the decomposition as advocated in my approach from them. This derivation is not meant to model a process in language, it merely serves to relate two linguistic representations.

For our example *to eat*, the Dowty-style decomposition would be something like (18):

(18) $\lambda y\, \lambda x\, \lambda t\, \mathbf{make-vanish'}(x,y)(t)$

Of course, (18) is only shorthand for an elaborate decomposition in terms of operators like CAUSE and BECOME [Dowty 1979, 91ff., 141]. As formulae that use open variables for eventualities tend to be quite unreadable as soon as such operators come into play, I will use 'decomposition skeletons' like (18) in the following. In addition, as it is not the purpose of this paper to spell out exactly what it means to eat something, I gloss over the details and represent *to eat* as 'to make vanish'.

The next step in our expository derivation consists in filling in the argument positions in the process of syntactic/semantic construction. Here the difference between aspectually relevant and aspectually not relevant positions is that the former is filled by an individual variable. After all positions but the highest have been filled in, λ-abstraction over this individual variable takes place. The result is a property P, viz., a relation between a time and an individual. For our example (17), P relates entities eaten by Max and the corresponding eventualities of eating:

(19) $\lambda y\, \lambda t\, \mathbf{make-vanish'}(\mathbf{m},y)(t)$

So far, there are two separate semantic representations, one for the meaning of the DP that fills in the aspectually relevant argument position, and one for the property P. These representations are then connected in terms of an iteration: The meaning of the sentence radical (17) can be glossed as an iteration of the predicate 'P applies to an

6. The main individual is defined as the one that is predicated over by the NP semantics and whose variable is bound by the application of the determiner semantics to the NP semantics.

element of the partition of I_1' (with I_1 being the main individual introduced by the aspectually relevant DP's semantics). Thus, (17) denotes an iteration of the predicate 'Max makes an element of the five-apple partition vanish'.

Additional restrictions apply:

- every member of the partition undergoes P exactly once during the iteration
- iterations are the minimal eventualities that contain all involved sub-eventualities

This makes the (un-)boundedness of the sentence radical semantics dependent on the (un-)boundedness of the semantics of the aspectually relevant DP. Note that the second restriction follows from the definition of iteration in (16). This machinery is part of the lexical entry of verbs like *to eat*. No such iteration is assumed for the lexical representation of verbs like *to see*.

Formally, all of the above is put into an operator '$ITER_N$'. Thus, *to eat* gets a decomposition like the following:

(20) $\lambda y\ \lambda x\ \lambda t\ ITER_N(\lambda z\ \lambda t'\ \mathbf{make-vanish'}(x,z)(t'),y)(t)$

In words: (20) maps two individuals y and x and one time t on the three-place relation $ITER_N$. The first argument of this relation is the property P as defined earlier, in our case, the property of being made vanish by the second of the above individuals (x, which corresponds to the main individual contributed by the subject DP). The first individual y appears as the second argument of $ITER_N$, which amounts to being singled out as an aspectually relevant argument. In (20), the semantic argument position which is picked as the one with aspectual influence is the one that is linked to the object DP. This second argument y of $ITER_N$, which corresponds to the main individual contributed by the object DP, is the one with the partition as described above. Finally, the third argument of $ITER_N$ is the time t. $ITER_N$ involves the iteration operator $ITER_S$ in a well-defined way, which will be spelt out in detail in the definition of $ITER_N$ in the next subsection.

This operator $ITER_N$ has been devised to solve the seeming contradiction between two trends in aspectual semantics as sketched in the introduction: The iteration itself models the dependency of the aspect on the aspectually relevant DP: It is not bounded itself, but can be bounded by additional restrictions (i.e., by the boundedness of the semantics of the aspectually relevant DP, see the next subsection for details). On the other hand, the intuitions on the aspectual class of verbs in isolation emerge in the iterated predicate. E.g., the iterated predicate P in the above semantic description of (17) is bounded and denotes changes of state. Thus, the proposed approach comprises the

values and insights of both trends in aspectual semantics.

In sum, the proposed approach puts down aspectual influence of DP arguments on their predicates to the semantics of the involved verbs. Their decomposition comprises an operator $ITER_N$ that singles out one argument position as the one that exerts aspectual influence and carries over the (un-)boundedness of the semantics of this argument to the whole predicate in terms of an iteration. This operator combines the insights both of semanticists that classify verbs in isolation as well as of those that assign no aspectual class to verbs themselves.

4.2 Formal definitions

After the rather informal presentation of $ITER_N$ in 4.1, this subsection will spell out the formal definition of this operator in detail and show how $ITER_N$ allows the (un-)boundedness of the aspectually relevant DP's semantics to carry over to the semantics of the whole predicate.

First, partition is defined in (21):

(21) $\forall x\ \forall X\ (X \in \mathbf{P}(x) \leftrightarrow x = \mathbf{FU}(X) \wedge$
$\forall x_1\ \forall x_2\ (x_1 \in X \wedge x_2 \in X \wedge x_1 \neq x_2 \rightarrow \neg x_1 \circ x_2 \wedge x_1 \neq \mathbf{0}))$

That is to say, X is a partition of x if x is the fusion of X and the elements of X are not empty and do not overlap mutually.

The empty element $\mathbf{0}$ is defined as the element that is part of all elements:

(22) $\forall x\ (\mathbf{0} \subseteq x)$

Overlap and fusion are defined as existence of a common part (23) and the supremum of a set (24), respectively:

(23) $\forall x\ \forall y\ (x \circ y \leftrightarrow \exists z\ (z \sqsubseteq x \wedge z \sqsubseteq y))$

(24) $\forall x\ \forall X\ (x = \mathbf{FU}(X) \leftrightarrow \forall y\ (y \in X \rightarrow y \sqsubseteq x) \wedge \forall z\ (\forall y\ (y \in X \rightarrow y \sqsubseteq z) \rightarrow x \sqsubseteq z))$

Finally, (25) and (26) model intuitions that will become of relevance later on. According to (25), the union of two non-overlapping individuals x and y which have partitions X and Y, respectively, likewise has a partition, viz., the set union of X and Y. (26) expresses the intuition that if there are two individuals x and y with a respective partition X and Y such that $X \subset Y$, then x is a proper part of y:

(25) $\forall x\ \forall y\ \forall X\ \forall Y\ (X \in \mathbf{P}(x) \wedge Y \in \mathbf{P}(y) \wedge \neg x \circ y \rightarrow X \cup Y \in \mathbf{P}(x \sqcup y))$

(26) $\forall x\ \forall y\ \forall X\ \forall Y\ (X \in \mathbf{P}(x) \wedge Y \in \mathbf{P}(y) \wedge X \subset Y \rightarrow x \sqsubset y)$

With the help of these definitions, $ITER_N$ can be defined as follows:

(27) $\forall P\ \forall x\ \forall t\ (ITER_N(P,x)(t) \leftrightarrow$
$\exists Y\ (Y \in \mathbf{P}(x)\ \wedge\ \forall y\ (y \in Y\ \rightarrow\ \exists! t'\ (t' \sqsubseteq t\ \wedge\ P(y)(t'))) \wedge$
$ITER_S(\lambda t''\ \exists y\ (y \in Y\ \wedge\ P(y)(t'')))(t)))$

In prose: $ITER_N$ holds for a triple of a property P, an individual x and a time t if and only if there is a partition of x such that each of its elements undergoes P exactly once during t and t belongs to the extension of an iteration of the predicate 'P holds for an element of the partition of x'.

