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Contents

Syntactic Generalizations in Compositional Grammars

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Abstract
In this paper I present an approach to the description of natural language in which two aspects are combined: (1) the grammars used are compositional, so that the semantics of natural language and the relation between syntax and semantics is adequately described, and (2) syntactic constructions are derived by the interaction of a conglomerate of construction-independent rules, so that syntactic generalizations can be adequately captured. Syntactic transformations play an important role here. The first aspect has been adopted from the *Montague-Grammar* tradition, the second aspect from the *Move α* program as pursued in the *Principles and Parameters Approach*. Both aspects are combined in a computationally viable grammatical framework, called *controlled M-grammar*, which has been used in the Rosetta3 experimental machine translation system developed at Philips Research Laboratories.

It is shown that for an adequate description of the syntax of auxiliaries and inversion in English, syntactic transformations, i.e. rules with identity as meaning, are required in a Montague-type compositional grammar. It is then shown that such rules are also useful to obtain an adequate description of passivization.

The optimal use of syntactic transformations yields as one of its consequences the fact that the relation between form and meaning is rather indirect, in the sense that the form aspects which are visible in the sentence are often accounted for by one set of rules, but the meaning aspects by a different set.

1 Introduction

In this paper I will sketch an approach to the description of natural language in which it is possible to express syntactic generalizations adequately in a compositional grammar. The approach makes it possible to combine insights from Montague Grammar[1] and from the Principles and Parameters Approach.[2]

Montague Grammar represents a framework for the description of natural language in which emphasis is put on semantic issues, and in which a specific idea on the relation between syntax and semantics (viz. compositionality) has been adopted as a guiding principle. The *Principles and Parameters* approach to the description of natural language focuses mainly on syntax, and one of its main objectives is the elimination of language-specific and construction-specific rules of grammar. This program is also known as the *Move α* program.

The framework adopted in this paper, *controlled M-grammar*, attempts to combine the virtues of these two frameworks. Controlled M-grammars are compositional grammars, but a special kind of rule, called *syntactic transformations*, is used optimally to express syntactic generalizations. These syntactic transformations make it possible to describe constructions as the result of the interaction of a conglomerate of construction-independent rules.[3] This will be discussed in more detail below.[4] In addition, controlled M-grammars are computationally viable, and *effectively reversible* (see Van Noord 1993, Rosetta 1993), so that it can be used in applications both for synthesis and for analysis of natural language. This framework

1. Thomason 1974
2. For a recent overview, see Chomsky and Lasnik 1991.
3. We will not deal with the language-specificity of rules in this paper.
4. See Appelo et al. 1987, Rosetta 1993 for a more elaborate discussion of this framework.

has been used as the framework of the experimental machine translation system *Rosetta3* developed at Philips Research Laboratories.

2 Compositional Grammars

I will consider a *language* to be a set of pairs < form, meaning >. A *(formal) grammar* is a formal description of a language and usually is a procedure which generates all and only the pairs which are members of the language.

A *compositional grammar* is a special kind of formal grammar in which the pair-like nature of the elements of language is taken very seriously and is reflected throughout the whole grammar. A compositional grammar consists of (1) basic expressions, i.e. pairs < basic form, basic meaning >, and (2) rules, i.e pairs < syntactic operation, meaning operation >. A compositional grammar can generate (or 'derive') the elements of a language in the following manner. The form part of these elements is derived by recursively applying the syntactic operations of rules, initially to basic forms. The meaning part of these elements is derived by recursively applying the meaning operations of rules, initially to basic meanings.

An example compositional grammar is given in Table 1. This grammar is extremely simple, and uses a syntactically quite primitive grammar. It is kept this simple for expository purposes, and no linguistic claims are made with it.

In the first part of table (1), the basic expressions of the grammar are enumerated. Each of these basic expressions is assigned a syntactic category which is given in the first column, followed by a number of example basic expressions and (between brackets) the names of the associated basic meanings. The basic meanings themselves have not been given here. In most cases they are not very interesting, but where required I will specify the relevant meaning.

In the second part of table (1) the rules are specified. The first column contains the rule names and (between brackets) the names of the meaning operations of the rules. The second column contains a description of the syntactic operations of the rules. These operations apply to tuples of strings which have been assigned a syntactic category (categorized strings) and turn these into a categorized string (Greek letters are used as variables for strings).[5] To illustrate a simple example, consider rule RNP1. This rule takes an arbitrary string (α) of syntactic category *N* (e.g. *book*) and an arbitrary string (β) of category *Det* (e.g. a) and turns them into their concatenation (*a book*) of syntactic category *NP*.

Rule RSUBST is slightly more complex. It is a parameterized rule which applies to a tuple of categorized strings. The first string must be of category *S* or *VP*, and it must be possible to partition the string into three parts: an arbitrary string which precedes a variable with index i (α), a variable with index i (this index is determined by the parameter of the rule), and an arbitrary string which follows the variable with index i (β). The second argument of the rule can be any string (γ) of category *NP*. Applying the rule results in a copy of the first string in which the variable with index i is replaced by γ.

The meaning operations associated to each rule are given in the last part of table 1. The symbols $\#i$ stand for the i^{th} argument of the relevant meaning operation.

Given this example compositional grammar, pairs < form, meaning > can be derived. The form *he bought a book* can be derived as indicated in (1), which represents its *syntactic derivation tree*. This is clear from (2), in which the results of applying the rules are represented on the nodes of the tree. The top node contains

5. I would like to emphasize again that this is a far too simplistic grammatical formalism for real grammars, but it is used here for expository convenience.

Basic Expressions	
category	basic expression (basic meaning)
TV	buy(buy'). see (see'),...
IV	leave (leave'), walk (walk'). ...
Det	a (a'). the (the'), which (which')....
N	book (book'). car (car'),...
Pron	I (I'). he (he'),...
Adv	never (never'). ...
Var	$x_1(X_1). x_2(X_2)$. ...

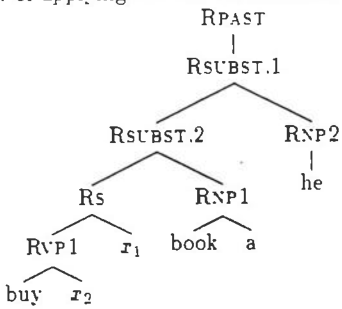
Rules	
rule name (meaning name)	syntactic operation
RNP1 (LNP1)	N[α] + Det[β] $\Rightarrow$ NP[Det[β] N[α]]
RNP2 (LNP2)	Pron[α] $\Rightarrow$ NP[Pron[α]]
RVP1 (LVP1)	TV[α] + x_i $\Rightarrow$ VP[TV[α] x_i]
RVP0 (LVP0)	IV[α] $\Rightarrow$ VP[IV[α]]
RS (LS)	VP[α] + x_i $\Rightarrow$ S[x_i α]
RSUBST.i (LSUBST)	S/VP[α x_i β] + NP[γ] $\Rightarrow$ S/VP[α γ β]
RPAST (LPAST)	S[α TV/IV[β]γ] $\Rightarrow$ S[α TV/IV{tense=past}[β]γ]

meaning name	meaning operation
LNP1	#2(#1)
LNP2	#1
LVP1	#1(#2)
LVP0	#1
LS	#1(#2)
LSUBST.i	#2(λx_i#1)
LPAST	**P**#1

Table 1: Example compositional grammar

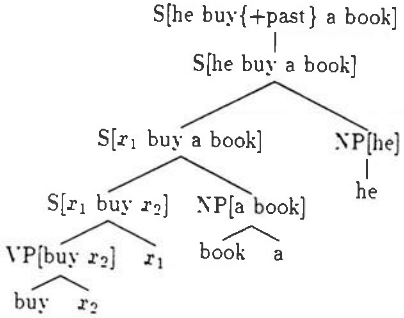
the result of applying all rules as described by this derivation tree.[6]

(1)



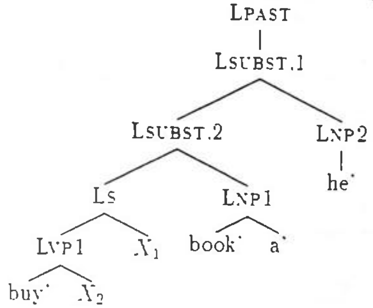
6. For simplicity. I ignore the fact that the form *buy{+past}* has to be spelled out as *bought*.

(2)

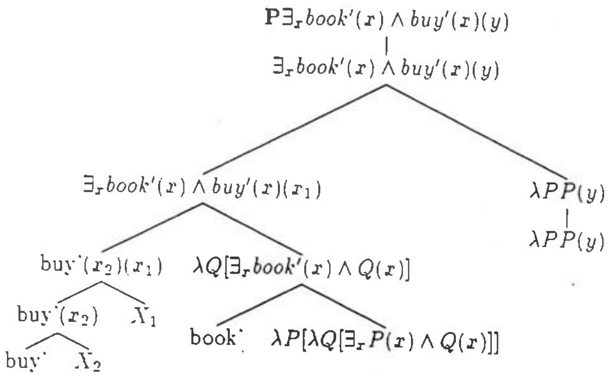


Since each basic expression also has a basic meaning, and each rule a meaning operation, the meaning associated to the form *he bought a book* can be derived as well. This is represented in (3), a *semantic derivation tree*. The related meaning is actually computed (in the form of a logical expression) in (4):[7]

(3)



(4)



This concludes our illustration of compositional grammars. In the next section I will take this example grammar as a basis, and I will try to extend it to cover a more substantial fragment of English. It will become clear soon that the compositional grammar will get quite complex and that it is necessary to duplicate several syntactic operations. In addition, I will sketch how this problem is solved in the controlled M-grammars.

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7. For this computation, I have to be explicit about the basic meaning of the determiner a. I assume its meaning to be $\lambda P[\lambda Q[\exists_x P(x) \wedge Q(x)]]$.

3 Auxiliaries and Inversion in English

In this section I will introduce the general problem using the peculiar properties of English auxiliaries as an example (section 3.1). A general characterization of the solution to this kind of problem, and an instantiation of this kind of solution, specific to the problem at hand, will be described in section 3.2.

3.1 The Problem

Let us start by considering the following two sentences

(5) a He bought a book.

b Did he buy a book?

It is our intention to extend the example grammar from the preceding section in such a way that both sentences in (5) can be derived.

At first sight, these sentences appear to be excellent examples for an analysis within a compositional approach. The two sentences clearly differ in meaning (declarative sentence versus yes-no question) and corresponding to this meaning difference there is a difference in form. Sentence (5a) contains a sentence-initial subject followed by an inflected main verb, while sentence (5b) contains a sentence-initial inflected auxiliary verb, followed by the subject, followed in its turn by the main verb in its basic form.[8] It is fairly straightforward how two different rules can be constructed, each with its own meaning, and each carrying out different formal changes to an input structure. We have seen that the rules from the example grammar can derive the structure S(*he buy{+past}* a *book*). The properties common to (5a,b) have been expressed in this structure, which will be the input for the rules to form declarative and yes-no interrogative sentences. One rule, which will be called RDECL, will take this structure as its input, and it simply resets the value of the attribute *mood* of the S-node dominating the structure from *unspecified* to *decl*. The rule is associated with the meaning of declarative sentences. A second rule, which will be called RYNQ, also takes this structure as input, and it effects the following changes: the attribute *mood* of S is changed from *unspecified* to *ynq*, the auxiliary verb *do* is introduced in a sentence-initial position, the tense features are copied from the main verb onto this auxiliary, and then deleted from the main verb. This rule is associated with the meaning of yes-no questions.

If we extend the grammar with additional rules, however, it soon becomes clear that the simple approach outlined above runs into severe trouble. I will extend the grammar with a number of rules using the compositional approach as indicated, to illustrate in what ways such an analysis is unsatisfactory.

Consider the following sentence:

(6) Which book did he buy?

This sentence differs from the sentences discussed above in several respects. Let us compare it with (5a). First, there is a semantic difference: (5a) has the meaning of a declarative sentence, (6) has the meaning of a wh-question. Second, corresponding to this semantic difference, there is a formal difference: sentence (6) has a sentence-initial NP, with the determiner *which* instead of the article a; this NP is followed by an inflected auxiliary verb, which is followed by the subject, and the main verb in its basic form is sentence-final. Again, it is quite straightforward to write a rule to form such sentences, and to associate this rule with its own meaning. If we assume that the input for the rule is S(*he buy{+past}* x_2),[9] the rule, which will be called RWHQ, has to perform the following formal changes: the attribute *mood* of the S-node must be reset from *unspecified* to *whq*: an NP containing a interrogative determiner (in this example, the NP *which book*) must be substituted for the variable x_2 and this phrase must be preposed to a sentence-initial position.

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8. Details relating to punctuation will be ignored.
9. This structure can be derived by the example grammar, as one can easily check.

the auxiliary verb *do* must be inserted, the tense features of the main verb must be copied onto this auxiliary verb, and then these tense features must be deleted from the main verb.

Similarly, we can formulate rules to form a certain kind of topicalized structures (7a), negative sentences (7b) and emphatic sentences (7c). I will call the relevant rules RTOP, RNEG and REMPH, without discussing their exact formulation in detail.[10]

(7) a Never did he buy a book.
    b He did not buy a book.
    c He did buy a book.

We have now postulated five rules: RYNQ, RWHQ, RTOP, RNEG and REMPH. Each of these rules has its own meaning, and each performs its own formal operations. They could readily be incorporated into a compositional grammar. Nevertheless, the approach adopted here has serious shortcomings. Certain operations occur in each of these rules:

- introduce the auxiliary verb *do*
- copy the tense features from the main verb onto the auxiliary verb *do*
- delete the tense features from the main verb.

These operations must be described separately in each of these rules. It is clear that this is an undesirable state of affairs. In fact, the situation is worse than described, because the operations were only defined for a limited variety of sentences. As soon as we try to extend the analysis to cover an interesting fragment of English, the common operations invariably become considerably more complicated: different operations must apply if the input already contains auxiliary verbs, the introduction of the auxiliary verb is optional if the main verb is *have*, and then the main verb *have* starts to occupy the position where otherwise the auxiliary verb is introduced, etc., etc. The important point here is that all these complications are the same for all the rules introduced. By formulating these rules separately in each rule given above, this fact is described as being accidental: a considerable redundancy is introduced into the system: the rule system is unnecessarily large, and maintaining and updating the system of rules is made considerably more complex. In short, it is clear that a number of linguistic generalizations have been missed.

3.2 A Solution

It is obvious how the problems described in the previous section can be solved: the operations common to all these rules should be factored out. Each individual rule can then be considerably simplified, the grammar as a whole becomes smaller, the common properties are explicitly identified and isolated, and maintainability and updating become easier. A future change in an operation that has been factored out will immediately have effects in each of the constructions mentioned, clearly a desirable result.

One way to factor out these operations is by writing these operations as separate rules of grammar. This is the method which will be illustrated here, and has actually been used in the Rosetta system. This method has several advantages. It turns out that most of these operations resemble syntactic rules to a high degree. The method adopted accounts for this fact immediately: it supplies one with a format and a notation to describe the operations, and the modes of interaction with existing rules are also immediately determined. An alternative method could consist of writing a set of functions which are called by rules, as suggested by Partee 1977, Partee 1979 and Partee et al. 1990, p. 318-9. Though I did not compare the methods in detail, the latter method requires additional clarification of the operations allowed, a format and a notation for them and a specification of the

10. See Odijk 1993 for an elaboration of the formulation of these rules.

possible modes of interaction with rules, and it appears that certain operations cannot be kept local under this approach. See Odijk 1993 for further discussion. To my knowledge, this latter method has never been applied in a real large-scale system, while the method adopted here has proven its usefulness in the Rosetta system.

There are, however, examples where it is advantageous to call functions from within rules, and where an analysis in terms of separate rules does not really achieve the desired result. Such examples have been described in Odijk 1993, p. 54. However, the functions called can perform only a limited set of well-defined operations which are independently required in the grammar.

Applying the solution, in which the common properties are factored out in separate rules, to the analysis of auxiliaries in English, we observe that the rule in which the common properties of the preceding rules are described performs operations on auxiliary verbs. It is not at all obvious that a meaning can be associated with this rule. The rule performs a number of the operations needed to form yes-no questions, wh-questions, topicalized constructions, negated sentences and emphatic sentences. Its semantics should be the 'intersection' of the meanings of these constructions, but this intersection is most probably empty. This rule must therefore be a *transformation*, i.e. a rule which is associated with the identity operation as its meaning. The rule will be called RAUX.

When we consider the rule RAUX in a realistic system, it proves desirable to split it up into two different rules. Note that RAUX introduces an auxiliary verb in all examples, but in certain cases it should do so to the right of the subject (in the constructions for which RNEG, REMPH have been proposed), and to the left of the subject in others (in the constructions for which RYNQ, RWHQ, RTOP have been proposed). When another auxiliary verb is present in the structure, no additional auxiliary verb should be introduced, but in the case of RYNQ, RWHQ and RTOP the auxiliary present has to change its position and be placed before the subject. By isolating and factoring out this operation of inverting the subject and the auxiliary verb, the complicated rule RAUX can now be broken down into two simpler rules: one which introduces the auxiliary verb *do* if no other auxiliary is present, and one which inverts the subject and the auxiliary in certain configurations. The rule of inversion is necessary anyway, to deal with cases of inversion involving auxiliaries other than the auxiliary verb *do*. This rule can now also be simplified, since it need no longer exclude the auxiliary verb *do*: it can simply state that in certain configurations any auxiliary verb must invert.

In fact, the rules and the whole grammar can be simplified still further if the auxiliary verb is introduced into *all* structures which do not already contain an auxiliary verb, and by postulating a rule which copies the tense features from the auxiliary *do* onto the main verb and deletes *do* if, at the end of the derivation, *do* and the main verb are adjacent. This simplifies the rule of auxiliary introduction, simplifies the rules assigning values to tense attributes (these rules can now refer in all cases to the auxiliary verb), and simplifies the rule of subject-verb agreement (which also need only refer to the auxiliary verb). The only problem it creates is that this solution will not work for REMPH (e.g. in *he did buy a book* the auxiliary verb *do* and the main verb are adjacent, but *do* cannot be deleted). This, however, can be solved in a very simple manner, e.g. by marking *do* in this construction as [+stressed], and formulating the rule of auxiliary deletion in such a way that it does not apply to [+stressed] elements, or by postulating an abstract element EMPH which occupies the position between *do* and the main verb, and is deleted later in the derivation. This abstract element could be a basic expression with the meaning of emphasis. The analysis of the English auxiliary system presented here is, of course, based to a large extent on the analysis given originally by Chomsky 1957 and in several studies since.

3.3 Summary

To summarize, the resulting analysis which has actually been implemented, can be described as follows. There is a transformation which introduces the auxiliary *do* to the right of the subject in all finite sentential structures, unless there is already an auxiliary or modal verb. The meaningful rules RNEG, REMPH, RYNQ, RWHQ and RTOP still apply, but they have been considerably simplified since the operations common to them have been factored out. There is an inversion transformation which inverts the auxiliary or modal verb and the subject in certain configurations created by the meaningful rules RTOP, RWHQ and RYNQ. And there is a transformation which copies the tense features of the auxiliary *do* onto the main verb and deletes this auxiliary if it is adjacent to a verb.

In the analysis, the relation between the meaning and the form of the sentence has now become very indirect. Though the sentences (8a,b,c) differ formally only with regard to the presence of an auxiliary and to the relative positions of the subject and the auxiliary, neither of these formal differences is accounted for by a rule which takes care of the semantic differences.

(8) a he bought a book

b he did buy a book

c did he buy a book?

This situation is typical of most phenomena in natural language. The kind of approach described here, in which common properties of rules are maximally factored out and written as separate transformations, turns out to be useful for many constructions.

This can be illustrated with a number of examples (see Odijk 1993). In all these cases, common properties are factored out of semantically motivated rules, and this leads to operations which operate in a purely formal, syntactic way, making the relation between meaning and form considerably more indirect.

In this paper, I will illustrate how the approach adopted here can be used to describe passive constructions in such a way that various insights from the Principles and Parameters Approach are incorporated.

4 Passives

There are basically two major types of analyses of the passive construction in modern formal theories of grammar. They are called the *syntactic* and the *lexical* analysis of passive, respectively. I will not discuss the differences between these approaches in detail here (see Odijk 1993 for a more elaborate discussion).

In the Rosetta grammars, the syntactic analysis of passivization of Government Binding (GB) theory has been taken as a basis for the analysis. The main reason for this is that the lexical analysis combines a number of operations which in my view are independent and should be described as independent operations. This will become clear below, where I will describe and justify the analysis adopted in Rosetta. Many of the arguments for certain assumptions have been derived directly from GB-like analyses of passive constructions.

The analysis of passive in the Government Binding Theory is an excellent example to illustrate what the *Move α* program has led to. I have attempted to incorporate some of the positive results of this approach into the controlled M-grammar framework, especially with respect to the fact that most rules involved in forming passive structures are not specific to the passive construction, but have independent motivation and are used to form completely different constructions as well.

I will first outline globally how passive structures are derived in the grammar, and then explain in more detail why this analysis has been adopted.

The first step to form a sentence consists of combining a verb with a number of variables into a propositional structure consisting of a predicate and optionally a subject, by so-called start rules. The position of the variables and the grammatical relations they bear in this structure correspond to their positions and relations when used in an active sentence. Thus, for instance, the two-place verb *kiss* can be combined with two variables, x_1 and x_2, where x_1 is the subject and x_2 is the direct object in the propositional structure headed by *kiss*.

Voice rules apply to determine the voice of the structure. The rule for active voice leaves the structure unchanged, and only changes the value of the voice attribute of the proposition node from *unspecified* to *active*.

The rule for passive voice (called *Rpassive*) changes the value of this voice attribute into *passive*, but a number of other changes are also performed. The rule *Rpassive*, however, is not really comparable to the traditional *passive transformation* (e.g. as in Chomsky 1957). The traditional passive transformation has been decomposed into a number of separate rules and transformations, which together form passive structures. This approach is inspired directly by the treatment of passives in Chomsky 1981.

In order to illustrate this, I will summarize the differences between a typical passive sentence and the corresponding active sentence, and indicate where and how the differences among them are accounted for:

Voice The attribute **voice** has the value *active* in active sentences, the value *passive* in passive sentences. As pointed out above, the value of this attribute is set in the voice rules called *Ractive*, *Rpassive*.

Subject The first argument of a verb in active sentences is expressed as a *subject*. In passive sentences it is expressed as a complement to the preposition *door* in Dutch, *by* in English or *por* in Spanish. This change is performed in a separate rule.

Verb Form In active sentences the form of the main verb is not fixed. It can be a finite verb, an infinitive, a present participle, a past participle (if an auxiliary verb *hebben* or *zijn* (in Dutch), *have* (in English) or *haber* (in Spanish) is present); in passive structures the main verb is a past participle. The form of the verb is set in the rule *Rpassive*.

Auxiliaries In active sentences no auxiliaries to express voice are present. In passive sentences, however, auxiliary verbs (Dutch: *worden* or *zijn*; English: *be*; Spanish: *ser*) are present. These auxiliary verbs are introduced by rules turning the propositional structure into a clause.

NP-movement One of the NP-arguments inside VP in an active sentence is usually realized as a subject in the corresponding passive sentence. I will call this NP the *moving NP*. It can be a *direct object*, (**The girl** *was kissed*), an *indirect object* (**The man** *was given a book*), a *prepositional object* (**The girl** *was looked at*), the subject of an embedded clause (**The man** *was believed to be ill*) or the subject of an embedded small clause (**He** *was considered a fool*). The operation to turn the moving NP into a subject will be called *NP-movement*. NP-movement is performed by a special transformation.

As is clear, the traditional effects of the passive transformation are performed by the interaction of voice rules, rules to form *by*-phrases (or their equivalents in other languages), rules to form clauses and transformations to turn NPs into subjects. The reasons for doing this in this way are:

***by*-phrase** This phrase is not formed in the rule *Rpassive*, because it is used in other constructions as well. Dutch has a construction which can occur only with the verb *laten* 'to let' in which the Dutch equivalent of a *by*-phrase (formed with the preposition *door*) occurs in a structure which is active in all other respects: *hij liet het huis* **door hen** *schoonmaken* lit. *he let the house clean by them* 'he had them clean the house'. As in passives, this phrase is optional (cf. *Hij liet het huis schoonmaken*). *By*-phrases also occur in nominal constructions, and in so-called 'modal passives', a construction in which *te* + infinitival V has the meaning 'can/must be V-ed' (e.g. *de door ons schoon te maken huizen* lit. *the by us clean to make houses* 'the houses which are to be cleaned by us'), though we did not deal with these systematically in the Rosetta system. It is clear then, that *by*-phrases are not specific to passive structures, and that their formation should not be carried out in the rule Rpassive.

NP-movement This operation is not performed inside the rule *Rpassive* because it is not essential to the passive voice at all:

- There are passive sentences with no VP-internal NP at all (impersonal passives, for instance *Er werd gedanst* (lit. *There was danced*) in Dutch, and passives of verbs that take a sentential complement, for instance *It was said that he was dishonest*). If NP-movement were part of the rule *Rpassive*, a separate rule for such passives would have to be stated.
- In personal passive sentences the moving NP sometimes remains inside VP, and is not moved into the subject position. Some relevant examples are: *De mannen werd* **het boek** *gegeven* (lit. *The men was the book given*). *Er werd door hem* **een boek** *gelezen* (*There was by him a book read*), for which it can be argued convincingly that the boldface NPs are in object position, not in subject position (cf. Den Besten 1981).
- A distinction is made between *ergative* and *non-ergative* verbs. Ergative verbs are verbs that take an NP as argument but do not realize any of its arguments as an *external argument*, i.e. none of the verb's arguments is made the subject of the verb when its arguments are combined with the it. For these verbs there must be a rule to turn an NP into a subject as well, though this involves only active structures. It is natural to use the same rule both in passives and in ergative structures.
- NP-movement must also be performed for NPs that are not yet present in the structure at the moment that the rule *Rpassive* applies. As pointed out above, the first rules to form sentences combine verbs with a number of variables, and it is the case that for a sentence such as *She was considered smart by us* the structure contains a variable for the subject (*we*), and a variable for the 'small clause' *she smart*, but nothing that corresponds directly to the NP *she* (which is the NP which should become the subject) at the moment that the rule *Rpassive* must apply.
- Control transformations dealing with *obligatory control* (in the sense of Williams 1980) can be simplified if the object is still an object at the moment that they apply. This makes it easier to account for the fact that a derived subject can still function as a controller, though an NP in the *by*-phrase cannot. I will not discuss this argument here. A more detailed discussion can be found in Odijk 1993.

Auxiliary Verb The auxiliary verb is not introduced in the rule *Rpassive* because there are passive structures where no such auxiliary occurs. This is the case in small clauses with a verb as their head, e.g. *Hij kreeg* **het boek afgeleverd** (*He had the book delivered*). *Hij wist* **zich door hem gesteund** (*He knew himself by him supported*). *They had* **him killed** .

So, the effects of passivization are achieved by the interaction of a number of rules, some of which are necessary on independent grounds. The analysis is very much in the spirit of analyses within the *Move α* program. The following properties of these analyses have been incorporated: (1) passivization is *syntactic*, not lexical: (2) no reference to a *by*-phrase is made inside a rule specific to passive structures: (3) no reference to a direct-object NP or any NP whatsoever is made; (4) movement of NP is necessary in certain cases, but (5) this movement is independent of passivization: (6) the presence of the auxiliary is independent of passivization.

Note that the lexicon contains no passive verbs (passive participles) as entries. Each verb is in the lexicon in its base form. The passive forms of a verb are created in syntax. Even verbs which can occur in passive only (e.g. Dutch *achten* 'expect'. English *rumor*) are in the lexicon in their base forms, though their inherent properties specify that they can occur in passive voice only. The analysis deviates considerably from analyses of the passive construction in most other computationally oriented frameworks (LFG, HPSG), in which the passive construction is usually accounted for by a lexical rule that creates a new lexical entry with new complementation properties from an existing lexical entry.

5 Conclusions

In this paper I presented an approach to the description of natural language in which two aspects are combined: (1) the grammars used are compositional, so that the semantics of natural language and the relation between syntax and semantics is adequately described, and (2) syntactic constructions are derived by the interaction of a conglomerate of construction-independent rules, so that syntactic generalizations can be adequately captured. Syntactic transformations play an important role here. The first aspect has been adopted from the *Montague-Grammar* tradition, the second aspect from the *Move α* program as pursued in the *Principles and Parameters Approach*. Both aspects are combined in a computationally viable grammatical framework, called *controlled M-grammar*, which has been used in the Rosetta3 experimental machine translation system developed at Philips Research Laboratories.

Starting from a purely compositional Montague-style grammar, I showed that for an adequate description of the syntax of auxiliaries and inversion in English, syntactic transformations, i.e. rules with identity as meaning, are required. I then showed the usefulness of such rules to obtain an adequate description of passivization.

The optimal use of syntactic transformations yields as one of its consequences the fact that the relation between form and meaning is rather indirect, in the sense that the form aspects which are visible in the sentence are often accounted for by one set of rules, but the meaning aspects by a different set.

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FLEXIBLE VARIABLE-BINDING AND MONTAGUE GRAMMAR

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1 Introduction

In [5] we introduced a version of predicate logic (PFO), with a new method of variable-binding, designed to handle some familiar anaphoric constructions in natural language compositionally. The idea to adapt a classical logical formalism to obtain a compositional account of certain linguistic binding phenomena is also the basis of Groenendijk and Stokhof's DPL [3], but their adaptation is different. DPL preserves the formulas of predicate logic, but changes the variable-binding mechanism and uses a dynamic semantics in the sense that the semantic value of a formula is a relation between assignments rather than a set of assignments. PFO on the other hand employs a new kind of formulas, with yet another variable-binding mechanism, but keeps essentially the standard semantics. The switch to a dynamic semantics is a substantial departure from classical predicate logic (PL), and we argued in [5] that whereas PFO is just a variant of PL, DPL is not.

The main interest of PFO, we feel, is that it is so simple and departs so little from PL. We will not here discuss the need for or the implications of a dynamic semantics (cf. [4] for a recent discussion). Regardless of this it is of interest to see how far one can handle the relevant anaphoric phenomena with a very slight change in the formal language.

PFO and DPL stop at sentence level. To obtain full compositionality à la Montague, one needs to extend the formalism to something like (intensional) type theory—Montague's IL. In this paper we present such an extension of PFO, called TFO. Here the difference with DPL becomes more drastic. The extension of the latter to Dynamic Montague Grammar (DMG) in [2] is not straightforward, since the dynamics and the DPL style variable-binding do not extend to the λ operator. Instead, explicit dynamic mechanisms are added to the formalism, via devices that were formerly used to handle intensions but now take on this further role. The extension of PFO to TFO, on the other hand, proceeds smoothly once one realizes how to do it. The basic PFO recipe for variable-binding works in the type theory too. The intensional apparatus has the same role as in IL. The expressive power is the same. The only thing that needs to be reviewed with some care is conversion—this is a syntactic phenomenon and TFO does have a different syntax than IL.

After some background on PFO style variable-binding we present the syntax and semantics of TFO, and then discuss conversion, in particular the Church-Rosser property and normalization. Finally we show how a standard Montague style translation into TFO handles familiar examples of donkey anaphora and cross sentence anaphora in a compositional way.

The paper is to be regarded as an extended abstract of a more detailed presentation. In particular, some definitions and all proofs have been omitted.

2 Background

The characteristics of PFO style variable-binding are that it is *unselective* and *reverses the binding order*: from the outside in, rather than the usual order from the inside out. Instead of standard connectives and quantifiers two binary variable-binding operators $[\cdot,\cdot]$ and $(\cdot,\cdot)$ are used, corresponding to universal quantification (implication) and existential quantification (conjunction), respectively. These bind (unselectively) *all common* variables of the two arguments, whether these variables were already bound in the arguments or not—such bindings are thus 'cancelled' (binding from the outside in). Details are given in [5], but a few sample formulas and their PL counterparts suffice to convey the idea:

$[Ax, By]$	$Ax \rightarrow By$
$[Ax, Bx]$	$\forall x(Ax \rightarrow Bx)$
$(Axy, Bxyz)$	$\exists x \exists y(Axy \wedge Bxyz)$
$[(Axy, Bx), Czu]$	$\exists x(Axy \wedge Bx) \rightarrow Czu$
$[(Axy, Bx), Dzx]$	$\forall x(Axy \wedge Bx \rightarrow Dzx)$

In PFO the sentence

(1) If a man encounters a lion he runs from it

can be translated as

(1′) $[(Mx, (Ly, Exy)), Rxy]$

and

(2) A man walks. He talks.

as

(2′) $((Mx, Wx), Tx)$

Both translations are compositional at sentence level.

The reverse binding order of PFO necessitates an adjustment of the usual inductive definition of satisfaction. One way to do this is to let a set of variables X be an argument of the satisfaction relation. When you start evaluating a formula this set is usually empty, but quantified variables are successively put in the set as subformulas, subformulas of subformulas, etc., are evaluated, to prevent variables from being quantified again. That is, the satisfaction relation

$$\mathcal{M}, X \models_f \varphi$$

between an assignment f, a formula ϕ, a model $\mathcal{M}$, and a set X of variables is defined so that the variables in X are never quantified. The ordinary ternary satisfaction relation is then obtained by letting $X = \emptyset$.

Before moving on to TFO we make two brief comments. The first is that it is primarily the reverse variable-binding of PFO, rather than the unselectivity, that makes anaphoric sentences come out right. To see this, consider a formalism which is exactly as PL *except* that variable-binding goes from the outside in. In such a system (1) could be translated

(1″) $\forall x \forall y(\exists x(Mx \wedge \exists y(Ly \wedge Exy)) \rightarrow Rxy)$

This has as subformulas the translations of *a man encounters a lion* and *he runs from it*, so it is compositional at sentence level. And, because of the reverse variable-binding, it gets the correct meaning—the existential quantifiers are 'cancelled' because they are within the scope of corresponding universal quantifiers.

Still, (1″) is rather ugly, and it is not quite obvious what the rule for 'if-then' would look like. The unselective PFO operators yield a more elegant system and more natural translation rules, and we will continue to use them in TFO.

Our second comment is that whereas with universal and existential quantification, selectivity vs. unselectivity is mere matter of style, this is not so with other quantifiers. In effect, unselective variable-binding allows quantification over *finite sequences* of individuals, which is an increase of expressive power with many generalized quantifiers. For example, suppose we add to PFO an operator $[_m\cdot,\cdot]_m$ corresponding to the determiner *most*. To evaluate a sentence $[_m\phi,\psi]_m$ relative to a model $\mathcal{M}$, we find the variables common to ϕ and ψ, say $x_1,\ldots,x_n$. Let R (S) be the set of n-tuples $(a_1,\ldots,a_n)$ such that the assignment of a_i to x_i satisfies ϕ (ψ) in $\mathcal{M}$. Then $[_m\phi,\psi]_m$ is true in $\mathcal{M}$ iff $|R\cap S| > |R-S|$. This PFO style generalized quantifier is stronger than the ordinary selective quantifier **most** which binds one variable only in each formula.[1] In the terminology of generalized quantifier theory, if you add a monadic generalized quantifier to PFO, you also obtain all the *resumptions* of that quantifier, because of the unselective variable-binding.

3 TFO

The terms of TFO are the same as those in IL, except that we use the two PFO operators instead of connectives and quantifiers. The *types* are the usual ones: basic types e and t, and complex types (a,b) and (s,a). So the *terms* T_a of type a are defined inductively as follows.

Definition 3.1 (TFO syntax)

(a) Variables and constants of type a are in T_a.
(b) $\bot\in T_t$.
(c) If $t,u\in T_a$ then $(t=u)\in T_t$
(d) If $\phi,\psi\in T_t$ then $[\phi,\psi],(\phi,\psi)\in T_t$.
(e) If $t\in T_{(a,b)}$ and $u\in T_a$ then $tu\in T_b$.
(f) If $t\in T_b$ and x is a variable of type a then $(\lambda x.t)\in T_{(a,b)}$.
(g) If $t\in T_a$ then $^\wedge t\in T_{(s,a)}$.
(h) If $t\in T_{(s,a)}$ then $^\vee t\in T_a$.
(i) If $\varphi\in T_t$ then $\Box\phi\in T_t$.

Thus, we use the unselective PFO operators as well as the selective λ operator in the syntax. However, for *both* operators, the binding direction is from the outside in. Roughly: let an *x-binder*, for a variable x of any type, be a term of the form $(\lambda x.u)$, or of the form $[\phi,\psi]$ or (φ,ψ) where x occurs in both φ and ψ. Then an occurrence of x in t is bound by the *outermost* x-binder in t within whose scope it occurs. For example, the term

$$(Ax,Bx)$$

(with A and B constants of type (e,t)) expresses, as in PFO, that $A\cap B\neq\emptyset$. But in

$$\lambda.x(Ax,Bx)$$

the PFO binding is not in force, and the term denotes the set $A\cap B$. And the λ binding can in turn be 'cancelled' by a PFO binding further out, as in

$$((\lambda.x(Ax,Bx))y\ ,\ Cx)$$

which expresses that $A\cap B\cap C\neq\emptyset$. Note that since the λ binding is not in force here, the application of the λ term to y has no (semantic) effect.

A precise syntactic definition of binding in TFO is easily given, but the above examples should make the idea clear. The meaning of TFO terms is given by the next definition, which with each term t, model $\mathcal{M}$, possible word w, $\mathcal{M}$-assignment f, and set of variables X associates a denotation $[\![t]\!]_{\mathcal{M},w,f,X}$. As usual, $\mathcal{M}$ consists of a domain $D=D_e$, a set of possible worlds W and an interpretation function I. $D_t=\{0,1\}$, and the domain is lifted to a function domain D_a for each type a in

1. A proof of this fact can be found in [6].

the usual way. I assigns functions from W to D_a to constants of type a, and an $\mathcal{M}$-assignment assigns a value in D_a to each variable of type a. If t is of type a, $[\![t]\!]_{\mathcal{M},w,f,X} \in D_a$.

Modulo the variable-set X, the clauses in the definition below are exactly the same as for IL, except the two clauses (d) and (f) dealing with variable-binding operators.

Definition 3.2 (TFO semantics)

(a) If x is a variable and C a constant of type a, then $[\![x]\!]_{\mathcal{M},w,f,X} = f(x)$ and $[\![C]\!]_{\mathcal{M},w,f,X} = I(C)(w)$.

(b) $[\![\perp]\!]_{\mathcal{M},w,f,X} = 0$.

(c) $[\![t = u]\!]_{\mathcal{M},w,f,X} = 1$ iff $[\![t]\!]_{\mathcal{M},w,f,X} = [\![u]\!]_{\mathcal{M},w,f,X}$.

(d) Suppose $(Var_\varphi \cap Var_\psi) - X = \{x_1, \ldots, x_n\}$, where x_i is of type a_i. Then

$$[\![(\varphi, \psi)]\!]_{\mathcal{M},w,f,X} = 1 \text{ iff there are } d_1 \in D_{a_1}, \ldots, d_n \in D_{a_n} \text{ such that}$$
$$[\![\varphi]\!]_{\mathcal{M},w,f(x_i/d_i),X\cup\{x_1,\ldots,x_n\}} = [\![\psi]\!]_{\mathcal{M},w,f(x_i/d_i),X\cup\{x_1,\ldots,x_n\}} = 1.$$

Similarly for $[\varphi, \psi]$, except that universal quantification and implication is used.

(e) $[\![tu]\!]_{\mathcal{M},w,f,X} = [\![t]\!]_{\mathcal{M},w,f,X}([\![u]\!]_{\mathcal{M},w,f,X})$.

(f) If $(\lambda x.t)$ is of type (a, b), then, for all $d \in D_a$.

$$[\![(\lambda x.t)]\!]_{\mathcal{M},w,f,X}(d) = \begin{cases} [\![t]\!]_{\mathcal{M},w,f,X} & \text{if } x \in X \\ [\![t]\!]_{\mathcal{M},w,f(x/d),X\cup\{x\}} & \text{if } x \notin X \end{cases}$$

(g) If $w' \in W$, then $[\![{}^\wedge t]\!]_{\mathcal{M},w,f,X}(w') = [\![t]\!]_{\mathcal{M},w',f,X}$.

(h) $[\![{}^\vee t]\!]_{\mathcal{M},w,f,X} = [\![t]\!]_{\mathcal{M},w,f,X}(w)$.

(i) $[\![\Box\varphi]\!]_{\mathcal{M},w,f,X} = 1$ iff for all $w' \in W$, $[\![\varphi]\!]_{\mathcal{M},w',f,X} = 1$

We also define

$$[\![t]\!]_{\mathcal{M},w,f} = [\![t]\!]_{\mathcal{M},w,f,\emptyset}$$

It is rather clear that TFO has the same expressive power as IL. To translate from TFO to IL, define for each TFO term t and each set X of variables an IL term $t^{+,X}$ inductively following Definition 3.1, distributing over the operators except in clauses (d) and (e) which read, respectively.

$$[\varphi, \psi]^{+,X} = \forall x_1 \ldots \forall x_n(\varphi^{+,X\cup\{x_1,\ldots,x_n\}} \rightarrow \psi^{+,X\cup\{x_1,\ldots,x_n\}})$$

(with $x_1, \ldots, x_n$ as in Definition 3.2 (d); $(\varphi, \psi)^{+,X}$ is similar).

$$(\lambda x.t)^{+,X} = (\lambda x.t^{+,X\cup\{x\}})$$

It then follows that $[\![t]\!]_{\mathcal{M},w,f,X} = [\![t^{+,X}]\!]_{\mathcal{M},w,f}$, so in particular

$$[\![t]\!]_{\mathcal{M},w,f} = [\![t^{+,\emptyset}]\!]_{\mathcal{M},w,f}$$

To translate in the other direction simply note, first, that logical symbols $\forall$, $\wedge$, etc. can be eliminated in IL, second, that every IL term is equivalent to a *strict* term, i.e., one where no variable is both free and bound, nor quantified more than once, and third, that strict IL terms without logical symbols are also TFO terms and moreover mean the same in both systems.

4 Conversion

The fact that TFO has both selectively and unselectively binding operators makes conversion a little more complex than in IL. Also, the reverse binding direction means that we should keep track of the set of variables X in conversion. Let $\rightsquigarrow_X$ be the conversion relation relative to X between a redex and the result of performing β conversion. Here are some case where β conversion can *not* be performed: *Unless* $y \in X$,

$$(\lambda x.[Ax, Bx])y \not\rightsquigarrow_X [Ay, By]$$

$$(\lambda x.[Ax, By])y \not\rightsquigarrow_X [Ay, By]$$

$$(\lambda x.[Az, (\lambda y.(By, Cx))z])y \not\rightsquigarrow_X [Az, (\lambda y.(By, Cy))z]$$

The reason, of course, is that the usual constraint that no new bindings must be created by the substitution is violated. It is just that such bindings can arise in more ways than one in TFO. In fact, they may arise in three ways, illustrated by the above examples. Therefore, the most straightforward approach to conversion in TFO is to formulate explicitly the variable constraints on conversion and then verify that conversion under these constraints is sound.

Of course, there is another kind of constraint on β conversion, due to the presence of intensional operators. But these constraints are exactly as in IL. They lead to the failure of the Church-Rosser property—a failure which can be overcome by treating s as a regular type of possible words (cf., for example, [1], ch. 5). Thus, for simplicity, and to bring out the characteristic binding features of TFO, we restrict attention in this section to the *extensional part* of TFO. That is, we only consider terms as defined by Definition 3.1 (a)–(f). Hence possible worlds are not needed, and the denotation of a term t can be written $[\![t]\!]_{\mathcal{M},f,X}$.

If t and u are terms and x is a variable, let

$$[x/u]t$$

be the result of replacing *all* occurrences of x in t by u.[2]

The next definition gives the condition for this substitution to be permissible.

Definition 4.1 $P(x, t, u, X)$ is the conjunction of (a)–(c) below:

(a) if x is PFO-bound in t then $Var_u \subseteq X$

(b) if x occurs in one component of a subterm (ϕ, ψ) or $[\phi, \psi]$ of t and $y \in Var_u$ in the other, then $y \in X$

(c) if x occurs in a subterm $(\lambda y.v)$ of t where $y \in Var_u$ then $y \in X$.

Now we say that

$$t \Rightarrow_X t'$$

if t' results from t by performing one β conversion: replacing an occurrence of a subterm $(\lambda x.u)v$ of t for which $P(x, u, v, Z)$ holds by $[x/v]u$, where Z results by adding to X the variables which become bound in the semantic evaluation process from t to the subterm $(\lambda x.u)v$. The following can now be proved.

Proposition 4.1 *If* $t \Rightarrow_X t'$, *then* $[\![t]\!]_{\mathcal{M},f,X} = [\![t']\!]_{\mathcal{M},f,X}$.

This approach to β conversion is a quite simple extension of the ordinary one, but it has one problem: the Church-Rosser property fails. Here is an example. We have

$$(\lambda z.((\lambda x.(Ax, Bz))z))y \Rightarrow_\emptyset (\lambda z.(Az, Bz))y$$

and

$$(\lambda z.((\lambda x.(Ax, Bz))z))y \Rightarrow_\emptyset (\lambda x.(Ax, By))y$$

2. Here "occurrence" is to be taken in its literal sense, except that x is *not* taken to occur in 'λx'.

but there is no way to continue the reduction to a common term. Note that changing bound variables (α conversion) will not help. In fact, whereas in ordinary λ calculus bound variables can always be chosen so that the variable constraint on substitution is satisfied, this is not so for TFO and the constraint $P(x, t, u, X)$.

In view of this one may try the following alternative approach. First, require, for $t \Rightarrow_X t'$, that the free variables of t are elements of X. This can always be achieved. More precisely, if Y is the set of free variables of t, one can α convert t to a term t_0 such that

$$[\![t]\!]_{\mathcal{M},f,X} = [\![t_0]\!]_{\mathcal{M},f,X \cup Y}$$

for all $\mathcal{M}$ and f.[3] Second, for such terms one may redefine substitution, incorporating α conversions in a way that always makes substitution permissible. Then there is no need for the condition P.

In the example above, both $(\lambda z.(Az, Bz))y$ and $(\lambda x.(Ax, By))y$ convert to (Ay, By) relative to a set of variables containing y. In general, it seems to us that the Church-Rosser property would hold with this approach to conversions. We hope to present the details in the full version of the paper.

Even if Church-Rosser fails on our first approach to conversion, normal forms always exist. This follows from the

Theorem 4.2 (Strong Normalization Theorem)
For every term t and every set of variables X: Every $\Rightarrow_X$-chain starting witht is finite.

5 Montague grammar

A few examples will suffice to illustrate how a fragment of English, containing sentences with donkey anaphora, can be compositionally translated into TFO. To begin we again skip intensions, since they are unimportant for the first examples, and are handled in exactly the same way as in IL. Also, the analysis trees will be implicit—they are just as in ordinary Montague grammar, except that we shall use *indexing* to indicate anaphoric links. Let * be the translation function. Also, let '$\Rightarrow$' stand for (repeated) applications of $\Rightarrow_\emptyset$ as well as standard meaning postulates and intension-extension cancellations.

$a_i^* = \lambda Y.\lambda X.(Yx_i, Xx_i)$
$man^* = M$
$walk^* = W$
$(a_i\ man)^* = a_i^* man^* = (\lambda Y.\lambda X.(Yx_i, Xx_i))M \Rightarrow \lambda X.(Mx_i, Xx_i)$
$(a_i\ man\ walks)^* = (a_i^* man^*)walk^* \Rightarrow (Mx_i, Wx_i)$
$he_i^* = \lambda X.Xx_i$

The next example, a donkey sentence, also illustrates a global constraint that the translation must satisfy: Always choose distinct bound variables in the translations. Otherwise unwanted binding may occur, due to the reverse binding order.

$(if\ a_i\ man\ walks\ he_i\ sings)^* = [(a_i\ man)^* walk^*, (he_i\ sings)^*]$
$\Rightarrow [(\lambda X.(Mx_i, Xx_i))W, (\lambda Y.Yx_i)S] \Rightarrow [(Mx_i, Wx_i), Sx_i]$

The following examples use the PTQ meaning postulates and notation for extensional transitive verbs.

$(a_1\ man\ encounters\ a_2\ lion)^* = (a_1\ man)^*(encounters^*(a_2\ lion)^*)$
$\Rightarrow (\lambda X.(Mx_1, Xx_1))(E(\lambda Y.(Lx_2, Yx_2))) \Rightarrow (Mx_1, (Lx_2, E_*(x_1, x_2)))$

3. In PFO, this holds without any α conversion, since no variable can occur both free and bound in a PFO formula. But TFO does not have the latter property.

(*if* a_1 *man encounters* a_2 *lion he*$_1$ *runs from it*$_2$)*
$\Rightarrow [(Mx_1, (Lx_2, E_*(x_1, x_2)))\ ,\ R_*(x_1, x_2)]$

Finally, we point out that we will also get donkey sentences with attitude verbs. For example,

(*if* a_1 *man believes he*$_1$ *owns* a_2 *donkey he*$_1$ *wants to beat it*$_2$)*
$\Rightarrow [\,(Mx_1, BEL(x_1, {}^\wedge (Dx_2, O_*(x_1, x_2))))\ ,\ WANT(x_1, {}^\wedge B_*(x_1, x_2))\,]$

Again we get universal quantification over (men and) donkeys. The intuitive *de dicto* reading of this sentence certainly appears to universally quantify over *something*. The best candidate seems to be so called objects of thought. The reading can hardly be captured by any informal analysis that would give wide scope to an attitude verb. Hence examples such as this present a rather strong case for a quantificational treatment and for an ontology of objects of thought.

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The Mereo-Topology of Event Structures

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Abstract. We hold that combining a mereological approach with a topological perspective provides a resourceful framework for the formal-ontological analysis of natural language semantics. In this spirit we present a general setting—using as primitives the relation of overlapping and a pure topological notion of boundary—which is meant to apply uniformly to as diverse domains as space, time, and the common-sense world. In particular, we focus on event-related issues and show how the temporal dimension can be reconstructed from the basic primitives. Illustrative examples include a discussion of some facts about present tense sentences and a tentative characterization of *Aktionsarten*-aspectual phenomena.

1. Introduction

Much recent work in philosophy, linguistics, and cognitive science has emphasized the role of mereology—the theory of parts and wholes—in providing a basis for formal-ontological investigations. In some cases, a mereological framework can even be viewed as a candidate alternative to set theory in laying down the foundations of semantic theorizing. However, reasoning about the common-sense world also shows that a purely mereological prospect may turn out to be too tight unless integrated with concepts and principles of a topological nature. There are in fact various reasons for this. One is that the notion of connectedness (or individual integrity) runs afoul of plain mereology, hence a theory of parts and *wholes* really needs to incorporate a topological machinery of some sort. A second reason becomes apparent particularly in connection with certain areas of artificial intelligence, most notably naive physics and qualitative reasoning about space and time: here mereology proves useful to account for certain basic spatio-temporal relations among the entities of ordinary discourse; but one needs topology to account e.g. for the fact that two events can be continuous with each other, or that something can be inside, abutting, straddling, or surrounding something else.

How exactly mereology and topology can be bridged and combined is in itself an interesting and rather unexplored issue. In this work we consider using two distinct primitives, a plain mereological relation of parthood and a pure topological notion of boundary. These two notions are axiomatized within the framework of a free quantification theory and the resulting system can be shown to subsume classical extensional mereology and to allow a rather natural reconstruction of a fair deal of standard topology. Moreover, the system supports a variety of derivative notions in terms of which several conceptual distinctions and taxonomies can be formulated that cannot easily be captured by means of standard set-theoretic tools.

The approach is meant to apply uniformly to as diverse domains as space, time, and the common-sense world. In particular, here we focus on events and event-related phenomena and point out how the temporal dimension can be reconstructed from the basic mereo-topological primitives. In the final part we argue that the framework thus advanced can be fruitfully applied to the analysis of natural language semantics and we provide some illustrative examples. These include a discussion of some facts about present tense sentences and a tentative characterization of *Aktionsarten*-aspectual phenomena.

2. Combining Mereology and Topology

As we mentioned, we sympathize with the view that mereology—as rooted in the work of Leśniewski [1916] and Leonard & Goodman [1940]—provides a resourceful alternative to set theory for the formal analysis of common-sense reality. Specifically with respect to events, this view goes back to Whitehead [1919] and has been defended e.g. by Thomson [1977] and, more recently, by Moltmann [1991] and Franconi *et al.* [1993]. At the same time, we urge that a mereological prospect need be supplemented with topological concepts and principles even for the purpose of very simple representations (Tiles [1981], Simons [1987, 1991a], Smith [1992, 1994]). With reference to Figure 1, for instance, it appears that reasoning exclusively in terms of parthood cannot do justice to the topological distinction between the two self-connected discs x and y (on the one hand) and the scattered sum z of their facing halves (on the other). Likewise, Figure 2 illustrates a simple case where mereological reasoning proves inadequate to account for relations that entail a step into the territory of topology. Mereologically the two patterns involve no difference, though of course we may want to keep track of the basic opposition in terms of inclusion of the square, x, inside the doughnut, y.

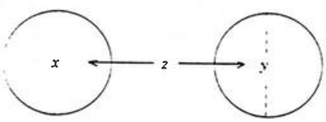

Figure 1

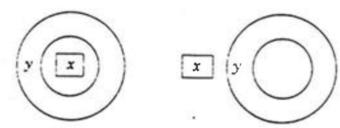

Figure 2

There are various ways in which the two domains of mereology and topology can be combined (see Eschenbach & Heydrich [1993] and Varzi [1994] for a first assessment). One can see them as two independent provinces (following in the footsteps of *inter alia* Tiles [1981], Lejewski [1982], and Smith [1994]); or one may grant priority to topology and characterize mereology derivatively, for instance defining parthood in terms of topological connection (as in Clarke [1981, 1985]). The latter approach is apparently more popular in artificial intelligence, and has been applied e.g. to spatio-temporal reasoning (Randell & Cohn [1989, 1992], Randell *et al.* [1992a, 1992b]) and to the analysis of spatial prepositions in natural language (Vieu [1991], Aurnague & Vieu [1993]). Indeed the approach proves fit to account for a fair deal of mereo-topological reasoning if we confine ourselves to an ontology of temporal intervals and/or spatial regions. If, however, we are to take an open-faced attitude towards real world entities and actual happenings (without identifying them with their respective spatio-temporal co-ordinates), then the reduction of mereology to a distinguished chapter of topology seems hardly tenable, as different entities can occupy exactly the same spatio-temporal regions (see Doepke [1982] or Simons [1986]). An object can be wholly located inside a hole, hence totally connected with it, without actually bearing any mereological relation to the hole (Casati & Varzi [1994]). Or two events may have exactly the same topological connections and yet be mereologically distinct, as with the rotating and the becoming warm of a metal ball that is simultaneously rotating and becoming warm (example from Davidson [1969]).

In general, therefore, we are inclined to favor the first option mentioned above, treating mereology and topology as conceptually independent domains. Formally this is reflected in our use of two distinct primitives, viz. a pure mereological notion of *part* and a pure topological notion of *boundary*. The basic principles axiomatizing these notions, along with some examples and illustrative comments, will be set forth in the following section. In the remaining of the paper we shall then consider in greater detail applications of the formalism to the study of events and event-related phenomena.

3. The Basic Formalism

3.1 Preliminaries. The entire mereo-topological machinery can be developed within a first-order language with identity and descriptions. We shall use '$\neg$', '$\wedge$', '$\vee$', '$\rightarrow$', and '$\leftrightarrow$' as connectives for negation, conjunction, disjunction, material implication, and material equivalence respectively; '$\forall$' and '$\exists$' for the universal and existential quantifiers; and 'ι' for the definite descriptor. To simplify readability, we shall rely on standard conventions to minimize the use of parentheses. In particular, we shall assume the five connectives to bind their arguments in decreasing order of strength as listed, so that negation binds the strongest and equivalence the weakest.

Note that the description operator will not have any use *per se*. However, it will play a crucial role in the definition of the term-forming operator of general sum in terms of which several basic mereological and topological notions will be characterized. (Alternatively one could use set variables along with a set fusion operator, as in Tarski [1937] or Leonard & Goodman [1940], but this method would introduce additional complications and would be in contrast with the above-mentioned foundational outlook on mereology.) Since this sum operator may be undefined for some arguments, the underlying logical apparatus requires therefore some means of accounting for the possibility of non-denoting expressions.

This of course can be done in a number of different ways. A preferred option is a free quantification theory with a supervaluational semantics (as in Bencivenga [1980] or Varzi [1983]). This would have the advantage of entertaining the unconditional acceptance of various "natural" principles that on a standard treatment *à la* Russell [1905], where descriptions are handled indirectly as improper symbols, would only be assertable under restriction. The following are some interesting cases in point (where 'Q' is any predicate and 'ϕ' an arbitrary formula):

(1) $Q(\iota x Q(x))$
(2) $y = \iota x(x=y)$
(3) $\iota x Q(x) = \iota x Q(x)$
(4) $\phi(\iota x Q(x)) \leftrightarrow \phi(\iota x Q(x))$
(5) $\exists y(y=\iota x\phi x) \leftrightarrow \exists y \forall x(\phi x \leftrightarrow x=y)$
(6) $\forall y(y=\iota x\phi x \leftrightarrow (\phi y \wedge \forall x(\phi x \rightarrow x=y))$

Each of these principles has been taken seriously by most free logicians ever since the pioneering work of Hintikka [1959] (see Lambert [1987] for an overview), and only a supervaluational semantics seems adequate to account for them in a systematic way. Unfortunately, however, this account is seriously defective from a computational perspective, as the set of valid principles is known to be not recursively enumerable.

In the face of this, among the several alternatives available (including Lesniewski's [1916] original approach) we find it convenient to rely instead on the minimal theory stemming from Lambert [1962], consisting in assuming (6) as the only specific principle for descriptive expressions. This theory is "minimal" insofar as it only captures the logic of 'ι' with respect to descriptions that are denotationally successful, leaving open the issue of what specific principles should continue to hold in the presence of referential gaps. It does, however, allow us to treat descriptive terms as genuine singular terms, and this is all we need for our present purposes. In fact, with slight modifications of the original argument of Van Fraassen & Lambert [1967], it can be shown that the system obtained by adding the principle in question to a free quantification theory, e.g. the one obtained from standard quantification theory by replacing the principle of Universal Instantiation with its universal closure

(7) $\forall y(\forall x\phi x \rightarrow \phi y)$,

is complete with respect to a semantics using partial models with classically saturated bivalent valuations. Thus, though too weak to validate (1), (2), or any other specific principle for descriptions, this system is strong enough to secure the validity of (3), (4), and any other propositional or quantificational laws for singular terms.

This minimalistic strategy has already been exploited in the context of mereological theorizing by Simons [1991b], and will prove sufficient also for our purposes in spite of failing to reveal the whole truth. In any case, to facilitate comparisons, we shall generally try to highlight those points where the choice of a different strategy may affect the theory.

3.2 Mereology. We symbolise the primitive mereological relation of parthood by 'P', so that 'P(x, y)' reads "x is (a) part of y". Derived notions, such as identity, overlapping, and the like, or the operations of sum, product, difference, etc. can be immediately defined:

DP1	$x=y$	$=_{df}$	$P(x,y) \wedge P(y,x)$	x is identical with y
DP2	$O(x,y)$	$=_{df}$	$\exists z\,(P(z,x) \wedge P(z,y))$	x overlaps y
DP3	$X(x,y)$	$=_{df}$	$O(x,y) \wedge \neg P(x,y)$	x crosses y
DP4	$PO(x,y)$	$=_{df}$	$X(x,y) \wedge X(y,x)$	x properly overlaps y
DP5	$PP(x,y)$	$=_{df}$	$P(x,y) \wedge \neg P(y,x)$	x is a proper part of y
DP6	$\sigma x \phi x$	$=_{df}$	$\iota x \forall y\,(O(y,x) \leftrightarrow \exists z\,(\phi z \wedge O(z,y)))$	sum of all ϕers
DP7	$\pi x \phi x$	$=_{df}$	$\sigma x\, \forall z\,(\phi z \rightarrow P(x,z)))$	product of all ϕers
DP8	$x+y$	$=_{df}$	$\sigma z\,(P(z,x) \vee P(z,y))$	sum of x and y
DP9	$x \times y$	$=_{df}$	$\sigma z\,(P(z,x) \wedge P(z,y))$	product of x and y
DP10	$x-y$	$=_{df}$	$\sigma z\,(P(z,x) \wedge \neg O(z,y))$	difference of x and y
DP11	$-x$	$=_{df}$	$\sigma z\,(\neg O(z,x))$	complement of x
DP12	υ	$=_{df}$	$\sigma z\,(z=z)$	universe

Note that each functor/term based on DP6 may be partially defined (i.e., correspond to an improper description) unless we go with the fiction of a null individual that is part of everything (as in Martin [1965, 1978]). For instance, non-overlapping entities will have no product and the universe will have no complement. This introduces a significant deviation in the obvious correspondence between mereological operations and the standard set-theoretical operations of union, intersection, etc.

As purely mereological axioms we assume the following two, along with the standard axioms for identity:

AP1 $\quad P(x,y) \leftrightarrow \forall z\,(O(z,x) \rightarrow O(z,y))$

AP2 $\quad \exists x \phi x \rightarrow \exists x \forall y\,(O(y,x) \leftrightarrow \exists z\,(\phi z \wedge O(z,y)))$

AP1 secures that parthood is an extensional partial ordering while AP2 guarantees that every satisfied condition ϕ picks out an entity consisting of all ϕers. This yields a classical mereology as usually understood, corresponding to a Boolean algebra with zero deleted (see Simons [1987]). A sample selection of specific theorems that will be relevant to the following is listed below:

TP1 $\quad P(x,x)$

TP2 $\quad P(x,y) \wedge P(y,z) \rightarrow P(x,z)$

TP3 $\quad x=y \leftrightarrow \forall z\,(P(z,x) \leftrightarrow P(z,y))$

TP4 $\quad x=y \leftrightarrow \forall z\,(P(x,z) \leftrightarrow P(y,z))$

TP5 $\quad x=y \leftrightarrow \forall z\,(O(x,z) \leftrightarrow O(y,z))$

TP6 $\quad PP(x,y) \rightarrow \exists z(P(z,y) \wedge \neg O(z,x))$

TP7 $\quad \neg P(x,y) \rightarrow \exists z(P(z,x) \wedge \neg O(z,y))$

TP8 $\quad \exists x \phi x \rightarrow \exists y(y = \sigma x \phi x)$

It may be worth recalling that none of these principles is uncontroversial. For instance, since Rescher [1955] several authors have had misgivings about such straightforward consequences of AP1 as TP2, expressing the transitivity of 'P' (see e.g. Cruse [1979], Winston *et al.* [1987], Moltmann [1990]), or TP3–TP5, expressing its extensionality (Wiggins [1979], Simons [1987]). In both cases, however, the objections involve reasoning about the variety of part-whole relations that may be distinguished (e.g., between components and complex, or quantity and mass) and may therefore be disregarded as long as we remain at a sufficiently general level of analysis. Even such an apparently innocent consequence of AP1 as the reflexivity of 'P' (TP1) might on some conditions be objected to, particularly insofar as the logical background that we are assuming allows for non-denoting terms. For instance, in this regard Simons [1991b] suggests applying the falsehood principle of Fine [1981] to deny that '$P(x,x)$' can be true when x does not exist. However, such a stance would introduce a disturbing asymmetry between parthood and other basic predicates such as identity—of which parthood is a generalization—unless we are also ready to make a self-identity statement '$x=x$' depend on the existence of x. This is in contrast with our general attitude towards free logic, which in fact follows the popular policy of assuming a standard identity theory. Therefore, also this type of objection will be disregarded in the following.

The second axiom is not uncontroversial either. For one thing, in the presence of AP1 it implies the so-called "supplementation" principles expressed by TP6–TP7, which some authors found reason to deny (see e.g. Chisholm [1978]). There are actually cases where restrictions on the domain of interpretation might involve violating such principles (a disc with a disc removed is not a disc), but this is already a matter of *material* ontology and need not concern us for the moment. As a matter of generality, the existence of a remainder between a whole and a proper part can hardly be denied: otherwise it would be possible for an object or event to have a single proper part, and that goes against any intuitive understanding of the very meaning of 'part'. Likewise, AP1 has often been disputed for having counter-intuitive instances when ϕ is true of scattered or otherwise ill assorted entities or events, such as the totality of red things, or Brutus' birth and his stabbing of Caesar (see e.g. the early criticisms of Lowe [1953]). From a purely mereological prospective, however, this criticism is also off target. If you already have some things, allowing for their sum is no further commitment: the sum *is* those things (Lewis [1991]; that the sum is always *uniquely* defined is guaranteed by TP8). In any case, one may feel uncomfortable with treating unheard-of mixtures as individual wholes; but which wholes are more "natural" than others is not a mereological issue. As noted above, the question of what constitutes a natural, integral whole cannot even be formulated in mereological terms: it is precisely here that topology—the theory of boundaries—comes in.

3.3 Topology. The primitive topological notion of boundary is symbolized by 'B', so that '$B(x, y)$' reads "x is a boundary in y". (Following Chisholm [1984], we say "boundary *in*" (rather than *of*) to avoid a reductive interpretation of boundaries as *maximal* boundaries. In general, any boundary *in* something is a boundary *of* some part thereof.) Some useful derived notions can be defined as follows:

DB1	$b(x)$	$=_{df}\ \sigma z\ (B(z, x))$	maximal boundary of x
DB2	$c(x)$	$=_{df}\ x+b(x)$	closure of x

DB3	$i(x)$	$=_{df}\ x - b(x)$	interior of x
DB4	$e(x)$	$=_{df}\ -x - b(x)$	exterior of x
DB5	$Cl(x)$	$=_{df}\ x = c(x)$	x is closed
DB6	$Op(x)$	$=_{df}\ x = i(x)$	x is open
DB7	$T(x, y)$	$=_{df}\ \exists z\ (P(z, x) \wedge B(z, y))$	x is a tangential part of y
DB8	$I(x, y)$	$=_{df}\ \exists z\ (Op(z) \wedge P(x, z) \wedge P(z, y))$	x is an interior part of y
DB9	$C(x, y)$	$=_{df}\ O(c(x), y) \vee O(c(y), x)$	x is connected with y
DB10	$EC(x, y)$	$=_{df}\ C(x, y) \wedge \neg O(x, y)$	x externally connects to y
DB11	$ST(x, y)$	$=_{df}\ \forall z(I(x, z) \rightarrow X(z, y))$	x straddles y
DB12	$Cn(x)$	$=_{df}\ \forall y \forall z\ (x = y + z \rightarrow C(y, z))$	x is self-connected
DB13	$k(x)$	$=_{df}\ \pi z(Cn(z) \wedge P(x, z))$	convex closure of x

Again, these functors, predicates, and relations may not be defined for certain arguments, as they may involve improper definite descriptions.

Note that nothing in these definitions implies that boundaries are always parts of the entities they bound. In fact, we accept the standard topological distinction between open and closed entities, allowing for events with external boundaries. (This marks a departure from Clarke [1981] and related AI work mentioned above, where boundary elements are not acknowledged thereby violating the supplementation principles TP6–TP7). We shall see below that this permits a natural characterization of the standard classification of event types (Vendler [1957]) by treating processes (such as *climbing the mountain*) as non-closed events, accomplishments (*having climbed the mountain*) as closed processes, and achievements (*reaching/having reached the top*) as parts of the corresponding boundaries.

In this regard, therefore, our basic axiomatization is in line with the familiar Kuratowski axioms, which we assume in the following form:

AB1	$P(x, c(x))$
AB2	$c(c(x)) = c(x)$
AB3	$c(x+y) = c(x) + c(y)$

This gives us a straightforward reformulation of standard topology based on mereology instead of set theory, provided only that we take care in handling undefined operators. In particular, it follows that the relation of connection is reflexive and symmetric and that boundaries are always symmetrical, in the sense that every boundary of an entity is also a boundary of the entity's complement. Here is a list of further theorems that can be proved from AB1–AB3 and that will be used in the following developments:

TB1	$B(x, y) \wedge B(y, z) \rightarrow B(x, z)$
TB2	$P(x, y) \wedge B(y, z) \rightarrow B(x, z)$
TB3	$I(x, y) \wedge P(y, z) \rightarrow I(x, z)$
TB4	$P(x, y) \wedge I(y, z) \rightarrow I(x, z)$
TB5	$B(x, y) \leftrightarrow \forall z(P(z, y) \rightarrow ST(z, y)$
TB6	$B(x, y) \leftrightarrow \forall z(P(z, y) \rightarrow T(z, y)$
TB7	$\forall x(\phi x \rightarrow B(x, y)) \rightarrow B(\sigma x \phi x, y)$

The last of these theorems is particularly important, as it shows that boundaries are closed under general sum and are therefore closed under all mereological properties. Following Smith [1994], however, we also wish to capture some further common-sense intuitions that go beyond the repertoire of standard topology. In particular, we need at least a rendering of the intuitive Brentanian idea that boundaries are "parasitic" entities, i.e., cannot exist independently of the things they bound (Brentano [1976]; cp. also Chisholm [1984], Smith [1992]). This stands in opposition to the standard set-theoretic conception of boundaries as sets of ordinary, ontologically independent points. More specifically, we assume that every self-connected boundary is a boundary part of some larger self-connected entity with a non-empty interior:

AB4 $\quad Cn(x) \wedge \exists y\, B(x, y) \rightarrow \exists z\, \exists w\, (Cn(z) \wedge B(x, z) \wedge P(x, z) \wedge I(w, z))$

It is understood that further principles would be needed to obtain at least a rough approximation of the folk theory of spatio-temporal continua. Here, however, we shall content ourselves with AP1–AP2 and AB1–AB4, regarding this as a minimal theory for the purpose of formal ontological representations.

Note that, within these limits, the main formal connection between our mereological and topological notions is neatly expressed by the following theorem:

TB8 $\quad P(x, y) \rightarrow \forall z\, (C(z, x) \rightarrow C(z, y))$

As already hinted at above, many systems in the tradition of Clarke [1981] also assume the converse of this principle, with the effect of reducing mereology to topology. By contrast, the possibility that topologically connected entities bear no mereological relationship to one another leaves room for a much richer taxonomy of basic mereo-topological relations than usually recognized. For instance, the customary relations of connection, overlapping, parthood, and interior parthood introduced above, and common to most known systems, can be integrated by the following (Varzi [1993]):

DB14	$E(x, y) =_{df} \forall z\, (C(z, x) \rightarrow C(z, y))$		x is enclosed in y
DB15	$S(x, y) =_{df} \exists z\, (E(z, x) \wedge E(z, y))$		x is superimposed on y
DB16	$A(x, y) =_{df} C(x, y) \wedge \neg S(x, y)$		x abuts y
DB17	$W(x, y) =_{df} E(x, i(y))$		x is within y

Evidently S is implied by O, E by P, and W by I. The resulting taxonomy of relations is depicted in Figure 3.