I will now briefly sketch how this operator models the aspectual influence of DP arguments. Consider for instance *to leak out*. This verb allows aspectual influence of its subject position, hence, a sentence radical like *water leak- out* is given the following decomposition in terms of $ITER_N$, with the main argument intoduced by the subject DP semantics being the second argument of $ITER_N$:

(28) $\lambda t\ \exists x\ (\mathbf{water}'(x)\ \wedge$
$ITER_N(\lambda z\ \lambda t'\ \mathbf{change-location}'(z)(t'),x)(t))$

Applying definition (27) to (28) yields the decomposition (29):

(29) $\lambda t\ \exists x\ (\mathbf{water}'(x)\ \wedge\ \exists Y\ (Y \in \mathbf{P}(x) \wedge$
$\forall y\ (y \in Y\ \rightarrow\ \exists! t'\ (t' \sqsubseteq t\ \wedge\ \mathbf{change-location}'(y)(t'))) \wedge$
$ITER_S(\lambda t''\ \exists y\ (y \in Y\ \wedge\ \mathbf{change-location}'(y)(t'')))(t)))$

If $ITER_N$ can model the aspectual influence of the DP *water* in *water leak- out*, the (un-)boundedness of the aspectually relevant subject DP's semantics must carry over to the whole predicate *water leak- out*, that is, (29) must be unbounded. Indeed, (29) is unbounded, it is even cumulative:[7] Take two adjacent times t and t' in the extension of (29) (with two non-overlapping involved water entities). Then, the same predicate (29) holds for $t \sqcup t'$, too, because

- due to the cumulativity of *water*, the two water objects involved in t and t' add up to a new object that is once again in the denotation of *water*.
- according to (25), the new water object already has a partition, viz., the union $X \cup X'$ of the partitions X and X' of the two water objects involved in t and t'
- as the predicate 'an iteration takes place in which an element of a certain partition changes its place' is in itself cumulative and holds for both t and t', $t \sqcup t'$ falls likewise under this predicate
- as each element of the partitions X and X' undergoes P exactly once during t and t', respectively, and t and t' do not overlap, each element of the new partition $X \cup X'$ undergoes P exactly once during $t \sqcup t'$.

For the sentence radical *50 litres of water leak- out*, the decomposition looks similar. Following [Krifka 1992], expressions like *litre* are interpreted as measure functions that map individuals on the cardinal that is their volume in litres.

(30) $\lambda t\ \exists x\ (\mathbf{water}'(x)\ \wedge\ \mathbf{litre}'(x) = 50\ \wedge\ \exists Y\ (Y \in \mathbf{P}(x) \wedge$
$\forall y\ (y \in Y\ \rightarrow\ \exists! t'\ (t' \sqsubseteq t\ \wedge\ \mathbf{change-location}'(y)(t'))) \wedge$
$ITER_S(\lambda t''\ \exists y\ (y \in Y\ \wedge\ \mathbf{change-location}'(y)(t'')))(t)))$

Intuitively, *50 litres of water leak- out* is a bounded predicate because the semantics of the subject DP is bounded. This intuition is borne out by its semantic representation (30).

Proof: Assume to the contrary two times t and t' with $t' \sqsubset t$ in the domain of (30). Then, every member of a partition Y' of a fifty-litre water object x' changes its location exactly once during t'. Similarly, every member of a partition Y of a fifty-litre water object x changes its location exactly once during t. As $t' \sqsubset t$, there are parts of t not in t', i.e., the complement $t \backslash t'$.[8] During $t \backslash t'$, however, the iteration 'an element of a partition changes its place' must hold, too (due to the assumption that t is in the extension of (30) and the divisivity of iterative predicates). But objects that change their location during $t \setminus t'$ cannot belong to Y', the fifty-litre water partition whose elements change their location during t', otherwise the condition that every element of the involved partition changes its location exactly once during t would be violated. Hence, the partition Y whose elements change their position exactly once during t has to comprise additional objects (in addition to Y'). Following (26), the fusion of this partition is an object x that has x' (the water object with the partition Y', all of whose elements change their location during t') as a proper part. Due to the boundedness of the semantics of *fifty litres of water*, however, this object x does not belong to the extension of *fifty litres of water*. This contradicts the assumption that t belongs to the extension of (30), hence, (30) is bounded.

As the discussion of (29) and (30) has shown, $ITER_N$ as part of certain verb decompositions captures the observation that these verb allow the (un-)boundedness of an aspectually relevant argument (for *to leak out*, the subject) to carry over to the semantics of the whole predicate.

In sum, in this section I have introduced the operator $ITER_N$ that models aspectual influence of generalized quantifiers as verbal arguments. The operator singles out the aspectually relevant argument

7. All cumulative predicates are unbounded, but not vice versa. For instance, *Max drink- less than two bottles of beer* is unbounded, yet not cumulative.

8. Formally, the complement can be defined as follows:
(i) $\forall x\ \forall y\ (x \sqsubset y\ \rightarrow\ \forall z\ (z = y \setminus x\ \leftrightarrow\ \neg z \circ x\ \wedge\ x \sqcup z = y))$

position and involves a partition of the object introduced in the semantics of the aspectually relevant DP and an iteration as defined in section 3. It has been shown that the aspectual class of verb decompositions that include $ITER_N$ depends on the (un-)boundedness of the aspectually relevant DP argument, hence, this operator is an adequate means of representing aspectual influence of DP arguments.

5 Solutions for aspectual puzzles

On the basis of this approach the coverage of aspectual theory can be extended. In this section, two hitherto problematic cases will be presented, whose explanation follows directly in the proposed approach, viz., sentence radicals like (31) that seem to be both bounded and unbounded and predicates with the quantifier *all the*, that have not been treated up to now.

(31) Max see- twenty zebras

5.1 Predicates that seem to be bounded and unbounded

The problem sentence radicals like (31) pose for aspectual semantics is the fact that they are compatible both with durative adverbials and time frame adverbials like *in two hours*. But, according to standard assumptions in aspectual semantics, the former property is a sign for unboundedness of verbal projections, while the latter signals their boundedness.

The solution for this problem rests on the assumption that aspectual influence of DP arguments is not always obligatory. There are cases of optional aspectual influence in which e.g. even the object DP of a verb like *to see* may exert aspectual influence.