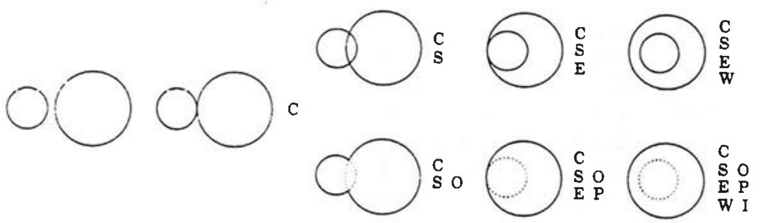

Figure 3

4. Event Structures

4.1 Preliminaries. The mereo-topological framework outlined above can be specialized for application to a variety of domains. Here we illustrate it in connection with some phenomena pertaining to event-based semantics.

One reason for this is that historically the first applications of mereology concern precisely this domain (Whitehead [1919]). As a matter fact, in recent times it has been rather popular to conceive of events as purely temporal entities. In forms ranging from the strong reductionist view that events are nothing but intervals *cum* description (in the spirit of Allen [1981, 1984]) to the weaker forms that take events as primitive entities connected by some primitive temporal relation (the strict ordering of Kamp [1979], van Benthem [1983], Bach [1986] or Link [1987]), this view has been predominant in AI, natural language semantics, and knowledge representation. By contrast, in the following we endeavour to show that events can be given an independent characterization that satisfies common-sense intuitions about their mutual relations.

Our characterization relies on the auxiliary concept of a *divisor*. Intuitively, a divisor may be thought of as a complex event whose maximal boundary is disconnected, i.e., an event that splits the entire event domain into two separate parts, thereby making it possible to choose one part as corresponding to the sum total of all events that temporally precede—and the other as the sum of all events that follow—the divisor itself. (The choice of which part is to count as preceding and which as succeeding will be arbitrary, as long as successive choices for different divisors be done in a consistent way: if we think of an event domain as comprising the totality of all happenings—past, present, and future—there is no *a priori* way to fix the temporal orientation.) Formally this is obtained by requiring every divisor to split every one of its neighbourhoods into disconnected parts:

DE1 $\quad D(x) =_{df} \forall y\, (I(x, y) \rightarrow \neg Cn(y{-}x))$

One may recognize here an analogue of the 1-codimensionality property that in set theory characterizes points with respect to the line, lines with respect to the plane, and so on (compare White [1993]).

Note of course that although every divisor is itself an event, not every event is a divisor. We may think of a divisor as a sort of cross event made up of all that happens during a certain "period". By contrast, an action such as Brutus' stabbing of Caesar, or an incident such as the sinking of the Titanic, are "localized" events that do not qualify as divisors: many other actions and events occurred at the same time, but in different places. This is of course a complication arising from our rejection of the identification of events with the intervals that they occupy. Events are fully-fledged entities, though we need not for this reason see them as endowed with a multidimensional part structure distinguishing for instance between spatial and temporal mereological relations (see Moltmann [1990] for a proposal in this direction).

We also want our construction to allow for a certain degree of control on its grain. Intuitively, the idea is that any two events that are parts of the same divisor should count as simultaneous. However, for that purpose we need some means of associating each event in the domain with the "right" divisor, as it were, i.e., the smallest one containing it. (Note in fact that for any two events x and y one can generally pick out a sufficiently large divisor z containing both.) Furthermore, the possibility of varying the grain itself may be a welcome one. For instance, it could be helpful to account for the various degrees of precision that natural language permits when talking about events and time. Thus, in general reference to the predicate 'D' will not be sufficient, for it only captures one (ideal) possible way of defining a consistent segmentation of the universe. We should rather like to rely on a more general notion of a divisor, one of which 'D' itself is a special case but which may very well yield different result for different cases. We shall, in other words, rely on what we call an *event structure*.

4.2 Characterization. We define an *event structure* quite generally as a triple $\langle E, \delta, f\rangle$ made up of a domain of events, E, and two functions. The conditions are as follows. First, we assume the domain E to be mereo-topologically self-connected:

AE1 $\forall z(\mathrm{O}(z,x) \vee \mathrm{O}(z,y)) \rightarrow \mathrm{C}(x,y)$

We are here taking E as a domain satisfying all mereological and topological axioms set forth in the previous section, taking variables to range over this domain. Thus, AE1 could also be written as

AE1' $\mathrm{SC}(\upsilon)$

which effectively corresponds to the statement that the universe is self-connected. This is stipulative, but reflects the idea that there are no gaps in history: there is always something happening, whether remarkable or not. (If the universe consisted of two or more separate parts, one could still apply the reasoning below to each of these parts, ending up with a family of disconnected worlds each having its own temporal ordering. Here we shall not pursue this possibility to keep things simple and intuitively straightforward.)

Second, δ is a divisor-specific condition incorporating the above-mentioned requirement of minimality. It satisfies the following axioms:

AE2 $\delta x \rightarrow \mathrm{D}(x)$
AE3 $\sigma x \delta x = \sigma x \mathrm{D}(x)$
AE4 $\forall x\,(\phi x \rightarrow \delta x) \rightarrow \delta(\mathrm{k}(\sigma x \phi x))$
AE5 $\forall x\,(\phi x \rightarrow \delta x) \rightarrow \pi(\sigma x \phi x)$

In other words, the events fulfilling δ are all divisors, covering the entire domain of divisors and such that the connected closure as well as the product of any number of them is itself a divisor. (We have assumed nothing concerning the possibility that the universe be bounded somewhere. If it is, for instance if it has a starting event—a sort of Big Bang—,then notice that such an event will not qualify as a divisor and will not be part of any divisor regardless of how we choose δ.)

Note that in set-theoretic terms the last condition on δ (AE5) means that divisors form a closure system. The corresponding closure operator associates each event x with the *smallest divisor* containing x (relative to the conditions specified by δ). We define it as follows:

DE2 $\quad d(x) =_{df} \pi z(\delta z \wedge P(x, z))$

That this is indeed a closure operator is guaranteed by the following facts, which can be regarded as the analogues of the usual increasing, idempotency, and monotonicity conditions:

TE1 $\quad P(x, d(x))$
TE2 $\quad d(d(x)) = d(x)$
TE3 $\quad P(x, y) \rightarrow P(d(x), d(y))$

Finally, the last term in the definition of an event structure is a (possibly partial) function $f: E \rightarrow E$ closed under the following four conditions:

AE6 $\quad f(x) = f(d(x))$
AE7 $\quad f(x)+f'(x) = -d(x)$
AE8 $\quad P(x, f(y)) \rightarrow P(f(x), f(y)))$
AE9 $\quad D(x) \wedge D(y) \wedge O(y, f(x)) \wedge O(y, f'(x)) \rightarrow P(x, y)$

where in general

DE3 $\quad f'(x) =_{df} \sigma z\ (P(x, f(z)))$.

Intuitively, our purpose is to regard f as a function of temporal orientation associating each event in the given domain with the totality of events that preceded it. Correspondingly, f' will map each event to its future events, and the above axioms are to secure that the mapping be coherent, as it were, throughout the entire domain of events. (In particular, AE6-AE7 relate this interpretation to the intuitive interpretation of the divisor operator.) More precisely, since f is only defined for events that have a minimal divisor, i.e., only events that are temporally bounded "on both sides", we shall for this purpose expand f and f' to a new pair of functions f^* and $f^{*\prime}$ as follows:

DE4 $\quad f^*(x) =_{df} \pi z\ \exists y\ (P(y, x) \wedge z=f(y))$
DE4' $\quad f^{*\prime}(x) =_{df} \pi z\ \exists y\ (P(y, x) \wedge z=f'(y))$

As is easily verified, these new functions coincide with f and f' whenever $d(x)$ is defined, and vice versa. Unlike f and f', however, reference to f^* and $f^{*\prime}$ allows us to express the fact that the infinitely extending sum of all future events follows each past event, or that the infinitely extending sum of all past events is followed by each future event. (In fact, note that in such cases, where $d(x)$ is undefined and $x \neq \upsilon$, $f^*(x)$ is defined iff $f^{*\prime}(x)$ is not defined.)

The behavior of these functions is schematically illustrated in Figure 4 below, which portrays a section of a simple event structure showing an event e together with its minimal divisor $d(e)$. (Events are indicated by line segments, boundaries by vertical thick traits.) Note that the event-sum k cannot qualify as a divisor of e', due to condition AE9.

Further constraints on E, δ, or f can possibly be added. For instance, we may want to rule out the possibility that there be atomic events (i.e., events with no proper parts) that are open, or we may want to impose stronger conditions than self-connectedness on E. Moreover, let us stress again that the operators and mappings introduced above may be not defined for some arguments, and that different models could therefore be obtained by changing the underlying logic and/or semantics.

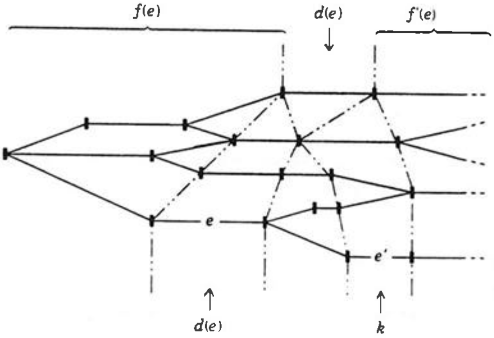

Figure 4

4.3 Remark. We can show that if $\langle E, \delta, f\rangle$ is an event structure, the following conditions are generally satisfied:

TE4 $\quad P(x, f^*(y)) \rightarrow P(f^*(x), f^*(y)))$
TE5 $\quad P(x, y) \rightarrow P(f^*(y), f^*(x))$
TE5' $\quad P(x, y) \rightarrow P(f^{*\prime}(y), f^{*\prime}(x))$
TE6 $\quad P(x, f^*(y)) \leftrightarrow P(y, f^{*\prime}(x))$
TE7 $\quad \neg O(f^*(x), f^{*\prime}(x))$

These theorems justify our suggested interpretation of f^* and $f^{*\prime}$. Moreover, we can prove the following:

TE8 $\quad P(x, f^{*\prime}(f^*(x)))$
TE8' $\quad P(x, f^*(f^{*\prime}(x)))$

which together with TE5 and TE5' ensure that f^* and $f^{*\prime}$ behave as a pair of Galois connection in E.

These facts allow us to intuitively think of E as a domain of events and f^* as a function of *temporal orientation*. In particular, we can define a relation of *temporal overlap* and a corresponding relation of temporal *precedence* as follows:

DE5 $\quad Ot(x, y) =_{df} \exists z \exists w(P(z, x) \wedge P(w, y) \wedge O(d(z), d(w))$
DE6 $\quad Pr(x, y) =_{df} P(x, f^*(y))$

(In DE5, reference to parts is necessary, since the minimal divisors of x and y may themselves be undefined).

The important thing to observe now is that these relations are can be shown to satisfy the usual conditions on temporal relations of Kamp [1979]:

TE9	$Ot(x, x)$
TE10	$Ot(x, y) \to Ot(y, x)$
TE11	$Pr(x, y) \to \neg Pr(y, x)$
TE12	$Pr(x, y) \wedge Pr(y, z) \to Pr(x, z)$
TE13	$Pr(x, y) \to \neg Ot(x, y)$
TE14	$Pr(x, y) \wedge Ot(y, z) \wedge Pr(z, w) \to Pr(x, w)$
TE15	$Pr(x, y) \vee Ot(x, y) \vee Pr(y, x)$

Thus, although the basic formal ontological machinery is silent on time, the temporal dimension can be easily regained within event structures, at least relative to the usual characterization: all we need to do is to take divisors as the counterparts of intervals.

5. Applications to Natural Language Semantics

It is understood that event structures are only a first, very rough indication of how the mereo-topological framework advocated here can be specialized for application to event-related phenomena. Arguably, they can already provide a useful perspective from which to address in a uniform fashion various issues that are typically associated with treating events as *bona fide* individuals, including philosophical issues concerning e.g. the characterization of adequate identity and individuation criteria for such entities (see Bennett [1988] for a general overview). Here, however, we shall confine ourselves to some illustrative applications to natural language semantics.

The idea that we are going to pursue is that eventive predicates can impose conditions that restrict the particular structures availabe for the purposes of semantic interpretation. Note in fact that event structures are entities strongly dependent on the choice of a specific condition δ: divisors are events that separate the past from the future; but what exactly is to count as a relevant divisor, i.e. what is the granularity of the temporal segmentation, is not fixed once and for all. Actually, one may observe that a structure's orientation function f—and, consequently, its extension f^*—is completely determined by δ, free choice being limited to associating it with the totality of preceding events or with the totality of events that follow. Thus, varying the conditions on the component δ is the main formal instrument at our disposal to approach different issues within the uniform framework provided by event structures. Our examples below will illustrate this with specific reference to some tense and aspect phenomena.

5.1 Present tense and imperfective readings. Our first application concerns some facts about present tense. As is well known, in this respect English differs from all other Germanic and Romance languages in that it does not allow for the *continuous* or *imperfective* reading of present tense sentences involving accomplishment and activity predicates (we do not consider progressive forms here). For instance, (8) below cannot mean that John is presently involved in an act of eating an apple; and likewise, (9) does not mean that John is presently involved in an act of running. By contrast, both readings are available in the other languages, as exemplified here by the Italian translations (8') and (9'):

(8) John eats an apple
(9) John runs
(8') Gianni mangia una mela
(9') Gianni corre

Furthermore, neither English nor the other Germanic and Romance languages—and we suspect no language *tout court*—allows for the *perfective* reading: for instance, (8) or (8') cannot mean that an act of eating an apple was performed and completed by the subject at the speech time (where we take the notion of completion to be a crucial one for defining perfectiveness).

This peculiar behaviour of the English present tense with accomplishments and activities can be related to the more generally manifested impossibility of present tense perfective readings (Giorgi & Pianesi [1992]). More precisely, it can be argued that the English verbal stem has the status of a full word, or, in X-bar theoretic terms, it is an X^0 (Roberts [1993]; see also Giorgi & Pianesi [199+]), whereas it is a bound morpheme (an X^{-1}) in the other languages. In a minimalist perspective (Chomsky [1992]) one can see these differences as fixed by processes operating before the insertion of a lexical item in the syntax. In English the lexical determination of the verbal stem as a full word is accompanied by the assignment of a *[Perf+]* feature to the stem itself; in the other languages the verbal stem remains perfectively neutral. Accordingly, English verbal stems always surface with a *[Perf+]* feature.

From this perspective, therefore, the reason why (8) and (9) cannot have a continuous interpretation is that the verbs 'eats' and 'runs' are perfective. But why is it that perfective readings cannot be obtained with the present tense? One possible account would be to exploit the alleged punctuality of the utterance time (Dowty [1979], Smith [1991]). Since the present tense requires the event time to be co-temporaneous with the utterance time, the fact that the latter is punctual makes it impossible to accommodate, so to speak, an event having duration within it. On this account, however, it is not clear why it is possible to have present tense imperfective readings (usually, the events they refer to have a duration). Furthermore, the account would predict that no similar restrictions should hold when the event in question is itself punctual. Achievements are aften regarded as good candidates for punctuality; however, it is a fact that when achievement predicates are involved, as in (10)–(10'), the imperfective reading of present tense is blocked not only in English, but also in Italian and the other Germanic or Romance languages:

(10) John finds a book
(10') Gianni trova un libro

Reference to the general mereo-topological framework set up in the previous section allows us to conjecture a different explanation. Generally speaking, our suggestion is that the reason why present tense does not support perfective readings has to do with the kind of restrictions that the *Aktionsarten* of eventive predicates and aspectual factors place on the choice of the divisor condition δ in the relevant event structure—that is, ultimately, on the temporal structure that is induced by the domain of events on which the discourse is being interpreted. In particular, some of these restrictions can be characterized in terms of a notion of punctuality which is made available by the mereo-topology of event structures. Building on previous work (Franconi *et al.* [1993]), we maintain that some sort of minimality is crucial for a proper understanding of punctuality, in the intuitive sense that a punctual event cannot accommodate more structured ones. This is in line with the account put forward in Kamp [1979]. However, here we do not consider the distinction between istants and intervals, and more generally any distinction based on some absolute notion as size or duration, to be the relevant parameters. We also differ from Kamp in not imposing any specific axioms for characterizing punctuality. Rather, the same properties proposed by Kamp will be derived from more basic aspects of our constructions.

As a first step, let us introduce the notion of a *minimal divisor* (relative to a given event structure $\langle E, \delta, f\rangle$):

DL1 $\mathrm{MD}(x) =_{\mathrm{df}} \delta x \wedge \forall y(\mathrm{P}(y,x) \rightarrow \neg\delta y)$

Thus, a divisor x is minimal iff it does not contain other divisors (relative to the same structure). As a consequence, every event which is part of such an x has x as its divisor:

TL1 $\mathrm{MD}(x) \wedge \mathrm{P}(y,x) \rightarrow \mathrm{d}(y)=x$

In a sense, "temporal" differences are neglected inside a minimal divisor; any two events that are parts of such a divisor are co-temporaneous:

TL2 $\mathrm{MD}(x) \wedge \mathrm{P}(y,x) \wedge \mathrm{P}(z,x) \rightarrow y =_t z$

where the relation '$=_t$' of temporal coincidence is defined in the obvious way using 'Ot' (compare TP5):

DL2 $y =_t z =_{\mathrm{df}} \forall w(\mathrm{Ot}(y,w) \leftrightarrow \mathrm{Ot}(z,w))$

More generally, we have

TL3 $\mathrm{MD}(x) \wedge \mathrm{Ot}(y,x) \wedge \mathrm{Ot}(z,x) \rightarrow \mathrm{Ot}(y,z)$
TL4 $\mathrm{MD}(x) \wedge \mathrm{P}(w,x) \wedge \mathrm{Ot}(y,w) \wedge \mathrm{Ot}(z,w)) \rightarrow \mathrm{Ot}(y,z)$.

Thus, if two events temporally overlap a minimal divisor (or a part thereof) they temporally overlap each other. Vice versa, we have that:

TL5 $\delta x \wedge \forall y \forall z(\mathrm{Ot}(y,x) \wedge \mathrm{Ot}(z,x) \rightarrow \mathrm{Ot}(y,z)) \rightarrow \mathrm{MD}(x)$.

Putting TL3 and TL5 together, we can see that the fundamental properties characterizing punctual events according to Kamp hold of minimal—and only minimal—divisors. We can then propose the following definition for punctual events:

DL3 $\mathrm{PE}(x) =_{\mathrm{df}} \mathrm{MD}(\mathrm{d}(x))$.

Thus, punctual events are not just—and not necessarily—atomic events, i.e. events with no proper parts (though of course every atomic event is punctual regardless of δ). Rather, they are events whose internal structure is irrelevant for the purpose of temporal distinctions. Moreover, a major advantage of this notion of punctuality over Kamp's is that it is relativized with respect to the particular event structure at hand (and, ultimately, with respect to the particular divisor conditin δ), while retaining Kamp's basic insights. Changing δ, previuosly punctual events may become non-punctual, in that their internal temporal structure is made available, and vice versa. On the other hand, once we declare a given event to be punctual, we place restrictions on the kind of event structures it can be an element of and, eventually, on the particular δ's available.

With this notion at hand, we can now address our question on present tense perfective readings. If, as we suppose, utterance events in a sentence are conceived of as punctual, then we propose that the unavailability of present tense perfective readings of eventive senteces is due to *conflicting requirements* introduced by punctuality and perfectiveness. On the one hand, punctuality needs that "temporal" structure be neglected; on the other hand perfectiveness, as applied to processes (activities and accomplishments), makes it available.

5.2 Perfectivity. We now proceed to a characterization of process-type events and, more generally, to a revisitation of the classical distinction between processes or activities (on the one hand) and accomplishments and achievements (on the other). This proceeds from the remark that perfectiveness does not add anything to the temporal structure of a given event; it rather forces considering the event itself as a whole. Temporal structure is introduced by the *Aktionsarten* of the predicate. For example, Dowty [1979] proposed the so called *activity postulate* to characterize activities. It states that if A is an activity and A is true at an interval i, then A is true at every reasonably large subinterval of i (see also Landman [1992]). This idea can be extended to processes in general in terms of event structures.

First of all, in order to make the internal structure available, we can state that a process cannot be punctual; by DL3 this means that an event x can only be a process if $d(x)$ is not a minimal divisor in the given event structure. In the second place, the grain of the process cannot be too fine. That is, if $d(x)$ contains a minimal divisor y, then either y does not overlap x or it contains a part of x that is of the same type as x, i.e., satisfies the same basic predicate as x. Accordingly, if ϕ is x's basic predicate, we can take the following two postulates as expressing two necessary conditions for an event x to qualify as a process:

PR1 $\neg PE(x)$
PR2 $MD(y) \wedge P(y, d(x)) \wedge O(y, x) \rightarrow \exists z\, (P(z, y) \wedge PP(z, x) \wedge \phi z))$

Note that, because of AE4, these postulates naturally allow for pauses during the process, capturing what the "reasonably large sub-interval" condition was intended to capture in Dowty's postulate. Thus, a process need not be a self-connected event: the vacuuming of the carpets (example from Thomson [1977]) may be a discontinuous event involving other activities.

Imperfective (neutral) aspect does not add anything to this picture. Therefore, it is possible for tense to pick out proper parts of an event. Given the crucial role played by the restrictions on divisors, we can make this explicit by assuming that eventive verbs have two Davidsonian variables: the first ranges over events, as in Davidson [1967]. The second variable ranges specifically over divisors and plays the role of the temporal variable in other approaches. Tense and aspect operate on this second variable. (In the syntax, Aspect is a category c-commanding V and c-commanded by tense (Cinque [1993]). By specifier head agreement, when Asp^0 has the feature *[Perf+]*, its specifier is occupied by a (possibly null) sum operator binding the divisor variable of the verb. Tense is conceived of as a monadic predicate which T-marks AspP (Giorgi & Pianesi [199+]). For our present purposes, we may assume that the eventive variable gets bound by means of the default existential closure operator. When aspect remains neutral (or imperfective, for our present purposes the difference does not matter), then no value for the *Perf* feature is specified and default existential closure intervenes.)

When aspect *is* perfective, then tense picks out the *whole* event. We can then represent a neutral (imperfective) reading of a present tense sentence as in (11), and a perfective reading as in (12), where '*u*' denotes the utterance event and ϕ is, as before, x's basic predicate:

(11) $\exists x \exists y\, (\phi x \wedge \delta y \wedge P(y, d(x)) \wedge d(u) =_t y)$
(12) $\exists x\, (\phi x \wedge d(u) =_t d(x))$

However, when x is a process there is no event structure that can satisfy (12) because the requirements in PR1 conflicts with those on u (punctuality) and this explains the non-availability of the perfective reading with present tense eventive sentences.

5.3 More on the mereo-topology of Aktionsarten. We only gave two basic postulates that an event must satisfy to qualify as a process. A more complete charaterization of *Aktionsarten* would require considering the topological properties of events a bit closer. In particular, the following requirements may be added to PR1–PR2 as postulates that an event x must satisfy in order for it to qualify as a process, capturing the intuition that processes must not be closed but only initially bounded:

PR3 $\neg Cl(x)$
PR4 $c(x) \times c(f^*(x)) = x \times b(x)$

Here the left term of the identity in PR4 may be thought of as the *left limit* of the process; its *right limit* would represent its culmination and can be defined symmetrically by reference to $f^{*\prime}$ instead of f^*:

DL4 $l(x) =_{df} c(x) \times c(f^*(x))$
DL5 $r(x) =_{df} c(x) \times c(f^{*\prime}(x))$

(Perfective events are always topologically closed. The importance of the closure properties for aspectual purposes would deserve more discussion than is possible here. There is evidence to suppose that closure plays the main role in perfectiveness, so that *[Perf+]* shoud be better understood as *[Closed+]*.)

One interesting consequence is that certain peculiarities of achievements could now be explained if we regard them as composed of two parts: a culmination, to which the achievement predicate directly refers, and a process leading to it, whose existence is simply entailed. On this view, accomplishments would then be closed events, whereas achievements could be characterized as the right limits of processes (and therefore as ontologically dependent entities in the sense of AB4). Using 'Pro' to indicate processes (as characterized by PR1–PR4), we obtain the following definitions:

DL6 $\mathrm{Acc}(x) =_{df} \exists y(\mathrm{Pro}(y) \wedge x=\mathrm{c}(y))$
DL7 $\mathrm{Ach}(x) =_{df} \exists y(\mathrm{Pro}(y) \wedge x=\mathrm{r}(y))$

The behaviour of achievements with the present tense (as exemplified by (10)–(10')) would then follow.

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Equality in Labelled Deductive Systems and the functional interpretation of propositional equality*

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Abstract

Within the context of natural deduction for *Labelled Deductive Systems*, we formulate what appears to be a middle ground solution to the 'intensional' vs. 'extensional' dichotomy which permeates most of the work on characterising propositional equality (as in, e.g., P. Martin-Löf's type theories). The intensional aspect is dealt with in the functional calculus on the labels, whereas the extensionality is kept to the logical calculus on the formulas. Equalities which are dependent on the deduction/computation path (context) are handled by the functional calculus on the labels. Those equalities are usually definitional, and may come from the 'geometry' of deduction (e.g. β, η, ζ, μ), and thus carry essentially 'intensional' information. On the other hand, equality in the logical calculus (propositional equality) is essentially 'extensional' as it refers to the 'existence' of a way of rewriting a referent into another one.

We look at propositional equality ('$\doteq$') as a 'Skolem-type' connective (such as disjunction and existential quantification), where notions like 'dependent variables' and 'choice' play a crucial role. This means that in the elimination rule for '$\doteq$' we need to introduce identifiers (new symbols) for compositions of equalities denoting arbitrary rewriting paths. We believe this provides a new perspective on the connections Gentzen–Herbrand (i.e. the 'sharpened Hauptsatz').

1 Introduction

The functional interpretation of logical connectives via deductive systems which use some sort of labelling mechanism (Martin-Löf 1984,Gabbay 1991) can be seen as the basis for a general framework characterising logics via a clear separation between a functional calculus on the *labels*, i.e. the referents (names of individuals, expressions denoting the record of proof steps used to arrive at a certain formula, names of 'worlds', etc.) and a logical calculus on the formulas. The key idea is to make these two dimensions as harmonious as possible, i.e. that the functional calculus on the labels matches the logical calculus on the formulas at least in the sense that to every abstraction on the variables of the functional calculus there corresponds a discharge of an assumption-formula of the logical calculus. One aspect of such interpretation which stirred much discussion in the literature of the past ten years or so, especially in connection with *Intuitionistic Type Theory* (Martin-Löf 1984), was that of whether the logical connective of propositional equality ought to be dealt

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with 'extensionally' or 'intensionally'. Here we attempt to formulate what appears to be a middle ground solution, in the sense that the intensional aspect is dealt with in the functional calculus on the labels, whereas the extensionality is kept to the logical calculus. We also intend to demonstrate that the connective of propositional equality (cf. (Aczel 1980)'s '≐') needs to be dealt with in a similar manner to 'Skolem-type' connectives (such as disjunction and existential quantification), where notions like *hiding*, *choice* and *dependent variables* play crucial rôles.

Our motivation: where did it all start?

The characterisation of a proof theory for *Labelled Deductive Systems* has been our concern for some time now.[1] Here we address two topics of special interest to logic and computation, namely *substitution* and *unification*. As a starting point, we posed ourselves two interrelated questions: how could we incorporate the handling of rewrites and function symbols into the proof theory, and how could we 'give logical content', so to speak, to the procedures coming from unification algorithms?

For those not familiar with the *LDS* perspective, it suffices at this stage to say that the declarative unit of logical systems is seen as made up of two components: a formula and a label. The label is meant to carry information which may be of a less declarative nature than that carried by the formulas. The introduction of such an 'extra' dimension was motivated by the need to cope with the demands of computer science applications.

Indeed, with the diversification of computer science applications to problems involving *reasoning*, there has been a proliferation of logics originated mainly from the need to tailor the logical system to the demands of the particular application area. If there were a number of 'logics' already developed and well established in the mathematical and philosophical logic literature (relevant, intuitionistic, minimal, etc.), the diversification was significantly increased with the contribution from computer science.

Gabbay observed that many of the distinctive features of most logics being studied by logicians and computer scientists alike, stemmed from 'meta-level' considerations: in order to consider a step to be a valid one, it was invariably the case that one had to take into account questions like: 'whether the assumptions have actually been used'; 'whether they have been used in a certain order'; 'whether the number of times an assumption was used has been in keeping with the need to take care of resources'; etc.

There are a number of inconveniences in having to cope with increasingly diverse logical systems, and Gabbay set out a research programme with at least the following desiderata:

- to find a unifying framework (sequent calculus by itself would not do, and we shall see why later on) *factoring out* meta- from object- level features;
- to keep the logic (and logical steps, for that matter) simple, handling meta-level features via a *separate*, yet *harmonious* calculus;
- to have means of structuring and combining logics;
- to make sure the relevant assumptions in a deduction are uncovered, paying more attention to the explicitation and use of resources.

The idea of *labelled deduction* seemed to be a natural evolution from the traditional logical systems. The development of a novel approach to logic, namely *Labelled Deductive Systems*, where the meta-level features would be incorporated into the deductive calculus in an orderly manner, looked general enough to be an appropriate candidate for such an unifying framework.

In summary, it seems fair to say that *Labelled Deductive Systems* offer a

1. (Gabbay and de Queiroz 1992; de Queiroz and Gabbay 1991; Gabbay and de Queiroz 1991; de Queiroz and Gabbay 1993).

new perspective on the discipline of *logic and computation*. Arising from computer science applications, it provides the essential ingredients for a framework whereby one can study:

- meta-level features of logical systems, by 'knocking down' some of the elements of the meta-level reasoning to the object-level, and allowing each logical step to take care of *what has been done so far*;
- the 'logic' of Skolem functions and substitution (dependencies, term declaration).

Why sequent calculus by itself will not do. Boole did manage to formalise the algebra of logical connectives, with the aspect of *duality* coming out very neatly. The sequent calculus follows on this quest for duality:

$$\begin{array}{lcl} \text{negative} & \vdash & \text{positive} \\ \text{conjunctive} & \vdash & \text{disjunctive} \end{array}$$

Nevertheless, since Frege, logic is *also* about quantifiers, predicates, functions, equality among referents, etc. In a few words: beyond *duality*, first order logic also deals with *quantification* in a *direct* fashion, instead of via, say, Venn diagrams. Thus, a proof theory for first order logic ought to account for the manipulation of function symbols, terms, dependencies and substitutions, as Herbrand already perceived.

We shall see a little more about this later on when we come to a brief discussion of the so-called 'sharpened Hauptsatz'. But it seems appropriate to add here that we are looking for strengthening the connections between Gentzen's and Herbrand's methods in proof theory. We believe that the two-dimensional approach of *LDS* is the right framework for the enterprise. This is because, if, on the one hand:

(+) Gentzen's methods come with a well defined mathematical theory of proofs,

and

(+) Herbrand's method show how to handle function symbols and terms in a direct fashion,

on the other hand:

(−) in Gentzen's calculi (plain natural deduction, sequent calculus) function symbols are not *citizens*,

and

(−) Herbrand's methods hardly give us means of looking at proofs (deductions) as the main objects of study.

By combining a functional calculus on the labels (which carry along referents, function symbols) with a logical calculus on the formulas, the *LDS* perspective can have the (+)'s without the (−)'s.

The generality of Herbrand base

Let us take the example which Leisenring uses to demonstrate the application of Herbrand's decision procedure to check the validity of the formula Leisenring 1969:

$$\exists x^{\mathbf{D}}.\forall y^{\mathbf{D}}.(\mathbf{P}(x) \rightarrow \mathbf{P}(y))$$

Herbrand's 'original' procedure. The first step is to find the Herbrand resolution (∃-prenex normal form), which can be done by introducing a new function symbol g, and obtaining:

$$\exists x^{\mathbf{D}}.(\mathbf{P}(x) \rightarrow \mathbf{P}(g(x)))$$

As this would be equivalent to a disjunction of substitution instances like:

$$\mathbf{P}(a) \rightarrow \mathbf{P}(g(a)) \quad \vee \quad \mathbf{P}(a') \rightarrow \mathbf{P}(g(a')) \quad \vee \quad \mathbf{P}(a'') \rightarrow \mathbf{P}(g(a'')) \quad \vee \quad \cdots$$

the second step is to find a p-substitution instance (p finite) which is a tautology. For that, we take the 'Herbrand base' to be $\{a, g\}$, where a is an arbitrary individual from the domain, and g is an arbitrary function symbol which can construct, out of a, further elements of the domain. Thus, the 1-substitution instance is:

$$\mathbf{P}(a) \rightarrow \mathbf{P}(g(a))$$

which is clearly not a tautology. Now, we can iterate the process, and find the 2-reduction as a disjunction of the 1-reduction and the formula made up with a 2-substitution (taking $a' = g(a)$), that is:

$$\mathbf{P}(a) \rightarrow \mathbf{P}(g(a)) \quad \vee \quad \mathbf{P}(g(a)) \rightarrow \mathbf{P}(g(g(a)))$$

which is a tautology.

In summary:

1. $\exists x^{\mathbf{D}}.\forall y^{\mathbf{D}}.(\mathbf{P}(x) \rightarrow \mathbf{P}(y))$
2. take g as a unary function
3. $\exists x^{\mathbf{D}}.(\mathbf{P}(g(x)) \rightarrow \mathbf{P}(y))$
4. $\mathbf{P}(a) \rightarrow \mathbf{P}(g(a)) \quad \vee \quad \mathbf{P}(a') \rightarrow \mathbf{P}(g(a')) \quad \vee \quad \mathbf{P}(a'') \rightarrow \mathbf{P}(g(a'')) \quad \vee \quad \dots$
5. 1^{st} substitution: $\mathbf{P}(a) \rightarrow \mathbf{P}(g(a))$
6. take $a' = g(a)$
7. 2^{nd} substitution: $\mathbf{P}(a) \rightarrow \mathbf{P}(g(a)) \quad \vee \quad \mathbf{P}(g(a)) \rightarrow \mathbf{P}(g(g(a)))$ (tautology)

In checking the validity of $\exists x^{\mathbf{D}}.\forall y^{\mathbf{D}}.(\mathbf{P}(x) \rightarrow \mathbf{P}(y))$ we needed the following extra assumptions:

1. the domain is non-empty (step 4).
2. there is a way of identifying an arbitrary term with another one (step 6).

As we shall see below, the labelled deduction method will have helped us 'to bring up to the surface' those two (hidden) assumptions.

Now, how can we justify the *generality* of the 'base' $\{a, g\}$? Why is it that it does not matter which a and g we choose, the procedure always works? In other words, why is it that *for any* element a of the domain and *for any* 'function symbol' g, the procedure always works?

In a previous report (de Queiroz and Gabbay 1991) we have already demonstrated the *universal force* which is given to Skolem functions by the device of *abstraction* in the elimination of the existential quantifier. The point was that although there was no quantification over function symbols being made in the logic (the logical calculus on the formulas, that is), an abstraction on the name for the Skolem function was performed in the functional calculus on the labels. The observation suggested that, as in the statement of Skolem's theorem, *for any* (new) function symbol f we choose when Skolemising $\forall x^{\mathbf{D}}.\exists y^{\mathbf{D}}.\mathbf{P}(x, y)$ to $\forall x^{\mathbf{D}}.\mathbf{P}(x, f(x))$, if an arbitrary statement can be deduced from the latter then it can also be deduced from the former, regardless of the choice of f. We shall come back to this point later on when we will then demonstrate that in our labelled natural deduction the Herbrand function gets abstracted away thus getting universal force.

1.1 Gentzen–Herbrand connections: the sharpened Hauptsatz

The connections between the proof theory as developed by Gentzen and the work on the proof theory of first order logic by Herbrand are usually seen through the so-called 'sharpened Hauptsatz'.

Theorem 1.1 (Gentzen's sharpened Hauptsatz) *Given $\Gamma \vdash \Delta$ (prenex formulae), if $\Gamma \vdash \Delta$ is provable then there is a cut-free, pure-variable proof which contains a sequent $\Gamma' \vdash \Delta'$ (the* midsequent*) with:*

1. *Every formula in $\Gamma' \vdash \Delta'$ is quantifier free.*
2. *No quantifier rule above $\Gamma' \vdash \Delta'$.*

3. *Every rule below $\Gamma' \vdash \Delta'$ is either a quantifier rule, or a contraction or exchange structural rule (not a weakening).*

The theorem relies on the so-called *permutability* lemma which says that quantifier rules can always be 'pushed down' in the deduction tree.

Lemma 1.2 (Permutability) *Let π be a cut-free proof of $\Gamma \vdash \Delta$ (with only prenex formulas): then it is possible to construct another proof π' where:*

every quantifier rule can be permuted with a logical or structural rule applied below it (with some provisos).

(The interested reader will find detailed expositions of the sharpened Hauptsatz in (Kleene 1967), (Gallier 1987) and (Girard 1987).)

Our example. For the sake of illustration, let us construct a sequent-calculus deduction of the formula used in our original example, i.e. let us build proof of the sequent

$$\vdash \exists x \forall y(\mathbf{P}(y) \to \mathbf{P}(y))$$

and see what the *midsequent* means in that case.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathbf{P}(c), \mathbf{P}(b) \vdash \mathbf{P}(a), \mathbf{P}(b)}{\mathbf{P}(c) \vdash \mathbf{P}(b) \to \mathbf{P}(a), \mathbf{P}(b)}}{\mathbf{P}(c) \vdash \mathbf{P}(b), \mathbf{P}(b) \to \mathbf{P}(a)}}{\boxed{\vdash \mathbf{P}(c) \to \mathbf{P}(b), \mathbf{P}(b) \to \mathbf{P}(a)}}}{\vdash \forall y(\mathbf{P}(y) \to \mathbf{P}(b)), \mathbf{P}(b) \to \mathbf{P}(a)}}{\vdash \exists x \forall y(\mathbf{P}(y) \to \mathbf{P}(b)), \mathbf{P}(b) \to \mathbf{P}(a)}}{\vdash \mathbf{P}(b) \to \mathbf{P}(a), \exists x \forall y(\mathbf{P}(y) \to \mathbf{P}(b))}}{\vdash \forall y(\mathbf{P}(y) \to \mathbf{P}(a)), \exists x \forall y(\mathbf{P}(y) \to \mathbf{P}(x))}}{\vdash \exists x \forall y(\mathbf{P}(y) \to \mathbf{P}(x)), \exists x \forall y(\mathbf{P}(y) \to \mathbf{P}(x))}}{\vdash \exists x \forall y(\mathbf{P}(y) \to \mathbf{P}(x))}$$

Note that every rule below the boxed sequent is either a quantifier rule, or a contraction or exchange rule (no weakenings). Due to the *eigenvariable* restrictions to the quantifiers rules ($\exists$ on the left, and $\forall$ on the right), we had to use contraction and choose the same b in different instantiations. (As we will see later on, when we use labelled natural deduction the 'assumption' that the firstly used b is the same as the other one is introduced as a logical formula using propositional equality.)

With such an example we intend to draw the attention to the fact that although the sharpened Hauptsatz brings Gentzen's methods closer to Herbrand's methods, it shows how less informative the former is with respect to the latter. The midsequent does not mention any function symbol, nor does it point to the inductive nature of the generation of the so-called 'Herbrand universe': the fact that the proof obtains is related to the 'meta-level' choice of the same instantiation constant b.

2 A proof theory for equality in perspective

The intention here is to show how the framework of labelled natural deduction can help us formulate a proof theory for the logical connective of propositional equality. The connective is meant to be used in reasoning about equality between referents (i.e. the objects of the functional calculus), as well as with a general notion of substitution which is needed for the characterisation of the so-called *term declaration logics* (Aczel 1991). The characterisation of propositional equality shall be useful for the establishment of a proof theory for 'descriptions'.

In order to account for the distinction between the equalities that are:

definitional, i.e. those equalities that are given as rewrite rules (equations), orelse originate from general functional principles (e.g. β, η, etc.),

and those that are:

propositional, i.e. the equalities that are supported (or otherwise) by an evidence (a composition of rewrites),

we need to provide for an equality sign as a symbol for *rewrite* (i.e. as part of the functional calculus on the labels), and an equality sign as a symbol for a *relation* between referents (i.e. as part of the logical calculus on the formulas).

Within the framework of the functional interpretation (à *la* Curry–Howard (Curry 1934,Howard 1980)), the definitional equality is often considered by reference to a judgement of the form:

$$a = b : \mathbf{D}$$

which says that a and b are equal elements from domain $\mathbf{D}$. Notice that the 'reason' why they are equal does not play any part in the judgement. This aspect of 'forgetting contextual information' is, one might say, the first step towards 'extensionality' of equality, for whenever one wants to introduce intensionality into a logical system one invariably needs to introduce information of a 'contextual' nature, such as, where the identification of two terms (i.e. equation) comes from.

We feel that a first step towards finding an alternative formulation of the proof theory for propositional equality which takes care of the intensional aspect is to allow the 'reason' for the equality to play a more significant part in the form of judgement. We also believe that from the point of view of the logical calculus, if there is a 'reason' for two expressions to be considered equal, the proposition asserting their equality will be true, regardless of what particular composition of rewrites (definitional equalities) amounts to the evidence in support of the proposition concerned. Given these general guidelines, we shall provide what may be seen as a middle ground solution between the intensional (Martin-Löf 1975b) and the extensional (Martin-Löf 1982) accounts of Martin-Löf's propositional equality. The intensionality is taken care by the functional calculus on the labels, while the extensionality is catered by the logical calculus on the formulas. In order to account for the intensionality in the labels, we shall make the composition of rewrites (definitional equalities) appear as indexes of the equality sign in the judgement with a variable denoting a sequence of equality identifiers (we have seen that in the Curry–Howard functional interpretation there are at least four 'natural' equality identifiers: β, η, ξ and μ). So, instead of the form above, we shall have the following pattern for the equality judgement:

$$a =_s b : \mathbf{D}$$

where 's' is meant to be a sequence of equality identifiers.

In the sequel we shall be discussing in some detail the need to identify the kind of definitional equality, as well as the need to have a logical connective of 'propositional equality' in order to be able to reason about the functional objects (those to the left hand side of the ':' sign). For example, one might wish to prove that for any two functional objects of $\rightarrow$-type, if they are equal then their application to all objects of the domain type must result in equal objects of the codomain type.

2.1 Definitional equality

Equality in mathematics is more complex to deal with formally than it might first appear. Prior to any attempt at a formalisation, one needs to consider the conceptual framework which is more appropriate. We have found Frank Ramsey's 'The Foundations of Mathematics' (Ramsey 1925) particularly attractive, especially in what concerns the defence of the idea that many equalities in mathematics are simply definitional (though not necessarily abbreviatory) equalities. So, in the functional interpretation we have β-equality (*reduction* rules), η-equality (*induction* rules) and

ξ-equality (second *introduction* rule, stating when two canonical elements are equal), ζ-equality (*permutative conversions* for Skolem-type connectives) and all are 'definitional' equalities, even if not abbreviatory equalities.

Clearly, there is a need for a careful analysis of equality in its various facets, especially in connection with the formalisation of mathematics in the lines of Frege's logic. Recall, for example, that ξ-equality is the formal counterpart to Bishop's constructive principle of definition of a set that says that not only the canonical elements must be shown (Bishop 1967), but also the condition under which two canonical elements are equal.

The methodological point about defining sets by not only showing how to build its elements, but also by saying when two elements are to be taken as equal, would seem partly to find a counterpart in Frege's idea that for an expression to be a *logical object* it has to have a criterion for identity (Frege 1884). Only, there seems to be more to the establishment of a minimum criterion for identity, as there are other kinds of equality than ξ-equality, which is to say that equality among non-canonical and canonical elements (the conversions) also plays an important rôle in the definition of a *logical* object. Here we are not taking a type to be identical to a set, as it is done in some accounts of constructive mathematics (Bishop 1967; Martin-Löf 1984), but we are trying to follow the general principles set out by Frege in the sense that the establishment of criteria for identity among the so-called *logical* objects ought to be of prime concern in formal logic.

2.1.1 Reckonable (inspectable) terms

The notion of '*definitionally equal* reckonable terms' was introduced in Gödel's *Dialectica* interpretation of intuitionistic arithmetic via the system T of functionals of finite type (Gödel 1958). Although the notion of *reckonable* (*inspectable*) terms is thoroughly discussed, there is little clarification concerning on what should be understood by equality among two *inspectable* terms. In his intensional interpretation of Gödel's system, Tait defines the notion of definitional equality *relative* to a definite collection of operations with their conversion rules (cf. opening paragraph of (Tait 1967)).

Starting from a clear separation of our logical system into a functional calculus on the labels and a logical calculus on the formulas, here we shall keep the term *definitional equality* for those equalities given by the conversion rules (i.e. those *immediate* conversions, such as β, η, ξ, μ, etc.), whereas the term *propositional equality* will be left for the counterpart of Tait's definitional equality: a sequence of conversions leading up to an equality of terms (in the functional calculus on the labels) constitutes a support (evidence) for them being considered propositionally equal (in the logical calculus).

Existential force. Observe that in the present formulation the connective of propositional equality embodies an existential force: the truth conditions will be such that, given that there is a sequence of rewrites to support the assertion, the propositional equality among two referents is taken to be true. The implications for the analysis of deduction (proof theory) will be such that the pattern of introduction rules will involve, as in the case of '$\vee$' and '$\exists$', witness hiding, and those of elimination rules will be concerned with the opening of local branches with the introduction of new local variables.

2.1.2 Equality in labelled natural deduction

Following our methodology of 'keeping track of proof steps' via a mechanism of labelling formulas with relevant information about the deduction, we take it that the reason for equality (e.g. β, η, ξ, μ, etc.) must be part of the information carried by the label. As the separation functional-logical is taken seriously in labelled systems, there will be no place for formulas in the label, so the equality sign used in the label will not be indexed by the domain (as in some accounts of intensional equality given as '$a =_{\mathbf{D}} b$').[2]

Obviously, there will be a place for the formula expressing the domain of both elements said to be equal, such as $\mathbf{D}$ in $a =_s b : \mathbf{D}$, in the logical side of calculus. That is to say, the logical connective of propositional equality will be relative to the given domain in much the same way the quantifiers are defined with respect to an explicitly stated domain of quantification. Thus, we would like to go from:

$$a =_s b : \mathbf{D}$$

which is the (functional) rewriting judgement where s stands for a composition of definitional equalities, to:

$$s(a,b) : \doteq_{\mathbf{D}} (a,b)$$

which is the (logical) assertion that a and b are propositionally equal due to the evidence '$s(a,b)$', i.e. 'a' and 'b' are identifiable via s.

Examples of definitional equalities come from λ-calculus and the functional interpretation:

(β): an *introduction* followed by an *elimination*

$$\dfrac{a:\mathbf{A} \qquad \dfrac{\begin{matrix}[x:\mathbf{A}]\\ b(x):\mathbf{B}\end{matrix}}{\lambda x.b(x):\mathbf{A}\to\mathbf{B}}\to\text{-}intr}{\mathsf{APP}(\lambda x.b(x),a):\mathbf{B}}\to\text{-}elim \qquad \triangleright_{\beta} \qquad \begin{matrix}[a:\mathbf{A}]\\ b(a/x):\mathbf{B}\end{matrix}$$

thus

$$\mathsf{APP}(\lambda x.b(x),a) =_{\beta} b(a/x)$$

(η): an *elimination* followed by an *introduction*

$$\dfrac{\dfrac{[x:\mathbf{A}] \qquad c:\mathbf{A}\to\mathbf{B}}{\mathsf{APP}(c,x):\mathbf{B}}\to\text{-}elim}{\lambda x.\mathsf{APP}(c,x):\mathbf{A}\to\mathbf{B}}\to\text{-}intr \qquad \triangleright_{\eta} \qquad c:\mathbf{A}\to\mathbf{B}$$

thus

$$\lambda x.\mathsf{APP}(c,x) =_{\eta} c$$

(ζ): preserving dependencies of local branches

$$\dfrac{\dfrac{p:\mathbf{A}_1\vee\mathbf{A}_2 \qquad \begin{matrix}[s_1:\mathbf{A}_1]\\ d(s_1):\mathbf{C}\end{matrix} \qquad \begin{matrix}[s_2:\mathbf{A}_2]\\ e(s_2):\mathbf{C}\end{matrix}}{\mathsf{CASE}(p,\acute{v}s_1.d(s_1),\acute{v}s_2.e(s_2)):\mathbf{C}}}{w(\mathsf{CASE}(p,\acute{v}s_1.d(s_1),\acute{v}s_2.e(s_2))):\mathbf{W}} \qquad \triangleright_{\zeta}$$

$$\dfrac{p:\mathbf{A}_1\vee\mathbf{A}_2 \qquad \dfrac{\begin{matrix}[s_1:\mathbf{A}_1]\\ d(s_1):\mathbf{C}\end{matrix}}{w(d(s_1)):\mathbf{W}} \qquad \dfrac{\begin{matrix}[s_2:\mathbf{A}_2]\\ e(s_2):\mathbf{C}\end{matrix}}{w(e(s_2)):\mathbf{W}}}{\mathsf{CASE}(p,\acute{v}s_1.w(d(s_1)),\acute{v}s_2.w(e(s_2))):\mathbf{W}}$$

thus

$$w(\mathsf{CASE}(p,\acute{v}s_1.d(s_1),\acute{v}s_2.e(s_2))) =_{\zeta} \mathsf{CASE}(p,\acute{v}s_1.w(d(s_1)),\acute{v}s_2.w(e(s_2)))$$

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2. Cf. (Troelstra and van Dalen 1988, page 593): "Let $\vdash \Gamma \Rightarrow A$, $A \equiv I(B,s,t)$ and abbreviate $I(N,t,s)$ as $t =_N s$."
and (Nordström et. al. 1990, page 62): "Instead of $\mathsf{Id}(A,a,b)$ we will often write $a =_A b$."

(ξ): stating when two canonical proofs are equal

$$\frac{\begin{matrix}[x:\mathbf{A}]\\ f(x) =_s g(x) : \mathbf{B}\end{matrix}}{\lambda x.f(x) =_{s\cdot\xi} \lambda x.g(x) : \mathbf{A} \rightarrow \mathbf{B}}$$

(μ): stating when two noncanonical proofs are equal

$$\frac{a : \mathbf{A} \qquad m =_s n : \mathbf{A} \rightarrow \mathbf{B}}{\mathsf{APP}(m, a) =_{s\cdot\mu} \mathsf{APP}(n, a) : \mathbf{B}}$$

Note also that whilst the equality belonging to the functional calculus (i.e. the equalities which are to be seen as *rewrites*) is always relative to a particular (sequence of) definitional equalities, the propositional equality (i.e. the equality belonging to the logical calculus) is always relative to a domain of individuals (similarly to the quantifiers).

3 Martin-Löf's Equality type

There has been essentially two approaches to the problem of characterising a proof theory for propositional equality, both of which originate in P. Martin-Löf's work on *Intuitionistic Type Theory*: the intensional (Martin-Löf 1975b) and the extensional (Martin-Löf 1982) formulations.

3.1 The extensional version

In his (Martin-Löf 1982) and (Martin-Löf 1984) presentations of *Intuitionistic Type Theory* P. Martin-Löf defines the type of *extensional* propositional equality '**I**' (here called '$\mathbf{I}_{ext}$') as:

$\mathbf{I}_{ext}$-*formation*

$$\frac{\mathbf{D}\ type \qquad a : \mathbf{D} \qquad b : \mathbf{D}}{\mathbf{I}_{ext}(\mathbf{D}, a, b)\ type}$$

$\mathbf{I}_{ext}$-*introduction*

$$\frac{a = b : \mathbf{D}}{\mathbf{r} : \mathbf{I}_{ext}(\mathbf{D}, a, b)}$$

$\mathbf{I}_{ext}$-*elimination*[3]

$$\frac{c : \mathbf{I}_{ext}(\mathbf{D}, a, b)}{a = b : \mathbf{D}}$$

$\mathbf{I}_{ext}$-*equality*

$$\frac{c : \mathbf{I}_{ext}(\mathbf{D}, a, b)}{c = \mathbf{r} : \mathbf{I}_{ext}(\mathbf{D}, a, b)}$$

Note that the above account of propositional equality does not 'keep track of all proof steps': both in the $\mathbf{I}_{ext}$-*introduction* and in the $\mathbf{I}_{ext}$-*elimination* rules there is a considerable loss of information concerning the deduction steps. While in the $\mathbf{I}_{ext}$-*introduction* rule the 'a' and the 'b' do not appear in the 'trace' (the label/term alongside the logical formula), the latter containing only the canonical element '$\mathbf{r}$', in the rule of $\mathbf{I}_{ext}$-*elimination* all the trace that might be recorded in

3. The set of rules given in Martin-Löf 1982 contained the additional *elimination* rule:

$$\frac{c : \mathbf{I}(\mathbf{D}, a, b) \qquad d : \mathbf{C}(\mathbf{r}/z)}{\mathsf{J}(c, d) : \mathbf{C}(c/z)}$$

which may be seen as reminiscent of the previous *intensional* account of propositional equality (Martin-Löf 1975b).

the label 'c' simply disappears from label of the conclusion. If by 'intensionality' we understand a feature of a logical system which identifies as paramount the concern with issues of *context* and *provability*, then it is quite clear that any logical system containing $\mathbf{I}_{ext}$-type can hardly be said to be 'intensional': as we have said above, neither its *introduction* rule nor its *elimination* rule carry the necessary *contextual* information from the premise to the conclusion.

And, indeed, the well-known statement of the extensionality of functions can be proved as a theorem of a logical system containing the $\mathbf{I}_{ext}$-type such as Martin-Löf's *Intuitionistic Type Theory* (Martin-Löf 1984). The statement says that if two functions return the same value in their common codomain when applied to each argument of their common domain (i.e. if they are equal pointwise), then they are said to be (extensionally) equal. Now, we can construct a derivation of the statement written in the formal language as:

$$\forall f, g^{\mathbf{A}\to\mathbf{B}}.(\forall x^{\mathbf{A}}.\mathbf{I}_{ext}(\mathbf{B}, \mathsf{APP}(f, x), \mathsf{APP}(g, x)) \to \mathbf{I}_{ext}(\mathbf{A} \to \mathbf{B}, f, g))$$

by using the rules of proof given for the $\mathbf{I}_{ext}$, assuming we have the rules of proof given for the implication and the universal quantifier.

3.2 The intensional version

Another version of the propositional equality, which has its origins in Martin-Löf's early accounts of *Intuitionistic Type Theory* (Martin-Löf 1975a; Martin-Löf 1975b), and is apparently in the most recent, as yet unpublished, versions of type theory, is defined in Troelstra and van Dalen 1988 and Nordström et al. 1990. In a section dedicated to the *intensional* vs. *extensional* debate, (Troelstra and van Dalen 1988, page 633) says that:

> "Martin-Löf has returned to an intensional point of view, as in Martin-Löf (1975), that is to say, $t = t' \in A$ is understood as "t and t' are definitionally equal". As a consequence the rules for identity types have to be adapted."

If we try to combine the existing accounts of the *intensional* equality type '$\mathbf{I}$' (Martin-Löf 1975b; Troelstra and van Dalen 1988; Nordström et al. 1990), here denoted '$\mathbf{I}_{int}$', the rules will look like:

$\mathbf{I}_{int}$-*formation*

$$\frac{\mathbf{D}\ type \qquad a : \mathbf{D} \qquad b : \mathbf{D}}{\mathbf{I}_{int}(\mathbf{D}, a, b)\ type}$$

$\mathbf{I}_{int}$-*introduction*

$$\frac{a : \mathbf{D}}{\mathsf{e}(a) : \mathbf{I}_{int}(\mathbf{D}, a, a)} \qquad \frac{a = b : \mathbf{D}}{\mathsf{e}(a) : \mathbf{I}_{int}(\mathbf{D}, a, b)}$$

$\mathbf{I}_{int}$-*elimination*

$$\frac{a : \mathbf{D} \quad b : \mathbf{D} \quad c : \mathbf{I}_{int}(\mathbf{D}, a, b) \quad \begin{array}{c}[x : \mathbf{D}]\\ d(x) : \mathbf{C}(x, x, \mathsf{e}(x))\end{array} \quad \begin{array}{c}[x : \mathbf{D}, y : \mathbf{D}, z : \mathbf{I}_{int}(\mathbf{D}, x, y)]\\ \mathbf{C}(x, y, z)\ type\end{array}}{\mathsf{J}(c, d) : \mathbf{C}(a, b, c)}$$

$\mathbf{I}_{int}$-*equality*

$$\frac{a : \mathbf{D} \quad \begin{array}{c}[x : \mathbf{D}]\\ d(x) : \mathbf{C}(x, x, \mathsf{e}(x))\end{array} \quad \begin{array}{c}[x : \mathbf{D}, y : \mathbf{D}, z : \mathbf{I}_{int}(\mathbf{D}, x, y)]\\ \mathbf{C}(x, y, z)\ type\end{array}}{\mathsf{J}(\mathsf{e}(a), d(x)) = d(a/x) : \mathbf{C}(a, a, \mathsf{e}(a))}$$

With slight differences in notation, the 'adapted' rules for identity type given in (Troelstra and van Dalen 1988) and (Nordström et al. 1990) resembles the one given in (Martin-Löf 1975b). It is called *intensional* equality because there remains no direct connection between judgements like '$a = b : \mathbf{D}$' and '$s : \mathbf{I}_{int}(\mathbf{D}, a, b)$'.

4 A labelled proof theory for propositional equality

Now, it seems that an alternative formulation of propositional equality within the functional interpretation, which will be a little more elaborate than the extensional $\mathbf{I}_{ext}$-type, and simpler than the intensional $\mathbf{I}_{int}$-type, could prove more convenient from the point of view of the 'logical interpretation'. It seems that whereas in the former we have a considerable loss of information in the $\mathbf{I}_{ext}$-*elimination*, in the latter we have an $\mathbf{I}_{int}$-*elimination* too heavily loaded with (perhaps unnecessary) information. If, on the one hand, there is an *over*explicitation of information in $\mathbf{I}_{int}$, on the other hand, in $\mathbf{I}_{ext}$ we have a case of *under*explicitation. With the formulation of our proof theory for equality via labelled natural deduction we wish to find a middle ground between those two extremes.

4.1 Identifiers for (compositions of) equalities

In the functional interpretation, where a functional calculus on the labels go hand in hand with a logical calculus on the formulas, we have a classification of equalities, whose identifications are carried along as part of the deduction: either β-, η-, ξ-, μ- or ζ- equality will have been part of an expression labelling a formula containing '$\doteq$'. There one finds the key to the idea of 'hiding' in the *introduction* rule, and opening local (Skolem-type) assumptions in the *elimination* rule. (Recall that in the case of disjunction we also have alternatives: either into the left disjunct, or into the right disjunct.) So, we believe that it is not unreasonable to start off the formalisation of our propositional equality with the parallel to the disjunction and existential cases in mind. Only, the witness of the type of propositional equality are not the 'a's and 'b's of '$a = b : \mathbf{D}$', but the actual (sequence of) equalities (β-, η-, ξ-, ζ-) that might have been used to arrive at the judgement '$a =_s b : \mathbf{D}$' (meaning '$a = b$' *because of* 's'), 's' being a sequence made up of β-, η-, ξ- and/or ζ-equalities, perhaps with some of the general equality rules of reflexivity, symmetry and transitivity. So, in the *introduction* rule of the type we need to form the canonical proof as if we were *hiding* the actual sequence. Also, in the rule of *elimination* we need to open a new local assumption introducing a new variable denoting a possible sequence as a (Skolem-type) new constant. That is, in order to eliminate the connective '$\doteq$' (i.e. to deduce something from a proposition like '$\doteq_{\mathbf{D}} (a, b)$'), we start by choosing a new variable to denote the reason why the two terms are equal: 'let t be an expression (sequence of equalities) justifying the equality between the terms'. If we then arrive at an arbitrary formula '$\mathbf{C}$' labelled with an expression where the t still occurs free, then we can conclude that the same $\mathbf{C}$ can be obtained from the $\doteq$-formula regardless of the identity of the chosen t, meaning that the label alongside $\mathbf{C}$ in the conclusion will have been abstracted from the free occurrences of t.

Observe that now we are still able to 'keep track' of all proof steps (which does not happen with Martin-Löf's $\mathbf{I}_{ext}$-type) (Martin-Löf 1982; Martin-Löf 1984), and we have an easier formulation (as compared with Martin-Löf's $\mathbf{I}_{int}$-type) (Martin-Löf 1975b) of how to perform the *elimination* step. Moreover, this will hopefully be coherent with the chosen conceptual framework, namely, Ramsey's idea that mathematical equalities are definitional (though not always abbreviatory) equalities (Ramsey 1925).

4.2 The proof rules

In formulating our propositional equality connective, which we shall identify by '$\doteq$', we shall keep the pattern of inference rules essentially the same as the one used for the other logical connectives (as in, e.g. (de Queiroz and Gabbay 1991)), and we shall provide an alternative presentation of propositional equality as follows:

≐-introduction

$$\frac{a =_s b : \mathbf{D}}{s(a,b) : \doteq_{\mathbf{D}} (a,b)}$$

≐-reduction

$$\frac{\dfrac{a =_s b : \mathbf{D}}{s(a,b) : \doteq_{\mathbf{D}} (a,b)}\doteq\text{-}intr \qquad \begin{matrix}[a =_t b : \mathbf{D}]\\ d(t) : \mathbf{C}\end{matrix}}{\mathsf{TEST}(s(a,b), \theta t.d(t)) : \mathbf{C}} \doteq\text{-}elim \qquad \twoheadrightarrow_\beta \qquad \begin{matrix}[a =_s b:\mathbf{D}]\\ d(s/t):\mathbf{C}\end{matrix}$$

≐-induction

$$\frac{e : \doteq_{\mathbf{D}} (a,b) \qquad \dfrac{[a =_t b : \mathbf{D}]}{t(a,b) : \doteq_{\mathbf{D}} (a,b)}\doteq\text{-}intr}{\mathsf{TEST}(e, \theta t.t(a,b)) : \doteq_{\mathbf{D}} (a,b)} \doteq\text{-}elim \qquad \twoheadrightarrow_\eta \qquad e : \doteq_{\mathbf{D}} (a,b)$$

where 'θ' is an abstractor which binds the occurrences of the (new) variable 't' introduced with the local assumption '$[a =_t b : \mathbf{D}]$' as a kind of 'Skolem'-type constant denoting the (presumed) 'reason' why 'a' was assumed to be equal to 'b'. (Recall the Skolem-type procedures of introducing new local assumptions in order to allow for the elimination of logical connectives where the notion of 'hiding' is crucial, e.g. disjunction and existential quantifier – in (de Queiroz and Gabbay 1991).)

Now, having been defined as a 'Skolem'-type connective, '≐' needs to have a conversion stating the non-interference of the newly opened branch (the local assumption in the *≐-elimination* rule) with the main branch. Thus, we have:

≐-(permutative) reduction

$$\frac{\dfrac{e : \doteq_{\mathbf{D}}(a,b) \qquad \begin{matrix}[a=_t b:\mathbf{D}]\\ d(t):\mathbf{C}\end{matrix}}{\mathsf{TEST}(e,\theta t.d(t)):\mathbf{C}}}{w(\mathsf{TEST}(e,\theta t.d(t))):\mathbf{W}}\,\mathsf{r} \qquad \twoheadrightarrow_\zeta \qquad \frac{e:\doteq_{\mathbf{D}}(a,b) \qquad \dfrac{\begin{matrix}[a=_t b:\mathbf{D}]\\ d(t):\mathbf{C}\end{matrix}}{w(d(t)):\mathbf{W}}\,\mathsf{r}}{\mathsf{TEST}(e,\theta t.w(d(t))):\mathbf{W}}$$

provided w does not disturb the existing dependencies in the term e (the main branch), i.e. provided that rule 'r' does not discharge any assumption on which '$\doteq_{\mathbf{D}} (a,b)$' depends. The corresponding ζ-equality is:

$$w(\mathsf{TEST}(e,\theta t.d(t))) =_\zeta \mathsf{TEST}(e,\theta t.w(d(t)))$$

The equality indicates that the operation w can be pushed inside the θ-abstraction term, provided that it does not affect the dependencies of the term e.

Since we are defining the logical connective '≐' as a connective which deals with singular terms, where the 'witness' is supposed to be hidden, we shall not be using direct *elimination* like Martin-Löf's $\mathbf{I}_{ext}$*-elimination*. Instead, we shall be using the following *≐-elimination*:

$$\frac{e : \doteq_{\mathbf{D}} (a,b) \qquad \begin{matrix}[a =_t b : \mathbf{D}]\\ d(t) : \mathbf{C}\end{matrix}}{\mathsf{TEST}(e,\theta t.d(t)) : \mathbf{C}}$$

The *elimination* rule involves the introduction of a new local assumption (and corresponding variable in the functional calculus), namely '$[a =_t b : \mathbf{D}]$' (where 't' is the new variable) which is only discharged (and 't' bound) in the conclusion of the rule. The intuitive explanation would be given in the following lines. In order to eliminate the equality ≐-connective, where one does not have access to the 'reason' (i.e. a sequence of 'β', 'η', 'ξ' or 'ζ' equalities) why the equality holds because '≐'

is supposed to be a connective dealing with singular terms (as are '$\vee$' and '$\exists$'), in the first step one has to open a new local assumption supposing the equality holds because of, say 't' (a new variable). The new assumption then stands for 'let t be the unknown equality'. If a third (arbitrary) statement can be obtained from this new local assumption via an unspecified number of steps which does not involve any binding of the new variable 't', then one discharges the newly introduced local assumption binding the free occurrences of the new variable in the label alongside the statement obtained, and concludes that that statement is to be labelled by the term '$\text{TEST}(c, \theta t.d(t))$' where the new variable (i.e. t) is bound by the 'θ'-abstractor.

Another feature of the $\doteq$-connective which is worth noticing at this stage is the equality under 'ξ' of all its elements (see second *introduction* rule). This does not mean that the labels serving as evidences for the $\doteq$-statement are all identical to a constant (cf. constant '$\mathbf{r}$' in Martin-Löf's $\mathbf{I}_{ext}$-type), but simply that if two (sequences of) equality are obtained as witnesses of the equality between, say 'a' and 'b' of domain $\mathbf{D}$, then they are taken to be equal under ξ-equality. It would not seem unreasonable to think of the $\doteq$-connective of propositional equality as expressing the proposition which, whenever true, indicates that the two elements of the domain concerned are equal under some (unspecified, *hidden*) composition of definitional equalities. It is as if the proposition points to the existence of a term (witness) which depends on both elements and on the kind of equality judgements used to arrive at its proof. So, in the logical side, one forgets about what was the actual witness. Cf. the existential generalisation:

$$\frac{\mathbf{F}(a)}{\exists x.\mathbf{F}(x)}$$

where the actual witness is in fact 'abandoned'. Obviously, as we are interested in keeping track of relevant information introduced by each proof step, in our labelled natural deduction system the witness is not abandoned, but is carried over as an unbounded name in the label of the corresponding conclusion formula.

$$\frac{a : \mathbf{D} \qquad f(a) : \mathbf{F}(a)}{\varepsilon x.(f(x), a) : \exists x^{\mathbf{D}}.\mathbf{F}(x)}$$

Note, however, that it is carried along *only* in the functional side, the logical side not keeping any trace of it at all.

Now, notice that if the functional calculus on the labels is to match the logical calculus on the formulas, than we must have the resulting label on the left of the '$\rightsquigarrow_\beta$' as β-convertible to the concluding label on the right. So, we must have the convertibility equality:

$$\text{TEST}(s(a, b), \theta t.d(t)) =_\beta d(s/t) : \mathbf{C}$$

The same holds for the η-equality:

$$\text{TEST}(c, \theta t.t(a, b)) =_\eta c :\doteq_{\mathbf{D}} (a, b)$$

Parallel to the case of disjunction, where two different **constructors** distinguish the two alternatives, namely '**inl**' and '**inr**', we here have any (sequence of) equality identifiers ('β', 'η', 'μ', 'ξ', etc.) as **constructors** of proofs for the $\doteq$-connective. They are meant to denote the alternatives available.

4.3 General rules of equality

Apart from the already mentioned 'constants' (identifiers) which compose the reasons for equality (i.e. the indexes to the equality on the functional calculus), it is reasonable to expect that the following rules are taken for granted:

$$\frac{x : \mathbf{D}}{x =_{\mathtt{refl}} x : \mathbf{D}} \quad \textit{reflexivity} \qquad \frac{x =_t y : \mathbf{D}}{y =_{\mathtt{symm}\cdot t} x : \mathbf{D}} \quad \textit{symmetry} \qquad \frac{x =_t y : \mathbf{D} \quad y =_u z : \mathbf{D}}{x =_{t\cdot u} z : \mathbf{D}} \quad \textit{transitivity}$$

4.4 Substitution without involving quantifiers

We know from logic programming, i.e. from the theory of unification, that substitution can take place even when no quantifier is involved. This is justified when, for some reason a certain referent can replace another under some condition for identifying the one with the other.

Now, what would be counterpart to such a 'quantifier-less' notion of substitution in a labelled natural deduction system. Without the appropriate means of handling equality (definitional and propositional) we would hardly be capable of finding such a counterpart. Having said all that, let us think of what we ought to do at a certain stage in a proof (deduction) where the following two premises would be at hand:

$$a =_g y : \mathbf{D} \qquad \text{and} \qquad f(a) : \mathbf{P}(a)$$

We have that a and y are equal ('identifiable') under some arbitrary sequence of equalities (rewrites) which we name g. We also have that the predicate formula $\mathbf{P}(a)$ is labelled by a certain functional expression f which depends on a. Clearly, if a and y are 'identifiable', we would like to infer that $\mathbf{P}$, being true of a, will also be true of y. So, we shall be happy in inferring (on the logical calculus) the formula $\mathbf{P}(y)$. Now, given that we ought to compose the label of the conclusion out of a composition of the labels of the premises, what label should we insert alongside $\mathbf{P}(y)$? Perhaps various good answers could be given here, but we shall choose one which is in line with our 'keeping record of what (relevant) data was used in a deduction'. We have already stated how much importance we attach to names of individuals, names of formula instances, and of course, what kind of deduction was performed (i.e. what kind of connective was introduced or eliminated). In this section we have also insisted on the importance of, not only 'classifying' the equalities, but also having variables for the kinds of equalities that may be used in a deduction. Let us then formulate our rule of 'quantifier-less' substitution as:

$$\frac{a =_g y : \mathbf{D} \qquad f(a) : \mathbf{P}(a)}{g(a, y) \cdot f(a) : \mathbf{P}(y)}$$

which could be explained in words as follows: if a and y are 'identifiable' due to a certain g, and $f(a)$ is the evidence for $\mathbf{P}(a)$, then let the composition of $g(a, y)$ (the label for the propositional equality between a and y) with $f(a)$ (the evidence for $\mathbf{P}(a)$) be the evidence for $\mathbf{P}(y)$.