Formally, this is represented in terms of an adequate operator. I assume $ITER_N$ to occur not only in lexical entries for verbs that allow aspectual influence of one of their DP arguments (as in *to eat*, *to sing*, *to paint* and numerous other examples). $ITER_N$ also constitutes the core of a whole family of operators that model optional aspectual influence of DP arguments. E.g., optional aspectual influence of the object in (31) is represented with this member of the family:

(32) $\lambda P\ \lambda x_1\ \lambda x_2\ \lambda t\ ITER_N(\lambda z\ \lambda t'\ P(x_2, z)(t'), x_1)(t)$

This operator maps a three-place relation P (between two individuals x_1 and x_2 and a time t) onto a relation with the same arity that involves $ITER_N$. The first argument of $ITER_N$ is a property that is derived from P by replacing its second argument position by x_2. The

second argument of $ITER_N$ (i.e, the aspectually relevant argument is x_1.

Applying this operator to the decomposition of *to see* yields the semantic representation of a (possibly contextually triggered) reading of *to see* in which the object argument is aspectually relevant:

(33) $\lambda x_1\ \lambda x_2\ \lambda t\ ITER_N(\lambda z\ \lambda t'\ \mathbf{see}'(x_2, z)(t'), x_1)(t)$

Following the above explanation of $ITER_N$, the object position 'x_2' is singled out as the one that exerts aspectual influence. In prose: (33) relates two objects x_2 and x_1, and a time if and only if during the time parts of x_2 are seen in succession by x_1 and these parts add up to x_2.

Then, (31) emerges as (34). **NU** maps properties of individuals on measure functions that map individuals on their number. The property determines the criterion for counting, so to speak, e.g., for living things, every single organism would be counted [Krifka 1992]:

(34) $\lambda t\ \exists x\ (\mathbf{zebra}'(x)\ \wedge\ \mathbf{NU}(\mathbf{zebra}')(x) =$
$20\ \wedge\ ITER_N(\lambda z\ \lambda t'\ \mathbf{see}'(\mathbf{m}, z)(t'), x)(t)$

This decomposition denotes the set of all times t such that during t Max successively sees groups of zebras, on the condition that these groups add up to 20 zebras.

The additional decomposition (33) for *to see* explains the puzzle of why expressions like (31) are compatible with both durative adverbials and time frame adverbials. If the lexical reading of *to see* is assumed ($\lambda y\ \lambda x\ \lambda t\ \mathbf{see}'(x, y)(t)$), no aspectual influence of arguments takes place, hence, predicates like (31) have an unbounded reading that is compatible with durative adverbials. On the other hand, if the reading (33) is chosen, the object DP exerts aspectual influence. Then, an object DP whose semantics is bounded (like *twenty zebras*) makes the whole predicate bounded, which in turn is compatible with time frame adverbials.

As argument positions with different depths of embedding may exert aspectual influence, a whole family of operators like (32) is necessary. If we assume functional composition[9], we need one operator for each depth of embedding. The general scheme for these operators is given in (35):

(35) $\lambda P\ \lambda x_1\ ...\ \lambda x_n\ \lambda t\ ITER_N(\lambda z\ \lambda t'\ P(x_n, ..., x_2, z)(t'), x_1)(t)$

Each member of this family of operators maps a n-ary relation P onto another relation of the same arity. This second relation involves the operator $ITER_N$, which selects the argument position with the

9. Functional composition combines a pair of expressions of type $< \alpha, \beta >$ and $< \beta, \gamma >$, respectively, into a new expression of type $< \alpha, \gamma >$

highest depth of embedding, viz., x_1, as its second argument, i.e., the one that exerts aspectual influence. The first argument of $ITER_N$ is a property derived from P by filling in all its individual arguments with the exception of x_1, which has the least embedding.

By the way, the optional unbounded reading of the well-known example (36) of [Dowty 1979] can be explained in terms of an optional application of the member of (35) where $n = 1$.

(36) Tourists detect- the quaint little village

In this section I have shown how the proposed approach can explain why there are predicates that seem to be both bounded and unbounded, because they are compatible with both durative adverbials and time frame adverbials. This phenomenon does not contradict standard assumptions about the aspectual selection restrictions of these adverbials, it is put down to optional aspectual influence of DP arguments. Formally, this is represented as an optional application of an operator (out of a whole family of operators) that makes an argument position of the involved verb exert aspectual influence. In other words, the lexicon allows but does not force the (un-)boundedness of an argument DP to carry over to the whole sentence radical in the case of verbs like *to see*.[10]

5.2 Aspectual influence of the quantifier *all the*

Another problem that can be accounted for in terms of the proposed analysis is the aspectual influence of DPs with the quantifier *all the*: These DPs may also cause the bounding of the verbal projection they appear in (e.g., in (37)).

(37) Max eat- all the apples

I assume Link's ([Link 1991]) interpretation of DPs like *all the apples* in terms of the property of being the smallest sum of all elements

10.The remarkable ease of understanding sentences like (31) in the sense in which the object exerts aspectual influence begs the question of whether this application of $ITER_N$ has to be interpreted as a (possibly quite conventionalized) reinterpretation of *to see*, as I have assumed here. The other option would be to argue that the lexical entry of *to see* is underspecified between a reading that allows and one that bars aspectual influence of the object argument. However, even if my assumption that the only reading of *to see* in the lexicon is the one without aspectual influence of DP arguments turned out to be false, the members of (35) would still be of relevance. They would have a different status, though: instead of being optionally applied operators, they would be building blocks of lexical semantics in terms of whose presence or absence the difference between the readings of many verbs could be described.

of the extension of the involved nominal.[11]

[Link 1991] assumes mereological join-semilattice structures to describe the denotations of nouns. For count nouns the plural operator '*' is defined as an operator that maps properties P on the complete join-semilattice whose atomic members are the members of the denotation of P. E.g., if there are three apples a, b, and c in the domain, **apple**$'$ denotes the set $\{a, b, c\}$, whereas ***apple**$'$ denotes the set $\{a, b, c, a \sqcup b, a \sqcup c, b \sqcup c, a \sqcup b \sqcup c\}$.

The denotation of *all the* in [Link 1991] is an operator that maps predicates on their supremum:

(38) $\lambda P\ \sigma{*}x(P(x))$

The sigma operator without the asterisk is defined as the supremum of the lattice induced by the plural operator '*' on P (the sigma operator with the star merely carries the additional presupposition that there is more than one element in the extension of P):

(39) $\sigma{*}x(P(x)) = \iota x\ (*P(x)\ \wedge\ \forall y\ (*P(y)\ \rightarrow\ y \sqsubseteq x))$

In the apple example given above, $\sigma{*}x(\mathbf{apple}'(x))$ would be $\{a \sqcup b \sqcup c\}$.

According to the proposed approach, the predicate (37) is assigned the following semantic representation:

(40) $\lambda t\ ITER_N(\lambda z\ \lambda t'\ \mathbf{make-vanish}'(\mathbf{m}, z)(t'), \sigma{*}x(\mathbf{apple}'(x)))(t)$

Following the reasoning employed to show the unboundedness and boundedness of (29) and (30) respectively, it can be shown that (40) is bounded.

Proof: Assume to the contrary that (40) contains t and t' with $t \sqsubset t'$. This entails that both t and t' belong to the extension of the iteration of 'Max eats an element of the apple supremum'. This predicate is divisive, hence, it holds for $t' \setminus t$, too. However, the apple supremum has already been eaten during t (assumption), hence, if some parts of the apple supremum are eaten during $t' \setminus t$, this entails that these parts are eaten twice during t' (no apple is available that does not belong to the apple supremum). This would contradict the assumption that t' belongs to the extension of (40), which entails that every apple supremum part is eaten exactly once during t'.

In sum, on the basis of the quantifier representations of [Link 1991], the proposed approach explains directly why DPs with the quantifier *all the* make their predicates bounded if they exert aspectual influence.

11.Universal quantifiers, which often appear in the semantics of these expressions (but not in expressions like *All the students gathered*), are attributed to distributivity of the verb.

6 Conclusion

In this paper, I have presented a representation of the aspectual influence of generalized quantifiers on the verbal projection they appear in, which is based on an appropriate notion of iteration. The domain of the semantics of iteration proper was delimited against the effects of interacting pragmatic principles. This approach reconciliates and unifies two seemingly diverging trends of aspectual semantics, viz., the trend to give aspectual classifications for all verbs in isolation (or with singular count noun DP arguments only) and the trend to refute the classifiability of certain verbs in isolation because they allow aspectual influence of certain DP arguments. This reconciliation then forms the basis for an extension of the coverage of aspectual theory. Without having to take recourse to additional machinery, the presented approach allows an explanation of the problem that some predicates are compatible with both durative and time frame adverbials (although these compatibilities signal conflicting aspectual classification). In addition, the aspectual influence of DPs with the quantifier *all the* follows directly from the presented approach.

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An Undecidability Result for Polymorphic Lambek Calculus

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1 Introduction

A precursor to linear logic was introduced in [Lambek 1958], who proposed a single conclusion sequent calculus which lacked the structural rules of *Contraction*, *Weakening* and *Permutation*. Three connectives are treated, two *directed* implications / (right to left),\ (left to right), and a product, which can be identified with linear logic's multiplicative conjunction. This calculus was introduced as a means for reasoning about sequences of linguistic objects, a domain in which the customary structural rules indeed are not valid. On grounds of linguistic expressivity, there are motivations for considering the calculus which results from the addition of second order quantifiers – the *Polymorphic* Lambek Calculus (PLC). For example, where [Pentus 1992] shows that grammars based on the Lambek calculus can recognise exactly the context-free languages, [Emms 1993a] shows that grammars based on PLC can recognise a strictly larger class of languages. By now a certain amount is known about PLC. Besides the above mentioned recognising power result, there is in [Emms 1993c], [Emms 1994b] discussion of several kinds of linguistic applications, in [Emms and Leiß 1993] a Cut elimination result, in [Emms 1994a] several completeness results, and in [Emms 1993b] a decidability result for a subset of all possible sequents.

The theme of this paper is embeddings of neighbouring logics into PLC. We describe a result[1] to the effect that PLC can embed the extension of PLC with the structural rules of *Permutation*, *Contraction* and *Weakening*. This enables us to settle, in the negative, the open question of the decidability of PLC. We will also show that the connectives (/,\,$\forall$) suffice to express a variety of other connectives, amongst which are the *product*, $\otimes$, the existential quantifier, and the *multiplicative unit*, $\mathbf{1}$. We conclude with some speculations as to the linguistic significance of these results.

2 Preliminaries

Assume a denumerably infinite set of variables. We specify various propositional languages by $\mathcal{L}$, followed by a series of connectives drawn from the set $\{\otimes, \mathbf{1}, \top, \mathbf{0}, /, \backslash, \forall, \exists\}$. The calculi of concern will be defined with reference to Figures 1 and 2. Thus we are concerned with calculi which may include 2nd order quantification, the binary multiplicatives $\otimes$, /,\, the multiplicative neutral $\mathbf{1}$, and the additive neutrals $\mathbf{0}$ and $\top$.

The rules derive intuitionistic sequents from intuitionistic sequents, where an intuitionistic sequent is an antecedent sequence of formulae, followed by ' $\Rightarrow$ ', and then a single formula. w, x, y range over formulae, U, V, T range over sequences of formulae. $x[y/Z]$ stands for the substitution of y for Z in x, defined to include a change of bound variable to avoid accidental capture. The rules ($\forall$R) and ($\exists$L) are subject to the side condition that Z is not free below the line, and X is not free in $QZ.x$, this latter part allowing $QX.x[X/Z]$ to be an alphabetic variant of $QZ.x$.

We will use $\mathrm{L}^{\langle c_i \rangle_{1 \leq i \leq n}}$ for the calculus obtained by taking the identity axiom scheme, together with the rules associated with each of the connectives c_i, with in the case

1. First reported to the Linear Logic mailing list in April 1995

$$x \Rightarrow x$$

$$\frac{U, y, V \Rightarrow w \quad T \Rightarrow x}{U, y/x, T, V \Rightarrow w} / \mathrm{L} \qquad \frac{T, x \Rightarrow y}{T \Rightarrow y/x} / \mathrm{R}$$

$$\frac{T \Rightarrow x \quad U, y, V \Rightarrow w}{U, T, x\backslash y, V \Rightarrow w} \backslash \mathrm{L} \qquad \frac{x, T \Rightarrow y}{T \Rightarrow x\backslash y} \backslash \mathrm{R}$$

$$\frac{U, x, y, V \Rightarrow w}{U, x \otimes y, V \Rightarrow w} \otimes \mathrm{L} \qquad \frac{T_1 \Rightarrow x \quad T_2 \Rightarrow y}{T_1, T_2 \Rightarrow x \otimes y} \otimes \mathrm{R}$$

$$\frac{U, x[y/Z], V \Rightarrow w}{U, \forall Z.x, V \Rightarrow w} \forall \mathrm{L} \qquad \frac{T \Rightarrow x}{T \Rightarrow \forall X.x[X/Z]} \forall \mathrm{R},\ Z!$$

$$\frac{U, x, V \Rightarrow w}{U, \exists X.x[X/Z], V \Rightarrow w} \exists \mathrm{L},\ Z! \qquad \frac{T \Rightarrow x[y/X]}{T \Rightarrow \exists X.x} \exists \mathrm{R}$$

$$\frac{U, V \Rightarrow w}{U, \mathbf{1}, V \Rightarrow w} \mathbf{1}\ \mathrm{L} \qquad \Rightarrow \mathbf{1}$$

$$U, \mathbf{0}, V \Rightarrow w$$

$$T \Rightarrow \top$$

Figure 1: Identity axiom, and rules for $/, \backslash, \otimes, \forall, \exists, \mathbf{1}, \mathbf{0}, \top$

$$\frac{T_1, T_2 \Rightarrow w}{T_1, x, T_2 \Rightarrow w}\mathrm{Weak} \qquad \frac{T_1, x, x, T_2 \Rightarrow w}{T_1, x, T_2 \Rightarrow w}\mathrm{Contr} \qquad \frac{\pi(T) \Rightarrow w}{T \Rightarrow w}\mathrm{Perm}$$

Figure 2: Structural Rules

of $\forall$L and $\exists$R, the obvious proviso that formula y substituted for the bound variable be drawn from the appropriate language: $\mathcal{L}(\langle c_i \rangle_{1 \leq i \leq n})$. A general parameter that may be varied is whether sequents with an empty antecedent sequence are admitted. With a subscript ϵ, we indicate a calculus permitting this (clearly the axiom scheme $\Rightarrow \mathbf{1}$ is omitted when there is no ϵ subscript). The calculi including the second-order quantifiers we will refer to as *polymorphic*. In this notational scheme, $\mathrm{L}^{/,\backslash,\otimes}$ is the calculus proposed in [Lambek 1958].