By having this extra rule of substitution added to the system of rules of inference, we are able to validate one half of the so-called 'Leibniz's law', namely:

$$\forall x, y^{\mathbf{D}}.(\dot{=}_{\mathbf{D}} (x, y) \rightarrow (\mathbf{P}(x) \rightarrow \mathbf{P}(y)))$$

4.5 Our initial example

Now, if we were to construct a deduction of the formula used to demonstrate Herbrand's procedure, i.e.

$$\exists x^{\mathbf{D}}.\forall y^{\mathbf{D}}.(\mathbf{P}(x) \rightarrow \mathbf{P}(y))$$

we would proceed by using the *introduction* rules backward. First of all, assuming the formula is valid, and knowing that its outermost connective is an existential quantifier, its labels must be of the form:

$$\varepsilon x.(f(x), a) : \exists x^{\mathbf{D}}.\forall y^{\mathbf{D}}.(\mathbf{P}(x) \to \mathbf{P}(y))$$

for some functional expression f, and witness a. Now, to have arrived at that conclusion, we must have come from two premises of the form:

$$a : \mathbf{D} \qquad \text{and} \qquad f(a) : \forall y^{\mathbf{D}}.(\mathbf{P}(a) \to \mathbf{P}(y))$$

The first premise is already reduced to an atomic formula so we cannot go any further, but the second one has an universal quantifier as its outermost connective. Thus, our $f(a)$ must be of the form:

$$\Lambda y.h(y, a) : \forall y^{\mathbf{D}}.(\mathbf{P}(a) \to \mathbf{P}(y))$$

for some h which depends on both y and a. In its turn, this h will have come from a deduction where we ought to have had:

$[y : \mathbf{D}]$ as an assumption, and $h(y, a) : \mathbf{P}(a) \to \mathbf{P}(y)$ as a premise

Now, the latter, having implication as its major connective, must be of the form:

$$\lambda u.m(u, y, a) : \mathbf{P}(a) \to \mathbf{P}(y)$$

for some expression m which depends on u, y and a, and must also have come from a deduction where:

$[u(a) : \mathbf{P}(a)]$ is an assumption, and $m(u, y, a) : \mathbf{P}(y)$ as a premise

(Recall that labels of predicate formulas will be such that they are expressions which depend on the object of predication.)

Now we have reached the atoms, and we now need to move backwards 'solving' the unknowns: we need to find the form of m as an expression depending on u, y and a, in order to find what h is, and so on, until we really obtain the whole label expression of our original formula. Thus, we have the assumptions:

$$a : \mathbf{D}, \quad y : \mathbf{D}, \quad u(a) : \mathbf{P}(a)$$

and we need to arrive at

$$m(u, y, a) : \mathbf{P}(y)$$

As we need to get $\mathbf{P}(y)$ from $\mathbf{P}(a)$, let us then recall our 'quantifier-less' substitution procedure. What extra assumption do we need to make in order to be able to apply our rule of quantifier-less substitution? It is not too difficult to guess:

$$a =_g y : \mathbf{D}$$

for some arbitrary sequence of equalities (rewrites) g. If we have that extra assumption we can apply the rule and get $\mathbf{P}(y)$ as follows:

$$\frac{a =_g y : \mathbf{D} \qquad u(a) : \mathbf{P}(a)}{g(a, y) \cdot u(a) : \mathbf{P}(y)}$$

Having done that, the fact that the validity of the formula must be independent of the choice of g, there must be a step where the g is 'abstracted' away from the label expression at some point down the deduction. This will happen if the assumption $a =_g y : \mathbf{D}$ will have been introduced in the context of an $\doteq$-*elimination* inference.

Prima facie, the degree of *explicitation* may look a little too excessive, but if for nothing else, we have certainly uncovered two extra assumptions on which the validity of the formula $\exists x^{\mathbf{D}}.\forall y^{\mathbf{D}}.(\mathbf{P}(x) \to \mathbf{P}(y))$ must depend. First, the domain must be non-empty, therefore if no specific element is mentioned we simply take an arbitrary one ('let a be an arbitrary element'). Secondly, there must be a way of performing substitutions even after we strip the formula of its quantifiers, which means that if there is no 'function symbol' already specified so that other elements can be built out of a, then take an arbitrary one ('let g be the key to the identifications/substitutions').

If we now reconstruct the whole deduction as a tree, we get the following:

$$
\dfrac{[a:\mathbf{D}] \qquad \dfrac{\dfrac{\dfrac{\begin{array}{c}[a:\mathbf{D}],[y:\mathbf{D}]\\ {[s(a,y):\doteq_{\mathbf{D}}(a,y)]}\end{array} \qquad \dfrac{\dfrac{[a =_g y:\mathbf{D}] \qquad [u(a):\mathbf{P}(a)]}{g(a,y)\cdot u(a):\mathbf{P}(y)}}{\lambda u.g(a,y)\cdot u(a):\mathbf{P}(a)\to\mathbf{P}(y)}}{\text{TEST}(s(a,y),\theta g.\lambda u.g(a,y)\cdot u(a)):\mathbf{P}(a)\to\mathbf{P}(y)}}{\lambda s.\text{TEST}(s(a,y),\theta g.\lambda u.g(a,y)\cdot u(a)):\doteq_{\mathbf{D}}(a,y)\to(\mathbf{P}(a)\to\mathbf{P}(y))}}{\Lambda y.\lambda s.\text{TEST}(s(a,y),\theta g.\lambda u.g(a,y)\cdot u(a)):\forall y^{\mathbf{D}}.(\doteq_{\mathbf{D}}(a,y)\to(\mathbf{P}(a)\to\mathbf{P}(y)))}}{\dfrac{\varepsilon x.(\Lambda y.\lambda s.\text{TEST}(s(x,y),\theta g.\lambda u.g(x,y)\cdot u(x)),a):\exists x^{\mathbf{D}}.\forall y^{\mathbf{D}}.(\doteq_{\mathbf{D}}(x,y)\to(\mathbf{P}(x)\to\mathbf{P}(y)))}{\lambda a.\varepsilon x.(\Lambda y.\lambda s.\text{TEST}(s(x,y),\theta g.\lambda u.g(x,y)\cdot u(x)),a):\mathbf{D}\to(\exists x^{\mathbf{D}}.\forall y^{\mathbf{D}}.(\doteq_{\mathbf{D}}(x,y)\to(\mathbf{P}(x)\to\mathbf{P}(y))))}}
$$

If we look at abstraction as a device which brings *universal* force to the reading of a functional expression, it is not difficult to see why our 'base', i.e. '$\{a, g\}$', has a generality aspect to it: in the final label:

$$\boxed{\lambda a.}\varepsilon x.(\Lambda y.\lambda s.\text{TEST}(s(x,y), \boxed{\theta g.}\lambda u.\boxed{g}(x,y)\cdot u(x)), \boxed{a})$$

both a and g are bound by the respective abstractors λ and θ.

5 Finale

When Frege set up a framework to formalise arithmetic, pursuing his programme on demonstrating that 'numbers are objects', it was inevitable that the theme of *equality* gained prominence. In order to have a precise characterisation of those objects, Frege needed to establish conditions for deciding when two objects would be considered equal or non-equal. In passing, it may be relevant here to remind ourselves of the contribution given by Peano with his axiomatisation of arithmetic: two numbers were equal or not depending on whether they would be provably equal in Peano arithmetic. Peano's arithmetic provided the means for the characterisation of equality among the objects called numbers.

Now, when we study deduction systems, and we wish to characterise proofs as objects, it may be wise to look back at Frege's attempt at treating numbers as objects. It is important to have adequate means for providing criteria for considering proofs as equal (or equivalent). The establishment of criteria for equality between proofs has in fact been a recurring question among proof-theorists, at least since Kreisel's investigation on the foundations of intuitionistic logic and the theory of constructions (see, e.g. (Kreisel 1968)).

The importance of the so-called 'internalisation' of the proof transformation rules has been our concern for some time now,[4] and it has remained an important issue in our programme of characterising a generalised functional interpretation for labelled deductive systems.[5] We believe it is an important aspect of any framework for systems of deduction which to some extent demonstrates the need for using a

4. (de Queiroz 1988),(de Queiroz 1990), (de Queiroz and Maibaum 1990),(de Queiroz 1994).
5. (de Queiroz and Gabbay 1991),(de Queiroz and Gabbay 1992).

separate calculus alongside the calculus on the formulas: the rewriting system arising out of the functional calculus on the labels provides the adequate and sufficiently fine-grained means for comparing proofs. The sequent calculus by itself does not seem to enjoy this useful feature, as the only proof transformation rule is the rule which allows one to transform a proof with cut into a proof without cut. It is obvious that the granularity of the calculus on proofs is far too coarse in a system with only one way of identifying two deductions. This again would seem to support the view that the sequent calculus needs some extra device to cope with the various kinds of equality among proofs.

A recent manuscript entitled 'What is a deductive system?' by J. Lambek rightly points out the need for the formalisation of equality among proofs when characterising deductive systems. We quote from the opening lines (Lambek 1992):

> 'A deductive system should deal with
> (a) formulas,
> (b) deductions, alias proofs,
> (c) equality between deductions.
> While most, if not all, logicians would agree with (a) and (b), they have often neglected (c), the importance of which is now becoming apparent under the influence of category theory and computer science. Computer scientists may even go further and replace the equality in (c) by a partial order called "reduction".'

We feel this is an important observation, and the previous accounts of our labelled natural deduction (de Queiroz and Gabbay 1991; de Queiroz and Gabbay 1992) go further in defining a framework for equality. They show how we provide fine-grain analysis of various kinds of equalities between proofs. This paper adds to the previous ones by demonstrating how we can reason with equality itself.

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Anaphoric Presuppositions and Zero Anaphora

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1 Introduction

The purpose of this paper is to use an anaphoric notion of presupposition for solving the problem of zero argument anaphora. Since Shopen (1973) it has been known that many missing arguments have an anaphoric interpretation, but it has not been known how this interpretation arises. I argue that these arguments are involved in presuppositions. On an anaphoric account of presuppositions as in van der Sandt (1992) or Kamp & Roßdeutscher (1992), it can be shown that the zero arguments acquire an anaphoric interpretation through the presuppositions. The analysis rests on the principle that the Discourse Representation Structure for the presupposition is proper, so that the discourse referents for the zero arguments are in its universe and must be anchored to discourse referents in the context.

1.1 The Problem

It has been noted many times that some zero arguments are regularly interpreted existentially while other zero arguments are regularly interpreted anaphorically. But although intuitions converge that this different behavior of zero arguments is related to the semantics of the predicates, it has not proved possible to determine the responsible semantic property. Consider some examples of zero anaphora.

(1) John wants to sell his house.
Sue has **offered** one million, but he isn't **satisfied**.

(2) A sheep has been killed in the mountains.
Environmentalists **suspect** a poacher, but farmers **disagree**.

(3) Scrooge had buried a hoard of nuggets at his claim on Agony Creek.
He had **forgotten** completely, but the pills **reminded** him.

(4) It's dangerous to hypnotize people. You might do it to somebody
with a gullible mind sometime, and that person would never **recover**.

Tim Shopen in his pioneer paper (1973) claimed that the definiteness is due to the semantic structure of the verb and that there is a significant relationship with the notion of presupposition (p 68). This remark has not been elaborated on, however, and he himself wrote the definiteness of the relevant argument into the lexical entry. The apparatus was not yet in place for analyzing the presuppositions in such a way that the anaphoric reading of the zero argument follows, but now I believe it is.

1.2 Anaphoric Presuppositions

There have been repeated suggestions in the literature – notably, Soames (1989) and Heim (1987), both referring to Kripke (unpublished) – that some, if not all, presuppositions have an anaphoric property that makes even a context-oriented notion as in (Karttunen 1974) inadequate. Relevant data include the following, where there is a strong tendency, on account of the adverb *again*, to infer that Norway is a Scandinavian country.

(5) After suffering humiliating 0 to 2 against Norway,
Taylor vowed never to underestimate a Scandinavian team **again**.

Such 'accommodation effects' show that when accommodating a presupposition, sometimes we do not add it totally but partially: We verify it as far as possible and only accommodate the rest. We tend to differentiate the discourse referents and the conditions of the presupposition, verifying the former and accommodating the latter. This requires a more fine-grained notion of presuppositions.

Such a fine-grained notion underlies Irene Heim's (1982) familiarity theory: For a definite description to be felicitous with respect to a file, the set of sequences satisfying it must include the set of sequences satisfying the file for every world. And Heim's (1983) strong definition of admittance conditions incorporates this: The context must entail the presupposition. In contrast to Karttunen's definition of pragmatic presupposition (1974), the context and the presupposition are both not propositions but sets of pairs of worlds and variable assignments.

Two proposals have been made for an anaphoric conception of presuppositions in Discourse Representation Theory. In the theory of van der Sandt every instance of anaphora is an instance of presupposition and vice versa. Kamp's & Roßdeutscher's (1992) 'principle of presupposition justification' mirrors van der Sandt's (1992) definition of presupposition resolution in all essentials. A sentence is represented provisionally in a bipartite structure, the presupposition and the assertion DRS, and as many as possible of the discourse referents of the former are mapped onto familiar discourse referents, respecting consistency of the merge of the context DRS and the so-called picture of the presupposition DRS under the mapping. Finally, the context DRS is merged with the so-called picture of the presupposition and the assertion DRS under the mapping. Since binding has priority over accommodation of referents, accommodation effects like the one observed are in effect predicted.

1.3 Zero Argument Anaphora

Given this notion of anaphoric presupposition, there is a straightforward way of predicting when a zero argument is anaphoric: If (and only if) the predicate triggers a presupposition involving it. The zero anaphor is then resolved in the same process in which the presupposition is justified. Consider an example:

(6) John is finally stepping down as coach despite efforts to **dissuade** him.

The zero argument of *dissuade* is interpreted as 'from stepping down as coach', and it is not necessary to postulate a zero anaphor associated with the verb to predict this. Instead, I assume, and this assumption must be made anyway, that the verb triggers the presupposition that John has been planning to do what there have been efforts to dissuade him from doing. More precisely, a sentence *Susan dissuades John* asserts that Susan dissuades John from P, and presupposes that there is some P such that John plans to P. On the anaphoric notion of presupposition, P must be anchored to some familiar (action) property Q such that John plans to Q; and once it is, Q replaces P in the assertion.

The argument involves the principle that the presupposition DRS is proper, containing no free occurrences of referents. Thus when there is a presupposition involving the zero argument, we can predict that the corresponding referent is introduced in the presupposition structure, yielding an anaphoric interpretation. However, to show that a zero argument is anaphoric *only* if a presupposition involves it, it is necessary to impose the converse principle that when a referent is introduced in the presupposition structure but does not occur in a condition in that structure, it originates in an overt anaphor. This depends on empirical investigations which I cannot carry out completely in this paper. One has to make a convincing case that every instance of zero anaphora is connected with a presupposition of the predicate, and some putative presuppositions are admittedly subtle, vague, and elusive. Still, it is my hope that further investigations can show that presuppositions do provide the key to the explanation of zero anaphora in general.

2 Anaphoric Presuppositions

Anaphoric presuppositions have been approached from several different angles. 1) For some ten years, Saul Kripke has been arguing that facts about certain triggers demonstrate the need for an anaphoric account of the associated presuppositions. Soames (1989) and Heim (1987) cite and discuss his observations, concerning, in particular, the focus particle *too*, the adverb *again*, and the aspectual verb *stop*. 2) Rob van der Sandt has devised an anaphoric account of presupposition invoking the concept of resolution, as a general theory intended to cover all presuppositions (1992). A general definition of an anaphoric conception of presupposition is also proposed by Kamp & Roßdeutscher (1992), in connection with work on the adverb *wieder* (*again*). 3) In a third category, Heim's (1982) analysis of definite descriptions in terms of the Familiarity Condition, characterized as a presupposition, is in reality an anaphoric interpretation of the classical existential presupposition of definite descriptions. Heim (1983) generalized this notion, and Heim (1992) applies the same mechanism in defining the presupposition of the focus particle *too*.

I devote section 2.1 to reviewing the intuitions which can be traced to Kripke. Section 2.2 represents the presupposition of Heim (1982) and (1983) and presents a 'translation' into Discourse Representation Theory, and section 2.3 discusses the definitions proposed by van der Sandt (1992) and Kamp & Roßdeutscher (1992). Section 2.4, finally, is designed to develop a definition of anaphoric presuppositions that can form an adequate basis for the analysis of zero anaphora in section 3.

2.1 Accommodation Effects

Soames (1989) credits Kripke for calling attention to a number of facts that show the necessity of an anaphoric concept of the presuppositions of *too*, *again*, and *stop*. According to Soames (1989: 613), Kripke has observed that the presuppositions of the sentences (7)(i)-(iii) should include (8)(i)-(iii), and that these presuppositions would not be forthcoming if the presuppositions of (7)(i)-(iii) were just (9)(i)-(iii).

(7) (i) If Herb's wife comes, then Francis will come too.
(ii) If Reagan criticizes Hart in his radio talk,
then he will criticize him again in his press conference.
(iii) If Bill watches the opera at 2 o'clock,
he will stop watching it when the Redskins' game begins.

(8) (i) Herb's wife will come → Francis is not Herb's wife.
(ii) Reagan criticizes Hart in his radio talk →
the radio talk will take place before the press conference.
(iii) Bill watches the opera at 2 o'clock →
the Redskins' game will begin after 2 o'clock.

(9) (i) Herb's wife will come → someone other than Francis will come.
(ii) Reagan criticizes Hart in his radio talk →
Reagan will have criticized Hart prior to the press conference.
(iii) Bill watches the opera at 2 o'clock → Bill will have been
watching the opera prior to the beginning of the Redskins' game.

> "Kripke suggests that ... the content of the presupposition of a sentence or clause containing *too*, *again*, or *stop* may vary with, and be dependent upon, the preceding discourse or conversational context. The idea is that these presupposition creating elements may, in some way, be anaphoric with other elements in the discourse or context." (Soames 1989: 614)

Heim (1987: 12ff) provides an interpretation of this situation. Consider (10).

(10) We will have pizza on John's birthday,
so we shouldn't have pizza again on Mary's birthday.

> "As Kripke observes, you will spontaneously infer from this utterance that John's birthday precedes Mary's. This inference seems due somehow to the presence of *again* in the sentence; if *again* had been left out, it would have been just as easy to imagine the birthdays in the opposite temporal order. [...] Suppose that the presupposition of *again* amounts to the requirement that a particular proposition among a certain set be entailed by the common ground. For the sentence at hand, suppose that [the second clause] calls for a context in which some occasion t is salient and the common ground entails about it that at t we have pizza and t precedes Mary's birthday. Under this analysis, we expect that the hearer will need to identify an appropriate contextually salient occasion of pizza eating, and of course the one that immediately comes to mind is the one mentioned in the first sentence, viz. John's birthday. This will do if only it can be presumed to lie before Mary's birthday, so what is more natural than to accommodate this missing bit of information."

Consider next (11) and Heim's comment (1987: 14).

(11) John is cooking.
He will stop (cooking) when tomorrow's football game starts.

> "This utterance invites the spontaneous inference that John is engaged in one single protracted cooking activity that started before the speech time and will continue through the night. Why should this inference, which isn't all that plausible pragmatically, arise? [...] Suppose what *John will stop cooking at t* really requires is a context with a salient cooking event by John about which it entails that it extends right up to t. Under this assumption, we can explain the inference: The obvious candidate for a salient event of John cooking is the one mentioned in the first sentence. The information that this event extends right up to the beginning of tomorrow's football game is then a natural one to accommodate to make the context fit the requirement of *stop* fully. This explains the inference and thereby receives empirical support."

Let us try to state as precisely as possible how the relevant inferences come about. The context should entail that there is somebody different from Francis who will come, that there is an event such that we have pizza prior to Mary's birthday, and that there is a state such that John is cooking prior to and adjacent to the start of tomorrow's football game. Now the linguistic context almost entails this, but not quite. What is lacking is the piece of information that Herb's wife is different from Francis, that the event such that we have pizza on John's birthday is prior to Mary's birthday, and that the state such that John is cooking now is adjacent to the start of tomorrow's football game. So this is accommodated. The most interesting point to note in this connection is that what is accommodated, then, is stronger than necessary, so to speak; the proposition that we'll have pizza on John's birthday and on Mary's and the former precedes the latter is stronger than the proposition that we'll have pizza on John's birthday and on Mary's and on some occasion preceding the latter. (This occasion might be John's birthday but it might be another.) This **accommodation effect** is a clear symptom of anaphoric presuppositions: Accommodation does not consist in taking the shortest logical route to recreating the presupposition but preserves the discourse referents already present, within limits of course. Accommodation effects of this sort support the conjecture of Kamp & Roßdeutscher (1992: 119) that accommodating new discourse referents carries a higher price than accommodating new conditions on old discourse referents. When this principle is effective, we can speak of **anaphoric accommodation**.

The conditions accommodated in the cases considered so far are what can be called structural conditions, coming from the general format of the presupposition itself, independently of the individual carrier sentence. The referent involved is what can be called the structural referent, that is, the alternative individual or event, or the state about to stop. Here, it seems, accommodation is routine. But accommodation effects do not stop here. Rather, the anaphoricity of the presupposition can be seen to affect other referents, in such a way that other conditions are accommodated.

(12) Jack broke his leg in July 1933, and Joe also broke his leg at the age of 15.

¿From (12), we spontaneously infer that Jack was 15 in July 1933. Again, it is the stronger proposition which is accommodated: Instead of accommodating that Jack broke his leg at the age of 15 (in addition, maybe, to the event in July 1933), we accommodate that the event of his breaking his leg at the age of 15 was the event of his breaking his leg in July 1933, thereby maximizing the sum of information.

As a consequence of anaphoric accommodation, presuppositions can provide new information by virtue of being presuppositions. The result is something over and above what it would be if the presupposition were an assertion. It is a well-known fact that notably factive presuppositions can carry new information; but this is different. The match is imperfect, but still close enough to license a 'unification' of two bits of information, and in this way, accommodation yields information that is new both with respect to the context and to the carrier sentence.

2.2 Anaphoric Definite Descriptions

In her landmark thesis (1982), Irene Heim reinterpreted the classical presupposition of definite descriptions in terms of the familiarity condition (p 369).

> "For φ to be felicitous w.r.t. F it is required for every NP_i in ϕ that:
> [...]
> (ii) if NP_i is [+definite], then
> [...]
> (b) if NP_i is a formula, F entails NP_i"
>
> "A file F entails a formula ϕ iff for every world w:
> $Sat_w(F) \subseteq Sat_w(F+\phi)$"

Informally, for the file to entail the logical form of the definite description, every 'sequence' (variable assignment) and every world satisfying the file must satisfy the update of the file by the logical form of the definite description. This means that the index i on NP_i must have been introduced into the file previously and that the information going with it (the content of the NP) must have been assembled. Heim calls the familiarity condition a presupposition, and I think it is useful to think of it as the existential presupposition traditionally associated with definite descriptions interpreted as an anaphoric presupposition. Let me illustrate this.

(13) A man wielding a crowbar outside the elementary school where his two children are enrolled was fatally shot by a police officer. The principal said the man had gone to the school to see his daughter's teacher and had been asked to leave. The police were called, and when an officer approached the man charged with the crowbar and was shot.

The last line of this discourse presupposes that there is a man (and that there is a crowbar). For the file to satisfy this presupposition it must contain that information. However, it is not sufficient that the file entails 'x is a man' for some x; rather, it must entail 'x_i is a man' for the variable x_i coming with the description. If it does, as it well may if it is correctly constructed from the discourse at hand, the assertion 'x_i ...was shot' updates an already established file card.

If anaphoric definite descriptions, analyzed as in Heim (1982) or equivalently, are instances of anaphoric presuppositions, this shows that anaphoric presuppositions are sensitive to variable assignments. The weak requirement that the file entails 'x

is a man' for some x corresponds to a traditional concept of entailment: For every world satisfying the file, there is an assignment satisfying it plus the presupposition; and to Karttunen's (1974: 181) definition:

> "Surface sentence A pragmatically presupposes a logical form L, if and only if ... A can be felicitously uttered only in contexts which entail L."

However, the stronger requirement that the file entails 'x_i is a man' for the variable x_i coming with the description corresponds to a strong concept of entailment: Every world and every assignment satisfying the file satisfy (it plus) the presupposition.

In her seminal paper (1983), Irene Heim generalized this concept of presupposition. Heim (1982) did not focus on presuppositions in general, but Heim (1983) did. On the other hand, that paper did not focus on definite descriptions, or on other typical instances of anaphoric presuppositions, so the capacity of the theory presented there to account for anaphoric presuppositions is only stated implicitly. The definition is

> "S presupposes p iff all contexts that admit S entail p." (p 117)

And as long as contexts and presuppositions are propositions, this is of course more or less the same as Karttunen's (1974) definition quoted above. However, in order to account for presuppositions of phrases with free variables, Heim reidentifies contexts and presuppositions, along the lines of (1982), as sets of pairs of variable assignments and possible worlds instead of just sets of possible worlds (pp 120f). Thus in (1983), Heim actually presents an anaphoric concept of presupposition. And, in fact, in (1992), Heim presents an analysis of *too* which parallels her (1982) analysis of definite descriptions (1992: 189):

> "The general rule for the interpretation of *too* is (21).
> (21) $\phi[\alpha_F]too_i$ presupposes $x_i \neq \alpha \wedge \phi[x_i]$."
> "... I assume that *too* is implicitly deictic or anaphoric, sort of like *in addition to* x, where the intended reference of x is disambiguated at Logical Form by means of a referential index."

It may be useful to provide a 'translation' into Discourse Representation Theory of Heim's (1983) strong admittance condition. This definition presupposes that a sentence gives rise to two provisional Discourse Representation Structures S and P, corresponding to S and p in Heim's own formulation.

> **Heim's strong admittance condition in procedural terms**
>
> Let C be any context DRS, S a sentence DRS and P the presupposition DRS of S.
> C + S is only defined if there is a function f:$U_P \rightarrow U_C$ such that $\langle U_C, Con_C \cup$ "$f(Con_P)$"$\rangle$ is a logical consequence of C, and if defined, it contains the conditions $x = f(x)$ for every $x \in U_P$ for some such f.

The presupposition DRS uses fresh discourse referents, just like any 'new' DRS. For such a DRS to be a consequence of the context DRS in the strong sense that for every model M and every embedding f verifying the context DRS in M, f verifies it in M as well. those discourse referents must be replaced by 'old' discourse referents. Under such a substitution, the requirement is sufficient that the merge of the context DRS and the presupposition DRS be a consequence of the former in the weaker sense that for every M and f verifying C in M, there is a g verifying the merge of C and "the picture of P" under the substitution.

2.3 The Discourse Representation of Anaphoric Presuppositions

The strong admittance condition modelled on Heim (1983) is very strong indeed: There is no way for the presupposition to introduce novel entities, nor is there any way for it to introduce novel information. In short, the condition does not allow for accommodation, either with respect to discourse referents or with respect to conditions. As it happens, the discourse (13) contains several definite descriptions where accommodation is necessary and possible:

(13) A man wielding a crowbar outside the elementary school where his two children are enrolled was fatally shot by a police officer. The principal said the man had gone to the school to see his daughter's teacher and had been asked to leave. The police were called, and when an officer approached the man charged with the crowbar and was shot.

But they all involve accommodation of discourse referents, conforming to Heim's **linking** condition (1982: 373): "When a new file card is introduced under accommodation, it has to be linked by cross-references to some already-present file card(s)." By contrast, the cases considered in Section 2.1 involve accommodation of conditions on old discourse referents: Anaphoric accommodation. Anaphoric accommodation occurs in connection with definite descriptions too. Zeevat (1992: 407) comments on (14) and (15) that "this is not accommodation proper, which would also create the antecedents themselves".

(14) A soldier entered the room. The man asked for a beer.
(15) A man died in a car crash yesterday evening. The Amsterdam father of four was found to have been drinking.

Section 2.1 has shown that in some cases at least, it is more difficult to accommodate discourse referents than conditions, so an account of anaphoric presupposition and accommodation should discriminate between the two forms of accommodation.

Rob van der Sandt has proposed an anaphoric account of presuppositions in a Discourse Representation Theory framework on which presuppositions are anaphora with descriptive material. Accommodation is treated not as a repair strategy but rather as another way of presupposition satisfaction: Resolution has two parts, binding and accommodation, where binding has a form of priority over accommodation. DRSs are defined as triples where the third member, encoding the presupposition(s), is itself a set of DRSs, the A-structure (A for anaphora).

van der Sandt's presupposition resolution (1992: 358):

"*Resolution*
Let K be a DRS and let K_s be the source of an anaphoric expression, that is an element of an A-structure of some sub-DRS of K and let $A(K_s)$ be empty. Let its target be a (sub)DRS K_t on K_s's projection line. Let K_s have the markers $y_1 \ldots y_m$ and $Acc(K_t)$ the markers $x_1 \ldots x_n$. Let f be a function from $U(K_s)$ to $Acc(K_t)$, such that the conditions of K_t are compatible with the conditions of K_s under the substitution of $y_1 \ldots y_m$ for $x_1 \ldots x_n$. The resolution of the anaphoric structure K_s with respect to K_t yields a DRS K′, which differs from K in the following respects.

Binding

(i) $U(K'_s)=Con(K'_s)=\emptyset$
(ii) $U(K'_t)=U(K_s)\cup U(K_t)$
(iii) $Con(K'_t)=Con(K_s)\cup Con(K_t)\cup\{x = y \mid x = f(y)\}$

[...] Accommodation of K_s into K_t is [...] just like binding with the one exception that no restrictions on compatibility are required and no anaphoric equations are added to $Con(K_t)$."

The definition of binding takes height for accommodation of information in that the conditions of K_t are only required to be compatible with the conditions of K_s under the relevant substitution, and accommodation of referents and information is covered under Accommodation. Let us go through an example to see how the theory works. Consider the NP *his daughter's teacher* in (16).

(16) A man came to a school. His daughter's teacher was inaccessible.

DRSs are constructed in two stages. First, a DRS is constructed for the incoming sentence. This DRS is then merged with the main DRS, resulting in a new DRS in which the anaphoric structures still await processing:

$$\langle\{u,v\},\{man(u), school(v), cometo(u,v), inaccessible(x)\},$$
$$\{\langle\{x\},\{teacher(x), poss(y,x)\},\{\langle\{y\},\{daughter(y), poss(z,y)\},$$
$$\{\langle\{z_{masc}\},\emptyset,\emptyset\rangle\}\rangle\}\rangle\}\rangle.$$

Resolution starts with the deepest embedded anaphor, the A-structure set up for the possessive pronoun. Going upwards along its projection line we check whether a suitable antecedent can be found. We do, and identify the anaphoric marker z with the established marker u. Next we repeat the maneuver for the A-structure set up for *his daughter*, but now we cannot find a suitable antecedent, so it will be accommodated, that is, the marker y and the conditions $daughter(y)$ and $poss(z,y)$ will be transferred to main DRS level. The same happens with the A-structure set up for *his daughter's teacher*, and the result is the A-structure-empty DRS

$$\langle\{u,v,z,y,x\},\{man(u), school(v), cometo(u,v), inaccessible(x),$$
$$z=u, daughter(y), poss(z,y), teacher(x), poss(y,x)\},\emptyset\rangle.$$

And in fact, since 'binding' has a form of priority over 'accommodation', this theory can account for anaphoric accommodation, where the presupposition provides novel information about familiar referents, as in (10)–(12) and (14)–(15). The conditions of the presupposition are compatible with the conditions of the main DRS, so binding can occur; and they are transferred to the new DRS by rule (iii).

Kamp & Roßdeutscher (1992) develop an anaphoric conception of presuppositions very similar to van der Sandt's theory. Their immediate aim is to account for the presuppositions of the German adverb *wieder*, corresponding to English *again*. Consider the 'restitutive' variant of *wieder* as in (17).

(17) Der Patient ist vor einigen Wochen an Typhus erkrankt.
Jetzt hat der Assistenzarzt ihn wieder vom Typhus geheilt.

This discourse displays an accommodation effect: We infer that the typhoid of which the intern has cured the patient is the same typhoid with which she came down a few weeks ago. Since *wieder* requires that the context contain a representation of the state which the described process turns into the result state, the presupposition by accommodation brings about the unification of the two typhoid states.

> "What we see here is a kind of mixture of presupposition verification and presupposition 'accommodation': part of the information ... in the presupposition is present in the context; the rest has to be assumed. [...] In general they [presupposition verification and accommodation] do not represent exclusive alternatives, with accommodation coming into play only when verification fails, and then wholesale. Rather ...they often go hand in hand. We will refer to such combinations of verification and accommodation as *presupposition justification*." (p 109)

The authors are not yet in a position to state a justification procedure precisely. They do, however, propose the following principle in the form of a conjecture:

> **"Principle of Presupposition Justification**
>
> To justify a pair (K,K') in a DRS K_0 find a function f from a subset of $U_{K'}$ into U_{K0}, such that $K_0 \cup f(K')$ is consistent and such that $Dom(f)$ is maximal among such functions. Extend f to a function g such that $Dom(g)=U_K$, which is 1-1 on $Dom(g)\backslash Dom(f)$ and which maps the discourse referents from this set onto discourse referents not occurring in K_0. Add $g(K)$ to K_0." (p 120)

A lengthy footnote fills in the picture.

> "The weak point in this formulation is the requirement that $K_0 \cup f(K')$ be 'consistent'. As it stands, this requirement allows for cases where the information that K' contains about a discourse referent x has very little to do with the information which K contains about $f(x)$; all that we required is that there be no logical inconsistency between these two bits of information. In general, however, this won't be enough to rule out presupposition justifications that are intuitively unacceptable. [...]
>
> (i) [...] Everyone in the Gertraudenkrankenhaus remembers some child's cure by an intern of a blood disease. Now an intern has once again cured a patient of some pernicious disease.
>
> (ii) [...] Everyone in the Gertraudenkrankenhaus remembers some patient's cure by an intern of a blood disease. Now an intern has once again cured a child of some pernicious disease.
>
> (i) is felicitous, (ii) is not. [...] ... mere consistency isn't good enough. At present we have no clear idea of a suitable stronger condition ..."

The Principle of Presupposition Justification as it stands is weak in another respect: The function f is not required to be total on the universe of the presupposition.

> "A second shortcoming of [the principle] is its failure to provide lower limits on the set of discourse referents that must find targets under f in the context DRS K_0. When the context is too meagre, so that too many discourse referents have to be accommodated, the discourse becomes unprocessable and will be rejected as incoherent."

And the principle is nondeterministic in that it does not require there to be a unique f. This reflects the resolution problem known from anaphora in the narrow sense.

> "The details of how presuppositions are justified in situations where several parts of the DRS offer themselves as candidates is a matter that will have to be looked into more closely than we have so far done."

All these features the Principle of Presupposition Justification shares with van der Sandt's definition of Presupposition Resolution (though in van der Sandt's theory, accommodation of discourse referents is constrained by admissibility conditions).

Evidently, the constraints on accommodation of conditions on discourse referents, that is, the constraints on anaphoric accommodation, are very difficult to generalize. The asymmetry between *patient* and *child* in the contrast quoted above suggests that the subset relation is relevant: A child to be cured of a disease is necessarily a patient, but not conversely. But as (18) and (19) demonstrate, the relation between a constant and a set is also unidirectional in this respect. In other words, entailment (consequence) is too strong, but compatibility (consistency) is too weak.

(18) The 5000 m race was won by Zandstra.
The 1500 m, too, will be won by a Dutch skater.

(19) ? The 5000 m race was won by a Dutch skater.
The 1500 m, too, will be won by Zandstra.

2.4 Anaphoric Presuppositions and Anaphoric Presupposition

To apply an anaphoric notion of presupposition to zero anaphora, it is practical to state a simple deterministic condition. The following definition of the contribution of a presupposition to the update of a discourse by a sentence presupposes that the sentence is represented in two separate provisional structures, one for the assertion and one for the presupposition.

Anaphoric Presupposition

Let K_0 be any context DRS, K_1 an assertion DRS and K the presupposition DRS of K_1. $K_0 + K_1 / K$ is defined if and only if there is a best function $f: U_K \rightarrow U_{K0}$ such that the merge of K_0 and "$f(K)$" is consistent; then it is the merge of K_0 and "$f(K_1)$" and "$f(K)$".

The cost of determinism is vagueness: The best function f is in principle partial but defined for as many as possible of the discourse referents of the presupposition, and the merge of K_0 and "$f(K)$" should come as close as possible to being a logical consequence of K_0. The definition thus abstracts away from the resolution problem and the accommodation problem.

The definition predicts that the presupposition is independent of the assertion. Specifically, there cannot be discourse referents free in the presupposition structure introduced in the assertion structure. Indeed, the principle that the presupposition DRS is **proper** will play a central role in section 3. Now both van der Sandt (1992) and Kamp & Roßdeutscher (1992) assume that the assertion is merged with the context prior to processing the presupposition. One reason is that the (repetitive) presupposition of *again* is supposed to include the condition $e' < e$ where e is the event introduced in the assertion. This is reasonable in many cases, but not when the sentence is negated; then e is inaccessible. Similarly, the presupposition of *too* is supposed to include the condition $x' \neq x$ where x is the focus referent. Again, this is reasonable in many cases, but not when there is a proportional NP in focus. In the general case, the presupposition must be represented without reference to the assertion. To be sure, in some cases it is necessary to process a part of the sentence first, in particular, when the presupposition is in the scope of a quantifier, as in (20): The quantifier and its restrictor must be merged with the context prior to processing the scope and its presupposition. I return to this issue in 3.4.

(20) Every man loves his wife.

Are all presuppositions anaphoric? The above definition is cast as a general principle, but, as Delin (1992) points out, factive presuppositions and cleft presuppositions are easy to accommodate. Also, they do not seem to display accommodation effects. I assume that these presuppositions are not anaphoric in the sense that there are discourse referents to be anchored to already-introduced discourse referents, but that they are still subject to the same rule. They are anaphoric in a wide sense but the anaphoricity does not show. This means that the universe of the presupposition DRS is empty and the content of the embedded clause is represented as a condition in the form of a sub-DRS. This reduces to Karttunen's pragmatic presupposition, where the presupposition is entailed by the context; but the same general definition applies. An anaphoric presupposition in the narrow sense is, then, a presupposition with a nonempty universe.

Anaphoric Presuppositions

An anaphoric presupposition in the narrow sense is a presupposition represented as a structure with a nonempty universe.

3 Zero Argument Anaphora

The empirical basis of anaphoric presuppositions would seem to be rather limited. The triggers examined so far form a closed class: The definite article, focus particles like *too*, adverbs like *again*, aspectual verbs like *stop*. But, as this section shows, the concept of anaphoric presuppositions naturally generalizes to a wide range of lexical presuppositions, triggered by words with a full meaning from open classes. However, it is not the main aim of this section to extend the domain of anaphoric presuppositions quantitatively; rather, the goal is a qualitative extension to the concept of anaphoric presuppositions. By this I mean that anaphoric presuppositions can be seen to cause more than we have considered so far, which is, basically, to impose strong admittance conditions, induce anaphoric accommodation effects, and bind definite descriptions. What they accomplish over and above this is to bind implicit variables in the carrier sentence, thus taking the load off overt anaphors. Specifically, what I have in mind is that the concept of anaphoric presuppositions can be used to solve the problem of definite ellipsis, as stated by Shopen (1973).

Since Shopen (1973) distinguished between indefinite and definite ellipsis, we have known that missing arguments sometimes have an existential interpretation but sometimes depend on context. The latter case gives rise to so-called zero anaphora. But it has remained a mystery why these zero arguments behave like anaphors, and it has seemed necessary to mark this behavior lexically. My hypothesis is that this phenomenon correlates with presuppositions and that the context dependence in a missing complement can be independently described as an effect of presupposition. The idea is that the corresponding referent is introduced in the presupposition and that in the update process, the substituted referent is propagated to the assertion. Just as with definite descriptions, there is variable sharing between the assertion and the presupposition, and the variable is instantiated through the presupposition. For instance, a sentence *Sue agrees* asserts that Sue thinks that p and presupposes that there is a proposition p such that somebody other than Sue thinks that p; this p must be mapped onto some previously mentioned proposition q, and once it is, q replaces p in the assertion. Thus zero anaphors are not really anaphors but epiphenomena of presuppositions, and there will be no need to distinguish null complements with a definite interpretation in any other way than through the presence of a presupposition, which must be marked in the lexicon anyhow.

The first subsection 3.1 reviews descriptions of zero argument anaphora, 3.2 presents hypotheses and principles of a presuppositional account, and 3.3 discusses a variety of applications. Subsection 3.4, finally, confronts a number of problems.

3.1 Definite Ellipsis

Tim Shopen (1973) observed that 'lexically-determined constituent ellipsis' can be definite as well as indefinite. Since, it has become common knowledge that when an optional complement is not realized syntactically, the semantic result is either that the corresponding variable is existentially quantified over, that is, the empty argument is interpreted as though it were an indefinite, or it remains free, that is, the empty argument is interpreted as though it were a definite, an anaphor. For the former case, Shopen cites (21), where the 'source' role is left unexpressed.

(21) - Bill received a letter today.
- Who did he get it from?

"It is a natural sequence for the second speaker to ask what he does because it does not conflict with any of the presuppositions of the initial statement .... The ellipsed SOURCE of *receive* is interrogated and the meaning of *receive* does not tell us that it should be uniquely identifiable. This is indefinite ellipsis." (p 67)

Thomas (1979) followed up with a similar test for indefinite ellipsis (p 57):

(22) - Have you been eating onions?
- I've been eating, but not onions.

Cases of definite ellipsis, on the other hand, are unacceptable in such sequences:

(23) - Do you expect to pass your driving test?
? - I expect to pass, but not my driving test.

(24) - When Mother told him to clean up his room, Bobby refused.

? - What did he refuse to do?

> "The question 'What did he refuse to do?' is an unnatural sequence because it rejects the presupposition of definiteness in 'Bobby refused.'. [...] The definiteness of the ellipsis with *refuse* is due to the semantic structure of that verb (there is a significant relationship at this juncture with the notion of presupposition), as revealed in its lexical entry."

Shopen offered many more examples of definite ellipsis, ia. featuring the predicates *agree*, *continue*, *disapprove*, *persuade*, *suspect*, and *surprised*. For a few instances of zero anaphora complete with a suitable discourse antecedent, cf. (1)-(4) in 1.1. In (Sæbø 1984) I gave many authentic examples of indefinite as well as definite ellipsis. Two descriptive generalizations made there are that many of the verbs that require a definite interpretation of their missing optional complements express reactions and that in many cases the complements are infinitival or sentential. Cases of definite ellipsis in an individual argument are rare (note, however, *offer*), as are cases of indefinite ellipsis in an abstract entity argument (note, however, *think*).

3.2 The Source of Zero Argument Anaphora

Shopen's remark that there is a significant relationship with the notion of presupposition (p 68) has not been elaborated on, and he himself writes the definiteness of the "activity proposed by a second party in an offer, a command or an invitation" into the lexical entry for *refuse*: "z = definite when ellipsed" (p 69).

It is unsatisfactory to have to indicate in every single case what intuitively appears as a function of some semantic property which the predicates in question have in common and which should be recoverable from the lexical entries anyhow. If it is correct, as Shopen claims, that the definiteness of the ellipsis is due to the semantic structure of the verb and that there is a significant relationship with the notion of presupposition, then that semantic structure and, specifically, its relationship with presupposition, should be explored. And I believe that with the notion of anaphoric presupposition at hand, there is a straightforward way of predicting when a zero argument is anaphoric and how zero anaphora works: It is anaphoric if (and only if) the predicate triggers a presupposition involving the argument, and the zero anaphor is resolved in the same process in which the presupposition is justified. When there is no presupposition involving the relevant referent, it is introduced in the assertion structure, and indefinite ellipsis evolves. When, on the other hand, there is a presupposition involving the relevant referent, that referent is introduced in the presupposition structure, and definite ellipsis evolves. The difference surfaces in where the referent is introduced; which universe it belongs to, but it originates in the absence or presence of a presupposition involving it.

> **Hypothesis** A zero argument is anaphoric iff the predicate triggers a presupposition involving it.

There are two directions to this hypothesis: The 'if' and the 'only if' implication. The 'if' claim is theoretical and can be shown on the basis of the following simple assumption about provisional DRS construction.

> **Principle 1** The presupposition DRS is proper.

That is, the presupposition DRS does not contain free occurrences of referents (cf. Kamp & Reyle 1993: 111). If the discourse referent representing a zero argument occurs in a condition in the presupposition, it is introduced in the presupposition, it is in its universe. In other words, the zero argument is an anaphoric element, to be instantiated in the process of presupposition justification.

The 'only if' implication, on the other hand, is empirical and must be defended piece by piece. To first see how the indefinite interpretation can come about, consider

Principle 0 The merge of the assertion DRS and the presupposition DRS is proper.

Thus if the zero argument referent is not involved in any presupposition, it can belong to the universe of the presupposition DRS or to the universe of the assertion DRS. In the latter case, it is essentially novel; the universe of the presupposition DRS is the only place for an anaphoric element. Now to force an indefinite interpretation of a zero argument not involved in a presupposition, we need the principle

Principle 2 If a referent is in the universe of the presupposition but does not occur in a condition in that structure, it originates in an overt anaphor.

That is, to be anaphoric a zero argument must be involved in a presupposition.

When a sentence with a zero argument is processed, it is initially represented as a bipartite structure assertion / presupposition where the zero argument is rendered as a dummy referent in the relevant argument places of the relevant relations. This referent must be introduced in the assertion or the presupposition DRS. If it occurs in a condition in the presupposition DRS, then as a consequence of Principle 1 it is introduced in this DRS. If it does not, as a consequence of Principle 2 it is not. On this account, in contrast to overt anaphors, zero anaphors do not trigger the introduction of a referent in the presupposition. In general, the zero argument does not trigger the introduction of a referent; it is only on account of general principles of provisional DRS construction that the effect is the same as with an indefinite or a definite pronoun. An anaphoric element, we might say, is an element which corresponds to a referent in the presupposition's universe, whether this correspondence is direct, as with overt anaphora, or indirect, as with zero, or covert, anaphora.

Principle 2, saying that only an overt anaphor is represented as a referent in the presupposition's universe without occurring in a condition in the presupposition, secures an indefinite interpretation of zero elements not involved in a presupposition. As noted, the 'only if' direction of the hypothesis, encoded in Principle 2, depends on showing piece by piece that there really is a presupposition involved, and it would go beyond the scope of this paper to deliver a complete defense of this half of the hypothesis. The following section is intended to show for some select cases that the presuppositional analysis is more than a mere stipulation.

3.3 Applications

Quite many predicates that can have anaphoric zero arguments describe reactions and generate presuppositions as to the stimulus. The verbs *agree* and *refuse* are clear cases in point. The class of emotive past participle predicates like *surprised*, describing how the subject is affected by a piece of information presupposed to reach the subject, can also be subsumed under reaction predicates. These, in turn, are closely related to factive predicates where the fact argument can be zero, like *remember*. Finally, the presuppositions of state transition predicates like *recover* involve optional arguments characterizing precondition states.

3.3.1 Reactions

Let us first consider the verb *agree*. This word has quite much in common with the focus particle *also* or *too*; in fact, it can almost be paraphrased as also believing. The relevant variant subcategorizes for a *that* clause.
We seek a principled way of deducing (26) from (25):

(25) A sheep has been killed in the mountains.
Farmers believe it was a wolf, and the Sheriff **agrees**.

(26) A sheep has been killed in the mountains.
Farmers believe it was a wolf, and the Sheriff **agrees it was a wolf**.

Let us first consider (26). Simplifying and abstracting away from inessentials, we take this to be a case of full match between the presupposition and the context:

context: $\langle\{x_1\}, \{believe(x_1, \langle\{w\}, \{wolf(w)\}\rangle)\}\rangle$
assertion: $\langle\{x_3\}, \{believe(x_3, \langle\{w\}, \{wolf(w)\}\rangle)\}\rangle$
presupposition: $\langle\{x_2\}, \{believe(x_2, \langle\{w\}, \{wolf(w)\}\rangle)\}\rangle$

For (25), eliminating the subDRS altogether, initially we have

assertion: $\langle\{x_3\}, \{believe(x_3, K)\}\rangle$
presupposition: $\langle\{x_2\}, \{believe(x_2, K)\}\rangle$

However, this structure violates the principle that a presupposition be proper, so the correct result is:

presupposition: $\langle\{x_2, K\}, \{believe(x_2, K)\}\rangle$

The context stays the same, of course, or equivalently (cf. Asher 1993: 225ff),

context: $\langle\{x_1, K'\}, \{believe(x_1, K'), K' = \langle\{w\}, \{wolf(w)\}\rangle\}\rangle$

Thus on anaphoric presupposition, K is replaced by K' and (25) says the same as (26).

For Shopen's paradigm verb *refuse*, consider

(27) The board instructs the management to reduce employment,
but it **refuses**.

context: $\langle\{x, y, P\}, \{instruct(x, y, P),$
$P = \lambda y\langle\ y\ reduce\ employment\ \rangle\}\rangle$
assertion: $\langle\ \emptyset, \{refuse^*(w, Q)\}\rangle$
presupposition: $\langle\{v, w, Q\}, \{ask(v, w, Q)\}\rangle$

Here and in the following, *predicate** is supposed to designate the predicate stripped of its presupposition. It should not come as a surprise that in the majority of cases, there is no straightforward way of identifying the separate presupposition and assertion components of a predicate. Rather, *agree* is a special case in that it is almost possible to transcribe the presupposition and assertion in natural language predicates, but even here, it represents an idealization. In general, what is required is in the presupposition a meta predicate just unspecific enough to cover the range of appropriate contexts, and in the assertion a meta predicate expressing the rest.

Implicative verbs, of course, do permit an unambiguous formulation of the assertion component:

(28) On Wednesday, the chief UN negotiator, Thorvald Stoltenberg, met
with Mr. Izetbegovic to try and ease those fears. He clearly **failed**.

assertion: $\langle\ \emptyset, \{\neg P(x)\}\rangle$
presupposition: $\langle\{x, P\}, \{try(x, P)\}\rangle$

3.3.2 Emotive Predicates and Factives

Zero anaphora occurs regularly with a number of past participle verb forms like *surprised, impressed, delighted, shocked, relieved*. The syntactic category of the optional complement varies, but its semantic type is basically propositional.

(29) Sue told Joe that she was pregnant.
He was **surprised** that she was pregnant.

The fact that these past participles subcategorize for i.a. a *that* clause indicate that they are not true passives. For ellipsis, the salient fact is that the proposition in the theme role is involved in the presupposition of the predicate. That Joe is surprised that Sue is pregnant presupposes that Joe learns that Sue is pregnant. Thus (simplified with respect to the pronouns):

context: $\langle\{s, j\}, \{tell(s, j, \langle pregnant(s)\rangle)\}\rangle$
assertion: $\langle\, \emptyset, \{surprised^*(j, \langle pregnant(s)\rangle)\}\rangle$
presupposition: $\langle\, \emptyset, \{learn(j, \langle pregnant(s)\rangle)\}\rangle$

Now when the *that* clause is omitted, the double occurrence of the constant propositional referent $\langle pregnant(s)\rangle$ is replaced by a variable propositional referent K. The presupposition must be proper, so K is introduced in its universe.

(30) John was surprised.

assertion: $\langle\, \emptyset, \{surprised^*(j, K)\}\rangle$
presupposition: $\langle\{K\}, \{learn(j, K)\}\rangle$

This means that it must be mapped onto some propositional referent in the context. Strictly, there is none in the universe of the context, but it must, for independent reasons of abstract entity anaphora (cf. Asher 1993: 225ff), always be possible to declare one for a constant abstract entity referent occurring in a condition.

Even presuppositions which are not anaphoric in the narrow sense that the universe is nonempty can become anaphoric in this sense once the predicate generating them occurs with an empty argument. In fact, this is the case with factive predicates in case the propositional arguments are optional. In English, *know* can be used without an overt propositional complement, along with a range of other factive verbs and adjectives such as *forget*, *remind*, and *remember* (cf. (3) in 1.1), *notice*, and *aware*. Many have presuppositions over and above pure factivity; *forget* and *remember*, for instance, appear to presuppose in addition that the subject has known the proposition or other object, and *notice* appears to presuppose that the proposition or other object (not) noticed is epistemically accessible to the subject. Let us see how the zero argument of *know* acquires its anaphoric interpretation.

(31) It is not necessary to tell them the climb is dangerous. Sue **knows**.

assertion: $\langle\{z\}, \{Sue(z), believes(z, K)\}\rangle$
presupposition: $\langle\{K\}, \{K\}\rangle$

The propositional variable K occurs in two different roles in the presupposition, both as a referent in the universe and as a condition, more accurately a referent in an empty condition encoding factuality. In the assertion it appears as a referent in a regular condition. Now the verb *know* shows a zero anaphor more often when the 'antecedent' is a question than when it is a definite proposition, and then the factive presupposition is in effect tautologous: cf. (32). The verb *ask* can reasonably be assumed to presuppose that the asker does not know the answer to the question, maybe even that she wants to know it:

(32) We were wondering which track to take to Goatteluobal,
so we **asked** a Lap woman, but she didn't **know**.

3.3.3 Phase Presuppositions

State transition verbs like *recover* or *return* and verb groups like *stop raining* have traditionally been analyzed in terms of a backward-looking presupposition and a forward-looking assertion, plus, as the case may be, some process of transition as part of the assertion. Thus *a recovers at t* would presuppose that a has been ill up to *t* and assert that a is well from *t* on, and that a process of recovery goes on at *t*. And *it stops raining at t* would presuppose that it rains some time up to *t* and assert that it does not rain some time from *t* on. Recall Heim's (1987) Kripke-inspired example (11) in 2.1. The interesting thing to note now is that the state type verb *cooking* is enclosed in parantheses in Heim's formulation. It is omissible, and in fact, its omission can be shown, using discourse referents at state type level, to be a case of zero anaphora via the backward-looking presupposition of *stop*.

We should be able to predict the validity of the inference from (4) in 1.1. to (33):

(33) It's dangerous to hypnotize people. You might do it to somebody with a gullible mind sometime, and that person would never **recover from the hypnosis**!

This is strongly reminiscent of a case treated by Kamp & Roßdeutscher (1992):

> "Our ultimate goal is to account for the validity of the following inference:
> The tourist came down with typhoid.
> After three weeks he was well again. A doctor from Izmir cured him.
> Conclusion:
> The doctor cured him of typhoid." (p 76)

They reach this goal only by invoking discourse (rhetorical) relations. Let us see how it can be reached with recourse only to presuppositions. Simplifying the example and abstracting away from inessentials:

(34) John comes down with typhoid at t. He recovers at t'.

context: $\langle\{e, s, y\}, \{ANT(HEILEN)(e, j, y), typhoid(y), e \text{ at } t, PRE(HEILEN)(s, j, y), e)(s\}\rangle$

assertion: $\langle\{e', s', z\}, \{HEILEN(e', j, z), e' \text{ at } t', RES(HEILEN)(s', j, z), e')(s'\}\rangle$

The predicate *HEILEN* is primitive, and there are the meaning postulates

1. $s : PRE(C)(u, v) \Leftrightarrow s : \neg RES(C)(u, v)$,
2. $\langle\{e, u, v\}, \{C(e, u, v)\}\rangle \Rightarrow \langle\{s\}, \{PRE(C)(s, u, v)\}\rangle$ and
3. $PRE(ANT(C)) \equiv RES(C)$ and $RES(ANT(C)) \equiv PRE(C)$.

By postulate 2, the precondition state, intuitively the state of having that from which the *HEILEN* event is a recovery, can be added to the assertion:

assertion: $\langle\{e', s', z, s''\}, \{HEILEN(e', j, z), e' \text{ at } t', RES(HEILEN)(s', j, z), e')(s', PRE(HEILEN)(s'', j, z), s'')(e'\}\rangle$

Thus Kamp & Roßdeutscher do not treat the precondition as a presupposition. However, if it is treated as one, we can conclude (35) from (34):

assertion: $\langle\{e', s'\}, \{HEILEN(e', j, z), e' \text{ at } t', RES(HEILEN)(s', j, z), e')(s\}\rangle$

presupposition: $\langle\{s'', z\}, \{PRE(HEILEN)(s'', j, z), s'')(e'\}\rangle$

(35) John comes down with typhoid at t. He recovers from the typhoid at t'.

The precondition state s'' is mapped onto the result state of the *ANT(HEILEN)* event and the referent z for the illness is mapped onto the referent y for the typhoid.

3.4 Problems

The hypothesis that a zero argument is anaphoric iff involved in a presupposition (3.2) rests on the principle that the presupposition DRS is proper (Principle 1) in one direction and on the principle that only an overt anaphor can trigger the introduction of a referent in the presupposition DRS (Principle 2) in the other. Now both principles are problematic. Cases where apparently, an argument involved in a presupposition is not anaphoric, are discussed in 3.4.1, and cases where apparently, an anaphoric zero argument is not involved in a presupposition, in 3.4.2.

3.4.1 Subsentential Presuppositions

At first sight, cases like (36) seem to contradict the assumption that a predicate like *recover* triggers a presupposition involving the state to recover from:

(36) John has just recovered from a serious illness.

The indefinite signals novelty, and novelty is incompatible with the presupposition. Referents for indefinite descriptions are introduced in assertion structures, not in presupposition structures. But these cases are reminiscent of cases like (37).

(37) John has written to the author of a book about Schubert.

That is, cases where, contrary to Heim's (1982: 373ff) linking hypothesis, a new file card is introduced under accommodation without being linked by cross-references to some already-present file card(s). Evidently, the indefinite outscopes the definite. The assertion consists of two parts: First, there is a book y about Schubert, and second, John has written to z; and the presupposition is that y has an author z. In this way, the referent y in the presupposition is bound in a part of the assertion processed in advance. To see more clearly that this is a sensible analysis, consider

(38) John knew the crew on every ship in the harbor.

This sentence must be processed as a tripartite structure: the quantifier 'every ship x in the harbor', the presupposition 'x has a crew', and the associated assertion. Quantificational NPs appear as complements of predicates that supposedly trigger presuppositions, too:

(39) John refuses to do everything he is asked to.

The reasonable approach to these cases is, then, to decompose the sentence into three parts, processing the indefinite or quantificational NP at once. The sentences will presuppose nothing because the presupposition is preempted in the NP. (40) will be analyzed as 'there is something K John has heard on the radio such that (presupposition) John has learnt of K; (assertion) John is shocked* at K'.

(40) John is shocked at something he has heard on the radio.

What this shows is, first, that it is too simple to identify the carrier sentence with the entire incoming sentence, second, that we must qualify the principle that the presupposition DRS is proper. It may contain free occurrences of discourse referents as long as those occurrences represent bound variables.

3.4.2 Elusive Presuppositions

The claim that zero argument anaphora depends on presupposition is, of course, vulnerable to facts. It should not be possible to locate an anaphoric zero argument without being able to formulate a presupposition going with it. Now predictably, many presuppositions will be subtle, vague, and difficult to specify. In some cases, positing a presupposition risks circularity and comes close to stipulation. Let me comment on a few problematic predicates.

Consider first a class of predicates describing how agents are ascribed to actions, like *suspect* and *blame*. It seems prima facie plausible that a sentence with *suspect* presupposes that the action property of the *of* complement holds of somebody. So (41) would presuppose that (the police believe) the bomb has been placed. However, this presupposition is difficult to maintain in cases like (42).

(41) The police suspect the mafia of having placed the bomb.

(42) The police suspect Neonazis of having started the fire.

This sentence does not appear to presuppose that (the police believe) the fire was started by somebody. In fact, it appears to have two readings, one presuppositional and one non-presuppositional, where an accent on *started* correlates with the latter. By comparison, the verb *blame* appears to only permit a presuppositional reading. Evidently, the zero argument construction is based on the presuppositional reading. The other reading might be accounted for by saying that the property description, the *of* object, is processed in advance (cf. 3.4.1), in the appropriate belief context, thus restoring presuppositionality:

(43) The police believe the fire was started by somebody, suspecting Neonazis

(of having started it).

Manfred Pinkal (1985: 76) noted that the German verb *verkaufen (sell)* shows indefinite ellipsis in its indirect (*an* or dative) object but definite ellipsis in its direct (accusative) object, as witnessed by:

(44) Kolumbus hat an Cortez verkauft.

(45) Kolumbus hat die Santa Maria verkauft.

The former sentence can only mean that Columbus has sold it to Cortez, while the latter sentence can only mean that Columbus has sold the Santa Maria to somebody. This asymmetry is prima facie mysterious. But note that the zero theme argument cannot be a zero anaphor with the same antecedence conditions as the anaphor *it*:

(46) Cortez hat ein Schiff. Kolumbus hat es ihm / an ihn verkauft.

(47) ? Cortez hat ein Schiff. Kolumbus hat ihm / an ihn verkauft.

Although the antecedent is available in the form of the indefinite *ein Schiff*, the zero anaphor is not resolved. This is indicative of a presupposition, and in fact, on the reasonable assumption that *x verkauft y an z* presupposes that *x* has had *y*, both the definite interpretation of the direct object zero argument and the indefinite interpretation of the indirect object zero argument are predicted, as the presupposition involves only the former. However, since (46) is felicitous, the presupposition is evidently to some degree dependent on the zero argument, and this, of course, complicates the analysis. Similarly, the verb *tell*, probably not inherently factive, appears to acquire a factive interpretation with a zero propositional argument:

(48) John believes that Susan is pregnant. She has told him so / that.

(49) John ?believes / knows that Susan is pregnant. She has told him.

This indicates that the zero argument itself helps bring about the presupposition. I am told the same holds for the German verb *gestehen* (*confess*).

Is it reasonable to describe zero anaphoric arguments of relational nouns, as in 'bridging' definite descriptions (*the captain*, *the crew*), in terms of presuppositions? Note that as long as the noun phrases are definite, it is not necessary to assume a presupposition for an anaphoric interpretation of the zero argument: given the presuppositional analysis of definite descriptions in general, the zero argument referent is in the presupposition.. The 'ellipsis rule' stated by Zimmermann (1991: 199) (if α is a relational noun and Ø an empty genitive NP the extension of αØ is the set of x such that (x, y) is in the extension of α where y is relevant in the utterance situation) is redundant as far as anaphoric definite descriptions are concerned. And in fact, as Zimmermann notes, there is in principle the possibility of an indefinite interpretation, when the definite article is replaced by the indefinite: *A captain*. However, if a relational noun shows definite ellipsis as an indefinite, as does *victim* in (50), there is reason to assume a presupposition, such as: There is some event to be described as an accident, an attack, a calamity, or the like.

(50) In the night of the American bombardment, Anna Braun,
a survivor, and her mother, a **victim**, were asleep in their beds.

To be sure, there are several cases of implicit anaphora remaining problematic on the account proposed; notably implicitly anaphoric adverbials like *2 miles away*, elliptic comparatives, implicitly deictic words like *come* and *go*, and nouns like *enemy*, where a presupposition is difficult to identify. It may ultimately turn out that some instances of zero argument anaphora are not presuppositionally but inferentially driven, depending on relevance, coherence, or informativeness considerations.

4 Conclusions

In a nutshell, I have tried to describe zero argument anaphors not as anaphors in their own right but simply as variables accidentally occurring in a presupposition. Unless the connection between definite ellipsis and presupposition is appreciated, covert anaphors are a variant of overt anaphors, which are presuppositional de se in that they introduce discourse referents in the presupposition structure. However, once the presuppositions of the predicates are taken seriously, the zero anaphors reduce to argument positions, so to speak; and via general properness principles, in particular prohibiting free occurrences in the presupposition structure, the referents surface in the position of anaphora, in the universe of the presupposition structure. Thus implicit anaphors come out as truly implicit in that they do not come with an instruction to find an antecedent, instead, such an instruction comes about indirectly through the presupposition involving them. In this way, it is not necessary to notate the anaphoricity of certain zero arguments as opposed to others.

Another advantage of this account is that it allows a uniform treatment of null anaphors which have no overt anaphor counterpart because the verb simply has no syntactic argument place to provide. Partee (1991) notes that "not all dependent elements take complements or otherwise offer a 'site' for a 'null pronoun'", as evidence against regarding (primarily deictic) null elements as concealed pronouns. The present account draws a sharp distinction between zero and overt anaphors, and this accords well with cases where it is not possible to substitute an overt anaphor. A clear example is the German verb *nachdrängen* (approximately two-place *replace* with an optional complement). This verb takes no complement, but semantically, it is two-place, and the second argument is invariably anaphoric. The anaphoricity comes about through the presupposition of *nachdrängen(x,y)*: *y* goes away.

I do not claim that every form of zero anaphora is presuppositionally conditioned, but I do claim that zero argument anaphors are presuppositionally conditioned – typically or generally. In any case, a presupposition is a sufficient condition, and this mechanism can be shown to account for a considerable subset of zero anaphors. Whether a presupposition is also a necessary condition is an empirical question. As elsewhere, there are a number of clear cases, but also quite a few less clear cases. In some, a case can be made that the predicate triggers a presupposition after all. But I leave the possibility open that the presuppositional account must ultimately be supplemented by some other principle.

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Polarity, veridicality, and temporal connectives

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> I am strongly inclined to maintain that the rules for our grass-roots employment of temporal conjunctions - not only "at the same time", but also "before" and "after" - belong to the domain of formal logic.
> Peter Geach, *Logic Matters*, 1972: 316.

1 Introduction

The purpose of this paper[1] is to draw attention to the semantical properties of *before*, *after*, and related elements. In particular, we shall raise the question whether the occurrence of negative polarity items in *before*-clauses can be described in terms of the semantic structure of the connective. In order to provide an answer, we adopt the analysis proposed by Landman (1991), which is based on Anscombe's (1964) discussion of *before* and *after*, and incorporates the findings of Heinämäki (1974), Hinrichs (1981), Partee (1984) and Oversteegen (1989). We then show that *before* is not only a monotone decreasing connective, but has the characteristic properties of an *n*-word.[2] This result will enable us to point out some unexpected connections between the phenomenon of negative polarity, on the one hand, and ontological assumptions about the flow of time, on the other. In particular, we will prove that *before* can only be analyzed as an *n*-word if the model of time underlying natural language is the model of linear time. We also discuss two other interesting features of Landman's account: *before* and *after* cannot be treated as converses, and *before* is what Montague (1969) calls nonveridical in that it doesn't force us to accept the truth of the clause it introduces.[3] Veridicality and monotonicity turn out to be

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2. The notion of an *n*-word is due to Laka Mugarza (1990), who uses the term to describe universal negatives like *nadie* 'no one', *nada* 'nothing' and *nunca* 'never' in Spanish. Though Laka herself regards these expressions as existential polarity items, Zanuttini (1991) argues that they should be treated as universal negatives. Van der Wouden and Zwarts (1993) maintain that Romance *n*-words are at times polarity items and at times universal negatives, a point of view which was advanced earlier in Zanuttini (1989). For present purposes, an *n*-word is simply a universal negative which has the semantic structure of what will hereinafter be referred to as an anti-additive expression.

3. We refrain from employing the terminology introduced by Heinämäki (1974), who calls *before* 'non-committal' instead.

related properties, since it can be shown that monotone decreasing connectives are nonveridical in nature.

2 Negative polarity items

The term polarity item, as applied to language, allows us to describe the behavior of certain words and phrases with respect to negation. One class of such expressions, usually referred to as the class of negative polarity items, requires the presence of a negative element in the sentence. As an illustration, consider the examples in (1).

(1) a. None of the children noticed anything
    b. *Each of the children noticed anything

The ungrammaticality of (1b) proves that the presence of the noun phrase *each of the children* is not sufficient to justify the occurrence of the polarity item *anything*. Apparently, it is only negative expressions such as *none of the children* that are capable of licensing such elements. We must not suppose that this is a peculiar feature of English. Similar patterns can be found in Dutch and German, as shown by the examples in (2) and (3).

Dutch

(2) a. Niemand zal zulk een beproeving hoeven te doorstaan
    No one will such an ordeal need to go through
    'No one need go through such an ordeal'
    b. *Iedereen zal zulk een beproeving hoeven te doorstaan
    Everyone will such an ordeal need to go through
    *'Everyone need go through such an ordeal'

German

(3) a. Keiner wird solch eine Prüfung durchzustehen brauchen
    No one will such an ordeal to go through need
    'No one need go through such an ordeal'
    b. *Jeder wird solch eine Prüfung durchzustehen brauchen
    Everyone will such an ordeal to go through need
    *'Everyone need go through such an ordeal'

Although the contrasts between these sentences may well seem perplexing at first, Ladusaw (1979) has shown that they can be explained in terms of the monotonicity properties associated with various words and phrases. By way of illustration, consider the conditional *At least one villager sang loudly* → *At least one villager sang*. Provided that the structure of the universe is such that the class of individuals associated with the verb phrase *sang loudly* (VP_1) is a subset of the class of individuals associated with the verb phrase *sang* (VP_2), we may legitimately pass from the proposition *At least one villager sang loudly* to *At least one villager sang*. What this means is that noun phrases of the form *at least n N* have are monotone increasing: if NP VP_1 and $VP_1 \subseteq VP_2$, then NP VP_2. The same test shows that expressions of the forms *some N*, *every N* and *both N* are also upward monotonic. For if the predicate *ate fish* applies only to what the predicate *ate* also applies to, then the following conditionals are all valid: *Some porters ate fish → Some porters ate*, *Every child ate fish → Every child ate*, *Both lawyers ate fish → Both lawyers ate*.

It turns out that monotone increasing noun phrases have a decreasing counterpart. To demonstrate this, we begin by considering the conditional *At most one villager sang → At most one villager sang loudly*. Whenever the state of affairs in the universe is such that the class of individuals associated with the verb phrase *sang* (VP_1) is a superset of the class of individuals associated with the verb phrase *sang loudly* (VP_2), we may legitimately pass from the proposition *At most one villager sang* to *At most one villager sang loudly*. This is important because it entails that noun phrases of the form *at most n N* are monotone decreasing: if NP VP_1 and $VP_2 \subseteq VP_1$, then NP VP_2. In a similar manner, one easily shows that expressions of the forms *not every N*, *no N* and *neither N* are also downward monotonic. For if the predicate *ate* applies to whatever the predicate *ate fish* applies to, then the following conditionals are all valid: *Not every woman ate → Not every woman ate fish*, *No attorney ate → No attorney ate fish*, *Neither connoisseur ate → Neither connoisseur ate fish*.

In the light of the distinction between upward and downward monotonic noun phrases, the contrasts in (1), (2), and (3) admit only one explanation: the class of elements which are capable of licensing the occurrence of negative polarity items is coextensive with the class of monotone decreasing expressions. This conclusion is corroborated by the contrasting examples in (4).

(4) a. At most five of the children noticed anything
b. *At least five of the children noticed anything

Of the two phrases *at most five of the children* and *at least five of the children*, it is only the first that can act as a licensing expression for the negative polarity item *anything* - a state of affairs which must be attributed to the circumstance that *at most five of the children* belongs to the class of monotone decreasing noun phrases, and *at least five of the children*, to the class of monotone increasing noun phrases.

2.1 Weak and strong polarity items

Negative polarity items can be either of the weak, or of the strong, type.[4] In order to get a clear view of the content of this distinction, one does well to take the following Dutch and German examples into consideration.

4. A more elaborate discussion of weak and strong forms of polarity can be found in Zwarts (1993) and van der Wouden (1994).

Dutch

(5) a. Hoogstens één kind zal zich hoeven te verantwoorden
At most one child will himself need to justify
'At most one child need justify himself'

b. Niemand zal zulk een beproeving hoeven te doorstaan
No one will such an ordeal need to go through
'No one need go through such an ordeal'

(6) a. *Hoogstens zes agenten hebben ook maar iets bemerkt
At most six cops have anything noticed
'At most six cops noticed anything'

b. Niemand heeft van de regenbui ook maar iets bemerkt
No one has of the rain anything noticed
'No one noticed anything of the rain'

German

(7) a. Höchstens eine Frau wird sich zu verantworten brauchen
At most one woman will herself to justify need
'At most one woman need justify herself'

b. Keiner wird solch eine Prüfung durchzustehen brauchen
No one will such an ordeal to go through need
'No one need go through such an ordeal'

(8) a. *Höchstens zehn Kinder haben auch nur irgendetwas bemerkt
At most ten children have anything noticed
'At most ten children noticed anything'

b. Keiner von diesen Leuten hat auch nur irgendetwas bemerkt
None of these people has anything noticed
'None of these people noticed anything'

The contrast between (5) and (7), on the one hand, and (6) and (8), on the other, proves that expressions such as *ook maar iets* and *auch nur irgendetwas* place stronger restrictions on their environments than the negative polarity items *hoeven* and *brauchen* ('need'). As the ungrammatical sentences in (6) and (8) show, neither Dutch *ook maar iets* nor German *auch nur irgendetwas* is satisfied with the presence of a monotone decreasing expression of the form *hoogstens* (*höchstens*) *n N* 'at most n N'. Instead, both seem to require an *n*-word like *niemand* (*keiner*) 'no one' or *keiner von diesen Leuten* 'none of these N'. As a matter of fact, the distinction between weak and strong forms of negative polarity appears to correspond with that between monotone decreasing and so-called anti-additive noun phrases.

Monotonic noun phrases are characterized by the fact that they are closed under supersets or subsets. If they are closed under supersets, they are monotone increasing; if they are closed under subsets, they are monotone decreasing. This does not exhaust the matter, for a closer look reveals that there are several alternative ways to determine whether a noun phrase is upward or downward monotonic. In fact, monotonic noun phrases can be given a number of logically equivalent characterizations. The next theorem provides the relevant details.

Fact

(9) a. A noun phrase is monotone increasing iff the following two schemata are logically valid:
(a) NP (VP_1 and VP_2) → (NP VP_1 and NP VP_2);
(b) (NP VP_1 or NP VP_2) → NP (VP_1 or VP_2).

b. A noun phrase is monotone decreasing iff the following two schemata are logically valid:
(a) NP (VP_1 or VP_2) → (NP VP_1 and NP VP_2);
(b) NP VP_1 or NP VP_2) → NP (VP_1 and VP_2).

On the basis of these tests one can arrive at fairly accurate judgments concerning the presence of monotonicity properties. It is readily established, for instance, that expressions of the forms *many N*, *most N*, and *several N* are all upward monotonic. The class of monotone decreasing noun phrases, on the other hand, can be shown to include expressions of the forms *few N*, *no N*, and *not all N*.

The foregoing result gives us yet another way of characterizing the behavior of monotonic expressions. If we regard the semantic value associated with noun phrases as a function, the typical monotonicity patterns can be represented as in (10).[5]

		Upward monotonic		**Downward monotonic**
(10)	a.	$f(x \cap y) \subseteq f(x) \cap f(y)$	c.	$f(x \cup y) \subseteq f(x) \cap f(y)$
	b.	$f(x) \cup f(y) \subseteq f(x \cup y)$	d.	$f(x) \cup f(y) \subseteq f(x \cap y)$

It should be noted that the formulas in (10c) and (10d) correspond to one half of the first, and one half of the second, law of De Morgan, respectively. Inasmuch as these laws can be said to characterize the use of negation, monotone decreasing phrases may be regarded as being weakly negative. We can now show what the difference is between a monotonic expression and one which is additive or anti-additive. An element which is additive displays the pattern in (11a); one which is anti-additive exhibits the pattern in (11b).

		Additive		**Anti-additive**
(11)	a.	$f(x \cup y) = f(x) \cup f(y)$	b.	$f(x \cup y) = f(x) \cap f(y)$

In other words, anti-additive phrases embody a stronger form of negation than downward monotonic ones in that they are governed by the first law of De Morgan as a whole. This logical difference is reflected in the behavior of the negative polarity items in (5)–(8). Whereas Dutch *hoeven* and German *brauchen* are content with a monotone decreasing expression like *hoogstens* (*höchstens*) *n N* as licensing element, *ook maar iets* and *auch nur irgendetwas* require the presence of an anti-additive phrase like *niemand*, *keiner* or *keiner von diesen Leuten*.

5. Monotone increasing functions are sometimes said to be isotone. Their monotone decreasing counterparts, defined in (11), are accordingly referred to as antitone functions. See Birkhoff (1967: 3) and Stoll (1974: 55), among others.

By way of illustration we give here a formulation of the laws which govern the distribution of negative polarity items.

Laws of negative polarity

(12) a. Only sentences in which a monotone decreasing expression occurs can contain a negative polarity item of the weak type.
b. Only sentences in which an anti-additive expression occurs can contain a negative polarity item of the strong type.

According to the first law, the presence of a monotone decreasing expression is a necessary condition for the appearance of negative polarity items of the weak type. The second law stipulates that negative polarity items of the strong type require the presence of an anti-additive expression as licensing element. To forestall any misunderstanding, we note that every anti-additive expression is also a monotone decreasing expression. It follows that negative polarity items of the weak type can also occur in sentences containing an anti-additive expression.

2.2 A hierarchy of negative expressions

Although the distinction between monotone decreasing and anti-additive expressions may not at first seem transparent, it finds its origin in the fact that phrases of the forms *no one*, *nothing*, *neither N* and *none of the N* embody a stronger type of negation than those of the forms *at most n N* and *few N*. This becomes apparent when we compare the logical behavior of such elements with that of the sentential prefix *it isn't the case that*. By way of illustration, we consider the biconditionals in (13).

(13) a. It isn't the case that Jack ate or Jill ran $\leftrightarrow$
It isn't the case that Jack ate and it isn't the case that Jill ran
b. It isn't the case that Jack ate and Jill ran $\leftrightarrow$
It isn't the case that Jack ate or it isn't the case that Jill ran

One sees immediately that the equivalences in (13a) and (13b) must both be accepted as valid - a state of affairs which admits of no other explanation than that the operation in question is governed by the laws of De Morgan. This observation is important because it has frequently been argued that the logical patterns in (13) characterize the use of negation. Although such a conclusion is correct with respect to sentential negation and similar expressions, it must be regarded as misleading when it comes to other forms of negation. Not only does natural language contain a variety of negative expressions, their logical behavior is also not the same. In order to convince ourselves of this fact, we consider the conditionals in (14).

(14) a. Few trees will blossom or will die $\rightarrow$
Few trees will blossom and few trees will die
b. Few trees will blossom and few trees will die $\not\rightarrow$
Few trees will blossom or will die

c. Few trees will blossom and will die $\not\rightarrow$
Few trees will blossom or few trees will die

d. Few trees will blossom or few trees will die $\rightarrow$
Few trees will blossom and will die

¿From these examples it is clear that the phrase *few trees*, though a negative expression, differs substantially from the prefix *it isn't the case that*. Of the four conditionals presented above, only two are valid: the one in (14a) and the one in (14d). In other words, the logical behavior of noun phrases of the form *few N* is governed by one half of the first law of De Morgan and one half of the second law of De Morgan. In this regard, they are by no means alone, for it requires little reflection to realize that monotone decreasing noun phrases of the forms *at most n N*, *not all N*, *only a few N* and *no more than n N* behave in much the same way. What this suggests is that such expressions embody a weak form of negation.

It turns out that there exists, in fact, a whole hierarchy of negative expressions. For not only do we have phrases of the forms *few N* and *at most n N*, but we also find anti-additive cases such as *no N*, *none of the N* and *no one*. The latter category differs from the former in that it expresses a stronger form of negation. The following conditionals provide a clear illustration.

(15) a. No man escaped or got killed $\rightarrow$ No man escaped and no man got killed

b. No man escaped and no man got killed $\rightarrow$ No man escaped or got killed

c. No man escaped and got killed $\not\rightarrow$ No man escaped or no man got killed

d. No man escaped or no man got killed $\rightarrow$ No man escaped and got killed

From these examples we may conclude that the noun phrase *no man*, regarded as a negative expression, differs considerably from *few trees*. Of the four conditionals presented above, no less than three must be counted as valid: the one in (15a), the one in (15b), and the one in (15d). What this means is that the logical behavior of noun phrases of the form *no N* is determined by the first law of De Morgan as a whole and one half of the second law of De Morgan. We must not suppose that this is a mere accident, for it is easy to see that the property in question also holds of anti-additive noun phrases of the forms *none of the N*, *neither N* and *no one*. The conclusion must therefore be that expressions of this type embody a stronger form of negation than monotone decreasing phrases like *few N* and *at most n N*, though not as strong as the type of negation expressed by the sentential prefix *it is not the case that*.

There is another class of expressions which represents a stronger form of negation than the monotone decreasing ones, but which is independent of the class of anti-additive expressions. These are the so-called antimultiplicative elements, which are typically associated with the semantic pattern in (16b).

(16) **Multiplicative** **Antimultiplicative**

a. $f(x \cap y) = f(x) \cap f(y)$ b. $f(x \cap y) = f(x) \cup f(y)$

It is easy to see that the antimultiplicative expressions differ from their antiadditive counterparts in that they validate not the first, but the second law of De Morgan as a whole. Well-known representatives of this group are phrases of the forms *not all N*, *not every N* and *not always*.

3 Temporal connectives

Negative polarity items such as ***anyone*** and ***ever*** can occur in several temporal environments, among them *before*-clauses. As is illustrated by the contrast between (17) and (18), *before* differs in this respect from *after*, which does not allow such elements.[6]

(17) a. The children left before anyone had arrived
b. The boys died before they ever reached Nice

(18) a. *The children arrived after anyone had left
b. *The boys died after they ever reached Nice

We must not suppose that this is a peculiar feature of English. Similar patterns can be found in Spanish and Dutch, as shown by the examples in (19) and (20), which feature the negative polarity items *mover un dedo* 'lift a finger' and *ooit* 'ever'.

Spanish

(19) a. Juan se fué antes que María moviera un dedo para ayudarle[7]
Juan left before Maria lifted a finger to help him
'Juan left before Maria lifted a finger to help him'
b. *Juan se fué después que María ha movido un dedo para ayudarle
Juan left after Maria had lifted a finger to help him
'Juan left after Maria had lifted a finger to help him'

Dutch

(20) a. De kinderen vertrokken voordat zij ooit een tempel gezien hadden
The children left before they ever a temple seen had
'The children left before they ever saw a temple'
b. *De kinderen vertrokken nadat zij ooit een tempel bezocht hadden
The children left after they ever a temple visited had

6. See Heinämäki (1974) and Ladusaw (1979), among others.
7. Like its French counterpart *avant que* (see de Swart 1991), *antes que* requires the presence of a subjunctive in the clause it introduces. In what follows, it will be argued that this is a consequence of the non-veridical nature of both connectives. Henriëtte de Swart informs us that *avant que*, as opposed to *après que*, licenses the use of paratactic negation (so-called expletive *ne*) as well.

'The children left after they ever visited a temple'

In view of the fact that expressions such as *anyone*, *ever*, *mover un dedo*, and *ooit* are typically restricted to downward monotonic contexts, this means that *before* must receive a monotone decreasing function as its semantic value. The temporal connective *after*, on the other hand, cannot be associated with such a function.

It is easy to establish that *before* is not only downward monotonic, but behaves like an anti-additive expression. As the Dutch examples in (21) clearly show, it is possible for the strong polarity item *ook maar iets* to appear in a clause which is introduced by *before*.

Dutch

(21) a. De kinderen vertrokken voordat zij ook maar iets ontdekt hadden
The children left before they anything discovered had
'The children left before they had discovered anything'

b. Wij zullen vertrokken zijn voordat zij ook maar iets ontdekken
We will left have before they anything discover
'We will have left before they discover anything'

Such patterns can be found in German as well. The strong polarity item *auch nur irgendetwas*, for example, may be part of a clause headed by *bevor* 'before'.

German

(22) a. Er ist abgefahren bevor sie auch nur irgendetwas bemerkt hatten
He has left before they anything noticed have
'He left before they noticed anything'

b. Er wird abgefahren sein bevor sie auch nur irgendetwas bemerken
He will left have before they anything notice
'He will have left before they notice anything'

This raises the question how the anti-additive behavior of elements like *voordat* and *bevor* can be derived from the associated semantic values.

4 Landman's analysis

At first sight, it is perfectly natural to regard *after* and *before* as converses. This way of portraying the matter entails that *p after q* (henceforth: pAq) should express the same as *q before p* (henceforth: qBp). The definitions which Landman (1991: 141) proposes are given in (23).[8] Since he restricts his attention to *after* and *before* as past tense operators, they only characterize the retrospective use of these connectives. The corresponding prospective definitions are given in (24).

Retrospective definitions

(23) a. $pAq(t_0)$ iff $\exists x[x < t_0 \wedge p(x) \wedge \exists y[y < x \wedge q(y)]]$

8. Note that t_0 is used to indicate an arbitrary moment of evaluation.

b. $pBq(t_0)$ iff $\exists x[x < t_0 \wedge p(x) \wedge \exists y[x < y < t_0 \wedge q(y)]]$

Prospective definitions

(24) a. $pAq(t_0)$ iff $\exists x[t_0 < x \wedge p(x) \wedge \exists y[y < x \wedge q(y)]]$
b. $pBq(t_0)$ iff $\exists x[t_0 < x \wedge p(x) \wedge \exists y[t_0 < x < y \wedge q(y)]]$

From this it follows immediately that the statement *Juan arrived before Maria arrived* can only be true if *Maria arrived after Juan arrived* is a true statement as well. It is also clear, however, that the different behavior of *before* and *after* with respect to negative polarity items cannot be explained in this way. For that reason, Landman proposes that the retrospective definition in (23b) be replaced by the somewhat more complex characterization in (25a). The corresponding prospective definition is given in (25b).

(25) a. $pBq(t_0)$ iff $\exists x[x < t_0 \wedge p(x) \wedge \forall y[(y < t_0 \wedge q(y)) \rightarrow x < y]]$
b. $pBq(t_0)$ iff $\exists x[t_0 < x \wedge p(x) \wedge \forall y[(t_0 < y \wedge q(y)) \rightarrow x < y]]$

Such an account is attractive in more than one respect. To begin with, it no longer forces us to infer from the truth of the whole sentence that the clause headed by *before* should also be true. That this is indeed the right approach is shown by the work of Heinämäki (1974), who points at the existence of sentences like (26).

(26) They left the country before anything happened

Here we have a clear example of the nonveridical use of *before*: one can accept the truth of the whole sentence without being forced to accept the truth of the *before*clause. Heinämäki (1974) speaks in such cases of 'non-committal' *before*. Following Anscombe (1964), she distinguishes two other uses as well: 'factual' and 'nonfactual' *before*. In the first case, the truth of the whole sentence implies the truth of the *before*-clause, as in (27).

(27) John checked the car carefully before he bought it

In the second case, we may legitimately pass from the truth of the whole sentence to the falsity of the *before*-clause. According to Heinämäki, a typical example is the sentence in (28).

(28) Max died before he saw his grandchildren

Landman's treatment of *before* is compatible with all three uses and consequently doesn't force us to distinguish more than one lexical element. The definition in (25) makes *before* a nonveridical connective whose characteristic feature is that pBq doesn't necessarily imply q. This is reflected in the linguistic behavior of the Romance counterparts of *before*, which require the presence of a subjunctive in the clause they introduce. *After*, on the other hand, must be regarded as belonging to the class of veridical connectives: by definitions (23a) and (24a), pAq unconditionally implies q. We assume that such elements always select the indicative mood.

Even more important is the fact that Landman's analysis entails that *before* and *after* cannot be treated as converses. If pBq doesn't necessarily imply q, then we aren't forced to infer qAp either. A closer look reveals that this property holds in the opposite direction as well: qAp doesn't necessarily imply pBq. As observed by Heinämäki (1974), and before her by Anscombe (1964), we are not always able to pass from the truth of (29a) to the truth of (29b).

(29) a. Doris travelled all over the world after she finished her studies
b. Doris finished her studies before she travelled all over the world

In view of Landman's analysis, this need not surprise us. By virtue of definition (23a), the sentence in (29a) is true if there is a moment t_1 at which *Doris travelled all over the world* is true and a moment t_2 preceding t_1 at which *Doris finished her studies* is true. Obviously, this is compatible with a situation in which *Doris travelled all over the world* is true both before and after the moment at which *Doris finished her studies* is true. However, in order to be able to infer (29b), every moment at which *Doris travelled all over the world* is true must be preceded by the moment at which *Doris finished her studies* is true. In other words, Landman's account not only predicts that pBq does not imply qAp, but it also predicts that qAp does not imply pBq.

The present treatment of *before* thus solves three problems: the observed lack of veridicality, the absence of a transition from pBq to qAp (as a result of the non-veridical nature of *before*) and the absence of a transition from qAp to pBq (as a result of the use of an existential quantifier over moments of time in the semantic characterization of *after*, but a universal quantifier over moments of time in the characterization of *before*). It remains to be seen how Landman's analysis deals with the fourth problem: the possibility of strong polarity items in *before*-clauses and the impossibility of negative polarity items in *after*-clauses.[9]

5 The main theorem

To prove that *before* is anti-additive it is enough, in virtue of definition (11b), to prove that it validates the biconditional in (30).

(30) $pB(q \vee r)(t_0) \leftrightarrow (pB(q)(t_0) \wedge pB(r)t(_0))$

9. Linebarger (1987) claims that the occurrence of the negative polarity item *budge an inch* in * *The mule sighed before it budged an inch* leads to an ungrammatical sentence. In her opinion, this should be contrasted with *The mule sighed piteously for hours before the heartless owner budged an inch*, which is perfectly acceptable. As Jack Hoeksema reminds us, however, it is by no means clear that the observed contrast is a matter of well-formedness. Pragmatic factors influencing acceptability may be responsible instead. See also von Bergen and von Bergen (1993). The OED (s.v. *soldier*) gives the following example (from Melville's *White Jacket*): *off Cape Horn some 'sogers' of sailors will stand cupping, and bleeding, and blistering before they will budge.*

We assume that the interpretation of disjunction in a temporal setting is the usual one. That is to say:

(31) $(q \vee r)(t_0) \leftrightarrow (q(t_0) \vee (r)(t_0))$

It can be shown that from this, together with the characterization of *before* in (25), it follows that (30) is valid from left to right. For the reverse, we must assume that the model of time underlying natural language is the model of linear time.

5.1 The easy part

In what follows, we will restrict our attention to the retrospective use of *before*. It should be emphasized, however, that everything we say about retrospective *before* holds for prospective *before* as well.

We begin by noticing that the formulas in (25) and (31) allow us to expand $pB(q \vee r)(t_0)$ into (32).

(32) $\exists x[x < t_0 \wedge p(x) \wedge \forall y[(y < t_0 \wedge (q(y) \vee r(y))) \rightarrow x < y]]$

It is easy to see that within this formula the disjunction $(q(y) \vee r(y))$ is part of the antecedent of a conditional. Because conditionals are monotone decreasing with respect to their antecedents, this means that the disjunction in question may be replaced by a stronger formula.[10] In particular, we wish to consider the formulas that result from replacing the occurrence of $(q(y) \vee r(y))$ in (32) by any of its two proper subformulas.

(33) a. $\exists x[x < t_0 \wedge p(x) \wedge \forall y[(y < t_0 \wedge q(y)) \rightarrow x < y]]$
b. $\exists x[x < t_0 \wedge p(x) \wedge \forall y[(y < t_0 \wedge r(y)) \rightarrow x < y]]$

By definition (25), the formulas in (33a) and (33b) may be replaced equivalently by $pBq(t_0)$ and $pBr(t_0)$, respectively. Clearly, these sentences may be conjoined by the introduction rule for $\wedge$, which gives us the result in (34).

(34) $pB(q \vee r)(t_0) \rightarrow (pBq(t_0) \wedge pBr(t_0))$

In view of definition (9b), this shows that *before* is monotone decreasing with respect to the clause it introduces, as desired.

5.2 A counterexample

The argument from right to left is more difficult. As a matter of fact, without additional assumptions about the flow of time the relevant formula, given in

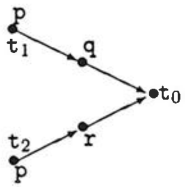
10. The relation of strength among formulas is usually defined in terms of entailment. We say that p is stronger than q if p entails q. In monotone decreasing contexts, this means that q may be replaced *salva veritate* by p. In monotone increasing contexts, on the other hand, we find the opposite to be the case: p may be replaced by the weaker formula q. See Kadmon and Landman (1993) for an interesting attempt to describe the distribution of *any* in terms of strength of propositions.

(26). permits the construction of a countermodel.

(35) $(pBq(t_0) \wedge pBr(t_0)) \rightarrow pB(q \vee r)(t_0)$

To see this, we consider the branching model in (36).

(36)



Notice, to start with, that according to the definition in (25) pBq holds at t_0 if

1. there is a p point and it lies earlier than t_0 itself; and
2. all points, if any, at which q is true lie between this p point and t_0 itself

Look at any of the two branches in this model, for instance, the upper one. The only potentially falsifying q point is located on this branch, but is preceded by a p point and so $p\ B\ q$ is verified by this branch and a fortiori by this model. Similarly, the only r point located on the lower branch is preceded by a p point and so there is no way of falsifying $p\ B\ r$. Consequently, we have shown that the model in (36) verifies $(pBq) \wedge (pBr)$.

On the other hand, notice that the q and r points in (36) are also $(q \vee r)$ points. But the upper p point does not precede the lower $(q \vee r)$ point. By the same token we can argue that the lower p point does not precede the upper $(q \vee r)$ point. Hence, there is no p point in this model of which we can truthfully say that it precedes all the $(q \vee r)$ points. This proves that $pB(q \vee r)$ does not hold at t_0.

5.3 Eliminating the counterexample

The the above argument rests essentially on the branching nature of the relation of temporal precedence. If we assume that the model of time underlying natural language is linear, it can be shown that *before* is anti-additive. To see this it is enough to check what happens when we adopt a non-branching perspective. Suppose we merge the two branches. Let us concentrate on the three possibilities that arise with regard to the p point t_1 and the only r point.

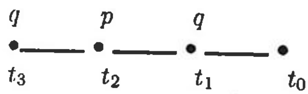
1. Suppose the r point is identified with the upper p point t_1. In this case the lower p point, t_2, will precede all the $(q \vee r)$ points.
2. Suppose the r point precedes t_1. Once more the lower p point, t_2, will precede all the $(q \vee r)$ points.
3. Suppose t_1 precedes the r point. In this case, the upper p point, t_1, will precede all the $(q \vee r)$ points.

Thus, we see that in such a setting our counterexample does not arise. In fact, it can be proven that no model in which the underlying precedence relation is transitive and connected falsifies the biconditional in (30).[11] This means that *before* is invariably anti-additive in linear models of time.

6 Restricted definitions

The truth definitions in (25) become inadequate when repetition is involved.[12] In order to see this, it is sufficient to take the situation in (37) into consideration.

(37)



It follows from the retrospective definition of *before* that if John lighted a cigarette and later he coughed and then lighted a cigarette again, we cannot truthfully say *John coughed before he lighted a cigarette*. The reason is that the existential and the universal quantifier in (25) are both unrestricted, ranging over the entire past, relevant or not. It should be clear that this leads to truth value evaluations which are not at all in accordance with our intuitions.

Interestingly, these problems were anticipated in Anscombe's (1964) discussion of *before* and *after*. One of the analyses of *before* which she considers is the following.

> "p before q" means "There was some time at which *p* such that every time at which *q* was after it" (1964: 10)

Although such an account yields the right truth conditions for sentences which contain an occurrence of the negative polarity item *ever* in the *before*-clause, Anscombe rejects this analysis because it doesn't adequately deal with certain uses of plain *before*. To quote her verbatim:

> Now this formulation is right for "I was in Greece before you were *ever* in Italy"; but "I was in Greece before you were in Italy" may be true, although "I was in Greece before you were ever in Italy" is false. Or again "He studied his appearance in the glass before he used the telephone" may well be a true piece of narrative; it does not at all suggest that he studied his appearance in the glass before he ever in his life used the telephone. (1964: 13)

The foregoing passage clearly shows that Anscombe was aware of the difficulties that repetition creates for the truth definitions in (25). But it is equally clear that the source of these difficulties is the use of unrestricted quantification. This forces us to take the whole time axis into consideration when evaluating a sentence of the form pBq. In particular, if we find a situation as depicted in the

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11. A standard first-order argument showing this is presented in Sánchez Valencia, van der Wouden, and Zwarts (1993).

12. We owe this observation to Henriëtte de Swart.

diagram in (37), we will have to conclude that pBq is false, even if the points at which q is true are far apart and in some cases contextually irrelevant.

There are other considerations which suggest that tensed sentences should not evaluated with respect to the whole time axis. One of the issues that Partee (1973) addresses is the problem of interpreting negated statements against the background of indefinite time. The sentence she discusses is *I didn't turn off the stove*, which illustrates the deictic use of the past tense morpheme. As Partee observes, when uttered halfway down the turnpike, such a sentence does not mean either that there exists some time in the past at which I did not turn off the stove or that there exists no time in the past at which I turned off the stove. The sentence clearly refers to a particular time whose identity is generally obvious from the context.

One way to obtain a correct semantics for Partee's example is to abandon the idea that sentences are to be evaluated at a single point. Instead, one evaluates at points with respect to a relevant time span. In the case at hand, this means that we propose that the unrestricted definition of retrospective *before* in (23b) be replaced by the restricted one in (38a), in which a definite time span I has been substituted for the indefinite past. The corresponding prospective definition is given in (38b).

(38) a. $pBq(I, t_0)$ iff $\exists x \in I[x < t_0 \wedge p(x) \wedge \forall y \in I[(y < t0 \wedge q(y)) \rightarrow x < y]]$
b. $pBq(I, t_0)$ iff $\exists x \in I[t_0 < x \wedge p(x) \wedge \forall y \in I[(t_0 < y \wedge q(y)) \rightarrow x < y]]$

It should be emphasized that such an approach solves the problem of repetition as well. To see this, it is enough to take the model in (37) into consideration. If the relevant time span I includes t_1, t_2, and t_3, then $pBq(I, t_0)$ is false. But if this time span is restricted to t_1 and t_2, then $pBq(I, t_0)$ is true, as desired.

Note that the restricted definitions in (38) do not affect our reasoning with regard to the relationship between anti-additivity and the structure of time.

7 Nonveridicality

It is easy to see that the definitions in (25) and (38) make *before* a nonveridical connective one of whose pronounced features is that pBq doesn't necessarily imply q. Other connectives with this property are *or*, *unless* and *without*, among others. In many cases, the absence of veridicality is not a coincidence. Since it can be shown that monotone decreasing connectives are always nonveridical, the observed lack of veridicality must often be regarded as a consequence of the downward monotonic nature of the element in question. In order to demonstrate this, we will record a simple, but useful fact.

Fact

(39) Let C be a connective which is both veridical and monotone decreasing with respect to its second argument. Then $pCq \rightarrow q \wedge \neg q$.

Proof. Suppose that pCq is true. Since $(q \wedge \neg q) \rightarrow q$ is a logical truth, it follows from the monotone decreasing nature of C that $pC(q \wedge \neg)$ is true as well. Therefore, by the veridicality of C, $q \wedge \neg$. □

The above result proves that no connective can be both veridical and downward monotonic with respect to a given argument place. In other words: every connective which is monotone decreasing in a given argument place is nonveridical in that argument place and every connective which is veridical in a given argument place is either monotone increasing or nonmonotonic in that argument place.

8 A comparison of 'before' and 'after'

If we assume that the model of time underlying natural language is the model of linear time, then *before* must be classified as a connective which is both anti-additive and nonveridical in its second argument. Matters are different, however, when we turn to the first argument position. The presence of the existential quantifier in the definitions (25) and (33) is sufficient to make the element in question additive and veridical in its first argument. What this means is that the logical behavior of *before* is characterized by the valid formulas in (40).

(40) a. $(p \vee q)Br \leftrightarrow (pBr \vee qBr)$
b. $pB(q \vee r) \leftrightarrow (pBq \wedge pBr)$
c. $pBq \rightarrow p$

Accordingly, we expect to find negative polarity items in the subordinate clause, but not in the main clause. The examples below show that this is the right prediction.

(41) a. The children left before anyone had arrived
b. *Anyone arrived before the children had left

In view of the fact that *before* is anti-additive in its second argument, we even expect to find strong polarity items in the subordinate clause. That this is indeed the case is shown by the Dutch and German examples in (21) and (22).

A closer look at the semantics of the temporal connective *after*, presented in (23b) and (24b), reveals that it is additive and veridical in both argument places. The logical behavior of the element in question can therefore be characterized by means of the valid formulas in (42).

(42) a. $(p \vee q)Ar \leftrightarrow (pAr \vee qAr)$
b. $pA(q \vee r) \leftrightarrow (pAq \vee pAr)$
c. $pAq \rightarrow p$
d. $pAq \rightarrow q$

Consequently, we do not expect polarity items in either the main, or the subordinate, clause. The ungrammatical sentences in (43) confirm this expectation.

(43) a. *Anyone left after the children had arrived
b. *The children arrived after anyone had left

9 A problem with 'since' and 'until'

The logical behavior of the temporal connectives *since* and *until* has been studied by Kamp (1968). He presents two truth definitions: one for the retrospective use of *since* and one for the prospective use of *until*.

(44) a. $pSq(t_0)$ iff $\exists x[x < t_0 \wedge q(x) \wedge \forall y[x < y < t_0 \rightarrow p(y)]]$
b. $pUq(t0)$ iff $\exists x[t_0 < x \wedge q(x) \wedge \forall y[(t_0 < y < x \rightarrow p(y)]]$

It is readily established that *since* and *until*, so defined, are multiplicative with respect to their first argument, and additive with respect to their second argument. Moreover, the existential quantifiers in (44) ensure that both connectives are veridical in the q position. If the relation of temporal precedence is not only linear, but dense, they will be veridical in the p position as well. This means that the logical behavior of the two connectives is characterized by the valid formulas in (45).

(45)

a.	$(p \wedge q)Sr \leftrightarrow (pSr \wedge qSr)$	e.	$(p \wedge q)Ur \leftrightarrow (pUr \wedge qUr)$
b.	$pS(q \vee r) \leftrightarrow (pSq \vee pSr)$	f	$pU(q \vee r) \leftrightarrow (pUq \vee pUr)$
c.	$pSq \rightarrow p$	g.	$pUq \rightarrow p$
d.	$pSq \rightarrow q$	h.	$pUq \rightarrow q$

For the sake of clarity the semantical properties of *after* and *before*, *since* and *until* have been listed in table 1.

Table 1: Semantical properties of four temporal connectives

	p after q	*p before q*	*p since q*	*p until q*
p position	additive veridical	additive veridical	multiplicative veridical	multiplicative veridical
q position	additive veridical	anti-additive nonveridical	additive veridical	additive veridical

In view of the monotone increasing nature of *since* and *until*, both in the *p* and in the *q* position, Kamp's analysis predicts that we will not find negative polarity items in either clause. It is interesting to see that there are several counterexamples. Bolinger (1977: 31), for example, reports that the sentence *It's been a week since I bought any is perfectly acceptable*. In the corpus of English texts that Hoeksema is collecting, we also find a number of sentences which appear to involve the polarity item *anyone*. The relevant cases have been collected in (46).

(46) a. It's two weeks since anyone was towed away from outside their door, the Computerland clerk tells me
b. 'You know, it's been a long time since anyone did that for me'. 'Did you like it?' I asked
c. 'W-what's that for?' 'It's been a while since anyone's been to the bathroom'

Curiously enough, Dutch does not allow polarity items at all in such examples. This might be taken to suggest that *any*-phrases differ substantially from other types of polarity items in their distributional properties. On the other hand, the Dutch grammarian Paardekooper (n.d.) tells us that *tot(dat)* 'until' is capable of licensing *ook maar iets* 'anything at all' in examples like (47):

(47) Het zal heel lang kunnen duren totdat er hier ook maar iets verandert
It will very long can last until there here anything changes
'It will take a long time before anything changes here'

Note, however, that sentence (48) which involves the perfect instead of the future is considerably worse:

(48) ?Het heeft heel lang geduurd totdat er hier ook maar iets veranderde
It has very long lasted until there here anything changed
'It took a long time before anything changed here'

This suggests that the polarity item *ook maar iets* in (47) is licensed by the future operator which is nonveridical, though not monotone decreasing.

10 A similar problem with 'as soon as'

The temporal connective *as soon as* and its Dutch equivalent *zodra* present us with a similar problem. When used retrospectively these expressions are clearly veridical in both argument places. As prospective connectives, however, they appear to be nonveridical. A number of negative polarity items seem to be sensitive to this distinction, as is clear from the examples in (49).

(49) a. *De kinderen vertrokken zodra zij ook maar iets ontdekt hadden
The children left as soon as they anything dicovered had
*'The children left as soon as they had discovered anything'[13]
b. De kinderen zullen vertrekken zodra zij ook maar iets ontdekken
The children will leave as soon as they anything discover
'The children will leave as soon as they discover anything'

We see that *ook maar iets* and *anything* are only compatible with *zodra* and *as soon as* if these elements are used as prospective connectives, with present

13. Greg Carlson informs us that the English equivalent is just as bad as the Dutch sentence.

or future tense. It is not clear, however, whether the polarity items are licensed by the connective or by the tense operator. It is also not clear whether the non-veridicality of prospective *as soon as* and *zodra* entails downward monotonicity.

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An algebraic appreciation of diagrams

Jerry Seligman*

1 Are diagrams terms?

At least since Frege, it has been widely acknowledged that the concepts of function and argument are indispensable tools in linguistics. This idea is most clearly and forcefully expressed by Montague in [3]. Simply put, Montague claims that algebraic semantics is completely general: the semantic values of syntactic parts of a linguistic symbol are related to the semantic value of the whole in the same way that the denotation of subterms of a term are related to the denotation of the whole term. This follows almost directly from the assumption that the symbol is unambiguous and Frege's principle of compositionality. Any unambiguous symbol is uniquely analysed as the result of composing its syntactic parts together in a way that determines the meaning of the whole from the meaning of the parts. Consequently, any symbol can be seen to have the abstract term structure '$F(S_1, \ldots, S_n)$', where $S_1, \ldots, S_n$ are the term structures of its principal components, and F stands for the mode in which they are composed. By the principle of compositionality, F can be given an algebraic interpretation, as the function mapping the semantic-values of $S_1, \ldots, S_n$ to the semantic-value of the whole symbol.

It is very tempting to suppose that the Frege-Montague view of semantics applies quite generally, not just to language but to all forms of symbolic representation. After all, the approach has a great track record. In computer science, the method of using algebraic specification-languages has proved a powerful tool in the analysis of data-structures and programming languages. In linguistics, especially in semantics, an allegiance to the Frege-Montague approach has inspired many of the advances of the last thirty years. Moreover, the approach is very robust—quite often apparent detractors can be brought back into the fold if a sufficiently abstract view is taken (see Janssen's [2]).

One would expect simple diagrammatic systems of representation, such as Venn diagrams and Euler circles, to be ideal candidates for an analysis along Frege-Montague lines. Each diagram is composed from a finite number of diagrammatic objects, such as circles, ellipses, crosses and shading, perhaps with a few simple annotations; and the meaning of the diagram is clearly composed from the meaning of its parts. Two such diagrams are shown in Figure 1.

Indeed, one might try to argue, as follows, that *all* diagrammatic systems can be given the abstract syntax of terms. First, fix a co-ordinate frame for each diagram—for instance, that provided by measurement in inches, vertically and horizontally, from the mid-point of the paper. Associate each component of the diagram with its position within that frame. For each integer n, and each n-tuple σ of co-ordinates, let F_σ stand for the syntactic operation of drawing a diagram by placing its ith argument at co-ordinate position σ_i. Together with a range of atomic symbols of various sizes, this is enough to specify an abstract term structure for every diagram.

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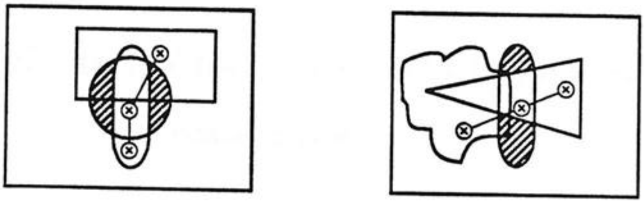


Figure 1: Homeomorphic diagrams

The proposal fails because it is manifestly at odds with our *use* of diagrams. Imagine that the diagram on the left of Figure1 were to be transformed imperceptibly; the above analysis would assign a distinct syntactic structure to the new diagram, even though we take it to be syntactically unchanged. The compulsion that the two diagrams are syntactically equivalent is as basic as the compulsion that two inscriptions of the same word are instances of a common syntactic type.

Clearly, there must be some partition of the uncountably many syntactic forms into those which we regard as syntactically equivalent. The crucial difference between diagrams and linear systems of representation lies in how this partition is to be made.

A characteristic feature of linear systems, such as written language, is that their symbols can be divided into *segments* in a way which is invariant across all instances. For example, this sentence can be divided into ten words. Any other inscription of the sentence can also be divided into ten words, and the sentences will be syntactically identical if and only if they match word-for-word.[1] Once such a segmentation is given, the possibility of following the Frege-Montague line is opened: at the very worst, one can regard each symbol as a term built from primitive symbols and the operation of concatenation.

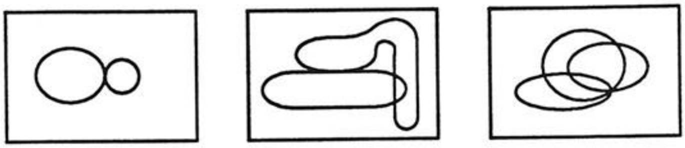
By contrast, the task of finding a useful segmentation of diagrams is quite hopeless in all part the most trivial cases. Indeed the fact that primitive components of a diagram overlap is often of the utmost importance in determining the meaning of the whole. The uncountably many arrangements of diagrammatic objects on the page must be partitioned in a different way.

A moment's thought suggests a viable alternative: the structure of most diagrams is invariant under many transformations of the plane, such as enlargement, rotation, reflection, and even more general topological transformations; so perhaps we can partition diagrams into classes which are closed under a given set of transformations. Put slightly differently, the proposal is that the syntactic type of a diagram is an *invariant* of some class of transformations.

Exactly which transformations are chosen will depend on the diagrammatic system we are analysing. In this paper, we will consider only the very simplest of diagrammatic systems, in which syntactic type is taken to be a topological invariant. Venn diagrams and Euler circles fall into this category, because their meaning depends only on facts about whether or not one diagrammatic objects overlap with another, and not on the size or shape of the object. However, we must not be too quick to *identify* syntactic type using semantic equivalence. Although—we claim—any adequate semantic theory of Venn diagrams should ensure that semantic-value is a topological invariant, it need not be the same invariant as syntactic type. Just as in language, there are many semantically equivalent diagrams which are not

1. Of course, this is an idealization. In phonology, the problem of providing a segmentation of natural speech is very difficult, and some argue for a "tiered" approach in which several stacked segments are required for a correct analysis. Nonetheless, it is almost universally accepted that spoken language has a segmental syntax at some level of analysis.

Figure 2: Basic diagrams: connected and not



(i) (ii) (iii)

Figure 3: Three deviant diagrams

naturally analysed as being of the same syntactic type.

We adopt the most liberal approach to diagram syntax by identifying syntactic type with invariance under homeomorphism. By this criterion, the diagrams in Figure 1 are syntactically equivalent.

2 Basic diagram syntax

Although we believe that the methods adopted in this paper can be applied to any diagrammatic system in which syntactic type is identified with invariance under homeomorphism, we will only consider the simplest of such system, in which the only diagrammatic objects are simple closed curves.[2]

Definition 2.1 A *(basic) diagram* $D = \langle \Box_D, \bigcirc_D \rangle$ consists of a rectangular region $\Box_D$ of the real plane, and a finite set $\bigcirc_D$ of simple closed curves inside $\Box_D$, such that the following conditions hold.
(i) No two curves intersect without crossing.
(ii) No two curves intersect at more than finitely many points.
(iii) No three curves intersect at the same point.
The region $\Box_D$ is called the *rectangle* of D, and each member of $\bigcirc_D$ is called a *curve* of D. D is said to be *connected* iff each pair of curves intersects at exactly two points or not at all.

Examples of connected, and disconnected basic diagrams are shown in Figure 2, on the left and right, respectively. The labels 'A', 'B' and 'C', are not considered part of the diagram itself; they are merely annotations which enable us to refer to the curves in the text. Diagrams which fail to satisfy conditions (i) to (iii) are shown in Figure 3.

Notation Given a function $f\colon A \to B$, we often need to refer to the associated function mapping each subset X of A to its image under f, namely $\{f(x) \mid x \in X\}$. To avoid notational clutter, we call this function f, and rely on the reader to discern which function is meant. Likewise, we use f^{-1} to refer both to the function mapping

2. A *simple closed curve* is a homeomorph of the unit circle.

each subset Y of B to its inverse image under f, namely $\{x \in A \mid f(x) \in Y\}$, and to the inverse of f, if it has one. To support this convention, we banish sets which contain a subset as a member.

Definition 2.2 Given a diagram D and a homeomorphism h of the plane, the *image of D under h*, written $h(D)$, is the pair $\langle h(\Box_D), \{h(c) \mid c \in \bigcirc_D\}\rangle$.

If D is a diagram and $h(\Box_D)$ is rectangular then hD is also a diagram; moreover, if D is connected, so is hD. Thus the class of diagrams (and the class of connected diagrams) is "almost" closed under homeomorphic images. We could lift the restriction to rectangular rectangles—and with it the "almost" of the previous statement—but the need for such restrictions is often present in more complicated diagrammatic systems of representation, and so it would be somewhat artificial to do so.

Definition 2.3 Diagrams D_1 and D_2 are said to be *syntactically equivalent* iff there is a homeomorphism h such that $D_2 = hD_1$.

3 Basic diagram semantics

Basic diagrams may be interpreted by taking each curve to represent a class of individuals. A diagram is true under an interpretation just in case the set-theoretic relationships between the classes are as portrayed. For example, the diagram shown on the right of Figure 2 is true under an interpretation just in case every member of the class represented by the curve labelled 'B' is also a member of the class represented by the curve labelled 'A'. The curve labelled 'C' may be interpreted as any class whatsoever, without effecting the truth-value of the diagram.

We can sharpen the account of how a diagram receives a truth-value with the aid of a simple thought-experiment. To see if a diagram is true under an interpretation, imagine placing each individual on the diagram in such a way that it is surrounded by a curve if and only if it is a member of the class represented by that curve. If this can be done, the diagram is true under the interpretation; if not, it is false. We invite the reader to check that this method agrees with common sense.

Interpretations of diagrams are modelled using structures called classifications.[3]

Definition 3.1 A *classification* $\boldsymbol{A}$ consists of two sets $tok(\boldsymbol{A})$ and $typ(\boldsymbol{A})$, whose elements are called *tokens* and *types*, respectively, and a binary relation of *classification* between them. We write $a :_{A} \alpha$ to mean that token $a \in tok(\boldsymbol{A})$ is classified by type $\alpha \in typ(\boldsymbol{A})$, dropping the subscripted '$\boldsymbol{A}$' when no ambiguity can arise. The *extension* $\underline{\alpha}$ of a type $\alpha \in typ(\boldsymbol{A})$ is defined by $\underline{\alpha} = \{a \in tok(\boldsymbol{A}) \mid a : \alpha\}$.

A generic example of a classification is a relational structure, all of whose relations are unary—indeed the reader may prefer to think of all classifications in this way, and may interpret our definitions and results accordingly.[4]

Definition 3.2 Given a diagram D, an *interpretation* of D consists of a classification $\boldsymbol{A}$, together with a function $f{:}\bigcirc_D \to typ(\boldsymbol{A})$. The diagram is *true* under the interpretation, written $\boldsymbol{A}, f \models D$, iff there is a function $g{:}\, tok(A) \to \Box_D$ such that for each $a \in tok(A)$, and each $c \in \bigcirc_D$,

3. The theory of classifications is developed by Barwise and Seligman in a series of papers, the most recent being [1]

4. The advantage of using classifications is that we will be defining structure-preserving maps between classifications which do not generalize easily to arbitrary relational structures.

$a :_A f(c)$ iff $g(a)$ is surrounded by c. [5]
A function g satisfying the above is called a *witnessing function* of the interpretation.

Our informal account of interpretation can be recaptured by regarding the type $f(c)$ as the class represented by the curve c of D, whose members are the elements of $\underline{f(c)}$. The function g witnesses the imaginary placement of individuals in our thought-experiment.

In support of our definition of syntactic equivalence, we show that interpretations commute with homeomorphisms.

Notation Given functions $f: A \to B$ and $g: B \to C$, the function $gf: A \to C$ is the composition of f and g. Given a set A, the function $\mathrm{id}(A): A \to A$ is the identity function on A.

Claim 3.3 *Given a diagram D and a homeomorphism h, if $\langle \boldsymbol{A}, f\rangle$ is an interpretation of D then $\langle \boldsymbol{A}, fh^{-1}\rangle$ is an interpretation of hD, and*

$\boldsymbol{A}, f \models D$ *iff* $\boldsymbol{A}, fh^{-1} \models hD$.

PROOF: Given a homeomorphism h, a simple closed curve c, and a point p, p is surrounded by c iff $h(p)$ is surrounded by $h(c)$. The rest follows from the definitions.
QED

4 Diagram classifications and links

We will now work towards an alternative characterization of the syntax and semantics of basic diagrams using some concepts from the theory of classifications and links. The first step is to see diagrams as classifications.

Definition 4.1 For any diagram D, $\boldsymbol{D}$ is the classification defined by: $tok(\boldsymbol{D}) = \square_D$, $typ(\boldsymbol{D}) = \bigcirc_D$, and for each point $p \in \square_D$ and each curve $c \in \bigcirc_D$, $p :_D c$ iff p is surrounded by c.

The next step is to see interpretations as links between a diagram classification and the classification into which the diagram is interpreted.

Definition 4.2 Given classifications $\boldsymbol{A}$ and $\boldsymbol{B}$, and functions $f: typ(\boldsymbol{A}) \to typ(\boldsymbol{B})$ and $g: tok(\boldsymbol{B}) \to tok(\boldsymbol{A})$, the pair $\langle f, g\rangle$ is an *S-link* from $\boldsymbol{A}$ to $\boldsymbol{B}$, written $f, g: \boldsymbol{A} \rightleftarrows \boldsymbol{B}$, iff for each $b \in tok(\boldsymbol{B})$ and each $\alpha \in typ(\boldsymbol{A})$, $g(a) :_A \alpha$ iff $a :_B f(\alpha)$. [6]

Claim 4.3 *$\boldsymbol{A}, f \models D$ iff there is a g such that $f, g: \boldsymbol{D} \rightleftarrows \boldsymbol{A}$.*
PROOF: Direct from the definitions. QED

This characterization puts the following, purely link-theoretic question in focus: given a function $f: typ(\boldsymbol{A}) \to typ(\boldsymbol{B})$, what property must f have for there to be a function $g: tok(\boldsymbol{A}) \to tok(\boldsymbol{B})$ such that $f, g: \boldsymbol{A} \rightleftarrows \boldsymbol{B}$, and when is the choice of g uniquely determined? For the reminder of this section, we will work towards an answer. First, we introduce a useful abbreviation.

5. A point in the plane is *surrounded by* a simple closed curve iff it lies in the (bounded) open region bounded by the curve.
6. In the language of [1], an 'S-link' is a Strongly Sound link with functionality '$\rightleftarrows$', i.e., it is 'S'-shaped.

Definition 4.4 Given classifications $\boldsymbol{A}$ and $\boldsymbol{B}$, a function $f: typ(\boldsymbol{A}) \rightarrow typ(\boldsymbol{B})$ is a *partial S-link* from $\boldsymbol{A}$ to $\boldsymbol{B}$ iff there is a $g: tok(\boldsymbol{B}) \rightarrow tok(\boldsymbol{A})$ such that $f, g: \boldsymbol{A} \rightleftarrows \boldsymbol{B}$.

To sharpen Claim 4.3, we would like to show that there is a one-one correspondence between true interpretations and S-links. In general, it is not true that a partial S-link $f: typ(\boldsymbol{A}) \rightarrow \boldsymbol{B}$ has a unique extension to an S-link $f, g: \boldsymbol{A} \rightleftarrows \boldsymbol{B}$. However, we can find another classification $\boldsymbol{A}_{\sim}$, such that if f uniquely determines and is determined by an S-link from $\boldsymbol{A}_{\sim}$ into $\boldsymbol{B}$.

Definition 4.5 Given a classification $\boldsymbol{A}$, elements $a, b \in tok(\boldsymbol{A})$ are *indistinguishable*, written $a \sim b$, iff for each $\alpha \in typ(\boldsymbol{A})$, $a :_{\boldsymbol{A}} \alpha$ iff $b :_{\boldsymbol{A}} \alpha$. The *indistinguishability class of* a, written $[a]_{\sim}$, is the set of elements of $tok(\boldsymbol{A})$ which are indistinguishable from a. The $\sim$-quotient of $\boldsymbol{A}$ is the classification $\boldsymbol{A}_{\sim}$ with $tok(\boldsymbol{A}_{\sim}) = \{[a]_{\sim} \mid a \in tok(\boldsymbol{A})\}$, $typ(\boldsymbol{A}_{\sim}) = typ(\boldsymbol{A})$, and $[a]_{\sim} :_{\boldsymbol{A}_{\sim}} \alpha$ iff $a :_{\boldsymbol{A}} \alpha$.

Theorem 4.6 *If f is a partial S-link from $\boldsymbol{A}$ to $\boldsymbol{B}$ then there is an S-link $f, f_{\bullet}: \boldsymbol{A}_{\sim} \rightleftarrows \boldsymbol{B}$, and for any S-link $f, g: \boldsymbol{A}_{\sim} \rightleftarrows \boldsymbol{B}$, f is a partial S-link from $\boldsymbol{A}$ to $\boldsymbol{B}$ and $f_{\bullet} = g$.*

To prove Theorem 4.6 we will need to establish a few elementary properties of links and $\sim$-quotients. First, note that S-links compose in the obvious way: if $f, g: \boldsymbol{A} \rightleftarrows \boldsymbol{B}$ and $f', g': \boldsymbol{B} \rightleftarrows \boldsymbol{C}$ then $f'f, gg': \boldsymbol{A} \rightleftarrows \boldsymbol{C}$. Also, for any classification $\boldsymbol{A}$, there is an *identity link*, $\mathrm{id}(typ(\boldsymbol{A})), \mathrm{id}(tok(\boldsymbol{A})): \boldsymbol{A} \rightleftarrows \boldsymbol{A}$. Finally, if $f, g: \boldsymbol{A} \rightleftarrows \boldsymbol{B}$ and both f and g are bijections, then $\langle f, g\rangle$ is called an *isoinfomorphism* and $\boldsymbol{A}$ and $\boldsymbol{B}$ are said to be *isoinfomorphic*, written $\boldsymbol{A} \rightleftarrows \boldsymbol{B}$. As expected, the compositions $\langle f^{-1}f, gg^{-1}\rangle$ and $\langle ff^{-1}g^{-1}g\rangle$ are the identity links on $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively.

A classification is related to its $\sim$-quotient by the following lemma.

Lemma 4.7 *Given classifications $\boldsymbol{A}$ and $\boldsymbol{B}$, there are functions μ_A and η_A such that*

(i) if $f, g: \boldsymbol{A}_{\sim} \rightleftarrows \boldsymbol{B}$ then $f, \eta_A g: \boldsymbol{A} \rightleftarrows \boldsymbol{B}$

(ii) if $f, g: \boldsymbol{A} \rightleftarrows \boldsymbol{B}$ then $f, \mu_A g: \boldsymbol{A}_{\sim} \rightleftarrows \boldsymbol{B}$

PROOF: Let $\mu_A: tok(\boldsymbol{A}) \rightarrow tok(\boldsymbol{A}_{\sim})$ be the function mapping each $a \in tok(\boldsymbol{A})$ to its indistinguishability class, and let $\eta_A: tok(\boldsymbol{A}_{\sim}) \rightarrow tok(\boldsymbol{A})$ be a function which selects a representative of each indistinguishability class, so that $\mu_A\eta_A$ is the identity function on $tok(\boldsymbol{A}_{\sim})$. From these definitions, it is easy to see that $\mathrm{id}(typ(\boldsymbol{A})), \mu_A: \boldsymbol{A}_{\sim} \rightleftarrows \boldsymbol{A}$ and $\mathrm{id}(typ(\boldsymbol{A})), \eta_A: \boldsymbol{A} \rightleftarrows \boldsymbol{A}_{\sim}$. The two parts of the lemma follow by composition of S-links. ·

QED

Unlike classifications in general, $\sim$-quotients have the useful property that any partial S-link from $\boldsymbol{A}_{\sim}$ is uniquely extendable to an S-link.

Lemma 4.8 *If $f, g_1: \boldsymbol{A}_{\sim} \rightleftarrows \boldsymbol{B}$ and $f, g_2: \boldsymbol{A}_{\sim} \rightleftarrows \boldsymbol{B}$ then $g_1 = g_2$.*

PROOF: By Lemma 4.7(i), $f, \eta_A g_1: \boldsymbol{A} \rightleftarrows \boldsymbol{B}$ and $f, \eta_A g_2: \boldsymbol{A} \rightleftarrows \boldsymbol{B}$. So for each $b \in tok(\boldsymbol{B})$ and each $\alpha \in typ(\boldsymbol{A})$, $\eta_A g_1(b) :_{\boldsymbol{A}} \alpha$ iff $b :_{\boldsymbol{B}} f(\alpha)$ iff $\eta_A g_2(b) :_{\boldsymbol{A}} \alpha$, showing that $\eta_A g_1(b) \sim \eta_A g_2(b)$. Thus $\mu_A\eta_A g_1(b) = \mu_A\eta_A g_2(b)$. But $\mu_A\eta_A = \mathrm{id}(tok(\boldsymbol{A}))$, and so $g_1(b) = g_2(b)$.

QED

PROOF OF THEOREM 4.6: Given that $\boldsymbol{A}, f \models D$, the existence of g follows from Claim 4.3 and Lemma 4.7(ii), the uniqueness from Lemma 4.8. The converse follows from Claim 4.3 and Lemma 4.7(i).

QED

5 Consistent and tautologous diagrams

The results of the previous section provide us with a way of investigating the logical properties of diagrams.

Definition 5.1 A diagram is *consistent* iff it is true under at least one interpretation, and *tautologous* iff it is true under every interpretation.

Observe that every diagram can be interpreted in itself: by Claim 4.3, the existence of the identity link on $\boldsymbol{D}$ establishes that $\boldsymbol{D}, \mathrm{id}(\mathrm{O}_D) \models D$. Thus:

Corrollary 5.2 *Every diagram is consistent.*

And, by composition of links, we can see that S-links between interpretations are truth-preserving.

Corrollary 5.3 *If $\boldsymbol{A}, f \models D$ and f' is a partial S-link from $\boldsymbol{A}$ to $\boldsymbol{B}$ then $\boldsymbol{B}, f'f \models D$.*

As promised, Theorem 4.6 yields a characterization of the true interpretations of diagram D as S-links from $\boldsymbol{D}_\sim$.

Corrollary 5.4 *If $\boldsymbol{A}, f \models D$ then $f, f_\ast: \boldsymbol{D}_\sim \rightleftarrows \boldsymbol{A}$ is the unique S-link from $\boldsymbol{D}_\sim$ to $\boldsymbol{A}$ extending f. Moreover, for any S-link $f, g: \boldsymbol{D}_\sim \rightleftarrows \boldsymbol{A}$, $\boldsymbol{A}, f \models D$.*

Thus, for any diagram D, the classification $\boldsymbol{D}_\sim$ contains all the information necessary to evaluate the truth of D under an interpretation. It is finite and very easy to inspect, especially when D is connected, because then the tokens of $\boldsymbol{D}_\sim$ are just the smallest regions bounded by curves of the diagram.

An important aspect of our analysis is that S-links can be used to study both the semantics and the syntax of diagrams in a uniform way. The bridge is made by exploring the idea that one diagram can be interpreted in the diagram-classification of another.

Definition 5.5 Given two diagrams D and D', a function $f: \mathrm{O}_D \rightarrow \mathrm{O}_{D'}$ is a *diagram-homomorphism* from D to D' iff f is a partial S-link from $\boldsymbol{D}$ to $\boldsymbol{D}'$. A diagram-homomorphism is a *diagram-isomorphism* iff it is a bijection and its inverse is also a diagram-homomorphism. Diagrams D and D' are *isomorphic* iff there is an isomorphism from D to D'.

The diagrams in Figure 2 are isomorphic, with an isomorphism given by associating curves with the same labels. To check whether two diagrams D and D' are isomorphic it is often advisable to look for an isoinfomorphism between $\boldsymbol{D}_\sim$ and $\boldsymbol{D}_\sim$. This is clearly sufficient, but the existence of such an link is also necessary.

Claim 5.6 *Diagrams D and D' are isomorphic iff $\boldsymbol{D}_\sim \leftrightarrows \boldsymbol{D}'_\sim$.*

PROOF: Suppose $f: \mathrm{O}_D \rightarrow \mathrm{O}_{D'}$ is an isomorphism from D to D'. By Theorem 4.6, $f, f_\ast: \boldsymbol{D}_\sim \rightleftarrows \boldsymbol{D}'$, and $f^{-1}, f_\ast^{-1}: \boldsymbol{D}'_\sim \rightleftarrows \boldsymbol{D}$. So, by Lemma 4.7 and composition of links, $f\mathrm{id}(\mathrm{O}_D)f^{-1}\mathrm{id}(\mathrm{O}_D'), f_\ast\eta_{D'}f_\ast^{-1}\eta_D: \boldsymbol{D}_\sim \rightleftarrows \boldsymbol{D}_\sim$. But $f\mathrm{id}(\mathrm{O}_D)f^{-1}\mathrm{id}(\mathrm{O}_D') = \mathrm{id}(\mathrm{O}_D)$ and so $\langle f\mathrm{id}(\mathrm{O}_D)f^{-1}\mathrm{id}(\mathrm{O}_D'), f_\ast\eta_{D'}f_\ast^{-1}\eta_D\rangle$ must be the identity link on $\boldsymbol{D}_\sim$, by Lemma 4.8. Thus $f_\ast\eta_{D'}f_\ast^{-1}\eta_D = \mathrm{id}(tok(\boldsymbol{D}_\sim))$. A similar argument shows that $f_\ast^{-1}\eta_D f_\ast\eta_{D'} = \mathrm{id}(tok(\boldsymbol{D}'_\sim))$, and so $f_\ast^{-1}\eta_D = (f_\ast\eta_{D'})^{-1}$. The required isoinfomorphism is therefore $\langle f, f_\ast\eta_{D'}\rangle$. The converse is trivial. QED

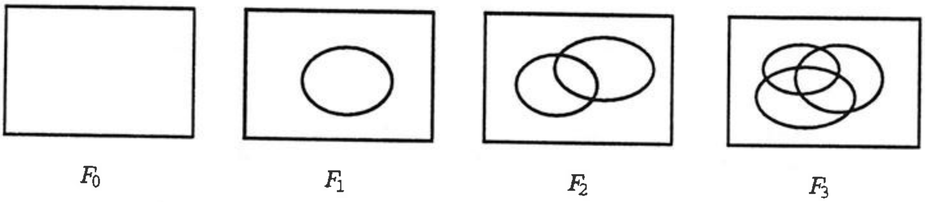


Figure 4: Free diagrams

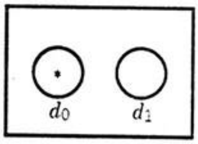


Figure 5: The diagram D_*

The following two Lemmas follow straight from Claim 4.3 and the definition of homomorphism.

Lemma 5.7 *If $A, f' \models D'$ and $f{:}\mathsf{O}_D \rightarrow \mathsf{O}_{D'}$ is a homomorphism then $A, f'f \models D$.*

Lemma 5.8 *$f{:}\mathsf{O}_D \rightarrow \mathsf{O}_{D'}$ is a homomorphism iff $D', f \models D$.*

Definition 5.9 A diagram D is *free* iff for each diagram D', every function from O_D to $\mathsf{O}_{D'}$ is a homomorphism. Examples of free diagrams are given in Figure 4.

Theorem 5.10 *A diagram is tautologous iff it is free.*
PROOF: Suppose D is a tautologous diagram. For any diagram D' and any function $f{:}\mathsf{O}_D \rightarrow \mathsf{O}_{D'}$, $D'. f \models D$. so f is a homomorphism, by Lemma 5.8. Thus D is free.

Conversely, suppose D is free and $\langle A, f\rangle$ is an interpretation of D. For each $a \in tok(A)$, let f_a be the function from O_D to the curves of the diagram D_* depicted in Figure 5, defined by

$$f_a(c) = \begin{cases} d_0 & \text{if} \quad a : f(c) \\ d_1 & \text{otherwise} \end{cases}$$

f_a is a homomorphism, because D is free, and so there is a function g_a such that $f_a, g_a{:}D \rightleftarrows D_*$. Now define the function $g{:}\ tok(A) \rightarrow tok(D)$ by $g(a) = g_a(*)$, where '$*$' is as marked in Figure 5. Noting that $g(a) :_D c$ iff $* :_{D_*} f_a(c)$ iff $a :_A f(c)$, we can conclude that $f, g{:}D \rightleftarrows A$. and so $A, f \models D$. QED

Corrollary 5.11 *Tautologous diagrams with the same number of curves are isomorphic.*
PROOF: QED
If D and D' are tautologous diagrams with the same number of curves then there is a bijection f from O_D to $\mathsf{O}_{D'}$. By Theorem 5.10. D and D' are both free, so f and f^{-1} are both homomorphisms, and so f is an isomorphism.

This shows that the diagrams F_0, F_1, F_2 and F_3, shown in Figure 4, are the unique tautologous diagrams with 0, 1, 2, and 3 curves, respectively, up to isomorphism. In fact, these diagrams are also the unique tautologous, connected diagrams in their size, up to *syntactic equivalence*. Unfortunately, we will see later that the series cannot be continued.

Definition 5.12 Diagram D' is an *extension* of diagram D, written $D' \supseteq D$, iff $\square_D = \square_{D'}$ and $\bigcirc_D \subseteq \bigcirc_{D'}$. We also say that D is a *subdiagram* of D'.

Diagram D' is a *free extension* of diagram D, written $D' \geq D$, iff $D' \supseteq D$ and for any diagram D'' and any function $f: \bigcirc_{D'} \rightarrow \bigcirc_{D''}$ if the restriction of f to $\bigcirc_D$ is a homomorphism then so is f. D is *simple* iff it is a free extension only of itself.

Corrollary 5.13 *D is tautologous iff $D \geq F_0$.*

PROOF: For any $f: \bigcirc_D \rightarrow \bigcirc_{D'}$, the restriction of f to $\bigcirc_{F_0} = \emptyset$ is just the empty function, and so is a homomorphism. Thus $D \geq F_0$ iff D is free. The result then follows from Theorem 5.10. QED

6 Covers

A different kind of characterization of partial S-links (and so of true interpretations and diagram homomorphisms) can be obtained by looking for the structural property they preserve.

Definition 6.1 Given a classification $\boldsymbol{A}$ and sets $\Sigma, \Sigma' \subseteq typ(\boldsymbol{A})$, let $[\Sigma, \Sigma'] = \bigcap_{\alpha \in \Sigma} \underline{\alpha} - \bigcup_{\alpha' \in \Sigma'} \underline{\alpha'}$. The pair $\langle \Sigma, \Sigma' \rangle$ is a *cover* iff $[\Sigma, \Sigma'] = \emptyset$. A *subcover* of a pair $\langle \Sigma, \Sigma' \rangle$ is any cover $\langle \Sigma_0, \Sigma'_0 \rangle$ such that $\Sigma_0 \subseteq \Sigma$ and $\Sigma'_0 \subseteq \Sigma'$. A function $f: typ(A) \rightarrow typ(B)$ is said to *preserve covers* from $\boldsymbol{A}$ to $\boldsymbol{B}$ iff the image under f of each cover in $\boldsymbol{A}$ is a cover in $\boldsymbol{B}$.

Theorem 6.2 *A function $f: typ(\boldsymbol{A}) \rightarrow typ(\boldsymbol{B})$ is a partial S-link iff it preserves covers.*

PROOF: If f is a partial S-link then there is a g such that $f, g: \boldsymbol{A} \rightleftarrows \boldsymbol{B}$. For any $\Sigma, \Sigma' \subseteq typ(\boldsymbol{A})$, we will show that if $b \in [f\Sigma, f\Sigma']$ then $g(b) \in [\Sigma, \Sigma']$, which is enough to show that f preserves covers. So, for any $b \in [f\Sigma, f\Sigma']$ and any $\alpha \in typ(\boldsymbol{A})$,

if $\alpha \in \Sigma$ then $b \in \underline{f(\alpha)}$, so $b \in \underline{\alpha}$, and

if $\alpha' \in \Sigma'$ then $b \notin \underline{f(\alpha')}$, so $b \notin \underline{\alpha'}$.

Conversely, if f preserves covers, we define a function $g: tok(\boldsymbol{B}) \rightarrow tok(\boldsymbol{A})$ as follows. For each $b \in tok(\boldsymbol{B})$, let $\Sigma_b^+ = \{\beta \in typ(\boldsymbol{B}) \mid b :_B \beta\}$ and let $\Sigma_b^- = typ(\boldsymbol{B}) - \Sigma_b^+$. Note that $b \in [\Sigma_b^+, \Sigma_b^-]$. Also note that $ff^{-1}\Sigma_b^+ \subseteq \Sigma_b^+$ and $ff^{-1}\Sigma_b^- \subseteq \Sigma_b^-$, and so $b \in [ff^{-1}\Sigma_b^+, ff^{-1}\Sigma_b^-]$. But then $\langle f^{-1}\Sigma_b^-, f^{-1}\Sigma_b^- \rangle$ is not a cover, because f preserves them. So pick any $a \in [f^{-1}\Sigma_b^-, f^{-1}\Sigma_b^-]$ and let $g(b) = a$. We leave it to the reader to check that, for each $\alpha \in typ(\boldsymbol{A})$, $g(b) :_A \alpha$ iff $b :_B f(\alpha)$. QED

The theorem may be applied to both interpretations and homomorphisms.

Corrollary 6.3 *$\boldsymbol{A}, f \models D$ iff f preserves covers.*

Corrollary 6.4 *$f: \bigcirc_D \rightarrow \bigcirc_{D'}$ is a homomorphism iff f preserves covers from $\boldsymbol{D}$ to $\boldsymbol{D}'$.*

Corollary 6.5 *D is a tautologous diagram iff $\boldsymbol{D}$ has no non-trivial covers.*
PROOF: For every interpretation $\langle \boldsymbol{A}, f\rangle$ of D, f preserves trivial covers, so the result follows from Corollary 6.3. QED

Covers are also useful for studying free extensions.

Lemma 6.6 *D' is a free extension of D iff each non-trivial cover in D' has a subcover in D.*

Theorem 6.7 *Every diagram is the free extension of a unique simple diagram.*

PROOF: Let core(D) be the subdiagram of D with $\circ_{\mathrm{core}(D)} = \bigcap \{\circ_{D'} \mid D' \leq D\}$. We show that core($D$) $\leq D$, and so establish that core(D) is the smallest subdiagram of D with this property. Suppose $\langle C, C'\rangle$ is a non-trivial cover in D with no proper subcover. We call such a cover a *minimal pair* of D. For all $D' \leq D$, $\langle C, C'\rangle$ has a subcover in D', by Lemma 6.6. By minimality, this subcover can only be $\langle C, C'\rangle$ itself; and so $\langle C, C'\rangle$ is also a cover in core(D), by construction of the latter. We have shown that every minimal cover in D is also a cover in core(D). The subcover-order is clearly well-founded, so every cover in D contains a minimal subcover in D, and hence also in core(D): thus core(D) $\leq D$, by Lemma 6.6 again. Now core(D) is clearly simple, and every $D' \leq D$ is a free extension of core(D), so core(D) is the only simple subdiagram of D having D as a free extension. QED

Definition 6.8 For each diagram D, let core(D) $= \langle \square_D, \bigcap \{\circ_{D'} \mid D' \leq D\}\rangle$ be the unique simple subdiagram of D having D as a free extension.

Finally, we leave the proofs of the following three claims to the reader.

Claim 6.9 *core(D) is the largest simple subdiagram of D.*

Claim 6.10 *Given diagrams D and D', the following are equivalent.*
(i) There is a homomorphism from D' to a free extension of D.
(ii) There is a homomorphism from core(D') to D.

Claim 6.11 *core(D) is isomorphic to core(D') iff there are homomorphisms from core(D) to D' and from core(D') to D.*

7 Constructing diagrams

The association of a diagram D with the classification $\boldsymbol{D}$ respects our notion of syntactic equivalence in the following sense.