For any of the calculi that may be defined in this way, the derivability of the following Cut rule can be shown (by an inductive argument on the sizes of the premise proofs of a Cut[2])

$$\frac{U,\ x,\ V\ \Rightarrow w \quad T\ \Rightarrow x}{U,\ T,\ V\ \Rightarrow w}\mathrm{Cut}$$

By adjoining various structural rules to these calculi we obtain versions of familiar calculi. Let $LJ2$ be $\mathrm{L}_{\epsilon}^{\mathbf{1},/,\otimes,\forall} + Weak + Contr + Perm$. This is a formulation of *2nd order intuitionistic propositional logic*. It is also the case that the Cut rule is derivable in $LJ2$, as is shown in [W.W.Tait 1966] (using a semantic argument). The undecidability of $LJ2$, which will be used below, is shown in [Löb 1976],[Gabbay 1981]. Let IMLL2 be $\mathrm{L}_{\epsilon}^{\mathbf{1},/,\otimes,\forall} + Perm$. This is a formulation of second-order intuitionistic multiplicative linear logic, referred to as IMLL2 in [Lincoln et al. 1995]. We will also refer to $\mathrm{L}_{\epsilon}^{\mathbf{1},/,\backslash,\otimes,\forall}$ as N-IMLL2, and $\mathrm{L}^{/,\backslash,\otimes,\forall}$ as LC2.

A simple property of any of the $\mathrm{L}^{/,\backslash,\cdots}$ or $\mathrm{L}_{e}^{/,\backslash,\cdots}$ calculi is[3]:

Lemma 1 (Unknown Elimination) *where* L *is one of the* $\mathrm{L}^{/,\backslash,\cdots}$ *or* $\mathrm{L}_{e}^{/,\backslash,\cdots}$ *calculi then*

$$\left.\begin{array}{l} \text{there is an } \overline{X} \text{ with } FV(\overline{X}) \subseteq \\ FV(U,V,w) \text{ such that} \\ \mathrm{L} \vdash U,\ \overline{X},\ V\ \Rightarrow w \\ \mathrm{L} \vdash T_1\ \Rightarrow \overline{X} \\ \quad \vdots \\ \mathrm{L} \vdash T_n\ \Rightarrow \overline{X} \end{array}\right\} \textit{iff} \begin{cases} \mathrm{L} \vdash U,\ T_1,\ V\ \Rightarrow w \\ \quad \vdots \\ \mathrm{L} \vdash U,\ T_n,\ V\ \Rightarrow w \end{cases}$$

Proof: for one direction use Cut and Cut Elimination, for other choose $\overline{X} = U\backslash w/V$ (shorthand for $u_n\backslash \ldots u_1\backslash w/v_n/\ldots/v_1$)

When we use this to infer the derivability of two premises $U, \overline{X}, V \Rightarrow w$ and $T \Rightarrow \overline{X}$ from $U, T, V \Rightarrow w$ we write:

$$\overbrace{\frac{U, T, V \Rightarrow w}{U, \overline{X}, V \Rightarrow w \quad T \Rightarrow \overline{X}}}$$

3 Polymorphic Reconstruction of Structural Rules

In this section we will be concerned with embeddings of logics with structural rules into logics without, using polymorphism to reconstruct the lost structural rules. We

2. Lambek's proof of Cut elimination for $\mathrm{L}^{/,\otimes,\backslash}$ by induction on the complexity of the Cut formula, does not work for the polymorphic calculi. The absence of the contraction rule, however, allows a similarly simple proof to be given by induction on proof size. See [Emms and Leiß 1993].

3. This is not the same as Interpolation, since we do not require the material of $\overline{X}$ to occur in both T and (U, V, w).

begin by recalling the well known fact that the absence of structural rules can be compensated for by adding new axiom schemes and Cut.

Example where L is $\mathrm{L}^{\mathbf{1},/,\otimes,\backslash,\cdots}$ or $\mathrm{L}_\epsilon^{\mathbf{1},/,\otimes,\backslash,\cdots}$,

$$\text{L}{+}\mathit{Perm} \qquad \text{is equivalent to L}{+}\mathit{Cut}+\left\{\begin{array}{l} x \Rightarrow (y\otimes x)/y, \text{ or} \\ x \Rightarrow y/(y/x), \text{ or} \\ \ldots \end{array}\right\}$$

Example where L is $\mathrm{L}^{\mathbf{1},/,\otimes,\backslash,\cdots}$ or $\mathrm{L}_\epsilon^{\mathbf{1},/,\otimes,\backslash,\cdots}$,

$$\text{L}{+}\mathit{Weak} \qquad \text{is equivalent to L}{+}\mathit{Cut}+\left\{\begin{array}{l} x \Rightarrow x/y, x \Rightarrow y\backslash x, \text{ or} \\ x \Rightarrow \mathbf{1}, \text{ or} \\ \ldots \end{array}\right\}$$

Example where L is $\mathrm{L}^{\mathbf{1},/,\otimes,\backslash,\cdots}$ or $\mathrm{L}_\epsilon^{\mathbf{1},/,\otimes,\backslash,\cdots}$,

$$\text{L}{+}\mathit{Contr} \qquad \text{is equivalent to L}{+}\mathit{Cut}+\left\{\begin{array}{l} x \Rightarrow y/(x\backslash x\backslash y), x \Rightarrow (y/x/x)\backslash x, \\ \text{or} \\ x \Rightarrow x\otimes x, \text{ or} \\ \ldots \end{array}\right\}$$

We note here the proof in the case of permutation, choosing the first formulation of the 'permutation' axiom. From right to left it suffices to note that the 'permutation' axiom is derivable in L + *Perm*. From left to right, it clearly suffices to show that $U, a, V \Rightarrow w$ implies $a, U, V \Rightarrow w$, and this we have, given Lemma 1, by the derivation:

$$\dfrac{a \Rightarrow (\overbrace{\overline{X}\otimes a}^{U,a,V \Rightarrow w})/\overline{X} \qquad \dfrac{\overline{X}, a, V \Rightarrow w \quad U \Rightarrow \overline{X}}{(\overline{X}\otimes a)/\overline{X}, U, V \Rightarrow w}\,/\mathrm{L},\otimes\mathrm{L}}{a, U, V \Rightarrow w}\,\mathrm{Cut}$$