Claim 7.1 *Syntactically equivalent diagrams are isomorphic.*
PROOF: If D and D' are syntactically equivalent then there is a homeomorphism h such that $hD = D'$. It is easy to see that $h^{-1}, h\colon \boldsymbol{D}' \rightleftarrows \boldsymbol{D}$ is an isoinfomorphism. QED

However, the converse of Claim 7.1 is not true, even if we restrict it to free diagrams which are connected. The isomorphic, free, connected diagrams shown in Figure 6 are not syntactically equivalent. This is a shame, because Corollaries 5.11

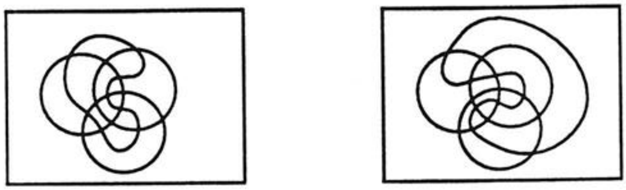
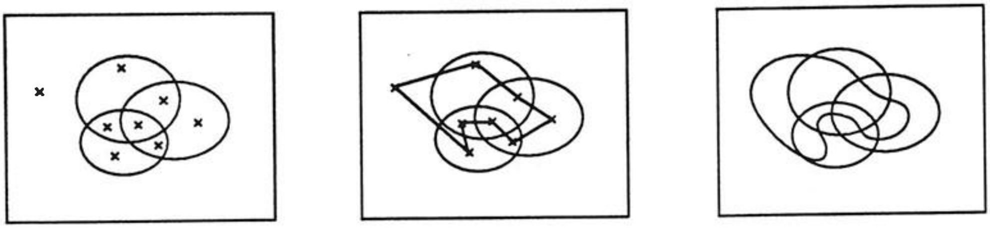


Figure 6: Isomorphic, free, connected diagrams, which are not syntactically equivalent.



(a) (b) (c)

Figure 7: Freely extending a diagram

and 6.5 suggest a strategy for constructing all free diagrams. If for each n we can construct one free diagram with n curves then we can be sure that it is the unique free diagram with n curves, up to isomorphism. Unfortunately, that is not enough to guarantee syntactic equivalence.

To construct the tautologous diagrams we must therefore be a little more cunning that one might have expected. First, we show that any diagram can be freely extended by one curve.

Construction 7.2 Recall that any finite number of points inside a rectangular region can be joined by a simple closed curve lying inside the region. The following is an algorithm for constructing such a curve. If there is only one point then a (small enough) circle passing through the point will do. If there is more than one point then there are two cases, depending on whether or not the points are collinear. If they are, then they lie on a line of a finite length, which forms the side of a (small enough) rectangle. If they are not collinear, draw a line through each pair of points in the collection. Pick one of the resulting minimal regions (a convex polygon) and call its centre c. No two points of the collection are collinear with c. Order the points according to the size of the angle a line drawn from the point to c makes with the horizontal. Connect the points with straight lines in the order just determined, to form a simple polygon—of course, any closed curve connecting the points in this order will do as well.

Now suppose D is a basic diagram, to which we wish to add a curve freely. Pick points in each $\sim$-class of D, none of which lie on one of the curves. Join the selected points them with a simple closed curve, as shown above. Because D is a basic, there are only a finite number of points of intersection, so we can always draw a curve which avoids them all and intersects the old curves in only a finite number of places: this guarantees that the resulting diagram is also basic.

Figure 7(a) shows a three-curve diagram with selected points marked by crosses. In (b), the crosses have been joined by straight lines, following the above algorithm. In (c), a smother curve is chosen.

Claim 7.3 *If D' is constructed from D in accordance with Construction 7.2 then it is a free extension of D.*

PROOF: By construction, each $\sim$-class E of D contains a point p lying on the new curve. The point p was selected so that it does not lie on any of the curves of D, and so it is contained in the interior of E, which is an open set. The new curve passes through p and so it divides the interior of E (and hence E also) into two non-empty regions. Thus if c is the new curve, neither $\underline{c} \cap E$ nor $E - \underline{c}$ are empty. Consequently, if $\langle \Gamma, \Gamma' \rangle$ is a cover of D' then either $c \in \Gamma \cap \Gamma'$ and so $\langle \Gamma, \Gamma' \rangle$ is trivial, or $\langle \Gamma - \{c\}, \Gamma' - \{c\} \rangle$ is a cover of D. The result follows by Lemma 6.6. QED

Construction 7.2 gives us a method of constructing free diagrams of each size, thus characterizing the free diagrams up to isomorphism. To improve on this result, we need to show that every free diagram is syntactically equivalent to one constructed in this way. To do this, we must look a little more closely at the construction.

Definition 7.4 Given a diagram D, a sequence $p_1, \ldots, p_n$ of points in $\Box_D$ is a *blueprint* of D iff

1. for each $i \neq j < n$, $p_i \neq p_j$
2. $p_1 = p_n$
3. for each $i < n$, there is a simple curve with endpoints p_i and p_{i-1}, and which crosses one curve of D exactly once.[7]

Blueprints $p_1, \ldots, p_n$ and $q_1, \ldots, q_m$ of D are *equivalent* iff $n = m$ and for each $i < n$, there is a curve with endpoints p_i and q_i which does not intersect any of the curves of D.

Given any simple closed curve c in $\Box_D$, we say that $p_1, \ldots, p_n$ is a *blueprint of* c iff for every segment s of c bounded by (but not containing) points of intersection with curves of D, there is an $i < n$ such that p_i lies on s.

Construction 7.5 Given a diagram D and a blueprint $p_1, \ldots, p_n$. For each $i < n$, there is a curve c_i with endpoints p_i and p_{i+1}, and which crosses exactly one curve of D exactly once. Draw a closed curve c by joining up the curves $c_1, \ldots, c_{n-1}$.

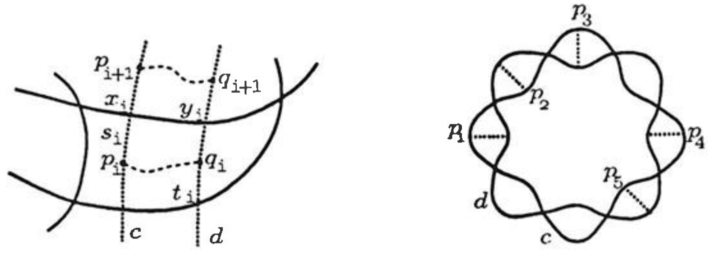
Construction 7.5 provides a finer degree of control than Construction 7.2, but it has the drawback that it may not produce a basic diagram, because the constructed curve may not be a simple closed curve—it may intersect itself. Nonetheless, it is more general.

Claim 7.6 *Given a diagram D and a curve $c \in \bigcirc_D$. D results from the diagram $D' = \langle \Box_D, \bigcirc_D - \{c\} \rangle$ by an application of Construction 7.5.*

PROOF: Divide c into segments $s_1, \ldots, s_n$ bounded by, but not containing, points of intersection between c and curves of D'. (There are only a finite number of segments, because c crosses any other curve of D at most finitely many times, and there are only a finite number of curves in D—both restrictions imposed by the definition of basic diagram.) For each $i < n$, select a point p_i on s_i. The sequence $p_1, \ldots, p_n$ is a blueprint for c, and so c can be constructed from it by an application of Construction 7.5.

QED

7. A simple curve is a homeomorph of the closed unit interval.



(i) (ii)

Figure 8: Equivalent extensions

The following theorem gives the condition under which successful applications of Construction 7.5 yield syntactically equivalent diagrams.

Theorem 7.7 *Let D be a diagram, and let c and d two simple closed curves in $\Box_D$, which are not contained in $\bigcirc_D$, and do not lie on any of the intersection points of D, but which have equivalent blueprints. If c and d also surround the same curves of D, then the diagrams $\langle \Box_D, \bigcirc_D \cup \{c\}\rangle$ and $\langle \Box_D, \bigcirc_D \cup \{c\}\rangle$ are syntactically equivalent.*

PROOF: Suppose $p_1, \ldots, p_n$ and $q_1, \ldots, q_n$ are equivalent blueprints of c and d, respectively. For each $i < n$, there are segments s_i of c and t_i of d which are bounded by, but do not contain, points of intersection with the curves of D. The points p_i and q_i lie on s_i and t_i, respectively; and there is a curve with endpoints p_i and q_i, which does not cross any of the curves of D. Figure 8(i) depicts the situation.

Let x_i be the point of intersection of c with a curve of D between p_i and p_{i+1}, whose existence and uniqueness is implied by the fact that $p_1, \ldots, p_n$ is a blueprint of c. Likewise, let y_i be the point of intersection of d with a curve of D, between q_i and q_{i+1}.

Claim 1 The points x_i and y_i lie on the same curve of D and no other curves of D intersect the region $p_i p_{i+1} q_{i+1} q_i$.

PROOF OF CLAIM: First note that x_i and y_i are the only points of curves of D to cross the boundary of the region. This follows from observations already made, stemming from the fact that the corner points of the region lie on equivalent blueprints of the curves c and d. Thus the curve of D which enters the region at x_i must either terminate inside the region, or else leave at either x_i or y_i. The first possibility is excluded because every curve of d is closed, and the second is excluded because no curve of D is self-intersecting. The only remaining possibility is that the curve entering at x_i leaves at y_i, which is just to say that x_i and y_i lie on the same curve. No other curve can enter the region, because x_i and y_i are the only possible entry points, and neither c nor d lies on an intersection point of D (by hypothesis). Finally, if there were any curve lying entirely within the region, it would be surrounded by either c or d, but not by both, which is forbidden by hypothesis.

If we now consider the whole curves c and d, it is clear that they are related in the manner depicted in Figure 8(ii). In other words, the region bounded by c and d is a "pinched" annulus crossed by a finite number of segments of curves of D. By a standard extension theorem (?), any homeomorphism of c onto d which maps p_i to q_i and x_i to y_i (for $0 < i < n$), can be extended to a homeomorphism of the plane which maps c to d, while keeping the rectangle and curves of D fixed. QED

Corrollary 7.8 *If D_c and D_d are free extensions of D which result from the addition of curves c and d, and c and d have equivalent blueprints, then D_c is syntactically equivalent to D_d.*
PROOF: If D_c and D_d are free extensions of D then c and d must *intersect* all the curves of D, and so there are none which they *surround*. QED

Corrollary 7.9 *There are countably many syntactically non-equivalent diagrams.*
PROOF: In any diagram D there are only countably many non-equivalent blueprints. So, by Theorem7.7, D has only countably many extensions with one extra curve. The result follows by induction on the number of curves in a diagram. QED

We will now see how to enumerate basic diagrams, up to syntactic equivalence.

Definition 7.10 Let D be a diagram. Blueprints $p_1, \ldots, p_n$ and $q_1, \ldots, q_m$ in D are *co-extensive* iff for each $i < n$ there is a $j < m$ and a curve with endpoints p_i and q_j, and which does not intersect any curve of D. We define the *order* of a blueprint in D by induction:

(0) A blueprint in D has order 0 iff there is no shorter, co-extensive blueprint in D.

(n) A blueprint in D has order $n+1$ iff it does not have order n but every shorter, co-extensive blueprint has order n or less.

An extension D' of D has *order n* iff it has one extra curve, and that curve has a blueprint of order n.

Note that if we restrict our attention to connected diagrams, every one-curve extension has order 0. Unfortunately, we do not yet know whether there are free connected diagrams of every size; but we can be certain that there are always extensions of order 0.

Corrollary 7.11 *Every diagram has a finite, positive number of syntactically non-equivalent, one-curve extensions of order n.*

Corrollary 7.12 *There are a finite number of syntactically non-equivalent, free diagrams with n curves.*

Thus, by enumerating blueprints, we can enumerate the tautologous diagrams, up to syntactic equivalence.

8 Valid diagrammatic arguments

Our next goal is a link-theoretic characterization of valid arguments using diagrams. It is not sufficient to represent a diagrammatic argument just as a sequence of premise diagrams and a conclusion. In addition, we need to know which curves in the conclusion are intended to represent the same class as curves in the premises. In other words, we must record the connections between curves in the premises and conclusion which establish co-reference. In informal use, these connections are indicated by gestures, labels, or simply by the fact that a concrete image standing for the conclusion is arrived at by a process of modifying concrete images of the premises in a way that leaves some of the original curves in tact.

The easiest way to model co-reference between curves is to pretend that all the curves in both premises and conclusion are labelled.[8]

Definition 8.1 Let L be a set. An *L-labelled diagram* is a pair $\langle D, \lambda \rangle$, consisting of a diagram D together with a function $\lambda : \bigcirc_D \to L$. It is *properly-labelled* iff λ is one-one. $\langle D, \lambda \rangle$ is *syntactically equivalent* to $\langle D', \lambda' \rangle$ iff there is a homeomorphism h such that $hD = D'$ and $\lambda = \lambda' h$.

Let L be a fixed countably infinite set of labels. We draw labelled diagrams by writing the label $\lambda(c)$ next to the curve c, in the expected way. Our decision not to regard the labels as part of the diagram itself is reflected in the definition of syntactic equivalence: we only require the curves with the same label to be preserved under homeomorphism, not the labels themselves. Two properly-labelled diagrams are shown in Figure 2. An improperly labelled diagram is shown in Figure 9.

Definition 8.2 An *L-interpretation* is a classification $\boldsymbol{A}$ with $typ(\boldsymbol{A}) = L$. An L-labelled diagram $\langle D, \lambda \rangle$ is *true* under an L-interpretation $\boldsymbol{A}$, written $\boldsymbol{A} \models \langle D, \lambda \rangle$ iff $\boldsymbol{A}.\lambda \models D$. An L-labelled diagram is *consistent* iff it is true under at least one L-interpretation, and *tautologous* iff it is true under all.

Given a set Δ of L-labelled diagrams and an L-labelled diagram $\langle D, \lambda \rangle$, we say that $\langle D, \lambda \rangle$ is a *consequence* of Δ, and write $\Delta \models \langle D, \lambda \rangle$, iff every L-interpretation under which each labelled diagram in Δ is true is one under which $\langle D, \lambda \rangle$ is also true. Labelled diagrams $\langle D, \lambda \rangle$ and $\langle D, \lambda' \rangle$ are *logically equivalent* iff $\langle D, \lambda \rangle \models \langle D, \lambda' \rangle$ and $\langle D', \lambda' \rangle \models \langle D, \lambda \rangle$

Claim 8.3 *Given an L-model* $\boldsymbol{A}$, *and an L-labelled diagram* $\langle D, \lambda \rangle$, *the following are equivalent:*

1. $\boldsymbol{A} \models \langle D, \lambda \rangle$.
2. $\lambda, \lambda_* : \boldsymbol{D}_\sim \rightleftarrows \boldsymbol{A}$.
3. λ *preserves covers.*

PROOF: From Theorems 4.6 and 6.2. QED

In order to use this theorem to extend the results of the previous sections, we need some way of relating arbitrary interpretations of a diagram to L-models. To do this, it is convenient to make use of another concept from the theory of links.

Definition 8.4 Given a classification $\boldsymbol{A}$, and a relation $r \subseteq typ(\boldsymbol{A}) \times B$, we define *the coherent projection of* $\boldsymbol{A}$ *along* r, written $r\boldsymbol{A}$, as follows. For each $a \in tok(\boldsymbol{A})$, define

$$a^+ = \{\beta \in B \mid \exists \alpha \in typ(\boldsymbol{A}) \quad a :_A \alpha \quad \text{and} \quad \langle \alpha, \beta \rangle \in r\}$$
$$a^- = \{\beta \in B \mid \exists \alpha \in typ(\boldsymbol{A}) \quad a \not:_A \alpha \quad \text{and} \quad \langle \alpha, \beta \rangle \in r\}$$

Let $r\boldsymbol{A}$ be the classifications with $tok(r\boldsymbol{A}) = \{a \in tok(\boldsymbol{A}) \mid a^+ \cap a^- = \emptyset\}$, $typ(r\boldsymbol{A}) = \mathrm{rng}(r)$, and $a :_{rA} \beta$ iff $\beta \in a^+$.

Lemma 8.5 *For any labelled diagram* $\langle D, \lambda \rangle$, *let* $\langle \lambda \rangle$ *be the graph of* λ. *Then* $\lambda, \mathrm{id}(\square_D) : \boldsymbol{D} \rightleftarrows \langle \lambda \rangle \boldsymbol{D}$

PROOF: By construction. QED

8. Another strategy is to represent the co-reference relation between curves (See Shin's [4]). It is fairly easy to see that this is equivalent.

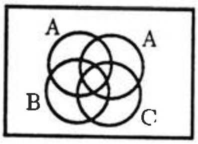


Figure 9: A tautologous labelled diagram

The classification $\langle\lambda\rangle \boldsymbol{D}$ does for labelled diagrams what $\boldsymbol{D}$ did for unlabelled diagrams.

Theorem 8.6 *Every labelled diagram is consistent.*
PROOF: Let $(\langle\lambda\rangle \boldsymbol{D})^+$ be any L-model extending $\langle\lambda\rangle \boldsymbol{D}$ (which must exist: assign arbitrary extensions to the labels not occurring in $\text{rng}(\lambda)$). The inclusion function is a partial S-link, and so by composition with the link in Lemma 8.5, we have $(\langle\lambda\rangle \boldsymbol{D})^- \models \langle D, \lambda\rangle$.
QED

Theorem 8.7 *A labelled diagram $\langle D, \lambda\rangle$ is tautologous iff $\langle\lambda\rangle \boldsymbol{D}$ has no non-trivial covers.*
PROOF: If $\langle\lambda\rangle \boldsymbol{D}$ has no non-trivial covers, then by Theorem 6.2, for any L-model $\boldsymbol{A}$, $\text{id}(\text{rng}(\lambda))$ is a partial S-link from $\langle\lambda\rangle \boldsymbol{D}$ to $\boldsymbol{A}$. By composition with the link in Lemma 8.5, $\lambda = \text{id}(\text{rng}(\lambda))\lambda$ is a partial S-link from $\boldsymbol{D}$ to $\boldsymbol{A}$. Then, by Claim 8.3, $\boldsymbol{A} \models \langle D, \lambda\rangle$.

Conversely, suppose $\langle D, \lambda\rangle$ is tautologous. For any function, $f: L \to typ(\boldsymbol{A})$, let $\boldsymbol{A}_f$ be the classification with types L, tokens $tok(\boldsymbol{A})$ and $a : l$ iff $a :_A f(l)$. Then $\langle \boldsymbol{A}_f, \lambda\rangle$ is an L-interpretation of D, and so $\boldsymbol{A}_f \models \langle D, \lambda\rangle$, because $\langle D, \lambda\rangle$ is tautologous; and so there is a function g such that $\lambda, g: \boldsymbol{D} \rightleftarrows \boldsymbol{A}_f$.

Claim 1 $f, g: \langle\lambda\rangle \boldsymbol{D} \rightleftarrows \boldsymbol{A}$

PROOF OF CLAIM: Given $a \in tok(\boldsymbol{A})$ and $l \in typ(\langle\lambda\rangle \boldsymbol{D}) = \text{rng}(\lambda)$, there is a $c \in \text{O}_D$ such that $l = \lambda(c)$. We have shown that $\lambda, g: \boldsymbol{D} \rightleftarrows \boldsymbol{A}_f$ and so $g(a) :_D c$ iff $a :_{A_f} \lambda(c) = l$, and this is the case iff $a :_A f(l)$, by the definition of $\boldsymbol{A}_f$, above. Now, by Lemma 8.5, $g(a) = \text{id}(\Box_D)g(a) :_D c$ iff $g(a) :_{\langle\lambda\rangle D} l$; and we are done.

From this it follows that every $f: L \to typ(\boldsymbol{A})$ is a partial S-link from $\langle\lambda\rangle \boldsymbol{D}$ to $\boldsymbol{A}$. By Theorem 6.2, $\langle\lambda\rangle \boldsymbol{D}$ can have no non-trivial covers.
QED

Corrollary 8.8 *If $\langle D, \lambda\rangle$ is a properly-labelled diagram, $\boldsymbol{D}$ is isoinfomorphic to $\langle\lambda\rangle \boldsymbol{D}$ and $\langle D, \lambda\rangle$ is a tautology iff D is a tautology.*
PROOF: By Lemma 8.5, if $\langle D, \lambda\rangle$ is a properly-labelled then $\langle \lambda, \text{id}(\Box_D)\rangle$ is an isoinfomorphism. The rest follows from the Theorem 8.7 and the fact that S-links preserve covers.
QED

Note that the corollary cannot be generalized to improperly-labelled diagrams. Figure 9 shows a tautologous labelled diagram which would cease to be tautologous were we to rub out the labels. We extend the notion of a diagram-homomorphism to labelled diagrams in the obvious way.

Definition 8.9 If $\langle D, \lambda\rangle$ and $\langle D', \lambda'\rangle$ are L-labelled diagrams, then f is a *labelled-diagram homomorphism* from $\langle D, \lambda\rangle$ to $\langle D', \lambda'\rangle$ iff f is a digram homomorphism from D to D' and $\lambda' f = \lambda$. A labelled-diagram homomorphism is a *labelled-diagram isomorphism* iff it is a bijection and its inverse is also a labelled-diagram homomorphism.

$\langle D'\lambda'\rangle$ is an *extension* of diagram $\langle D,\lambda\rangle$, written $\langle D',\lambda'\rangle \supseteq \langle D,\lambda\rangle$ iff $D' \supseteq D$ and λ is the restriction of λ' to O_D. It is a *free extension*, written $\langle D',\lambda'\rangle \geq \langle D,\lambda\rangle$ iff, in addition, for any labelled diagram $\langle D'',\lambda''\rangle$ and any function $f{:}O_{D'} \rightarrow O_{D''}$ such that $\lambda'' f = \lambda'$, if the restriction of f to O_D is a homomorphism then so is f. D is *simple* iff it is a free extension only of itself.

Lemma 8.10 *If $\langle D',\lambda'\rangle$ is a free extension of $\langle D,\lambda\rangle$ then $\langle D,\lambda\rangle \models \langle D',\lambda'\rangle$*
PROOF: Left to the reader. QED

Theorem 8.11 *(For properly-labelled diagrams only.) $\langle D,\lambda\rangle \models \langle D',\lambda'\rangle$ iff there is a free extension $\langle D^*,\lambda^*\rangle$ of $\langle D,\lambda\rangle$ and a labelled-diagram homomorphism from $\langle D',\lambda'\rangle$ to $\langle D^*,\lambda^*\rangle$.*
PROOF: Suppose $\langle D,\lambda\rangle \models \langle D',\lambda'\rangle$. Let $L' = \mathrm{rng}(\lambda') - \mathrm{rng}(\lambda)$. Construct the free extension D^* of D by adding a new curve c_l for each $l \in L'$, according to Construction 7.5. Define $\lambda^*{:}O_{D^*} \rightarrow L$ by

$$\lambda^*(c) = \begin{cases} \lambda(c) & \text{if} \quad c \in O_D \\ l & \text{if} \quad c = c_l \text{ for some } l \in L' \end{cases}$$

This makes $\langle D^*,\lambda^*\rangle$ a free extension of $\langle D,\lambda\rangle$. From the proof of Theorem 8.6, we have $(\langle\lambda^*\rangle \boldsymbol{D}^*)^+ \models \langle D^*,\lambda^*\rangle$, and so $(\langle\lambda^*\rangle \boldsymbol{D}^*)^+ \models \langle D',\lambda'\rangle$, by hypothesis. Thus there is a g such that $\lambda', g{:}\, \boldsymbol{D}' \rightleftarrows (\langle\lambda^*\rangle \boldsymbol{D}^*)^+$. In fact, because $\mathrm{rng}(\lambda') \subseteq L' \cup \mathrm{rng}(\lambda) = typ(\langle\lambda^*\rangle \boldsymbol{D}^*)$, we have $\lambda', g{:}\, \boldsymbol{D}' \rightleftarrows \langle\lambda^*\rangle \boldsymbol{D}^*$. Now, we use the fact that $\langle D,\lambda\rangle$ and hence $\langle D^*,\lambda^*\rangle$ is properly-labelled: by Corollary 8.8, $\lambda^{*-1}, \mathrm{id}(\Box_D){:}\, \langle\lambda^*\rangle \boldsymbol{D}^* \rightleftarrows \boldsymbol{D}^*$. By composition, $\lambda^{*-1}\lambda', g\mathrm{id}(\Box_D){:}\, \boldsymbol{D}' \rightleftarrows \boldsymbol{D}^*$, and so $\lambda^{*-1}\lambda'$ is a homomorphism from $\langle D',\lambda'\rangle$ to $\langle D^*,\lambda^*\rangle$, as required.

Conversely, suppose we have a homomorphism f from $\langle D',\lambda'\rangle$ to a free extension $\langle D^*,\lambda^*\rangle$ of $\langle D,\lambda\rangle$. If $\boldsymbol{A} \models \langle D',\lambda'\rangle$ then by Lemma 8.10 $\boldsymbol{A} \models \langle D^*,\lambda^*\rangle$. So $\boldsymbol{A}, \lambda^* \models D^*$, and thus $\boldsymbol{A}, \lambda^* f \models D'$ by Lemma 5.7. Finally, $\lambda^* f = \lambda'$, and so $\boldsymbol{A} \models \langle D',\lambda'\rangle$, as required. QED

Definition 8.12 If $\langle D,\lambda\rangle$ is properly-labelled, we define $\mathrm{core}(\langle D,\lambda\rangle)$ to be the diagram $\mathrm{core}(D)$, labelled with the restriction of λ to $O_{\mathrm{core}(D)}$.

Corollary 8.13 *(For properly-labelled diagrams only.) $\langle D,\lambda\rangle \models \langle D',\lambda'\rangle$ iff there is a homomorphism from core($\langle D',\lambda'\rangle$) to $\langle D,\lambda\rangle$.*
PROOF: From the theorem and Lemma 6.10. QED

Corollary 8.14 *(For properly-labelled diagrams only.) $\langle D,\lambda\rangle$ is logically equivalent to $\langle D',\lambda'\rangle$ iff core($\langle D,\lambda\rangle$) is isomorphic to core($\langle D',\lambda'\rangle$)*
PROOF: From Corollary 8.13 and Lemma 6.11. QED

Theorem 8.11 and its corollaries do not apply to the case in which the premise is improperly labelled. A counterexample is shown in Figure 10. The diagram on the right is a consequence of the one on the left and both are simple, but there is no homomorphism from the right to the left.

We would like to extend Theorem 8.11 to a characterization of valid arguments with an arbitrary number of premises. Clearly it would be sufficient to find a way of combining diagrams $\langle D_1,\lambda_1\rangle$ and $\langle D_2,\lambda_2\rangle$ into a single diagram $\langle D_3\lambda_3\rangle$, such that

Figure 10: Improper consequence

for each $\langle D, \lambda \rangle$

$$\langle D_3 \lambda_3 \rangle \models \langle D, \lambda \rangle \text{ iff } \langle D_1, \lambda_1 \rangle, \langle D_2, \lambda_2 \rangle \models \langle D, \lambda \rangle$$

In fact, this can be done, but it is convenient to consider first a diagrammatic device which makes the construction of the combined diagram much easier: shading. But that would take us beyond the scope of this paper.

9 Depiction and Denotation

Our initial departure from the algebraic thoroughfare was motivated by syntactic considerations: the fact that syntactic equivalence between diagrams is best described using geometric concepts. But all roads lead to algebra, and we should say something about how to navigate the rest of the journey.

First, we should recall the role played by terms in algebra. It will suffice to restrict our attention to Boolean algebras, although the point is quite general. The Boolean algebras constitute a variety BA of algebras of type $\langle \wedge, \vee, \neg, 0, 1 \rangle$, which can be characterized either equationally, by the usual axioms, or as the class of algebras embeddable in a powerset algebra $\langle \mathcal{P}(S), \cap, \cup, -, \emptyset, S \rangle$, for some set S.

A *term-algebra* of type $\langle \wedge, \vee, \neg, 0, 1 \rangle$ is an algebra $\mathcal{T}[X]$ whose elements are terms built from the Boolean connectives with elements of X taken as atomic symbols, and whose operations are the corresponding *syntactic* functors, e.g., the function mapping terms t_1 and t_2 to the term '$(t_1 \wedge t_2)$'. The term-algebras play an essential role in algebra for various reasons.

First, terms are *segmentable*: they can be written down in a linear notation. This is an obvious point, but a very important one. So close is the concept of an abstract term to the concept of a concrete symbol that it is sometimes difficult to imagine one without the other.

Second, terms are *inductive*. To put it another way, terms wear their inductive structure on their sleeves. This plays an essential role in shaping the way we think about the manipulation of terms, when designing logical calculi, for example. Rules which are defined in terms of the inductive structure of terms will have a special character.

Third, terms are *free*: for any algebra A of the same type type as $\mathcal{T}[X]$, and any function $f: X \to A$, there is a unique extension of f to a homomorphism $\hat{f}: \mathcal{T}[X] \to \mathcal{A}$. This means that terms provide an extremely versatile means of representing the elements of another algebra; no structural information is built into a term, apart from its arity.

Fourth, terms *denote*. The sole representational role of a term is to denote an element of an algebra. On its own, a term does not make any claim about the element it denotes. To achieve go beyond denotation, one needs to combine terms in more complicated expressions, the simplest being the equations.

Equations provide the means by which the denotational powers of terms are turned into classificatory powers. An equation '$t_1 = t_2$' classifies functions $f: X \to A$ into those that do and those that do not satisfy the condition: $\hat{f}(t_1) = \hat{f}(t_2)$.

Moreover, equations allow us to find a *free* Boolean algebra, and thereby characterize the variety BA, by purely algebraic means. We define a congruence '$\approx$'

on $\mathcal{T}[X]$ by $t_1 \approx t_2$ iff for each Boolean algebra B and each function $f: X \to B$, $\hat{f}(t_1) = \hat{f}(t_2)$. Now the quotient $\mathcal{T}[X]/\approx$ is still free with respect to the Boolean algebras—each $f: X \to B$ is uniquely extendable to a homomorphism $\hat{f}: \mathcal{T}[X]/\approx \to \mathcal{B}$—but unlike $\mathcal{T}[X]$ the quotient $\mathcal{T}[X]/\approx$ is itself a Boolean algebra.

Elements of the free Boolean Algebra $\mathcal{F}[X] = \mathcal{T}[X]/\approx_{BA}$ are a perfect compromise between freedom and expressivity. All and only the equations which are satisfied by all Boolean algebras are satisfied by $\mathcal{F}[X]$. However, one important ingredient is gone: the wearing of structure on the sleeve. One cannot usually divine the structure of a congruence simply by looking at an equation; at least not in the same way that one can see the structure of a term.

By moving away from terms, we have shifted the balance from syntax to semantics—or so it would seem, if we were to make the mistake that congruence-classes of terms are the only way to represent the elements of $\mathcal{F}[X]$. However, there is another way. As mentioned earlier, the class of Boolean algebras can be characterized either using equations, or as subalgebras of concrete powerset algebras, also called "fields of sets". This result due to Birkhoff was improved by Stone, who made the remarkable discovery that the structure of these fields of sets can be specified *by entirely geometric means*.

To be a little more precise, Stone's Duality Theorem states that a Boolean algebra can be uniquely represented as a certain kind of topological space (now called a Stone space), in such a way that homomorphisms between Boolean algebras are also uniquely representable as *continuous functions* between the corresponding spaces (in the opposite direction).

The potential for diagrams would be apparent, even had they not been invented first. It only remains to fill in the details.

As was noted earlier, the essential structure of a basic diagram D is contained in the classification $\boldsymbol{D}_\sim$. This classification retains all the geometric structure of the diagram, without inessential details about the exact arrangement of curves in the plane. Such a classification can be used to generate both a Boolean algebra and its corresponding Stone space.

Definition 9.1 Given a classification $\boldsymbol{A}$, we let $\mathcal{B}(\boldsymbol{A})$ to be the Boolean set algebra of $\sim$-closed sets, and let $\mathcal{S}(\boldsymbol{A})$ be the topological space with points $tok(\boldsymbol{A})$ and whose open sets are the $\sim$-closed sets.

For a digram D, the space $\mathcal{S}(\boldsymbol{D}_\sim)$ is (homeomorphic to) the Stone space of $\mathcal{B}(\boldsymbol{D}_\sim)$. What's more, the duality between Boolean homomorphisms and continuous functions is reflected in the two components of an S-link.

Claim 9.2 *Given functions $f: typ(\boldsymbol{D}_\sim) \to typ(\boldsymbol{D}'_\sim)$ and $g: typ(\boldsymbol{D}'_\sim) \to typ(\boldsymbol{D}_\sim)$, $f, g: \boldsymbol{D}'_\sim \rightleftarrows \boldsymbol{D}_\sim$ iff*

(i) f can be uniquely extended to a homomorphism $\hat{f}: \mathcal{B}(\boldsymbol{D}_\sim) \to \mathcal{B}(\boldsymbol{D}'_\sim)$
(ii) g is a continuous function from $\mathcal{S}(\boldsymbol{D}'_\sim)$ to $\mathcal{S}(\boldsymbol{D}_\sim)$, and
(iii) $\hat{f}$ and g are Stone duals.

The correspondence can be extended to much of what has gone before. In particular, D is a free diagram iff $\mathcal{B}(\boldsymbol{D}_\sim)$ is a free Boolean algebra. The addition of labels makes the correspondence easier to state:

Claim 9.3 *If $\langle D, \lambda \rangle$ is a properly-labelled, free diagram, then $\hat{\lambda}: \mathcal{B}(\boldsymbol{D}_\sim) \to \mathcal{F}[L]$ is an embedding.*

This shows precisely how diagrams are analogous to terms: both provide a way of concretely representing finite information about the free Boolean algebra. It also shows how they differ: terms do it by denoting elements of the algebra and use them to define congruences; whereas diagrams do it by depicting a finite subalgebra.

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Definite and indefinite generics

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Abstract

In this paper I develop a unified analysis of generic and non-generic readings of definite and indefinite NPs in sentences expressing characteristic predication. I build on examples from English and French in order to show that genericity is not cross-linguistically related to the presence of indefinite NPs but can be expressed by means of definite NPs as well. An analysis of adverbs of quantification as generalized quantifiers over events combined with an interpretation of indefinite NPs as dynamic existential quantifiers and of definite NPs as context-dependent quantifiers is shown to yield the right interpretation of generic sentences if we make a number of independently motivated pragmatic assumptions. As far as truth conditions are concerned, the pragmatic account developed here is equivalent to proposals made by Chierchia (1992) and Dekker (1993), which appeal to type-shifting mechanisms in the semantics. The main advantage of the event-based approach is that it does not need type shifting because of the appeal to lexical-semantic properties of the predicate involved. It therefore offers a unified semantics of both adverbs of quantification and (in)definite NPs in generic and non-generic sentences.

1 Definite and indefinite generic NPs

1.1 Introduction

In English, we typically use indefinite singulars or bare plurals to generalize over individuals having a certain characteristic property as in (1):[1]

(1) a. A potato contains vitamin C, amino acid, protein and thiamin
b. Italians drink wine with their dinner

In a Romance language such as French, this characteristic predication is expressed by either indefinite singulars or definite plurals. Indefinite plurals cannot be used to provide the NP we generalize over as (2b) shows:[2]

(2) a. Un Italien boit (généralement) du vin à table
An Italian drinks (generally) wine-INDEF-MASS at table

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[2]There is also a bare plural in French, which occurs for instance after the preoposition *de*:

(i) La maison est entourée d'arbres
The house is surrounded by trees

I will not give an analysis of bare plurals in French here.

b. *Des Italiens boivent (généralement) du vin à table
INDEF-PL Italians drink (generally) wine-INDEF-MASS at table

c. Les Italiens boivent (généralement) du vin à table
DEF-PL Italians drink (generally) wine-INDEF-MASS at table

Other than in generic contexts, the indefinite plural *des* N behaves pretty much like the English bare plural. For instance, it allows discourse anaphora and donkey anaphora, as in the following examples:

(3) a. Hier soir, *des terroristes basques* ont essayé d'enlever le Premier Ministre. *Ils* n'ont pas eu de chance: *ils* ont été arrétés ce matin
Yesterday evening, *Basque terrorists* tried to kidnap the Prime Minister. *They* were not very lucky: *they* have been arrested this morning

b. Tous les paysans qui ont *des ânes têtus* les battent
All farmers who have *stubborn donkeys* beat them

The rule of thumb is then: bare plurals that have an existential reading translate as *des* N in French, generic bare plurals translate as *les* N.[3] In the literature, we find several proposals which intend to account for indefinite NPs in sentences which express characteristic predication, but not much attention has been paid to definite NPs in these contexts. In this paper I develop a unified analysis of generic and non-generic readings of indefinite NPs, building on examples from English and French. I will then extend the approach to definite NPs.

Note that I will only be concerned with generic sentences like (1) and (2). I will not discuss reference to kinds as in (4):

(4) a. The potato was first cultivated in South America
b. Potatoes were introduced in Ireland by the end of the 17th century

There are a number of linguistic differences between reference to kinds and characteristic predication. For instance, singular indefinites are excluded in contexts in which kind-level predication is involed:

(5) a. The dodo is extinct
b. *A dodo is extinct

In contradictinction to (5a), (5b) is unacceptable, unless we assign the sentence a taxonomic reading and claim that a particular subspecies of dodos is extinct. (cf. Krifka e.a. 1992 for extensive discussion of the two varieties of genericity).

Standard wisdom has it that the kind of genericity which plays a role in characteristic predication is a property of sentences rather than of NPs. One

[3]In colloquial French, there is an alternative construction in which *des* occurs in relation with so-called generic *ça*, e.g.:

(i) Des chiens, ça aboie
INDEF-PL dogs, that barks

(ii) Beaucoup d'arbres, ça attire des insectes
Many trees, that attracts insects

As (ii) shows, other determiners can also be used in this construction, which suggests that it is quite different from the generic sentences treated in this paper, cf. Auger, 1993.

argument is that indefinite singulars cannot be kind-referring (cf. 5b), but they can certainly get a generic reading when they occur in a characterizing sentence (cf. 1a, 2a). Therefore, the source of genericity is not in the indefinite NP itself, but rather in the sentence. This has led many authors to introduce a generic operator, which functions at the sentential level. This generic operator is not phonologically realized, but it is closely related to adverbs of quantification like *always*, *usually*, *generally*, as the near-synonymy of the following examples shows:

(6) a. Italians drink wine with their dinner
 b. Italians usually drink wine with their dinner

Both the adverbs and the generic operator can be treated as two-place quantifiers, relating a restrictor and a matrix.

(7) Q [Restrictor] [Matrix]

For reasons of exposition, I will tend to use explicit adverbs, rather than rely on a phonologically null generic operator.

1.2 Existential and generic readings of indefinite NPs

The bare plural in (6) is clearly a generic NP: we express a generalization about Italians. This doesn't mean that any indefinite singular or bare plural which occurs in a generic sentence is itself a generic NP. A well-known counterexample is the kind of sentence in (8), which has the readings listed under (8a) and (8b):

(8) Typhoons often arise in this part of the Pacific

 a. Typhoons have a common origin in this part of the Pacific
 b. There arise typhoons in this part of the Pacific

The (a)-reading expresses a generalization about typhoons, and the bare plural is intuitively characterized as a generic NP. In the (b)-reading, the bare plural has an existential reading, rather than a generic interpretation, and the generalization expressed holds for this part of the Pacific. We can capture this difference by having the bare plural in (8a) end up as part of the restrictor, whereas it would be in the matrix in (8b):

(9) a. OFTEN [typhoons] [arise in this part of the Pacific]
 b. OFTEN [in this part of the Pacific] [typhoons arise]

If only the bare plural in the (a)-reading is generic, we expect the French translation of the sentence to solve the ambiguity. This turns out to be the case, as shown by (10):

(10) a. Les ouragans violents naissent souvent dans cette partie du Pacifique
 DEF-PL typhoons arise often in this part of the Pacific
 b. Dans cette partie du Pacifique naissent souvent des ouragans violents
 In this part of the Pacific arise often INDEF-PL typhoons

(10a) corresponds with the interpretation in which typhoons have a tendency to arise in this part of the Pacific, whereas (10b) only has the interpretation in which there arise typhoons in this part of the Pacific. In other words, *des* N can occur in a generic sentence, if it is part of the matrix, rather than the restrictor, because there it will get an existential reading. The NP about which a certain generalization is stated cannot be *des* N, so if the sentence is about typhoons in general, we switch to a definite plural, as in (10a).

We find more cases of non-generic indefinite NPs in generic sentences if we extend our discussion to NPs in object position. Consider the examples in (11):

(11) a. Anne always knits Norwegian sweaters
b. Anne tricote toujours des chandails norvégiens

Although these sentences are habitual rather than generic, because they describe generalizations which hold for individuals, rather than kinds, they illustrate the problem rather well. (11a) is ambiguous between (at least) two readings, which can be roughly spelled out as in (12):

(12) a. ALWAYS [Anne knits sweaters][Anne knits Norwegian sweaters]
b. ALWAYS [Anne knits something][Anne knits Norwegian sweaters]

¿From (12a) we can infer that all sweaters that Anne knits will be Norwegian sweaters. In (12b), all things that she knits will be Norwegian sweaters. We expect the bare plural in the (b)-reading to be existential, because it ends up in the matrix, and this is indeed what happens. Although *sweaters* is part of the restrictor in the (12a)-reading, the bare plural also gets an existential, rather than a generic interpretation. That is, it is not a generalization over sweaters that they are Norwegian sweaters when knitted by Anne, but a generalization over situations in which Anne knits some sweater or other. This intuition is confirmed by the French example (11b). *Des* N can only be existential, but (11b) is ambiguous in exactly the same way as the English example (11a). This suggests that only a subset of the indefinite NPs that are interpreted as part of the restrictor actually gets a generic interpretation.

2 Bound variable readings

Given that the indefinite plural *des* always gets an existential reading in the kind of quantificational contexts under consideration here, I think we can take the French data to confirm the intuition that indefinites that occur in the matrix get an existential reading, whereas generic indefinites are a (not necessarily proper) subset of the indefinite NPs that are interpreted as part of the restrictor. The only NPs which can be properly called generic NPs are those for which the generalization described by the sentence holds. That is, the generalization is in some sense felt to be 'about' that NP. Generic NPs are the only ones which are felt to be somehow bound by the (generic) quantifier. Given that all other indefinites are felt to be existentially quantified, we can unify all the cases of generic indefinite NPs by describing them as 'bound variable' readings.

I use the term 'bound variable' readings in a pretheoretic sense, without claiming that the indefinite NP is actually bound by the generic quantifier. Strictly speaking, binding of the indefinite in the semantics is just one possible point of view, although it seems to be the one that dominates in the field right now (cf. Krifka et al., 1992) and is know as the 'unselective binding theory'. This view is often - though not always (cf. Chierchia, 1992; Dekker, 1993) - related to an interpretation of indefinite NPs as free variables (cf. Kamp, 1981; Heim, 1982).

Instead of adopting an unselective binding analysis of genericity, I will adopt a more indirect approach to bound variable readings by interpreting indefinite NPs uniformly as (dynamic) existential quantifiers. As we will see later, this approach also has the advantage of providing a natural extension to generic readings of definite NPs.

2.1 Dynamic existential quantifiers

Groenendijk and Stokhof (1991, 1992) introduce dynamic existential quantifiers which differ from their static counterparts of classical predicate logic in that they provide an "anchor" for variable assignments in subsequent sentences to attach to. This allows for binding relations beyond the sentence boundaries. Interpreting the indefinite NP in terms of a dynamic existential quantifier gives (13) the representation under (13a). It is translated as in (13b), which can be rewritten as (13c):

(13) A dog came in. It lay down under the table

a. $\mathbf{E}d\ [\uparrow \text{dog}(d)\ ;\uparrow \text{come-in}(d)]\ ;\uparrow \text{lay-down}(d)$

b. $\lambda p \exists x\ [\text{dog}(x) \wedge \text{come-in}(x) \wedge \{x/d\}^{\vee}p]^{\wedge}(\uparrow \text{lay-down}(d))$

c. $\lambda q \exists x\ [\text{dog}(x) \wedge \text{come-in}(x) \wedge \text{lay-down}(x) \wedge \{x/d\}^{\vee}q]$

The uparrows signal the dynamic character of the proposition: the dynamic meaning or the context change potential of a sentence is taken to be its ability to constrain subsequent discourse. (13b) shows that in the first sentence the variable assignment anchors the discourse marker d to the individual x. This variable assignment is carried on to the next sentence, which is attached to it by means of dynamic conjunction: $\{x/d\}$ tells us that subsequent occurrences of d will also be attached to x. The pronoun *it* can then be interpreted as referring to the same individual x (13c). This approach accounts for discourse anaphora, but also for donkey sentences.

Chierchia (1992) introduces the notion of dynamic generalized quantifier, which establishes a relation between dynamic properties. The general definition of dynamic and conservative generalized quantifiers is given in (14):

(14) $\mathbf{D}^{+}\ (P)(Q) = \uparrow D(\lambda\, x \downarrow P(x)))\ (\lambda x \downarrow [P(x)\ ;\ Q(x)])$

P and Q are dynamic properties, which are just sets with an additional place holder for discourse continuations. Applying the downarrow brings us back from context change potentials to type t expressions. D^{+} is thus defined in terms of its static counterpart D, which just relates sets of individuals as in standard generalized quantifier theory. The context change potential of a donkey sentence like (15) is represented as follows:

(15) Every farmer who owns a donkey beats it

a. **Every**$^{+}$(x is a farmer that owns a donkey)(x beats it) =
↑ EVERY (λx ↓ [x is a farmer that owns a donkey])
(λx ↓ [x is a farmer that owns a donkey *and* beats it])

b. ↑ EVERY (λx ↓ [↑ Farmer(x) ; Ed [↑ Donkey(d) ; ↑ Own(x,d)]])
(λx ↓ [↑ Farmer(x) ; **Ed** [↑ Donkey(d) ; ↑ Own(x,d)] ; ↑ Beat(x,d)])

c. ↑ $\forall x$[[Farmer(x) $\wedge \exists y$[Donkey(y) $\wedge$ Own(x,y)]] $\rightarrow$
$\exists y$[Donkey(y) $\wedge$ Own(x,y) $\wedge$ Beat(x,y)]]

The conservative dynamic quantifier **Every**$^{+}$ relates the dynamic property of 'farmers that own a donkey' to 'farmers that own a donkey *and* beat it'. The binding capacities of the dynamic existential quantifier extend to the dynamically conjoined 'beat it'. Dynamic binding thus gives us the right anaphoric relation.

2.2 Quantification over events and individuals

We can incorporate adverbs of quantification (Q-adverbs) in dynamic Montague grammar by interpreting them as dynamic generalized quantifiers over (minimal) events or situations:

(16) **A**$^{+}$ (P)(Q) = ↑ A(λe ↓ P(e))) (λe ↓ [P(e) ; Q(e)])

We obtain the desired event variable by enriching the argument structure of the predicate with a Davidsonian argument. For example:

(17) When John is in the bathtub, he always sings

a. **Always**$^{+}$ (e is an event of John in the bathtub)
(e is an event of John singing) = ↑ ALWAYS
(λe ↓ [↑ In-bathtub(j,e)]) (λe ↓ [↑ In-bathtub(j,e) ; ↑ sing(j,e)])

b. ↑ $\forall e$ [[In-the-bathtub(j,e)] $\rightarrow$ [In-the-bathtub(j,e) $\wedge$ Sing(j,e)]]

(17) gets the interpretation that all events of John being in the bathtub are such that they are events of John being in the bathtub and of him singing.[4] The treatment extends to sentences containing an indefinite in a straightforward way, namely by iteration of bindings:

(18) When John invites a friend, he always cooks dinner for her

a. **Always**$^{+}$ (e is an event of John inviting a friend)
(e is an event of John cooking dinner for her) =
↑ ALWAYS (λe **Ed** [↑ Friend(d,j) ; ↑ Invite(j,d,e)])
(λe **Ed** [↑ Friend(d,j) ; ↑ Invite(j,d,e)] ; ↑ Cook-dinner-for(j,d,e))

[4] Compare De Swart, 1991 and 1993 for a more sophisticated treatment of quantified sentences introduced by a temporal connective, which extends to *before*- and *after*-clauses.

b. $\uparrow \forall e[[\exists x\ \text{Friend}(x,y) \wedge \text{Invite}(j,x,e)] \rightarrow$
$[\exists x\ \text{Friend}(x,y) \wedge \text{Invite}(j,x,e) \wedge \text{Cook-dinner-for}(x,j,e)]]$

Each alternative event generated by the *when*-clause encodes a choice of a friend as the value of the indefinite *a friend.* Because of the quantification over events, there is no need to suppress the existential import of indefinites within the restrictive clause. The general mechanism of carrying forward the value assigned to the indefinite guarantees the desired anaphoric binding relation.

One of the objections which has been raised against the event-based analysis of Q-adverbs concerns non-episodic sentences such as (19):

(19) a. *When Minouche has blue eyes, it is often intelligent
b. When a cat has blue eyes, it is often intelligent

The question is whether it makes sense to talk about quantification over events in (19), because a property such as *having blue eyes* is an individual-level property which is not bound to particular occasions. The unacceptability of (19a), which contrasts with (17) provides a strong argument in favor of the view that what (19b) really quantifies over is cats.

This has led to the suggestion (Kratzer, 1989) that only stage-level predicates as in (17) and (18) come with a Davidsonian event argument, whereas individual-level predicates such as the one in (19) do not have a spatio-temporal location argument. In De Hoop and De Swart (1991) and De Swart (1991) it has been argued that this line of reasoning cannot be correct, for it would make it hard to account for similar contrasts in the following examples:

(20) a. *When Anil died, his wife usually killed herself
b. When an Indian died, his wife usually killed herself

(21) a. *When Mary built Jim's house, she always built it well
b. When Mary builds a house, she always builds it well

Intuitively, it is quite clear why the (a)-sentences are ungrammatical: although the predicate is stage-level (because the event describes a short period in time, the predicate occurs in existential sentences, etc.), the same action cannot be repeated with respect to the same individual. That is, they are 'once-only' predicates which come with the following presupposition of uniqueness on the Davidsonian argument:

(22) Uniqueness presupposition on the Davidsonian argument
The set of events that is associated with a 'once-only' predicate is a singleton set of all models and each assignment of individuals to the arguments of the predicate

With respect to a particular assignment function then, the proposition denotes a singleton set of events. The correct generalization for both once-only stage-level and individual-level predicates in (19)-(21) is that Q-adverbs do not quantify over singleton sets. They need a plurality of situations, otherwise the quantification is vacuous in some sense. This rules out the unacceptable sentences in (19a), (20a) and (21a). In this perspective, (19b), (20b) and (21b) are all right, because the indefinite NP creates a plurality of situations the Q-adverb can quantify over. Compare for instance the representation of (20b) in (23):[5]

(23) When an Indian died, his wife usually killed herself
$\uparrow$ USUALLY $(\lambda e \exists x$ Indian(x) $\wedge$ die(x,e))
$(\lambda e \exists x$ Indian(x) $\wedge$ die(x,e) $\wedge$ the y wife-of$_e$(y,x) $\wedge$ Kill(y,y,e))

The set of situations such that there is an Indian who dies in that situation is not a singleton set, so quantification is possible. Note that the representations in (18a) and (23) are very similar. The main difference between the two examples resides in the relation between individuals and events. In (18), John can invite the same friend several times, so indirect binding of the existential quantifier does not induce a bound variable reading on the indefinite NP. In (23) on the other hand, quantification over events becomes indistinguishable from quantification over individuals. The reason is that individuals die only once, so there is a uniqueness presupposition on the Davidsonian argument. As a result, the set of (minimal) events in which an Indian dies is co-extensive with the set of dying Indians. That is, knowledge of the world tells us that there are as many dying Indians as there are (minimal) events in which an Indian dies:

(24) $|\{e | \exists x$ Indian(x) $\wedge$ die(x,e) $\}| = |\{x|$ Indian(x) $\wedge \exists e$ Die(x,e) $\}|$

It is thus not suprising that (23) is "felt" to express quantification over dying Indians (or over pairs $< x, e >$ of Indians x dying in a situation e). This induces a bound variable reading on the indefinite NP: it is felt to be quasi-bound along with the event variable. In the event-based analysis of adverbs of quantification we have adopted here, there is of course no real binding of the individual variable, because quantification is over events. Thus (18) and (23) can have the same representation in the semantics. The meaning effects of existential quantification in (18) and a generic interpretation in (23) don't have anything to do with unselective binding, they are just a consequence of the pragmatics of the sentence.

We can handle examples involving individual-level predicates in the same way, for they share the uniqueness presupposition with once-only predicates. For instance:

(25) When a cat has blue eyes, it is often intelligent
$\uparrow$ OFTEN $(\lambda e \exists x$ Cat(x) $\wedge$ Have-blue-eyes(x,e))
$(\lambda e \exists x$ Cat(x) $\wedge$ Have-blue-eyes(x,e) $\wedge$ Intelligent(x,e))

Given that individual-level predicates are once-only, no two (minimal) events in which there is a cat with blue eyes will involve the same cat. For every event there will be a different cat, which means that the set of these events is co-extensive with the set of cats having blue eyes. Again, this explains why quantification is "felt" to be about blue-eyed cats, although, formally, it is about events containing a blue-eyed cat. The bound variable reading of the indefinite NP falls out as an effect of the pragmatics of the sentence.

[5]For the analysis of the definite NP, see below.

2.3 Existential disclosure

The analysis developed in the previous section preserves a unified treatment of Q-adverbs as generalized quantifiers over events by pushing the explanation of generic readings of indefinite NPs into the pragmatics. A question one could ask at this point is how this pragmatic account of bound variable readings compares with analyses which adopt the unselective binding perspective and have the indefinite NP to be actually bound by the generic quantifier in the semantics. In a framework such as DMG, which treats indefinite NPs as dynamic existential quantifiers, this requires a mechanism to turn the indefinite NP into a variable. Type shifting mechanisms which realize this are discussed by Partee (1987). A good candidate would be Partee's ***ident*** or BE operation. *ident* is defined in (26a) and turns type e expressions into type $< e, t >$ expressions by means of identity and λ-abstraction:

(26) a. *ident* : a $\rightarrow \lambda x[\ x = a\ ]$

b. BE: $\lambda\mathcal{P}\lambda x[\mathcal{P}(\lambda y[y = x])]$

BE is an operator which maps type $<< e, t >, t >$ expressions onto type $< e, t >$ expressions, with the semantics in (26b). Partee argues that BE(A(P)) = P:

(27) a. $\|$ a P $\| = \lambda Q[\exists x[P(x) \wedge Q(x)]]$

b. $\|$ BE a P $\| = \lambda\mathcal{P}\lambda x[\mathcal{P}(\lambda y\ [y = x])]\ (\lambda Q[\exists x\ [P(x) \wedge Q(x)]])$

c. $\|$ BE a P $\| = \lambda x\ P(x)$

In (27b), we apply the type-shifting operator BE to the generalized quantifier denotation of *a* N, as defined in (27a). After successive λ-conversion, we end up with (27c), which is just the denotation of N. In this way, we map existentially quantified NPs onto the set of individuals which satisfy the property referred to by the common noun.

In DMG, a type shifting mechanism much like Partee's *ident* or BE operation is defined and called 'existential disclosure' (cf. Dekker, 1993; Chierchia, 1992). Existential disclosure transforms an existentially quantified NP into a predicate by means of identity and λ-abstraction. For instance, the antecedent of (23) repeated here as (28a) denotes a set of events. (28a) can be transformed into (28c), which denotes a set of pairs of an event and an individual via conjunction with a vacuous identity statement in (28b):

(28) a. $[\lambda e \downarrow [\mathbf{E}d \uparrow \text{Indian}(d)\ ;\ \uparrow \text{die}(x,e)]]$

b. $[\lambda y \lambda e \downarrow [\mathbf{E}d \uparrow \text{Indian}(d)\ ;\ \uparrow \text{die}(x,e)\ ;\ \uparrow d = y]]$

c. $[\lambda y \lambda e\ \text{Indian}(y) \wedge \text{die}(y,e)]$

d. $[\lambda y\ \text{Indian}(y) \wedge \exists e\ \text{die}(y,e)]$

The Q-adverb accordingly has to shift from a quantifier over events to a quantifier which binds pairs of events and individuals as in (28c). Given that every individual dies only once, (28c) is in fact equivalent to (28d), so we can say that existential disclosure gives us quantification over dying Indians as desired. In the case of (29) (repeated from 25) the predicate is so loosely tied to occasions that we may even be tempted to drop the event argument altogether and reduce (29d) further to (29e):

(29) When a cat has blue eyes, it is often intelligent

a. $[\lambda e \downarrow [\mathbf{E}d \uparrow \text{Cat}(d) \; ; \uparrow \text{Have-blue-eyes}(x,e)]]$

b. $[\lambda y \lambda e \downarrow [\mathbf{E}d \uparrow \text{Cat}(d) \; ; \uparrow \text{Have-blue-eyes}(x,e) \; ; \uparrow d = y]]$

c. $[\lambda y \lambda e \; \text{Cat}(y) \wedge \text{Have-blue-eyes}(y,e)]$

d. $[\lambda y \; \text{Cat}(y) \wedge \exists e \; \text{Have-blue-eyes}(y,e)]$

e. $[\lambda y \; \text{Cat}(y) \wedge \text{Have-blue-eyes}(y)]$

Given that the event variable does not really contribute much to the semantics in such cases, quantification is "felt" to be about blue-eyed cats rather than events in which exists a blue-eyed cat. Formally, of course, the inferences from (29c) to (29d) and further to (29e) are not legitimated by the semantics, but they follow from the pragmatics.

Chierchia suggests that existential disclosure gives us the 'topic', that is, what the quantification is about. This seems to be a suitable way to restrict the application of this operation, because generic NPs are topics in the sense that the generalization expressed by the sentence is 'about' that NP.

The operation extends in a natural way to cases like (30), in which the restriction is not given by an *if/when*-clause, but via association with focus (cf. Krifka, 1992):

(30) An intelligent cat always likes milk

a. $[\lambda e \downarrow [\exists X \; \mathbf{E}d \uparrow \text{Cat}(d) \; ; \uparrow \text{Intelligent}(d,e) \; ; \uparrow X(d,e)]]$

b. $[\lambda y \lambda e \downarrow [\exists X \; \mathbf{E}d \uparrow \text{Cat}(d) \; ; \uparrow \text{Intelligent}(d,e) \; ; \uparrow X(d,e) \; ; \uparrow d{=}y]]$

c. $[\lambda y \lambda e \exists X \; \text{Cat}(y) \wedge \text{Intelligent}(y,e) \wedge X(y,e)]$

d. $[\lambda y \exists X \; \text{Cat}(y) \wedge \exists e \; \text{Intelligent}(y,e) \wedge X(y,e)]$

e. $[\lambda y \; \text{Cat}(y) \wedge \text{Intelligent}(y)]$

Reconstructing Krifka's (1992) analysis of focus in generic sentences in an event-based semantics gives us (30a) as a representation of the antecedent. The variable X stands for a contextually determined alternative of liking milk (for instance, hating milk, liking yogurt). Given a suitable constraint on alternatives, which requires them to be members of the same comparison set, we can argue that only individual-level predicates will be able to fill this position. Basically, then, we have the same situation here as we had in (29), and we can reduce the antecedent to a quantification over intelligent cats by dropping reference to the event variable and the focus part of the restrictor altogether (30e).

So we can use type shifting mechanisms to turn the Q-adverb into a quantifier over pairs $< x, e >$ of an individual x and an event e, but even such a semantic approach heavily relies on an appropriate characterization of regular stage-level predicates on the one hand and once-only predicates (including individual-level predicates) on the other hand. In particular, existential disclosure will only apply in cases in which the set of events is co-extensive with the set of individuals which constitutes the topic of the generalization. This result considerably weakens Chierchia's (1992) claim that cases like (29) and

(30) are a problem in an event-based analysis of adverbs of quantification. In Chierchia's approach, the burden of the explanation for the unselective binding approach lies in the semantics (picking out the right variable for the operator to bind), whereas the event-based approach relies on pragmatic principles concerning uniqueness presuppositions and partitioning of situations, which can be said to 'simulate' type shifting principles. There is no reason then why an event-based semantics cannot be a viable alternative to an unselective binding approach. As far as the truth conditions of generic sentences are concerned, the pragmatic, event-based approach is equivalent to Chierchia's way of handling bound variable readings of indefinite NPs by means of a type-shifting mechanism in the semantics. Given that even in the semantic approach, pragmatic considerations are crucial in the interpretation of generic quantifiers, I prefer to keep the semantics "clean", so I adopt a unified treatment of Q-adverbs as generalized quantifiers over events and I defer the bound variable readings of indefinite NPs to the pragmatics. In the rest of this paper, I will loosely talk about existential disclosure when I refer to cases in which the pragmatics allows us to freely move back and forth between quantification over individuals and quantification over events.

In English, existential disclosure does not only apply to singular indefinites (31a), but also to bare plurals as in (31b):

(31) a. An Italian usually drinks wine at dinner
b. Italians usually drink wine at dinner

It seems that in such cases, the plurality marker is completely ignored: the property of drinking wine at table is distributively predicated of Italians in general, so (31b) ends up as equivalent to (31a).[6] In such cases we let bare plurals denote dynamic existential quantifiers and we apply existential disclosure as usual.

2.4 French *des* N

The interesting generalization to make here is of course that the contexts in which existential disclosure applies (namely those in which the NP in question is a topic) are exactly the cases in which we get bound variable readings of the indefinite NP, and in which the NP seems to be truly generic. This also allows us to formulate an appropriate constraint on the interpretation of the French indefinite plural *des* N. Just like other indefinite NPs, it denotes a dynamic existential quantifier. However, unlike the indefinite singular *un* N, the English indefinite singular *a* N and the bare plural, *des* N does not allow existential

[6]Krifka et al. 1992 point out that in generic sentences which involve collective predication, a bare plural rather than an indefinite singular is to be used:

(i) a. ??An antelope gathers near the waterhole
b. Antelopes gather near the waterhole

In order to account for such collective generic sentences, a more sophisticated version of existential disclosure would need to be formulated. I will abstract away from such cases in the present paper.

disclosure in the presence of an adverb of quantification. This rules out examples like (32a) and (33a):

(32) a. *En général, des Indiens meurent jeunes
In general, INDEF-PL Indians die young

b. En général, les Indiens meurent jeunes
In general, DEF-PL Indians die young

(33) a. *Des Italiens boivent généralement du vin à table
INDEF-PL Italians drink usually wine at table

b. Les Italiens boivent généralement du vin à table
DEF-PL Italians drink usually wine at table

The predicates in (32) and (33) are once-only. This means that the only way to satisfy the plurality condition on quantification would be to apply existential disclosure, but as we observed earlier, *des* N does not allow this in the presence of an adverb of quantification. For some reason, existential closure of indefinite plurals only arises under the influence of modal operators with a strong deontic or prescriptive flavor as in (34a), an example borrowed from Carlier (1989):

(34) a. Des agents de police ne se comportent pas ainsi dans une situation d'alarme
INDEF-PL police officers do not behave like that in an emergency situation

b. Les agents de police ne se comportent pas ainsi dans une situation d'alarme
DEF-PL police officers do not behave like that in an emergency situation

As Carlier points out, (34a) would be uttered to reproach a subordinate with his behavior. (34b) does not have the same normative value, but gives us a descriptive generalization which could possibly be refuted by providing a counterexample.

We can speculate that the restrictions on existential disclosure have to do with the origin of *des* as a partitive determiner. As Galmiche (1986) points out, *des* N allows - at least marginally - a partitive reading in contexts like (35):

(35) Des fauteuils sont bancals
INDEF-PL armchairs are shaky

According to Galmiche, the plural indefinite selects a subset of the contextually relevant armchairs as the referent of the NP. This suggests that the behaviour of *des* N is in fact closer to that of numerals and *many, few* than to singular indefinites and bare plurals. Just like numerals, *many* and *few, des* is unsuitable for the expression of genericity, but it does take up a partitive interpretation in the presence of an individual-level predicate.

In fact, I will not make very strong claims as to the reason why *des* N behaves this way. For the purposes of this paper it is more interesting to determine how French manages to get by without a generic reading of the indefinite plural.

As (32b), (33b) and (34b) show, we use definite plurals to express inductive generalizations. The question we will investigate in the next section is how the analysis developed so far can be extended in order to account for the behavior of definite generic NPs in sentences expressing characteristic predication.

3 Definite NPs

In the unselective binding framework, the question of definite generics has not been studied in much detail. This is not surprising, for in English, and other Germanic languages, these expressions are rare in sentences expressing characteristic predication. But in French and other Romance languages, inductive generalizations are typically expressed by either the indefinite singular, or the definite plural. According to Krifka et al., this implies that, in Romance languages, the definite article may be used with semantically indefinite NPs. It would indicate the position of the NP in the partition of the sentence, namely that it occurs in the restrictor. This means that definite NPs in characterizing sentences can also display variable behavior, and are therefore bound by an unselective quantifier. Although it is rather unclear to me what it means for a definite NP to be semantically indefinite, I take the intuition seriously that, in certain contexts, definite NPs also get bound variable readings. That requires a more precise description of the relation between bound variable and non-bound variable readings of definites. In order to make this work in the unselective binding framework, we would have to treat definite NPs as variables, but then we also have to make sure that, if the definite does not occur in the restrictor (of the restrictor) it gets a definite, rather than an indefinite interpretation:

(36) a. When Mary receives a letter, she often throws away the envelope
b. Quand Marie reçoit une lettre, elle jette souvent l'enveloppe

In both the English and the French sentence, the definite NP is interpreted in the matrix, where it gets a non-generic interpretation.[7] This means that definite NPs in general are not semantically indefinite, even in Romance languages. To account for these data in a framework which treats definites as variables, we would have to come up with a version of 'definite' closure, in order to avoid these variables to be captured by existential closure. Although this would certainly be feasible, my intuition is that this line of reasoning is somehow missing the point. Instead, I will argue that a treatment of definite NPs as (context-dependent) quantifiers provides a unified analysis of both the generic and non-generic readings of definite NPs.

3.1 Context-dependent quantifiers

Building on the original definition of Russell (1905), many analyses of definite descriptions make use of the iota-operator as in (37a):

[7] Of course the definite NP carries a presupposition of existence, so we quantify over situations in which Mary receives letters that come with an envelope (cf. Karttunen and Peters, 1979 for discussion of the inheritance of presuppositions in complex sentences).

(37) a. || the king ||: ιx [king'(x)]

b. || the king ||: $\lambda Q[\exists x$ king'(x) $\wedge \forall y$[king'(y) $\rightarrow y = x] \wedge$ Q(x)]

The iota-operator combines with an open sentence to give an entity-denoting expression, denoting the unique satisfier of that open sentence if there is just one, and failing to denote otherwise. The iota-operator gets a generalized quantifier interpretation in (37b), which shows that there is a uniqueness claim about the referent of the definite NP. Unless further restrictions are imposed, this uniqueness claim is formulated with respect to the model as a whole. In many cases this is too strong: we can felicitously use a definite NP like *the table*, even although there are many tables in the universe E. Uniqueness should be relativized to the context of use: in the context there is a unique relevant individual that satisfies the description. Neale (1990) calls this the problem of 'incomplete' descriptions. He points out that context-dependency arises with other quantifiers as well:

(38) Everyone arrived late

(38) certainly does not refer to everyone in the whole world but could for instance cover the set of people invited to yesterday night's party. One way of capturing this phenomenon is to add a contextual restriction on the domain of quantification. This leads to the truth conditions in (39a) (cf. Westerståhl, 1984), where the predicate Rel picks out the contextually relevant kings:

(39) (a) || the king || = $\lambda Q[\exists x$ king'(x) $\wedge$
$\forall y$[[king'(y) $\wedge$ Rel(y)] $\rightarrow$ y = x] $\wedge$ Q(x)]

(b) Rel = $\{< x, e > \mid$ x is relevant in e $\}$

In the event-based semantics used here, predicates always come with a Davidsionian argument. This means that the predicate Rel will be interpreted as the set of pairs $< x, e >$ of relevant individuals in a particular event e (39b). So definite NPs will pick out the unique individual that is relevant with respect to a particular event e.

Van Eijck (1993) shows that embedding the Russellian definition in a dynamic logic allows us to account for the fact that definite descriptions can be used both as anaphors and as antecedents for other anaphoric expressions. We add the relevance predicate here and define definite NPs as externally dynamic context-dependent quantifiers:

(40) ||**the** d_1 $P^e(d_1)$|| with respect to an event e =
λQ [Ed_1 P(d_1); **A**d_2 [[P(d_2) ; $\uparrow$ Rel(d_2,e)] $\Rightarrow\uparrow$ $d_2 = d_1$] ; Q(d_1)] =
$\lambda Q \lambda p[\exists x$ [P(x) $\wedge \forall$ y [[P(y) $\wedge$ Rel(y,e)] $\rightarrow$ y = x] $\wedge$ Q(x) $\wedge \{x/d_1\}^{\vee}p$]]

We can use the interpretation schema in (40) to analyze discourse anaphora built on a definite NP as in (41):

(41) The dean came in. She sat down and talked for an hour.

a. **E**d_1 [$\uparrow$ Dean(d_1) ; **A**d_2 [[$\uparrow$ Dean(d_2) ; $\uparrow$ Rel(d_2,e)] $\Rightarrow\uparrow$ $d_2 = d_1$] ;
$\uparrow$ Come-in(d_1]; $\uparrow$ Sit-down(d_1) ; $\uparrow$ Talk(d_1)

b. $\lambda p[\exists x$ [Dean(x) $\wedge \forall y$ [[Dean(y) $\wedge$ Rel(y,e)] $\rightarrow$ y = x] $\wedge$ Come-in(x) $\wedge\{x/d_1\}^\vee p]]$; $\uparrow$ Sit-down(d_1,e) ; $\uparrow$ Talk(d_1,e)]

c. $\lambda q[\exists x$ [Dean(x) $\wedge \forall y$ [[Dean(y) $\wedge$ Rel(y,e)] $\rightarrow$ y = x] $\wedge$ Come-in(x) $\wedge$ Sit-down(x) $\wedge$ Talk(x) $\wedge\{x/d_1\}^\vee q]]$

The event variable remains free in these representations. I assume that it gets a value from the context of use. As before, I will often rely on less formal representations if no confusion can arise. In the representations given below, I will highlight the context dependency of definite NPs by writing **the** d_1 $N^e(d_1)$ as short-hand for [**E**d_1 N(d_1) ; [**A**d_2 [[N(d_2) ; $\uparrow$ *Rel*(d_2,e)] $\Rightarrow$ $d_2 = d_1$]]] with respect to a certain event e.

Unlike indefinite NPs, definite descriptions do not impose a novelty condition on their discourse referent. Van Eijck (1993) exploits this fact to account for the anaphoric use of definite descriptions as in (42):

(42) A customer entered. The woman sat down. She smiled
Ed_1 [$\uparrow$ Customer(d_1) ; $\uparrow$ Enter(d_1)] ; **the** d_2[$\uparrow$ $d_2 = d_1$; $\uparrow$ Woman^e(d_2)] ; $\uparrow$ Sit-down(d_2) ; $\uparrow$ Smile(d_2)

The identification of the two discourse referents d_1 and d_2 guarantees that we are talking about the same individual.

The treatment of definite NPs as context-dependent quantifiers also gives us a straightforward interpretation of the examples in (36), repeated here as (43):

(43) Quand Marie reçoit une lettre, elle jette souvent l'enveloppe
When Mary receives a letter, she often throws away the envelope
$\uparrow$ OFTEN ($\lambda e \exists x$ Letter(x) $\wedge$ Receive(mary,x,e)) ($\lambda e \exists x$ Letter(x) $\wedge$ Receive(mary,x,e) $\wedge$ the y Envelope^e(y,x) $\wedge$ Throw-away(mary,y,e))

Adverbs of quantification denote relations between sets of events as usual and definite NPs are not treated as variables, but as context-dependent quantifiers. The desired interpretation of (43) now falls out naturally, and we do not need a special rule of 'definite closure'. There is no reason to assume a difference in behavior between definite NPs in English or in Romance languages. Also, in neither of these examples would we intuitively characterize the definite NP as generic.

3.2 Definite generics

The interesting step is of course to see how this interpretation allows us to account for generic readings of definite NPs. I will treat such cases as an extension of examples like (45):

(45) When the king dies, he is usually succeeded by his eldest son
$\uparrow$ USUALLY (λe the x King^e(x) $\wedge$ Die(x,e))
(λe the y eldest Son^e(y,x) $\wedge$ Succeed(y,x,e))

The predicate *to die* in (45) is a once-only predicate, so the set of events in which the individual that is the unique king in a particular context dies is a singleton set of events. As a result, quantification over events in which the unique king in that context dies is equivalent to quantification over pairs $< x, e >$ of an individual x and an event e such that x is the unique king in e and x dies in e. If we can make sure that for every king there is a context with respect to which the uniqueness condition is satisfied, this boils down to quantification over kings that die. In the world as we know it, it seems not unreasonable to assume this. It is thus not surprising that the quantification expressed by (45) is felt to be 'about' dying kings.

Similar observations can be made about examples which concern individual-level predicates such as (46):

(46) When the king has blue eyes, he is often popular
    ↑ OFTEN (λe the x King^e(x) $\wedge$ Have-blue eyes(x,e)) (λe Popular(x,e))

Individual-level predicates are always once-only, so quantification over events is equivalent to quantification over pairs $< x, e >$ of the unique king x in the context who is such that he has blue eyes in e. Given that individual-level predicates are only loosely tied to particular occasions, and we assume that every king is unique with respect to a particular context, this boils down to quantification over blue-eyed kings.

Note that the context-dependent character of definite NPs remains strongly present, even if we embed the NP under quantification. Although it is reasonable to assume that for every individual involved we can find a context with respect to which the uniqueness condition is satisfied, the fact that there is such an additional condition accounts for the difference felt between (45) and (46) on the one hand and (47a, b) on the other hand, where no extra assumptions are necessary for the set of individuals that satisfy the common noun to become available:

(47) a. When a king dies, he is usually succeeded by his eldest son
    b. When a king has blue eyes, he is often popular

The contrast extends to plural generic NPs as in the following pair of sentences, discussed by Hawkins (1978):

(48) a. Generals usually get their way
    b. The generals usually get their way

In (48a), the most probable intention of the speaker is to refer to all generals. The indefinite can cover all those in existence and all those to be. The definite article in (48b) singles out the object mentioned against the background of a more inclusive whole, which could be something like 'officers' or 'people in government'. If there is no larger whole that is held in mind, the article is omitted.

As far as French is concerned, we note that the difference between the singular indefinite and the singular definite NP is preserved. However, given that French does not use the indefinite plural as a generic NP, the contrast between

(48a) and (b) disappears, and we only use the context-dependent definite NP to express generalizations:

(49) Le plus souvent, les généraux imposent leur volonté
Usually, the generals impose their will

It is not surprising that the context-dependent character of definite NPs is felt less strongly here. In fact, it seems easy to rely on a flexible notion of contextual relevance which allows every individual that satisfies the predicate to satisfy the uniqueness condition with respect to one context or another. Such a weak interpretation of the uniqueness condition blurs the distinction between definite and indefinite closure and allows French to get by without a generic indefinite plural. Arguably, such a weak notion of context dependency does not apply to English, because there we have the possibility of switching to an indefinite NP if we want to capture the full general reference of the NP. For both indefinite and definite NPs then, we can explain bound variable readings by adopting a number of reasonable pragmatic assumptions concerning the relation between individuals and events. In both cases, then, we can preserve a unified treatment of Q-adverbs as generalized quantifiers over events.

3.3 Definite disclosure

For those who wish to express bound variable readings as variables actually bound in the semantics, I will show that we can reflect this meaning effect in the semantics of definite NPs if we adopt suitable type shifting operations. By applying Partee's BE operator to indefinite NPs, we were able to recover the set corresponding to the common noun interpretation. As Partee (1987) points out, applying the BE operator to a definite NP gives us the singleton set of the unique individual satisfying the description if there is one, the empty set otherwise:

(50) a. $BE(THE_{sg}(P)) = P$ if $|P| \leq 1$
b. λx [king'(x) $\wedge \forall y$[king'(y) $\rightarrow$ y = x]]

That is, the predicative reading of a definite singular like *the king* as given in (50) is the same as the common noun, since both pick out the empty set if there is no king and a singleton set if there is exactly one king.

If we relax the uniqueness presuppositions to uniqueness-in-context, as we have argued for above, the type shifting operator BE gives us the set of pairs $< x, e >$ such that x is the unique x satisfying the definite description in e. Conjunction with a vacuous identity statement thus does not only give us existential disclosure, it also provides for 'definite disclosure'. (51a) is the DMG representation of the restriction on the quantifier. In (51b) this formula is dynamically conjoined with a vacuous identity statement. This reduces in the regular way to (51c):

(51) When the king has blue eyes, he is often popular

a. $[\lambda e \downarrow [\mathbf{E}d_1[\uparrow$ King(d_1) ; $\mathbf{A}d_2[\uparrow$ King(d_2) ; $\uparrow$ Rel$(d_2,e) \Rightarrow \uparrow d_2 = d_1]]$; $\uparrow$ Have-blue-eyes$(d_1,e)]]$

b. $[\lambda y \lambda e \downarrow [\uparrow \text{King}(y) \ ; \ \mathbf{A}d_2[\uparrow \text{King}(d_2) \ ; \uparrow \text{Rel}(d_2,e) \Rightarrow \uparrow d_2 = y]] \ ;$
$\uparrow \text{Have-blue-eyes}(y,e)]]$

c. $\lambda y \lambda e \ [\text{King}(y) \wedge \forall x \ [\text{King}(x) \wedge \text{Rel}(x,e) \rightarrow x = y] \wedge \text{HBE}(y,e)]$

d. $\lambda y \exists e \ [\text{King}(y) \wedge \forall x \ [\text{King}(x) \wedge \text{Rel}(x,e) \rightarrow x = y] \wedge \text{HBE}(y,e)]$

e. $\lambda y \ [\text{King}(y) \wedge \text{Have-blue-eyes}(y)]$

Conjoining the restrictor with a vacuous identity statement as in (51b) yields the set of pairs $< y, e >$ such that y is the unique king in the context and y has blue eyes in e (51c). Because the individual-level predicate carries a uniqueness presupposition, this is equivalent to quantification over the set of blue-eyed kings that are unique with respect to a certain context (51d). Assuming that such a context can be found for every king, we end up expressing quantification over blue-eyed kings (51e). The comparison of (51e) and (29e) makes it understandable why Krifka et al. (1992) claim that generic definite NPs in Romance can be considered as semantically indefinite NPs. Of course we can only infer (51d) from (51c) if we appeal to lexical-semantic properties of the predicate involved, and further reduction to (51e) is also dependent on assumptions about the real world, so even in this semantic approach we crucially need to appeal to pragmatic considerations in order to obtain the right set of individuals to quantify over.

In the light of these observations, I again conclude that there is no reason why we would not keep the semantics "clean" altogether and handle the generic reading of definite NPs as an effect of "quasi-binding" of individual variables along with quantification over events. The pragmatics can be said to simulate type shifting, so again we can loosely speak about definite disclosure to refer to cases in which quantification over events is interchangeable with quantification over individuals.

4 Conclusion

We may conclude that it is neither necessary nor desirable to account for the generic reading of indefinite NPs by treating them semantically as variables. The interpretation of indefinites as dynamic existential quantifiers allows us to treat *des* N as a regular indefinite as far as discourse and donkey anaphora are concerned. English and other Germanic languages have a preference for the use of indefinite NPs in characteristic generic sentences. A semantic operation of existential disclosure or a pragmatic simulation of it makes it possible to fix certain indefinite NPs as the 'topic', and to determine what the quantification is about. The NPs which undergo existential disclosure are the only ones which can be properly called generic NPs. These are the only bound variable readings left in our theory. This approach then accounts for the generic character of the indefinite NPs in (52a) and (52b):

(52) a. An Italian usually drinks wine at dinner
b. Italians usually drink wine at dinner
c. The Italian usually drinks wine at dinner
d. The Italians usually drink wine at dinner

Although a similar operation of definite disclosure can be used in certain cases to turn (52c) and (52d) into generic sentences, this is not the usual way to express characteristic predication in English.

Not all indefinite NPs in all languages undergo disclosure operations. French and other Romance languages do allow existential disclosure for singular indefinite NPs (53a), but indefinite plurals do not usually occur as topics of generic sentences (53b):

(53) a. Un Italien boit généralement du vin à table
b. *Des Italiens boivent généralement du vin à table
c. L'Italien boit généralement du vin à table
d. Les Italiens boivent généralement du vin à table

We have formulated a constraint on the operation of existential disclosure, and argued that it does not apply to *des* N. Instead, Romance languages seem to have a preference for the use of definite NPs for the expression of inductive generalizations. (53c) has about the same status in French and English, but (53d) is clearly the typical way to express characteristic predication. The difference between Germanic and Romance languages then reduces to a contrast between existential and definite disclosure.

We can speculate that there is an overall preference for the expression of genericity by means of indefinites rather than definites because the relation between the NP-denotation and the CN-denotation is more straightforward in the case of indefinites. Definite disclosure is dependent on uniqueness-in-context and is as such a more indirect way of associating sets of events with the correlating sets of individuals. We would then predict that, only if existential disclosure is somehow prohibited for independent reasons languages would use definite NPs as a general means to express genericity. Whether this speculation turns out to be on the right track or not, I hope the paper has made it clear that for the treatment of genericity there is no need to assume that indefinite and definite NPs translate as variables and that an event-based semantics of adverbs of quantification is a viable alternative to the unselective binding approach.

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Quantifiers in Pair-list Readings and the Non-uniformity of Quantification

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This paper addresses two main issues. First, it examines the role of quantifiers in pair-list readings. Second, placing the results in the context of current work on scope, it argues that "quantification" is a semantically diverse phenomenon.

The term PAIR-LIST reading will be applied to both types (1) and (2):

(1) What did every man read?
'For every man, what did he read?'

(2) What did six men read?
'For six men of your choice, what did each read?'

Type (1) will be referred to as a FIXED DOMAIN reading and type (2) as a CHOICE reading, when the distinction is necessary. The latter term covers type (3), too, which may or may not be regarded as a pair-list question:

(3) What did John read? Or, what did Mary read?

Pair-list readings arise when the interrogative contains a quantifier; the issue to be addressed is WHAT ROLE THIS QUANTIFIER PLAYS. The standard view is that the quantifier here does not have the same kind of quantificational force as in other, "normal" contexts; instead, it contributes A RESTRICTION ON THE DOMAIN OF THE QUESTION. I will argue that, at least in the case of the pair-list readings of complement interrogatives, QUANTIFICATION IS BOTH NECESSARY AND HARMLESS. The argument is based on empirical data concerning exactly what quantifiers support pair-list readings.

Section 1 shows that the dilemma of quantification versus domain restriction arises ONLY IN COMPLEMENT INTERROGATIVES. In matrix questions only universals support pair-list readings, whence the simplest domain restriction treatment suffices. Apparent examples with indefinites probably have cumulative (more precisely: distributed group) readings.

Section 2 argues that in the case of complements, the domain restriction treatment is inadequate for at least two independent reasons. One has to do with the fact that NOT ONLY UPWARD MONOTONIC quantifiers support pair-list readings, and the other with the derivation of "APPARENT SCOPE OUT" readings.

This establishes the need for quantification, so the question arises how the OBJECTIONS explicitly enlisted in the literature against quantification can be answered. Section 3 considers the de dicto reading of the quantifier's restriction, quantificational variability, and the absence of pair-list readings with <u>whether</u>-questions, and argues that they need not militate against quantification here.

Section 4 turns to the most interesting objection, which is conceptual. Both Groenendijk & Stokhof and Chierchia argue that since quantifiers do not behave uniformly in this context, the creation of pair-list readings must involve a process other than standard quantification. This argument rests on the assumption that in other contexts, quantifiers do behave uniformly. I will argue that this assumption is empirically false, and conclude that "STANDARD QUANTIFICATION" INVOLVES A VARIETY OF DISTINCT, SEMANTICALLY CONDITIONED PROCESSES. That is, the harmony in the universe that quantification into interrogatives would allegedly disrupt does not exist anyway.

I will take Groenendijk & Stokhof (1984) [henceforth G&S] as my point of departure. The proposals in Higginbotham (1991) and Chierchia (1993) appear to be equivalent to G&S in the respects I am concerned with.

1. No need for quantification into matrix questions

The first, empirical point to be made is that in weighing the arguments for and against quantification into interrogatives, we have to focus on complement contexts. It turns out that, contrary to popular belief, matrix pair-list questions are supported only by universals. But these cases, as G&S point out, can be accommodated using the simplest possible domain restriction terms, either (6a) or (6b), analogously to multiple interrogation:

(4) Who did every dog bite? Fido bit John, King bit Bill, ...

(5) Who did which dog bite? Fido bit John, King bit Bill, ...

(6) a. $\underline{\lambda x \in dog(a)}\ \lambda y[bit(a)(x,y)]$
b. $\lambda i[\underline{\lambda x \in dog(a)}\ \lambda y[bit(a)(x,y)] = \underline{\lambda x \in dog(a)}\ \lambda y[bit(i)(x,y)]]$

In (6) the quantifier ‖every dog‖ contributes its unique witness, the set of dogs, to serve as a restriction on lambda abstraction. (6a) asks for the set of pairs consisting of a dog and a person it bit in the actual world a. This formulation is modelled after Hausser's (1979) and, as G&S (1989) note, affords a very transparent relation between questions and linguistic answers. (6b) is G&S's own general formulation; it seeks to identify the unique (smallest) world i in which the dog/bitten person pairs are the same as in a.

G&S point out that the same treatment does not carry over to questions that do not have a fixed domain, e.g.,

(7) Who did two dogs bite? $Answer_1$: Fido bit X and King bit Y.
$Answer_2$: Spot bit Z and King bit Y.

Here the answerer first needs to make a choice as to what question he wishes to answer (hence the name choice question). This amounts to choosing one of the witnesses of ‖two dogs‖. To accommodate this step, (7) must denote a generalized quantifier over individual questions:

(8) $\lambda P \exists W[witness\ (W, \|two\ dogs\|)\ \&$
$P(\lambda i[\lambda x \in W \lambda y[bit(a)(x,y)] = \lambda x \in W \lambda y[bit(i)(x,y)]])]$

My claim in 1.1 and 1.2 will be that the need for this kind of interpretation may only arise in complement interrogatives. What is the significance of this claim?

The most important consequence is purely data-minded. As I will point out in 2.1, exactly what quantifiers support pair-list readings has theoretical significance. But then we must make sure that we are looking at a context that provides representative data as to what these quantifiers are.

Secondly, a basic objection against analyzing pair-list readings in terms of standard quantification into interrogatives may be that it presupposes that interrogatives, just like declaratives, denote propositions. (Quantification is defined for expressions of type t.) As (6b) shows, this is in fact the analysis that G&S adopt; however, this is not the most attractive feature of their theory and is not shared by all others. To what extent can the treatment of pair-list readings be made independent of this feature?

Notice that this problem is relevant only in the matrix. It is quite natural to interpret all complement interrogatives as generalized quantifiers, irrespective of whether the presence of an indefinite forces this. Now suppose we prefer a non-propositional analysis of interrogatives, expressed in terms of some ? quantifier. What we get then is roughly as follows:

(9) It is known who John visited. λP[P(?x[visit(john,x)])](known)

Here the fact that the expression ?x.φ is not of type t is made irrelevant by the fact that P(?x.φ) is of type t anyway. Thus quantification, if otherwise motivated, is type-wise unproblematic:

(10) It is known who everyone visited. λP∀y[P(?x[visit(y,x)])](known)

On the other hand, in the matrix one can have (6a) or one's favorite ?-analysis:

(11) Who did every dog visit? ?y∈dog ?x[visit(y,x)]

This means that once we know that quantification into interrogatives does not arise in the matrix, we are free to combine it with a non-propositional analysis.

It may be unsettling to assign so divergent analyses to matrix and complement cases but, in fact, they may have to diverge in even further respects. Recall May's (1985) claim that a pair-list reading is possible only if the QP c-commands the trace of the fronted wh-phrase. May subsumes this under the Empty Category Principle, and Chierchia (1993), under Weak Cross-over. There have always been doubts concerning the predicted contrasts in the linguistic folklore; see also Williams (1988). T. Stowell (p.c.) judges, quite specifically, that matrix pair-list readings are supported only by a subject QP (and not e.g. by a direct object QP that c-commands an indirect object wh-trace), and in complements there are no contrasts at all. If Stowell is correct, even the syntax of matrix and complement pair-list readings must diverge significantly.

Finally, pair-list readings supported by indefinites have a very different intuitive status in the complement and in the matrix. A sentence like John found out where more than five friends got their degrees does not involve any "choice." It means that there are more than five friends about whom John found out something, that is, it involves simple quantification.

1.1 Choice questions with indefinites

The basic observation that indefinites only support pair-list readings in complements was made in the course of joint work with Frans Zwarts in 1992. A study by St.John (1993) confirmed this and revealed the significance of cumulative readings; Doetjes (1993) independently made consonant suggestions. The more detailed data the present paper rests on come from ongoing field work, for assistance with which I thank S. Spellmire.

Consider first (12) versus (13):

(12) Who / Which boys / Which boy did every dog bite?
- Who: ok Fido bit X, Spot bit Y, ...
- Which boys: ok Fido bit X, Spot bit Y, ...
- Which boy: % Fido bit X, Spot bit Y, ...

(13) Who / Which boys / Which boy did more than two dogs bite?
- Who: * Fido bit X, Spot bit Y,...
- Which boys: * Fido bit X, Spot bit Y,...
- Which boy: * Fido bit X, Spot bit Y,...

Although the availability of pair-list answers to (12) with every dog seems more restricted than is acknowledged in the literature (see the %), it clearly contrasts with (13) with more than two dogs, which no speaker is tempted to answer with a list of pairs. Similar to (13) are all "modified numerals," e.g. two or more dogs, exactly two dogs, fewer than two dogs, many/few dogs. As to Who did at least two dogs bite?, some speakers are willing to answer it with a pair-list, but this is probably a pragmatic "mention some" effect induced by a non-logical use of at least. The reason to believe this is that (i) logically equivalent two or more dogs never elicits pair-list answers, and (ii) speakers who answer the at least two dogs question with a pair-list tend pick just two dogs, rather than three or eleven.

The belief that choice questions do arise in the matrix may be based on the fact that the literature typically considers only "bare numeral indefinites" like two dogs. But do these support genuine choice questions? I claim that they do not. The acceptable examples, as in (7), have cumulative readings in the sense of Krifka (1991) and Srivastav (1991), who discuss questions with definites, e.g.,

(14)	Who	ok Fido bit X and Spot bit Y.
	Which boys did the dogs bite?	ok Fido bit X and Spot bit Y.
	Which boy	* Fido bit X and Spot bit Y.

They argue that the "real answer" here would be The dogs, Fido and Spot, bit X and Y (between them), and the apparent pair-list answers are just more cooperative ways of spelling out how exactly the bitings were distributed. (The same basic observation had been made in Szabolcsi (1983:128), in response to Haïk (1982).) Crucial here is the fact that the expressly singular wh-phrase, which boy does not allow for such an answer. I suggest that the indefinites data in (15) are to be interpreted in the same way:

(15)	Who	ok Fido bit X and King bit Y = They bit X and Y
	Which boys did two dogs bite?	ok Fido bit X and King bit Y = They bit X and Y
	Which boy	?? Fido bit X and King bit Y ≠ They bit X and Y

The claim that two dogs can indeed support such cumulative readings is confirmed by the fact that plain X and Y is itself an acceptable answer to Who/Which boys did two dogs bite? in the cumulative situation where one dog bit X and the other bit Y.

As to the absence of a genuine pair-list reading, the need to choose a plural or at least not overtly singular wh-phrase is unfortunately of less diagnostic value than Krifka and Srivastav think, since many speakers of English reject the singular even with fixed domain readings. However, I have found reliable informants who do accept the singular in the case of every dog (hence the % in (12)) and nevertheless reject it in the case of the dogs and two dogs.

Why is the cumulative option unavailable to modified numeral indefinites, cf. (13)? The term "cumulative" may be a little misleading here, since Scha (1981) introduced it in connection with cardinalities, and indeed, More than two dogs bit fewer than six boys between them is fine. The readings in (14) and (15) should rather be called "distributed group" readings. I suggest that more than two dogs and its brothers do not participate in such readings because, unlike the/two dogs, they are not potential group denoters in the relevant sense. This accords with Kamp & Reyle's (1993) observations; for further discussion, see section 4.

Notice now that the asymmetries between universals and indefinites (as well as the mysterious unacceptability of singular wh-phrases) vanish in complements:

(16) John found out which boy every dog bit. cf. (12)
'John found out about every dog which boy it bit'

(17) John found out which boy more than two dogs bit. cf. (13)
'John found out about more than two dogs which boy each bit'

In other words, the absence of choice readings is restricted to the matrix.

Why should choice readings be unavailable in the matrix? A straightforward explanation may be derived from G&S's argument that choice readings cannot denote less than generalized quantifiers, cf. (8). It seems that generalized quantifiers are genuinely looking for a property, in combination with which they can yield a truth value, hence their natural habitat is the "argument" position. In other words, they are just not good denotations for matrix sentences.

1.2 Supporting and potentially problematic data

The assumption that choice readings are unavailable in the matrix because they must denote generalized quantifiers is supported by the fact that question disjunctions, which are also choice questions though not pair-list questions, exhibit comparable matrix vs. complement asymmetries.

Question disjunctions that illustrate the choice reading in the literature invariably come in an inter-sentential format, as in (3). If this were an irrelevant detail, the <u>or</u> connecting the two sentences could easily be moved into intra-sentential position. But it cannot:

(18) a. Who did you marry? <u>Or</u>, where do you live?
b.?? Who did you marry <u>or</u> where do you live?

This suggests that the <u>or</u> in (18a) does not really offer a choice but, instead, is an idiomatic device that allows one to cancel the first question and replace it with the second. This idiomatic character is corroborated by the fact that the Hungarian equivalent of (18a) is entirely unacceptable unless <u>inkább</u> 'rather, instead' is added; something that we do not expect if the connective acts as a standard Boolean operator. The marginality of (18b) indicates, then, that questions cannot be directly disjoined.

Just as pair-list readings with indefinites are perfect in complements, disjunction becomes impeccable in complements, too. But the claim that questions cannot be directly disjoined is confirmed by the fact that (19) only has a wide scope <u>or</u> (distributive) interpretation obtained by lifting both disjuncts:

(19) John found out who you married <u>or</u> where you live.
'John found out who you married or found out where you live'
* 'John found out (who you married or where you live)'

Naturally, for this distinction to make sense, the two readings must be distinct. According to G&S (1989), <u>know wh-ϕ and/or wh-ψ</u> is logically equivalent to <u>know wh-ϕ and/or know wh-ψ</u>. I disagree with this in the case of <u>or</u>. Take (20):

(20) a. Bill knows where John lives or knows who Sue married.

b. Bill knows (where John lives or who Sue married).

If Bill never heard of Sue, (20a) may be true but (20b), if grammatical at all, seems implausible.

The claim that interrogatives must be first lifted to become disjoinable is corroborated by data from Hungarian and Korean (S. Nam, p.c.). In these languages, even wh-complements are introduced by a subordinator morpheme. The Hungarian subordinator is <u>hogy</u>, and the counterpart of (19) is unacceptable unless both disjuncts contain a <u>hogy</u>:

(21) János megtudta, hogy kit vettél feleségül vagy *(hogy) hol laksz.
John found-out <u>that</u> whom you married <u>or</u> *(<u>that</u>) where you live

It is important to note now that all these contrasts are semantic, not (logico-) syntactic: they disappear if disjunction is replaced by conjunction:

(18)' a. Who did you marry? <u>And</u>, where do you live?
b. Who did you marry <u>and</u> where do you live?

(21)' János megtudta, hogy kit vettél feleségül és (hogy) hol laksz.
John found-out <u>that</u> whom you married <u>and</u> (<u>that</u>) where you live

The crucial difference is, of course, that question conjunctions may, but need not, denote generalized quantifiers: they have a unique complete, true answer.

In sum, question disjunction supports the explanation I am offering for the observed absence of choice readings from the matrix. On the other hand, I am aware of three types of data that may be problematic. I briefly review them below and indicate why I think they need not be disastrous.

First, as the ?? in (15) indicates, a genuine pair-list reading is to some extent available with <u>two dogs</u> (for some reason, much less so in Dutch). I suspect that <u>two dogs</u> here is interpreted specifically (as a filter, cf. Ben-Shalom (1993)). Then we really have a fixed domain question, not a choice question.

Second, disjunctive questions like (22) allow for a choice-style answer:

(22) Who did Fido or King bite? ok King bit John.

I suggest that this is not a choice reading but, rather, a "mention some" answer to the question 'Who is such that either Fido or King bit him?,' presented in a co-operatively explicit format.

Third, all the observations made in connection with matrix questions are replicated by complements of <u>wonder</u>:

(23) a. John wonders who every dog bit. cf. (12)
b.% John wonders which boy every dog bit. cf. (12)
c.?? John wonders which boy more than two dogs bit. cf. (13)
d.?? John wonders where you live or who you married. cf. (19)
'John would be happy to know either'

This paradigm does not follow from the "type infelicity" story, but a compatible informal explanation may be suggested. On G&S's theory, these sentences are interpreted as 'John stands in the wonder-relation with the question...,' as opposed to 'John found out the answer to the question...' Apparently, you cannot stand in the wonder-relation to a question which, not being a possible matrix question, cannot be asked on its own right. (G. Carlson (p.c.) points out to me another respect in which <u>wonder</u>-complements are like matrix clauses: in some AmE. dialects they exhibit inversion (together with sequence of tenses).)

To conclude, the claim that matrix choice questions do not exist because matrix clauses do not denote generalized quantifiers seems tenable. Note, however, that the rest of the paper relies on the facts themselves, not on their proposed explanation.

2. The necessity of quantification into complement interrogatives
2.1 Domain restriction and monotonicity

In what follows I will assume that all complement interrogatives denote generalized quantifiers. The question, then, is whether the domain restriction schema underlying (8) is an adequate representation of complement pair-list readings:

(24) a. ... who QP bit

b. $\lambda P \exists W[\text{witness } (W, \|QP\|)\ \&$

$P(\lambda i[\lambda x \in W \lambda y[\text{bit}(a)(x,y)] = \lambda x \in W \lambda y[\text{bit}(i)(x,y)]])]$

$=_{abbr}\ \lambda P \exists W[\text{witness } (W, \|QP\|)\ \&\ P(\text{which } x \in W \text{ bit whom})]$

I argue that it is not adequate, for at least two independent reasons. The first has to do with monotonicity. The second has to do with "apparent scope out" readings, to be discussed in 2.2.

The simple point to be made in this subsection is that domain restriction requires upward monotonicity. Why? "Domain restriction" means that we pick a set and restrict our attention to its members, ignoring whatever happens outside. But we can only safely do so if that set is determined by an increasing quantifier. To illustrate with non-interrogative examples,

(25)a. <u>(At least) two men</u> walk = There is a set of (at least) two men who walk

b. <u>Exactly two men</u> walk ≠ There is a set of exactly two men who walk

c. <u>Less than two men</u> walk ≠ There is a set of less than two men who walk

The schema in (24b) faces exactly the same problem as the paraphrases in (25). For instance, if P is replaced by <u>John knows</u>, we get that there is a witness W of QP about whose members John knows who they bit, ignoring whatever else John knows. (24b) misinterprets any sentence in which the QP inducing the pair-list reading is not upward monotonic.

Thus, at this point the empirical question of exactly what quantifiers support pair-list readings becomes crucial.

It is agreed in the literature that decreasing quantifiers do not support pair-list readings; the standard example is like (26):

(26) Who did no dog bite?

* 'For no dog, tell me who it bit – that is, remain silent'

If this judgment is replicated in the complement case (and it seems it is), decreasing quantifiers are no threat to (24b). But no one ever considers non-monotonic quantifiers, although their theoretical significance is exactly the same.

One type of context I used to elicit the relevant judgments is as follows. We are in the business of finding out how dangerous each neighborhood dog is and get together to compare notes. This context simply makes the competing non-pair-list reading of the complement irrelevant, without being either pragmatically or syntactically too special to produce representative judgments. The results I obtained are as follows:

(27) a. I found out who three dogs bit.
b. I did a lot better! I found out who more than five dogs bit!
c. John is not here but I have glanced at his list, and I estimate that he found out who more than five but certainly fewer than ten dogs bit.
d. And I know that Judy found out who exactly four dogs bit.
e.?? Bill is very lazy: he only found out who at most three dogs bit.
f.* Mary is even worse: she found out who no dog bit.
g. Don't worry; I think we now know who every dog bit.

That is, in this context non-monotonic examples (27c,d) are as acceptable as upward monotonic ones. On the other hand, no N remains excluded, although some speakers are tempted to accept at most n, which is similarly decreasing but has non-empty witnesses. This last nuance may be relevant in the precise description of the data but does not choose between types of analyses, so I will ignore it. The significant point is that not only increasing quantifiers support pair-list readings.

The conclusion is that (24b) needs to be amended. I will consider three alternatives. The first, (28) just adds an ad hoc maximality condition to (24b), so that it will not go wrong if QP is not upward monotonic.

(28) $$\lambda P \exists W[\text{witness}(W, \|QP\|)\ \ \&\ \ P(\text{which } x \in W \text{ bit whom})$$
$$\&\ \ \forall x[x \notin W \rightarrow \neg P(\text{whom } x \text{ bit})]]$$

The second version, (29) departs from this most radically: it is standard quantification, assigning wide scope to Q dogs over the wh-phrase.

(29) $$\lambda P Q x[\text{dog}(x), P(\text{which person } y[x \text{ bit } y])]$$

The third version, (30) is an interesting intermediate case. If we read the original (24b) as a noble, though empirically incorrect, attempt to express that only increasing quantifiers support pair-list readings, then (30) just expresses, in the same spirit, that in fact increasing and non-monotonic quantifiers do so. This is how the first line works. QP is required to have a non-empty minimal witness B.[1] But we cannot stick with B: the minimal witnesses of exactly three dogs are the same as those of three or more dogs and more than two dogs, but sentences containing these QPs are not synonymous. We must be allowed to pick an appropriately big "enlargement" A of B to do the real work:

(30) $$\lambda P \exists A \exists B[\text{non-}\varnothing \text{ minimal witness}(B, \|QP\|)\ \&\ \text{witness}(A, \|QP\|)\ \&\ B \subseteq A$$
$$\&\ \ \forall x[P(\text{whom } x \text{ bit}) \text{ iff } x \in A]]$$

The second line then ensures that all and only the members of A count. At first sight this, too, seems like an innocent improvement over (28): the maximality condition is no longer added like an afterthought. But the new formulation makes a crucial difference. In (28), both reference to the relevant witness and universal quantification over its members took place inside the argument of the property variable P (cf. which $x \in W$); in (30), both take place outside P. This has the consequence that (30) is every bit as "quantificational" as (29) is.

General methodological considerations aside, is there empirical reason to prefer any of these alternatives? In 2.2 I argue that there is.

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1. This also excludes non-continuous quantifiers with a decreasing component, e.g. fewer than two or more than six dogs. Datawise this does not seem problematic.

2.2 "Apparent scope out" phenomena

It is generally agreed that whatever rule assigns scope to QPs like <u>every student</u>, it operates within the boundaries of one clause. A typical example is (31):

(31) Some librarian or other found out that every student needed help.
* 'every > some'

It is striking, then, that a comparable reading of (32) is entirely natural:

(32) Some librarian or other found out which book every student needed.
'every > some'

Should we allow <u>every student</u> to scope out of its own clause? Moltmann & Szabolcsi (1993) argue that the answer is No. It is proposed that the apparent scope out reading arises when (i) the complement clause has a pair-list reading and (ii) it is assigned scope over the matrix subject. This latter is of course a clause-internal step. That is, the derivation is not (33) but (34):

(33) * [every student]$_i$ [some librarian found out which book x_i needed]

(34) [$_{pair\text{-}list}$ which book every student needed]$_i$ [some librarian found out v_i]

The question is, then, what formal interpretation the pair-list reading must have for (34) to yield the "apparent scope out" effect. Let's see. In (35) through (37) I quantify (28) through (30) into <u>some librarian found out v_i</u>:

(35) $\lambda P \exists W[\text{min.witness}(W, \|\text{every student}\|)$ &
$P(\text{which } x \in W \text{ needs which book})$ & MAXIMALITY$]$
$(\lambda v[\exists z[\text{librarian}(z)$ & $\text{found-out}(z, v)]]) =$
$\exists W[\text{min.witness}(W, \|\text{every student}\|)$ &
$\exists z[\text{librarian}(z)$ & $\text{found-out}(z, \text{which } x \in W \text{ needs which book})]$ &
$\forall x[x \notin W \rightarrow \neg\exists z[\text{librarian}(z)$ & $\text{found-out}(z, \text{which book } x \text{ needs})]]]]$

(36) $\lambda P \forall x[\text{student}(x) \rightarrow P(\text{which book } y[x \text{ needs } y])]$
$(\lambda v[\exists z[\text{librarian}(z)$ & $\text{found-out}(z, v)]]) =$
$\forall x[\text{student}(x) \rightarrow \exists z[\text{librarian}(z)$ & $\text{found-out}(z, \text{which book } y[x \text{ needs } y])]]$

(37) $\lambda P \exists A \exists B[\text{non-}\varnothing \text{ min.witness } (B, \|\text{every st.}\|)$ & $\text{witness}(A, \|\text{every st.}\|)$
& $B \subseteq A$ & $\forall x[P(\text{which book } x \text{ needs}) \text{ iff } x \in A]]$
$(\lambda v[\exists z[\text{librarian}(z)$ & $\text{found-out}(z, v)]]) =$
$\exists A \exists B[\text{non-}\varnothing \text{ min.witness } (B, \|\text{every st.}\|)$ & $\text{witness}(A, \|\text{every st.}\|)$
& $B \subseteq A$ &
$\forall x[\exists z[\text{librarian}(z)$ & $\text{found-out}(z, \text{which book } x \text{ needs})] \text{ iff } x \in A]]$

Recall that (28) is G&S's original domain restriction interpretation of the pair-list reading, supplemented by an ad hoc maximality condition to take care of non-monotonic QPs. (35) shows that quantifying (28) into the matrix clause does not make the librarians vary with the students. It is easy to see why: as was mentioned in 2.1, in (28) all quantificational action takes place inside the argument of P that matrix material will replace. Thus matrix and complement quantifiers cannot interact scopally.

On the other hand, both (29) and (30) give the desired result: the librarians vary with the students. This confirms that they are variations on the same quantificational theme. Also, we now have an independent argument for preferring a quantificational analysis over (28).

(How shall we choose between (29) and (30)? As it stands, (29), just like (28), presupposes that the failure of (some or all) decreasing QPs to support pair-list readings has an independent explanation. G&S and Higginbotham (1991) offer such an explanation in pragmatic terms: a question that asks you to remain silent is not felicitous (see (26)). This explanation does not extend to complement cases like (27e,f): both would make perfect sense. Likewise, it does not account for Moltmann's (1992) and Schein's (1994) observation that parallel readings are absent from other wh-constructions:

(38) a. John is taller than OP no other student is. [OP = how tall]
* 'John isn't taller than any other student'
b. John read what no student wrote.
* 'John didn't read any student's writing'

Moltmann (1992) proposes that the reason is that decreasing quantifiers do not take inverse scope. But "modified numeral indefinites" do not take inverse scope, either (see section 4) and they nevertheless support pair-list readings. Pending a general syntactic or semantic explanation, we may for the time being simply build the restriction into the pair-list schema itself, as in (30).)

3. Specific empirical objections to quantification into interrogatives

Section 2 will have established the need for quantifying into interrogatives. But G&S and Chierchia did not merely propose a domain restriction theory; they also argued explicitly against quantification. The conceptual objections pertain to the uniformity of quantification and will be discussed in detail in section 4. The present section attempts to briefly comment on issues that arise in connection with specific empirical properties of pair-list readings: the de dicto reading of the quantifier's restriction, quantificational variability, and the absence of pair-list - readings with whether-questions. I wish to thank U. Lahiri and F. Moltmann for discussions on these matters.

One important reason why G&S object to Karttunen's (1977) treatment of pair-list readings in terms of quantification into interrogatives is that this does not account for the fact that the common noun part of the QP is interpreted "de dicto." That is, G&S claim that unless the QP is scoped out on its own (which they always allow!), it is not sufficient in (39) for John to know about every criminal which candy he craves; he must also know that the guys are criminals:

(39) John knows which candy every criminal craves for.

This objection carries over to my (29) and (30) in the following sense. If the complement clause is interpreted as an extensional object of know, know is part of what replaces the variable P. Thus reference to the common noun or witness set of QP is made only outside the scope of know. It is of course also possible to interpret the whole generalized quantifier that stands for the complement as an intensional object, in which case the problem does not arise.

Now, it appears to me that G&S's own stronger claim is in fact too strong, in two respects. First, compare (39) with (40):

(40) John has just discovered which candy every criminal craves for.

This need not mean that John has just discovered that the guys are criminals, although he may need to be independently aware of them being criminals. That is, there is a difference between presupposed awareness and an entailment expressible strictly in terms of whatever the matrix verb happens to be (here: discover). The fact that G&S consistently use know in their examples masks this difference.

Second, even presupposed awareness is restricted to cases when the matrix subject is an intelligent being acting knowingly. Consider:

(41) This experiment will reveal which candy every criminal craves for.

Here the experiment will neither reveal that the guys are criminals, nor does it have any awareness of this. Presumably the same holds for John in case he informs us about something inadvertently, in an indirect way.

Third, it seems that on the "varying librarians" reading (which I argued involves quantifying the whole complement, not merely its QP, into the matrix clause) librarians need not be aware that the person whose book needs they found out about is a student:

(42) Some librarian or other found out which book every student needed.

All in all, it appears that the data do not compel us to adopt G&S's specific formulation. It is not my aim in this paper to develop an alternative proposal. Let me assume that some theory of presuppositions and intensionality will be able to handle the facts that are undoubtedly there.

Another objection may be derived from a point made in Chierchia (1993). He mentions that one important advantage of his treatment of pair-list readings, which is in many respects like G&S's, is that Lahiri's (1991) proposal for the treatment of the "quantificational variability effect" straightforwardly extends to it. To recap, Lahiri interprets (43) roughly as follows:

(43) Mary knows, for the most part, which students came.
'Mary knows most parts of the complete answer to the question which students came = For most students, Mary knows whether they came'

Chierchia (1993: 218) comments on the extension, "In a situation with three people a, b, and c, where a loves b, b loves c, and c loves a, if Mary knows that a loves b and b loves c, sentence [44] would be true."

(44) Mary knows, for the most part, who everyone loves.

On the other hand, if the pair-list reading is supported by an indefinite, there is no unique complete answer, so Lahiri's algorithm -- correctly -- cannot apply.

How can this result be possibly replicated if the pair-list reading is derived using quantification? Here life is easy, because G&S's (1993) general proposal concerning quantificational variability incorporates a trick that does the job. G&S point out that in dynamic semantics $\forall x[\phi \rightarrow \psi]$ is equivalent to $\exists x\phi \rightarrow \psi$, even if x is free in ψ. $\exists x\phi \rightarrow \psi$ can then be subjected to existential disclosure, which makes x available for further quantification. Thus most can effectively quantify over the variable originally bound by the universal. (Fortunately, these equivalences do not hold if we replace every with, say, more than five.) So, (44) will be directly interpreted as (45):

(45) 'For most persons, Mary knows (completely) who that person loves'

With the main job thus done, let us ask whether this result is exactly the same as Lahiri's and Chierchia's. This is not easy to answer because they do not spell out what count as parts of a pair-list answer, but it seems they would quantify over pairs, as in (46), not over loving persons:

(46) 'For most $person_1$/$person_2$ pairs, Mary knows whether p_1 loves p_2'

My judgment is that models that distinguish the two interpretations show that in fact (45) is correct. This means that (if the assumptions of dynamic semantics are generally tenable) the quantificational approach to pair-list readings can be married with a fully satisfactory treatment of quantificational variability.

Finally, as is noted already in Karttunen and Peters (1980), quantifying into interrogatives incorrectly predicts that whether-questions have pair-list readings:

(47) John found out whether everyone left.
* 'John found out about everyone whether he left'

It is argued in Moltmann & Szabolcsi (1993) that this is really part of a bigger problem of why quantification into clauses lacking a variable binding operator is not attested:

(48) a. whether every girl walks * $\lambda P \forall x[girl(x) \rightarrow P(\text{whether } x \text{ walks})]$
b. that every girl walks * $\lambda P \forall x[girl(x) \rightarrow P(\text{that } x \text{ walks})]$

How do we know that (48b) is not available? If it were, then, assuming that complement clauses can be quantified into the matrix, as is suggested in 2.2, quantifiers in the complement would systematically appear to scope over quantifiers in the matrix. But this is not the case, cf. (31). M&Sz offer preliminary speculations, but this particular problem remains largely open for the time being.

To summarize, I have argued that quantification into interrogatives is not necessary in matrix clauses but it is in complements. The need for quantification was motivated by monotonicity and by "apparent scope out." I also discussed, and eliminated, various objections to quantification, in sections 1 and 3. Having completed this case study, in the rest of the paper I wish to place the phenomenon into a more general perspective.

4. Scope taking: a non-unitary phenomenon

The fact that not all generalized quantifiers support pair-list readings and that different cases require logical analyses of different complexity is well-known from the literature; my own findings confirm this global picture, although many of the details are revised. Let us now shift our attention from the details and ask, How shall we evaluate this diversity?

Both G&S and Chierchia (1993) make important comments on diversity. Since generalized quantifiers do not contribute to pair-list readings in a uniform manner, their contribution should not be described as quantification. It should be placed under some different umbrella. (Their proposal is domain restriction.) That is, diversity provides an independent conceptual objection to quantification into interrogatives. This objection remains in force, it would seem, even in the face of the arguments I have accumulated. (Those arguments may perhaps prompt one to look for an umbrella other than domain restriction.)

Rather than engaging in a terminological debate, the issue that I wish to address is whether this objection is well-founded on its own. I believe that this issue is substantial and important.

The objection obviously rests on the assumption that in the "standard" cases, quantifiers do behave uniformly. What I would like to show in this section is that this assumption is empirically false; "standard quantification" must involve a variety of distinct, semantically conditioned processes. I offer empirical data that illustrate the claim, as well as some preliminary considerations as to what the processes might be.

4.1 Quantifiers do not behave alike

Standard theories of scope are semantically blind. They employ a single logico-syntactic rule of scope assignment (quantifying in, Quantifier Raising, storage, or type change, etc.) which roughly speaking "prefixes" an expression α to a domain D and thereby assigns scope to it over D, irrespective of what α means, and irrespective of what operator β may occur in D:

(49) The semantically blind rule of scope assignment:
α $[_D \ldots \beta \ldots]$ => α scopes over β

There are two basic ways in which (49) turns out to be incorrect: the resulting interpretation may be incoherent, or the resulting interpretation may be coherent but not available for the string it is assigned to.

To illustrate the first case, take a version of (49) that is assumed to operate in surface syntax: WH-fronting. In a sizable class of cases, called "weak island violations," this rule yields unacceptable results. For instance:

(50) a. Who do you think that I mentioned this rumor to?
b. Who do you regret that I mentioned this rumor to?
c. Who didn't you mention this rumor to?

(51) a. How do you think I solved this problem?
b.* How do you regret that I solved this problem?
c.* How didn't you solve this problem?

(52) a. Who do you think that I got the ring I am wearing from?
b.* Who do you regret that I got the ring I am wearing from?
c.* Who didn't you get the ring that you are wearing from?

Szabolcsi & Zwarts (1993) argue that the violation is semantic in nature. <u>How</u> in (51b,c) and <u>who</u> in (52b,c) ought to scope over domains D that they are unable to. As will be explained in 4.3, the reason is that they range over elements of join semi-lattices for which meet and complement are not defined. For the moment, let it suffice that the $\alpha > \beta$ scope relation, pace (49), is not semantically unconstrained.

To illustrate the second case, consider the fact that quantifiers in English often scope over operators that are higher in the surface syntactic hierarchy. These cases are attributed to the covert operation of (49). This account predicts, however, that all quantifiers α interact uniformly with all operators β. But they do not. E.g., some but not all direct objects can scope over the subject (53), and some but not all can scope over negation (54):

(53) a. Two men saw every film.
'every N > two N'
b. Two men saw few films.
* 'few N > two N'

(54) a. John didn't see many films.
'many N > not'
b. John didn't see few films.
* 'few N > not'

It will turn out that these contrasts have to do with semantics, too; however, they pertain to the syntax/semantics interface, rather than pure semantics. That is, the starred examples are not incoherent; simply, the given form cannot carry the intended meaning. Proof is that the same α's are able to scope over the same β's in English when they are originally higher in syntactic structure (55) or when they acquire such a higher position via overt fronting (56):

(55) a. Few men saw two films.
'few N > two N'
b. Few men didn't call John.
'few N > not'

(56) Few men did no one / every woman / two women call.
'few N > no N / every N / two N'

I do not find it desirable to develop a theory that maintains the omnivorous rule (49) and supplements it with a variety of filters on its overt or covert application. Such a strategy would simply not be explanatory. Instead, I will argue for an approach that is as constructive as possible. The assumption is that "quantification" involves a variety of distinct, semantically conditioned processes. Each kind of expression participates in those processes that suit its particular semantic properties. Thus the heuristic principle is this:[2]

(57) What range of quantifiers actually participates in a given process is suggestive of exactly what that process consists in.

In the light of these general observations, the fact that quantifiers contribute to pair-list readings in more than one way appears more like the norm than the exception.

4.2 "Compute" and "look up"

Szabolcsi & Zwarts (1993) propose that there are two basic scopal mechanisms: "compute" and "look up." I will adopt these informal terms and extend their coverage. I begin by giving some global impression.

Take the following examples:

(58) Who doesn't walk?

(59) a. More than two men walk.
b. (These) two men walk.

(58) can be answered in two quite different ways. One is to form the set of walkers, take its complement, and list its elements. Another is to check every individual for the 'not walk' property and list the ones that have it. The first option is "compute," the second is "look up." Now consider (59). The truth of (59a) is

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2. The reasoning applied to branching quantification in Beghelli & Ben-Shalom & Szabolcsi (1993) illustrates the same principle.

established by forming the set of walkers and counting the men in it. The truth of (59b) is established by taking these two men and checking whether 'walk' can be predicated of them, distributively or collectively. The first procedure is "compute," the second is "look-up."

In the spirit of (57), there is a correlation between how compute and look-up work and what quantifiers they are available to.

Compute meticulously performs every possible operation on the denotations of the participating expressions. Consequently, scope taking is shown to be semantically sensitive: as will be discussed in connection with (60), it may be the case that not all the requisite operations can be performed.

Look-up checks a designated domain for some, potentially complex, property. One consequence is that, while it cannot be incoherent, either, look-up is only available to scope takers that range over, or define, an appropriate domain. The data I have investigated so far indicate that scope takers that denote a principal filter and/or a group are prototypically the ones that define an appropriate domain for look-up.

We may note that from this perspective, pair-list interpretations involving domain restriction are also instances of look-up. Thus it may not be surprising that such an analysis turns out to be tenable only when the quantifier denotes a principal filter (every man). This matches the intuition in Chierchia (1992).

4.3 "Compute" and "look up": scope and semantic coherence

As was mentioned in 4.1, Sz&Z argue that so-called weak island violations are due to the inability of the wh-phrase to scope over some element of the extraction domain D. Specifically, it is proposed that scope taking is a semantically sensitive procedure that obeys principle (60):

(60) For α to scope over some β means that the Boolean operations associated with β are performed in α's denotation domain.
A derivation stops if a step requires performing some operation which, however, cannot be performed because it is not defined in the relevant denotation domain.

Concretely, an intermediate step in the derivation of, say, (50c), (51c) and (52c), is as follows:

(61) Who didn't you mention this rumor to?
form complement of ‖[the person(s)] you mentioned this rumor to‖

(62)* How didn't you solve this problem?
form complement of ‖[the way] you solved this problem‖

(63)* Who didn't you get the ring you are wearing from?
form complement of ‖[the person(s)] you got this ring from‖

The derivation in (61) can proceed because ‖[the person(s)] you mentioned this rumor to‖ is a set, so its complement can be formed. In the jargon of (60), the denotation domain of who here is a Boolean algebra (of sets of individuals), and complements are defined in this domain. The unacceptability of the other examples is due to the fact that neither ‖[the way] you solved this problem‖ nor ‖[the person(s)] you got this ring from‖ is a set: manners and collectives are best viewed as semi-lattice objects. But complement is not defined in semi-lattices, so the requisite operation cannot be performed. The derivations stop.[3]

Examples in which the β that α is unable to scope over is a universal are analyzed analogously, with reference to the fact that meet (the operation corresponding to universal quantification) is not defined in a semi-lattice; and so on.

Of course, problems like in (62) and (63) are not confined to wh-constructions. Consider the following examples:

(64) More than ten soldiers passed by every house.
'more than ten > every' or 'every > more than ten'

(65) More than ten soldiers surrounded this house.
'the collective that surrounded this house has more than ten atoms'

(66) More than ten soldiers surrounded every house.
(i)* 'there is a collective with more than ten atoms which surrounded every house (one house after the other)'
(ii) 'for every house, a possibly different collective of more than ten surrounded it'

The unacceptability of (66i) is striking. This reading would have direct S>O scope, which is usually unproblematic. Moreover, (65) shows that $\|$more than ten$\|$ is capable of counting the atoms of a collective (see also fn. 4). But (60) explains why (66i) is bad. $\|$Surrounded every house$\|$ should be obtained by via $\wedge_{i\in I}\|$surrounded house$_i\|$, which is impossible, since these collectives are not elements of a structure with meets.

How do we know that the derivation involves precisely the steps assumed above? For instance, the derivations of (50c)-(52c) might proceed as below, where x is a variable over atomic or plural individuals, and M is a higher order variable:

(67) Who didn't you mention this rumor to? $\lambda x[\neg mention(r)(to\ x)(you)]$
(68)* How didn't you solve this problem? $\lambda M[\neg M(solve(p))(you)]$
(69)* Who didn't you get this ring from? $\lambda x[\neg get(r)(from\ x)(you)]$

In the case of (67), the result would be the same as above. On the other hand, (68) and (69) suggest that the questions are perfectly good and can be answered as Elegantly, for instance and My parents, for instance, respectively. According to the heuristic of (57), the fact that the questions are not acceptable and answering them in the said way can at best be a witticism is diagnostic of the fact that the actual derivation does not proceed in this way.

Sz&Z label (61)-(63) "compute" and suggest that (67) exemplifies the "look up" procedure. "Look up" is a faster, and thus pragmatically preferable, alternative to "compute" -- but it is not always available. When dealing with entities that are "pegs" in Landman's (1986) sense (and, possibly, are also contextually salient, as suggested in Dobrovie-Sorin (1993a)), we can proceed in a top-down fashion, inspecting pegs one by one and "looking up" whether they

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3. Note that $\|$[the person(s)] you got the ring from$\|$ must be a collective, since it contains a 'once only' predicate, which does not distribute. I chose this specific example here to demonstrate that logical type (entity vs. higher order) cannot be the distinctive property in these contrasts, contra what is stipulated in some current work. Collectives are commonly analyzed as plural individuals. Thus who in both the good and the bad sentences could be said to "bind an e-type variable."

exhibit a particular, simple or complex, property. But while it makes sense to assume a model or knowledge representation where atomic individuals have pegs associated with them, unless context exceptionally individuates certain manners, collectives, etc., it does not make sense to postulate pegs for the latter. In the absence of pegs, wide scope taking by these α's must resort solely to compute.

Note that in cases like this, where surface structure determines what α (here: the wh-phrase) should scope over, look-up does not create any new readings; it only makes some of the otherwise available ones more felicitous.

4.4 "Compute" and "look up" at the syntax/semantics interface

Let us now turn to one facet of inverse scope. Although it has for long been known that the scope behavior of quantified expressions is not uniform, the first systematic study of the data was carried out by Feng-hsi Liu (1990). Her crucial generalizations are as follows:

(70) a. QPs in object position whose determiner is a universal, a bare non-focused numeral (two), some, or most of the easily make the subject dependent.

b. QPs in object position whose determiner is a modified numeral (more than two, at least two, exactly two, between two and five, fewer than two), a focused bare numeral, most, or is plain decreasing (no, few), do not (easily) make the subject dependent.

c. All the above QPs, as well as some others, easily make the object dependent when they are in subject position.

Our interest here is in cases where, as (70c) indicates, there is no inherent semantic obstacle to the QP taking scope over another and making it referentially dependent. (E.g., we are not looking at mass QPs.) Yet, QPs in (70a) can, and QPs in (70b) cannot, do this when it would amount to taking "inverse scope."

Recall that the covert operation of (49) freely rebrackets a string, hence such contrasts are not predicted. Therefore we should abandon (49) and replace it with a set of more specific procedures that are available only to the right quantifiers. For all I know as of date, the crucial property of the QPs in (70a) is that their operation can be explicated in terms of look up, whereas the operation of the QPs in (70b) is necessarily explicated in terms of compute. Which category a QP belongs to is an essentially semantic matter. On the other hand, the fact that this semantic difference gives rise to a subject/object asymmetry is due to its interaction with the syntax of English. I wish to thank D. Ben-Shalom for stimulating discussions on these matters.

As has been anticipated in connection with (59), I claim that the interpretation of sentences like (71) is built differently than those of (72) and (73):

(71) More than two / exactly two / fewer than two men lifted this table.

(72) Every man lifted this table.

(73) Two men lifted this table.

In the case of (71), the set ‖lifted this table‖ is constructed and ‖more than two men‖ operates on it in the traditional manner of generalized quantifiers. In the case of (72), however, the same set is not constructed. Instead, lifting this table is predicated of the members of the set of men. Similarly for (73) with an even intonation: lifting this table is predicated of two particular men.

(72) resembles (71) in that both have only distributive readings. With (71) this follows from more than two men being a (traditional) generalized quantifier. (72) resembles (73) in that both involve predication. They differ in that every man ensures that predication is distributive, whereas two men supports both collective and distributive readings, depending on the predicate. This follows if two men with non-focused two denotes a group/plural individual.

The differential analysis of modified and bare (non-focused) numerals is supported by the observation in 1.1 that only the latter allow distributed group readings (cf. (13) vs. (14)-(15)). It is also fully consonant with the analysis in Kamp & Reyle (1993), to which I refer for detailed arguments.[4]

Do the similarities with K&R mean that I am simply reinventing DRT? I believe not. First, there are differences in the claims. E.g., K&R treat both every man and more than two men as generalized quantifiers (associate duplex conditions with them), whereas I do not: every man works by "predicate and distribute." See also the comment in fn. 4. Secondly, following Beghelli (1993), Beghelli & Stowell (1994), and Dobrovie-Sorin (1993a,b), I currently assume that the procedures that give rise to differential scope behavior belong to English syntax proper (albeit at the level of Logical Form), not to an independent level of representation between syntax and semantics.

With minimal commitment to details, the most salient aspects of this syntax are as follows. Since there is no rule like (49) that freely grabs QPs and assigns scope to them, by default all QPs that are left in argument position in surface structure are interpreted in observance of the argument hierarchy. In practice, however, only the QPs in (70b), more than two men and its brothers, end up always having narrower scope than QPs higher in the hierarchy. Some of the assumptions about LF that account for this are as follows:

(74) a. A QP that provides a set/group as a target of predication moves to a very high position, within its own clause or further on.
b. Distributive operators are hosted by specifically positioned heads Dist.
c. Predication is distributive with respect to the target QP if QP passes through the specifier of an available DistP. This process is agreement-like and, unlike (a), strictly clause-internal.
d. QPs like every man must, QPs like two men may, pass through DistP.

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4. One apparent problem for this analysis of modified numerals is More than six men gathered, which K&R explain satisfactorily. Another is (65)-(66ii), where the predicate is unambiguously collective, and More than two men lifted this table in the sense 'It took more than two men to lift this table' (a datum also observed by P. Jacobson (p.c.)). My generalization is that when the predicate is potentially ambiguous, more than two insists on a distributive interpretation, or else it invokes the 'It took...' reading. When the predicate is unambiguously collective or we have the 'It took...' reading, ‖more than two‖ counts the atoms of the group denoted by the predicate. That is, ‖more than two‖, in distinction to ‖two‖, is always a counter, either of set elements or of group atoms. (The duplication seems necessary since, as (61) vs. (63) show, predicates come in these two varieties.) This analysis is in the spirit of K&R, although it is technically not available to them. For them, the group cannot come from the predicate, it would have to be a referent introduced by the noun phrase itself.

e. A QP not involved in the semantically significant processes (a) or (c) may be reconstructed, from its surface structure case position, to its deep structure position inside VP. (Typically, (70b)-type QPs like <u>more than two men</u> are such.)

To illustrate just one of these assumptions, (c), consider the following:

(75) More than three men thought that two women had left.

This sentence has a reading 'there are two women of whom more than three men thought...' – but the men cannot vary with individual women! According to Beghelli & Stowell (1994), the reason is that while the target of predication may move up, distributivity (the Dist head) stays put. This property of Dist will eventually explain the standard clause-boundedness of quantifier scope.

Bare numeral QPs (70a) differ from modified numeral QPs (70b) in at least two respects. (i) BNQPs may introduce groups, and (ii) BNQPs may be "specific" in the sense that they can denote principal filters. Property (ii) is shown to be crucial in explaining their behavior in branching, cf. Beghelli & Ben-Shalom & Szabolcsi (1993). The question arises which property is crucial in explaining their ability to take inverse scope. Here (i) has a better chance, since BNQPs have intermediate inverse scope readings:

(76) Every professor assigned more than two problems to **ten students**.
'every prof > **ten students** > more than two problems,' where groups of students vary with individual professors, and problem sets may or may not vary with individual students

On the other hand, whether and how properties (i) and (ii) are logically related remains an open question for the time being.

To summarize, I have argued that quantification cannot be taken care of using a single semantically blind rule. Scope taking is a semantically sensitive phenomenon in at least two senses. One, certain scope interactions are impossible because they would be incoherent. To explain this, the semantic operations involved in evaluating the sentence need to be made explicit, cf. (60). Two, the match between coherent readings and syntactic forms is rather constrained. To explain this, the best strategy seems to be to invoke a variety of genuinely syntactic mechanisms, whose operation however is contingent on certain semantic properties of the input quantifiers. In more general terms, I proposed that the pertinent processes fall into two intuitive categories, "compute" and "look up."

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BABY-SIT: A Computational Medium Based on Situations

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1 Introduction

Following its inception (Barwise and Perry, 1983), *situation theory* has quickly matured (Cooper et al., 1990; Devlin, 1991) and under the familiar name of *situation semantics* has been applied to a number of linguistic issues (Barwise, 1987; Barwise, 1989; Barwise and Etchemendy, 1987; Cooper, 1986; Cooper, 1991; Cooper et al., 1990; Fenstad et al., 1987), including quantification and anaphora (Gawron and Peters, 1990). In the past, the development of a 'mathematical' situation theory has been held back by a lack of availability of appropriate technical tools. But by now, the theory has assembled its mathematical foundations based on intuitions basically coming from set theory and logic (Aczel, 1988; Barwise, 1989; Cooper et al., 1990). With a remarkably original view of information (which is fully adapted by situation theory) (Dretske, 1981), a 'logic,' based not on truth but on information, is being developed (Devlin, 1991). This logic will probably be an extension of first-order logic (Barwise, 1977) rather than being an alternative to it.

While situation theory and situation semantics provide an appropriate framework for a realistic model-theoretic treatment of natural language, serious thinking on their 'computational' aspects has just started (Black, 1992; Nakashima et al., 1988). Existing proposals mainly offer a Prolog- or Lisp-like programming environment with varying degrees of divergence from the ontology of situation theory. In this paper, we introduce a computational medium (called BABY-SIT) based on situations. The primary motivation underlying BABY-SIT is to facilitate the development and testing of programs in domains ranging from linguistics to artificial intelligence in a unified framework built upon situation-theoretic constructs.

2 Situations and Natural Language Semantics

Activities pertaining to language include talking, listening, reading, and writing. These activities are *situated*; they occur in situations and they are about situations (Austin, 1961). What is common to these situated activities is that they convey information (Devlin, 1991; Dretske, 1981). When uttered at different times by different speakers, a statement can convey different information to a hearer and hence can have different meanings.[1]

Situation semantics makes simple assumptions about the way natural language works. Primary among them is the assumption that language is used to convey information about the world (the so-called *external significance* of language). Even when two sentences have the same interpretation, i.e., they describe the same situation, they can carry different information.[2]

[1] Consider the sentence "That really attracts me." Depending on the reference of the demonstrative, interpretation (and hence meaning) would change. For example, this sentence would be uttered by a boy referring to a cone of ice cream or by a cab driver referring to fast driving, meaning absolutely different things (Grice, 1968).

[2] For example, "Bob went to the theater" and "The father of Carol went to the theater" both describe the same situation in which Bob (an individual) went to the theater, assuming that Bob is Carol's father. However, while the first sentence says that this individual is Bob, the second

Classical approaches to semantics underestimate the role played by *context-dependence*; they ignore pragmatic factors such as intentions and circumstances of the individuals involved in the communicative process (Austin, 1961; Grice, 1968). But, indexicals, demonstratives, tenses, and other linguistic devices rely heavily on context for their interpretation and are fundamental to the way language conveys information (Akman and Tın, 1990). Context-dependence is an essential hypothesis of situation semantics. A given sentence can be used over and over again in different situations to say different things (the so-called *efficiency* of language). Its interpretation, i.e., the class of situations described by the sentence, is therefore subordinate on the situation in which the sentence is used. This context-providing situation, *discourse situation*, is the speech situation, including the speaker, the addressee, the time and place of the utterance, and the expression uttered. Since speakers are usually in different situations, having different causal connections to the world and different information, the information conveyed by an utterance will be relative to its speaker and hearer (the so-called *perspectival relativity* of language).

Besides discourse situations, the interpretation of an utterance depends on the speaker's connections with objects, properties, times and places, and on the speaker's ability to exploit information about one situation to obtain information about another. Therefore, context supports not only facts about speakers, addressees, etc. but also facts about the relations of discourse-participants to *resource situations*. Resource situations are contextually available and provide entities for reference and quantification (Barwise and Perry, 1983).[3]

Another key assumption of situation semantics is the so-called *productivity* of language: we can use and understand expressions never before uttered. Hence, given a finite vocabulary, we can form a potentially infinite list of meaningful expressions. The underlying mechanism for such an ability seems to be *compositionality*.

Situation semantics closes another gap of traditional semantic approaches: the neglect of *subject matter* and *partiality of information*. In traditional semantics, statements which are true in the same models convey the same information (van Benthem, 1986). Situation semantics takes the view that logically equivalent sentences need not have the same subject matter: they need not describe situations involving the same object and properties. The notion of partial situations (partial models) leads to a more fine-grained notion of information content and a stronger notion of logical consequence that does not lose track of the subject matter.

The *ambiguity* of language is taken as another aspect of the efficiency of language. Natural language expressions may have more than one meaning. There are factors such as intonation, gesture, the place of an utterance, etc. which play a role in interpreting an utterance (Fenstad et al., 1987). Instead of throwing away ambiguity and contextual elements, situation semantics tries to build up a full theory of linguistic meaning by initially isolating some of the relevant phenomena in a formal way and by exploring how the rest helps in achieving the goal (Barwise and Perry, 1983).

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sentence conveys the information that Carol (another individual) has a father who went to the theater.

[3] Consider the following example adapted from (Barwise and Etchemendy, 1987). There are two card games going on, one across town from the other: Max is playing cards with Emily and Claire is playing cards with Dana. Suppose Bob watching the former game mistakes Emily for Claire, and utters the sentence "Claire has the three of clubs." According to the classical (Russelian) theories (Evans, 1991), if Claire indeed has 3♣, this claim would be true since the definite noun phrases "Claire" and "the three of clubs" are used to pick out, among all the things in the world, the unique objects satisfying the properties of being an individual named Claire and being a 3♣, respectively. In contrast, situation semantics identifies these objects with respect to some limited situation—the resource situation exploited by Bob. The claim would then be wrong even if Claire had 3♣ across town.

According to situation semantics, meaningful expressions convey information not only about the external world but also about our minds (the so-called *mental significance* of language). Situation semantics differs from other approaches in that we do not, in attitude reports, describe our mind directly (by referring to states of mind, ideas, senses, thoughts, etc.) but indirectly (by referring to situations that are external).

With these underlying assumptions and features, situation semantics provides a fundamental and appropriate framework for studying the semantics of natural language (Barwise and Etchemendy, 1989). The ideas emerging from research in situation semantics have also been coalesced with well-developed linguistic theories such as *lexical-functional grammar* (Sells, 1985) and led to rigorous formalisms (Fenstad et al., 1987). On the other hand, situation semantics has been compared to other influential mathematical approaches to the theory of meaning, viz., *Montague grammar* (Cooper, 1986; Dowty et al., 1981; Rooth, 1986) and *Discourse Representation Theory* (DRT) (Kamp, 1981).

3 Constructs for Situated Processing

Intelligent agents generally make their way in the world as follows: pick up certain information from a situation, process it, and react accordingly (Devlin, 1991; Dretske, 1981; Israel and Perry, 1990). Being in a (mental) situation, such an agent has information about situations it sees, believes in, hears about, etc. Awareness of some type of situation causes the agent to acquire more information about that situation as well as other situation types, and to act accordingly. Assuming the possession of prior information and knowledge of some constraints, the acquisition of an item of information by an agent can also provide the agent with an additional item of information.[4]

In situation theory, *infons* are the basic units of information. Abstraction can be captured in a primitive level by allowing parameters in infons. Parameters are abstractions or generalizations over classes of non-parametric objects (e.g., individuals, spatial locations). Parameters of a parametric object can be associated with objects which, if they were to replace the parameters, would yield one of the objects in the class that the parametric object abstracts over. The parametric objects actually define types of objects in that class. For example, $\langle$*see*, X, *Alice*; 1$\rangle$ and $\langle$*see*, X, Y; 1$\rangle$ are parametric infons where X and Y are parameters over individuals. *Parameter-free infons* are the basic items of information about the world (i.e., 'facts') while parametric infons are the basic units that are utilized in a computational treatment of information flow.

To construct a computational model of situation theory, it is convenient to have available abstract analogs of objects. As noted above, by using parameters we can have abstracts which are parametric objects, including parametric situations, parametric individuals, etc. This yields a rich set of data types. Abstract situations can be viewed as models of real situations. They are set-theoretic entities that capture only some of the features of real situations, but are amenable to computation. We define abstract situations as structures consisting of a set of parametric infons. Information can be partitioned into situations by defining a hierarchy between situations. A situation can be larger, having other situations as its subparts. For example, an utterance situation for a sentence consists of the utterance situations

[4] Reaping information from a situation is not the only way an agent processes information. It can also act in accordance of the obtained information to change the environment. Creating new situations to arrive at new information and conveying information it already had to other agents are the primary functions of its activities.

for each word forming the sentence. The *part-of* relation of situation theory can be used to build hierarchies among situations and the notion of nested information can be accommodated.

Being in a situation, one can derive information about other situations connected to it in some way. For example, from an utterance situation it is possible to obtain information about the situation it describes. Accessing information both via a hierarchy of situations and explicit relationships among them requires a computational mechanism. This mechanism will put information about situation types related in some way into the comfortable reach of the agent and can be made possible by a proper implementation of the *supports relation*, $\models$, of situation theory.[5]

Constraints enable one situation to provide information about another and serve as links. (They actually link types of situations.) Constraints can be treated as inference rules. When viewed as a backward-chaining rule, a constraint can provide a channel for information flow between types of situations, from the antecedent to the consequent. This means that such a constraint behaves as a 'definition' for its consequent part. Another way of viewing a constraint is as a forward-chaining rule. This approach enables an agent to alter its environment.

4 Computational Situation Theory

4.1 PROSIT

PROSIT (PROgramming in SItuation Theory) is a situation-theoretic programming language (Nakashima et al., 1988). PROSIT is tailored more for general knowledge representation than for natural language processing. One can define situation structures and assert knowledge in particular situations. It is also possible to define relations between situations in the form of constraints. PROSIT's computational power is due to an ability to draw inferences via rules of inference which are actually constraints of some type. PROSIT can deal with self-referential expressions (Barwise and Etchemendy, 1987).

One can assert facts that a situation will support. For example, if the situation S1 supports the fact that Bob is a young person, this can be defined in the current situation S as:

S: ($\models$ S1 (young "Bob")).

Note that the syntax is similar to that of Lisp and the fact is in the form of a predicate. The supports relation, $\models$, is situated so that whether a situation supports a fact depends on within which situation the query is made. Queries can be posed about one situation from another, but the results will depend on where the query is made.