3.1 Embedding Order-1 *LP* in $\mathrm{L}_\epsilon^{(/,\backslash,\forall)}$

In [Emms 1993a], in proving that there are 2nd order Lambek grammars for the permutation closure of any CF language, it was shown how one can in a certain sense, trade an axiom scheme expressing permutation for a polymorphic formula. We begin by essentially reformulating this result as an embedding. With reference to $\mathcal{L}(/,\backslash)$, we say a sequent $U \Rightarrow w$ is *order 1*, if w is atomic and no formula in U has complex arguments. We can simulate the possible applications of the 'permutation' axiom, $x \Rightarrow y/(y/x)$, by replacing x with $\forall X.X/(X/x)$. This leads to the following:

Theorem 1 (Emms,1993) *Where U, x is any sequence of formulae from $\mathcal{L}(/,\backslash)$,*

$$\begin{array}{rrl} & LP^1 & \vdash \; U \Rightarrow w \\ \textit{iff} & \mathrm{L}_\epsilon^{(/,\backslash,\forall)} & \vdash \; U^* \Rightarrow w \end{array}$$

where $x^* = \forall X.X/(X/x)$, $LP^1 = (\mathrm{L}_\epsilon^{/,\backslash} + \mathit{Perm})\lceil(\mathit{order}\ 1)$

Proof of Theorem 1 (sketch)

Left to Right

First we note a 'topicalisation' property for x^* categories:

$$\begin{array}{rrll} & \mathrm{L}_\epsilon^{(/,\backslash,\forall)} & \vdash & U, x_i, V \Rightarrow w \quad : \\ \Rightarrow & \mathrm{L}_\epsilon^{(/,\backslash,\forall)} & \vdash & x_i^*, U, V \Rightarrow w \end{array} \qquad \dfrac{\dfrac{\overline{X}, V \Rightarrow w \quad \dfrac{\overbrace{U, x_i, V \Rightarrow w}^{U, x_i \Rightarrow \overline{X}}}{U \Rightarrow \overline{X}/x_i}\,/\mathrm{R}}{\overline{X}/(\overline{X}/x_i), U, V \Rightarrow w}\,/\mathrm{L}}{\forall X.(X/(X/x_i)), U, V \Rightarrow w}\,\forall\mathrm{L}$$

Then we argue:

$$\begin{array}{rrll} & LP^1 & \vdash & x_1, \ldots, x_n \Rightarrow w \\ \Rightarrow & \mathrm{L}_\epsilon^{/,\backslash} & \vdash & \pi(x_1, \ldots, x_n) \Rightarrow w \quad \text{for some } \pi\text{, because } \textit{order 1} \\ \Rightarrow & \mathrm{L}_\epsilon^{(/,\backslash,\forall)} & \vdash & \pi(x_1, \ldots, x_n) \Rightarrow w \\ \Rightarrow & \mathrm{L}_\epsilon^{(/,\backslash,\forall)} & \vdash & x_1^*, \ldots, x_n^* \Rightarrow w \qquad \text{by 'topicalisation'} \end{array}$$

The *order 1* restriction is used in the first step, allowing one to assume that the LP^1 proofs consist of an initial permutation step, followed by purely $\mathrm{L}_\epsilon^{/,\backslash}$ steps.

Right to Left

$$\begin{array}{rrll} & \mathrm{L}_\epsilon^{(/,\backslash,\forall)} & \vdash & x_1^*, \ldots, x_n^* \Rightarrow w \\ \Rightarrow & \mathrm{L}_\epsilon^{(/,\backslash,\forall)} + (\text{Cut,Perm}) & \vdash & x_1, \ldots, x_n \Rightarrow w \quad \text{since } \mathrm{L}_\epsilon^{(/,\backslash,\forall)} + \text{Perm} \vdash x_i \Rightarrow x_i^* \\ \Rightarrow & \mathrm{L}_\epsilon^{(/,\backslash,\forall)} + \text{Perm} & \vdash & x_1, \ldots, x_n \Rightarrow w \qquad \text{by Cut elimination} \\ \Rightarrow & \mathrm{L}_\epsilon^{/,\backslash} + \text{Perm} & \vdash & x_1, \ldots, x_n \Rightarrow w \qquad \text{since the } x_i \text{ are } \forall \text{ free} \end{array}$$

End of Proof

What is noticeable is that the use of permutation in LP^1 is recovered within $\mathrm{L}_\epsilon^{(/,\backslash,\forall)}$, where $\mathrm{L}_\epsilon^{(/,\backslash,\forall)}$ itself lacks the permutation structural rule. The access to permutation is encoded by the $()^*$ mapping which replaces x with (the closure of) the result of applying a permutation axiom scheme (namely $x \Rightarrow y/(y/x)$) to it.

This is a first instance of the use of polymorphism to recover a structural rule. Given the fact that each structural rule matches up with an axiom scheme, there is reason to think that there may be other ways to embed structurally more relaxed logics into the polymorphic calculus.

In [Lincoln et al. 1995] there is a proof of such an embedding. The target logic is IMLL2 (= $\mathrm{L}_\epsilon^{\mathbf{1},/,\otimes,\forall}$ + *Contr* + *Weak* + *Perm*), and the source is LJ2 (= $\mathrm{L}_\epsilon^{\mathbf{1},/,\otimes,\forall}$ + *Perm*), and polymorphism is used to recover the lost *Contr* and *Weak* rules. In the next section, we carry this strategy one step further, embedding *LJ*2 not into IMLL2, which has *Permutation*, but N-IMLL2 (= $\mathrm{L}_\epsilon^{\mathbf{1},/,\otimes,\backslash,\forall}$), which has no *Permutation*.

This will then leave us with a picture of how, to date, polymorphism allows embeddings of structurally richer into structurally weaker logics, as follows:

- $(\mathrm{L}_\epsilon^{/,\backslash} + \text{Perm})\lceil_{\text{order 1}} \quad \rightsquigarrow \quad \mathrm{L}_\epsilon^{(/,\backslash,\forall)}$

  ... *thereby* $\mathrm{L}_\epsilon^{(/,\backslash,\forall)}$ *can* recognise the Perm closore of any CF language

- $\mathrm{L}_\epsilon^{(/,\backslash,\forall)} + \text{Perm} + \text{Contr} + \text{Weak} \quad \rightsquigarrow \quad \mathrm{L}_\epsilon^{(/,\backslash,\forall)} + \text{Perm}$

  ... *thereby* $(\mathrm{L}_\epsilon^{(/,\backslash,\forall)} + \textit{Perm})$ undecidable

- $\mathrm{L}_\epsilon^{(/,\backslash,\forall)} + \text{Perm} + \text{Contr} + \text{Weak} \quad \rightsquigarrow \quad \mathrm{L}_\epsilon^{(/,\backslash,\forall)}$