Constraints can be specified as forward chaining constraints, backward chaining constraints, or both. Backward chaining constraints are activated at query-time while forward-chaining constraints are activated at assertion-time. By default, all the tail facts of an activated forward-chaining constraint are asserted to the situation, which may in turn activate other forward-chaining constraints recursively.

For a constraint to be applicable to a situation, the situation must be declared to 'respect' the constraint. Constraints in PROSIT are about local facts within a situation rather than about situation types. That is, the interpretation of constraints does not allow direct specification of constraints between situations, only between infons within situations.

[5] Given an infon σ and a situation s, this relation holds if σ is made true by s, i.e., $s \models \sigma$.

Situated constraints offer an elegant solution to the treatment of *conditional constraints* which only apply in situations that obey some condition.[6] This is actually achieved in PROSIT since information is specified in the constraint itself. Situating a constraint means that it may only apply to appropriate situations and is a good strategy to achieve *background conditions*. However, it might be required that conditions are set not only within the same situation, but also between various types of situations. Because constraints have to be situated in PROSIT, not all situations of the appropriate type will have a constraint to apply. PROSIT does not provide an adequate mechanism for specifying *conventional constraints*, i.e., constraints which can be violated.[7]

Parameters, variables, and constants are used for representing entities in PROSIT. Variables match any expression in the language and parameters can be equated to any constant or parameter. That is, the concept of *appropriateness conditions* is not exploited in PROSIT. Appropriateness conditions, in fact, specify restrictions on the types of arguments a relation can take, and any restrictions between these arguments (Devlin, 1991). It is more useful to have parameters that range over various classes rather than to work with parameters ranging over all objects. Some treatment of parameters is given in PROSIT with respect to *anchoring*. Given a parameter of some type (individual, situation, etc.), an anchor is a function which assigns an object of the same type to the parameter (Devlin, 1991). Hence, parameters work by placing restrictions on anchors. There is no appropriate anchoring mechanism in PROSIT since parameters are not typed.

Set operations are possible on sets of facts supported by a situation. As mentioned before, situations are closed under constraints and rules of inference. PROSIT has been used to show how problems involving cooperation of multiple agents can be solved, especially by combining reasoning about situations (Nakashima et al., 1987).

4.2 ASTL

Black's ASTL (A Situation Theoretic Language) is another programming language based on situation theory (Black, 1992). ASTL is aimed at natural language processing. The primary motivation underlying ASTL is to figure out a framework in which semantic theories such as situation semantics and DRT (Kamp, 1981) can be described and possibly compared. One can define in ASTL constraints and rules of inference over the situations. An interpreter passes over ASTL definitions to answer queries about the set of constraints and basic situations.

ASTL ontology incorporates individuals, relations, situations, parameters, and variables. These form the basic terms of the language. Complex terms are in the form of i-terms (to be defined shortly), situation types, and situations. Situations can contain facts which have those situations as arguments. Sentences in ASTL are constructed from terms in the language and can be constraints, grammar rules, or word entries.

An *i-term* is simply an infon[8] $\langle rel, arg_1, \ldots, arg_n, pol \rangle$ where *rel* is a relation of arity n, arg_i is a term, and *pol* is either 0 or 1. A *situation type* is given in the form $[param|cond_1 \ldots cond_n]$ where $cond_i$ has the form $param \models i\text{-}term$. If S1

[6]For example, when Alice throws a basketball, she knows it will come down—a constraint to which she is attuned, but which would not hold if she tried to play basketball on the moon.

[7]An example of this sort of constraint is the relation between the ringing of the bell and the end of class. It is not necessary that the ringing of the bell should mean the end of class.

[8]We use Black's notation almost verbatim rather than adapting it to the 'standard' notation of our paper.

supports the fact that Bob is a young person, this can be defined as:

S1: [S | S $\models$ ⟨young, bob, 1⟩].

The single colon indicates that S1 supports the situation type on its right-hand side. The supports relation in ASTL is global rather than situated. Consequently, query-answering is independent of the situation in which the query is issued.

Constraints are actually backward-chaining constraints. For example, the constraint that every man is a human being can be written as follows:

*S: [S | S $\models$ ⟨human, *X, 1⟩] $\Leftarrow$ *S: [S | S $\models$ ⟨man, *X, 1⟩].

*S, *X are variables and S is a parameter. Another interesting property of ASTL is that constraints are global. Thus, a new situation of the appropriate type need not have a constraint explicitly added to it. Grammar rules are yet another sort of constraints with similar semantics.

Although one can define constraints between situations in ASTL, the notion of a background condition for constraints is not available. Similar to PROSIT, ASTL cares little about coherence within situations. This is left to the user's control. Accordingly, there is no mechanism in ASTL to specify constraints that can be violated.

Declaring situations to be of some type allows abstraction over situations to some degree. But, the actual means of abstraction over objects in situation theory, viz., parameters, carry little significance in ASTL.

As in PROSIT, variables in ASTL have scope only within the constraint they appear. They match any expression in the language unless they are declared to be of some specific situation type in the constraint. Hence, it is not possible to declare variables as well as parameters to be of other types such as individuals, relations, etc. Moreover, ASTL does not allow definition of appropriateness conditions for arguments of relations.[9]

ASTL does not have a mechanism to relate two situations so that one will support all the facts that the other does. This might be achieved via constraints, but there is no built-in structure between situations (as opposed to the hierarchy of situations in PROSIT).

4.3 Situation Schemata

Situation schemata have been introduced (Fenstad et al., 1987) as a theoretical tool for extracting and displaying information relevant for semantic interpretation from linguistic form. A situation schema is an attribute-value system which has a choice of primary attributes matching the primitives of situation semantics. In this sense, it is just another knowledge representation mechanism. The boundaries of situation schemata are however flexible and, depending on the underlying theory of grammar, are susceptible to amendment.

A simple sentence φ has the situation schemata shown in Figure 1(a). Here r can be anchored to a relation, and a and b to objects; $i \in \{0,1\}$ gives the polarity. LOC is a function which anchors the described fact relative to a discourse situation d, c. LOC will have the general format in Figure 1(b). IND.α is an indeterminate for a location, r denotes one of the basic structural relations on a relation set R, and loc_0 is another location indeterminate. The notation $[\ ]_\alpha$ indicates a repeated

[9] 'Speaking' relation, for example, might require its speaker role to be filled by a human. Such a restriction could be defined only by using constraints of ASTL. However, this requires writing the restriction each time a new constraint about 'speaking' is to be added.

$$
\text{SIT}.\varphi \begin{bmatrix} \text{REL} & r \\ \text{ARG.1} & a \\ \text{ARG.2} & b \\ \text{LOC} & - \\ \text{POL} & i \end{bmatrix}
\qquad
\begin{bmatrix} \text{IND} & \text{IND}.\alpha \\ \text{COND} & \begin{bmatrix} \text{REL} & r \\ \text{ARG.1} & [\,]_{\alpha} \\ \text{ARG.2} & loc_d \\ \text{POL} & i \end{bmatrix} \end{bmatrix}
$$

(a) (b)

Figure 1: (a) A prototype situation schema, (b) the general format of LOC in (a).

reference to the shared attribute value, IND.α. A partial function g anchors the location of SIT.φ, viz. SIT.φ.LOC, in the discourse situation d, c if

$$g(loc_0) = loc_d \text{ and}$$
$$c(r), g(\text{IND}.\alpha), loc_d; 1$$

where loc_d is the discourse location and $c(r)$ is the relation on R given by the speaker's connection c. The situation schema corresponding to "Alice saw the cat" is given in Figure 2.

Situation schemata can be adopted to various kinds of semantic interpretation.[10] One could give some kind of operational interpretation in a suitable programming language, exploiting logical insights. But in its present state, situation schemata do not go further than being a complex attribute-value structure. They allow representation of situations within this structure, but do not use situation theory itself as a basis. Situations, locations, individuals, and relations constitute the basic domains of the structure. Constraints are declarative descriptions of the relationships holding between aspects of linguistic form and the semantic representation itself.

5 BABY-SIT

5.1 Computational Model

The computational model underlying the current version of BABY-SIT consists of nine primitive domains: *individuals* (I), *times* (T), *places* (L), *relations* (R), *polarities* (O), *parameters* (P), *infons* (F), *situations* (S), and *types* (K). Each primitive domain carries its own internal structure:

- Individuals: Unique atomic entities in the model which correspond to real objects in the world.
- Times: Individuals of distinguished type, representing temporal locations.
- Places: Similar to times, places are individuals which represent spatial locations.

[10] Theoretical issues in natural language semantics have been implemented on pilot systems employing situation schemata. The grammar described in (Fenstad et al., 1987), for example, has been fully implemented using a lexical-functional grammar system (Fenstad, 1987) and a fragment including prepositional phrases has been implemented using the DPATR format (Colban, 1987).

$$\text{SIT.1}\begin{bmatrix} \text{REL} & \text{`see'} \\ \text{ARG.1} & \begin{bmatrix}\text{IND} & \text{`Alice'}\end{bmatrix} \\ \text{ARG.2} & \begin{bmatrix} \text{IND} & \text{IND.2} \\ \text{SPEC} & \text{`the'} \\ \text{COND} & \begin{bmatrix} \text{REL} & \text{`cat'} \\ \text{ARG.1} & [\,]_2 \\ \text{POL} & 1 \end{bmatrix} \end{bmatrix} \\ \text{LOC} & \begin{bmatrix} \text{IND} & \text{IND.3} \\ \text{COND} & \begin{bmatrix} \text{REL} & \text{`<'} \\ \text{ARG.1} & [\,]_3 \\ \text{ARG.2} & loc_d \\ \text{POL} & 1 \end{bmatrix} \end{bmatrix} \\ \text{POL} & 1 \end{bmatrix}$$

Figure 2: Situation schemata for "Alice saw the cat."

- Relations: Various relations hold or fail to hold between objects. A relation has argument roles which must be occupied by appropriate objects.
- Polarities: The 'truth values' 0 and 1.
- Infons: Discrete items of information of the form $<< rel, arg_1, \ldots, arg_n, pol >>$, where *rel* is a relation, arg_i, $1 \leq i \leq n$, is an object of the appropriate type for the *i*th argument role, and *pol* is the polarity.
- Parameters: 'Place holders' for objects in the model. They are used to refer to arbitrary objects of a given type.
- Situations: (Abstract) situations are set-theoretic constructs, e.g., a set of *parametric infons* (comprising relations, parameters, and polarities). A parametric infon is the basic computational unit. By defining a hierarchy between them, situations can be embedded via the special relation *part-of*. A situation can be either (spatially and/or temporally) *located* or *unlocated*. Time and place for a situation can be declared by *time-of* and *place-of* relations, respectively.
- Types: Higher-order uniformities for individuating or discriminating uniformities in the world.

The structure of the model, M, is a tuple $<I,T,L,R,O,P,F,S,K>$. This structure is shared by all components of the system. *Description* of a model, D_M, consists of a definition of the structure M and a set of *constraints*, C. The computational model is then defined as a tuple $<D_M, A, A', U>$ where A is an anchor for parameters, A' is an assignment for variables, and U is an interpretation for D_M. A is provided by the anchoring situation while A' is obtained through unification.

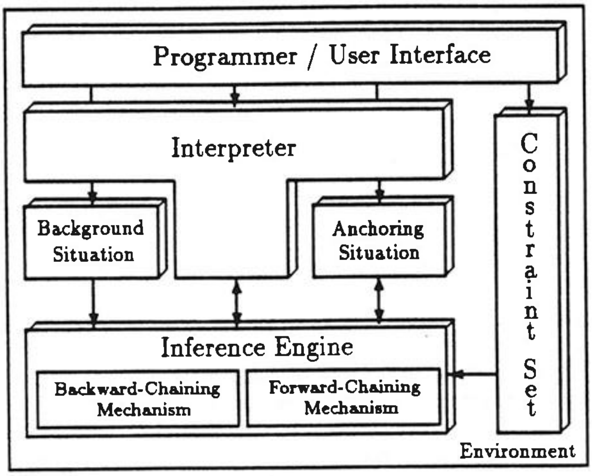

Figure 3: The architecture of BABY-SIT.

U will be defined by the operational semantics of the computation. Each object in the environment must be declared to be of some type.

5.2 Architectural Considerations

The architecture of BABY-SIT is composed of seven major parts: *programmer/user interface*, *environment*, *background situation*, *anchoring situation*, *constraint set*, *inference engine*, and *interpreter* (Figure 3).

The interface allows interaction of the user with the system. The environment initially consists of static situation structures and their relationships. These structures can be dynamically changed and new relationships among situation types can be defined as the computation proceeds. Information conveyance among situations is made possible by defining a *part-of* relation among them. In this way, a situation s can have information about another situation s' which is part of s. The background situation contains infons which are inherited by all situation structures in the environment. However, a situation can inherit an infon from the background situation only if it does not cause a contradiction in that situation.

A situation in the environment can be realized if its parameters are anchored to objects in the real world. This is made possible by the anchoring situation which allows a parameter to be anchored to an object of appropriate type—an individual, a situation, a parameter, etc. A parameter must be anchored to a unique object. On the other hand, more than one parameter may be anchored to the same object. Restrictions on parameters assure anchoring of one parameter to an object having the same qualifications as the parameter.

In addition to the *part-of* relation among situations, constraints are potent means of information conveyance between and within situations. They link various types of situations. Constraints may be physical laws, linguistic rules, law-like correspondences, conventions, etc. In BABY-SIT, they are realized as forward-chaining constraints or backward-chaining constraints, or both. Assertion of a new object into BABY-SIT activates the forward-chaining mechanism. Once their antecedent parts are satisfied, consequent parts of the forward-chaining constraints are asserted into BABY-SIT, unless this yields a contradiction. In case of a contradiction, the backward-chaining mechanism is activated to resolve it. The interpreter is the control authority in BABY-SIT. Anchoring of parameters, evaluation of constraints, etc. are all controlled by this part of the system.

5.3 Modes of Computation

A prototype of BABY-SIT is currently being developed in KEE^{TM} (Knowledge Engineering Environment) (KEE, 1993) on a SPARCstation^{TM}. Some of the available modes of computation in this evolving project are described below.

5.3.1 Constraints

Barwise and Perry identify three forms of constraints (Barwise and Perry, 1983). *Necessary constraints* are those by which one can define or name things, e.g., "Every dog is a mammal." *Nomic constraints* are patterns that are usually called natural laws, e.g., "Blocks drop if not supported." *Conventional constraints* are those arising out of explicit or implicit conventions that hold within a community of living beings, e.g., "The first day of the month is the pay day." They are neither nomic nor necessary, i.e., they can be violated. All types of constraints can be *conditional* and *unconditional*. Conditional constraints can be applied to situations that meet some condition while unconditional constraints can be applied to all situations.

Some constraints can be defined as forward-chaining constraints, some as backward-chaining constraints, others as both forward- and backward-chaining constraints. In BABY-SIT, conditional constraints come with a set of *background conditions* which must be satisfied for the constraint to be applied. Each background condition is in the form of a $\models$ relation between a situation and an infon.

In BABY-SIT, a constraint becomes a candidate for activation when its background conditions, if any, are satisfied. A candidate forward-chaining constraint is activated whenever its antecedent part is satisfied. All the consequences are asserted if they do not yield a contradiction in the situation into which they are asserted. New assertions may in turn activate other candidate forward-chaining constraints. If consequences cause contradictions within themselves, backward-chaining constraints are used to decide which one(s) will be successfully asserted. Candidate backward-chaining constraints are activated either when a query is entered explicitly or is issued by the forward-chaining mechanism.

In BABY-SIT, the following classes of constraints can be easily modeled (Black, 1991):

- Situation constraints: Constraints between situation types.
- Infon constraints: Constraints between infons (of a situation).
- Argument constraints: Constraints on argument roles (of an infon).

5.3.2 Assertions

Assertion mode provides an interactive environment in which one can define objects and their types. There are nine basic types corresponding to nine primitive domains: ~IND (individuals), ~TIM (times), ~LOC (places), ~REL (relations), ~POL (polarities), ~INF (infons), ~PAR (parameters), ~SIT (situations), and ~TYP (types). For instance, if *l* is a place, then *l* is of type ~LOC, and the infon <<*of-type*, *l*, ~LOC, 1>> is a fact in the background situation. Note that type of all types is ~TYP. For example, the infons <<*of-type*, ~LOC, ~TYP, 1>> and

<<*of-type*, ~TYP, ~TYP, 1>> are facts in the background situation by default. The syntax of the assertion mode is the same as in (Devlin, 1991) (cf. Table 1).

Suppose ***aynur*** is an individual, ***syntactic-entity*** is a relation, and ***u1*** is an utterance situation for the word 'aynur.' Then, these objects can be declared as:

I> ***aynur***: ~IND
I> ***syntactic-entity***: ~REL
I> ***u1***: ~SIT

The definition of relations includes the *appropriateness conditions* for their argument roles. Each argument can be declared to be from one or more of the primitive domains above. Consider ***syntactic-entity*** above. If we like it to have only one argument of type individual, we can write:

I> <***syntactic-entity*** | ~IND>

In order for the parameters to be anchored to objects of the appropriate type, parameters must be declared to be from only one of the primitive domains. It is also possible to put restrictions on a parameter in the environment. Suppose we want to have a parameter E that denotes any syntactic entity. This can be done by asserting:

I> E = IND1 ^ <<***syntactic-entity***, IND1, 1>>

IND1 is a default system parameter of type ~IND. E is considered as an object of type ~PAR such that if it is anchored to an object, say ***obj1***, then ***obj1*** must be of type ~IND and the background situation (denoted by w) must support the infon <<***syntactic-entity***, ***obj1***, 1>>.

Parametric types are also allowed in BABY-SIT. They can be formed by obtaining a type from a parameter. Parametric types are of the form $[P \mid s \models I]$ where P is a parameter, s is a situation (i.e., a *grounding* situation), and I is a set of infons. The type of all syntactic entities can be defined as follows:

I> ~CATEGORY = [IND1 | $w \models$ <<***syntactic-entity***, IND1, 1>>]

~CATEGORY is seen as an object of type ~TYP and can be used as a type specifier for declaration of new objects in the environment. For instance:

I> noun: ~CATEGORY

yields an object, ***noun***, which is of type ~IND such that the background situation supports the infon <<***syntactic-entity***, ***noun***, 1>>.

Infons can be added into situations in BABY-SIT. The following sequence of assertions adds <<***category***, ***u1***, ***noun***, 1>> into ***u1***:

I> ***category***: ~REL
I> <***category*** | ~SIT, ~CATEGORY>
I> ***u1*** $\models$ <<***category***, ***u1***, ***noun***, 1>>

5.3.3 Querying

Query mode enables one to issue queries about situations. BABY-SIT's response depends on its understanding of the intention of the user. There are several possible actions which can be further controlled by the user:

```
< proposition > ::= <situation-proposition> | <parameter-type-proposition> |
                    <situation/object-type-proposition> |
                    <infon-proposition> | <type-of-type-proposition> |
                    <relation-proposition>

<situation-proposition> ::= <constant> "|=" <infonic-set>

<parameter-type-proposition> ::= <parameter> "="
                                 {<basic-type>, <type-name>,
                                  <restricted-parameter-type>}

<situation/object-type-proposition> ::= <constant> ":"
                                 {<basic-type>, <type-abstraction>,
                                  <type-name>} ["[" <constant> "]"]

<infon-proposition> ::= <constant> "=" <infon>

<type-of-type-proposition> ::= <type-name> "="
                               {<basic-type>, <type-abstraction>}

<relation-proposition> ::= "<" <relation> ["|" <type-specifier>
                                ("," <type-specifier>)*] ">"

<type-specifier> ::= <basic-type> | <type-name> |
                     "{" {<basic-type>, <type-name>}
                         ("," {<basic-type>, <type-name>})*] "}"

<type-abstraction> ::= "[" <parameter> "|" { <constant>, <parameter>}
                                   "|=" <infonic-set> "]"

<restricted-parameter-type> ::= <parameter> " ^ " <infonic-set>

<basic-type> ::= "~LOC" | "~TIM" | "~IND" | "~REL" | "~SIT" |
                 "~INF" | "~TYP" | "~PAR" | "~POL"

<infonic-set> ::= "{" <infon> ("," <infon>)* "}" | <infon>

<infon> ::= "<<" <relation> ("," <argument>)* ["," <polarity>] ">>"

<relation> ::= <special-relation> | <constant>

<argument> ::= <constant> | <parameter> | <basic-type> | <type-name>

<polarity> ::= "0" | "1"

<constant> ::= {<digit>, <lower-case-letter>}
               ({<lower-case-letter>, <digit>})*

<parameter> ::= <upper-case-letter> ({<upper-case-letter>, <digit>})*

<type-name> ::= "~" <upper-case-letter> ({<upper-case-letter>, <digit>})*

<lower-case-letter> ::= "a" | "b" | ... | "z" | "-"

<upper-case-letter> ::= "A" | "B" | ... | "Z"

<digit> ::= "0" | "1" | ... | "9"
```

Table 1: Syntax of the assertion mode.

```
< query > ::= <situation-query> | <oracle-query>
<situation-query> ::= <situation> {"|=", "|=/="} <query-infonic-set>
                      ("," <situation> {"|=", "|/="} <query-infonic-set>)*
                      ["[" <anchoring-situation> "]"]
<oracle-query> ::= <constant> "=" "@" "(" <constant> ")" [<issue-set>]
<situation> ::= <constant> | <query-variable>
<issue-set> ::= "{" <issue-infon> ("," <issue-infon>)* "}"
<query-infonic-set> ::= "{" <query-infon> ("," <query-infon>)* "}" |
                        <query-infon>
<query-infon> ::= "<<" {<relation>, <query-variable>}
                  ("," {<argument>, <query-variable>})* ","
                  {<polarity> ">>"
<issue-infon> ::= "<<" <relation> ("," <argument>)* ["," <polarity>] ">>"
<query-variable> ::= "?" <parameter>
<anchoring-situation> ::= <constant>
```

Table 2: Syntax of the query mode.

- Replacing each parameter in the query expression by the corresponding individual if there is a possible anchor, either partial or full, provided by the given anchoring situation for that parameter.
- Returning solutions. (Their number is determined by the user.)
- Returning all solutions.
- Displaying a solution with its parameters replaced by the individuals to which they are anchored by the given anchoring situation.
- For each solution, displaying infons anchoring any parameter in the solution to an individual in the given anchoring situation.
- Displaying a trace of anchoring of parameters in each solution.

The computation upon issuing a query is done either by direct querying through situations or by the application of backward-chaining constraints. This is also under the control of the user. A situation, s, *supports* an infon if the infon is either explicitly asserted to hold in s, or it is supported by a situation s' which is part of s, or it can be proven to hold by application of backward-chaining constraints.[11] The syntax of the query expressions is given in Table 2. Given *anchor1* as the anchoring situation (Figure 4), a query and the system's response to it are as follows:

Q> u1 $\models$ {<<?X, ?Y, *nominative*, 1>>, <<*time-of*, u1, ?Z, 1>>},
u2 $\not\models$ {<<*category*, u2, *pronoun*, 1>>

u1 $\models$ {<<*case*, u1, *nominative*, 1>>, <<*time-of*, u1, T1, 1>>},
u2 $\not\models$ {<<*category*, u2, *pronoun*, 1>>

[11]Remember that given an infon σ and a situation s, if s supports σ, then this is denoted by $s \models \sigma$. Otherwise, we say $s \not\models \sigma$.

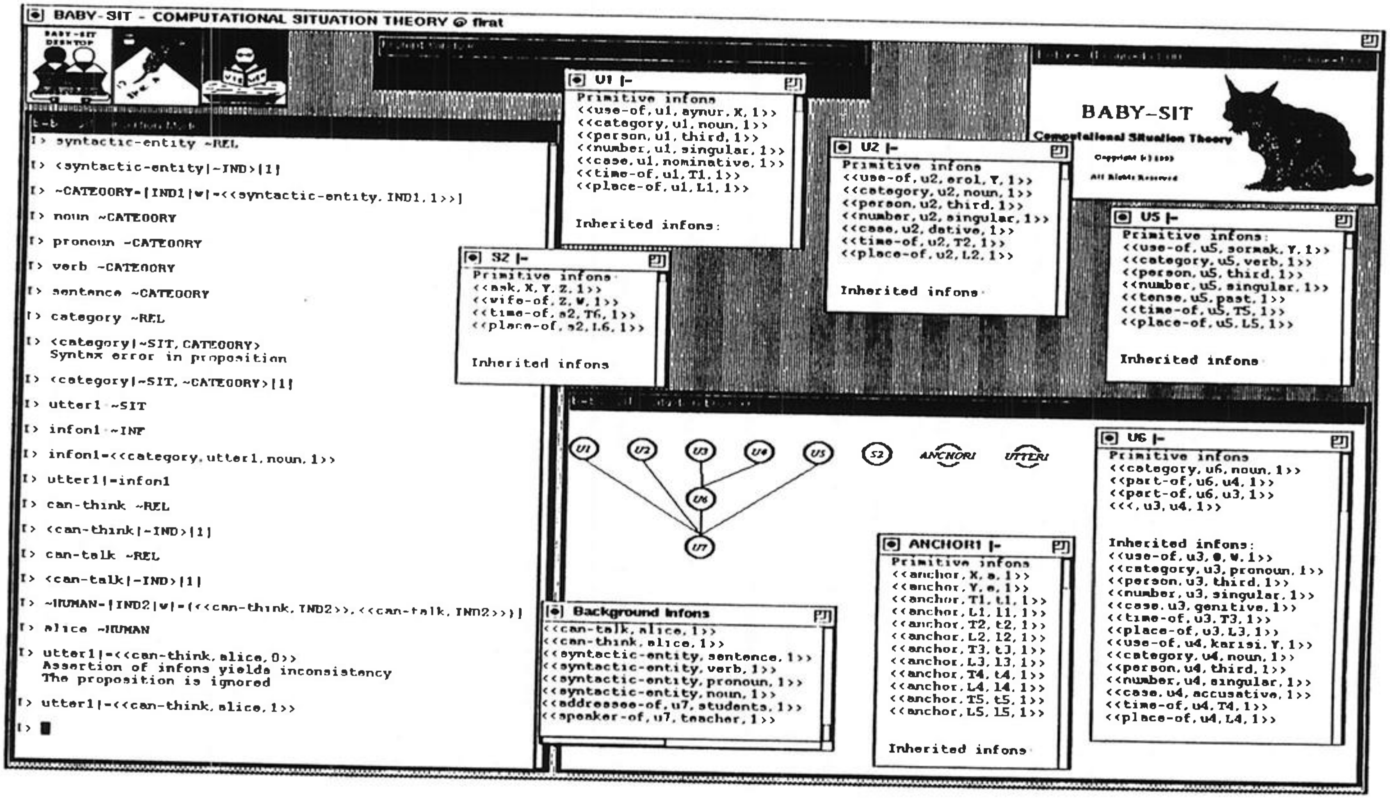

Figure 4: A view from the BABY-SIT desktop.

with the anchoring on parameters:

anchor1 $\models$ {<<*anchor*, T1, t1, 1>>}

In addition to query operations, a special operation, *oracle*, is allowed in the query mode. An *oracle* is defined over an object and a set of infons (*set of issues*) (Devlin, 1991). The oracle of an object enables one to chronologically view the information about that object from a particular perspective provided by the given set of infons. One may consider oracles as 'histories' of specific objects. Given an object and a set of issues, BABY-SIT anchors all parameters in this set of issues and collects all infons supported by the situations in the system under a specific situation, thus forming a 'minimal' situation which supports all parameter-free infons in the set of issues.

6 Conclusion

BABY-SIT accommodates the basic features of situation theory. The world is viewed as a collection of objects. The basic objects include individuals, times, places, labels, situations, relations, and parameters. Situations are 'first-class citizens' which represent limited portions of the world. Infons can be made true or false, or may be left unmentioned by some situation. A situation cannot support an infon and its dual at the same time. A situation can contain infons which have the former as arguments. Information flow is made possible via coercions that link various types of objects.

BABY-SIT enhances these features. Situations are viewed at an abstract level. This means that situations are sets of parametric infons, but they may be non-well-founded (Aczel, 1988; Barwise and Etchemendy, 1987). Parameters are place holders, hence they can be anchored to unique individuals in the anchoring situation. A situation can be realized if its parameters are anchored, either partially or fully, by the anchoring situation. Each relation has 'appropriateness conditions' which determine the type of its arguments. Situations (and hence infons they support) may have spatio-temporal dimensions. A hierarchy of situations can be defined both statically and dynamically. A built-in structure allows one situation to have information about another which is part of the former. Grouping of situations provides a computational context. Partial nature of situations facilitates computation with incomplete information. Constraints can be violated. This aspect is built directly into the computational mechanism: a constraint can be applied to a situation only if it does not cause an incoherence.

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Accenting phenomena, association with focus, and the recursiveness of focus-ground*

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1 Introduction

In an attempt to systematize and generalize the phenomena referred to as 'association with focus', a number of recent papers have argued that the quantificational structure of so-called focus-sensitive operators is crucially determined by the traditional pragmatic focus-ground partition. Research in this area has concentrated on providing a semantic description of focus-ground and then defining the semantics of focus-sensitive operators in terms of the semantics of focus-ground. In other words, focus-sensitive operators take as arguments semantic objects that are structured in accordance with an independent focus-ground partition. From this perspective, one needs to assume that sentences with more than one focus-sensitive operator contain multiple focus-ground partitions (overlapping or recursive) within a simplex sentence, something which is at odds with the traditional view of focus-ground.

This paper argues that focus-ground does not necessarily determine the quantificational structure of focus-sensitive operators. It shows that these operators may express their semantics on partitions other than the focus-ground partition. This means that recursive or overlapping focus-ground partitions are not required in sentences with more than one focus-sensitive operator. The belief that more than one focus-ground partition per sentence may be available appears to rest in part on the assumption that every pitch accent is correlated with a focus in a focus-ground partition. Since sentences may have more than one pitch accent, that means they contain more than one focus. This assumption, however, is unwarranted. Accenting is used as a structural resource in natural language for a number of different reasons. It is shown below that there is no one-to-one correspondence between accent and focus and between accent and an 'operator-associated' element.

2 Background

Let us first introduce some background notions and terminology concerning focus-ground, accent, and association with focus. The term 'focus' has (at least) three uses: a phonological one, a semantic one, and a pragmatic one. Clarification of the three uses of 'focus' is important in approaching the issues raised below.

2.1 Focus-ground

Focus-ground is found, under a number of different names and guises (e.g. focus-presupposition, theme-rheme, topic-comment, dominance), in a wide variety of works in the pragmatic and discourse-analytic literature (see Hockett 1958, Kuno 1972, Gundel 1974, Erteschik-Shir 1986, Prince 1986, Rochemont 1986, Ward 1988, among others). There are significant differences between these approaches, but they all share the view that focus-ground is an expression of the structuring of sentences according to informational or communicative requirements, i.e. it indicates how information conveyed by linguistic means is added to a (hearer's mental model of the)

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context or discourse. The focus constitutes actual information and the ground is what anchors this information to the context.

There is a wealth of characterizations of focus-ground, but, for expository purposes, let us settle on the following definitions adapted from Vallduví 1992. Let ϕ_s be the proposition conveyed by a sentence S and K_h (the relevant subset of) the hearer's model of the common ground at the time of utterance (t_u). The focus and ground can be defined thus:

- FOCUS: the part of the sentence that encodes *information* (I_s), i.e. the only augmentation or modification to be made to the hearer's model of the common ground (the update potential of ϕ_s in a particular context).
- GROUND: what is already established in the hearer's model at t_u: it ushers I_s to the right location (from the speaker's viewpoint) in the hearer's model.

If one adopts a Heimian file-like view of the context (Heim 1982) and K_h is thought of as a file F_1, then the function of the ground consists in ushering I_s to a particular file card and to a particular record on that file card which I_s is meant to augment or modify. If I_s can be appropriately added to F_1 without ushering, i.e. if specification of a file card and/or record is inherited from a previous utterance, no explicit ground is needed and an all-focus sentence may occur.[1]

Let us illustrate focus-ground with some examples adapted from Jackendoff (1972:248). In (1) the focus of an assertion is identified using the well-known, although not infallible, question test: the focus of the answer corresponds to the wh-phrase in the question.

(1) a. What did John do?
John [$_F$ gave his daughter a new BICYCLE].

b. What did John give his daughter?
John gave his daughter [$_F$ a new BICYCLE].

c. Who did John give a new bycicle?
John gave [$_F$ his DAUGHTER] a new bicycle.

d. Why don't we drive there?
[$_F$ The ROAD's dangerous.]

In these sentences focus is delimited by the F-labeled square brackets. Small caps identify the word receiving nuclear stress. In sentence (1a) the focus is the VP, (1b) shows a narrow focus on the direct object, and (1c) a narrow focus on the indirect object. These three sentences are truth-conditionally equivalent and differ only in their focus-ground structure. Sentence (1d) is a thetic all-focus sentence (see Lambrecht 1987 and Sasse 1987 for theticity).

Speaking of recursive focus-ground structures within a simplex sentence has little conceptual motivation from the informational or communicative perspective. It is unclear what it would mean to say, for example, that within the part of the sentence that ushers I_s, call it G, there is a second I_G with its own ushering ground. Simplex sentences have only one top-level (informational) focus-ground structure that determines what part of the sentence encodes their update potential and what part acts as an usher. Finally, let us note that the structural realization of focus-ground is subject to crosslinguistic variation. English, as shown in (1), tends to

[1] Vallduví 1992 argues that the informational or communicative structuring of sentences serves to optimize the process of information update. To this purpose, a sentence S encodes not only a proposition ϕ_s but also an instruction that indicates how I_s is best added to K_h in the terms just outlined. There are four possible instruction-types and each is the result of different combinations of focus and ground. Focus-ground is defined so as to encompass both the traditional focus-ground and topic-comment partitions in one single articulation. See Vallduví 1992 for further discussion.

resort to prosodic contrasts to express differences in focus-ground structure. Other languages, like Catalan or Hungarian, must resort to syntactic means to express the exact same differences.

2.2 Association with focus

Jackendoff 1972, among others, notices that sentences (1a-c) cease to be truth-conditionally equivalent in the presence of some operators like *even* and *only*:

(2) a. John even [$_F$ gave his daughter a new BICYCLE].

b. John even gave his daughter [$_F$ a new BICYCLE].

c. John even gave [$_F$ his DAUGHTER] a new bicycle.

The VP-external adverb *even* in (2a-c) is interpreted as being construed with, i.e. as associating with, the constituents enclosed in brackets. Having identified the bracketed constituents as foci, Jackendoff concludes that *even* associates with focus, with the pertinent truth-conditional effects. Since then, *even* and *only* have been known as focus particles or focus-sensitive operators.

Jacobs 1984, 1991 is a reformulation of the association-with-focus phenomenon. Jacobs argues that the pragmatic focus-ground partition discussed in § 2.1 is in fact the argument of an implicit illocutionary operator. In declaratives the illocutionary operator is ASSERT. Jacobs' view of the communicative import of focus-ground is not unlike Vallduví's (1992) in that he takes focus to be the element that represents the only augmentation of the context (what he calls the focus of the assertion). When ASSERT takes the focus-ground partition in (1b) as an argument, it yields the interpretation that, given a context where John gave his daughter something, the speaker asserts it is a bicycle. This is represented in (3a):

(3) a. ASSERT(John gave his daughter [$_F$ a new BICYCLE])

b. ASSERT(EVEN(John gave his daughter [$_F$ a new BICYCLE]))

Given the focus-ground partition used by ASSERT, *even* can parasitize on it and use the structure it provides to express its meaning, as in (3b). Both (3a) and (3b) have the same structure, but in (3a) the focus-ground partition remains truth-conditionally inert whereas in (3b) the partition is exploited not only by the original illocutionary operator but also by a logico-semantic operator, thus giving rise to truth-conditional effects. In addition to *even* and *only*, quantificational adverbs, negation, modals, and other elements have also been observed to be capable of associating with focus.

2.3 Quantificational partition

Recently, a stronger and more general approach to association with focus has evolved. The focus-ground partition is not a pragmatically motivated notion anymore, but rather its primary function is to provide a quantificational structure for the use of focus-sensitive operators. The communicative use of focus-ground is merely one of the uses this quantificational structure has. Jacobs' ASSERT operator is merely another focus-sensitive operator, without any special status. The exact characterization of the quantificational structure that focus-ground provides varies from work to work. Following work by von Stechow (1981, 1991), Krifka 1991-92 suggests that focus-ground provides a structured meaning ($<\alpha,\beta>$), which in later work (Krifka 1992, 1993) he equates to the <RESTRICTOR, NUCLEUS> partition of quantification structures that Partee 1991 defends. In contrast, for Rooth 1985, 1992, focus on an element x provides a set of alternatives for x (ALT(x)) which determines the domain of quantification for the operator that associates with x.

Given this view of focus-ground, it is imperative that focus-sensitive operators undergo association with focus, since they crucially depend on the structure provided by focus-ground to express their meaning.

This view of focus-ground has important consequences in examples like (4):

(4) a. Even [$_F$ JOHN] drank only [$_F$ WATER].

b. John even [$_F$ only [$_F$ drank WATER]]

These sentences, from Krifka 1991-92, have more than one focus-sensitive operator. Viewing association with focus as a mere tendency, the examples in (4) are quite uninteresting. Perhaps one operator does associate with focus (i.e. is parasitic on ASSERT), but the other operator associates with some other element in the sentence. But if we assume that focus-sensitive operators crucially depend on the structure provided by focus-ground to be able to express their meaning, we must conclude that these sentences have more than one focus-ground partition, as represented by the labeled bracketing.

As noted above, this presumption is the main concern of this paper. We argue that focus-ground is primarily an informational or communicative (or illocutionary) notion, which can be parasitized on by focus-sensitive operators. However, the presence of more than one of these operators does not entail the existence of more than one focus-ground partition, since focus-sensitive operators may associate with elements other than foci.

2.4 Pitch accent

Before moving on to the main body of the paper, a word about pitch accent and the phonological use of the word 'focus'.

Following the models of prosodic phonology in Pierrehumbert 1980 and Pierrehumbert & Hirschberg 1990, and Ladd 1980, 1988, 1990, we take perceived prominence to be correlated with pitch accent. A pitch accent is a simplex or complex tone—there are up to six types in Pierrehumbert's theory—associated with a stressed syllable. Sentences consist of one or more intonational phrases and each phrase has a single most prominent pitch accent, which is called the nucleus of 'focus'. Thus, a single sentence may contain multiple prosodic foci.

This multiplicity of pitch accents (for the sake of clarity, we will refrain from using athe term 'prosodic focus') is illustrated in the text in (5). All the items in small caps in (5) were associated with a pitch accent when the text was read out loud by an informant:

(5) It would have been unusual if Catherine Malfilano had NOT become involved somehow in THEATER and MUSIC, considering that her FATHER is a VIOLINIST in the orchestra at the Met and her MOTHER, a DANCER, was a member of the Met's BALLET company. (*The Opera 1992*, J. Ryder, 1991)

This text also illustrates two accent types that will be of special relevance in the discussion below: Jackendoff's (1972) A accent and B accent. *Father* and *mother* are associated with a B accent and *violinist* and *ballet* are associated with an A accent. In Pierrehumbert's terms, the A accent corresponds to a simplex H* tone (generally followed by an L boundary tone), whereas the B accent corresponds to a complex L+H* fall-rise (Pierrehumbert (1980:35) expresses some reservations about identifying Jackendoff's B accent to L+H*, but in Pierrehumbert & Hirschberg (1990:296-7) these reservations are overcome). The contour formed by an L+H* followed by an H*(L) has been called the 'suspension bridge contour' (Bolinger 1961) and the 'hat pattern' (Cohen & 't Hart 1967). The B accent is generally not correlated with a focus in a focus-ground partition. Nevertheless, it will be shown that focus-sensitive particles may associate with a B-accented item.[2]

3 Claims

We are now in a position to review the assumptions and the claims about focus made in the literature on association with focus. This claims concern the three notions of focus mentioned above and the relations that exist between them: how are phonological pitch accents, semantic nuclei in a nucleus-restrictor structure, and pragmatic foci in a focus-ground partition related? It must be noted that some of these claims are made implicitly rather than explicitly when a distinction fails to be made. These claims are summarized in Figure 1 and listed in (6):

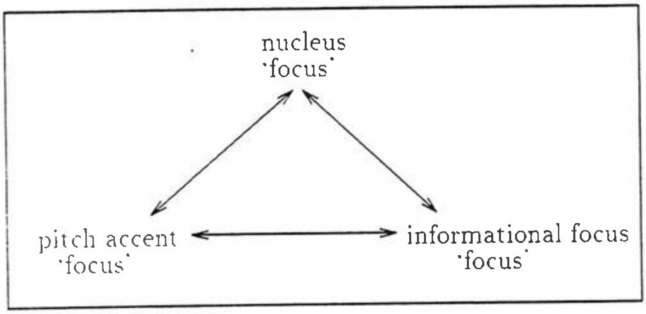

Figure 1: **Claims**

(6) a. If x is a nucleus, then x is [+pitch accent].
b. Every pitch accent is correlated with a nucleus.
c. If x is an info focus, then x is [+pitch accent].
d. Every pitch accent is correlated with an info focus.
e. If x is a nucleus, then x is info focus.
f. If x is an info focus, then x is a potential nucleus.

In what follows each one of these claims and assumptions will be examined. First, the relationship between pitch accents and quantificational nuclei will be considered (lefthand side of Figure 1). Then, we will show and discuss examples of nonfocal pitch accents, i.e. accents associated with an element in the ground, and

[2] Focal fall-rise accents do occur in contexts in which speakers wish to convey uncertainty. Consider (i), from Ward and Hirschberg (1985:774) (\/ enclose the item associated with the fall-rise accent):

(i) a. Do you have jello?
b. We have \PIE/.

The fall-rise in (ib) conveys uncertainty about whether some sweet other than jello, i.e. pie, is of interest to the querier. The fall-rise associated with uncertainty, however, is not a B accent: it is not an L+H* but rather an L*+H (Ward and Hirschberg 1985:750). Apparently, when a focal accent and an uncertainty accent compete for the same structural slot, the latter takes preeminence over the former. In addition, Pierrehumbert & Hirschberg (1990:296-7) mention some examples where focus appears to be associated with an L+H* accent. It is unclear to us what the semantic value of this type of focal accent is, although Pierrehumbert & Hirschberg talk about 'correction'.

to accents associated with a nonstandard subsegment of the focus (bottom of Figure 1). Finally, the implication that quantificational nuclei are necessarily foci in a focus-ground partition will be countered (righthand side of Figure 1). After due consideration of all the evidence, it seems that the only implications that can be maintained are those in (6′):

(6′) c. If x is an info focus, then x is [+pitch accent].

f. If x is an info focus, then x is a potential nucleus.

Implication (f) is a weak implication, in that it entails the nonidentity of foci and nuclei. This is precisely the position defended here: the arguments in this paper suggest that while nuclei may be foci, they may be part of the ground as well. Implication (c) is quite uncontroversial and has actually been heralded as a universal (e.g. Sgall *et al.* 1986, Lambrecht 1987). It seems to hold both for languages that realize focus largely by phonological means and for languages that do it largely syntactically. In the former language type, nuclear stress may shift to different positions in the clause to associate with different focus assignments, while the syntactic structure remains constant. In the latter type, the intonational structure remains constant and syntactic operations are needed to bring the focal constituent and the nonshifting nuclear stress together.[3]

4 Pitch accents and nuclei

This section examines the nature of the relationship between pitch accent and quantificational nuclei, i.e. claims (a) and (b) in (6).

4.1 Nucleus ⟶ [+pitch accent]

In standard cases of association with focus, differences in the placement of nuclear stress appear to be correlated with different <RESTRICTOR, NUCLEUS> structures. Sentences (7a) and (7b) differ not only prosodically but also in their truth value. In their default readings, (7a) means that John introduced no one other than Bill to Sue and (7b) that John introduced Bill to no one but Sue. In other words, *Bill* is the quantificational nucleus in (7a) and *Sue* is the quantificational nucleus in (7b):

(7) a. John only introduced BILL to Sue.

b. John only introduced Bill to SUE.

It is readily observable that in both cases the quantificational nucleus is realized with nuclear stress. The claim that nuclei must be spelled out by a pitch accent is quite standard in the semantic literature on association with focus, as illustrated by the following two quotes:

> 'Thus a pitch accent is the phonological interpretation of the focus feature.' (Rooth 1985:19)
>
> 'In phonology, the focus feature is spelled out by sentence accent.' (Krifka 1991:17)

The fact that both these quotes use the term 'focus' to denote a nucleus is potentially confusing, but it is clear they refer to a nucleus i a quantificational <RESTRICTOR,

[3] It is unclear whether the validity of (6c) extends to languages like Navajo (Schauber 1978), which realize focus-ground morphologically. Japanese, which makes at least partial use of morphology to realize focus-ground, appears to have some prosodic manifestation of focus, although focal accents in Japanese are less distinguishable than their English counterparts (see Lambrecht 1987:380).

NUCLEUS> partition, since for both Krifka and Rooth the location of the nucleus is determined by the presence of a focus feature.

There are some immediate problems for the implication that a nucleus must be correlated with a pitch accent. Sentences where the nucleus does not receive a pitch accent are, in fact, not deviant in any sense, given proper contextualization. Both Partee 1991 and Krifka 1992 notice this, but nevertheless decide to maintain the implication that nuclei are accented and suggest an explanation for the 'deviant' behavior of these nonaccented nuclei. Take examples (8) and (9), from Partee (1991:179) and Vallduví (1992:163), respectively:

(8) a. Eva only gave xerox copies to the GRAduate STUdents.

b. (No.) PETR only gave xerox copies to the graduate students.

(9) a. Who always took JOHN to the movies?

b. MARY always took John to the movies.

In (8b) the nucleus, i.e. the element associated with *only* is the unaccented *graduate students*. Likewise in (9b), where the element associated with *always* is the unaccented *John* (the meaning of (9b) is that at every time interval where Mary takes someone to the movies, she takes John to the movies). Nuclear stress in both (8b) and (9b) is associated with the pragmatic foci of these sentence, not with the nuclei of the focus-sensitive operators they contain.

Both Partee and Krifka argue that the associated nuclei in (8b) and (9b) are foci. Thus, (8b) and (9b) are sentences with two analogous foci: one of them is associated with an overt semantic operator, the other is associated with an implicit pragmatic CONTRAST operator, for Partee, or an ASSERT operator, for Krifka. To account for the fact that the nuclei *graduate students* in (8b) and *John* in (9b) are not associated with a pitch accent, Krifka (1992:233), commenting on an example practically identical to (9b), stipulates that, in sentences with more than one (pragmatic or semantic) focus-sensitive operator, it is always the nucleus associated with the highest operator which is realized with nuclear stress. In (9b) the illocutionary operator ASSERT (see discussion of Jacobs 1984, 1991 in § 2.2) is higher than the quantificational adverb and, therefore, accent is associated with the nucleus of the former. Presumable, the same analysis can be given for examples like (10) (Krifka 1991-1992:22), where the nucleus of the lower operator is unaccented:

(10) [Most people drank water at some time during yesterday's party.]
John even$_1$ drank [$_{F1}$ ONLY$_2$] [$_{F2}$ water].

The reason for expecting nuclei to be always accented rests on the assumption that nuclei are necessarily foci. Since foci are uncontroversially thought of as being always realized with nuclear stress, nuclei should too. If, however, nuclei are not identified with foci, the lack of a pitch accent on the nuclei in (8b) and (9b) ceases to be a problem. In fact, as elements within the pragmatic ground of (8b) and (9b), the lack of accent on these nuclei is not unexpected at all.

Sentences (8b) and (9b), then, contain unaccented nuclei that belong to the ground. It is also possible to encounter unaccented nuclei within a wide complex-category informational focus. Consider example (11), from Nevalainen (1987:148):

(11) [$_F$ There's only a month till CHRISTMAS now]

This sentence, as is the case with all canonical existential sentences (see Sasse 1987, Vallduví 1992) is an all-focus sentence with a null ground. The nuclear stress on *Christmas* is correlated with this informational focus. The nucleus of *only* in (11) is clearly *a month*. Nevalainen's data comes from a corpus of speech carefully coded for prosody. Nevalainen's transcription reveals that *a month* is realized with no accompanying pitch accent. In fact, the only pitch accent in this sentence is the one on *Christmas*, which as expected marks informational focushood. It is worth mentioning that a full 10% of the nuclei in Nevalainen's corpus are, like the nucleus in (11), realized with no pitch accent whatsoever.

Thus, we must reject claim (a) above. It is simply not the case that nuclei must be phonologically marked with a pitch accent. Krifka's (1992) stipulation that only the nucleus associated with the highest operator is accented appears to provide an escape hatch, but a counterexample will be discussed in § 6.

4.2 [+pitch accent] ⟶ nucleus

The assumption that every pitch accent is (partially) correlated with a nucleus is manifest in examples like (12), from Rooth (1992:80):

(12) In [$_F$ MY] opinion, in the [$_F$ OLD] days, in [$_F$ THIS] country...

Rooth 1992 argues that all the pitch accents in (12) are pragmatic foci associated with an F feature that gives rise to a set of alternatives used, in this case, merely for contrast. Calling all the accented items in (12) foci is stretching the notion of informational focus excessively, although it may be the case that they are nuclei associated with some abstract 'contrast' operator. Still, although it may be argued that all the accented items in (12) are contrastive, it is not entirely clear that all pitch accents are connected with a contrastive interpretation. Take example (13):

(13) That Ann—she's such an INteresting PERson.
She dances the taranTELla with a PASsion;
she grew up in SOUTH DaKOta,
and she studied classical ChiNESE at HARvard.

The informational focus in each of the four sentences in (13) is the verb phrase. In the last of these sentences, despite the fact that the pitch accent on *Harvard* suffices as an expressor of (pragmatic) focushood, the pitch accent on *Chinese* is obligatory. What motivates the pitch accent on *Chinese*? *Chinese* is clearly not the nucleus of any overt operator (there is none), nor it seems feasible to suggest that there is some 'abstract' contrast operator associating with it. In the context of (13) there is no explicit contrast between (classical) Chinese and any other language.[4]

The actual reason for the presence of the pitch accent on *Chinese* is hard to pin down. Most likely, it has nothing to do with focus-ground, quantificational nuclei, or contrast. Bolinger (1989:357) argues that words are accented if they are informative and interesting. Zacharski 1993, where this notion of informativeness or interestingness is pursued, points out that items than depart from some semantic or cultural stereotype, attract phrasal accentuation. If we replace *Classical Chinese* with something that is relatively uninteresting, the accentuation changes. None of the direct object NPs in (14) requires association with a pitch accent:

(14) a. She took courses at HARvard.
b. She studied English at HARvard.
c. She got a degree from HARvard.

[4] Of course, (classical) Chinese stands in contrast with, let us say, other members of the set of languages that can be studied in an American university. However, this contrast is not any more explicit here than the contrast that would arise from the unaccented constituents in (i):

(i) The middle-income woman bought the average-sized PICK-UP.

Evidently, unaccented *the middle-income woman* stands in contrast with women of other income brackets, although such contrast remains latent (or as latent as the contrast on *Chinese* in (13)).

In addition to this interestingness dimension, there are metalinguistic and emotive factors involved in accent determination. Marked accenting patterns may be used to carry out metalinguistic corrections even in languages that do not have a flexible intonation contour like Catalan and Hungarian (see Vallduví 1992). Pitch accents on certain structural positions can also be used to express solidarity (McLemore 1991) or to assist in the choreography of discourse, e.g. getting the floor, using turn-taking devices, beginning and ending a topic or discourse (Lehman 1977).

Accent placement in English is determined by a number of different independent factors, which include focushood (as in focus-ground), interestingness or informativeness, emotiveness, and others. In addition, as we will see below in § 5.2, English uses yet another (de)accenting strategy to express givenness or repetition of an item. In sum, there is no simple one-to-one relationship between semantics or pragmatics and the location of pitch accents.

Before moving on to other claims in (6), a qualification is in order. It is not the case that every pitch accent must constitute by itself a quantificational nucleus. Sometimes two pitch accents may correspond to a single complex-nucleus. This occur in examples like (15), from Krifka (1991-92:21):

(15) John only introduced BILL to SUE.

In (15) *Bill* and *Sue* from, as a pair, one single nucleus. Rooth (1985:60) describes the meaning of (15) as 'if John has the property of the form 'introduce x to y', then it is the property 'introduce Bill to Sue'. It is because of examples like this that one must say that every pitch accent is *partially* correlated with a nucleus.

5 Pitch accents and foci

5.1 [+pitch accent] ⟶ info focus

Let us move to the bottom half of Figure 1 and discuss implication (6d), i.e. the claim that pitch accents must be informational foci. In § 4.2 it was pointed out that some pitch accents appear to be unrelated to focus-ground structure, which, if correct, is an obstacle to the truth of (6d). The claim, however, does not hold even if we restrict our attention to information structure: pitch accents may appear within the ground of a focus-ground structure.

There is ample reference to this fact in the literature. Bardovi-Harlig 1983 emphasizes that the claim that ground elements must be unaccented is empirically unmotivated and cites examples from over a dozen sources. Bolinger (1986:47) states that 'since theme and rheme [= ground and focus] are fundamental to the meaning of the utterance, each is separately highlighted'. Jackendoff 1972 points out that 'topics', which appear within the ground in focus-ground partitions are associated with a fall-rise accent (B accent) and notes that topicalized elements (not focus-preposed elements) are always associated with such an accent. Finally, Steedman 1991 clearly treats L+H*-accented items as being part of the ground, which he calls theme.

It is not every item within the ground that is amenable to accenting. The L+H* or fall-rise accents we have been discussing here are associated with a specific ground element which is generally referred to as (shifted) topic, anchor, or link (see Jackendoff 1972, Gundel 1974, Ronat 1979, among others). In Vallduví's (1992) system, briefly discussed in § 2.1, this element is called link and this is the term we will use here. As noted, the function of the ground is to usher I_s to the right location in the hearer's model of the common ground (F_1). The ground is articulated into two parts: link and tail. Each performs a particular task within the general ushering function of the ground.[5]

Ann and *Clara* in (16) are typical subject links. They are associated with an accent, although they are part of the ground in any traditional pragmatic sense of the word. The foci in the answers to the question in (16) are the direct objects:

(16) What are people wearing to the concert?
Well, [$_L$ ANN] is wearing [$_F$ a black PANT suit]
and [$_L$ CLARA] is wearing [$_F$ a long black DRESS.]

When links are nonsubjects, there are two strategies to express linkhood. One is syntactic: the link XP may be fronted in a topicalization configuration. The other is purely prosodic: a link phrase may be left in situ and marked with an L+H* accent. This two strategies are illustrated in (17) and (18) respectively, where *this drawer* and *that one* are realized as links:

(17) What about the drawers? What do you keep in them?
In [$_L$ THAT drawer] I keep [$_F$ my SOCKS]
and in [$_L$ THIS one] I keep [$_F$ my SHIRTS.]

(18) What about the drawers? What do you keep in them?
I keep [$_F$ my SOCKS] in [$_L$ THAT drawer]
and [$_F$ my SHIRTS] in [$_L$ THIS one.]

The fact that *this drawer* and *that one* in (18) appear in sentence final position and are associated with a pitch accent should not lead us to think they are foci. The pitch accent associated with them is L+H* and the total information-structure equivalence of (17) and (18) show they are not foci but links.[6]

At this point it is interesting to compare English to Catalan, since Catalan is a language that does not allow realization of linkhood by prosodic means. In Catalan, all links must be topicalized. Example (19) is the equivalent of English (16), and (20) is the only equivalent of both (17) and (18), since (21) is an infelicitous sequence:[7]

(19) Què es posaran, per anar al concert?
[$_L$ L'Anna$_1$] es posarà [$_F$ un tern NEGRE] t_1.
i [$_L$ la Clara$_1$] es posarà [$_F$ un vestit de gala NEGRE] t_1.
'What will they wear to the concert?
'Anna will wear a black suit and Clara will wear a long black dress.'

(20) I als calaixos què hi guardes?
En [$_L$ aquest CALAIX$_1$] hi$_1$ guardo [$_F$ els MITJONS]
i en [$_L$ aquell CALAIX$_1$] hi$_1$ guardo [$_F$ les SAMARRETES.]
'And the drawers, what do you keep in them?
In that drawer I keep my socks and in this drawer I keep my shirts.'

(21) # I als calaixos què·hi guardes?
Guardo [$_F$ els MITJONS] en [$_L$ aquest CALAIX]
i [$_F$ les SAMARRETES] en [$_L$ aquell CALAIX.]

[5] Suffice it to say here that links designate a file card *fc* in F_1 (the hearer's model of the common ground) as the locus of information update with I_s. As noted in § 2.1, *fc* for S_n may be inherited from S_{n-1}, in which case no link is necessary. Tails, on the other hand, designate a given record R (a relation or attribute) already listed on *fc*. If a tail is present in the ground, I_s updates F_1 by completing or altering R.

[6] It must be noted that the prosodic difference that generally exists between links and foci in English appears to be entirely optional in languages like German. Féry 1992 shows that in German either an L+H* accent and an H*(L) falling accent may be used to realize linkhood. This second accent is phonologically identical to the pitch accent used for the realization of focushood.

[7] As can be deduced from the position of the subject traces in (19), Catalan is taken to be underlyingly VOS (see Solà 1992, Vallduví 1993).

For whatever reason Catalan does not exploit intonational resources as much as English does. Rather it must resort to syntax to express focus-ground structure. In languages like Catalan failure to distinguish between link and focus is less likely to happen, since their structural realizations are so drastically different. In English and German, however, where both may be realized exclusively by means of pitch accent, we must be careful not to treat the two elements in the same way. In these languages, a pitch accent need not be associated with a focus, but rather may be associated with a subpart of the ground, i.e. the link.

5.2 Info focus ⟶ [+pitch accent]

This implication is generally viewed as uncontroversial: foci are associated with a pitch accent. However, the exact identity of the focal constituent associated with this pitch accent has sometimes been mistaken. The exact placement, within a focal constituent, of the focal pitch accent may be the result of the interaction of a number of independent processes. Consider Clara's response in (22b):

(22) a. Ann: What did you get Ben for Christmas?
Clara: I got him [$_F$ a blue SHIRT.]

b. Ann: What did you get Diane?
Clara: I got her [$_F$ a RED shirt.]

In Clara's response in (22b), nuclear stress is associated with the lexical item *red*. Often, *red* is taken to be the pragmatic focus of this sentence as well. However, as the question contextualization shows, the focus is not *red* but rather *a red shirt*. Whereas the placement of a pitch accent on the focal constituent is indeed a correlate of focus-ground structure, the exact location of that accent within this focal constituent is determined not by focus-ground structure, but rather by an independent process of deaccenting (see Ladd 1980, 1983). *Red* ends up with a pitch accent because *shirt* is deaccented. This deaccenting is triggered by the presence of the phrase *a blue shirt* in the answer in (22a). If (22a) were not present, the answer in (22b) would be realized with an accent on *shirt*. The reason d'être behind this deaccenting phenomenon is a matter of debate. One can appeal to some anaphoric device like concept-givenness (van Deemter 1992) or to the notion of informativeness (or interestingness) in Zacharski 1993: in the context of (22), the interesting or informative bit within the focus of the answer in (22b) is the fact that the shirt Clara got Diana is red, since this is the information that distinguishes this shirt from the one Clara gave to Ben.

Interestingly, the deaccenting strategy shown in (22b) is not available in all languages. In languages like Catalan and Italian, the factor that triggers deaccenting of *shirt* in (22b) is not at work, or perhaps manifests itself differently. Examples (23) and (24) are the Catalan and Italian equivalents of English (22) (the examples have been slightly modified to get around the fact that Catalan and Italian are NAdj instead of AdjN). In both (a) and (b), the focal pitch accent falls on the last accentable item within the focus constituent. The fact that in the (b) sentences this item has already been mentioned does not affect accent placement in the least:

(23) a. Anna: Què li vas regalar, al Benjamí, per Nadal?
Clara: Li vaig regalar [$_F$ una camisa NEGRA.]
iobj 1s-past-give a shirt black
'I got him a black shirt.'

b. Anna: I a la Diana, què li vas regalar?
Clara: Li vaig regalar [$_F$ uns pantalons NEGRES.]
'I got her black pants'

(24) a. Anna: Cosa hai regalato a Benjamin per Natale?
Clara: Gli ho regalato [F una camicia NERA.]
iobj 1s-past-give a shirt black
'I got him a black shirt.'
b. Anna: E cosa hai regalato a Diana?
Clara: Le ho regalato [F dei pantaloni NERI.]
'I got her black pants.'

The contrast between English on the one hand and Catalan and Italian on the other is quite striking. To get a full grasp of what English would look like without the deaccenting rule, or, equivalently, to see what Catalan and Italian do, it is illuminating to pretend that the dialogue in (25) is made up of grammatical sentences, i.e., that English is NAdj:

(25) a. Ann: What did you get Ben for Christmas?
Clara: I got him [F a shirt BLACK.]
b. # Ann: What did you get Diane?
Clara: I got her [F pants BLACK.]

If the sentences in (25) were grammatical, Clara's response in (25b) would clearly be infelicitous. Native speakers of English cringe at the thought that this sentence could be felicitous after the previous mention of *black* in (25a).

Accented items like *red* in (22b) are indeed correlated with a focus constituent, in this case *a red shirt*. They do not, however, constitute narrow foci by themselves. They are associated with a pitch accent simply as a result of the deaccenting of a neighboring item. Both a careful analysis of the English facts and a contrastive look at Catalan and Italian suggest that this is the right analysis for patterns like (22b). Therefore, assigning focus semantics or pragmatics to an item like *red* is an unmotivated step, despite the presence of a focal pitch accent on this item.[8]

6 Nucleus ⟶ info focus

Perhaps the most important claim found in the literature on association with focus under consideration is that all nuclei are foci in the traditional sense, i.e. foci in an informational focus-ground partition. This claim corresponds to implication (6e) and is reflected on the righthand side of Figure 1. Identifying nuclei as foci is a necessity only if it believed that the quantificational structure of focus-sensitive operators is necessarily determined by focus-ground. This is view defended in, for example, Krifka 1991-92, 1992 and in Rooth 1985, 1992. From this standpoint, operators like *only* and *always* need to associate with a pragmatic focus because only focal constituents have the right semantics to act as their nuclei.

In this section we show that focus-sensitive operators may associate with elements other than a focus in a focus-ground partition. This mere possibility shows that nuclei with the kind of semantics required by operators like *only* become available through sources other than focus. First, we examine nuclei that are subsegments of the informational focus and then we discuss nuclei that are part of the ground.

[8] Other (de)accenting effects interfere with the default mapping between focus-ground and prosody. Accenting constraints seem to play a role in the following contrast, noted by R. Ladd:

(i) [F JOHNSON died.]
(ii) * [F Former president JOHNSON died unexpectedly.]

Thetic all-focus utterances like (i) are realized in one prosodic phrase with the pitch accent associated with the subject. However, when the subject and the predicate are made 'heavier', as in (ii), inclusion of all the material within one phrase is impossible. The predicate must be associated with an additional pitch accent.

6.1 Subfocal nuclei

Consider example (11) from § 4 again:

(11) [$_F$ There's only a month till CHRISTMAS now]

It was noted above that this is an all-focus example, where the nucleus of *only*, *a month*, is realized with no pitch accent. *A month* is clearly not the focus of (11), but rather a phrase within it. Nevertheless, it is capable of acting as a nucleus. Somehow, it provides the correct <RESTRICTOR, NUCLEUS> partition (or the required p-set) for *only*. Another case in point, although prosodically distinct, is (26b). Here, again, the nucleus is subfocal, but, unlike (11), it is associated with a pitch accent (the meaning of (26b) is that you will dream about your man but will do nothing else to him or with him):

(26) How will we relieve our libido?

a. Well, I will [$_F$ go to bed with my MAN].

b. but you'll [$_F$ only DREAM about YOURS].

[H.B. in conversation (23/11/93)]

The focus in (26b) is not the verb *dream*, but rather the entire verb phrase, as the question contextualization and the parallelism with (26a) indicate. Nevertheless, *only* associates with *dream* without any difficulty, i.e. *dream* is able to act as nucleus despite the fact that it is not a focus on its own.

Of course, if it is assumed that there may be focus-ground partitions within the focus of a simplex sentence, then it could be claimed that *dream* is a focus within the larger verb-phrase focus. This idea, however, is not unproblematic. If *dream* were a focus one would expect it to be realized as such in all languages, independently of whether these languages use prosody or syntax in their structural treatment of focus-ground. But in Catalan, for instance, *dream* is not realized as a focus in the Catalan equivalent of (26). In Catalan, focal constituents are, as expected, associated with a pitch accent. Unlike English, however, Catalan cannot shift prominence along the sentence. Rather, prominence is necessarily associated with the righthand boundary of the core clause. Therefore, a focal constituent must necessarily appear in core-clause-final position. If the focus-ground partition of a sentence is such that a narrow focus is required on the verb, all the elements that would otherwise appear to the right of the verb within the core-clause must be removed via left- or right-detachment. In the case at hand, for *dream* to be realized as focus, the prepositional phrase *about yours* would have to be detached. Example (27) is the Catalan equivalent of (26):

(27) Com ens ho farem, per satisfer el nostre desig sexual?
'What will we do to quench our sexual craving?'

a. Bé, jo [$_F$ me n'aniré al llit amb el meu HOME].
'Well, I will go to bed with my man,'

b. i tu [$_F$ només somniaràs amb el TEU].
'and you will only dream about yours.'

c. # i tu només hi$_1$ [$_F$ SOMNIARÀS,] amb el teu$_1$.

As shown in (27), this context allows the verb phrase *només somniaràs amb el teu* 'you'll only dream about yours' to be realized as focal, as in (27b). However, the use of the structure that would identify the verb *somniaràs* 'will dream' as focal is infelicitous in this context (27c).

Example (26b) is also a counterexample to Krifka's (1992) stipulation, discussed in § 4, that only the nucleus associated with the highest operator is realized with a pitch accent. In (26b), *only*, the operator that associates with *dream*, is lower than the putative ASSERT operator that associates with the entire verb phrase. Nevertheless, *dream* is realized with a pitch accent as or more prominent than the pitch accent associated with the larger pragmatic focus (the accent on *yours*).

6.2 Links as potential nuclei

In § 5 we emphasized the distinction between link—an element of the ground—and focus. It was noted that in languages like English and German both informational notions may be realized exclusively by prosodic means (unlike languages like Catalan or Hungarian, where the structural differences between the two are largely syntactic). This has lead to some confusion and to the conflation of the two notions in many cases.

The fact that links are also potential nuclei, i.e. focus-sensitive operators are able to associate with links adds to the confusion. Consider the following example from Vallduví (1992:143):

(28) John and Mary know the Amazon quite well,
but only John's [F been to the CITIES in Brazil.]

In the second sentence in (28) the (most natural) informational focus is the verb phrase. *John* acts as link and it is realized as such by being associated with an L–H* accent.

Hoeksema and Zwarts (1991:67) contend the force of example (28) and state that *John* is in some sense a 'focus' as well because it is associated with a pich accent. However, it was shown in § 5 that the existence of nonfocal pitch accents is beyond doubt and that links are precisely ground elements that are typically associated with a fall-rise pitch accent. Thus, the fact that *John* in (28) is accented does not make it a focus, other than in the trivial sense of being a phonological focus (a pitch accent).

Again, the comparison of (28) with its Catalan equivalent is rather suggestive. In Catalan postverbal (in-situ) subjets are focal, whereas preverbal (topicalized) subjects are ground elements (links). Consider (29):

(29) Who has been to the cities in Brazil?

a. Només hi_1 ha estat el JOAN, a les ciutats del $Brasil_1$.

b. # Només el Joan hi_1 ha ESTAT, a les ciutats del $Brasil_1$.
'Only JOHN's been to the cities in Brazil.'

In (29) the question contextualization requires *John* to be the focus of the answer, since it corresponds to the wh-phrase in the question. In (29a) *John* appears in the focal postverbal position, while in (29b) it appears in the preverbal position characteristic of links (*a les ciutats del Brasil* 'to the cities in Brazil' is also part of the ground and is right-detached in both potential answers). Indeed, only (29a), which encodes *John* as a focus, is a felicitous answer in this context. Realizing *John* as a link, as in (29b), is out. Now compare (28) to its Catalan equivalent in (30):

(30) El Joan i la Maria coneixen l'Amazones bastant bé
però només el Joan ha estat a les ciutats del BRASIL.
'John and Mary know the Amazon quite well,
but only John's been to the cities in Brazil.'

The *John* in the second conjunct in (30) felicitously appears in a preverbal position, a position that is associated with a link interpretation. The contrast between (29b) and (30) can be accounted for only if the link status of *John* in the latter is recognized. Therefore, it must be concluded that ground elements—links, at least—can act as nuclei to focus-sensitive operators like *only*. The assumption that only foci can provide the necessary semantic structure that nuclei require is not well motivated.

In fact, it is not only links that may act as nuclei. Other ground elements, realized with no pitch accent whatsoever, can be nuclei as well. Examples of this sort were already discussed in § 4. In § 4, (8b) and (9b), repeated here, were discussed as examples of unaccented nuclei:

(8) a. Eva only gave xerox copies to the GRAduate STUdents.

b. (No,) PETR only gave xerox copies to the graduate students.

(9) a. Who always took JOHN to the movies?

b. MARY always took John to the movies.

From the standpoint defended in this paper, the fact that the nuclei in (8b) and (9b) are unaccented is unproblematic. These nuclei are plain ground elements and there is no informational reason for their association with a pitch accent. Since nuclei do not need to be foci, there is no need for any additional explanation to account for the fact that the nuclei of *only* and *always* are not realized with typical focal prosody. These nuclei are simply not informational foci. Once again, we are compelled to conclude that nonfocal elements can act as quantificational nuclei.

Krifka 1991-92 explicitly treats some link nuclei as focal nuclei in examples like (31). He claims that *youngest* is a focus within the ground (or within the link or topic) of the sentence. Thus, he argues, sentences like (31), which appear to have a 'focus' within the link (for him, within the topic), constitute evidence for the recursive status of the informational partition of sentences:

(31) What did Bill's sisters do?
$[_L$ Bill's $[_F$ YOUNGEST] sister] $[_F$ kissed JOHN.]

Although this would provide support for the view that more than one focus-ground structure is allowed in a simplex sentence, it also represents, as noted, a significant departure from the traditional conception of focus in the pragmatic literature. However, on closer inspection, the evidence for the focal status of *youngest* fades away. Example (31) is, in fact, analogous to example (32):

(32) What did Bill's siblings do?
$[_L$ Bill's SISTER] $[_F$ kissed JOHN.]