  ... *thereby* $\mathrm{L}_\epsilon^{(/,\backslash,\forall)}$ undecidable

3.2 Embedding LJ2 in N-IMLL2

In [Lincoln et al. 1995] there is a proof of an embedding of LJ2 into the structurally stricter IMLL2. They note that certain quantified formulae seem to describe the action of the Contraction and Weakening rules, namely

$$\left.\begin{array}{rcl} C & := & \forall X.(X \otimes X)/X \\ W & := & \forall X.\mathbf{1}/X \text{ as } W \end{array}\right\} \text{cf. axioms for 'contraction', and 'weakening'}$$

C and W license *single* steps of Contraction and Weakening in the sense,

$$\begin{array}{rl} & \mathrm{L} \vdash \ U,\ a,\ a,\ V \Rightarrow w \\ \Rightarrow & \mathrm{L} \vdash \ U,\ C,\ a,\ V \Rightarrow w \end{array} \qquad \begin{array}{rl} & \mathrm{L} \vdash \ U,\ V \Rightarrow w \\ \Rightarrow & \mathrm{L} \vdash \ U,\ W,\ a,\ V \Rightarrow w \end{array}$$

$$\dfrac{\dfrac{\dfrac{U,\ a,\ a,\ V \Rightarrow w}{U,\ a{\otimes}a,\ V \Rightarrow w}\otimes L \qquad a \Rightarrow a}{U,(a{\otimes}a)/a,\ a,\ V \Rightarrow w}/\mathrm{L}}{\underbrace{U,\ \forall X.X{\otimes}X/X}_{C},\ a,\ V \Rightarrow w}\forall \mathrm{L} \qquad \dfrac{\dfrac{\dfrac{U,\ V \Rightarrow w}{U,\ \mathbf{1},\ V \Rightarrow w}\mathbf{1}L \qquad a \Rightarrow a}{U,\ \mathbf{1}/a,\ a,\ V \Rightarrow w}/\mathrm{L}}{\underbrace{U,\ \forall X.\mathbf{1}/X}_{W},\ a,\ V \Rightarrow w}\forall \mathrm{L}$$

Expressed as it is by a formula of a *linear* calculus, IMLL2, such a license to apply a structural rule is gone as soon as it has been used — in contrast to formulae marked with linear logic's exponentials, for which the repeated application of structural rules is licensed. However, the surprising fact pointed out in [Lincoln et al. 1995] is:

> *that when C, C, C, W is prefixed to a sequence T, one has license to apply Contraction and Weakening to T* indefinitely often.

Illustration for Contraction case:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{C,C,C,W,U,a,a,V \Rightarrow x}{C,C,C,W,U,a{\otimes}a,V \Rightarrow x}\otimes \mathrm{L} \qquad a \Rightarrow a}{C,C,C,W,U,C,a,V \Rightarrow x}\forall \mathrm{L},/\mathrm{L}}{C,C,C,C,W,U,a,V \Rightarrow x}\text{Perm} \qquad C \Rightarrow C \quad C \Rightarrow C}{(C{\otimes}C){\otimes}(C{\otimes}C),W,U,a,V \Rightarrow x}\otimes \mathrm{L},\otimes \mathrm{L},\otimes \mathrm{L} \qquad C,C \Rightarrow C{\otimes}C}{C,C,C,W,U,a,V \Rightarrow x}\otimes \mathrm{R}\forall \mathrm{L},/\mathrm{L}$$

This is the key to their embedding:

Theorem 2 (Lincoln et al., 1995) *Where U, x is any sequence of formulae from $\mathcal{L}(\mathbf{1},/,\otimes,\forall)$,*

$$\begin{array}{rll} & \mathrm{L}_\epsilon^{\mathbf{1},/,\otimes,\forall} + \text{Contr} + \text{Weak} + \text{Perm} & \vdash \ U \Rightarrow w \\ \textit{iff} & \mathrm{L}_\epsilon^{\mathbf{1},/,\otimes,\forall} + \text{Perm} & \vdash \ C,C,C,W,U \Rightarrow w \end{array}$$

We will prove an embedding of LJ2 in N-IMLL2, the version without with *Permutation*. In addition to C and W we need the following formula for encoding permutation:

$$P \quad := \quad \forall X \forall Y.(X \otimes Y)/X/Y \quad \} \text{cf. axiom for 'permutation'}$$

Then we observe P licenses *left-ward shift*, in the sense that where L is any of the $\mathrm{L}_\epsilon^{1,/,\otimes,\backslash,\forall,\cdots}$ calculi:

$$\begin{array}{rl} & \mathrm{L} \vdash \ U,\ a,\ V \Rightarrow w \\ \Rightarrow & \mathrm{L} \vdash \ P,\ a,\ U,\ V \Rightarrow w \end{array} \quad : \quad \dfrac{\dfrac{\dfrac{\dfrac{\overbrace{U,a,V \Rightarrow w}}{\overline{X},a,V \Rightarrow w}\otimes \mathrm{L}}{\overline{X}\otimes a,V \Rightarrow w} \qquad U \Rightarrow \overline{X}}{(\overline{X}\otimes a)/\overline{X},U,V \Rightarrow w}/\mathrm{L} \qquad a \Rightarrow a}{(\overline{X}\otimes a)/\overline{X}/a,a,U,V \Rightarrow w}/\mathrm{L}}{\underbrace{\forall X\forall Y.(X\otimes Y)/X/Y}_{P},a,U,V \Rightarrow w}\forall \mathrm{L}$$

Clearly, where T is a sequence of formulae of length n, any permutation, $\pi(T)$, can be generated from T by applying n leftward shifts:

Lemma 2 *where $P = \forall X.\forall Y.(X \otimes Y)/X/Y$, L any of the $\mathrm{L}_\epsilon^{1,/,\otimes,\backslash,\forall,\cdots}$ calculi, the following is a derivable rule of L*

$$\dfrac{x_1,\ldots,x_n \Rightarrow w}{\underbrace{P,\ldots,P}_{n \text{ times}},\pi(x_1,\ldots,x_n) \Rightarrow w}$$

A useful lemma concerning the structural rule encoding formulae, C, W and P is the following:

Lemma 3 *where L is any of the $\mathrm{L}_\epsilon^{1,/,\otimes,\backslash,\forall,\cdots}$ calculi, weakening in L is admissible for $x \in \{C, W, P\}$.*

Proof: by the following derivation schemes:

$$\dfrac{\dfrac{\dfrac{U,\ V \Rightarrow w}{U,\ \mathbf{1}\otimes\mathbf{1},\ V \Rightarrow w}\otimes L,\mathbf{1}L,\mathbf{1}L \qquad \Rightarrow \mathbf{1}}{U,(\mathbf{1}\otimes\mathbf{1})/\mathbf{1},V \Rightarrow w}/\mathrm{L}}{\underbrace{U,\ \forall X.X\otimes X/X}_{C},\ V \Rightarrow w}\forall \mathrm{L} \qquad \dfrac{\dfrac{\dfrac{U,\ V \Rightarrow w}{U,\ \mathbf{1},\ V \Rightarrow w}\mathbf{1}L \qquad \Rightarrow \mathbf{1}}{U,\mathbf{1}/\mathbf{1},V \Rightarrow w}/\mathrm{L}}{\underbrace{U,\ \forall X.\mathbf{1}/X}_{W},\ V \Rightarrow w}\forall \mathrm{L}$$

If we replace the permutation steps by left-ward shifts, the formula $a/b_2/b_1$ must be *twice* left-ward shifted. Correspondingly, we do not get N-IMLL2 derivability under the earlier defined: $(a/b_2/b_1)^*$, $(c/(a/b_1/b_2))^* \Rightarrow c$. Nor will we get N-IMLL2 derivability of $(a/b_2/b_1)^*$, $(c/(a/b_1^*/b_2^*))^* \Rightarrow c$. Rather we need: $((a/b_2/b_1)^*)^*$, $c/(a/b_1^*/b_2^*) \Rightarrow c$. It remains to be explored, however, whether variations on this strategy could yield further embeddings between logics which both lack contraction. A further line of investigation is a comparison with the work of [Kurtonina and Moortgat 1994], [Kurtonina and Moortgat 1995], who also consider embeddings between (implicational) logics with differing associated structural rules. Most particularly they also consider embeddings from weaker into stronger logics. This direction of embedding has not be considered at all in the 2nd order case.

How to react to this undecidability result

We consider now what the reaction to our undecidability result for $L_\epsilon^{\mathbf{1},/,\otimes,\backslash,\forall}$, and $L_\epsilon^{(/,\backslash,\forall)}$should be. The strongest possible reaction would be to conclude that these calculi should now be ruled out as playing a part in the linguist's toolkit for describing languages. In response to this, one should note that there are some notable precedents for not responding to an undecidability result in this way. [Lincoln et al. 1992] prove purely propositional linear logic (i.e quantifier-free) to be undecidable. [Schieber 1986] and [Johnson 1988], each with respect to their own notion of feature-structure based grammars, prove that the *universal recognition problem* is undecidable: there can be no algorithm which, given a feature-structure based grammar and a string, decides whether the string belongs to the language generated by the grammar. In neither of these cases has this undecidability stifled interest in the systems. It is also germane to note examples of decidable but provably intractable systems which are in wide spread use, such as the type-checking algorithm for the polymorphic functional programming language ML [Mairson 1990].

In these cases, the continuing interest in the systems involved reflects a switch of emphasis from the general situation to some restriction of it. Such a move seems the apt reaction to the undecidability for the 2nd order Lambek calculus, with an investigation of *decidable fragments* moving onto the agenda. [Emms 1993b] contains a preliminary investigation of this, in which a subset of all possible sequents is considered which have the property that, however a proof is unfolded, a set of subproblems is generated having the form of (i) in the Unknown Elimination lemma above. This forms the basis of a decision procedure for such sequents. Sequents involving the polymorphic categories that have been proposed to have linguistic applications (eg. $\forall X.X/(a\backslash X)$ (quantification), $\forall X.X\backslash X/X$ (coordination), $\forall X \forall Y.c/(X\backslash a)/(X/b)$ (extraction)) have this nature, and their derivability *can* be decided. It is interesting to note that of all the polymorphic formulae involved in the above embeddings, all but the formula expressing Contraction have this nature as well. Clearly the question of decidable fragments is one requiring further attention. The decidability of one oft-mooted candidate, namely a fragment allowing only outermost quantifiers, remains an open question.

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Appendix

The following sequent is derivable in $\mathrm{L}^{/,\otimes,\backslash,\forall}$, but not in $\mathrm{L}^{/,\backslash,\forall}$, where a, b, c, d are distinct atoms.

$$a/d/c,\ \forall X.X/(X\backslash b)/X,\ c,\ d,\ d\backslash c\backslash b\ \Rightarrow a$$

We must first show that the sequent will only have a proof if we can find an $\overline{X}$ that make the leaves of the following proof derivable:

$$\dfrac{\dfrac{a/d/c, \overline{X} \Rightarrow a \qquad \dfrac{\overline{X}, d\backslash c\backslash b \Rightarrow b}{d\backslash c\backslash b \Rightarrow \overline{X}\backslash b}\backslash\mathrm{R} \qquad c, d \Rightarrow \overline{X}}{a/d/c, \overline{X}/(\overline{X}/b)/\overline{X}, c, d, d\backslash c\backslash b \Rightarrow a}/\mathrm{L}/\mathrm{L}}{a/d/c, \forall X.X/(X/b)/X, c, d, d\backslash c\backslash b \Rightarrow a}\forall\mathrm{L}$$

One can quite quickly eliminate other possible proofs shapes by applying the following observations (whose proofs are here omitted). 1) Let the *spine* of a product free category be recursively defined $spine(x/y) = \langle /, spine(x)\rangle$, $spine(y\backslash x) = \langle \backslash, spine(x)\rangle$, $spine(\forall X.x) = \langle \forall, spine(x)\rangle$, with $spine(x) = \langle\rangle$ otherwise. One can easily show that when a series, ζ, of inferences belonging to { Slash L, Slash R, $\forall$L, $\forall$R }, separates a pair of inference steps # and \$ belonging to { Slash L, $\forall$L }, associated with two consecutive connectives on the spine of a category, then there is an alternative proof, ordering ζ before # and \$, and with # and \$ consecutive. 2) Slash (and $\forall$) Right inferences can be ordered before all others without loss of generality. 3) Quantifier-free sequents must satisfy *count-invariance* [van Benthem 1986].

The derivability of the sequent thus reduces to the problem of finding an $\overline{X}$ making the following derivable:

1. $a/d/c,\ \overline{X}\ \Rightarrow a$
2. $\overline{X},\ d\backslash c\backslash b\ \Rightarrow b$
3. $c,\ d\ \Rightarrow \overline{X}$

Clearly, in the case of $\mathrm{L}^{/,\otimes,\backslash,\forall}$, $c \otimes d$ is such a value for $\overline{X}$. We will show that there is no product-free value.

It is clear that no atomic $\overline{X}$ can be a solution. It also clear that the principal connective of $\overline{X}$ cannot be '/', for this will not solve 1., and cannot be '\', as this will not solve 2. This leaves just the possibility that $\overline{X}$ is quantified. So assume $\overline{X}$ has the form $\forall Y_1 \ldots \forall Y_n.y$, where y is either atomic or has a slash as principal connective. In case y is atomic, it is clear $\overline{X}$ is not a solution for 3. So suppose y has a slash as principal connective. Clearly the proofs of 1 and 2 must end with a succession of n ($\forall$L) inferences, transforming $\forall Y_1 \ldots \forall Y_n.y$ to y_1 in the proof of 1, and to y_2 in the proof of 2. The two proofs will therefore contain the premises:

$1'$: $a/d/c,\ y_1\ \Rightarrow a$
$2'$: $y_2,\ d\backslash c\backslash b\ \Rightarrow b$

In order to solve $1'$, y_1 must have a principal left slash, and in order to solve $2'$, y_2 must have a principal right slash, but both y_1 and y_2 must also have the same principal slash as y.

In this way we have eliminated all possibilities for $\overline{X}$. □