In both (31) and (32) we have a sentence with a link pitch accent (L+H*) and a focal pitch accent (H*(L)) in a typical 'suspension bridge' pattern. The pitch accent on *sister* in (32) is clearly a characteristic link-associated accent of the type discussed in § 5 and so is the pitch accent on *youngest* in (31). The only difference between (31) and (32) is that in the former sentence the pitch accent is not realized on the rightmost item within the link but rather appears to have shifted to the left.

This, however, is not enough motivation to argue that *youngest* is a focus within the link. In fact, the leftbound prosodic shift in the link in (31) is not the reflection of any informational or communicative effect, but rather another instance of the deaccenting phenomenon discussed in § 5.2. The deaccenting process discussed in § 5.2 took place within the focus constituent, but, as (31) shows, deaccenting can take place within the link constituent as well. The pitch accent on *youngest* in (31) is the same pitch accent associated with *sister* in (32). It expresses the linkhood of the subject noun phrase and it does not indicate that *youngest* is a focus in any sense. Thus, (31) should not be informationally partitioned as Krifka indicates, but rather as in (33)

(33) What did Bill's sisters do?
$[_L$ Bill's YOUNGEST sister] $[_F$ kissed JOHN.]

Steedman 1991 notices the different prosodic pattern that one may encounter within focus and within ground. However, he does not suggest that the accenting shifts within these larger units imply that embedded focus-ground partitions exist within the top-level partition. Rather, he relates them to a notion of (de)emphasis, akin to the notions of concept-givenness and interestingness or informativeness that we discussed in § 5.

Of course, given that languages like Catalan and Italian do not possess a deaccenting strategy, we would expect these languages not to display prosodic patterns like the one within the link in (31). This is indeed the case. Consider the following examples ((34) is from Steedman 1991):

(34) A: I know Mary's undergraduate degree is in physics,
but what subject is her doctorate in?
B: [$_L$ Mary's DOCTORATE] [$_F$ is in CHEMISTRY.]

(35) A: I didn't know both of them have a doctorate...
B: Yes but [$_L$ MARY's doctorate] [$_F$ is in CHEMISTRY]
and [$_L$ ANNA's doctorate] [$_F$ is in LAW.]

Between these two English examples there is a prosodic contrast within the link triggered by deaccenting. In (35) the item *doctorate* within the link is deaccented because of a previous mention of that same item. If the view that the pitch accent on *Mary* in (35) is due to deccenting of *doctorate* is correct, we should expect the Catalan equivalents of (34) and (35) to be identical, since Catalan lacks the deaccenting strategy. They are. Example (36) corresponds to English (34) and example (37) to English (35):

(36) A: Ja sé que la Maria té una llicenciatura de física,
però el seu doctorat de quina especialitat és?
B: [$_L$ El doctorat de la Maria] [$_F$ és de QUÍMICA.]

(37) A: No ho sabia, que totes dues fossin doctores...
B: Sí, però [$_L$ el doctorat de la Maria] [$_F$ és de QUÍMICA]
i [$_L$ el (doctorat) de l'Anna] [$_F$ és de DRET]

If *Mary* in (35) were a focus, Catalan (37) would have to structurally reflect that somehow. However, Catalan (36) and (37) are identical. The reason is that the focus-ground partition of these sentences (and of English (34) and (35)) is identical. The prosodic contrast observe in the English examples is the reflection of an orthonogonal factor.

7 Conclusion

The data discussed in this paper show that nuclei need not be informational foci. Elements within the ground, both links and nonlinks can act as the nuclei of focus-sensitive operators like *only*. These findings are in agreement with Koktová 1987 and Nevalainen 1987, where it is also noted that nuclei can belong to either focus or ground.

The claim in Rooth 1985, 1992 and Krifka 1991-92, 1992 is that only foci have the appropriate semantic structure to be able to act as nuclei. Given that focus-sensitive operators are capable of associating with elements other than focus, this claim is found to be unmotivated. The semantic structure that characterizes nuclei (e.g. the introduction of ALT(x)) can be made available through means that have nothing to do with informational focus. In this sense, for instance it is wrong to argue that there is a one-to-one mapping between the focus-ground partition and the quantificational <RESTRICTOR, NUCLEUS> structure of a sentence. We have shown that, while it is true that a focus-sensitive operator may parasitize, piggyback style, on a focus-ground partition, they can also utilize partitions other than focus-ground to express their meaning. In other words, focus does not necessarily determine the identity of the nucleus.

One way around the problems posed by the data discussed above is to stretch the notion of informational focus so as to incorporate under this label all attested nuclei. In fact, this is precisely what these authors have to do to describe the meaning of focus-sensitive operators, since from their perspectives only focus-ground provides the necessary semantic structure that these operators require to express their semantics. Such a move, however, is not uncontroversial. The analysis of deaccenting and the comparison of English to Catalan and Italian above indicate that nonfocal nuclei are a fact which any account of the meaning of focus-sensitive operators should take into account.

Accepting that nonfocal nuclei exist frees us from having to posit multiple focus-ground partitions for sentences with more than one focus-sensitive operator. Obviously, from a nuclei-must-be-foci point of view, one must argue for recursive focus-ground structures within a simplex sentence. However, if nuclei can be specified otherwise, this is not so anymore, since the presence of a focus-sensitive operator does not entail the presence of a focus. This approach naturally accommodates the many examples discussed above where the informational focus-ground partition differs from the <NUCLEUS, RESTRICTOR> partition. For Krifka, for instance, these examples are all cases with more than one focus-ground partition. He then has to provide an additional account for the fact that some of these foci are not structurally realized as such. No such additional mechanism is needed if the position defended here is the correct one. Of course, if the structured meaning approach to quantificational structure is correct, sentences with more than one operator would still require more than one <NUCLEUS, RESTRICTOR> partition. We have shown, however, that this does not entail that they have more than one focus-ground partition.[9]

The relationship between quantificational structure, informational structure and prosody needs to be accounted for with a more modular approach. The focus-ground partition is present in sentence structure for communicative purposes, probably having to do with the way in which information is presented to an updating agent. Pitch accents are available as a structural resource and they appear to be exploited for a number of different uses. Some of them have to do with the realization of focushood and linkhood, while other accenting phenomena are linked to other independent factors. Finally, an independent quantificational structure must accompany each operator in a sentence. The interaction among these components, however, is complex. Pitch accents do not appear to have a unique interpretation, focus-ground is realized in different ways in different languages, and nuclei may be accented or deaccented and maybe focal or ground.[10]

[9] Embedded sentences may have their own focus-ground structure, so in some sense focus-ground is indeed recursive. This type of recursiveness, however, is acknowledged in the informational literature and will not be discussed here, although the fact that embedded sentences may display informationally motivated structural properties deserves a more detailed analysis.

[10] Current work within DYANA-2 is looking at how this modular approach to quantificational structure, informational structure, and prosody can be spelled out from the perspective of sign-based syntactic formalisms.

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Tree Models and (Labeled) Categorial Grammar

Yde Venema

Abstract. This papers studies the relation between some extensions of the non-associative Lambek Calculus NL and their interpretation in tree models. We give various examples of sequents that are valid in tree models, but not derivable in NL. We argue why tree models may not be axiomatizable if we add finitely many derivation rules to NL, and proceed to consider labeled calculi instead.

We define two labeled categorial calculi, and prove soundness and completeness for interpretations that are 'almost' the intended one, namely for tree models where all resp. some trees may be infinite. Extrapolating from the experiences in our quite simple systems, we briefly discuss some problems involved with the introduction of labels in categorial grammar, and argue that many of the basic questions are not yet understood.

1 Introduction

For a long time, the associative Lambek calculus has been the predominant formalism in categorial grammar, and language models (free semigroups, string models) its standard model-theoretic interpretation. Recent years however have seen a proliferation of both alternative calculi and alternative interpretations. The reasons for this development stem from both logic and linguistic origins. In logic for instance Lambek's calculus has found itself surrounded by a whole landscape of of so-called *substructural logics* (cf. DOŠEN & SCHRÖDER-HEISTER [6]), and also connections with modal logic have been investigated (cf. VAN BENTHEM [1]); in linguistics, it was realized that the Lambek calculus is not a suitable device for studying phenomena like discontinuous constituency or head dependency (cf. MOORTGAT [16]).

The aim of this paper is to contribute to both model theory and proof theory of categorial grammar by studying a very simple example in detail. In order to formulate the motivation for writing this paper more precisely, let us start with a formal definition of this problem:

Definition 1 *Given a set Pr of primitive types, the set $Tp(Pr)$ of types is formed by closing Pr under the binary connectives $\circ$ ('times'), $/$ ('over') and $\backslash$ ('under'). A sequent is of the form $X \longrightarrow A$ with X a term and A a type; here the set of* **terms** *is defined as the closure of $Tp(Pr)$ under the structural connective $(\cdot,\cdot)$.*

We are interested in the following semantics for this language. Consider a set L of elements called **leaves**. $\mathtt{Tree}(L)$*, the set of* **trees over** L*, is defined as follows: any leaf is a tree, and if s and t are trees, then so is (st). A* **finite-tree model** *is a pair $\mathfrak{M} = ((\mathtt{Tree}(L), V)$ where V is an* **interpretation** *mapping basic types to subsets of $\mathtt{Tree}(L)$. V can be extended to types and terms as follows:*

$$
\begin{array}{rcl}
V(A \circ B) & = & \{(st) \in \mathtt{Tree}(L) \mid s \in V(A), t \in V(B)\} \\
V(A/B) & = & \{s \in \mathtt{Tree}(L) \mid (st) \in V(A) \text{ for all trees } t \text{ with } t \in V(B)\} \\
V(A\backslash B) & = & \{s \in \mathtt{Tree}(L) \mid (ts) \in V(B) \text{ for all trees } t \text{ with } t \in V(A)\} \\
V(X,Y) & = & \{(st) \in \mathtt{Tree}(L) \mid s \in V(X), t \in V(Y)\}.
\end{array}
$$

We usually denote $s \in V(A)$ by $\mathfrak{M}, s \Vdash A$, or if no confusion arises, by $s \Vdash A$. We also use terminology from modal logic, like 'A is **true** *at s' for '$s \Vdash A$'. A sequent $X \longrightarrow A$* **holds** *in a model $\mathfrak{M}$, notation: $\mathfrak{M} \models X \longrightarrow A$, if $V(X) \subseteq V(A)$; it is* **valid** *in the class of finite-tree frames, notation: $\mathsf{T_f} \models X \longrightarrow A$, if it holds in every finite-tree model.*

These models have occurred under various names in the literature, like *bracketed strings, free groupoids, non-associative category hierarchies*, etc. The central problem of the paper can now be formulated concisely:

Problem 1 *Can we find a 'nice' calculus recursively enumerating all sequents* Γ *for which* $\mathsf{T_f} \models \Gamma$*?*

where 'nice' refers to properties like cut-elimination or decidability.

Let us hasten to remark that this problem itself will be too simple to be of *direct* linguistic interest. Nevertheless, we do think it to be relevant for research on the linguistic side of the categorial grammar framework, witness the example of discontinuous constituency. To overcome the difficulties of the traditional Lambek calculus in handling this phenomenon, extensions of the categorial language with new type constructors have been proposed, cf. MOORTGAT [16]. It turned out that these new connectives do not find a natural surrounding in a string-based approach, cf. VERSMISSEN [28]. For instance, the associativity of the structural connective in Lambek's calculus seem to make it impossible to formulate a pair of natural left and right operational rules for Moortgat's infixation ($\downarrow$) and extraction ($\uparrow$) operators. The basic semantic problem seems to be that a *string* of words (`Mary rang up`) does not have a unique point where a second string (`John`) can be inserted. However, if we study finite-tree models in which every node of a tree has a distinguished *head* daughter, we can equip any tree with such a unique insertion point, viz. immediately before or immediately after the head of the tree. In [20], Moortgat and Oehrle give a nice inductive definition of a head wrapping operation on trees, thus providing a unified categorial framework for headedness and discontinuous constituency. Note that a solution to our Problem 1 would be a first step towards a proof calculus for Moortgat & Oehrle's system.

However, the above mentioned problem is not the only source of inspiration for studying Problem 1. There are in fact *two* more *kinds* of motivation.

The first one is a purely mathematical one, inspired by developments in the model theory of categorial grammars. For a long time it has been one of the outstanding open questions in this field whether the Lambek calculus is not only sound but also complete with respect to the interpretation in language models. Recently, this question has been answered affirmatively by M. Pentus (cf. [24]). The obvious counterpart of this question is whether a similar completeness result holds for the non-associative Lambek calculus NL with respect to tree models (free groupoids). Now *this* problem has been solved already a few years ago — in the negative, cf. DOŠEN [7]), but this negative answer now triggers the question whether we can *extend* NL with some simple axioms and/or derivation rules in order to obtain completeness. Note that this really is an instance of our Problem 1.

The last kind of motivation takes us to an area of logical proof theory which has has become rather active lately, viz. that of labeled deductive systems. The basic idea of a labeled deductive system is that the *structure* of the 'database' of assumptions $A_1, \ldots, A_n$ in a consequence relation

$$A_1, \ldots, A_n \longrightarrow B$$

is made *explicit* by labeling the types:

$$x_1 : A_1, \ldots, x_n : A_n \longrightarrow y : B.$$

This idea has been around in categorial grammar for some time already (cf. BUSZKOWSKI [2]), but seems to be taking off after Gabbay introduced his Labeled Deductive Systems as a general framework for reasoning with labels (cf. Gabbay [8]), and Oehrle suggested a multi-dimensional approach to formal linguistics (cf. [22]), in which linguistic objects are represented as tuples, with each coordinate providing information on a specific aspect like prosodic form or semantic meaning. In a categorial grammar framework labels seem to be the perfect vehicles to carry information other than the syntactic type, as was observed by Moortgat (cf. [18]). Grosso modo, there is the following important distinction to be made here as to the impact of the labels in the calculus.

If the labels just *follow* the proof, for instance in order to generate the meaning of a sentence fragment, we are just confronting (a generalization of) the well-known Curry-Howard isomorphism. On the other hand, in most of the recently developed systems the labels play a far more active role. For instance, MORRILL & SOLIAS [21] and HEPPLE [10] intend to solve precisely the above-mentioned problem of discontinuous constituency by formulating, in the equational theory of the label algebra, side conditions on the application of operational rules. The logical aspects of *such* applications of Labeled Deductive Systems are as yet largely unknown, although some first exercises have been carried out, witness (besides the papers cited above), CHAU [4], KURTONINA [13], ROORDA [25, 26]. The main aim of this paper is to put a few more steps in this new area: we will treat some model-theoretical and proof-theoretical investigations of our toy example.

Overview In the next section we will approach our Problem from a naive point of view. The basic idea in this section is to investigate whether a simple extension of the non-associative Lambek calculus might yield the desired completeness result. First we will define a hierarchy of frame classes generalizing the class of fintree models, for instance T (tree frames, i.e. where some trees may have infinite branches) and T_∞ (inftree frames, i.e. where all branches of a tree are infinite). In the first part of the section we will review some nice completeness results for NL itself, but then we give various examples as to why NL and some of its intuitive finite extensions will not be complete with respect to any class of tree frames. We leave it as an open problem whether our classes of tree frames allow a *finite* axiomatization in a 'pure' sequent calculus.

In section 3 we turn to *labeled* categorial grammars instead. For both the classes T_∞ and T we will develop sound and complete labeled calculi LC_∞ and LC_t. LC_∞ being nice in the sense that it allows a cut elimination theorem. Our main problem, viz. whether the class of frames of *finite* trees has a (nice) complete axiomatization, remains open.

We finish with a short section discussing some problems involved with the introduction of labels in categorial grammar. Our main conclusion here is that the logical foundations of the area seem to be unexplored yet.

2 Incompleteness for calculi without labels

In this section we will start looking for a complete calculus for (fin)tree frames by various 'naive' adaptations of the non-associative Lambek calculus NL. In the first subsection we will give some positive results concerning NL, in the last part of the section we will argue why this naive approach is unlikely to work.

2.1 The non-associative Lambek calculus

As we already mentioned in the introduction, the non-associative Lambek Calculus seems to be the natural starting point to look for a complete calculus.

Definition 2 *The non-associative Lambek Calculus NL is given by the following* **logical axiom** *and* **logical rule***:*

$$\frac{}{A \longrightarrow A}\,[Id] \qquad \frac{X \longrightarrow A \quad Y[A] \longrightarrow B}{Y[X] \longrightarrow B}\,[Cut]$$

and the following **operational rules** *for the three connectives:*

$$\frac{X[B] \longrightarrow C \quad Y \longrightarrow A}{X[Y, A\backslash B] \longrightarrow C}\,[\backslash L] \qquad \frac{(A, X) \longrightarrow B}{X \longrightarrow A\backslash B}\,[\backslash R]$$

$$\frac{X[B] \longrightarrow C \quad Y \longrightarrow A}{X[B/A, Y] \longrightarrow C}\,[/L] \qquad \frac{(X, A) \longrightarrow B}{X \longrightarrow B/A}\,[/R]$$

$$\frac{X[A, B] \longrightarrow C}{X[A \circ B] \longrightarrow C}\,[\circ L] \qquad \frac{X \longrightarrow A \quad Y \longrightarrow B}{(X, Y) \longrightarrow A \circ B}\,[\circ R]$$

Note that NL has **no structural rules***. Finally, notions like* **derivability** *and* **theorems** *are defined as usual.*

NL is the weakest logic in the landscape of so-called substructural logics, cf. DOŠEN [6] (at least, if one does not take systems like the head-dependency calculus of MOORTGAT & MORRILL [19] or ZIELONKA [29] into account). In fact it can be seen as the pure system of *residuation*, cf. the algebraic inequalities below:

$$A \longrightarrow C/B \text{ iff } A \circ B \longrightarrow C \text{ iff } B \longrightarrow A\backslash C.$$

It is well-known that the above schema has a natural reading in the power set algebra of relational structures. This gives an easy completeness result for NL, but an interesting one, as it forms the basis for our further investigations in this section.

Definition 3 *A* **(relational) frame** *is a pair $\mathfrak{F} = (W, R)$ with R a ternary* **accessibility relation** *on W. Adding an* **interpretation** *$V : Pr \mapsto \mathcal{P}(W)$, we obtain a* **(relational) model**. **Truth** *of types (and terms) is defined as follows:*

$$\begin{array}{lcl} V(A \circ B) & = & \{x \in W \mid (\exists yz)\ Rxyz, s \in V(A)\ \&\ t \in V(B)\} \\ V(A/B) & = & \{y \in W \mid (\forall xz)\ Rxyz\ \&\ z \in V(B) \text{ imply } x \in V(A)\} \\ V(A\backslash B) & = & \{z \in W \mid (\forall xz)\ Rxyz\ \&\ y \in V(A) \text{ imply } x \in V(b)\} \\ V(Y, Z) & = & \{x \in W \mid (\exists yz)\ Rxyz, y \in V(y)\ \&\ z \in V(Z)\} \end{array}$$

The class of relational models is denoted by R.

The trivial but crucial connection with finite-tree models is that a finite-tree frame becomes a relational frame by putting

$$Rstu \iff s = (tu).$$

So, the following theorem states that NL is at least sound with respect to $\mathsf{T_f}$ and complete with respect to a *superclass* of it:

Theorem 1 *NL is sound and complete with respect to* R.

Proof.
Soundness is left to the reader. The rather easy proof of the completeness direction goes by a canonical model construction. Let W be the set of deductively closed

sets of types (i.e. $\alpha \in W$ iff $A' \in \alpha$ whenever $A \in \alpha$ and $NL \vdash A \longrightarrow A'$); the accessibility relation R is defined by

$$R\alpha\beta\gamma \iff A \in \alpha \text{ whenever } B \in \beta, C \in \gamma \text{ and } NL \vdash B \circ C \longrightarrow A,$$

and the interpretation V by $V(p) = \{\alpha \mid p \in \alpha\}$. By induction to the complexity of A it is proved that $A \in \alpha \iff \alpha \Vdash A$. This implies that the canonical model is a counter model for every non-theorem of NL. □

So, to find a calculus for tree models, it might be a useful strategy to try and bridge the gap between R and $\mathsf{T_f}$. Let us define some new classes of frames:

Definition 4 *A* **groupoid** *is a pair* $\mathfrak{G} = (G, \cdot)$ *with* $\cdot$ *a binary operation on* G. *As before with tree frames, we may see* $\mathfrak{G}$ *as a special kind of relational frame by putting* $Rstu \iff s = tu$. *The class of groupoid frames is denoted by* G. *If in a groupoid frame,* $x = yz$, *we call* y *a* **left** *and* z *a* **right daughter** *of* x.

A **tree frame** *is a groupoid frame satisfying* **unique splittability**

$$(US) \qquad (Rwuv \ \&\ Rwu'v') \Rightarrow (u = u' \ \&\ v = v')$$

and **acyclicity**

$$(AC) \qquad \text{for no distinct } x_0, \ldots, x_n \text{ do we have } x_0Ex_1 \ldots x_{n-1}Ex_nEx_0$$

where E *is the relation defined by* xEy *iff* x *is a daughter of* y, *or* y *of* x.

A tree frame is a **fintree frame** *if it satisfies* **converse wellfoundedness** (CW): *there are no infinite paths* $x_0Mx_1Mx_2M\ldots$ *where* xMy *if* y *is a daughter of* x. *A tree frame is an* **inftree frame** *if every node has daughters, i.e. if it satisfies* (S): $\forall x \exists yz\ (x = yz)$.

The classes of tree frames, fintree frames and inftree frames are denoted by resp. T, $\mathsf{T_f}$ *and* $\mathsf{T_\infty}$.

We leave it to the reader to verify that the class of fintree frames defined above coincides with the class of finite-tree frames, up to isomorphism.

In a Venn-diagram we can depict these classes of relational frames as follows:

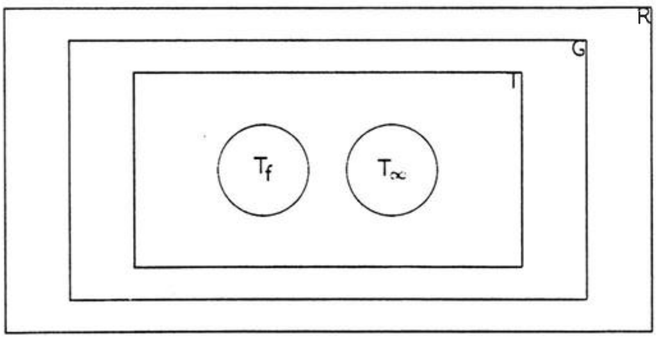

Figure 1.

Of course, these classes need not all have different theories — in fact, a consequence of the following result is that a sequent is valid in all groupoids iff it is valid in all relational frames:

Theorem 2 *NL is sound and complete with respect to* G.

The proof of this theorem is a straightforward adaptation of Theorem 2 in BUSZKOWSKI [2]), viz. that the (associative) Lambek Calculus is complete with respect to semigroup semantics. The basic idea in both proofs is to use an intermediate *labeled* natural deduction system (cf. also KANDULSKI [12]).

R and G form the only pair of frame classes in our list with identical categorial theories. In the next subsection we will give sequents separating the other classes. All these examples witness the incompleteness of NL with respect to (fin)tree semantics. Before moving to these negative results, let us mention a few more positive facts concerning NL.

Note that the existence of two notably different frame classes having identical theories is an indication of the weakness of a language. Dropping connectives from the language may leave us with a formalism of even less discriminating power. For instance, we have an interesting result for the language without product:

Proposition 5 *Let $\mathfrak{F}, \mathfrak{F}'$ be two groupoid frames such that $\mathfrak{F}'$ is a homomorphic image of $\mathfrak{F}$. If $\circ$ does not occur in the sequent $X \longrightarrow A$, then*

$$\mathfrak{F} \models X \longrightarrow A \text{ implies } \mathfrak{F}' \models X \longrightarrow A.$$

Proof.
We reason by contraposition: assume that $X \longrightarrow A$ is *not* valid in $\mathfrak{F}'$. Then there are an interpretation V' and a world w' in $\mathfrak{F}'$ such that $\mathfrak{F}', V', w' \Vdash X$ and $\mathfrak{F}', V', w' \not\Vdash A$. As a representative example, let X be of the form $(A_0, (A_{10}, A_{11}))$: then there are worlds w'_0, w'_1, w'_{10} and w'_{11} such that $w' = w'_0 \cdot w'_1$, $w'_1 = w'_{10} \cdot w'_{11}$ and $w'_i \Vdash A_i$. Now let w_0, w_{10} and w_{11} in $\mathfrak{F}$ be such that $fw_0 = w'_0$, etc. Define $w_1 := w_{10} \cdot w_{11}$ and $w := w_0 \cdot w_1$, then by the fact that f is a homomorphism, $fw_1 = w'_1$ and $fw = w'$.

Finally, define the following interpretation V on $\mathfrak{F}$:

$$V(p) := \{x \in W \mid fx \in V'(p)\}.$$

We now prove by induction on the complexity of $/, \backslash$-types that

$$(*) \qquad x \Vdash B \iff fx \Vdash B.$$

The base step of $(*)$ is immediate by definition of V.

For the induction step, we only consider the case where B is of the form C/D. First assume that $x \Vdash C/D$; to show that $fx \Vdash C/D$, let y' be such that $y' \Vdash D$. As f is surjective, $y' = fy$ for some y in $\mathfrak{F}$. The induction hypothesis gives that D is true at this y. Then $xy \Vdash C$, so by the induction hypothesis again, we find $f(xy) \Vdash C$. But $f(xy) = fx \cdot fy = fx \cdot y'$. As y' was arbitrary, this gives $fx \Vdash C/D$.

For the other direction, let x be such that $fx \Vdash C/D$. Take an arbitrary y in W with $y \Vdash D$; then $fy \Vdash D$ by the induction hypothesis. The truth definition of $/$ gives $f(xy) \Vdash C$, so by the induction hypothesis we get $xy \Vdash C$. This implies $x \Vdash C$.

To finish the proof, $(*)$ gives that $w \Vdash X$ while $w \not\Vdash A$. So $\mathfrak{F} \not\models X \longrightarrow A$. □

The above proposition allow us to give a new proof of the following theorem. It states that if we confine ourselves to the $/, \backslash$-language, NL is strong enough to capture the sequent logic of $\mathsf{T_f}$:

Theorem 3 (Kandulski) *$NL(/, \backslash)$ is sound and complete with respect to $\mathsf{T_f}$.*

Proof.
Although the original proof of this result (Theorem 1.1 in KANDULSKI [12]) is

quite easy, we enjoy showing it to be an immediate corollary of Theorem 2 and Proposition 5. The key fact is that fintree frames precisely constitute the class of *free* groupoids, whence every groupoid frame is a homomorphic image of a (sufficiently large) fintree frame. □

2.2 Tree models

In this subsection we will go into some detail as to why the non-associative Lambek calculus is not complete with respect to tree models. The following example was given by Došen in [7]: define the sequent

$$(\Gamma_0) \qquad p, p\backslash(q \circ r) \longrightarrow p \circ r.$$

It will be clear that this sequent is not derivable in NL. However, it is valid in T, as a simple but instructive argument shows:

Let $\mathfrak{M}$ be a model based on a tree frame, and assume that $t \in V(p, p\backslash(q \circ r))$. Then t has daughters t_0 and t_1 such that $t_0 \Vdash p$ and $t_1 \Vdash p\backslash(q \circ r)$. By the truth definition, $t \Vdash q \circ r$, so by the truth definition again, *and the fact that t_0 and t_1 are uniquely determined as the daughters of t*, we find that $t_0 \Vdash q$ and $t_1 \Vdash r$. But then we have $t \Vdash p \circ r$.

Clearly, the essential property used here is that of Unique Splittability (cf. Definition 3).

Following a suggestion by ZIELONKA [30], we could try to capture the fact that Γ_0 should be derivable in the logic we are heading for, by adding the following proof rule to NL:

$$\frac{X \longrightarrow A \circ B \quad X \longrightarrow A' \circ B'}{X \longrightarrow A \circ B'} \; [S]$$

and indeed, it is easy to show that in the resulting calculus NL_S, Γ_0 is derivable. Zielonka raises the question, whether NL_S is complete with respect to tree models.

Unfortunately, we have to answer this question in the negative, viz. the following sequent:

$$(\Gamma_1) \qquad p\backslash(q \circ r), p \longrightarrow r \circ p.$$

Γ_1 is not an NL_S-theorem:

An easy proof shows that NL_s is *sound* with respect to the class of R-frames satisfying (US). However, Γ_1 is not valid in every US-frame, as the following counter example (W, R, V) witnesses: $W = \{a, b, c\}$, $R = \{(a, b, c)\}$ and $V(p) = \{c\}$, $V(q) = V(r) = \varnothing$. It is immediate that $b \Vdash p\backslash(q \circ r)$, so $a \in V(p\backslash(q \circ r), p)$, while $a \nVdash r \circ p$.

On the other hand, Γ_1 is valid in T:

Let $\mathfrak{M}$ be a model based on a tree frame, and assume that $t \in V(p\backslash(q \circ r), p)$. Then t has daughters t_0 and t_1 such that $t_0 \Vdash p\backslash(q \circ r)$ and $t_1 \Vdash p$. Now the tree $(t_1 t_0)$ exists as well, and for this tree we have $(t_1 t_0) \in V(p, p\backslash(q \circ r))$. But we already showed that in this situation, ($t_1 \Vdash q$ and) $t_0 \Vdash r$. So we find that indeed $t \Vdash r \circ p$.

The essential property of tree models that we used in this argument (besides Unique Splittability) is the fact that the tree forming operation is a *total* function.

Heading for a calculus complete with respect to tree models, we might follow a naive idea and add the following proof rule to the calculus NL_S:

$$\frac{(Y, X) \longrightarrow B \circ A}{(X, Y) \longrightarrow A \circ B} \; [F_2]$$

It will be clear how to prove Γ_1 from the resulting calculus NL_{SF_2}:

$$\dfrac{\dfrac{p \longrightarrow p \quad p\backslash(q \circ r) \longrightarrow p\backslash(q \circ r)}{p, p\backslash(q \circ r) \longrightarrow p \circ (p\backslash(q \circ r))}[\circ R] \quad \dfrac{p \longrightarrow p \quad q \circ r \longrightarrow q \circ r}{p, p\backslash(q \circ r) \longrightarrow q \circ r}[\backslash L]}{\dfrac{p, p\backslash(q \circ r) \longrightarrow p \circ r}{p\backslash(q \circ r), p \longrightarrow r \circ p}[F_2]}[S]$$

However, NL_{SF_2} is not complete with respect to tree models. To see this, let us look with a bit more care at the proof as to why Γ_1 is valid in tree models. The crucial observation was that the two daughters t_0 and t_1 of the 'current' tree t, can be combined to form a new tree. However, the totality of the tree forming operation implies that *every* two elements of the frame can be combined to form a tree. This means that for instance, the sequent

(Γ_1') $$(p\backslash(q \circ r), s), p \longrightarrow (r \circ s) \circ p$$

is valid in tree frames too :

To show why this is so, one now combines t's *granddaughter* t_{00} (for which $t_{00} \Vdash p\backslash(q \circ r)$) with t's daughter t_1 (where p is true) to a new tree $(t_1 t_{00})$, and proceeds with the earlier argument.

On the other hand, Γ_1' is not derivable in NL_{SF_2}:

One proves this by first showing NL_{SF_2} to be sound with respect to the class of frames satisfying (US) and (FC_2): $\forall xy(\exists z Rzxy \leftrightarrow \exists z Rzyx)$. Then one inspects the following '$(US)\&(FC_2)$-model': $W = \{a, b, c, (ab), (ba), (ab)c, (ba)c, c(ab), c(ba)\}$; R is defined in the obvious way and V is given by $V(p) = \{c\}$, $V(s) = \{b\}$ and $V(q) = V(r) = \varnothing$. Then we find $a \Vdash p\backslash(q \circ r)$, so $(ab) \in V((p\backslash(q \circ r), s))$, $(ab)c \in V((p\backslash(q \circ r), s), p)$ while $(ab)c \nVdash (r \circ s) \circ p$.

In the end, it seems that one would have to add *infinitely* many derivation rules to the system before even coming to think of completeness [1]. Putting it differently, the essential difficulty seems to be that the following proposition holds for any tree t:

$(*)$ $t \Vdash p\backslash(q \circ r)$ implies $t \Vdash r$,
provided that *somewhere* in the frame there is a tree s with $s \Vdash p$.

Unfortunately, the sequent format of the calculus does not seem to be adequate to express this 'somewhere' concisely. There are several ways to try and solve this problem: for instance, one might think of *adding* an explicit 'somewhere' operator to the language, like in recent approaches to modal logic, cf. GORANKO & PASSY [9]. A disadvantage of this approach is that it does not fit nicely in the particular resource-sensitive paradigm of NL. It might be a better idea to go even further along the line of making information explicit that is already present in the sequent's antecedent. Note that if we evaluate a term (A, B) at a tree t, we know *exactly* where A and B have to hold: at the left resp. right daughter of t. So why not replace the antecedent (A, B) where this information is implicit, by a database $\{a_0 : A, a_1 : B\}$ where we have syntactic entities to refer explicitly to these daughters? In this way, $(*)$ can at least be *formulated* in the language, viz. as

$$\{a : p\backslash(q \circ r), b : p\} \longrightarrow a : r$$

1. We state it as an open problem whether the sequents valid in tree frames are axiomatizable with a *finite* set of axioms and rules or not. We conjecture that the answer to this problem is negative. It may turn out to be difficult to prove this conjecture, since the usual methods to prove non-finite axiomatizability (like the use of ultra-products) do not seem to apply. We suspect that every finite derivation system only captures valid sequents up to a certain *depth* or *bracket complexity* (suitably defined), but we have not been able to formalize this idea.

This move takes us to the area of Labeled Deductive Systems (cf. Gabbay [8]), and will be worked out in detail in the following section.

Let us finish this section by giving some sequents discriminating the other frame classes of Fig. 1. To start with, the attentive reader may have noticed that we have been speaking about *tree* semantics in this section rather than about *finite-tree* frames. The reason for this is that even if we had found a calculus that is sound and complete with respect to T, this system would not do the job for $\mathsf{T_f}$. For, consider the following sequent

$$(\Gamma_2) \qquad p, p\backslash((p \circ \top) \circ \top) \longrightarrow \bot$$

which is valid in fintree frames, but not in every tree frame. (We use $\top$ and $\bot$ to indicate that *any* type may be substituted; so, the $\bot$ in the succedent says that the antecedent can not be true in any tree.)

To show that $\mathsf{T_f} \models \Gamma_2$, suppose that t is a tree in a fintree frame, such that the antecedent X of Γ_2 holds at t. Clearly $t_0 \Vdash p$ and $t_1 \Vdash p\backslash((p \circ \top) \circ \top)$, so by the truth definition, $t \Vdash (p \circ \top) \circ \top$. Unique Splittability gives $t_0 \Vdash p \circ \top$, whence t_0 has daughters t_{00} and t_{01}. Now we let the tree $t_{00}t_1$ take the place of t, observing that X is true at $t_{00}t_1$. We find that t_{00} has daughters too ... An inductive argument yields an infinite path $t_0, t_{00}, t_{000}, \ldots$, contradicting the fact that t should be *finite*. We leave it to the reader to give a counter example to the validity of Γ_2 in the class of arbitrary tree frames.

Note that in all the earlier examples, the types in the succedent appeared in the antecedent as well; however, the information used to prove that $\mathsf{T_f} \models \Gamma_2$ is far more implicit. Given this example, we fear that it may be hard to find a sequent axiomatization for $\mathsf{T_f}$, even in the labeled approach. Therefore, we decided to aim a bit lower, viz. a calculus for T instead of one for $\mathsf{T_f}$. Even this problem turned out to be harder than expected. The problem is that in a tree frame, leaves have a different behavior than trees with daughters. This is well illustrated by the following sequent

$$(\Gamma_3) \qquad q\backslash(q \circ ((p/q) \circ (q\backslash p))) \longrightarrow (p/q) \circ (q\backslash p)$$

which distinguishes inftree frames from tree frames.

To see why Γ_3 is valid in an arbitrary inftree frame, assume that for an inftree t, $t \Vdash q\backslash(q \circ A)$, where A abbreviates the type $(p/q) \circ (q\backslash p)$. Now make a case distinction as to whether $V(q)$ is empty or not. If $V(q) = \varnothing$, then any tree s in the frame satisfies $s \Vdash p/q$ and $s \Vdash q\backslash p$. In particular, the daughters of t do. So $t \Vdash (p/q) \circ (q\backslash p)$. If on the other hand $V(q)$ has an element s, then by a now familiar argument, $t \Vdash q\backslash(q \circ A)$ implies $t \Vdash A$. Again, it is left to the reader to give a counter example to the validity of Γ_3 in the class of arbitrary tree frames.

So inftree frames seem to be simpler to axiomatize because they are more homogeneous. Therefore, we will first give a sound and complete labeled calculus for the class T_∞.

3 Labeled categorial grammar

In this section we will define two extensions LC_∞ and LC_t of the non-associative Lambek calculus with a labeling discipline and show that these derivation systems are sound and complete for respectively the classes of inftree frames and tree frames. For LC_∞ we also give a cut elimination result.

3.1 LC_∞: a complete labeled calculus for inftree frames

The basic idea of a labeled categorial calculus is that the *structure* of the 'database' of assumptions $A_1, \ldots, A_n$ in a consequence relation

$$A_1, \ldots, A_n \longrightarrow B$$

is made *explicit* by labeling the types:

$$x_1 : A_1, \ldots, x_n A_n \longrightarrow y : B.$$

The basic idea behind our labeling algebra is that somehow, labels will refer to trees in the model — this is our instantiation of Gabbay's slogan 'bringing semantics into the syntax'. So let us start with defining the label algebra:

Definition 6 *Assume that we have been given a set M of elements that we will call* **markers**. *Let S be the set of strings over the alphabet $\{0, 1\}$ (inductively defined: S is the smallest set containing the empty string λ which contains the strings $s0$ and $s1$ whenever it contains the string s). Elements of the set $M \times S$ are called* **atomic labels**: *the atomic label (a, s) is denoted as a_s, a_λ as a. A* **label over** M *is either an atomic label over M or it has the form (xy) where x and y are labels over M. The set of labels over M is denoted as* $\mathsf{Lab}(M)$.

As abbreviations[2] *we will use the functions l and r over* $\mathsf{Lab}(M)$ *defined as follows:*

$$\begin{array}{ll} l(a_s) = a_{s0} & l(xy) = x \\ r(a_s) = a_{s1} & r(xy) = y \end{array}$$

Now we can give a definition of our labeled language:

Definition 7 *Let Pr be a set of primitive types, and M a set of markers. Elements of the Cartesian product* $\mathsf{Lab}(M) \times Tp(Pr)$ *are called* **formulas (in M and Pr)** *and denoted as $x : A$ where $x \in \mathsf{Lab}(M)$ and $A \in Tp(Pr)$. A* **sequent (in M and Pr)** *is a pair $X \longrightarrow \varphi$ where φ is a formula and X a finite set of formulas (in M and Pr).*

Turning to the calculus, one of the first questions that we have to answer is whether we want sequents of the form

$$x : A \longrightarrow y : A$$

to be theorems, if x and y refer to the same trees in the model. Note that we have to be careful here: what about the theoremhood of $a : p \longrightarrow a_0 a_1 : p$? Although we have not introduced a semantics for labeled sequents yet, it will be clear that the answer to these questions depends on whether we want the tree referred to by a to have daughters or not. As we have an inftree semantics in mind for LC_∞, we will accept such sequents as theorems. For the precise formulation of the rule that takes care of these 'label shifts', we need some terminology:

Definition 8 *Define the relation $\rightarrow_l$ on* $\mathsf{Lab}(M)$ *by: $a_{s0}a_{s1} \rightarrow_l a_s$. Let $\equiv$ be the congruence relation generated by $\rightarrow_l$, i.e. $\equiv$ is the smallest equivalence relation R containing $\rightarrow_l$ such that $((xz), (yz)) \in R$ and $((zx), (zy)) \in R$ whenever $(x, y) \in R$. We denote the congruence class of x by $\bar{x}$.*

2. Note that l and r are not part of the label algebra; their introduction has the sole purpose of providing a uniform way of referring to the left resp. right daughter of a label x (whether x is atomic or not), thus enabling a concise formulation of for instance the left rule for $\circ$.

Note that it is easy to show that $\equiv$ is decidable.

As for the structural rules of the system, note that our databases are *sets*; therefore, the rules of associativity, permutation and contraction are implicit. The only rule that we need to add explicitly is Weakening (Monotonicity). The operational rules will be discussed after the definition.

Definition 9 *LC_∞ is defined as follows. Its* **logical axioms** *are sequents of the form*

$$x : A \longrightarrow x : A.$$

Its **logical rule** *is the [Cut]-rule given by*

$$\frac{X \longrightarrow x : A \quad Y, x : A \longrightarrow \phi}{X, Y \longrightarrow \phi}\ [Cut]$$

Its **label rule** *has the following form:*

$$\frac{X \longrightarrow \varphi}{(X \longrightarrow \varphi)[x \leftarrow x']}\ [\equiv]\dagger$$

where $(X \longrightarrow \varphi)[x \leftarrow x']$ is the sequent $X \longrightarrow \varphi$ with one occurrence of x replaced by x'. The rule $[\equiv]$ is only licensed if the side condition ($\dagger$) is met that $x \equiv x'$.

For every connective LC_∞ has two **operational rules***:*

$$\frac{X, l(x) : A, r(x) : B \longrightarrow \varphi}{X, x : A \circ B \longrightarrow \varphi}\ [\circ L] \qquad \frac{X \longrightarrow l(x) : A \quad Y \longrightarrow r(x) : B}{X, Y \longrightarrow x : A \circ B}\ [\circ R]$$

$$\frac{X, yx : B \longrightarrow \varphi \quad Y \longrightarrow y : A}{X, x : A\backslash B, Y \longrightarrow \varphi}\ [\backslash L] \qquad \frac{X, a : A \longrightarrow ax : B}{X \longrightarrow x : A\backslash B}\ [\backslash R]*$$

$$\frac{X, xy : B \longrightarrow \varphi \quad Y \longrightarrow y : A}{X, x : B/A, Y \longrightarrow \varphi}\ [/L] \qquad \frac{X, a : A \longrightarrow xa : B}{X \longrightarrow x : B/A}\ [/R]*$$

In the rules marked with $*$, there is a side condition on the rule stating that the marker a should not occur in x or X.

Finally, LC_∞ has the **structural rule** *of Weakening:*

$$\frac{X \longrightarrow \varphi}{X, Y \longrightarrow \varphi}\ [W]$$

Most of these rules seem to be pretty obvious[3]. For instance, the side condition in $[\backslash R]$ and $[/R]$ (ensuring a totally hypothetical introduction of $a : A$) is quite familiar. However, there are some subtleties in the rule $[\circ L]$, as will be discussed in the proof of soundness, and in the right rules for / and \, as we will see in the next subsection. Let us first define what it means for an NL-sequent (i.e. without labels) to be derivable in LC_∞:

Definition 10 *Let X be a term of NL (i.e. a tree over types): the* **formula representation of** *X, notation: $X^\bullet$, is given by the usual inductive definition: $A^\bullet = A$, $(X, Y)^\bullet = (X^\bullet) \circ (Y^\bullet)$.*

Now let $X \longrightarrow A$ be an NL-sequent. We say that $X \longrightarrow A$ is **LC_∞-derivable***, notation: $LC_\infty \vdash X \longrightarrow A$, if there is (for an arbitrary marker a) an LC_∞-derivation for the LC_∞-sequent $a : X^\bullet \longrightarrow a : A$.*

---

3. Note that it is a fairly easy exercise to turn the calculus into a classical one by adding boolean type constructors and replacing the intuitionistic one-formula succedents into finite sets of formulas. It seems that the three major results (soundness, cut elimination and completeness) that we are about to prove for LC_∞, will still hold for this extended calculus.

As an example, we show how the sequent Γ_1 (discussed in the previous section) can be derived:

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{a_0 : r \longrightarrow a_0 : r}{a_1 : q, a_0 : r \longrightarrow a_0 : r}\,[W]}{a_1 a_0 : q \circ r \longrightarrow a_0 : r}\,[\circ L] \qquad a_1 : p \longrightarrow a_1 : p}{a_0 : p\backslash(q \circ r), a_1 : p \longrightarrow a_0 : r, a_1 : p}\,[\backslash L]}{a_0 : p\backslash(q \circ r), a_1 : p \longrightarrow a : r \circ p}\,[\circ R]}{a : (p\backslash(q \circ r)) \circ p \longrightarrow a : r \circ p}\,[\circ L]}{}$$

Now we turn to the semantics of LC_∞:

Theorem 4 *LC_∞ is sound and complete with respect to inftree semantics, i.e. for any NL-sequent*

$$\mathit{LC}_\infty \vdash X \longrightarrow A \iff \mathsf{T}_\infty \models X \longrightarrow A.$$

Proof.

Let us first consider soundness. Here we arrive at the subtlety involved in the left rule for product. The point is that it allows us to define a sound interpretation for arbitrary *labeled* sequents. Let $\mathfrak{M} = (\mathfrak{G}, V)$ be an inftree model. An *assignment* to $\mathfrak{G}$ is a map $f : \mathsf{Lab}(M) \mapsto G$ satisfying $f(x) = f(lx)f(rx)$. We leave it to the reader to verify that this implies that for any assignment f, $x \equiv y$ implies $fx = fy$.

Now we say that a labeled sequent $X \longrightarrow y : b$ *holds in $\mathfrak{M}$ under f*, notation: $\mathfrak{M}, f \models X \longrightarrow y : B$, if

$$(\forall x : A \in X \;\; fx \Vdash A) \Rightarrow fy \Vdash B.$$

Clearly then, for an NL-sequent $X \longrightarrow A$ we have

$$\mathfrak{M} \models X \longrightarrow A \qquad \text{iff} \qquad \mathfrak{M}, f \models a : X^\bullet \longrightarrow a : A \text{ for all assignments } f,$$

so to prove the theorem it suffices to show that $\mathit{LC}_\infty \vdash X \longrightarrow y : B$ implies that for any inftree model $\mathfrak{M}$ and any assignment f, we have $\mathfrak{M}, f \models X \longrightarrow y : B$. We do this by a straightforward induction on LC_∞-proofs. As an example, we treat the induction step for $[\circ L]$.

Assume as an inductive hypothesis, that $X, l(x) : A_0, r(x) : A_1 \longrightarrow y : B$ holds at every inftree model, and let f be an assignment to an inftree model $\mathfrak{M}$ such that for all $z : C$ in X, $fz \Vdash C$, and $fx \Vdash A_0 \circ A_1$. The latter fact implies that fx has daughters u_0 and u_1 such that $u_i \Vdash A_i$. Now the crucial point is that $fl(x) \cdot fr(x) = fx$, so by Unique Splittability we find $f(lx) = u_0$ and $f(rx) = u_1$. But then the induction hypothesis gives $fy \Vdash B$.

The completeness direction is relatively easy, after we have introduced some terminology: a *description* is a triple $D = (M, P, N)$, where M is a set of markers, and P and N are sets of requirements. Elements of P resp. N are called *positive* resp. *negative* requirements. A description is called *consistent* if for no finite $\Pi \subseteq P$ and $\varphi \in N$ we have a derivation $\vdash \Pi \longrightarrow \varphi$, *complete* if $P \cup N$ is the set of all formulas (in a given set M of markers and a given set Pr of primitive types). A formula $x : C$ is a *$\circ$-defect* of a description D if C is of the form $A \circ B$ and $x : C$ is in P, but we do not find both $l(x) : A$ and $r(x) : B$ in P. A formula $x : C$ is a */-defect* of a description D if C is of the form A/B and $x : C \in N$, but we do not have a $y \in L$ with both $y : A \in P$ and $(yx) : B \in N$; \-defects are defined analogously. A description is called *saturated* if it does not have any defects, *perfect* if it is consistent, complete and saturated.

Let D, D' be two descriptions; D' is an *extension* of D, notation: $D \subseteq D'$, if $L \subseteq L'$, $P \subseteq P'$ and $N \subseteq N'$.

Perfect descriptions give rise to inftree models in a natural way: the trees of the model will be the equivalence classes of $\mathbf{Lab}(M)$ under $\equiv$; note that if $x \equiv y$, the rule $[\equiv]$ ensures that $x : A \in P$ iff $y : A \in P$ and likewise for N. Therefore, the following definition is correct: for a perfect description D, the groupoid model *generated by* D, notation: $\mathfrak{M}^D$, is given as $(\mathfrak{G}, V)$ where $\mathfrak{G}$ is the quotient algebra of the labeling algebra over $\equiv$, and V is given by

$$V(p) = \{\bar{x} \in L \mid x : p \in P\}.$$

Now it is easy to prove by induction on the complexity of types, that for any perfect description D and any formula $x : C$, we have

$$\mathfrak{G}^D, \bar{x} \models C \iff x : C \in L. \tag{1}$$

After these preliminaries, we can start to prove the completeness result. We will show that any sequent which is not derivable in LC_∞, can be satisfied in an inftree model. Let $X \longrightarrow \phi$ be such a sequent. By definition then, $D_0 = (\{a\}, \{a : X^\bullet\}, \{a : \phi\})$ is a consistent description. Suppose that we can extend D_0 to a perfect extension D, then it is easy to show by (1) that $\mathfrak{M}^D \not\models X \longrightarrow A$, as $\bar{a} \in V(X) - V(A)$. It is also immediate that $\mathfrak{M}^D$ is an inftree model.

So the only thing left is to prove the following crucial extension lemma:

$$\text{any consistent description can be extended to a perfect description.} \tag{2}$$

To prove (2), one shows by a straightforward procedure that

1. If D is a consistent description with a defect δ, then D has a consistent extension D' of which δ is not a defect.
2. If D is a consistent description, and $x \in L$, then at least one of $(L, P, N \cup \{x : A\})$ or $(L, P \cup \{x : A\}, N)$ is consistent.

Then by a standard step-by-step method, one can extend any consistent description to a perfect one. This proves (2) and thus the theorem. □

Finally, we show that the cut rule is not really needed in derivations of LC_∞:

Theorem 5 *Let $X \longrightarrow \varphi$ be a theorem of LC_∞. Then there is a cut-free derivation of $X \longrightarrow \varphi$.*

Proof.
We will need the usual notions like *derivations* or *proof trees*, the *depth* of a proof, the *main formula* in the application of a rule, and the *cut formula* in the application of the $[Cut]$-rule. The statement that $\mathcal{D}$ is a derivation of the sequent $X \longrightarrow \phi$ is denoted by: $X \xrightarrow{\mathcal{D}} \varphi$.

Now as usual, the essential idea in the proof of the theorem is to concentrate first on derivations in which the $[Cut]$-rule is applied only once. To be more precise, we will prove the following claim:

$$\begin{array}{l}\text{If } X \xrightarrow{\mathcal{D}} \phi, \text{ where } \mathcal{D} \text{ has only one cut,} \\ \text{then there is a cut-free derivation } \mathcal{D}' \text{ with } X \xrightarrow{\mathcal{D}'} \phi.\end{array} \tag{3}$$

After establishing this claim, we can prove the theorem by an easy induction on the number of cuts in the proof of $X \longrightarrow \varphi$.

So let us set out to prove (3). Note that it is sufficient to confine ourselves to treating derivations $\mathcal{D}$ which *end* in an application of $[Cut]$. Assume that the daughters of $\mathcal{D}$ are $\mathcal{D}_0$ and $\mathcal{D}_1$, and that $x : A$ is the cut formula, i.e. $\mathcal{D}$ has the following form:

$$\dfrac{\dfrac{\mathcal{D}_0}{X \longrightarrow x : A} \qquad \dfrac{\mathcal{D}_1}{Y, x : A \longrightarrow y : B}}{X, Y \longrightarrow y : B}\,[Cut]$$

Then the *degree* of the cut is defined as the pair consisting of the number of connectives occurring in (the type of) the cut formula and the sum of the depths of $\mathcal{D}_0$ and $\mathcal{D}_1$; assume that we impose a lexicographical ordering on cut-degrees. Now (3) is proved by induction on the cut-degree of $\mathcal{D}$. For the inductive step, we make the following case distinction:

I First, assume that the cut-formula is main in both $\mathcal{D}_0$ and $\mathcal{D}_1$. We distinguish cases as to whether a connective was introduced in the main formula, or a new label:

$\boxed{[\circ]}$ In this case the derivation looks like

$$\dfrac{\dfrac{\dfrac{\mathcal{D}_{00}}{X_0 \longrightarrow l(x) : A_0} \quad \dfrac{\mathcal{D}_{01}}{X_1 \longrightarrow r(x) : A_1}}{X_0, X_1 \longrightarrow x : A_0 \circ A_1}\,[\circ R] \qquad \dfrac{\dfrac{\mathcal{D}_1}{Y, l(x) : A_0, r(x) : A_1 \longrightarrow y : B}}{Y, x : A_0 \circ A_1 \longrightarrow y : B}\,[\circ L]}{X_0, X_1, Y \longrightarrow y : B}\,[Cut]$$

and can be replaced by $\mathcal{D}''$ of the form

$$\dfrac{\dfrac{\mathcal{D}_{00}}{X_0 \longrightarrow l(x) : A_0} \qquad \dfrac{\dfrac{\mathcal{D}_{01}}{X_1 \longrightarrow r(x) : A_1} \quad \dfrac{\mathcal{D}_1}{Y, l(x) : A_0, r(x) : A_1 \longrightarrow y : B}}{Y, l(x) : A_0, X_1 \longrightarrow y : B}\,[Cut]}{X_0, X_1, Y \longrightarrow y : B}\,[Cut]$$

Note that both cut formulas in $\mathcal{D}''$ have a smaller complexity than the original one, so by the induction hypothesis the two cuts can be removed (one by one).

$\boxed{[/]}$ Here $\mathcal{D}$ has the following form:

$$\dfrac{\dfrac{\dfrac{\mathcal{D}_0}{X, a : A_1 \longrightarrow xa : A_0}}{X \longrightarrow x : A_0/A_1}\,[/R] \qquad \dfrac{\dfrac{\mathcal{D}_{10}}{Y_0, xy : A_1 \longrightarrow y : B} \quad \dfrac{\mathcal{D}_{11}}{Y_1 \longrightarrow y : A_1}}{Y_0, x : A_0/A_1, Y_1 \longrightarrow y : B}\,[/L]}{X, Y_0, Y_1 \longrightarrow y : B}\,[Cut]$$

We leave it to the reader to verify that there is a cut free derivation $\mathcal{D}_0'$ of $X, y : A_1 \longrightarrow xy : A_0$. (Here one needs the side condition on $[/R]$ that a does not occur in X.)

So, if we replace $\mathcal{D}$ by

$$\dfrac{\dfrac{\dfrac{\mathcal{D}_{11}}{Y_1 \longrightarrow y : A_1} \quad \dfrac{\mathcal{D}_0'}{X, y : A_1 \longrightarrow xy : A_0}}{X, Y_1, \longrightarrow xy : A_0}\,[Cut] \qquad \dfrac{\mathcal{D}_{10}}{Y_0, xy : A_1 \longrightarrow y : B}}{X, Y_0, Y_1 \longrightarrow y : B}\,[Cut]$$

we are dealing with a proof tree to which we can apply the inductive hypothesis (twice, just like in the case above).

$\boxed{[\equiv]}$ Here we may replace

$$\dfrac{\dfrac{\dfrac{\mathcal{D}_0}{X \longrightarrow x' : A}}{X \longrightarrow x : A}\,[\equiv] \qquad \dfrac{\dfrac{\mathcal{D}_1}{Y, x'' : A \longrightarrow y : B}}{Y, x : A \longrightarrow y : B}\,[\equiv]}{Y, X \longrightarrow y : B}\,[Cut]$$

by

$$\dfrac{\dfrac{\dfrac{\mathcal{D}_0}{X \longrightarrow x' : A}}{X \longrightarrow x'' : A}[\equiv] \quad \dfrac{\mathcal{D}_1}{Y, x'' : A \longrightarrow y : B}}{Y, X \longrightarrow y : B}[Cut]$$

where the application of $[\equiv]$ is justified by the transitivity of $\equiv$. The proof depth of the cut has decreased, so the induction hypothesis applies.

II Now, assume that the cut formula is side formula in the last step of $\mathcal{D}_0$ or $\mathcal{D}_1$. In this case, we will permute the $[Cut]$-rule upwards; again, the particular action we take will depend on the last applied rule of the subtree in which the cut formula was not main. As most of these cases are standard, we only give a few examples:

- Suppose that the cut formula is side formula in the last step of $\mathcal{D}_0$, where we applied the rule $[W]$. Then the derivation $\mathcal{D}$

$$\dfrac{\dfrac{\dfrac{\mathcal{D}_0'}{X_0 \longrightarrow x : A}}{X_0, X_1 \longrightarrow x : A}[W] \quad \dfrac{\mathcal{D}_1}{Y, x : A \longrightarrow y : B}}{X_0, X_1, Y \longrightarrow y : B}[Cut]$$

  is replaced by

$$\dfrac{\dfrac{\dfrac{\mathcal{D}_0'}{X_0 \longrightarrow x : A} \quad \dfrac{\mathcal{D}_1}{Y, x : A \longrightarrow y : B}}{X_0, Y \longrightarrow y : B}[Cut]}{X_0, X_1, Y \longrightarrow y : B}[W]$$

  This proof has a smaller cut-degree than the original one (as the depth of the left subtree has decreased), and can thus be replaced by a cut-free derivation.
- If the last applied rule of $\mathcal{D}_1$ was $[/R]$, and the cut formula was not the main formula of this rule, $\mathcal{D}$ looks like

$$\dfrac{\dfrac{\mathcal{D}_0}{X \longrightarrow x : A} \quad \dfrac{\dfrac{\mathcal{D}_1'}{Y, x : A, a : B_1 \longrightarrow ya : B_0}}{Y, x : A \longrightarrow y : B_0/B_1}[/R]}{X, Y \longrightarrow y : B_0/B_1}[Cut]$$

  Note that we may replace the marker a by a marker b that does not appear in X, Y, x, y, obtaining a derivation $\mathcal{D}_1''$ for $Y, x : A, b : B_1 \longrightarrow yb : B_0$ of the same depth as $\mathcal{D}_1'$. We can then show that the following derivation $\mathcal{D}'$ may replace $\mathcal{D}$:

$$\dfrac{\dfrac{\dfrac{\mathcal{D}_0}{X \longrightarrow x : A} \quad \dfrac{\mathcal{D}_1''}{Y, x : A, b : B_1 \longrightarrow yb : B_0}}{X, Y, b : B_1 \longrightarrow yb : B_0}[Cut]}{X, Y \longrightarrow y : B_0/B_1}[/R]$$

  Again we have found a derivation to which the inductive hypothesis applies.

□

It follows from the cut elimination theorem that any theorem $X \longrightarrow \phi$ has a proof using only subtypes of the types occurring in X and ϕ. Note however that this does *not* imply decidability of the calculus, as the rules $[\equiv]$, $[\backslash L]$ and $/L]$ presume an *infinite* search space.

3.2 LC_t: a complete labeled calculus for tree frames

In this subsection we will transform the system LC_∞ into a calculus that works for arbitrary tree frames. Obviously, it is crucial to avoid as theorems sequents like

$$a : p \longrightarrow a_0 a_1 : p$$

as it would imply that every tree is branching. Clearly, we have to change the rule $[\equiv]$:

$$\frac{X \longrightarrow \phi}{(X \longrightarrow \phi)[x \leftarrow x']}\ [\equiv]$$

Let us concentrate on the case where we replace the label of the *succedent*, i.e. ϕ is of the form $x : A$. We need to install a side condition permitting the rule only when the atomic labels in x' are 'presupposed by' the ones in X. To formalize this condition, we set

Definition 11 *Let a_s and b_r be two atomic labels; we call a_s* **presupposed by** *b_r if $a = b$ and s is an initial segment of s. For a set X of formulas, the label y is presupposed by X, notation: $X \rhd y$, if every atomic label in y is presupposed by some atomic label in one of the labels of X.*

We can now formulate a *right* rule $[\equiv R]$ as

$$\frac{X \longrightarrow x : A}{X \longrightarrow x' : A}\ [\equiv R]\$$$

with the side condition ($) that $X \rhd x'$. We have not been able to formulate an appropriate left label rule, in the sense that the arising calculus allows a cut elimination theorem. Therefore we confine ourselves to this one right rule.

Note however, that this change is not sufficient: here we arrive at one of the subtleties mentioned in the previous subsection. The sequent Γ_3, true only in inftree frames (cf. section 2), would still be derivable, witness the derivation below:

$$\dfrac{\dfrac{\dfrac{b : q \longrightarrow b : q \qquad \dfrac{\dfrac{\dfrac{\dfrac{b : q \longrightarrow b : q \quad a_0 b : p \longrightarrow a_0 b : p}{b : q, a_0 : p/q \longrightarrow a_0 b : p}\,[/L]}{b : q, a_0 : p/q, a_1 : q\backslash p \longrightarrow a_0 b : p}\,[W]}{b : q, a : A \longrightarrow a_0 b : p}\,[\circ L]}{ba : q \circ A \longrightarrow a_0 b : p}\,[\circ L]}{a : q\backslash(q \circ A), b : q \longrightarrow a_0 b : p}\,[\backslash L]}{a : q\backslash(q \circ A) \longrightarrow a_0 : p/q}\,[/R] \qquad \dfrac{\text{likewise}}{a : q\backslash(q \circ A) \longrightarrow a_1 : q\backslash p}\,[\backslash R]}{a : q\backslash(q \circ A) \longrightarrow a : (p/q) \circ (q\backslash p)}\,[\circ R]$$

A close inspection of this derivation shows that the problem lies in the right rule for $/$: where from $\{a : q\backslash(q \circ A)\}$ one cannot conclude *semantically* that the tree referred to by a has daughters, this conclusion is justified from the database $\{a : q\backslash(q \circ A), b : q\}$. A solution is to replace $[/R]$ and $\backslash R]$ by

$$\frac{X, a : A \longrightarrow xa : B}{X \longrightarrow x : B/A}\ [/R']\ddagger \qquad \text{resp.} \qquad \frac{X, a : A \longrightarrow ax : B}{X \longrightarrow x : A\backslash B}\ [\backslash R']\ddagger$$

where we impose the side condition (‡) that x is presupposed by X.

Definition 12 *The calculus LC_t is defined just like LC_∞, with the rules $[/R]$ and $[\backslash R]$ replaced by $[/R']$ and $\backslash R']$, and $[\equiv]$ by $[\equiv R]$.*

We can now give the desired soundness and completeness results:

Theorem 6 *LC_t is sound and complete with respect to tree frames. i.e. for all NL-sequents*

$$LC_t \vdash X \longrightarrow A \iff \top \models X \longrightarrow A.$$

Proof.
To prove soundness, we again introduce an interpretation for *arbitrary* sequents. Here we have to be more careful in our formulation however.

Let X be a set of formulas. An X-assignment f is a *partial* map from labels to elements of a tree model such that (i) $f(x)$ is defined (notation: $f(x)\downarrow$) for all atomic labels x occurring in X, and (ii) for all labels x: if $f(lx)\downarrow$ and $f(rx)\downarrow$, then $f(x)\downarrow$ and $f(x) = f(lx)f(rx)$. Now we say that a labeled sequent $X \longrightarrow x : A$ *holds* in $\mathfrak{M}$ under f, if f can be extended to an X-assignment g satisfying $g(x) \Vdash A$ whenever $f(u) \Vdash B$ for every $u : B$ in X. With this adaptation we can follow the strategy of the old proof and show soundness of LC_t for tree frames. We give one crucial example: the right rule for $/$.

Assume that $X, x : A \longrightarrow xa : B$ holds in every tree model (i.e. under every $X, x : A$-assignment), and that f is an X-assignment into a tree model $\mathfrak{M}$ such that $\mathfrak{M}, f(u) \Vdash C$ for all $u : C$ in X. The key observation is that $x \in Dom(f)$, as by our new side condition on $[/R']$, X presupposes x . Now distinguish two cases: if $V(A) = \varnothing$, then B/A is true at every tree in $\mathfrak{M}$, so in particular $f(x) \Vdash A/B$. But then indeed $\mathfrak{M}, f \models X \longrightarrow x : B/A$. If on the other hand $V(A) \neq \varnothing$, take an arbitrary element $s \in V(A)$. It is left to the reader to verify that there is an extension g_s of f such that $g_s(u) = f(u)$ if $X \rhd u$, and $g_s(a) = s$ (here we crucially use the fact that a is *fresh*). But then g_s is an $X, a : A$-assignment, so by the inductive hypothesis, $g_s(xa) \Vdash B$. Observing that $g_s(xa) = g_s(x)g_s(a) = f(x)s$, we find $f(x) \Vdash A/B$, as s was arbitrary.

For the completeness direction we copy the proof of the previous section, only indicating the places where changes have to be made. The main adaptation is in the definition of a description: a description will here be a triple (L, P, N) with L an upward closed set of labels, and P and N sets of formulas with labels in L. (A set I of labels is *upward closed* if $y \in I$ whenever there is an x such that $x \in I$ and $x \rhd y$.)

Furthermore, we impose the 'finite-presupposition' constraint that every label in L is presupposed by a finite subset of P. In the definition of a complete description, we now take labels in L into account only.

The universe of the groupoid model generated by a perfect description D will now consist of equivalence classes over L. To show that the definition of the interpretation map V is correct, it suffices to prove that for all $x, y \in L$ such that $x \equiv y$, $x : A \in P$ implies $y : A \in P$. Here we need the new constraint that there is a finite subset P_0 of P such that $P_0 \rhd y$. For, the following derivation ensures that $x : A \in P \Rightarrow y : A \in P$:

$$\dfrac{\dfrac{x : A \longrightarrow x : A}{P_0, x : A \longrightarrow x : A}\,[W]}{P_0, x : A \longrightarrow y : A}\,[\equiv R]$$

In the remainder of the proof, we only have to take care to drag the finite-presupposition condition with us along the construction of the desired perfect description; this is relatively straightforward. □

4 Evaluation: labels in categorial grammar

Compared to other formalisms studied in the literature (cf. the references given in the introduction), the labeled categorial calculi presented here are of a very basic nature. We believe that it may be useful to put the problems that we encountered here in a wider context, since these problems will by no means vanish in more involved systems. Putting it bluntly, we have the feeling that the introduction of labels in a categorial logic causes as many problems as it solves[4]. One reason for this seems to be the following. Usually, the *motivation* for converting an ordinary calculus into a labeled one, is a mismatch between syntax and semantics. For instance, in the introduction we mentioned the problem concerning Moortgat's infixation and extraction operators, that allow a clear-cut definition of a semantic interpretation, but for which operational derivation rules cannot be expressed. Now, implementing Gabbay's slogan 'bringing semantics into the syntax', one runs the risk of importing this mismatch too...

Let us try to be a bit more precise. When labeling a categorial sequent calculus, one finds a number of areas where radical changes are brought about:

language The most obvious change is that the new 'declarative unit' (to use the LDS terminology) is no longer the pure type, but a type-with-a-label-attached-to-it. There are two points to be made here. First, let us assume that the labels will refer to elements of the intended interpretation. The problem is that the intuitions rising from this intended interpretation may also be quite confusing. For instance, suppose that the interpretation is some kind of *free* algebra, like the fintree models in our example, or the language models for the associative Lambek Calculus. Now, should the language have different kinds of labels referring to generators (leaves), complex terms (trees with daughters) and arbitrary elements (trees), respectively? Note that an affirmative answer may lead to a very complicated syntax, while a negative answer may cause problems for the soundness proof.

Second, in general the label set will have structure. It is not a trivial matter of how to represent this structure formally, and it may even become necessary to add more kinds of 'declarative units' to the language: for instance in a (substructural) *modal* calculus one may need *symbols* referring to the accessibility relation.

calculus It is by no means a trivial task to arrive at a perspicuous *formal* definition of a labeled calculus, even if (or perhaps precisely because) one is guided by sound semantical considerations. The main issue of course is how to receive the new bookkeeping forced by the introduction of the labels.

A relatively simple matter is formed by the *structural rules* of the calculus. Where the old set of structural rules is more or less determined by the implicit structuring of the database (antecedent), switching to an explicit representation in general will take one to a different level in, or even outside the substructural hierarchy.

Less clear is the problem how to *adapt* the *existing* axioms and derivation rules to the new formalization — there may be venomenous subtleties here. Obviously, this issue is crucially dependent on the system's logical properties that one wants to establish. In particular, it is a quite non-trivial matter how to find and state proper side conditions for the operational rules. First of all, in order to have a decidable set of rules, one should stay away from formulations involving undecidable problems like the quasi-equational theory of semigroups (the word problem). The main problem

4. One should not read this statement as a denunciation of labeled categorial grammars. A labeled approach *can* be a solution to problems, witness MOORTGAT [18], or KURTONINA [13]. And besides this, some of the problems introduced by labels are quite interesting, and solving them might lead to a better understanding of the issues involved.

however seems to be to avoid undesirable side effects of a too naive implementation of semantical intuitions (cf. our discussion preceding Definition 12).

Finally, one has make clear whether the calculus needs *label rules*, i.e. rules that only involve a re-labeling of the types (as an example, we mention our ≡-rules). Note how tricky the matter may become here: small changes in the labeling rules may have tremendous effects on the properties of the calculus (like cut-elimination). Note too that the decidability problem pops up again.

logical properties Obviously, the motivation to introduce labels in a calculus stems from the desire to obtain a system with some nice properties. Concerning the logical properties, *soundness* seems to be the minimal constraint for a system. Unfortunately, in some cases, it is quite difficult to give an *interpretation* of a labeled sequent in the intended semantics. (In the case of LC_∞ we were in some sense 'lucky' with our semantics, cf. the soundness proof in Theorem 4.) Note that even the *notion* of soundness may have various interpretations: in our examples LC_∞ and LC_t, one might demand validity either for all *labeled* theorems, or for (indirectly) derivable NL-sequents only.

At the moment, *completeness* seems to be out of reach for most systems, and as for *cut elimination*, the symmetry of the old calculus may be disturbed by the side conditions on the operational rules or the label rules.

Finally, *decidability* is no longer an easy consequence of a cut elimination result: the complications involved with the label management may blow up the size of the search space — one is likely to get mixed up with some non-trivial unification problems.

In short, the logical surroundings of labeled categorial grammars differ in almost all fundamental aspects from the substructural landscape that one has some familiarity with. Although we are not implying that labels form a Trojan horse for categorial grammars, it seems to us that the logical foundations of the area of labeled categorial grammars are not established as yet.

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